UNIVERSITY OF OSLO

Betatron Radiation and Self-Corrected Energy Spread in Plasma-Wakefield Accelerators

ALEGRO workshop 2025

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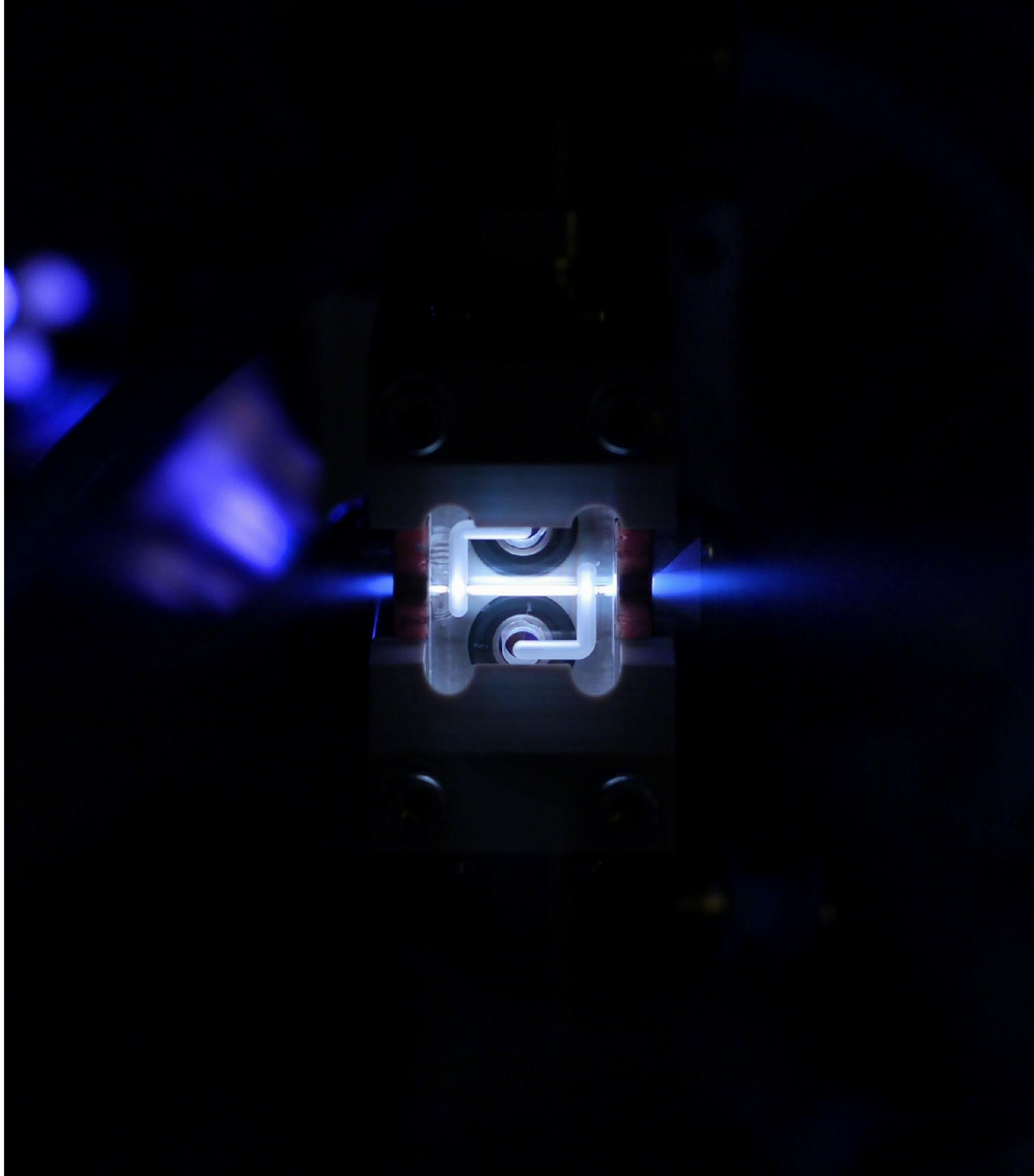
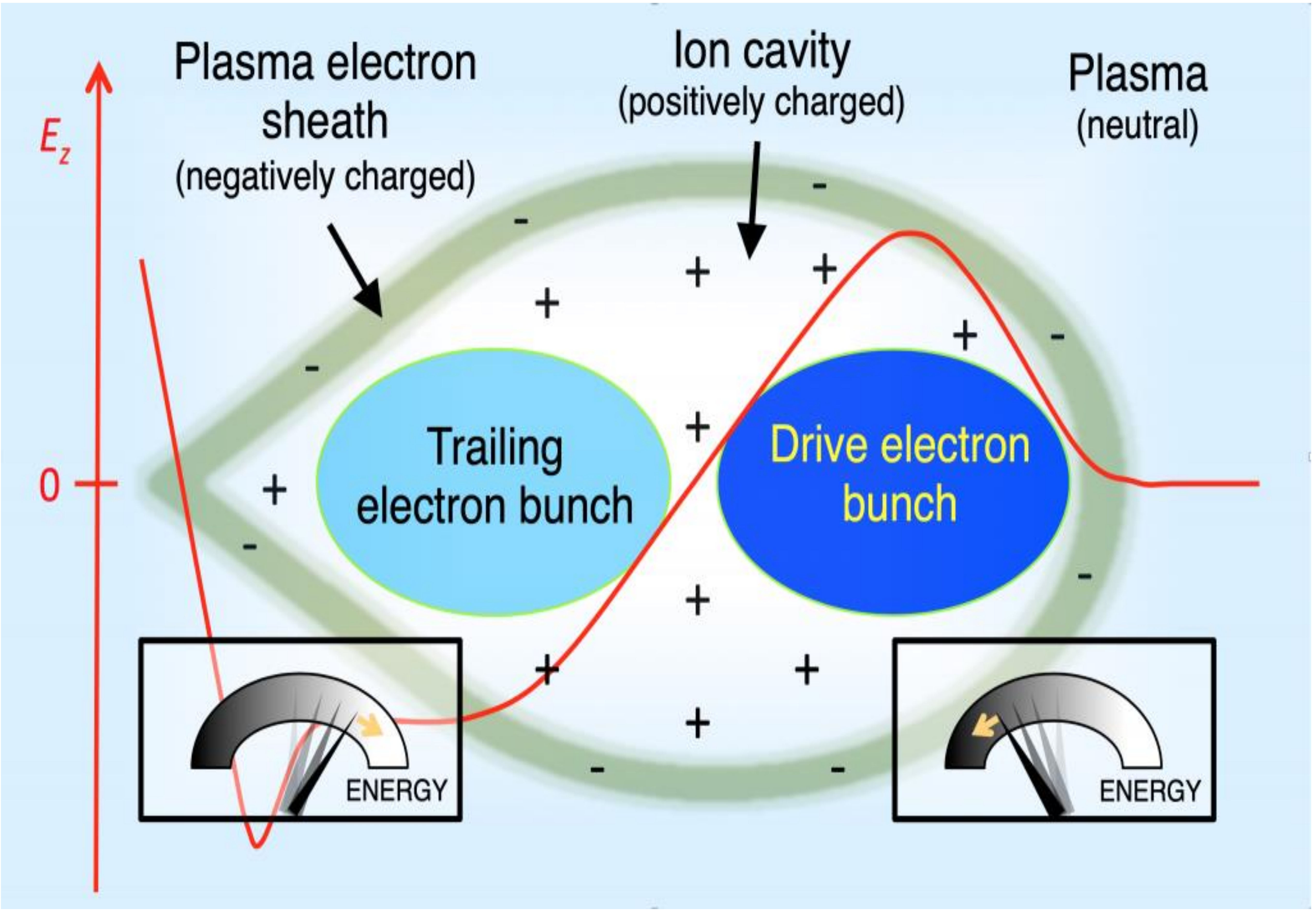
12.06.24



Induced energy spread

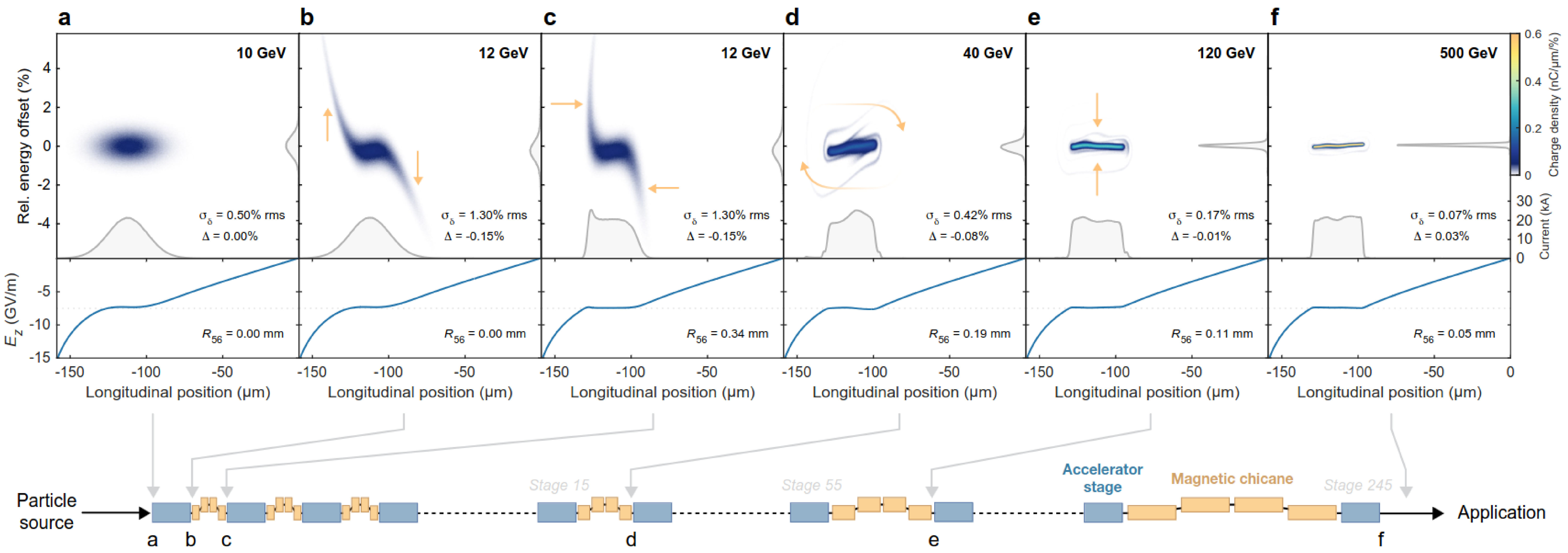
- The longitudinal electric field generates a large energy spread.



Laboratoire d'Optique Appliquée (LOA), 2024, https://loa.ensta-paris.fr/research/upx-research-group/particle-beam-driven-plasma-wakefield-acceleration/

Self-correction mechanism

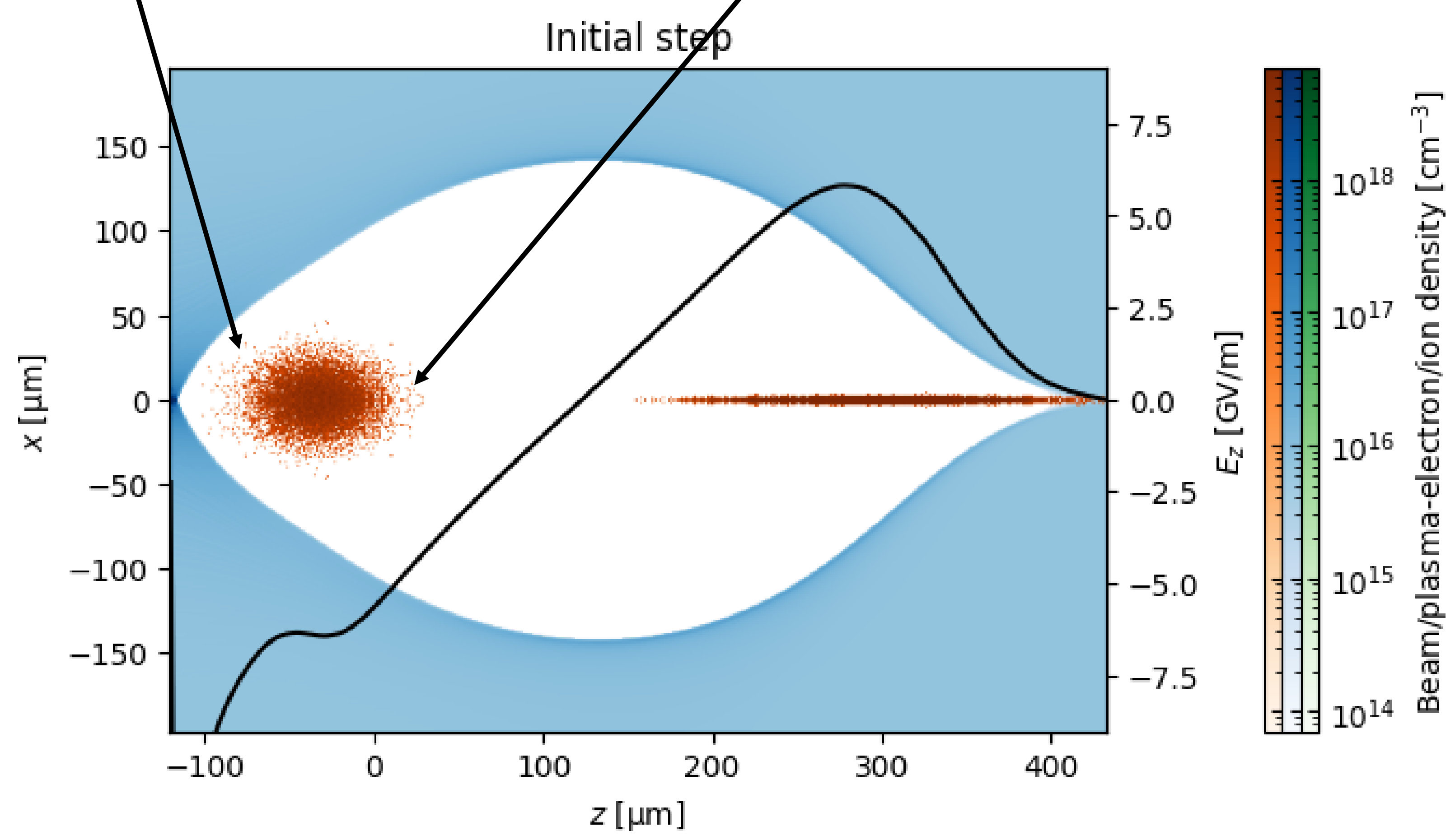
How to reduce the energy spread



Carl A Lindstrøm. 'Self-correcting longitudinal phase space in a multistage plasma accelerator'. eng. In: *arXiv (Cornell University)* (2021). ISSN: 2331-8422.

Stronger electric field

Weaker electric field

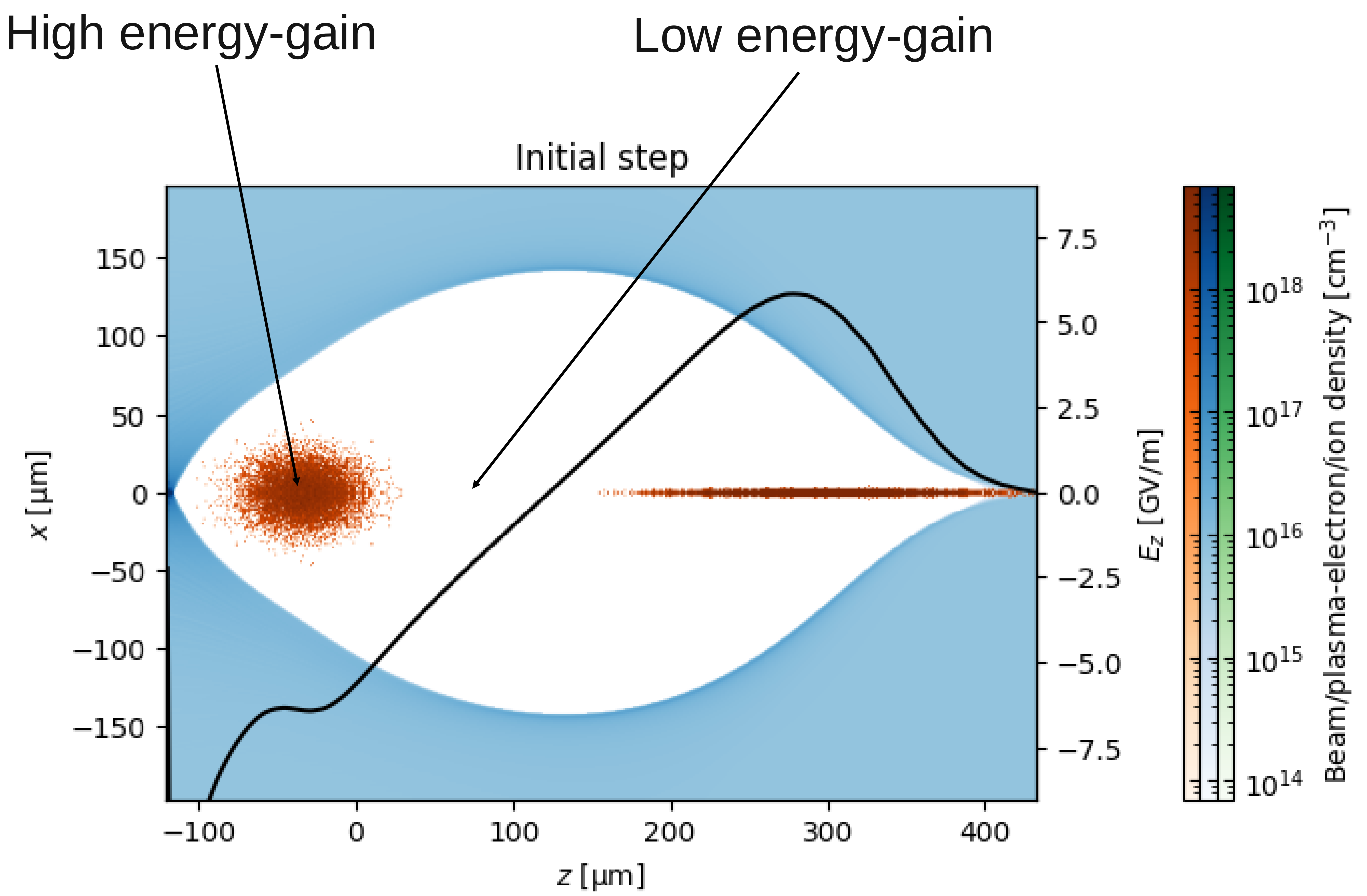


- > The longitudinal dynamics in bending magnets can reduce the energy spread.
- > High energy particles are bent less (they take a longer path), low energy particles are bent more, they take a longer path.

Self-correction mechanism

How to reduce the energy spread

- The strength of the correction is tuned to a specific location in the wake.
- I.e. the energy gain (decided by location of the beam) is predetermined.



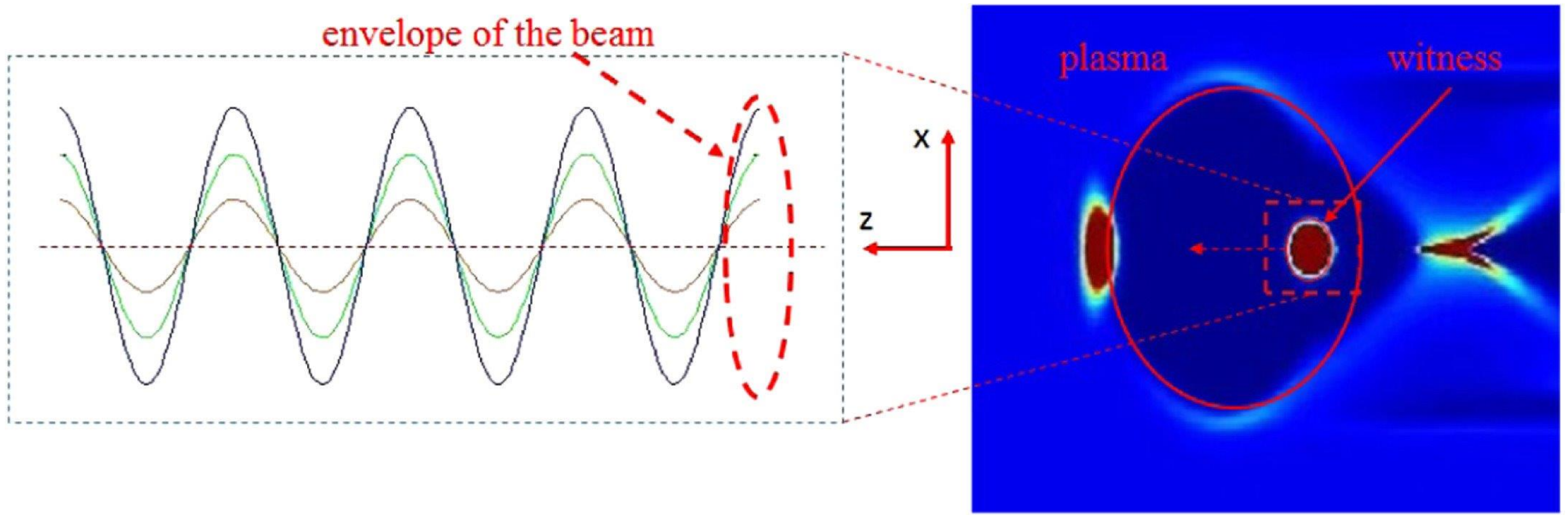
Particle dynamics

Motion in an ion cavity

- Newton's second law gives the equations of motion

$$\boldsymbol{a} + \frac{1}{\gamma}\frac{d\gamma}{dt}\boldsymbol{v} - \frac{\boldsymbol{F}}{m\gamma} = 0$$

- If the Lorentz factor is increasing, the oscillations are damped. This is known as adiabatic damping.
- In an ion cavity, the equation of motion becomes a harmonic oscillator

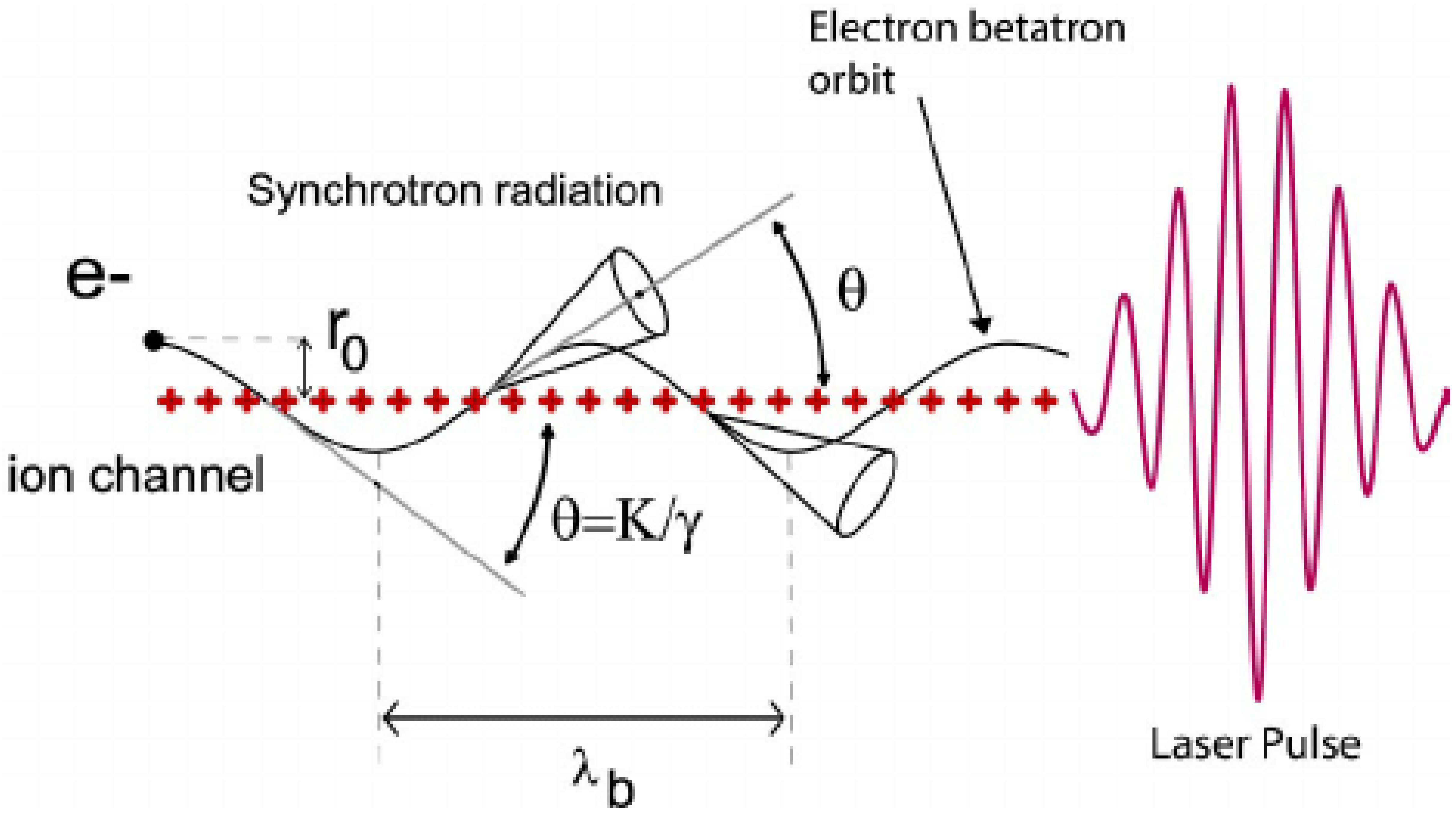


V. Shpakov et al. 'Plasma acceleration limitations due to betatron radiation'. In: *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* 909 (2018). 3rd European Advanced Accelerator Concepts workshop (EAAC2017), pp. 463–466. ISSN: 0168-9002. DOI: https://doi.org/10.1016/j.nima.2018.02.058. URL: https://www.sciencedirect.com/science/article/pii/S0168900218302109.

The radiation reaction

Interaciton between a charged particle and its own electromagnetic field

- As charged particles oscillate, a force is exerted on the particle by its own electromagnetic field; hence it is a self-force.
- This self-force is called the radiation reaction force, and its effect, the radiation reaction.
- Particles of different amplitudes radiate differently and contributes therefore to energy spread.



Ta, Phuoc & Burgy, frédéric & Rousseau, Jean-Philippe & Malka, Victor & Rousse, Antoine & Shah, Rahul & Umstadter, Donald & Pukhov, A. & Kiselev, Sergei. (2005). Laser based synchrotron radiation. Physics of Plasmas. 12. 10.1063/1.1842755.

Particle dynamics and the Larmor formula

Radiation for an oscillating charged particle

> The Larmor formula, $P = -\frac{e^2}{6\pi\varepsilon_0}\frac{c}{(m_e c^2)^2}\left(\frac{dp^\mu}{d\tau}\right)^2$ *, tells us how much energy is radiated by accelerating charges.

Amplitude of oscillations

> Simplifies to $P = \frac{e^2}{6\pi\varepsilon_0 c}\gamma^4\left(\frac{d\boldsymbol{\beta}}{dt}\right)^2$, which gives $\bar{P} = r_e m_e c^3 \gamma^2 k_p^4 r_\beta^2/12$ **, which is the radiated power, averaged over one betatron period.

> The radiation emitted as the particles oscillate, is betatron radiation.

* 'LXIII. On the theory of the magnetic influence on spectra; and on the radiation from moving ions'. eng. In: *The London, Edinburgh and Dublin philosophical magazine and journal of science* (). ISSN: 1941-5982.

* E. Esarey et al. 'Synchrotron radiation from electron beams in plasma-focusing channels'. eng. In: *Physical review. E, Statistical, nonlinear, and soft matter physics* 65.5 (2002), pp. 056505/15–056505. ISSN: 1539-3755.

Radiation in both transverse dimensions

Radiation for oscillating charged particles

> Expanding to 2D oscillations the new mean radiated power is $\bar{P} = \frac{e^2 c}{48\pi\varepsilon_0}\gamma^2 k_p^4 (r_x^2 + r_y^2)$. The equation is no longer dependent on the betatron amplitude. The amplitude in x/y give independent contributions.

> By averaging over all the betatron amplitudes for a bunch of particles, the total radiated power is

$$\langle \bar{P} \rangle = \frac{e^2 c}{6\pi\varepsilon_0} k_p^3 \left(\frac{\gamma}{2}\right)^{3/2} (\varepsilon_{nx} + \varepsilon_{ny}) \longleftarrow \frac{1}{\beta_m} = \frac{k_p}{\sqrt{2\gamma}}, \text{ and } \frac{\sigma_x^2}{\beta_m} = \frac{\varepsilon_{nx}}{\gamma}$$

> And the spread in the radiation is $\sigma_{\bar{P}} = \frac{e^2 c}{24\pi\varepsilon_0}(\sqrt{\gamma} k_p)^3 \sqrt{\varepsilon_{nx}^2 + \varepsilon_{ny}^2}$ (for no energy spread).

Self-consistent motion for accurate simulations

Including the radiation reaction and adiabatic damping

$$K^2 = \frac{k_p^2}{2}$$

- The equations of motion for each individual particle are (Deng et al.)

$$\ddot{x} + \left(\frac{\omega_p}{\gamma}\frac{E_z}{E_0} + \tau_R c^2 K^2\right)\dot{x} + \frac{c^2 K^2}{\gamma} x = 0$$

$$\ddot{y} + \left(\frac{\omega_p}{\gamma}\frac{E_z}{E_0} + \tau_R c^2 K^2\right)\dot{y} + \frac{c^2 K^2}{\gamma} y = 0$$

$$\dot{\gamma} = \omega_p \frac{E_z}{E_0} - \tau_R c^2 K^4 \gamma^2 \left(x^2 + y^2\right)$$

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$$\dot{\gamma} = \omega_p \frac{E_z}{E_0} - \tau_R c^2 K^4 \gamma^2 \left(x^2 + y^2\right)$$

Adiabatic damping

Damping from radiation reaction force

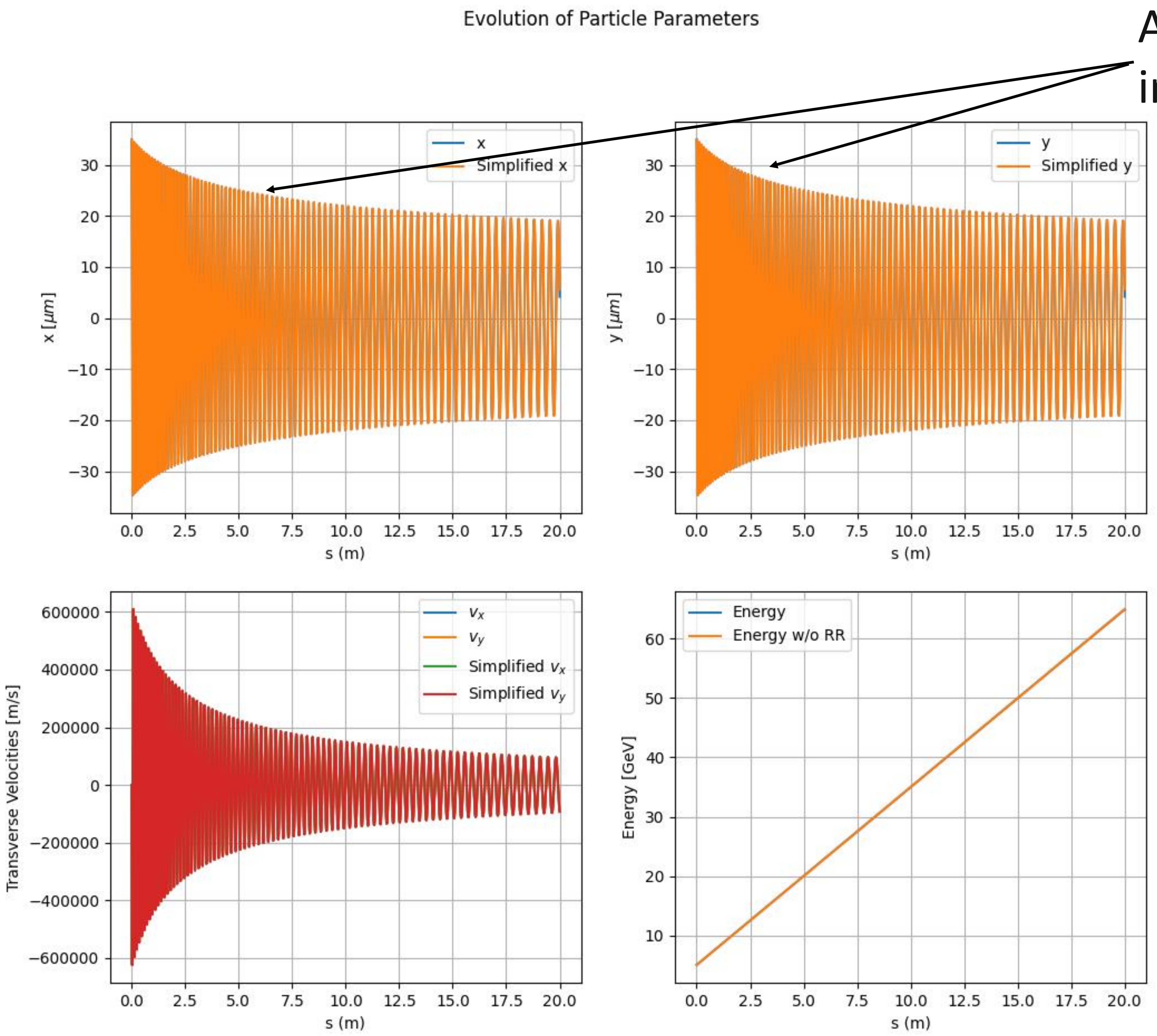
> My results are obtained by solving this set of equations, using a multistep method.

Single-particle motion

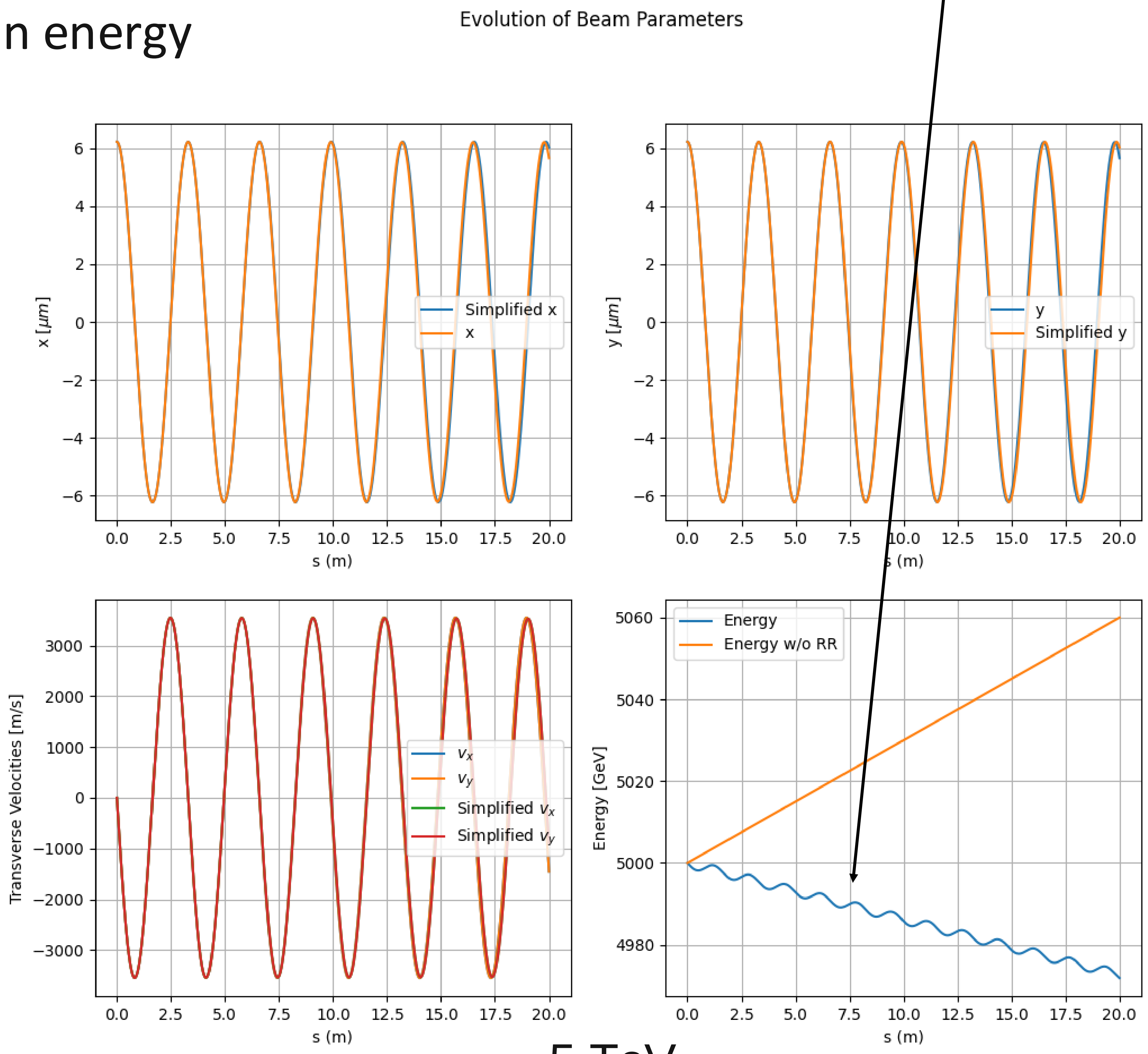
A diagonal oscillation with and without radiation

Adiabatic damping due to increase in energy

Energy loss due to radiation reaction



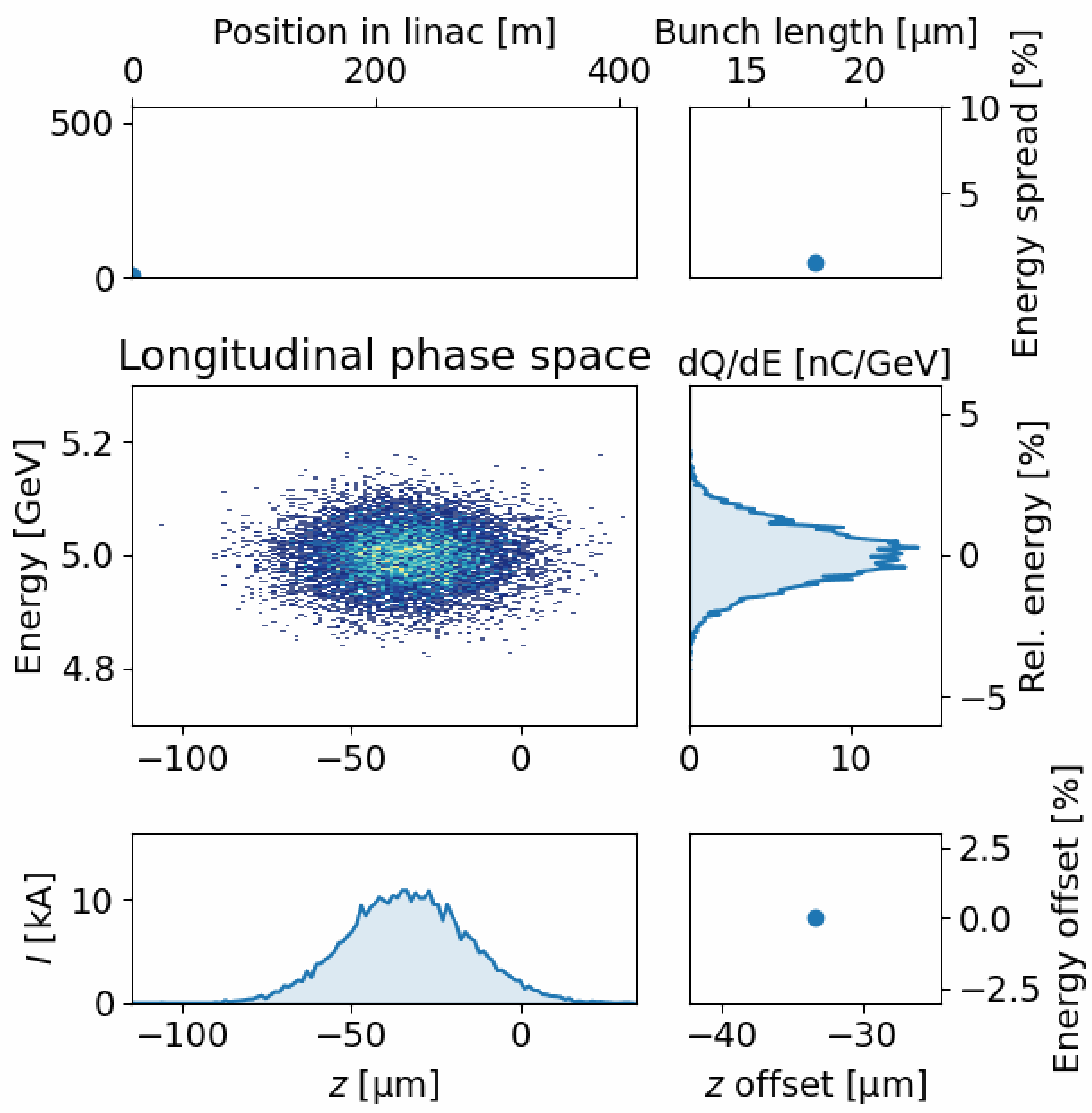
5 GeV



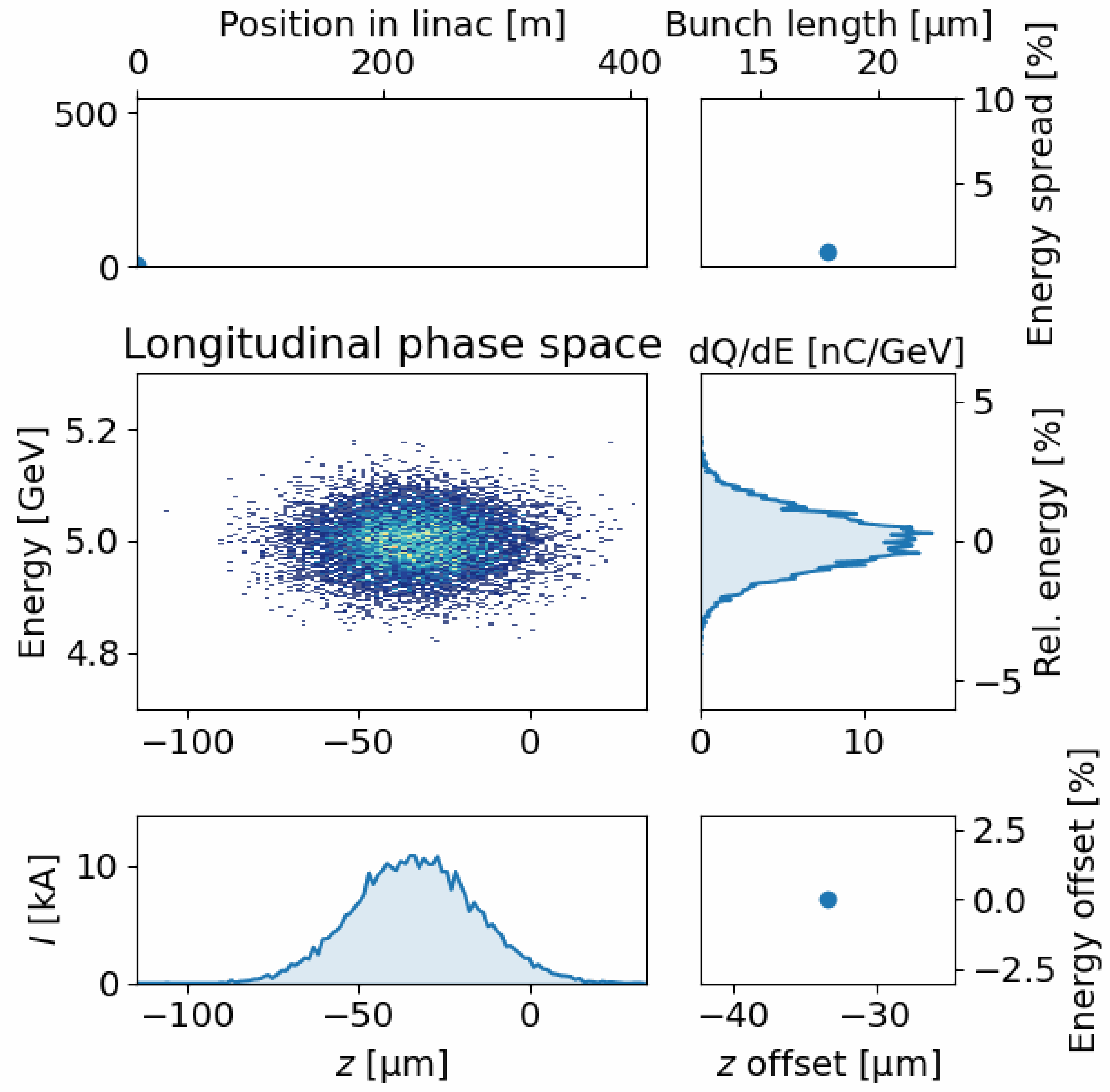
5 TeV

Beam energy loss

What does the effects of radiation reaction look like in the longitudinal phase space?



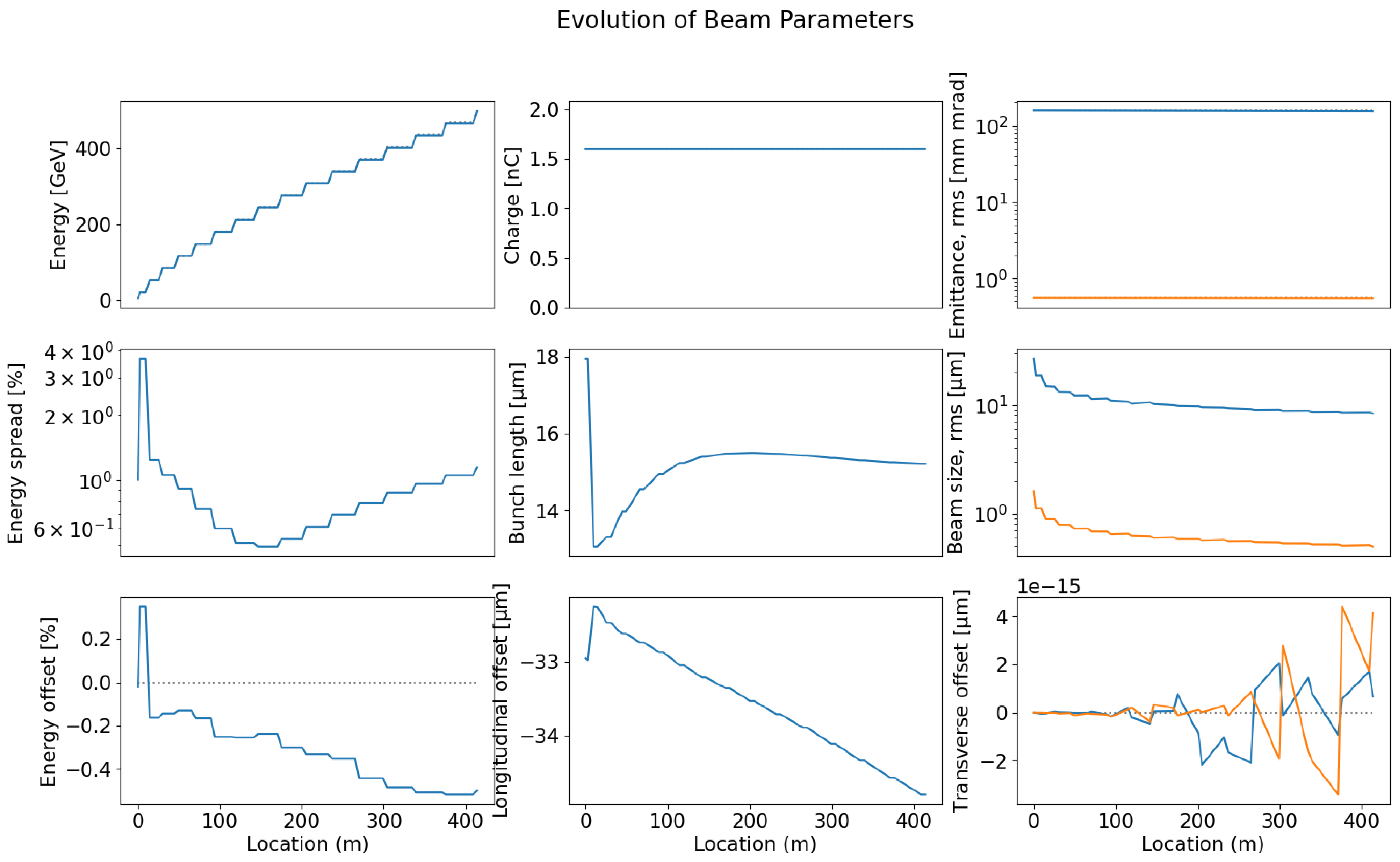
- The particles with large betatron amplitudes lose more energy.
- This causes a "Bleed" in energy, and an increase in energy spread.



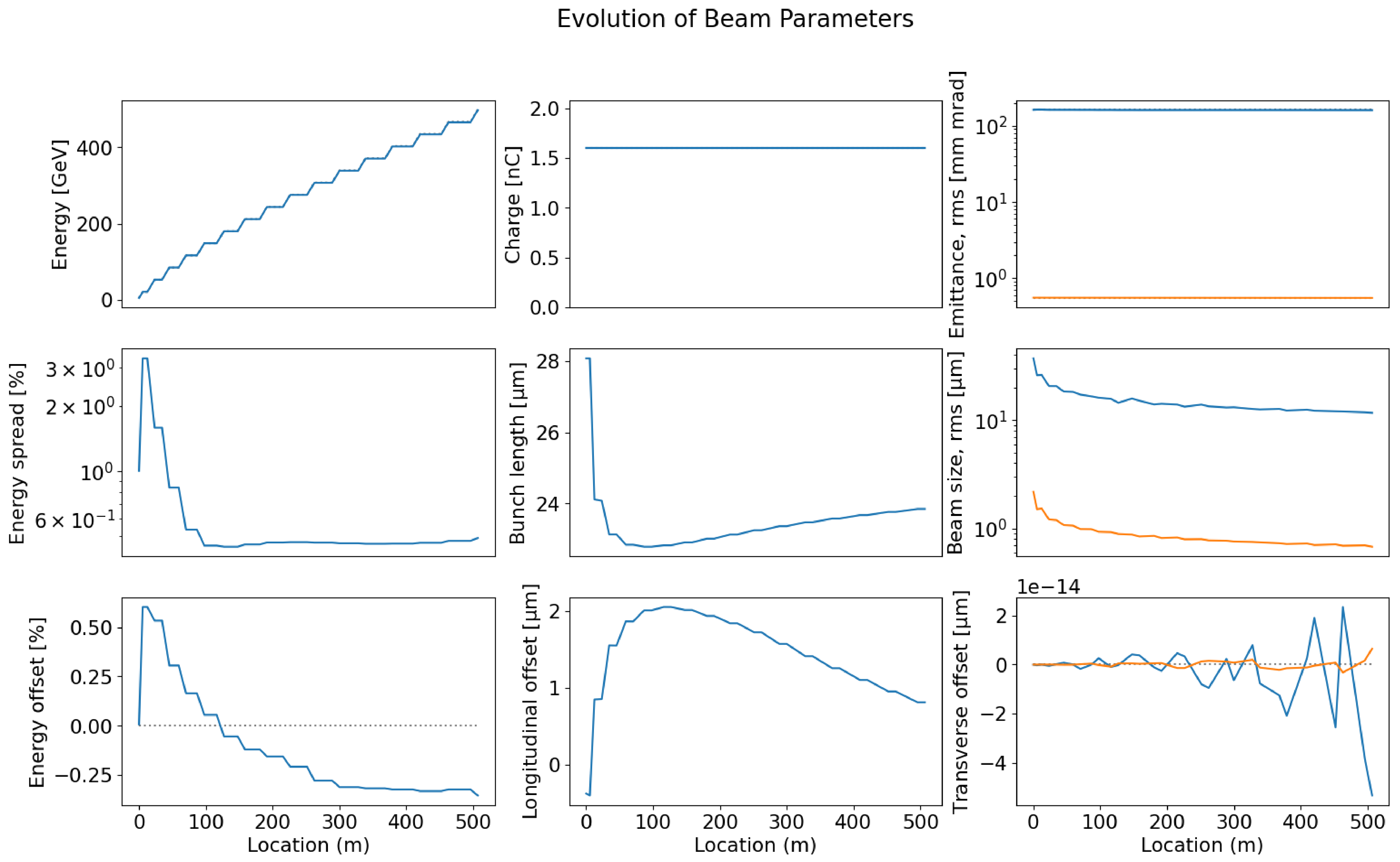
HALHF simulations

Tracking a beam through 16 stages

> Already at 500 GeV, we have had to lower the density



n_0 = 7e21 m^{-3}



n_0 = 2e21 m^{-3}

Goes as $n_0^{3/2}$

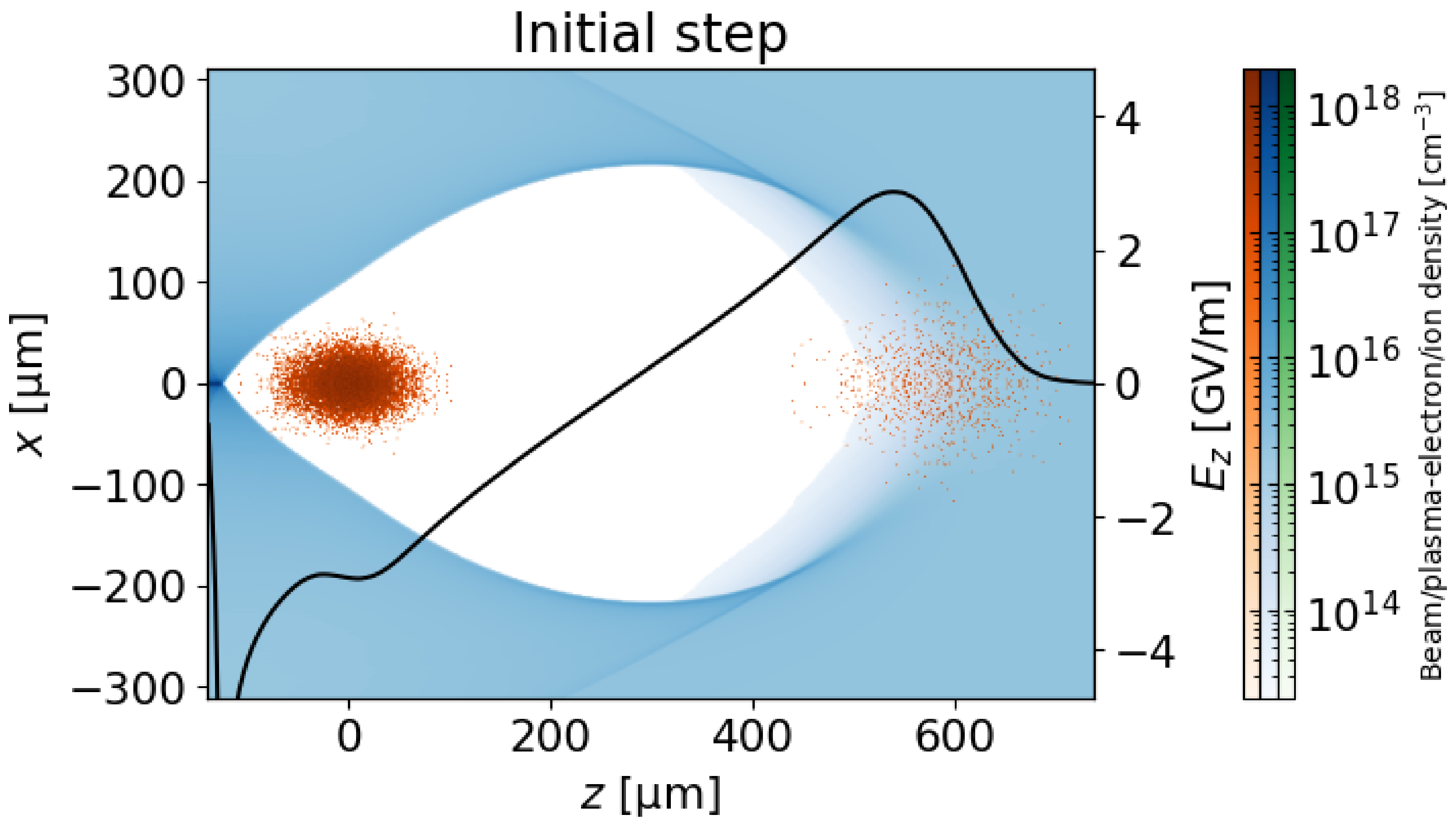
$$\sigma_{\bar{P}} = \frac{e^2 c}{24\pi\varepsilon_0}(\sqrt{\gamma}k_p)^3\sqrt{\varepsilon_{nx}^2 + \varepsilon_{ny}^2}$$

Plasma wakes

The effects of the radiation reaction + self-correction mechanism

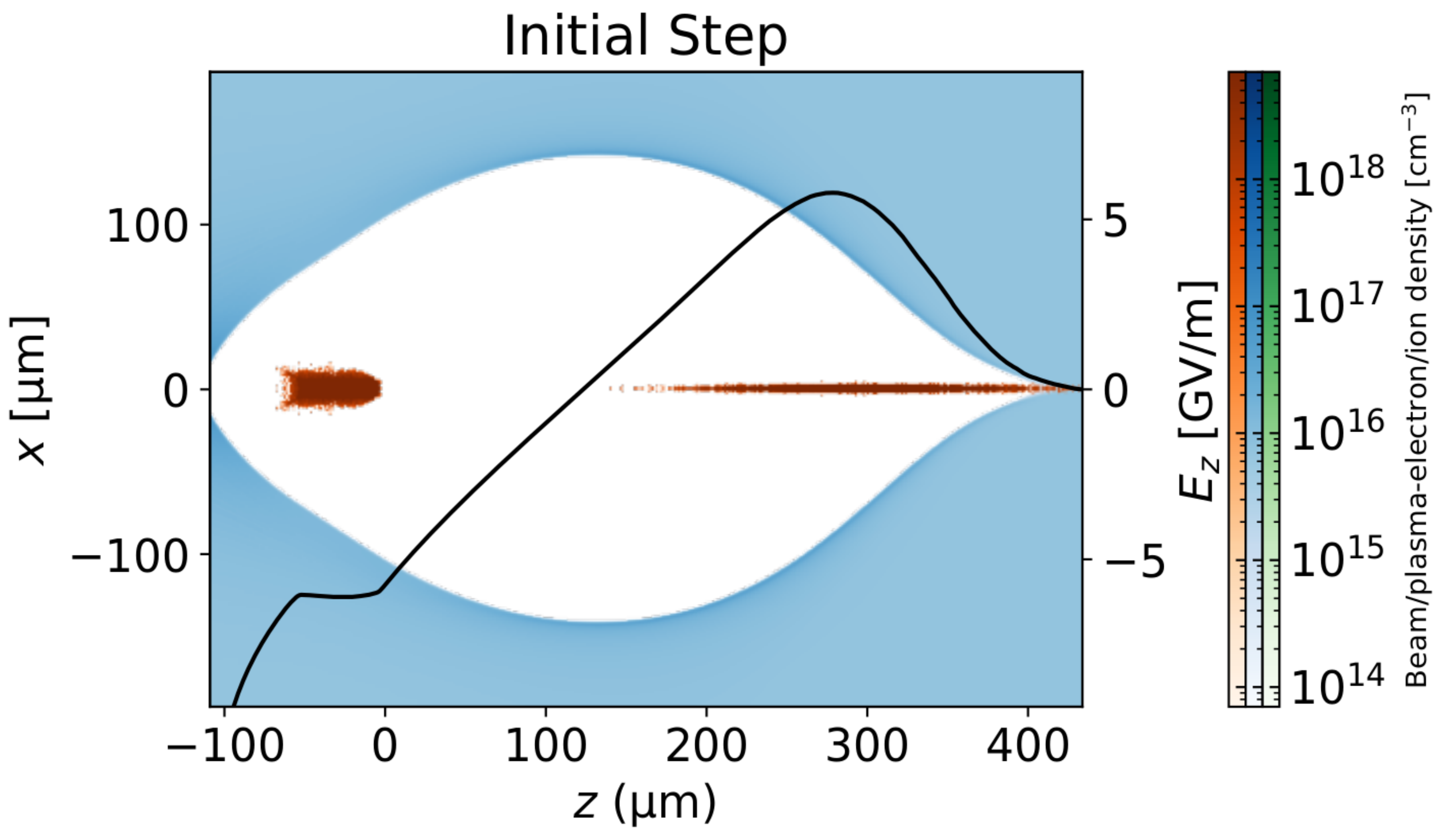
> Along with the self-correction mechanism, the beam shapes into a bullet.

1st stage



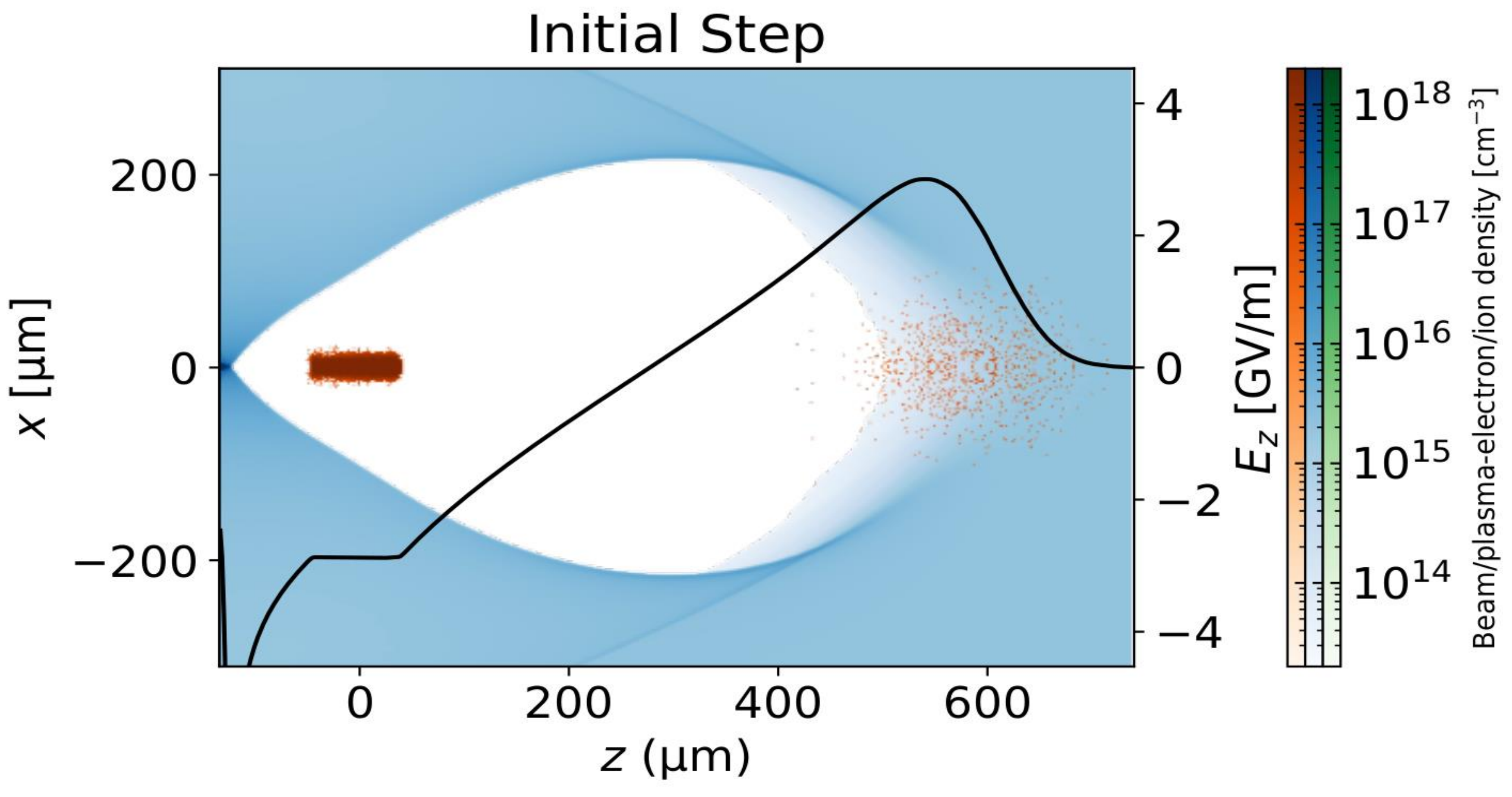
n_0 = 2e21 m^{-3}

16th stage



n_0 = 7e21 m^{-3}

16th stage



n_0 = 2e21 m^{-3}

Goes as $n_0^{3/2}$

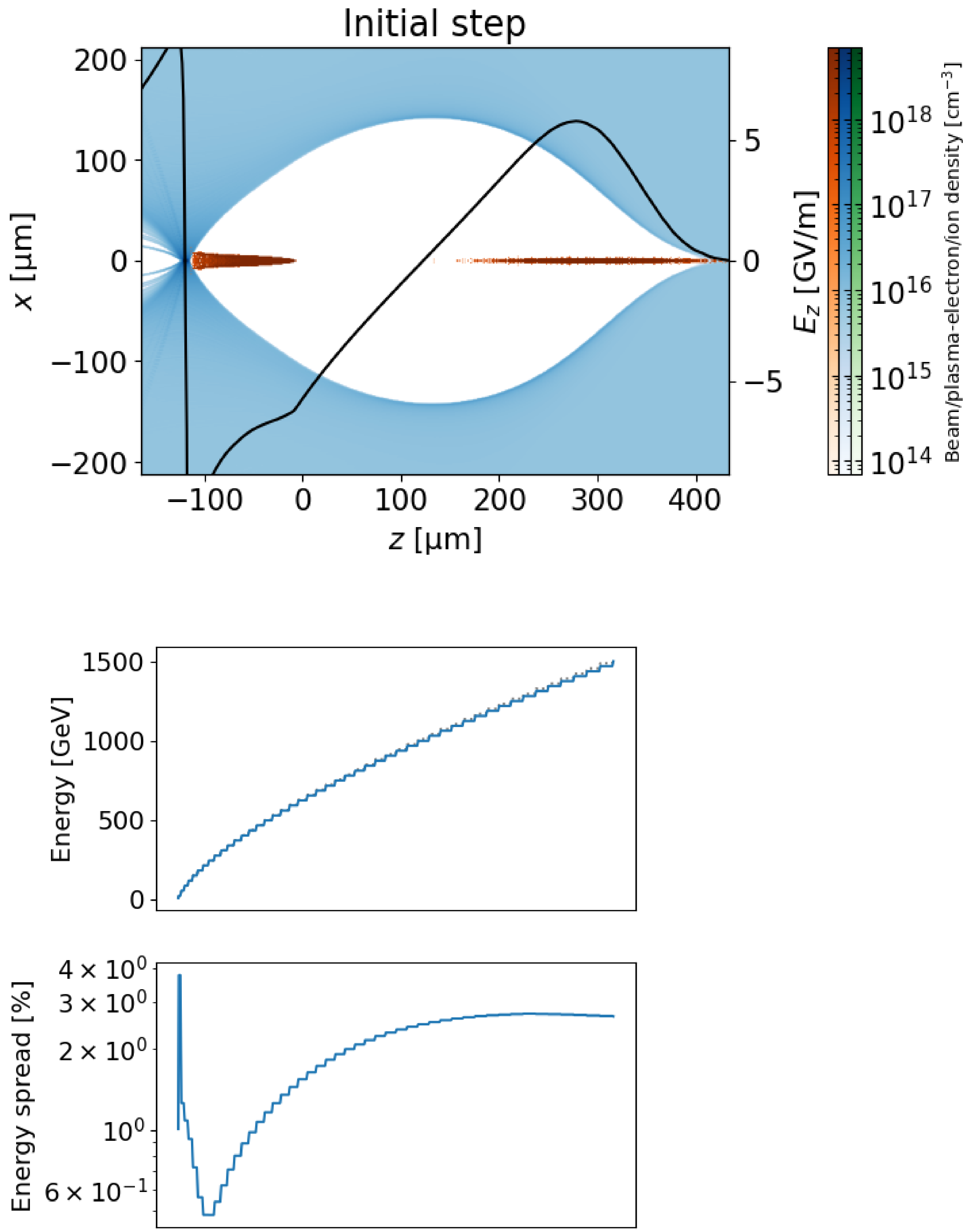
$$\sigma_{\bar{P}} = \frac{e^2 c}{24\pi\varepsilon_0}(\sqrt{\gamma}k_p)^3\sqrt{\varepsilon_{nx}^2 + \varepsilon_{ny}^2}$$

Reaching TeV energies

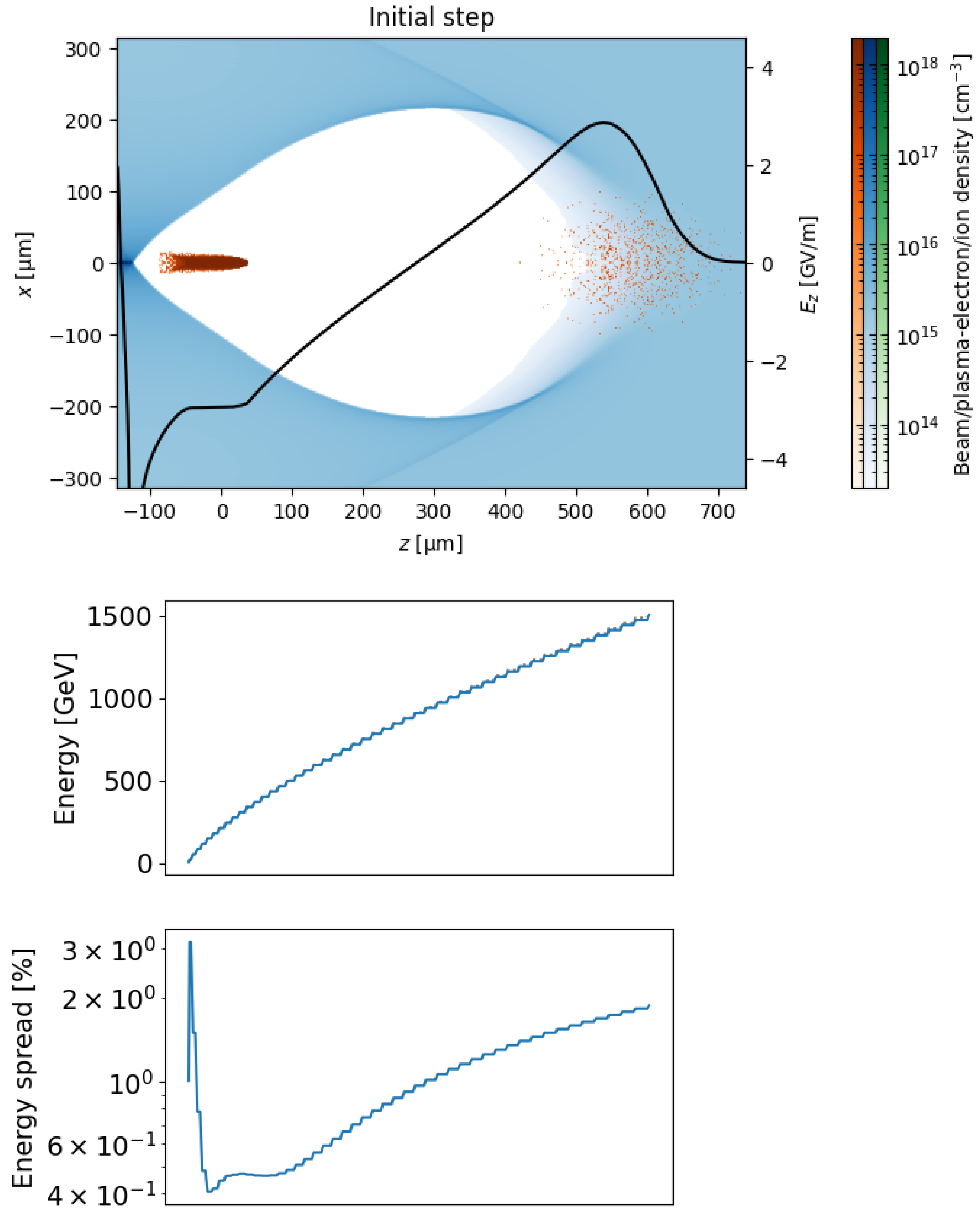
Can we take the self-correction further?

> The energy spread becomes too large.

> The beam reaches the end of the wake.



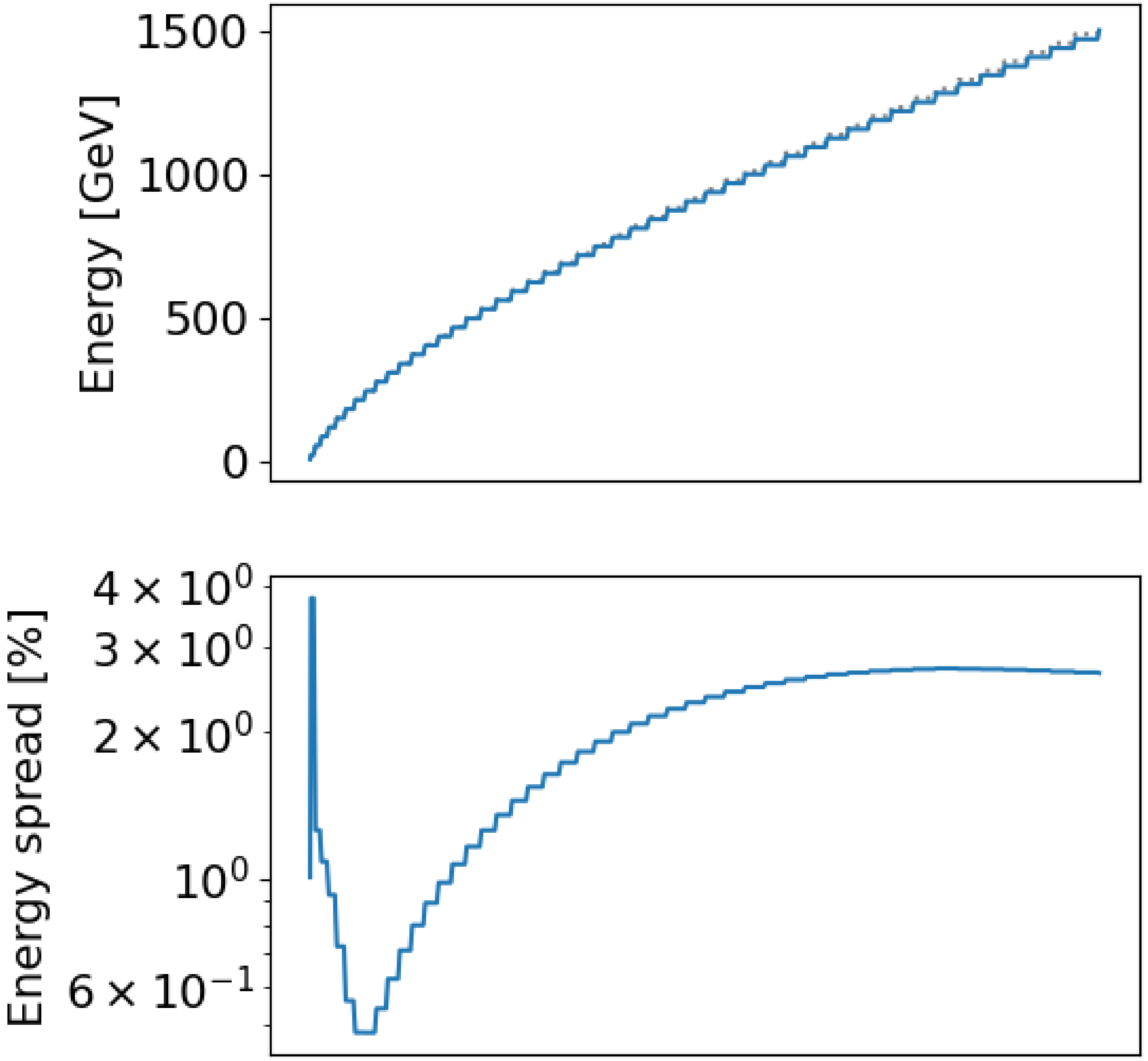
n_0 = 7e21 m^{-3}



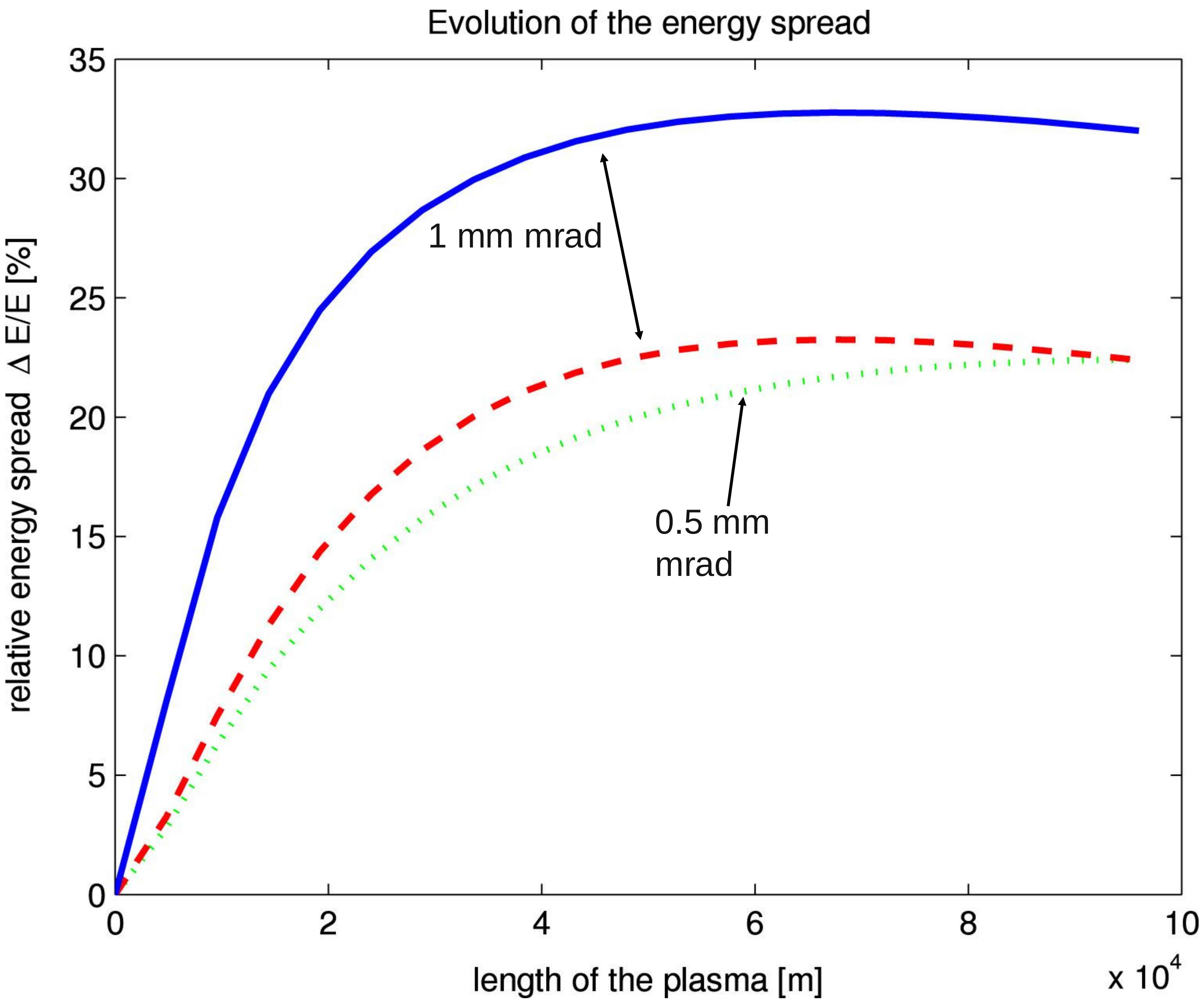
n_0 = 2e21 m^{-3}

Maximum energy-spread

Shpakov et al. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 909 (2018)



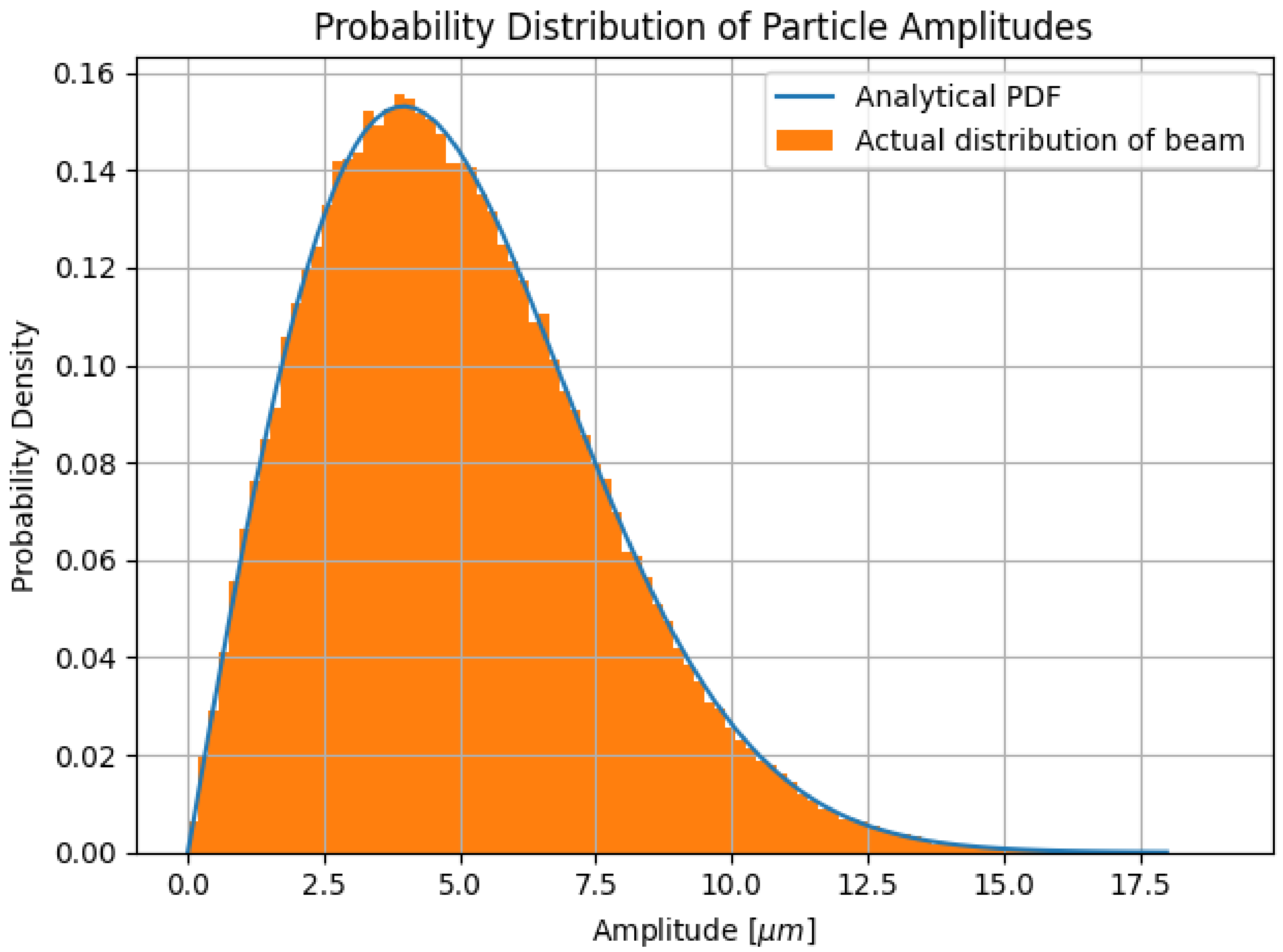
n_0 = 7e21 m⁻³

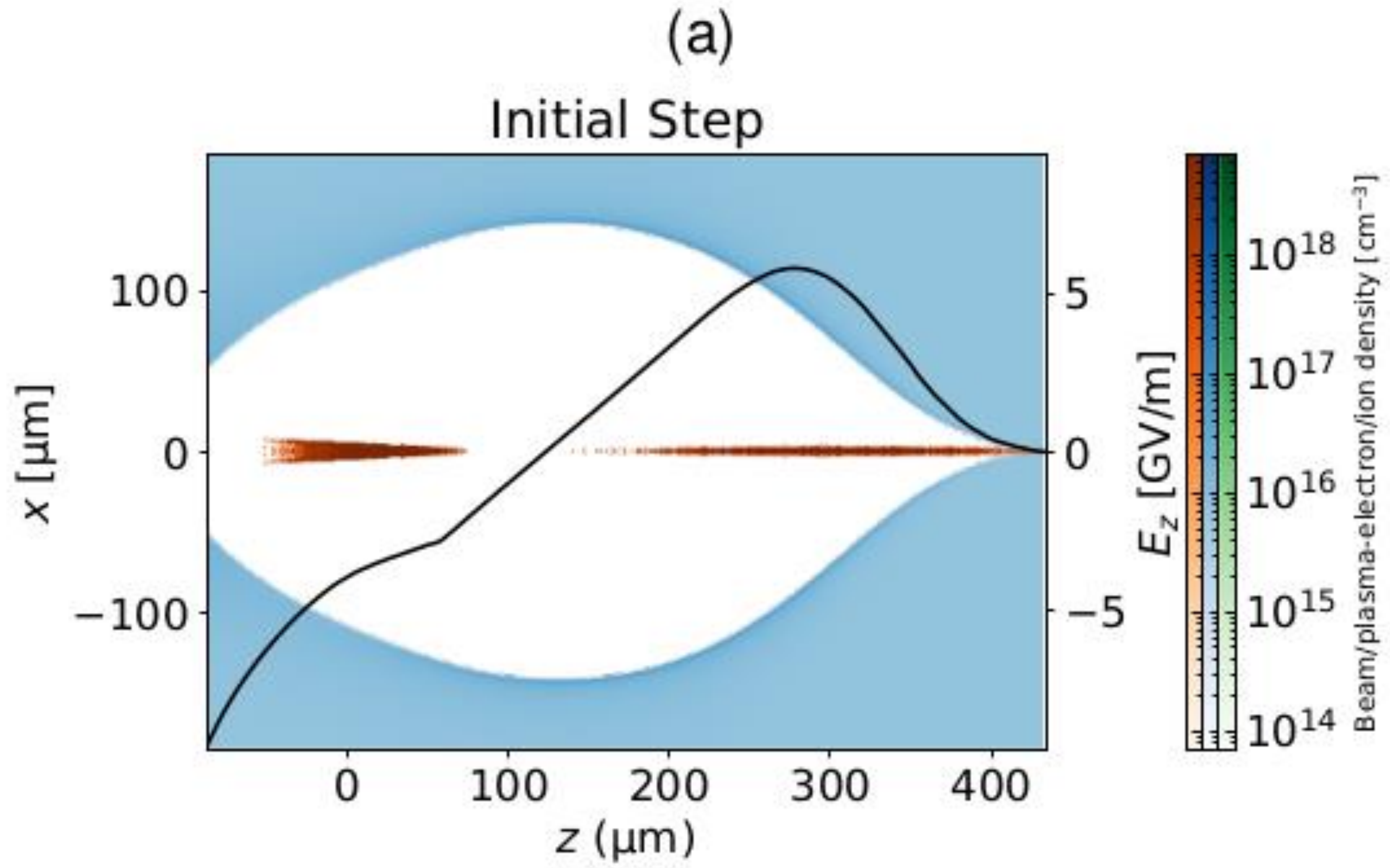


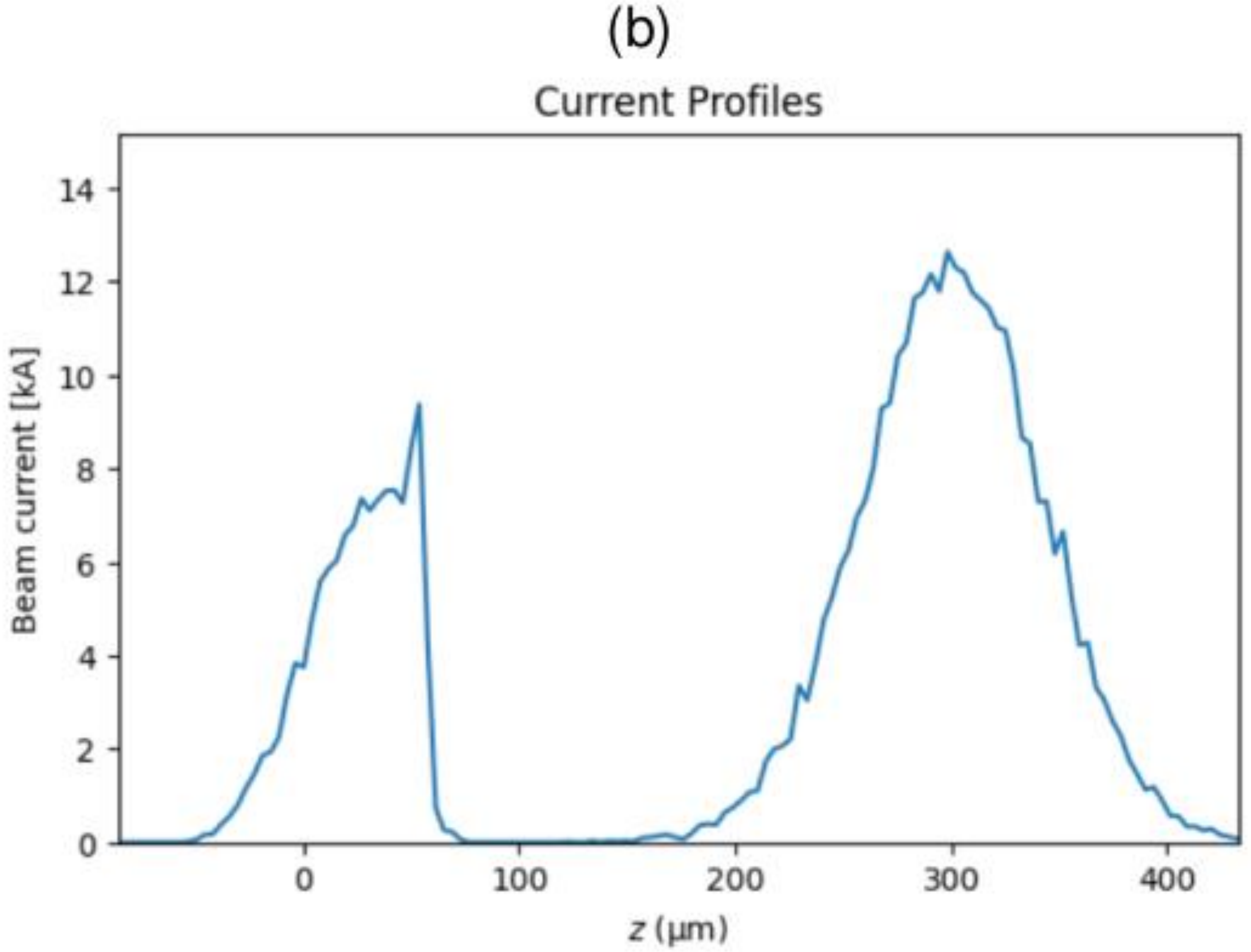
n_0 = 1e22 m^{-3} (blue/red), n_0 = 5e21 m^{-3} (green). no self-correction

Reaching TeV energies

> There is now a strong correlation between longitudinal position and betatron amplitude!



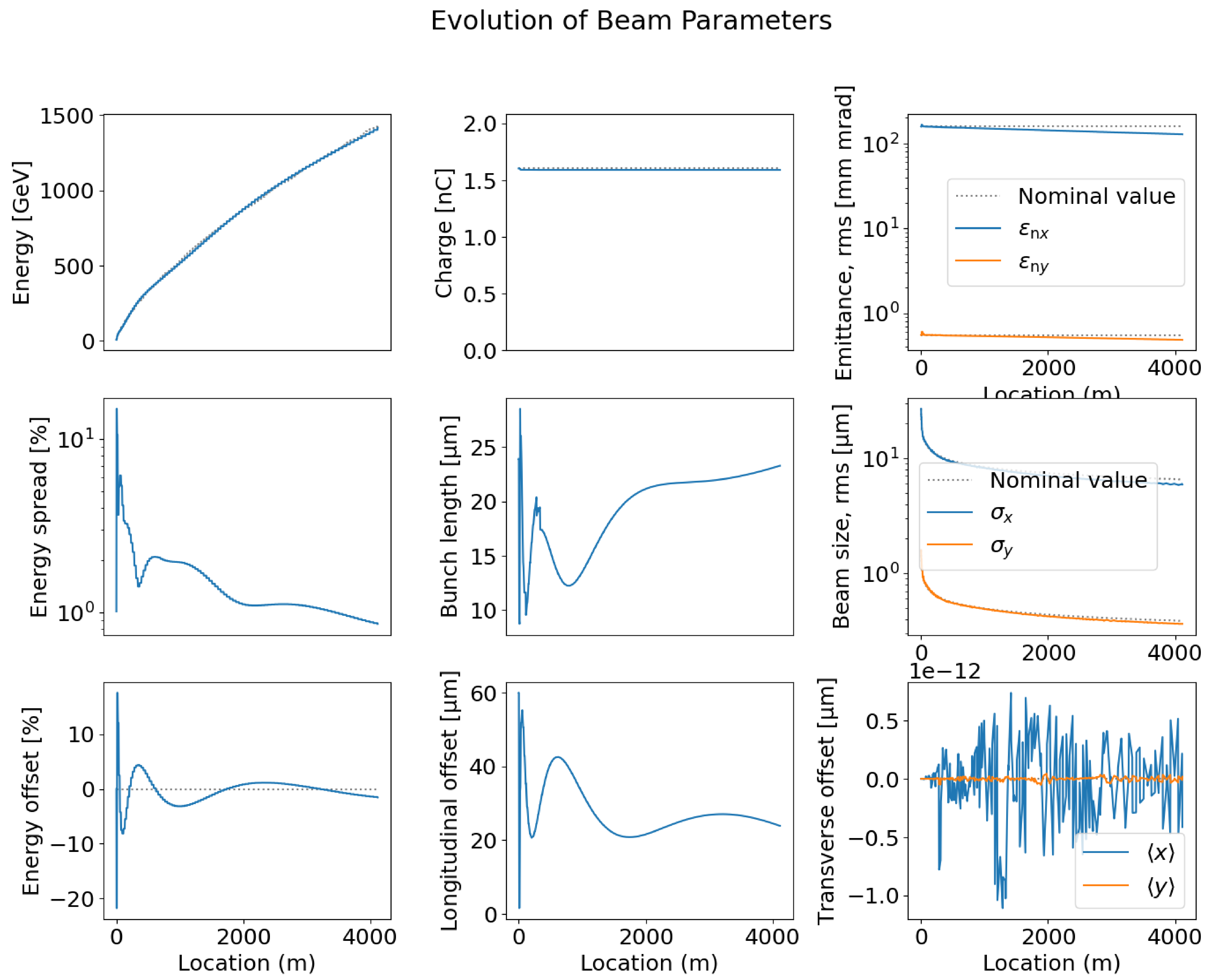


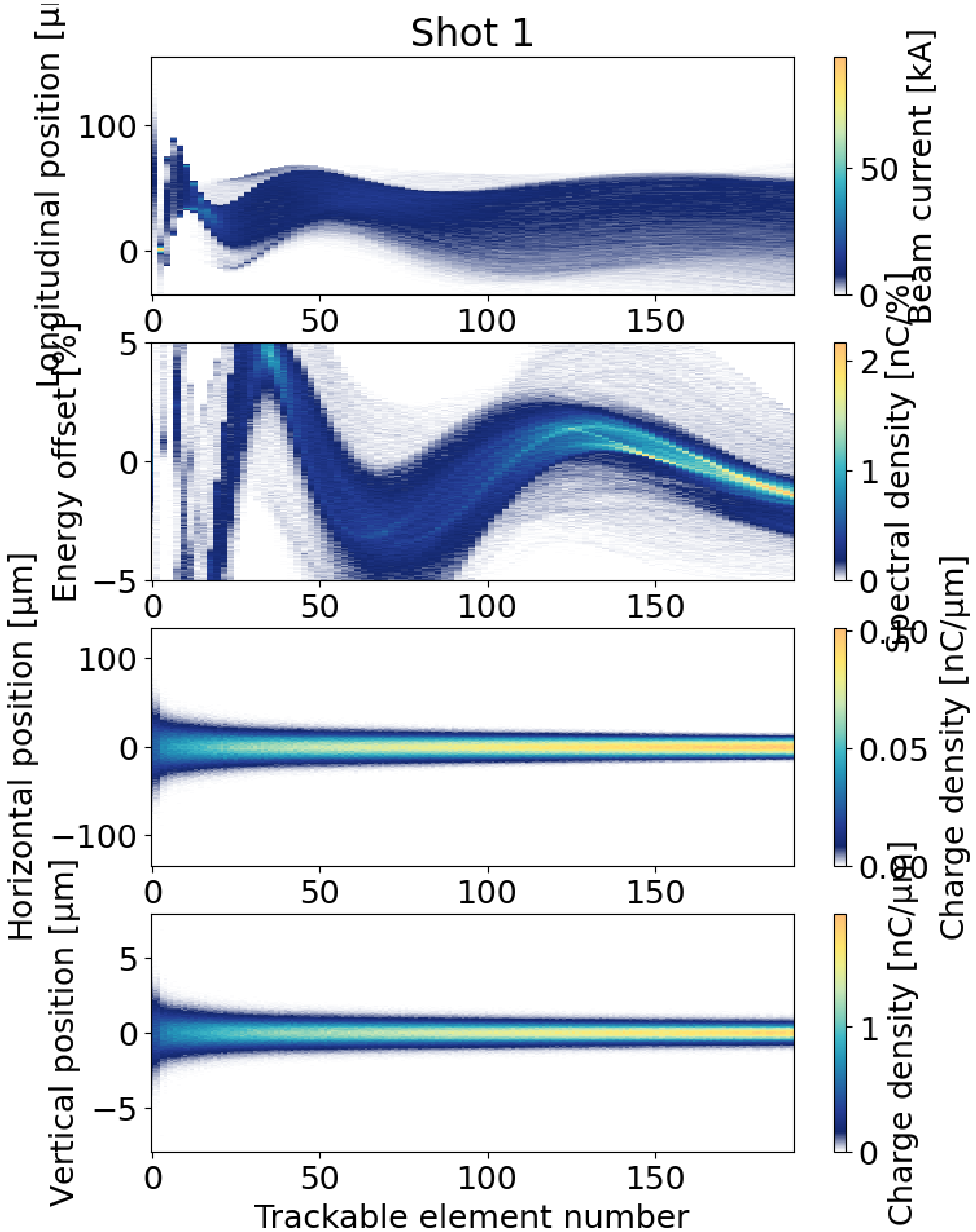


n_0 = 7e21 m^{-3}

Reaching TeV energies

Adjusting the parameters





n_0 = 7e21 m^{-3}

Reaching TeV energies

The self-correction can't be applied forever. The beam size will increase in the magnetic chicanes.

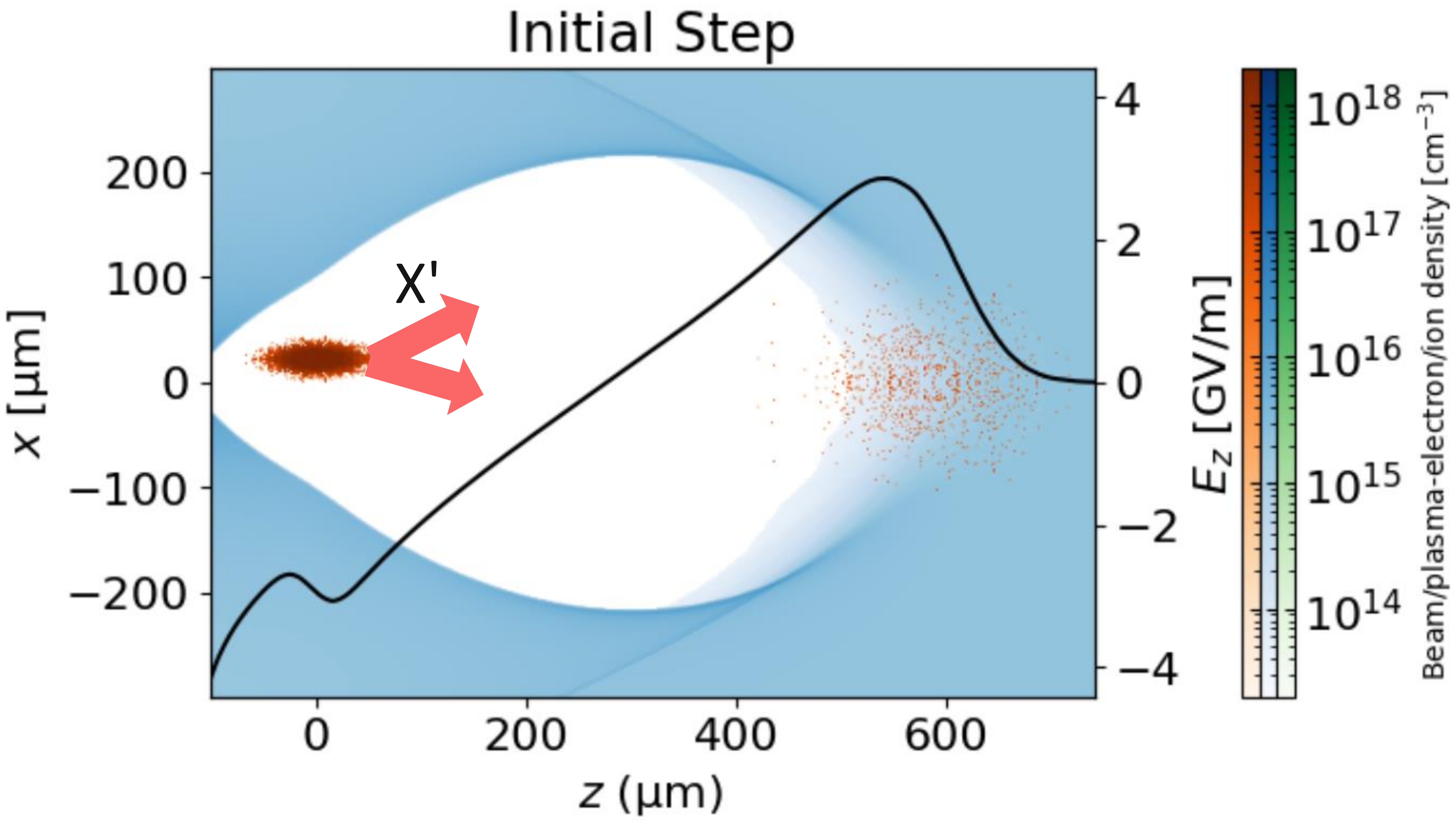
What About Jitter?

Radiation due to small offsets in staging

> When the beam enters the plasma, it will have a small random offset (jitter). In x, this means the new energy loss will be

$$\langle \bar{P} \rangle_x = \Lambda \left(1 - \frac{1}{2}\frac{\Delta x^2}{\sigma_x^2}\right)\left(2\sigma_x^2 + 2\Delta x^2 + \frac{15}{16}\frac{\Delta x^4}{\sigma_x^2} + \frac{5}{8}\frac{\Delta x^6}{\sigma_x^4}\right)$$

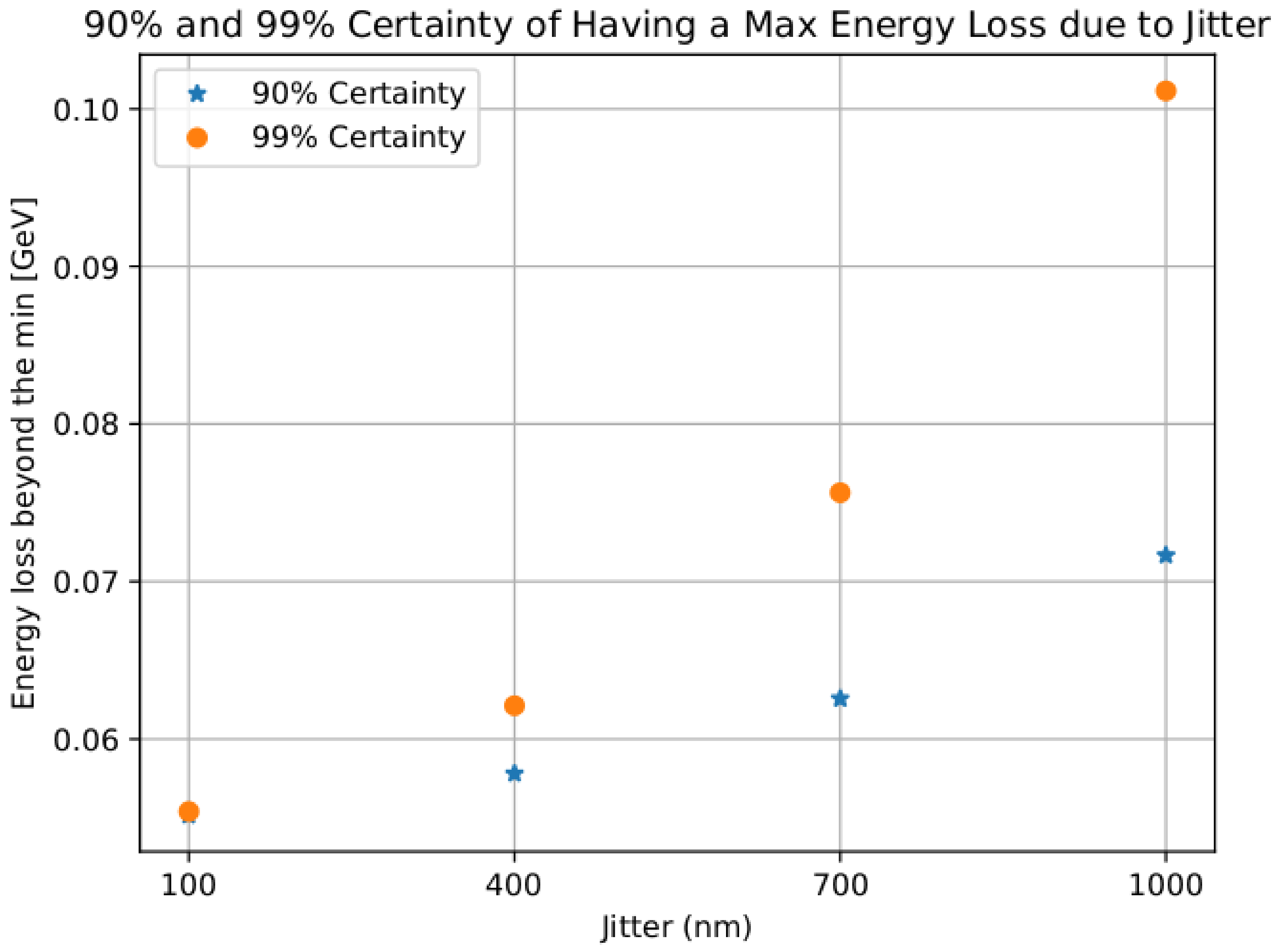
$$\Lambda = \frac{e^2 c}{12\pi\varepsilon_0}(\gamma k_\beta)^4$$



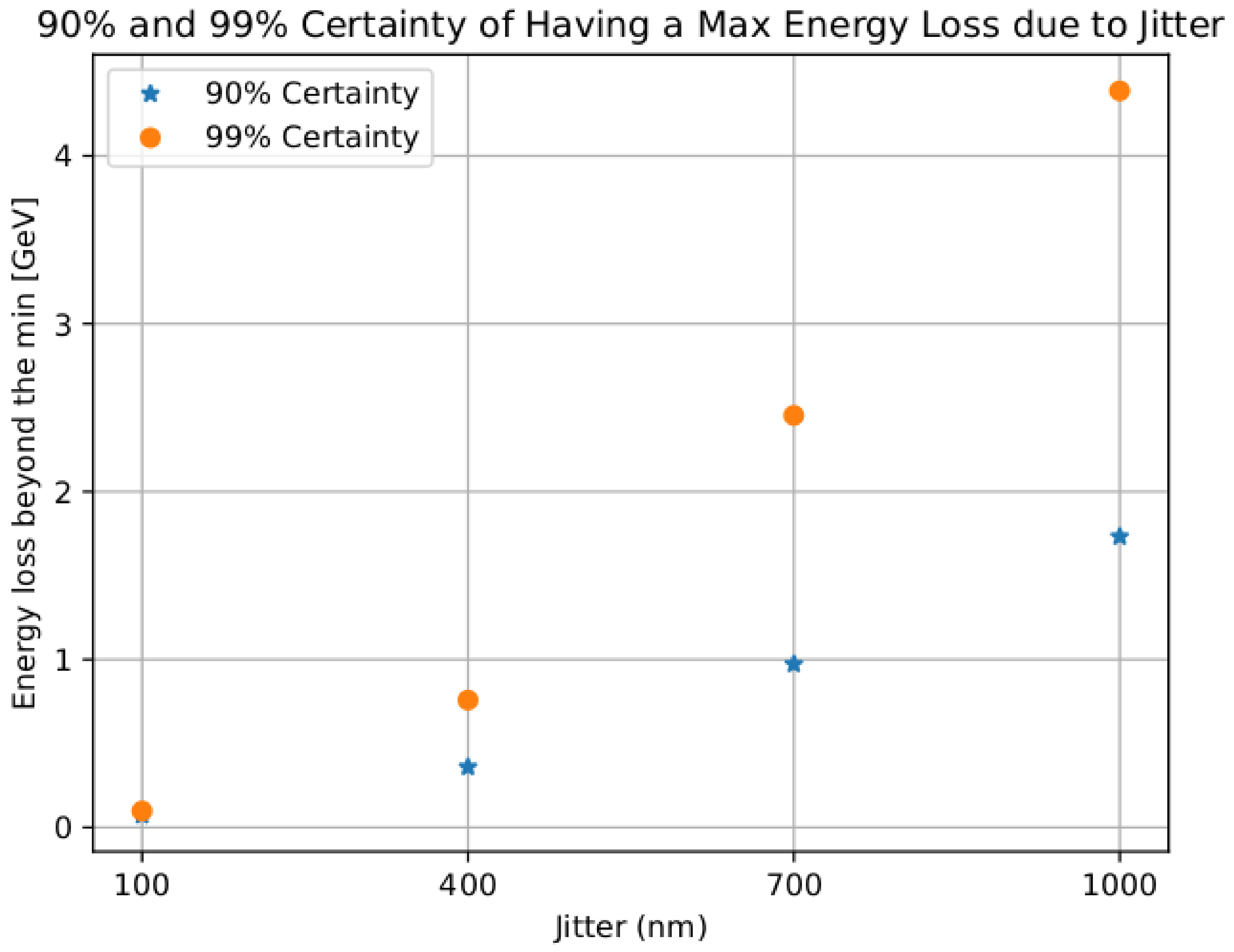
What about jitter?

Extra energy loss (beyond what the beam would lose anyway) due to jitter

500 GeV



5 TeV

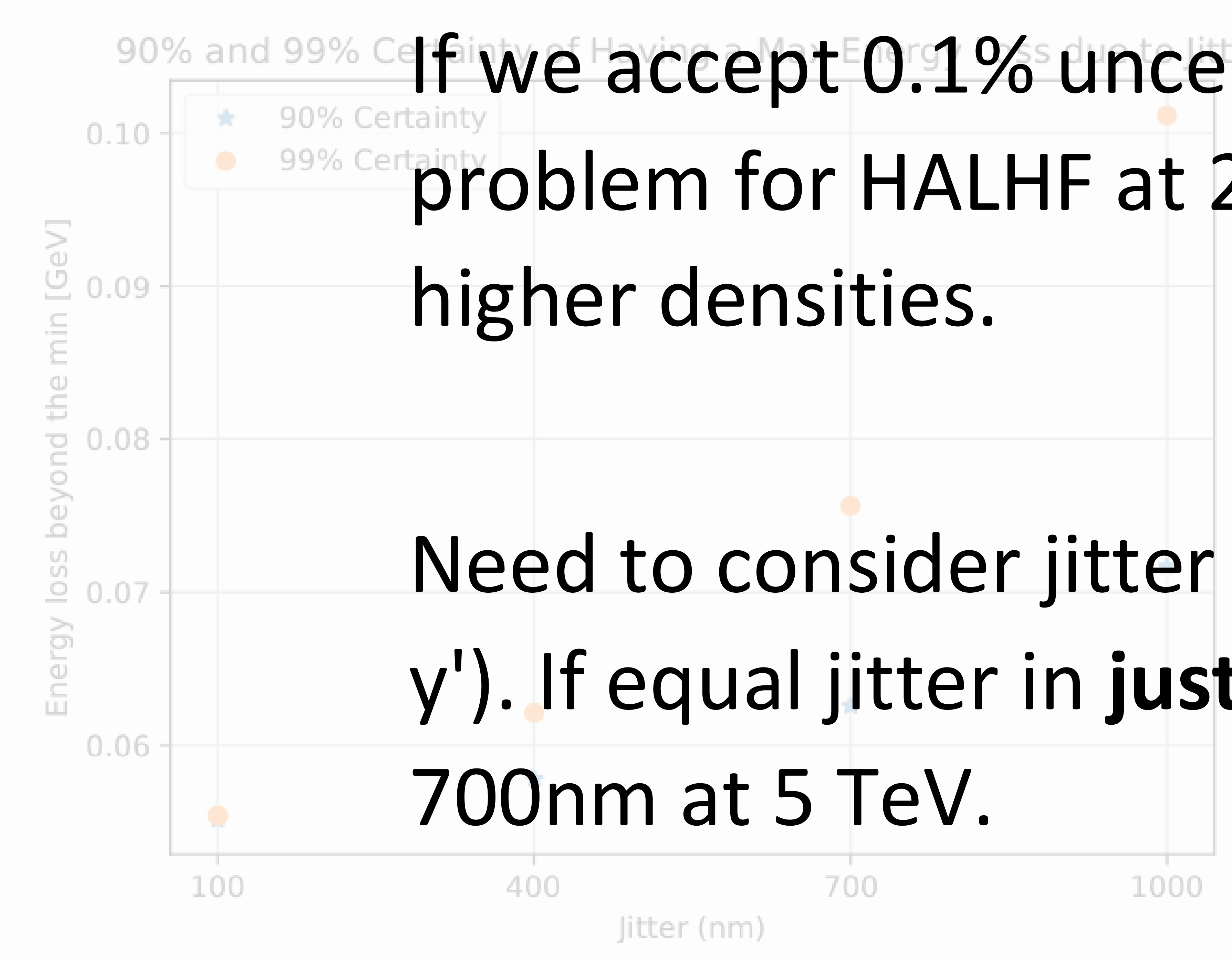


> Maximum energy loss with 90/99 % certainty.

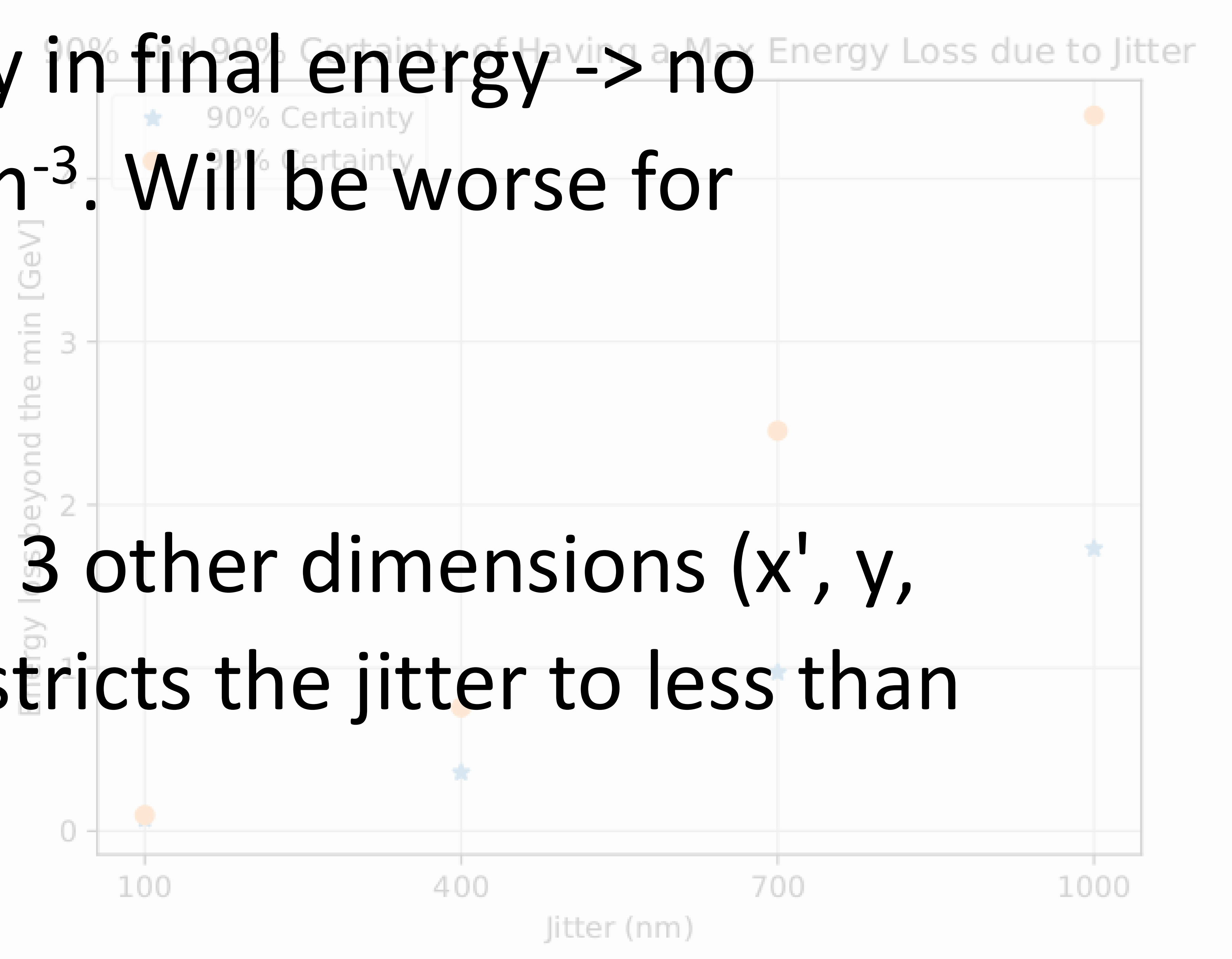
What About Jitter?

Extra energy loss (beyond what the beam would lose anyway) due to jitter

500 GeV



5 TeV



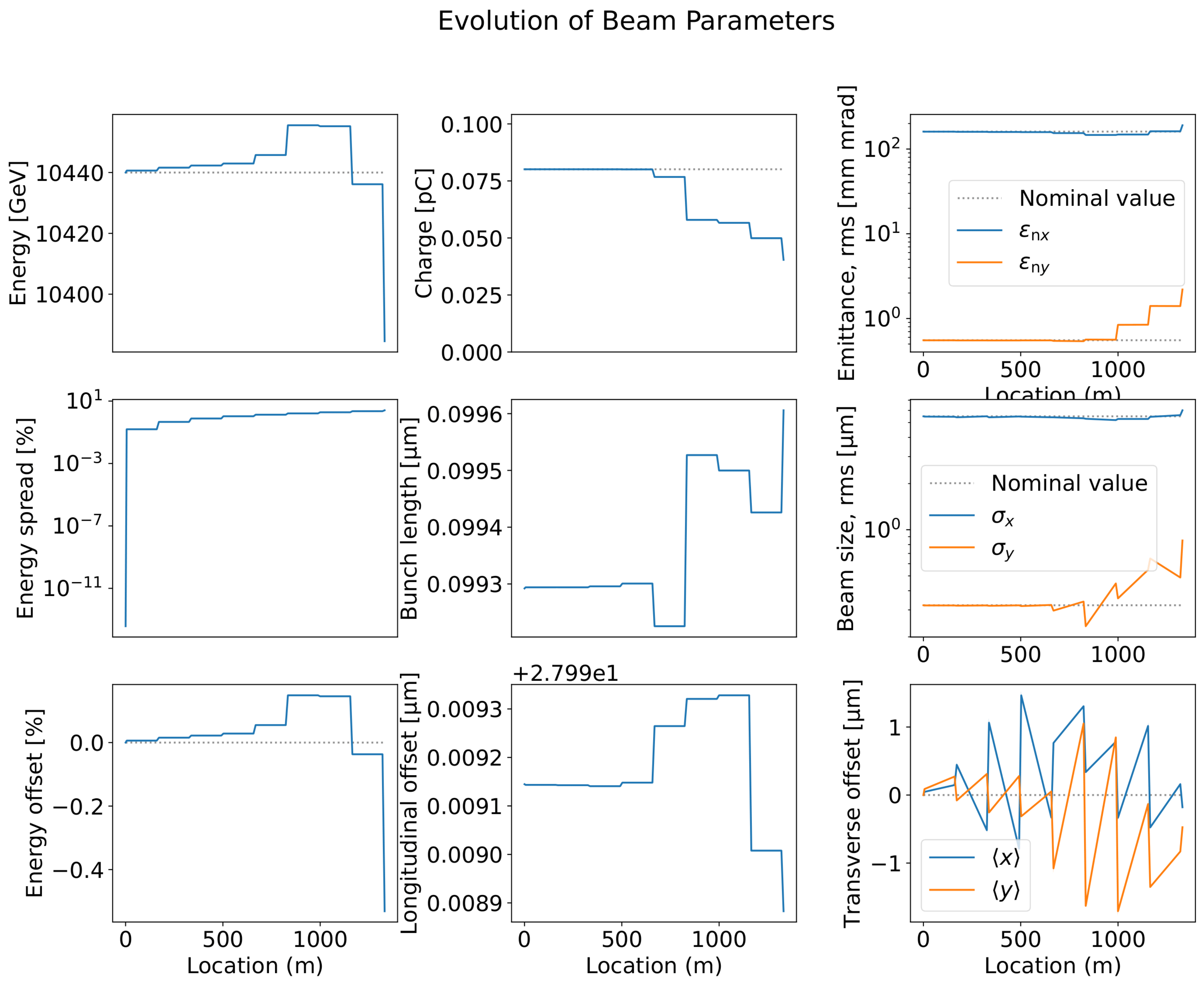
If we accept 0.1% uncertainty in final energy -> no problem for HALHF at 2e21 m^{-3}. Will be worse for higher densities.

Need to consider jitter in the 3 other dimensions (x', y, y'). If equal jitter in **just y,** restricts the jitter to less than 700nm at 5 TeV.

What about jitter?

Maximum achievable energy

- With staging, there is a maximum achievable energy.
- This maximum is probabilistic, and depends on the offset.



Jitter = 250 nm in x and y.

Summary

> The radiation reaction causes energy loss and energy spread. The self-correction mechanism helps correct for the energy spread, but will it work in practice at 5 TeV?

> These simulations show that TeV acceleration is possible with regards to energy spread. Still need to examine the radiation reaction in the correcting chicanes.

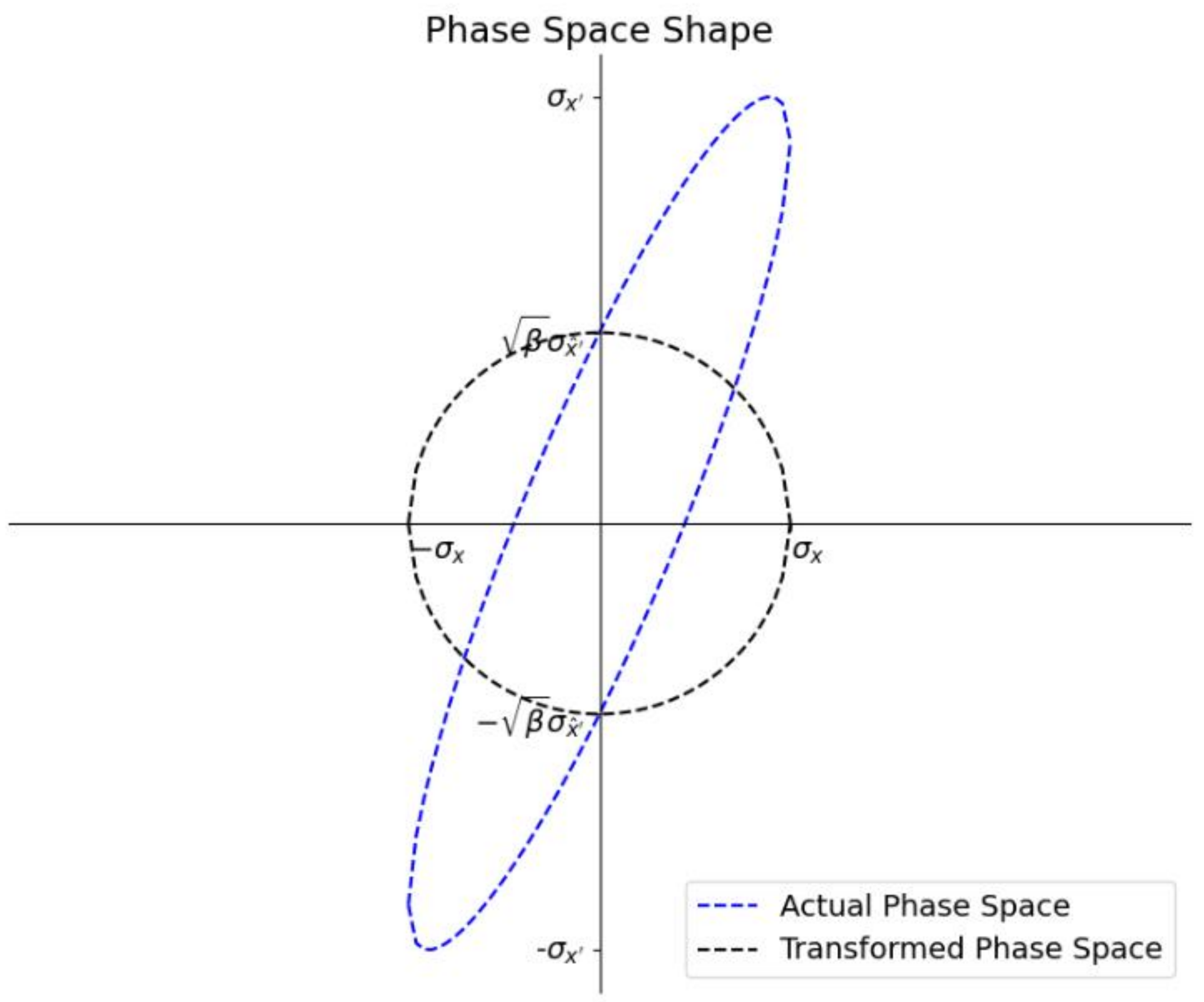
> Jitter contributes to energy loss, and random energy offsets. Seems not to be a problem for HALHF but seems to impose restrictions at 5 TeV.

Extra

Matching and Betatron Amplitude

> Normalized phase space (from ellipse eq.)

$$x^2 + \beta \hat{x}^2 = \varepsilon\beta \quad \text{where} \quad \hat{x}' = \alpha x + \sqrt{\beta} x'$$



> We also have $x^2 + x'^2 = r_x^2(cos^2(k_\beta ct + \phi) + k_\beta^2 sin^2(k_\beta ct + \phi))$

Which means $x^2 + \bar{x}'^2 = r_x^2$ where $\bar{x}' = \frac{1}{k_\beta} x'$

Caption

Extra

Matching and Betatron Amplitude

> For a matched beam; α=0, and the single particle trajectories must follow the shape of the phase space.

> I.e $x^2 + \bar{x}'^2 = r_x^2$ must match the shape of $x^2 + \beta \hat{x}^2 = \varepsilon \beta$, or the energy spread will cause emittance increase.

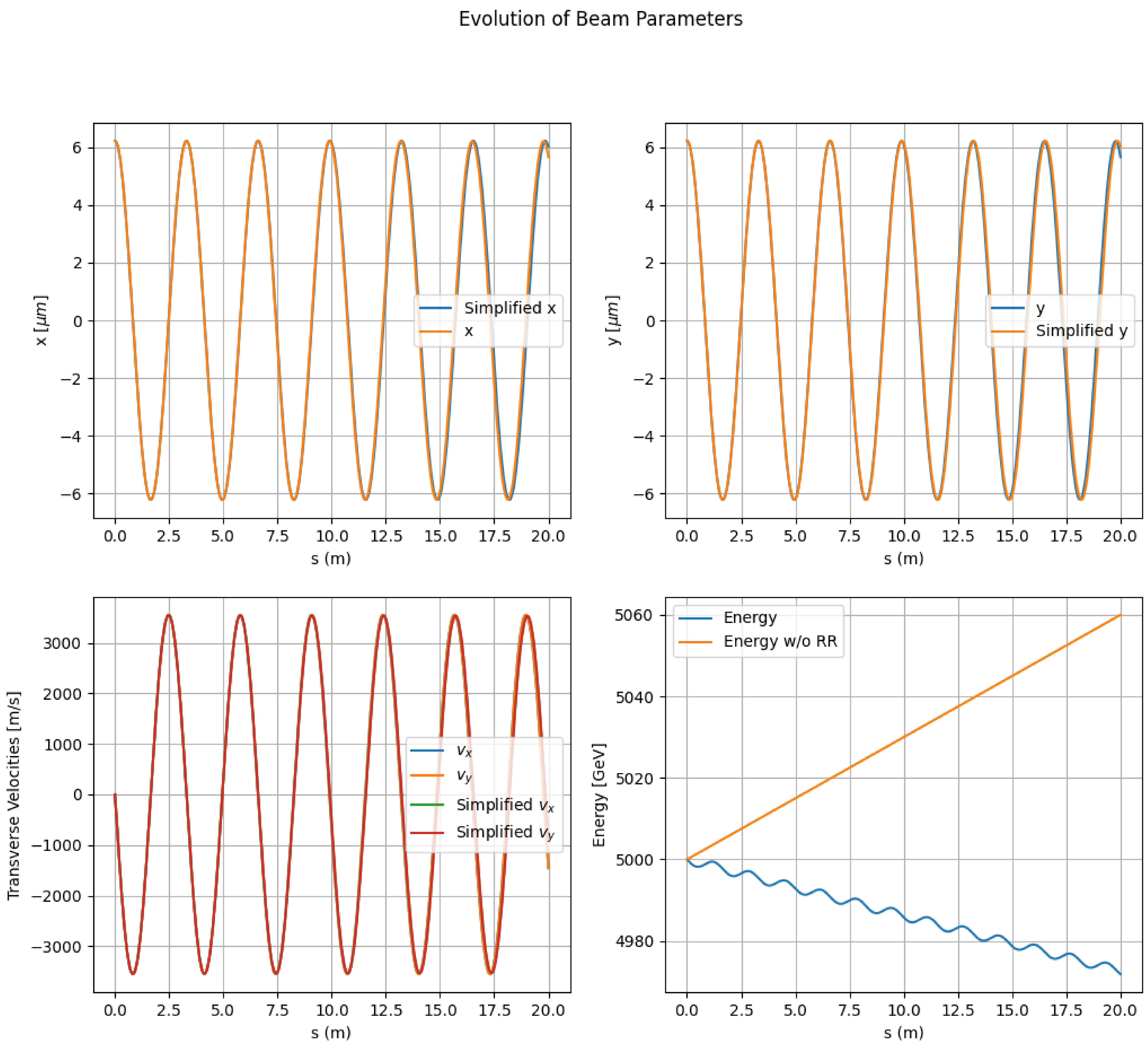
> We therefore know that $r_x = \sqrt{x^2 + (x'\beta_m)^2}$

Caption

Extra

Betatron Motion at 5 TeV

> Even at 5 TeV, the motion w/wo radiation reaction is almost the same



Extra

Jitter energy loss at 7.5 TeV

> The plot that led me to believe my initial expression for the jitter energy loss was correct