Beamstrahlung at 10 TeV e+e- Colliders

Thanks to Jaden He,
Arianna Formenti,
Spencer Gessner

M. E. Peskin
March 2025

We would like to have a better physical understanding of beam-beam effects at 10 TeV electron colliders.

The first step is understanding beamstrahlung. The most important beam-beam effects are QED interactions of electrons and photons. The initial energy distributions of these particles is generated by beamstrahlung. So the first step in understanding the full picture is to understand the energy distributions of electrons and photons as a function of time through the bunch-bunch collision as they are modified by beamstrahlung.

This time-dependent picture can be obtained through CAIN, Guinea-Pig, and, now, WARP-X simulations. These give the 3-dimension structure of the bunch-bunch collision in detail.   However, for physical understanding, it is useful to have a simpler picture represented by analytic formulae. That is what I am striving for here.

At the 1989 PAC, Yokoya and Chen wrote an analytic description of beamstrahlung appropriate for 1 TeV colliders. To obtain this, they considered beamstrahlung as only a function of electron energies evolving in time through the bunch-bunch collision. They simplified the underlying equations to the point where they could solve them, and they supplemented this solution with numerical parametrizations. That description compared well with simulations of linear collider beam-beam interactions up to 1 TeV, with Y parameters of order 1.

To think about the 10 TeV regime, with much larger values of Y, we should reconsider their work. We may need new approximations for the higher-energy regime.

Last summer, the SLAC summer student Jaden He carried out a series of Guinea-Pig simulations for a series of e+e- colliders at CM energies of

1, 3, 5, 10, 15, 20, 30 TeV

holding transverse beta function x normalized emittance fixed. This leads to constant disruption parameter along each series. Jaden studied two series, one with flat beams, one with round beams, both tuned to a luminosity of about 5×10^{35} integrated over collisions within up to 20% of the nominal CM energy. The scaling is

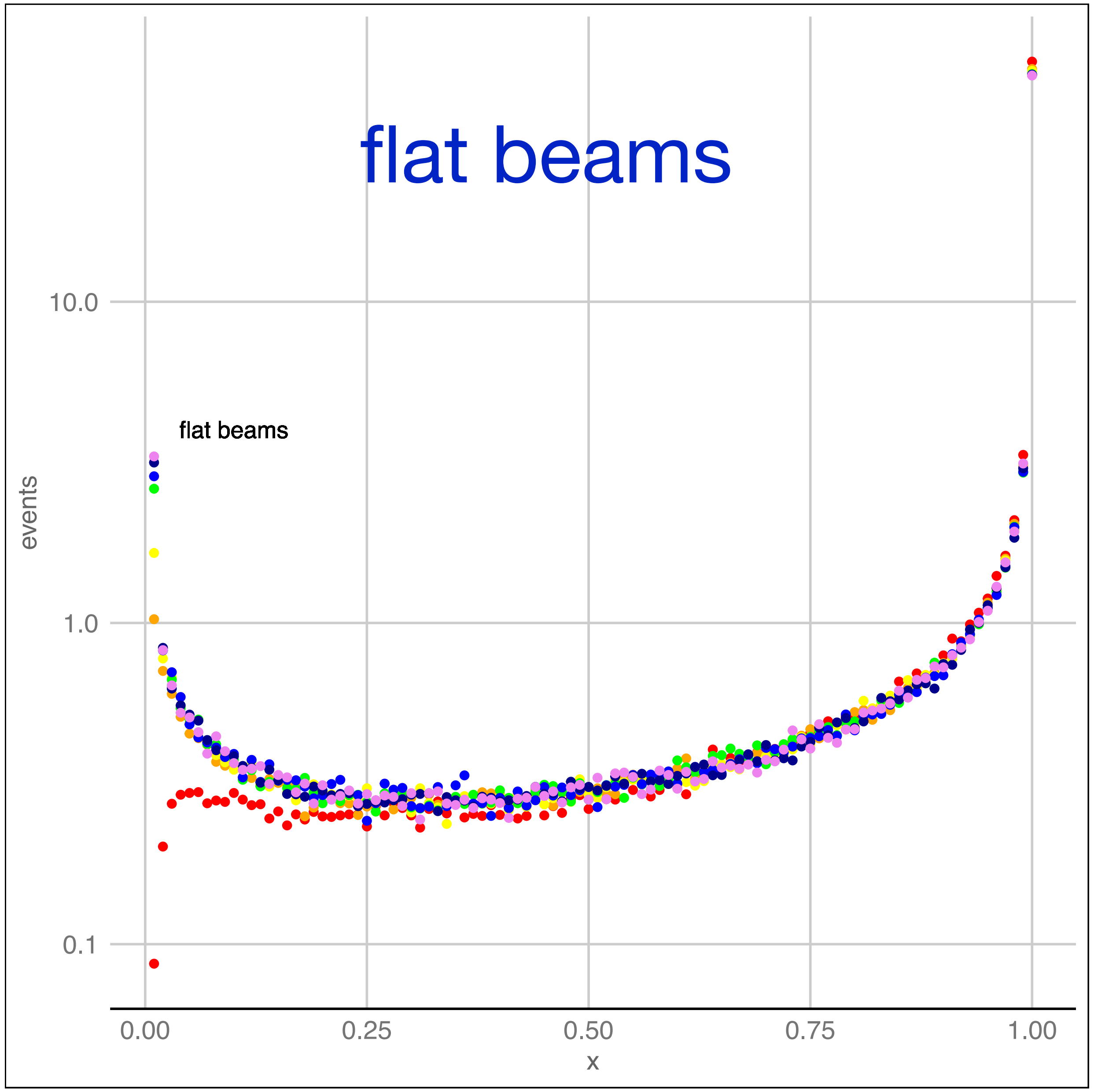
$$\mathcal{L} \sim E \qquad \Upsilon \sim E^{3/2} \qquad N_\gamma \sim \text{const.}$$

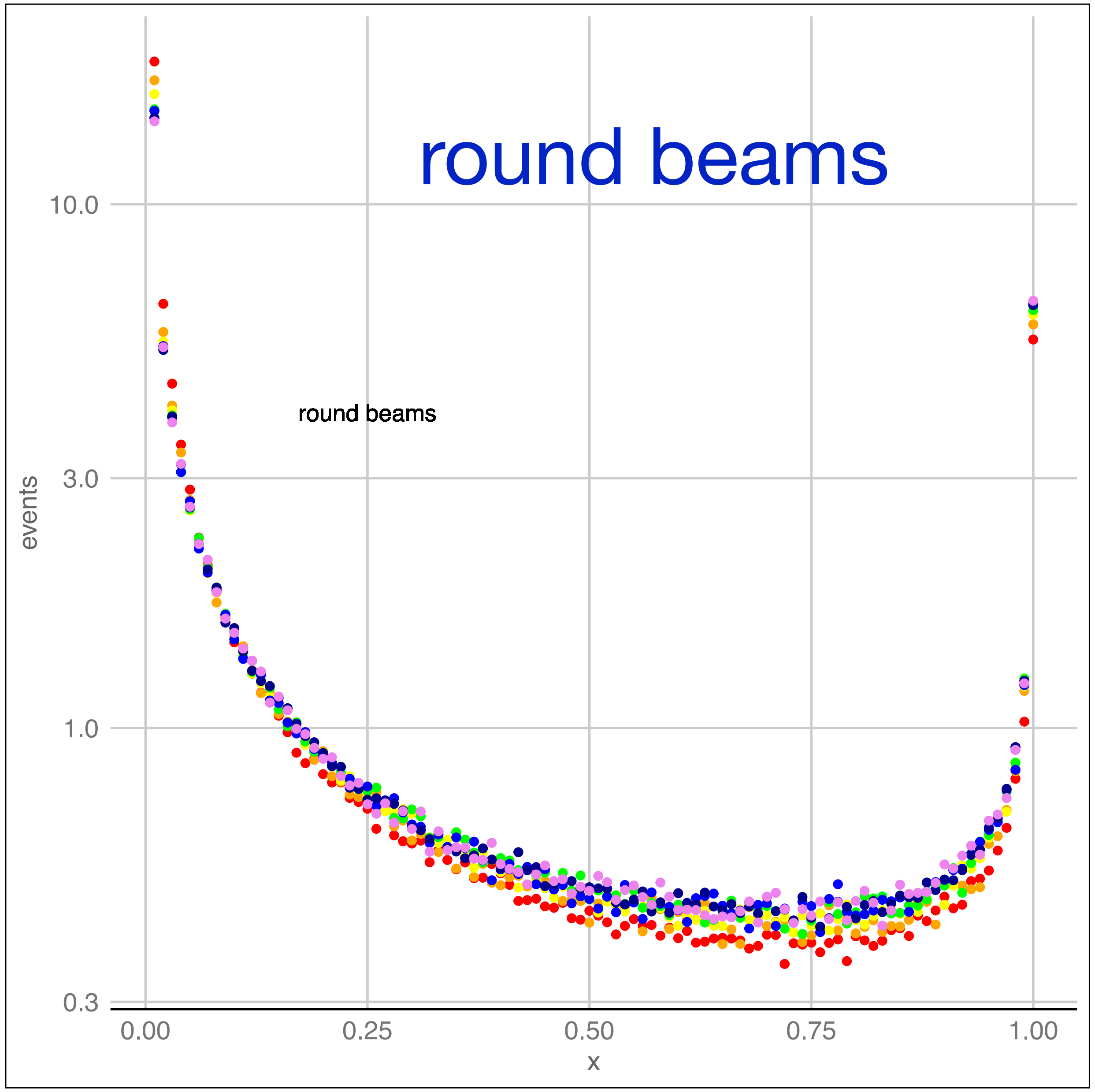
On the next slide, I show the normalized luminosity distribution for e+e- annihilations as a function of

$$z = E_{CM}/E_{CM,0} = \sqrt{x_e x_p} \qquad \text{where} \qquad x = E/E_0$$

normalized luminosity distribution as a function of z.

red to violet :   1 TeV -> 30 TeV





hmm, there is an asymptote in Υ !

Yokoya-Chen theory of multiphoton beamstrahlung:

Begin for the Sokolov-Ternov formula for the energy distribution in synchroton radiation: electrons at energy $\overline{E}$ radiating to energy E

$$\frac{d}{dE}P(E) = \frac{3\nu_{cl}}{5\pi\xi_0}\int_0^{\overline{E}} \frac{dE E_0}{E\overline{E}}\left[\frac{1}{1+\bar{\xi}y}\int_y^\infty dy' K_{5/3}(y') + \frac{(\bar{\xi}y)^2}{(1+\bar{\xi}y)^2}K_{2/3}(y)\right]$$

where ν_{cl} is the classical photon radiation rate and

$$\xi_0 = \frac{3}{2}\Upsilon \qquad \xi = \xi_0(E/E_0) \quad \bar{\xi} = \xi_0(\overline{E}/E_0) \qquad y = (1/\xi_0)(E_0/E - E_0/\overline{E})$$

Electrons are conserved, so the energy distribution of electrons is normalized

$$\int_0^1 dx\ y_e(x,t) = 1 \qquad \text{initial condition:} \quad y_e = \delta(x-1)$$

If electrons are subject to, on average, constant fields in the bunch-bunch collision, the distribution obeys the YC equation

$$\frac{d}{dt}y_e(x) = -\nu\, y_e(x)$$
$$+\frac{3\nu_{cl}}{5\pi\xi_0}\int_E^{E_0} \frac{d\overline{E}E_0}{E\overline{E}}\left[\frac{1}{1+\bar{\xi}y}\int_y^\infty dy' K_{5/3}(y') + \frac{(\bar{\xi}y)^2}{(1+\bar{\xi}y)^2}K_{2/3}(y)\right] y_e(\bar{x})$$

Notice that $y \sim 1/\xi_0 \to 0$ as $\Upsilon \to \infty$. Take the limit of y small in the Bessel functions. In this regime, the total quantum rate of photon emission is $\nu = 1.30\,\nu_{cl}\Upsilon^{-1/3}$

The nominal number of photons created is $N_\gamma = \nu \cdot \sqrt{3}\sigma_z$

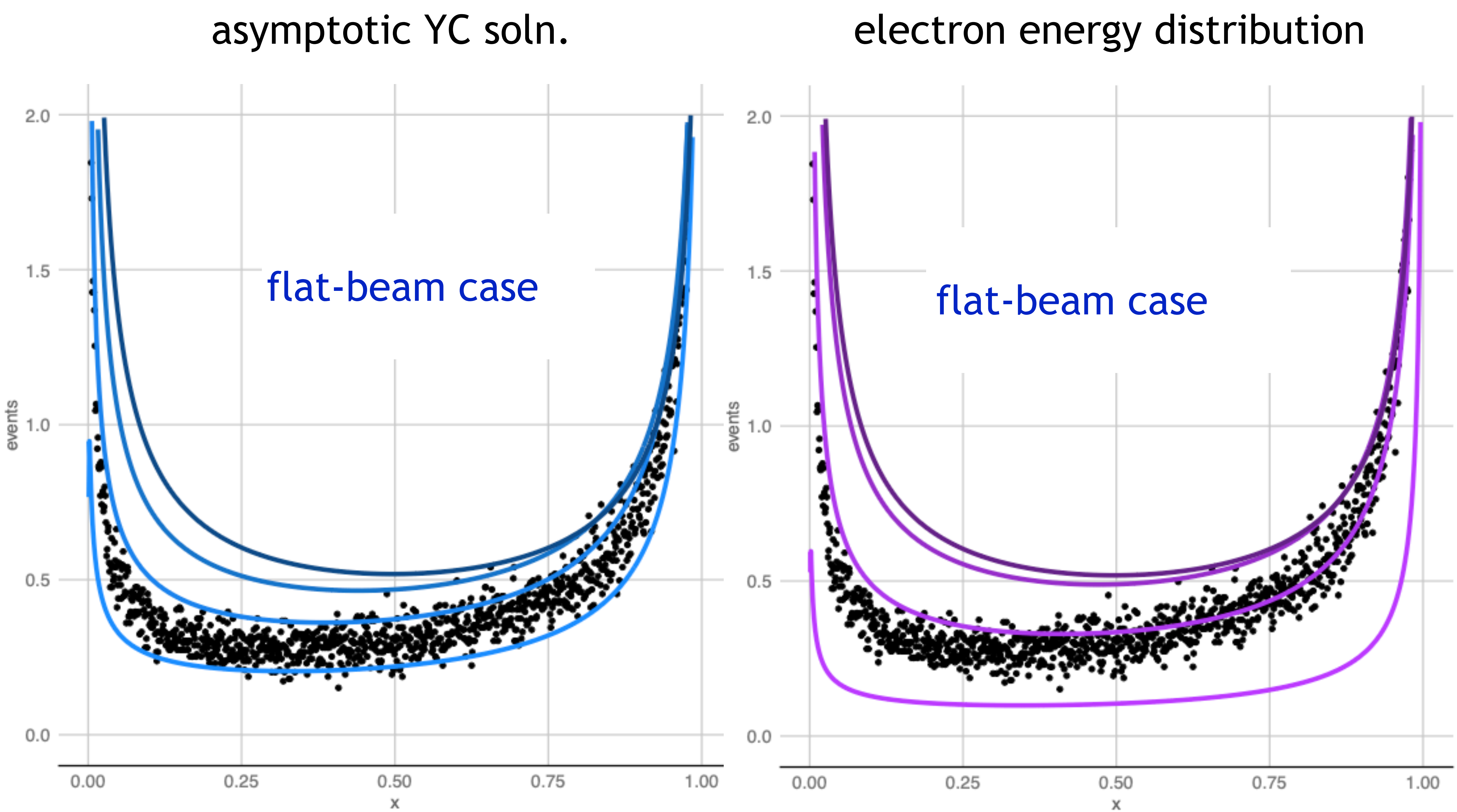
Rescale time to τ so that the bunch-bunch collision takes place over the interval [0,1] in τ. Then the YC equation reduces to

$$\frac{d}{d\tau} y_e(x,\tau) = -\frac{N_\gamma \tau}{x^{1/3}} y_e(x,\tau) + \frac{N_\gamma}{B}\int_x^1 \frac{d\overline{x}}{x^{1/3}(\overline{x}-x)^{2/3}\overline{x}^{1/3}}\; y_e(\overline{x},\tau)$$

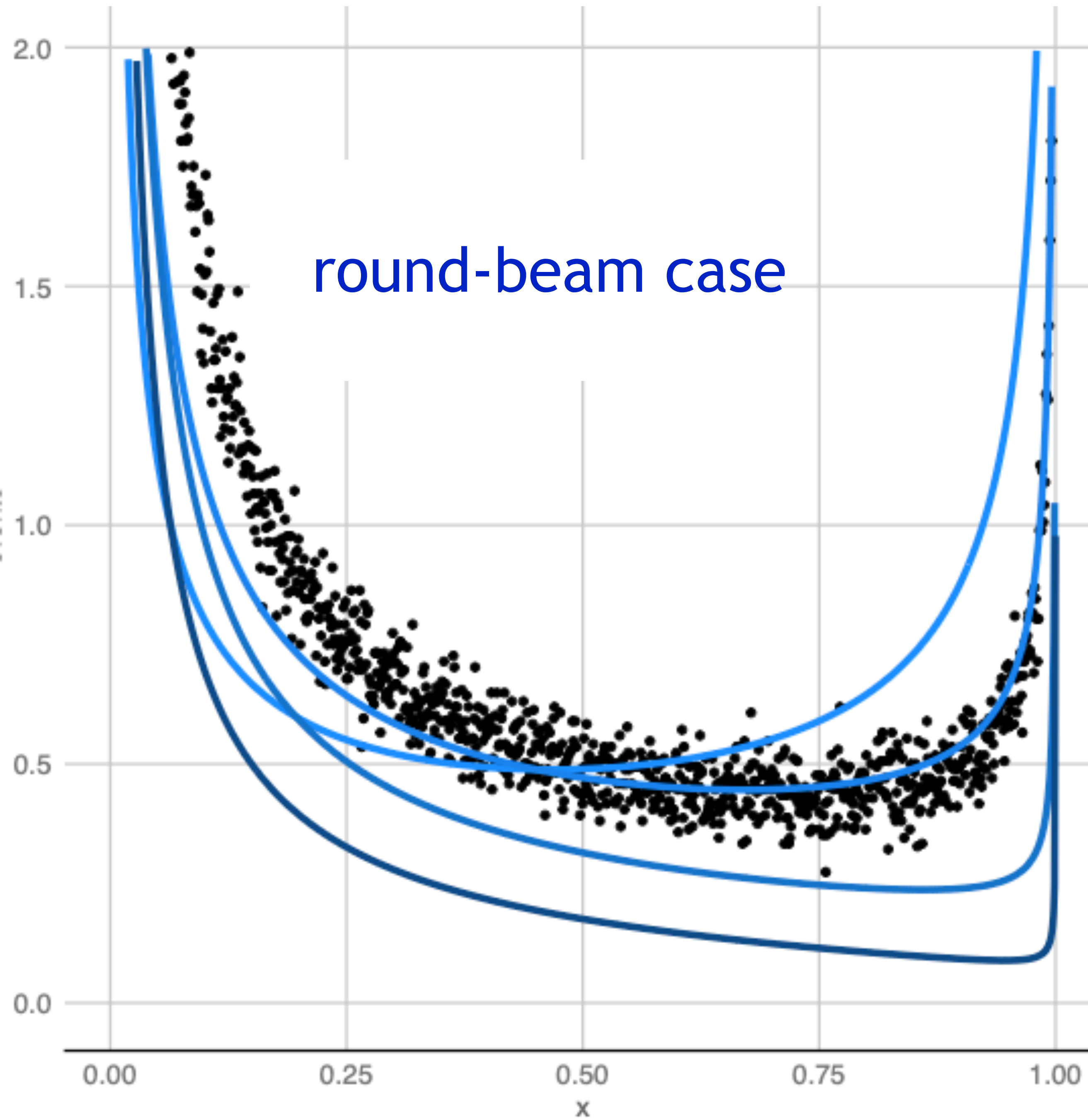
Even N_γ scales out with τ ! So there is a universal function that satisfies this equation that describes the time evolution of beamstrahlung in the large Υ regime. There is a similar universal function for the photon distribution.

We can solve this equation numerically. What does this look like relative to Jaden's data ?

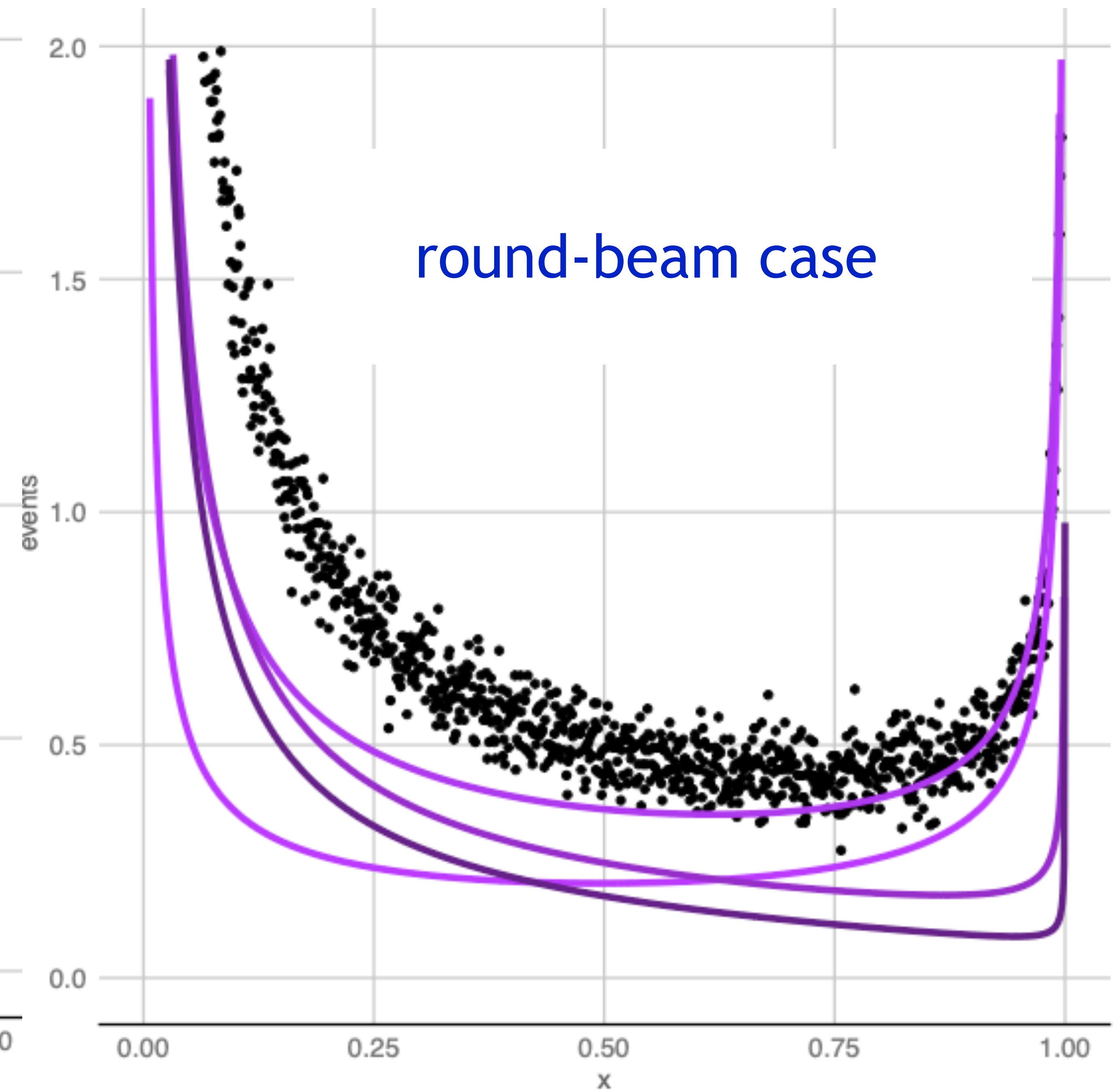
Plot the solutions at τ = 0.25, 0.5, 0.75, 1.0 vs. Jaden's data for the electron energy spectrum in annihilation reactions:


asymptotic YC soln.
electron energy distribution
flat-beam case
flat-beam case
events
events
x
x
2.0
1.5
1.0
0.5
0.0
0.00
0.25
0.50
0.75
1.00

asymptotic YC soln.

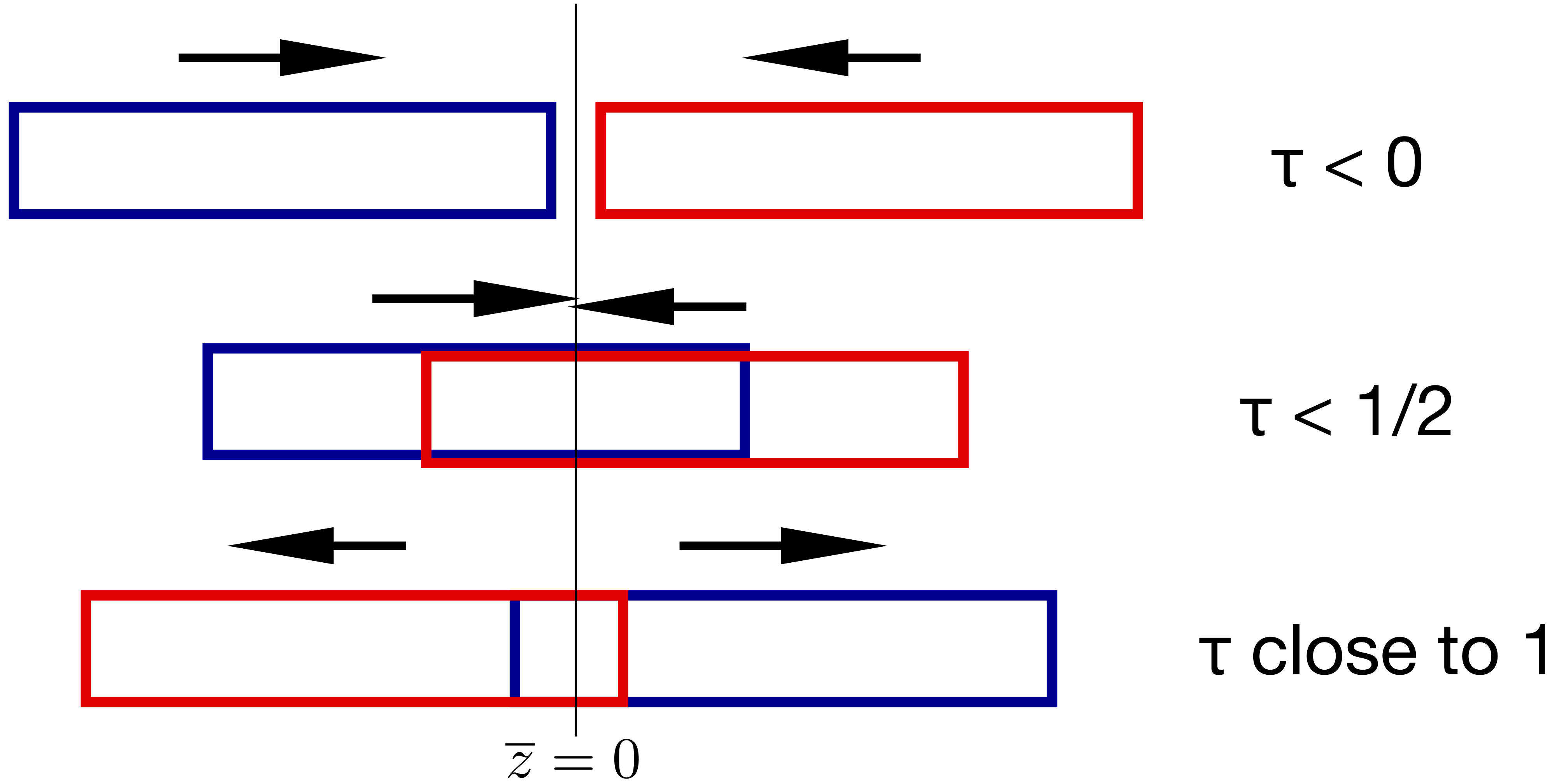


electron energy distribution



What function is actually supposed to predict the simulation data?

I will make the crude approximation of the bunch-bunch interaction as an interaction of colliding cylinders. Looking at simulation data, these cylinders are much thinner than the nominal transverse bunch size due to bunch-bunch attraction.



$\tau < 1/2$

$$\mathcal{L}(\tau)/\mathcal{L}_{tot} = 2z \int_{z^2}^{1} \frac{dx}{x} \int_{-\tau}^{\tau} d\overline{z}\ y_e(x, \tau + \overline{z})\ y_e(\frac{z^2}{x}, \tau - \overline{z})$$

$\tau > 1/2$

$$\mathcal{L}(\tau)/\mathcal{L}_{tot} = 2z \int_{z^2}^{1} \frac{dx}{x} \int_{-(1-\tau)}^{(1-\tau)} d\overline{z}\ y_e(x, \tau + \overline{z})\ y_e(\frac{z^2}{x}, \tau - \overline{z})$$

It turns out that, integrating through the whole collision, the electron energy distribution in collisions is

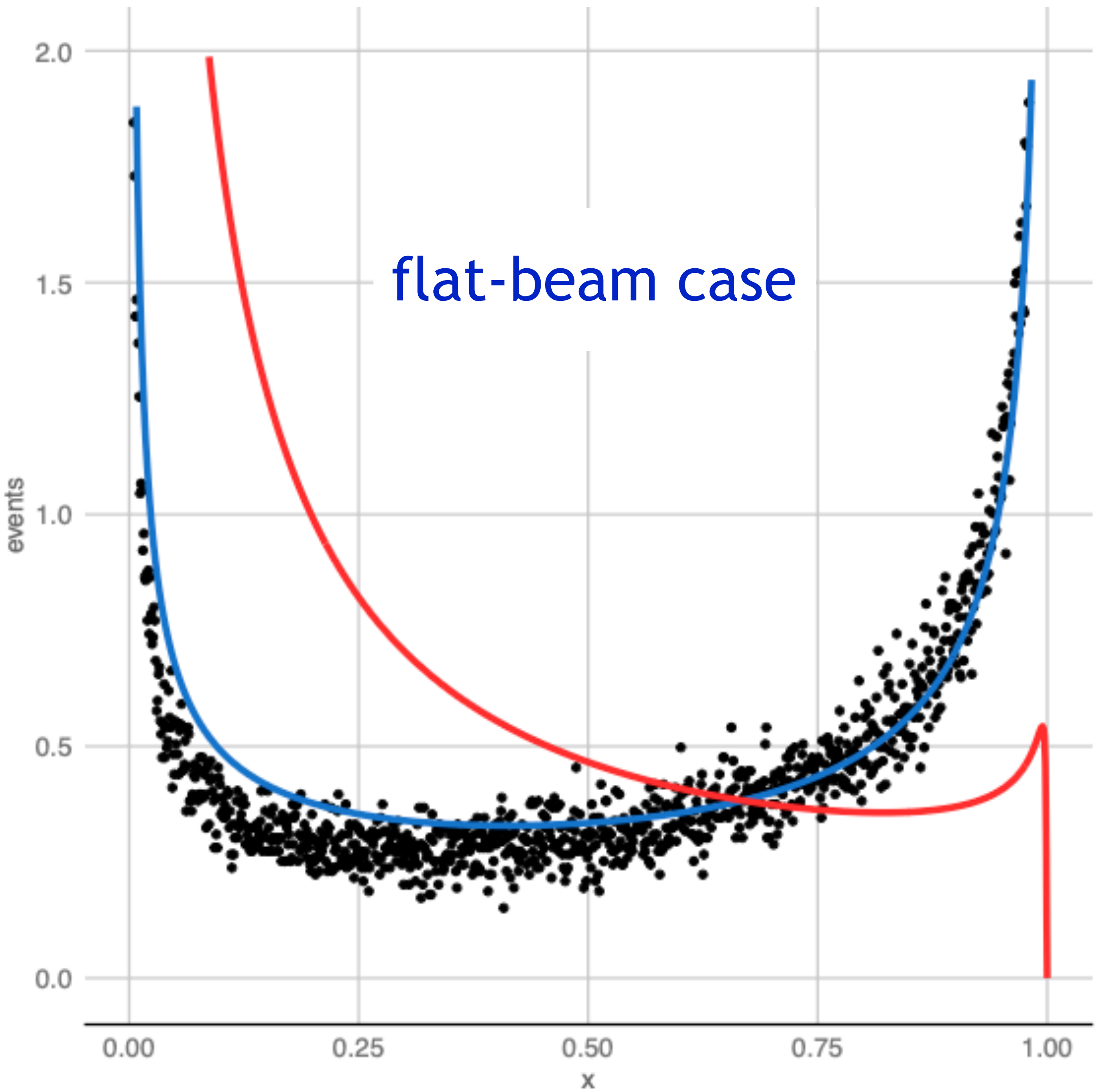
$$f_e(x) = \int_0^1 d\tau \ y_e(x, \tau)$$

and

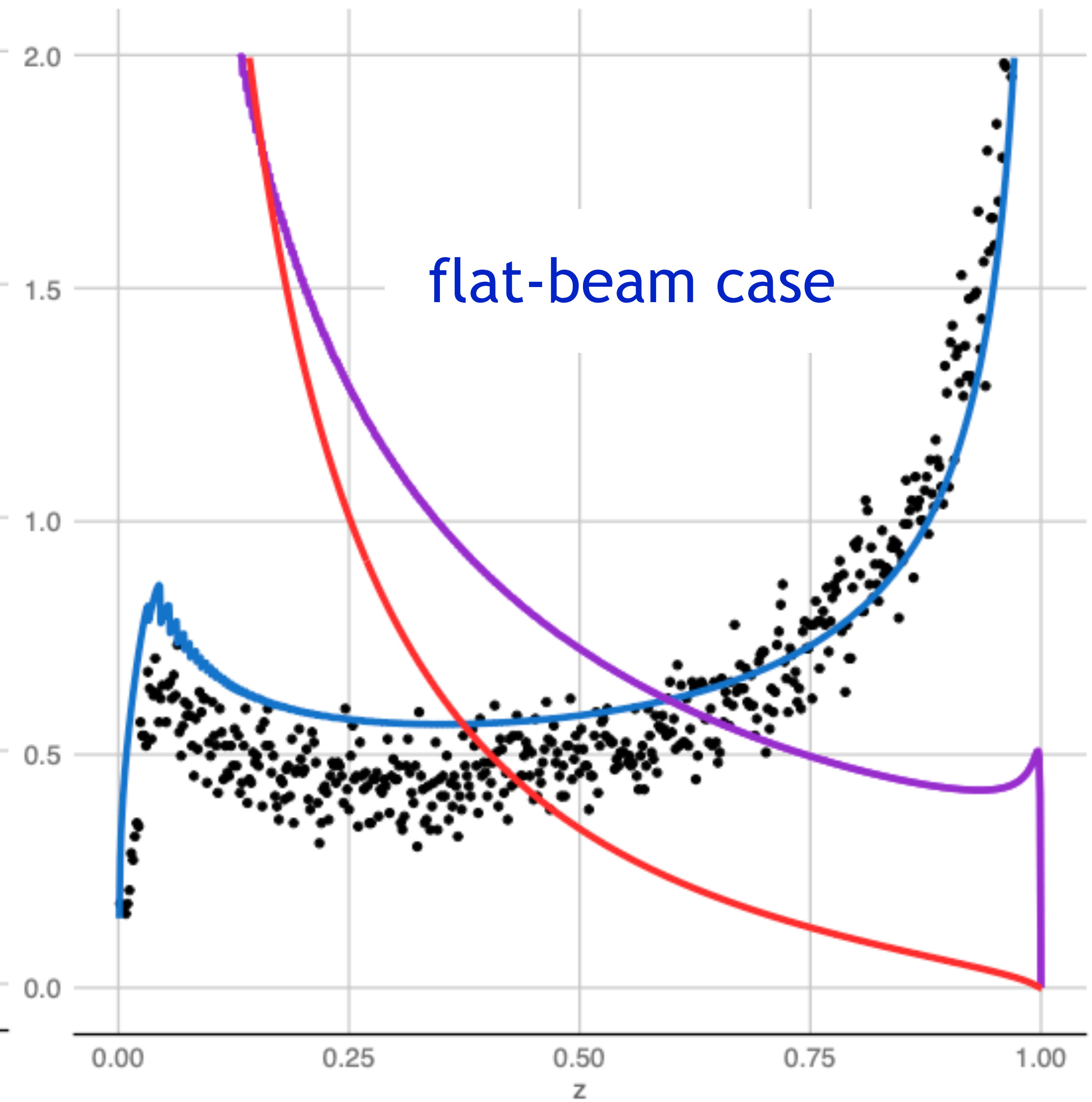
$$\mathcal{L}(\tau)\mathcal{L}_{tot} = 2z \int_{z^2}^1 \frac{dx}{x} \ f_e(x) \ f_e(\frac{z^2}{x})$$

How does this work ?

electron energy spectrum



luminosity spectrum

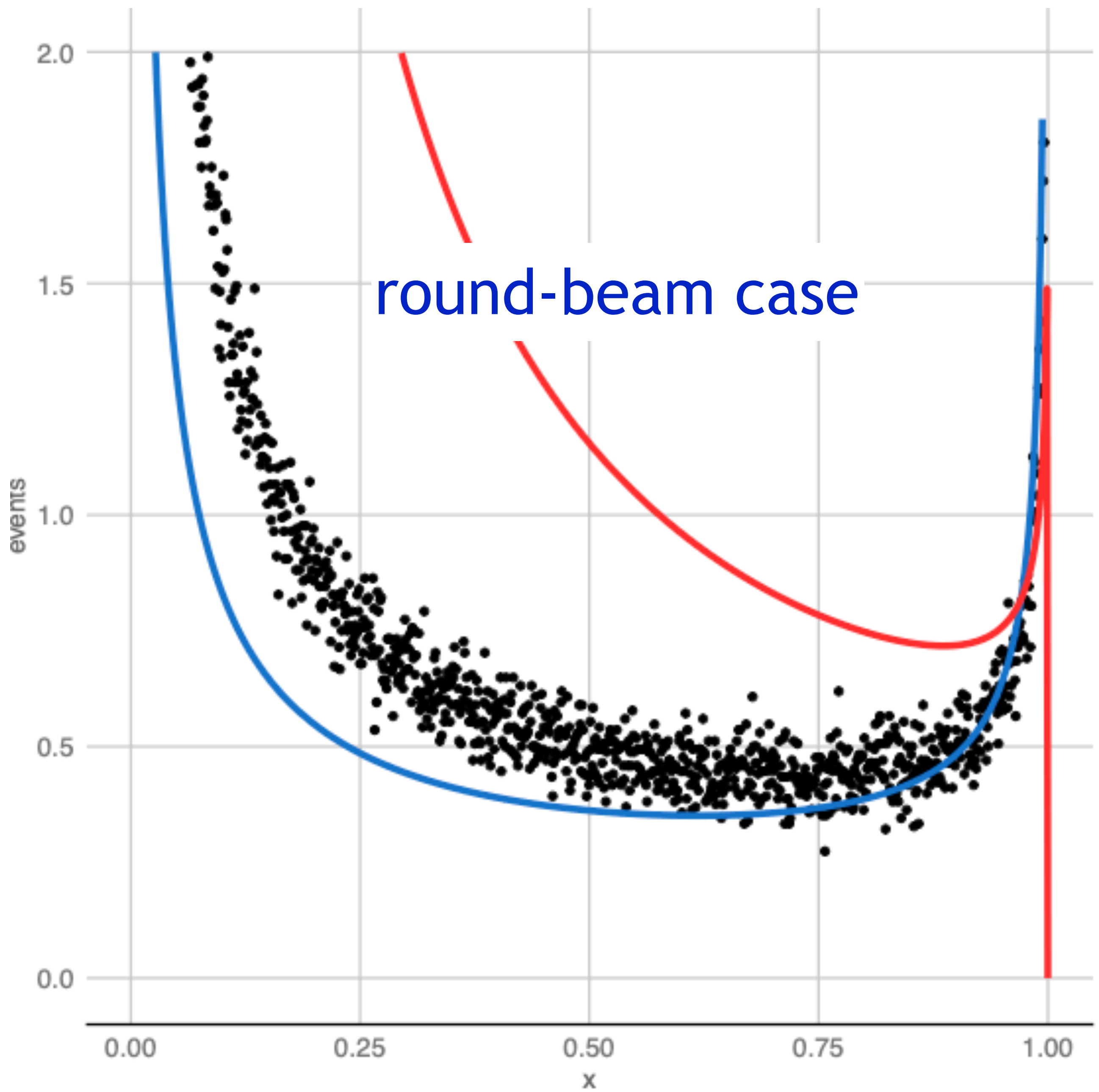


blue: e+, e- ; red: γ

blue: e+e- ; violet: eγ

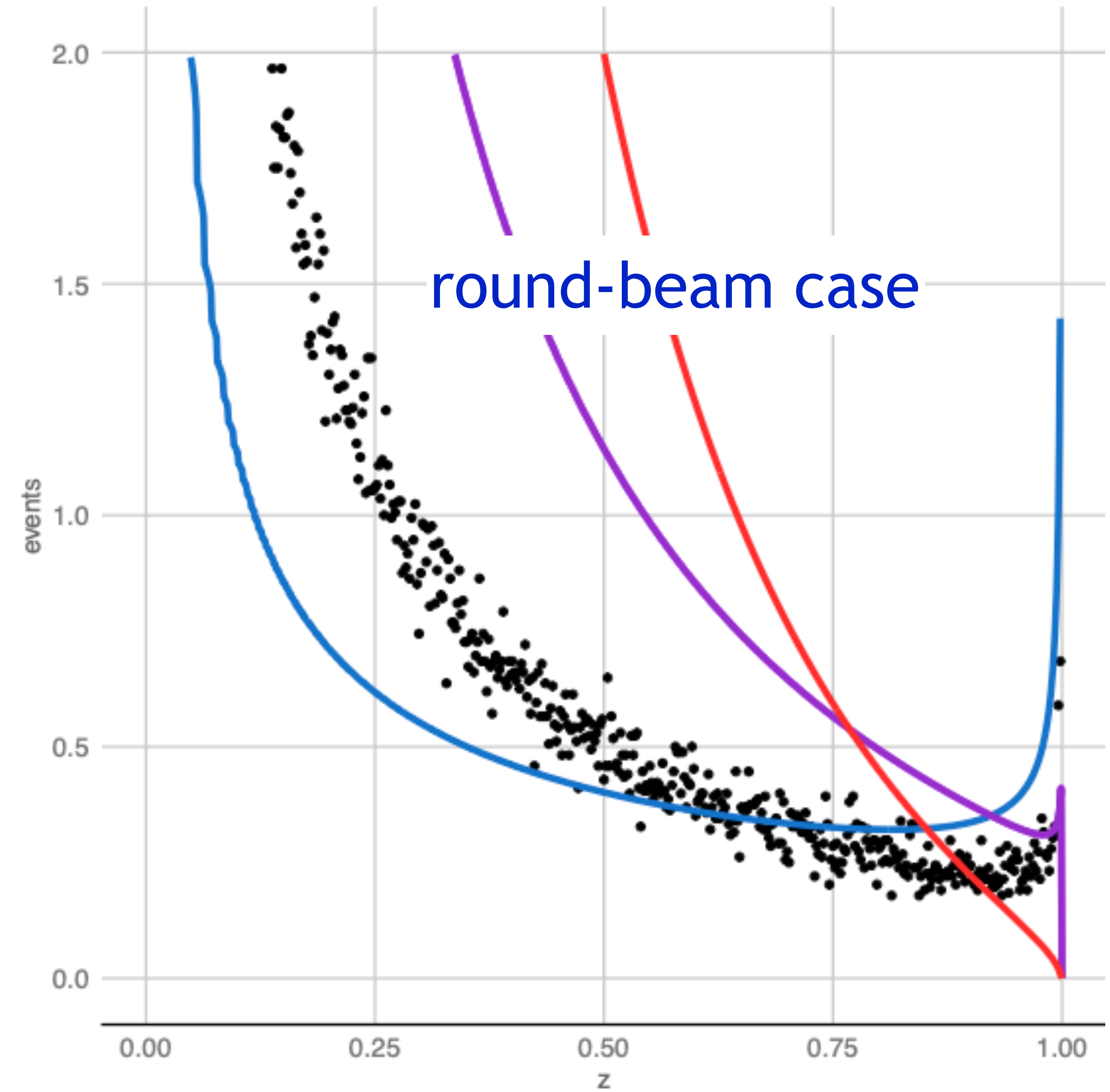
red: γγ

electron energy spectrum



blue: e+, e- ; red: γ

luminosity spectrum



blue: e+e- ; violet: eγ

red: γγ

One more thing:

It would be good to have an analytic approximation to the solution of the asymptotic YC equation. Then we can play with the various spectrum, including working out the time- and position-dependence of the luminosities.

I try to be as exact as possible for the large-x peak in the distribution, have the correct power law for the small-x peak, and preserve (as well as is convenient) the overall normalization constraint.

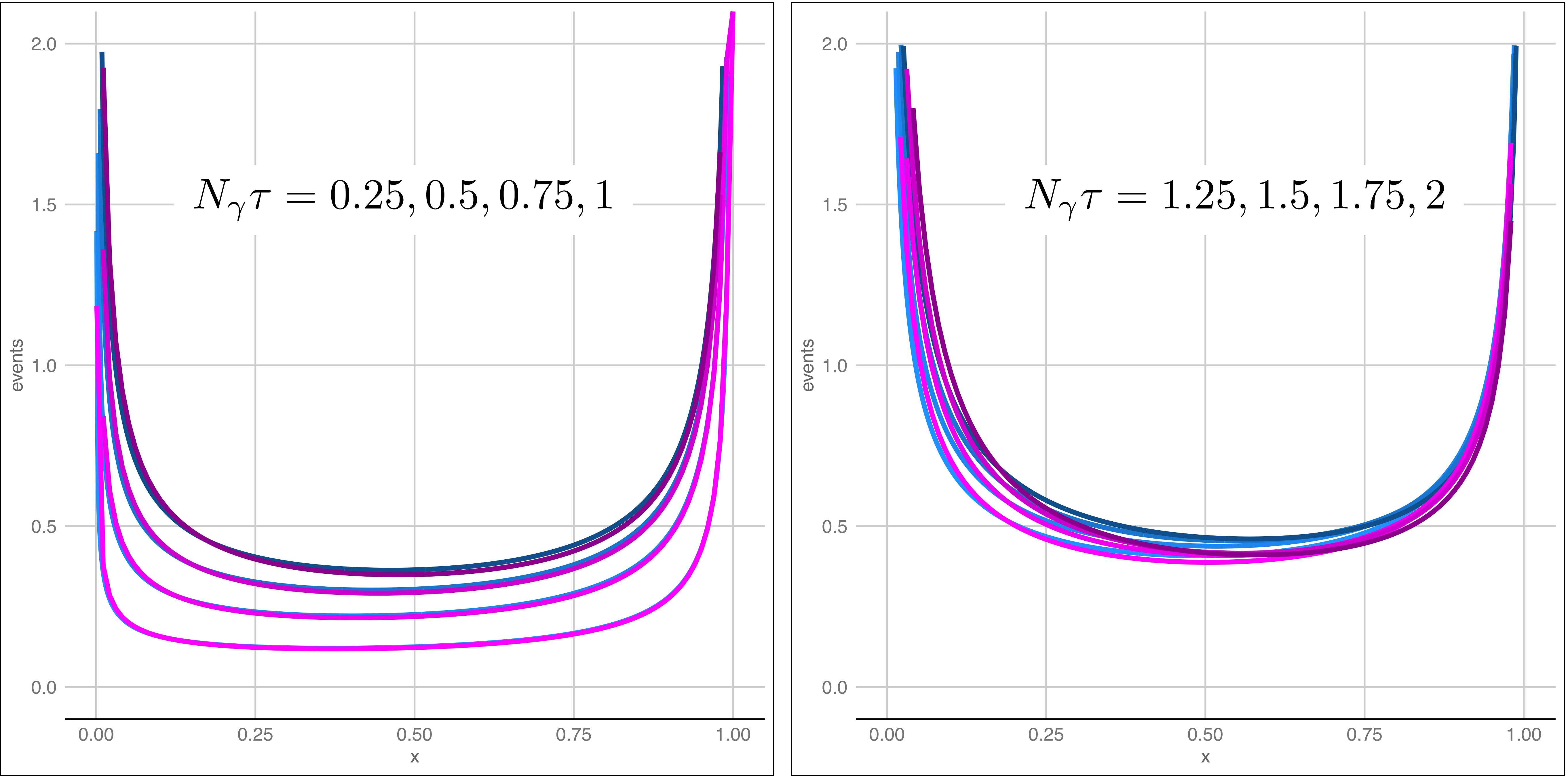
$$y_e(x,T) = e^{-T}\left[\delta(x-1) + \frac{T}{B}\frac{1}{x^{1/3}(1-x)^{2/3}} + \frac{T^2}{2B^2}B'\frac{1}{x^{2/3}(1-x)^{1/3}}\right]$$
$$+(1-e^{-cT}(1+cT))\frac{aT}{3x^{2/3}}e^{-aTx^{1/3}}$$

$$T = N_\gamma \tau$$

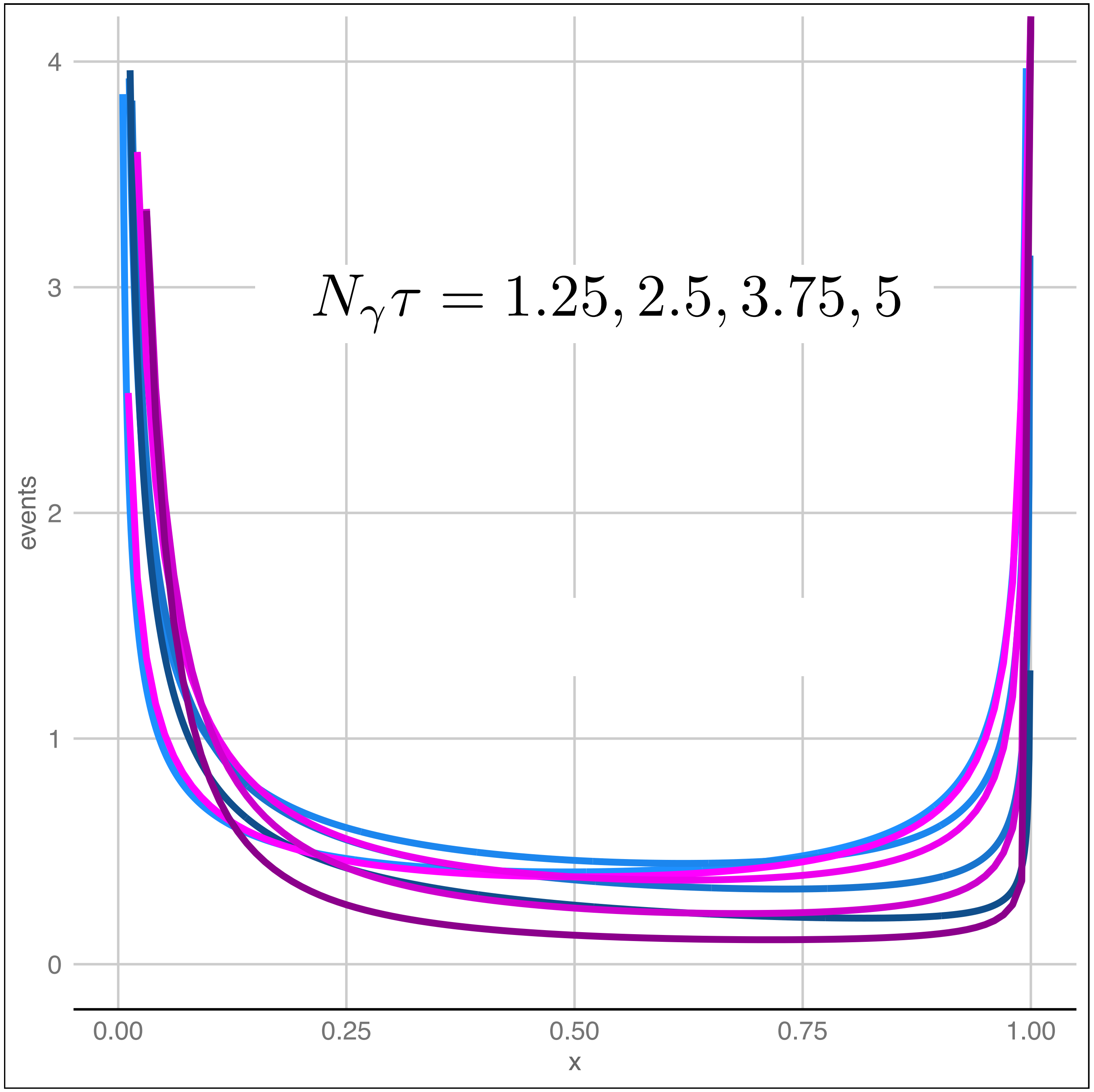
$$B = 3.6276 \qquad B' = 5.2999 \qquad c = 0.7 \qquad a = 0.5$$

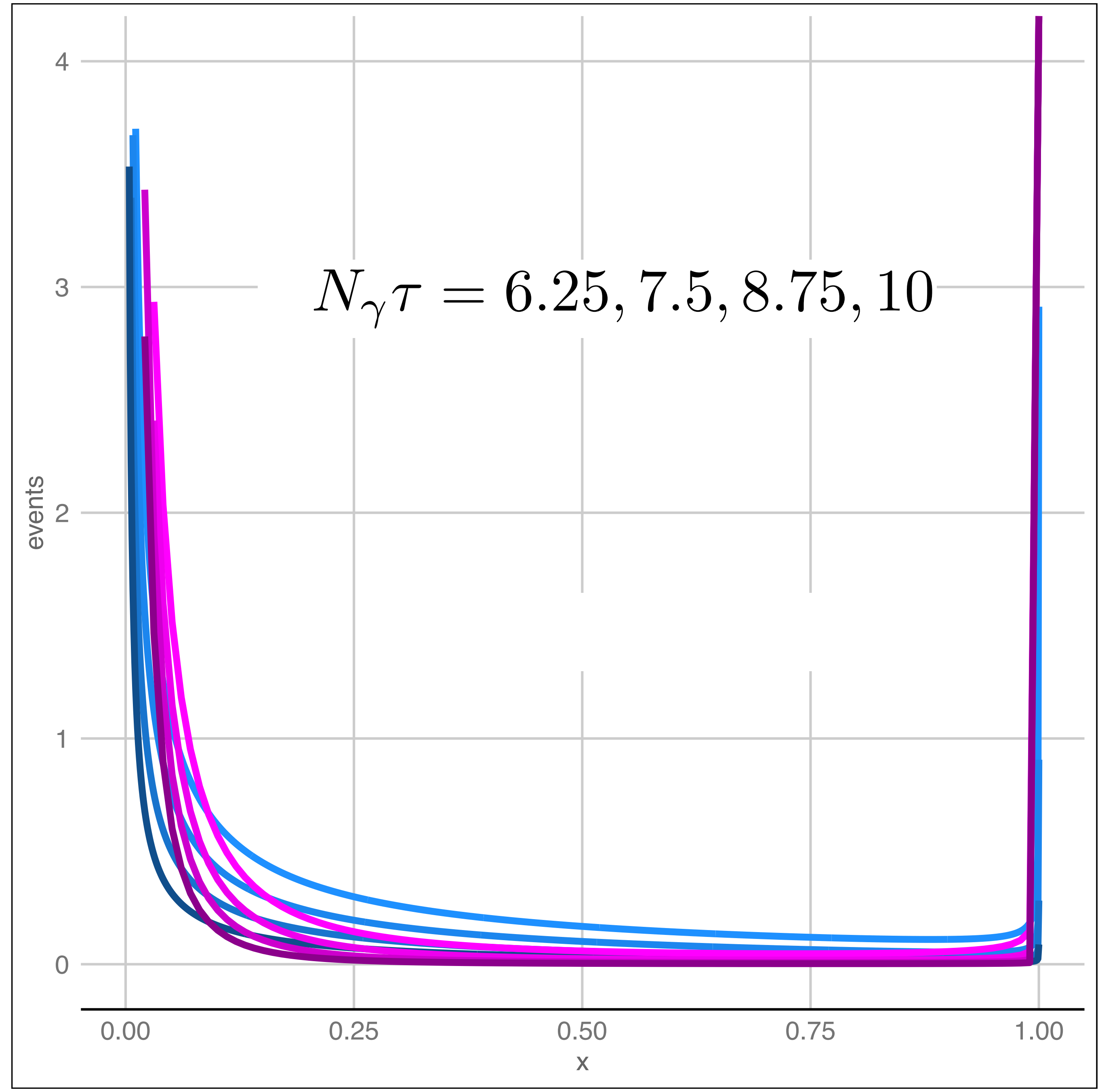
Applications of this formula, and the corresponding one for the photon distribution, are still a work in progress.

comparison of the analytic approximation to the numerical solution



comparison of the analytic approximation to the numerical solution





This is the current state.   There is more to come. Thank you for your attention.

Thanks again to Jaden, Arianna, and Spencer.