TOWARDS AN EXPRESSION OF UNCERTAINTY IN ACCELERATOR ALIGNMENT

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Abstract

When we are asked to align something in an accelerator, on a beamline or in a detector, there is always a desired tolerance associated with the alignment. Typically, this tolerance describes how precisely the object, such as a quadrupole magnet should be aligned with respect to another object, for example, the adjacent quadrupole magnet.

A problem often arises when the person giving the alignment tolerance isn't communicating about the same specifics as the person who is receiving the tolerance information and expected to perform the alignment. Naturally, this can lead to misunderstandings.

Tolerances are a vital part of designing an accelerator, beamline, detector components, and any other engineering products. Inadequately defined or incomplete tolerances in designs and drawings can lead to ambiguity, resulting in misinterpretation, issues and potentially costly errors. Unfortunately, misunderstandings between what is wanted and what is delivered occur frequently.

This problem has at least two aspects. One involves speaking a common language to define tolerances to ensure there is no ambiguity in their interpretation. This is addressed by Geometrical Product Specification (GPS). Geometrical Product Specification, for example as defined as outlined by the ISO GPS standards, is the system used to define the geometrical requirements of workpieces in engineering specifications, and the requirements for their verification (ISO 14638:2015) (Nicquevert B, 2022) (Nicquevert B., 2024). The ISO GPS standards provide clear and widely accepted practices that can be used to define tolerances and what they mean.

Once tolerances are explicitly defined, the object can be measured and compared to the theoretical design tolerances. When we have several objects, such as magnets on girders, or when an object is handled several times independently, for example when a magnet is first fiducialised, then installed on a girder, and later placed in an accelerator tunnel, each step results in independent comparisons to the theoretical design tolerances. Ultimately, we need to determine how well the tolerances are maintained by analysing several independent measurement results.

This is accomplished with an estimation, or statement of uncertainty in measurement.

The estimation of measurement uncertainty provides clear rules to calculate how well the tolerances are respected. The rules for the estimation of uncertainty in measurement are given in the Guide to the Uncertainty in Measurement (GUM). This Guide (and its supplements) establishes general rules for evaluating and expressing uncertainty in measurement ... (BIPM, 2008) The GUM is the first, and best known of a series of guides, each one addressing different aspects of establishing a statement of uncertainty in measurement*.

TOLERANCES AND UNCERTAINTY IN ACCELERATOR ALIGNMENT

Geometrical Product Specifications can and should define the geometrical requirements for fabricating accelerator components, such as a vacuum chamber. In general, design tolerances are determined with the component's intended use in mind. In an accelerator, these tolerances are ultimately based on the physics of the accelerator – specifically its lattice. For example, the EBS accelerator alignment tolerances for the magnet families are given in Table 1 (ESRF - Extremely Brilliant Source Technical Design Report)

Tolerances for the manufacture and installation, including the alignment of various accelerator components (e.g., magnets, vacuum chambers, absorbers, BPMs, front ends, etc.), are often influenced or dictated by specific requirements, such as those provided in Table 1. Notably, many different components contribute to the final alignment tolerances. For example, the design and separation of a quadrupole magnet's poles affects the design and tolerance of the vacuum chambers that need to fit inside it.

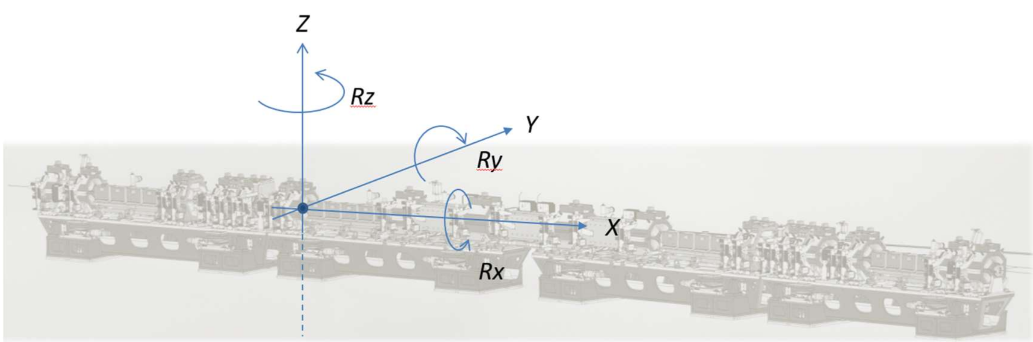
When multiple components compete for a share of the tolerance budget, a method is needed to allocate their portions and measure whether the apportionment is respected. This allocation must be decided before any manufacturing begins. Ensuring that the apportionment is respected after installation involves understanding and calculating measurement uncertainty.

* See https://www.bipm.org/en/committees/jc/jcgm/publications

Table 1 ESRF EBS tolerance specifications. Note; All tolerances are given at 1σ with a maximum acceptable error of 2.5σ. Note the directions (x, z, s) defined in machine system coordinates correspond to (Y, Z, X) in the Survey and Alignment Group coordinate system. x and Y (survey system) indicate the directions perpendicular to the beam in the horizontal plane, z and Z (survey system) are the directions perpendicular to the beam in the vertical direction orthogonal to the horizontal plane, and s and X (survey system) denote the direction along the beam.

	$\boldsymbol{\Delta x}$ (*Y*†) [μm]	$\boldsymbol{\Delta z}$ (*Z*†) [μm]	$\boldsymbol{\Delta s}$ (*X*†) [μm]	$\boldsymbol{\Delta\psi}$ [μrad]	$\boldsymbol{\Delta L/L}$
Long. Varying field dipoles	>100	>100	1000	500	10^{-3}
High gradient quadrupoles, Combined function dipoles	60	60	500	200	$5x10^{-4}$
Medium gradient quads	100	85	500	500	$5x10^{-4}$
Sextupoles	70	50	500	1000	$3.5x10^{-3}$
Octupoles	100	100	500	1000	$5x10^{-3}$

† X, Y and Z are directions defined in the alignment reference frame – see below



Rotations Rx, Ry and Rz are positive clockwise looking from the origin along the X, Y and Z axes

What does a Tolerance Mean?

To ensure tolerances are respected, we first need to understand and agree on what the tolerance means.

For example, for an ESRF EBS high gradient quadrupole magnet, the alignment tolerance is $(x, z, s) = (60, 60, 500)$ μm (1σ). This means the alignment uncertainty relative to its nominal or theoretical position must be less than or equal to this value at 1σ significance; and this uncertainty statement must include all possible alignment errors.

Typically, the nominal position aligns with the theoretical position. However, at the ESRF, the nominal and theoretical differ because the new machine was aligned based on the old machine's position to avoid realigning existing beamlines. This alignment required the new source points to be positioned exactly where the old ones were. It also constrained the new beamline, and straight section directions to be coincident with the old ones.

At the ESRF there are 27 quadrupole, sextupole, octupole and combined function dipoles, along with four assemblies of five dipoles each, distributed across four girders in each of the 32 cells. While the magnets are aligned to their theoretical positions on the girders, the girders themselves must be positioned in such a way as to respect the alignment constraints of the old machine.

This is an important nuance but it does not alter the alignment tolerances or the requirement that a quadrupole magnet, for example, must be aligned within $(x, z, s) = (60, 60, 500)$ μm (1σ) of its nominal position, including all alignment errors.

Note, we will use the survey reference system henceforth. Therefore, a quadrupole magnet, for example, must be aligned within $(x, y, z) = (500, 60, 60)$ μm (1σ) of its nominal position, including all alignment errors.

Smoothing, or magnet to magnet alignment, is an important concept in accelerator alignment. For example, the smoothing between a high gradient quadrupole (hgq) and a combined function dipole (cfd) can be expressed as:

$$(dx, dy, dz) = \left(\left(x_{hgq}^2 + x_{cfd}^2\right)^{1/2}, \left(y_{hgq}^2 + y_{cfd}^2\right)^{1/2}, \left(z_{hgq}^2 + z_{cfd}^2\right)^{1/2}\right)$$

In this example, the smoothing tolerance between these two magnets would be $(dx, dy, dz) = (710, 85, 85)$ µm (1σ).

Although this smoothing example is independent of measurement uncertainty per se, it illustrates how errors are combined in uncertainty calculations i.e. the square root of the quadratic sum of errors.

The expression of uncertainty in measurement

Measurement uncertainty as defined in the International Vocabulary of Metrology (VIM) by organisations including (BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, and OIML.) is a non-negative parameter characterizing the parameter that characterises the dispersion of the **quantity values**† being attributed to a **measurand**‡ being measured, based on the information used.

For many, this definition is probably a little pedantic, or overly complex, with some terms in the VIM being unfamiliar. Despite this, the core principles of measurement uncertainty are well known and well understood in the accelerator alignment community.

For example, consider measuring a distance five times, resulting in values: 12499.933, 12500.152, 12500.036, 12499.997, and 12500.036. The mean value is the best estimate of the distance, and the standard deviation represents the measurement uncertainty. The precise expression of uncertainty involves additional details (see the section *An example statement of uncertainty* below).

An expression of uncertainty includes four elements: a value, a unit (e.g., mm), an uncertainty, and a coverage factor, designated as k. If the uncertainty is expressed as one standard deviation, then $k = 1$ means the uncertainty covers one standard deviation (1σ), $k = 2$ means the uncertainty covers two standard deviations (2σ).

Therefore, the best estimate for the five distances measured above would be:

$$12500.031\ mm \pm 0.080\ mm;\ k = 1$$

For most in the accelerator alignment community, these concepts are familiar, indeed trivial. However, they serve to illustrate the somewhat obscure definition of measurement uncertainty in the VIM§. The expression of uncertainty in measurement, however is more complex and extends beyond this simple example.

Coming back to our example of aligning a high gradient quadrupole magnet, which must be positioned within $(x, y, z) = (500,60,60)$ µm $(k = 1)$ of its nominal position and including all alignment errors., several key factors contribute to magnet alignment errors including:

- The fiducialisation of the magnet;
- The preparation of the girder and its reference frame;
- The opening and closing of the magnet, sometimes several times;
- The installation of the magnet on the girder;
- The rectitude of the girder;
- Transport of the girder;
- The effect of bakeout of the vacuum chamber assemblies installed in the magnet;
- The uncertainty of the tunnel reference network used to position the magnet and the girder;
- Instrument errors. At the ESRF, the AT40x laser tracker. Importantly, these errors must be counted every time the magnet is measured;
- etc…

The alignment uncertainty for this magnet is the combined contribution of all of the errors. This is discussed in detail in the section: *Uncertainty in the installation and alignment of the EBS accelerator at the ESRF*.

Type A and Type B uncertainty contributions

The GUM provides two types of measurement uncertainty: Type A and Type B. Type A: evaluation of uncertainty uses statistical analysis of a series of observations such as the distance measurements cited previously. Type B evaluation of uncertainty provides an estimate of uncertainty derived from sources other than repeated measurements. Possible sources of Type B uncertainty include:

- previous measurement data;
- experience with or general knowledge of the behaviour and properties of relevant materials and instruments;
- manufacturer's specifications;
- data provided in calibration and other certificates;
- uncertainties assigned to reference data taken from handbooks.

The final overall uncertainty $U(x)$ is the combination of all Type A and Type B sources:

$$U(x) = \sqrt{TypeA^2 + TypeB^2}$$

An example statement of uncertainty

Before trying to establish an uncertainty statement for the alignment of the magnets in the ESRF Extremely Brilliant Source (EBS) Storage Ring (SR), it is worthwhile starting with a simple example to illustrate how to create a statement of uncertainty. We will build on the example of the distances measured above. These fictitious distances were measured with a TDA5005 Robotic Total Station (RTS).

What elements might contribute to the uncertainty of the measured distance? Consider the following elements:

- The manufacturer's quoted instrument precision;
- A distancemeter calibration certificate;
- Temperature, pressure and humidity conditions at the time of measurement;

† Quantity values refer to what is being measured, for example the distance between two points

‡ The measurand is the quantity e.g. distance intended to be measured.

§ A very useful tool for understanding and cross-referencing vocabulary is provided at: https://jcgm.bipm.org/vim/en/

- The stability of the instrument and reflector supports over the time the measurement was taken.

Keep in mind that in uncertainty statements, large errors tend to contribute disproportionately to the overall uncertainty.

To illustrate this, consider two made-up uncertainty contributions to a given measurement: the first is 0.001 mm, and the second 0.100 mm combining these quadratically results in contributions of 0.000001 and 0.010000 respectively. Adding and taking the square root gives a result of 0.100005 mm, or more reasonably 0.100 mm. The 0.001 mm contribution is negligible**. This doesn't mean that significant factors should be excluded if they are important for certain reasons; such as demonstrating that they were considered. However, there are limits to what can be considered a sensible contribution to a statement of uncertainty.

A useful rule of thumb is that if a contribution is considered, even if its impact is minimal, it should be included in the statement of uncertainty to show that it was considered. This approach can also address potential "what about ?... questions.

Now, back to the statement of uncertainty for the five measured distances.

The Type A contribution is:

$$u_A = 0.080 \text{ mm}$$

Recall that this refers to the standard deviation of the five measurements. The Type B contributions are given in the table below, with detailed explanations provided in the paragraphs that follow:

Ref	*Quantity*	σ
B_{d1}	Instrument uncertainty	1.025 mm
B_{d2}	Uncertainty in the correction of the distance due to temperature, pressure and humidity conditions: Δp = ± 1 hPa Δt = ± 0,5 °C Δh = ± 3 %	0.007 mm
B_{d3}	Uncertainty in the instrument and reflector support	0.010 mm

The combined uncertainty for the Type B contributions is:

$$u_B = \sqrt{1.025^2 + 0.007^2 + 0.010^2} = 1.025 \text{ mm}$$

B_{d1}: the manufacturer's instrument precision for the TDA5005 is 1 mm + 2ppm at 1σ over the full measurement range. The uncertainty of a measurement of 12500.052 mm is:

$$1 + 2 \times 12500.052/10^6 = 1 + 0.025 = 1.025 \text{ mm}$$

B_{d2}: we use the following formula that is derived from the Barrel-Sears formula and provided by the manufacturer Leica:

$$C = 281.8 - \left[\frac{0.29065P}{1+\frac{T}{273.16}} - \frac{4.126 \times 10^4 H}{1+\frac{T}{273.16}} \times 10^{\frac{7.5\,T}{237.3+T}+0.7857}\right]$$

Which gives us the following coefficients:

$$dc/dp = 0.2708;\ dc/dt = 0.9390;\ dc/dh = 0.0090$$

with uncertainties of 1 hPa for pressure, 0.5 °C for temperature and 10% for the humidity giving:

$$U(C) = \sqrt{(0.2708 \times 1)^2 + (0.9390 \times 0.5)^2 + (0.0090 \times 10)^2} = 0.55 \times 10^{-6} D_{dist}$$

So, for the distance 12500.052 mm the uncertainty due to temperature, pressure and humidity conditions is $12500.052 \times 0.55^{-6} = 0.007$ mm

** It would take two hundred equivalent 0.001 m contributions to increase the uncertainty from 0.100 mm to 0.101 mm.

B_{d2}: The stability of 0.010 mm between the instrument and reflector supports is derived from operator experience.

The combined Type A and Type B uncertainties is given by:

$$U(C) = \sqrt{0.080^2 + 1.025^2} = 1.069 \text{ mm}$$

So, we can say the distance that was measured is:

- (12500.03 ± 1.07) mm at $k = 1$; or,
- (12500.03 ± 2.14) mm at $k = 2$.

Here, we observe that the instrument uncertainty contribution specified by the manufacturer dominates the uncertainty statement for the measured distances. Although the temperature, pressure, humidity and instrument/reflector support uncertainties do not affect the overall uncertainty, they are included in the statement because they are well recognised sources of measurement error.

We know the manufacturer's specified precision for the TDA5005 distancemeter is conservative representing the Maximum Permissible Error (MPE) across the entire working range of the instrument. For shorter distances, such as those less than 100 m, the instrument's performance can be improved significantly though calibration. There is a well-known characteristic cyclic error in distance measurements made by the TDA5005. By identifying and correcting this error for a given distance, the precision of the measurements can be improved (Figure 1).

In the late 1990s, the ESRF was aware of this and developed the distancemeter calibration laboratory specifically to improve TDA5005 distancemeter uncertainty (Gatta G, 2024). Eventually the calibration bench had an ISO 17025 COFRAC accreditation for the calibration of distancemeters in instruments such as the TDA5005.

We will briefly recalculate the uncertainty by substituting the manufacturer's instrument precision with the ESRF calibration certificate uncertainty which was 0.21 mm at $k = 2$.

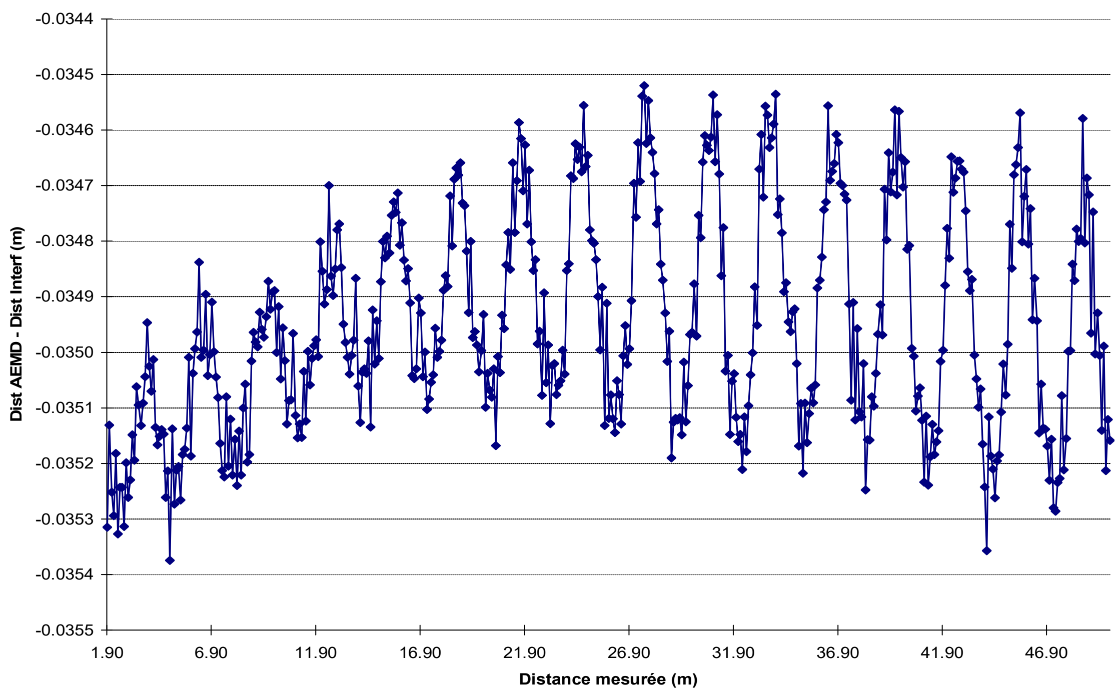


Figure 1 The calibration curve for TDA5005 serial number 438122 issued from calibration certificate 10102045DC-1 made on 3 February 2005 (Chaine Nationale d'Etalonnage BNM-COFRAC Métrologie Dimensionnelle Laboratoire D'Etalonnage Accrédité Accréditation N° 2-1508)

Repeating the Type B calculation without redoing B_{d2} and B_{d3}:

Ref	*Quantity*	σ
B_{d1}	Instrument uncertainty	0.105 mm
B_{d2}	Uncertainty in the correction of the distance due to temperature, pressure and humidity conditions: $\Delta p = \pm 1$ hPa $\Delta t = \pm 0{,}5$ °C $\Delta h = \pm 3$ %	0.007 mm
B_{d3}	Uncertainty in the instrument and reflector support	0.010 mm

The combined uncertainty for the Type B contributions is now:

$$u_B = \sqrt{0.105^2 + 0.007^2 + 0.010^2} = 0.106 \text{ mm}$$

The combined Type A and Type B uncertainties is given by:

$$U(C) = \sqrt{0.080^2 + 0.106^2} = 0.132 \text{ mm}$$

Using the calibrated instrument, we can say the best estimate of the distance that was measured is:

- (12500.03 ± 0.13) mm at $k = 1$; or,
- (12500.03 ± 0.26) mm at $k = 2$.

UNCERTAINTY IN THE INSTALLATION AND ALIGNMENT OF THE EBS ACCELERATOR AT THE ESRF

In the previous section we established a framework for developing a statement of uncertainty for a distance measurement. It provides a simple example to develop the more complex uncertainty statement for the alignment of magnets in the ESRF EBS SR.

We have tried to include all of the major alignment error contributions, and it is unlikely that any significant issues have been overlooked. However, there may be smaller contributions that are missing. Nonetheless, as we saw in the last section, large errors have a disproportionately significant effect on the overall uncertainty statement, and smaller contributions rapidly become negligible.

It is important to note that everything we will consider has a coverage factor $k = 1$ i.e. 1σ

Context - a brief discussion of the EBS alignment procedure

In the remainder of this chapter we will discuss all of the major contributions to the statement of uncertainty in the EBS alignment. This section will provide context for these contributions, specifically detailing how the accelerator was installed and aligned.

The first thing to consider is the magnet fiducialisation:

- 64 dipole magnet assemblies,
- 392 medium gradient quadrupoles,
- 132 high gradient quadrupoles,
- 100 combined function quadrupoles,
- 196 sextupoles, and
- 66 octupoles.

Fiducialisation occurred from summer 2017 through 2019, with the majority of the magnets being fiducialised before mid-2018.

The EBS girders were assembled in 2018. The assembly of a girder took three weeks. In theory, we could assemble 12 girders simultaneously, with three lines of four girders in at various stages of assembly. However, in practice, the process was less organised due to parts not always available when needed.

Fridays were dedicated to the installation of the girders. Since this process required opening doors to the assembly hall, and as the girders were not stored in an air-conditioned environment, they were given nearly 70 hours to acclimatise to the air-conditioned assembly hall before installation.

Note the air conditioning was relatively relaxed at ±1°C.

First the girders were levelled and a local reference frame was established. This initial step included measuring the planarity of the girder surfaces. The girder rectitude is important for the medium gradient quadrupoles, octupoles and sextupoles because they are shimmed and assumed to be installed on a perfectly flat horizontal surface. Subsequently, the magnets were installed and aligned to within 0.5 mm of their nominal positions. Fine alignment was then performed to achieve a precision of 0.05 mm. As the assembly progressed, and the teams gained experience, the distinction between the installation and fine alignment began to blur, with the process evolving into a single, precise fine alignment step.

In parallel, the vacuum chambers were assembled, but before this could proceed, all of the Beam Position Monitors (BPMs) needed to be fiducialised. The BPM fiducialisation process was performed by one team member and took several months to complete. The vacuum chamber assembly required one full-time alignment team to ensure that all mechanical tolerances were respected.

In the second week, all of the magnets were opened and the vacuum chamber assembly was inserted and aligned, paying particular attention to the 0.1mm BPM alignment. At the same time, the correctors were installed and aligned. Finally, all of the magnets were closed, final alignment corrections were made, and a comprehensive survey was conducted of the girder, magnets, BPMs, absorbers and other components.

The third week of the girder assembly was dedicated to tests. It did not concern alignment.

Due to space constraints, not all of the 128 assembled girders could be stored at the ESRF. Ninety were stored offsite in a warehouse. This required transporting them by lorry, a total distance of 60 km – 30 km to the warehouse for storage and another 30 km back to the ESRF for installation in the tunnel.

In October 2018 the old machine was decommissioned and dismantled. Once the removal was complete, the tunnel floor was prepared for the girder installation. The ESRF SR constructed in 1990 had experienced height variations both from original floor construction, and from site changes over the subsequent 28 years. To ensure all of the girders to be aligned at the same height, ideally close to the centre of the jacks' stroke, it was necessary to address the floor's unevenness. The peak to peak floor height variations were ±25 mm with local variations reaching up to 10 mm. These local variations were greater than the maximum permitted glue thickness of 7 mm under a girder support plate.

The girder support plates had to be carefully aligned and shims machined and installed on the plates to respect our height constraints.

Once the plates were installed, the girders were brought into the tunnel and installed on the plates. As they entered the tunnel, the girders were aligned in their designated positions.

All the girders including the magnets, BPMs, absorbers and other components were remeasured as they had been in the assembly hall. The installation positions of the girders in the tunnel were compared to their positions in the assembly hall to check if the magnets had shifted during transportation and installation.

All the fluids, electricity and services were then installed and the vacuum chambers baked out.

All the girders were measured again and compared between their assembly hall positions and their tunnel installation positions to determine if the bakeout process had affected their alignment.

Finally, the girders were precisely aligned to their designated positions and a comprehensive survey of the SR was conducted. This was the final step before the tunnel was closed for the commissioning of the new machine.

Beam was injected into the new EBS machine and the beamlines became operational shortly afterwards. Notably, "the EBS X-ray beam at distances ranging from 45 to 160m was found within fractions of millimetres from its position in December 2018 "††.

Magnet Fiducialisation

We aim to position the magnet's magnetic axis accurately, but since the axis itself is not visible, we need to materialise it and reference it with respect to something we can observe. At the ESRF, magnet fiducialisation is performed using a magnet measuring bench as illustrated in Figure 2. (Le Bec G, 2019)

The fiducialisation uncertainty calculation is provided in Table 2.

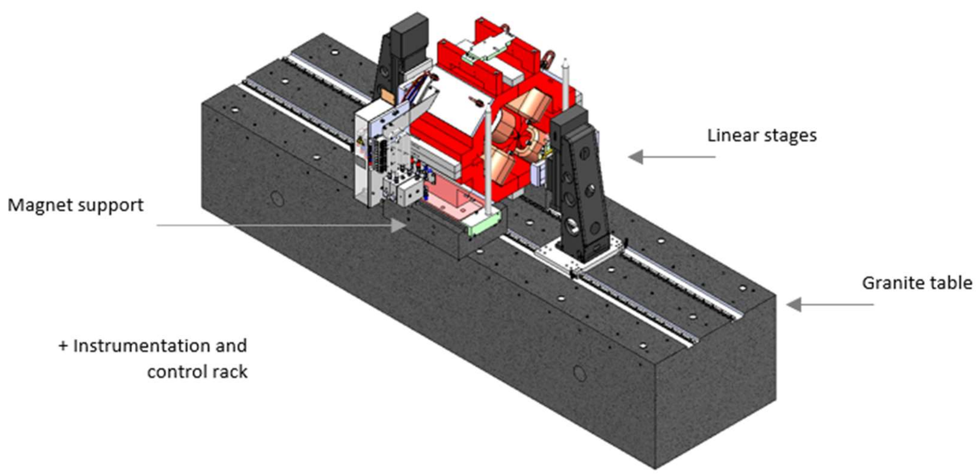


Figure 2 The magnet measurement bench where the magnet fiducialisation is made.

†† *E-mail Francesco Sette, ESRF General Director to all staff on 31/01/2020.*

Table 2 Fiducialisation uncertainty.

Ref	*Quantity*	*U(x) [µm]*	*U(y) [µm]*	*U(z) [µm]*
$B_{fidBench1}$	Magnet measurements		4	4
$B_{fidBench2}$	Shim determination			29†
B_{fidLT1}	Wire block reference repeatability	5	5	5
B_{fidLT2}	Wire position determination repeatability	20	20	24
B_{fidLT3}	Measurement	9	10	9
B_{fidLT4}	Repeatability	3	3	12
†Low gradient quadrupoles, octupoles and sextupoles use shims for the vertical alignment. This contribution reflects that the whole fiducialisation procedure needs to be done twice				

The combined uncertainties for the Type B contributions are:

$$u_x = \sqrt{5^2 + 20^2 + 9^2 + 3^2} = 23 \ \mu m$$

$$u_y = \sqrt{4^2 + 5^2 + 20^2 + 10^2 + 3^2} = 24 \ \mu m$$

$$u_z = \sqrt{4^2 + 5^2 + 24^2 + 9^2 + 12^2} = 29 \ \mu m$$

$$u_{z\_shimm} = \sqrt{4^2 + 29^2 + 5^2 + 24^2 + 9^2 + 12^2} = 41 \ \mu m$$

$B_{fidbench1}$: the 4 µm uncertainty of a magnet measurement using the ESRF magnet measuring bench is discussed in detail in (Le Bec G, 2019).

$B_{fidbench2}$: Low gradient quadrupoles, octupoles and sextupoles use shims for vertical alignment. The entire fiducialisation process is performed twice, which means that an additional vertical alignment contribution is included for the second fiducialisation i.e. 29 µm.

B_{fidLT1}: During the fiducialisation process, the positions of the two ends of the wire are measured in two independent steps. First, the positions of the four reflector sockets (with two visible in this picture) are determined with respect to the plane surfaces that define the wire's position (Figure 3).

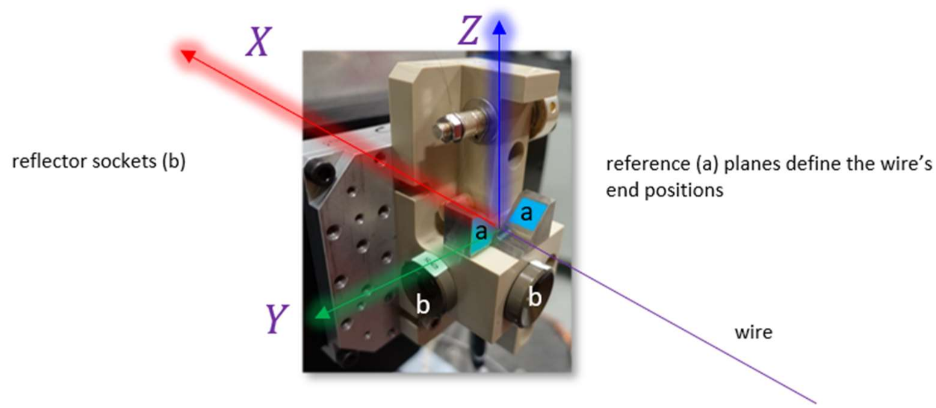


Figure 3 The wire reference assembly displays the reference planes that define the wire's position, along with the reflector sockets used for quickly measuring the assembly.

To facilitate rapid measurement, the planes defining the wire end positions were not measured during every fiducialisation. Instead, four easily measurable reference sockets were used for the fiducialisation. Periodically, the planes were remeasured using an AT40x laser tracker to re-determine the reflector socket positions. B_{fidLT1} are the standard deviation of 14 repeated sets of measurements made in 2017 and 2018.

B_{fidLT2}: All of the reference points on the benches were measured periodically to provide an estimate of the repeatability of the determination of the wire end points. This is the standard deviation $U(x), U(y), U(z)$ of the 11-determinations made each on Bench 3 and Bench 4 i.e. a total of 22 determinations, made between July 2017 and May 2018. Note these values have been revised from earlier publications (e.g. (Martin D, 2022) where the values used $U(x) = 13$ µm, $U(y) = 15$ µm, $U(z) = 18$ µm were based on fewer determinations.

B_{fidLT3}: The uncertainties come from the results of a typical Spatial Analyser (SA) USMN fiducialisation calculation. (Figure 4)

B_{fidLT4}: The repeatability comes from six independent fiducialisations of the same magnet on the same bench over two-day period. This is essentially the repeatability of the B_{fidLT2} contributions.

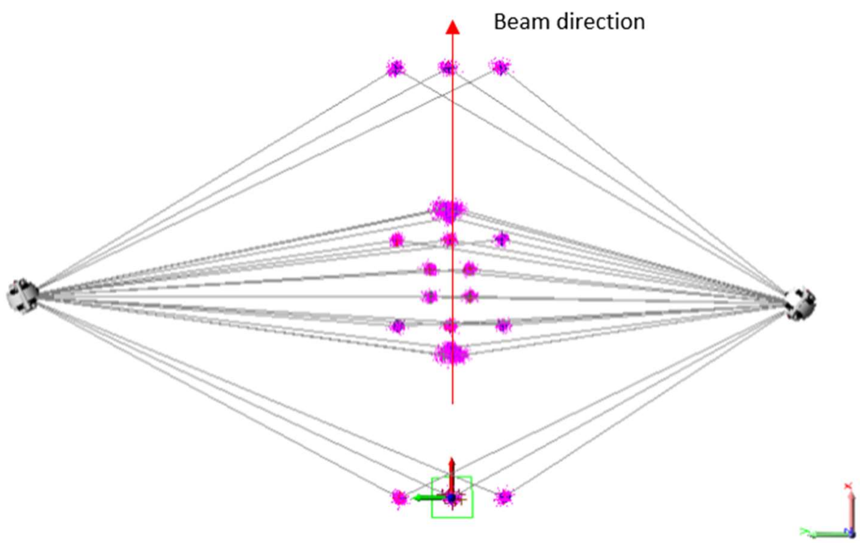


Figure 4 Measurements performed on a magnet during a fiducialisation. Note there are two instrument stations. Two independent measurements were made from each of the instrument's stations for a total of four independent measurements.

The opening and closing of the magnets

All of the magnets had to be opened and closed at least once to insert the vacuum chambers. For nine of the first magnets delivered to the ESRF, the coordinates of the reference points were compared before and after opening and closing. The standard deviation of these differences represents a Type A uncertainty:

$$u_{x\ opening} = 8\ \mu m;\ u_{y\ opening} = 5\ \mu m;$$
$$u_{z\ opening} = 7\ \mu m;$$

The girder rectitude

Twenty-two points were measured along the keyed reference surface of the girder (Figure 5). To determine the rectitude, a best fit plane was calculated using the measured x, y, z coordinates of the points.

The Type B uncertainty is defined as the standard deviation of all 2816 residuals between the measured z coordinates on the girder reference surfaces and the best fit plane at the x, y coordinates of each measured point for the 128 girders.

$$u_z = 8\ \mu m$$

The installation of the magnets and assembly of the girder

The alignment of the magnets on the girders was a complex task that took two weeks to complete. A third week was required to assemble the vacuum chambers and fiducialise the BPMs. The magnet assembly uncertainty estimation is given in Table 3

Figure 5 On the left the girder reference frame. On the right measuring the girder rectitude along the keyed surface.

Table 3 Magnet assembly uncertainty.

Ref	*Quantity*	*U(x) [µm]*	*U(y) [µm]*	*U(z) [µm]*	*U(z)† [µm]*
B_{ass1}	Measurements	7	6	7	7
B_{ass2}	Difference to nominal	126	24	25	25
B_{ass3}	Overall uncertainty		14	17	17
B_{ass4}	Magnet opening and closing	8	5	7	7
B_{ass5}	Girder rectitude				8
†Low gradient quadrupoles, octupoles and sextupoles use shims for the vertical alignment. This means the girder rectitude must be also included.					

The combined uncertainties for the Type B contributions are:

$$u_x = \sqrt{7^2 + 126^2 + 8^2} = 126\ \mu\text{m}$$

$$u_y = \sqrt{6^2 + 24^2 + 14^2 + 5^2} = 29\ \mu\text{m}$$

$$u_z = \sqrt{7^2 + 25^2 + 17^2 + 7^2} = 32\ \mu\text{m}$$

$$u_{z\_shimmed} = \sqrt{7^2 + 25^2 + 17^2 + 7^2 + 8^2} = 33\ \mu\text{m}$$

B_{ass1}: The uncertainties come from the results of a typical Spatial Analyser (SA) USMN calculation. (Figure 6)

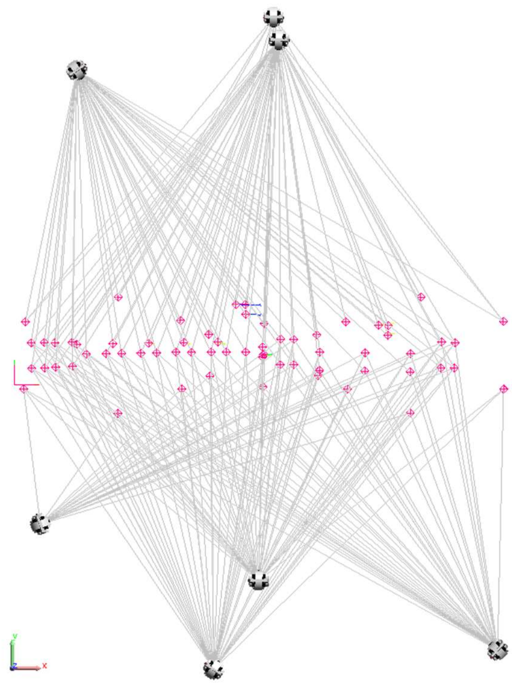


Figure 6 SA image showing instrument observations made for the final survey after assembly.

B_{ass2}: A transformation of the final measured coordinates determined in B_{ass1} was made to the nominal coordinates. These uncertainties are the residuals from a typical transformation.

B_{ass3}: The overall uncertainty is the standard deviation between all of the actual measured coordinates of the magnets on the girders and their nominal positions (Figure 7).

B_{ass4}: See the section: *The opening and closing of the magnets* above

B_{ass5}: See the section: *The girder rectitude* above.

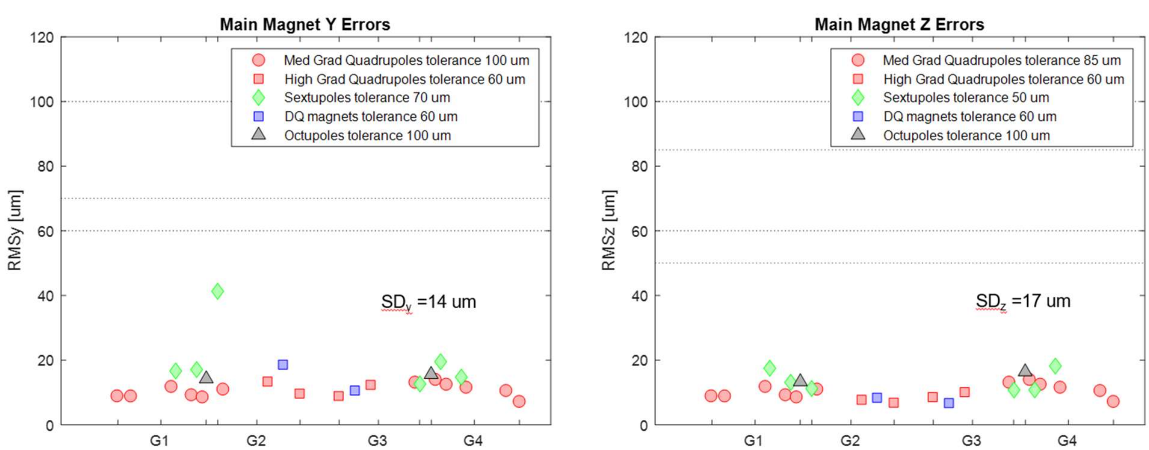


Figure 7 Summary statistics for all of the magnets on the 128 girders with respect to their nominal positions.

Transport of the girder

Once all 128 girders had been assembled, they were loaded by crane, then moved onto a lorry and offloaded and transported to a designated storage area (Figure 8). Out of these girders, 38 were stored on-site, while the remaining 90 were stored offsite in a warehouse that was 30 km away from the ESRF. Before installation in the tunnel, these girders had travelled a total distance of 60 km.

During installation, all 128 girders were loaded onto a lorry using a crane. From there they were transported to the Experimental Hall, offloaded from the lorry by another crane onto a special transport unit that took them near to the tunnel wall where they were lifted over the wall using a gantry crane and lowered onto a special transport unit that took them to where they were finally installed.

There was considerable concern that the transport and storage of the girders would impact the magnet alignment. Once the girders were installed in the tunnel, they were immediately aligned to their intended positions, and a detailed survey made (Figure 9). This survey was nearly identical to the one made after the girder was assembled (Figure 6). To determine if the magnets had moved on the girders during transport and storage, the magnet coordinates determined in the tunnel (Figure 9) were adjusted to those determined when the girders were assembled (Figure 6). Histograms of the residuals of the measured points in x, y and z are shown in Figure 10.

The uncertainty in the magnet positions due to transport is therefore considered as:

$$U(x) = 29\ \mu\text{m},\ U(y) = 13\ \mu\text{m},\ U(z) = 12\ \mu\text{m}.$$

Bakeout of the girder vacuum chambers

Once the girders were installed and aligned, the next step was to install the services (fluids and electricity). Following that, the vacuum chambers were baked out. Once the bake out was finished, a survey similar to those shown in Figure 6 and Figure 9 was conducted.

The coordinates from the final bakeout survey were then adjusted on the initial assembly coordinates, resulting in the data presented in Figure 11. These results incorporate the previous transport measurements and will be used in the final uncertainty calculation.

The uncertainty in the magnet positions after transport, installation of the services and bakeout is therefore considered as:

$$U(x) = 42\ \mu\text{m},\ U(y) = 17\ \mu\text{m},\ U(z) = 20\ \mu\text{m}.$$

Alignment in the tunnel

Just before the tunnel was closed and the new machine turned on for commissioning, a final alignment was performed. Immediately after this, a final survey was conducted with the results shown in Figure 12 to Figure 15. Note, to avoid realigning all the beamlines, the new machine was aligned to the position of the old machine, which explains the unusual curves seen in Figure 12 and Figure 13. These curves reflect the machine's evolution over time and indicate where the old SR machine was located before it was dismantled.

There are two contributions to the uncertainty associated with the final alignment. First there are the relative error ellipsoids between adjacent girders, which result from the least squares' calculation. These ellipsoids depend on the survey configuration, the calculation process, and the number of observations taken for each point. The vertical uncertainty is larger because there are considerably fewer observations made. Although distances up to 35 m are used for the planimetric calculation, we limit the distances greater to 15 m for the vertical z calculations.

The values for the relative errors are:

$$U(x) = 6\ \mu\text{m},\ U(y) = 6\ \mu\text{m},\ U(z) = 19\ \mu\text{m}.$$

The source of uncertainty comes from the smoothing of the machine or the differences between adjacent girders (Figure 14 and Figure 15):

$$U(x) = 219\ \mu\text{m},\ U(y) = 39\ \mu\text{m},\ U(z) = 36\ \mu\text{m}.$$

Figure 8 On the left (a), the assembled girder being offloaded from the lorry before final installation. In the middle (b), the girder being lifted over the tunnel wall. On the right (c), the girder being transported to its installation point using the specialised transportation unit.

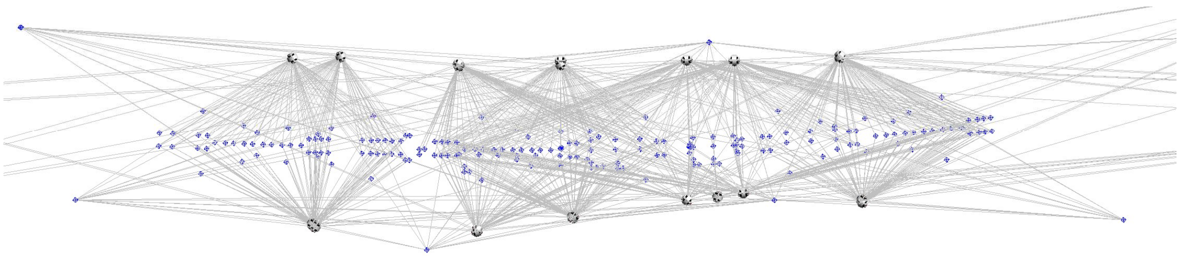


Figure 9 Measurements made of the four girders in a typical cell when they were first installed and aligned in the tunnel.

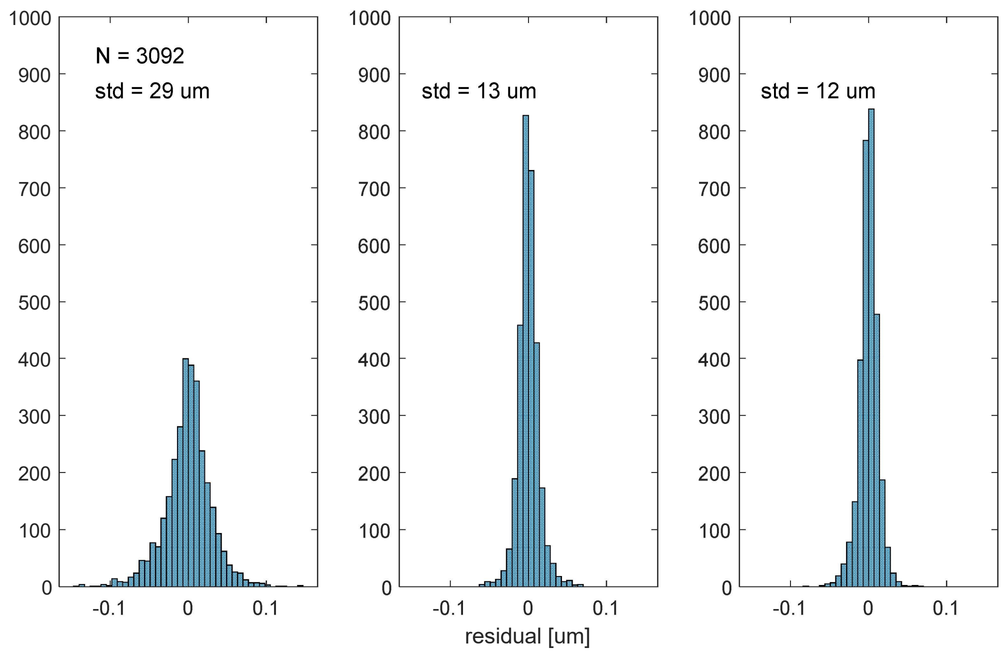


Figure 10 Histograms of residuals for the transformation of the girders that were measured after their installation in the tunnel, onto their measured positions when they were first assembled.

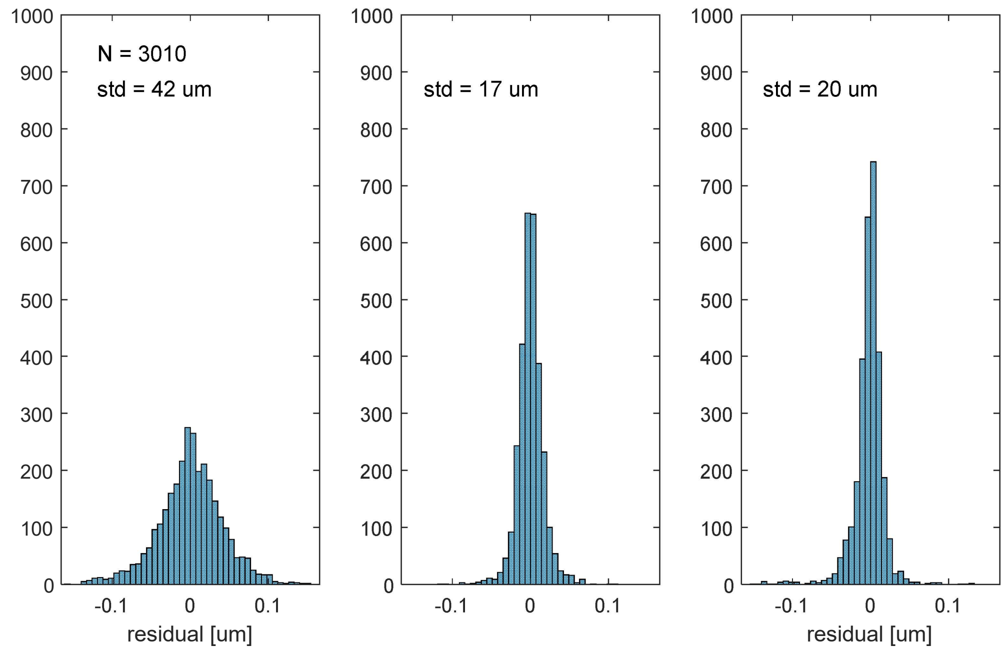


Figure 11 Histograms of residuals for the transformation of the girders that were measured after bakeout in the tunnel, onto their measured positions when they were first assembled.

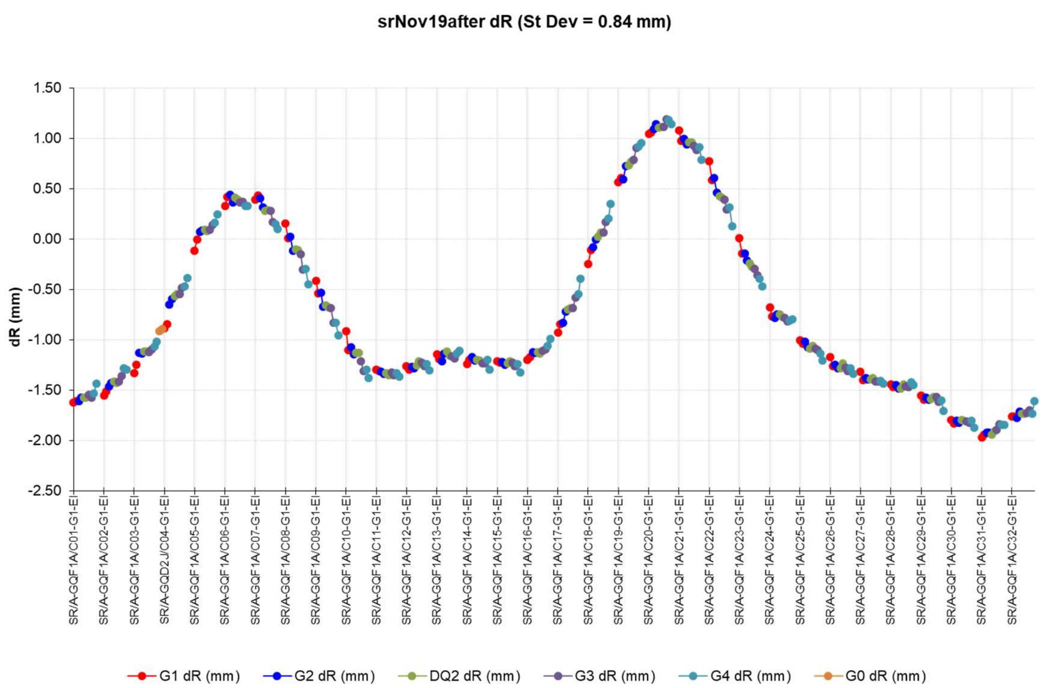


Figure 12 The final absolute radial position of the new EBS machine just before commissioning.

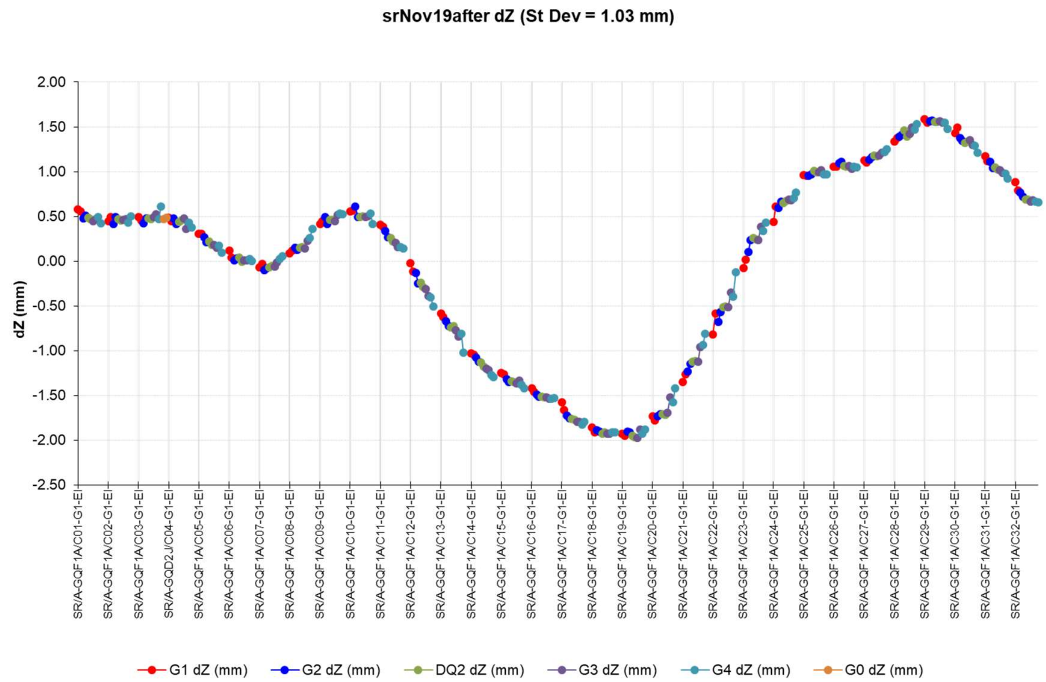


Figure 13 The final absolute vertical position of the new EBS machine just before commissioning.

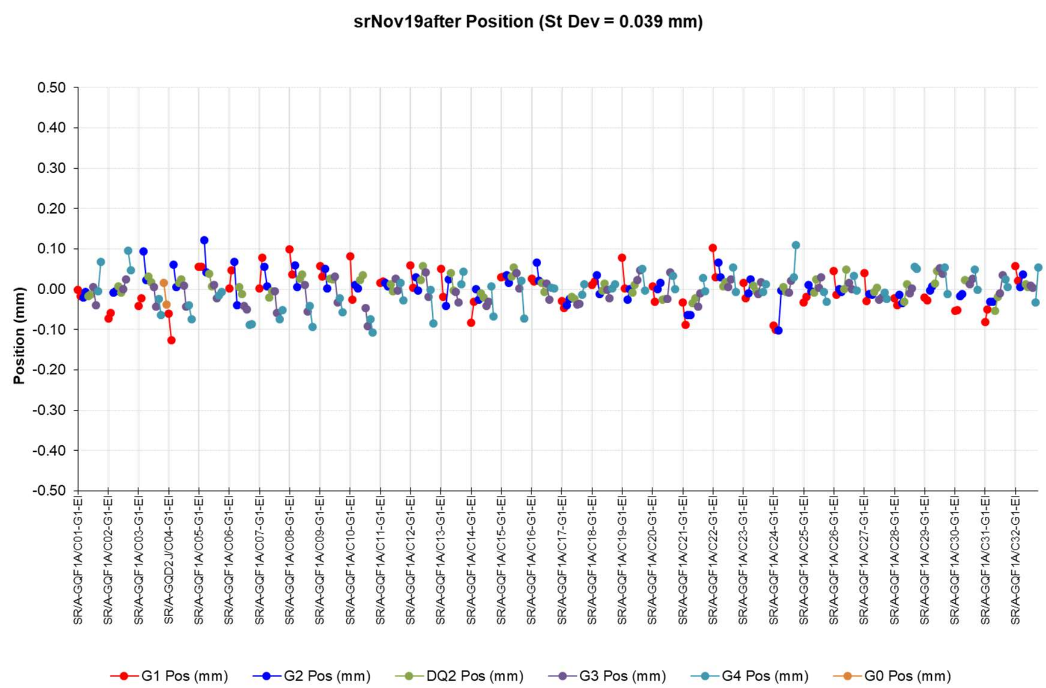


Figure 14 The final smoothing radial position of the new EBS machine just before commissioning.

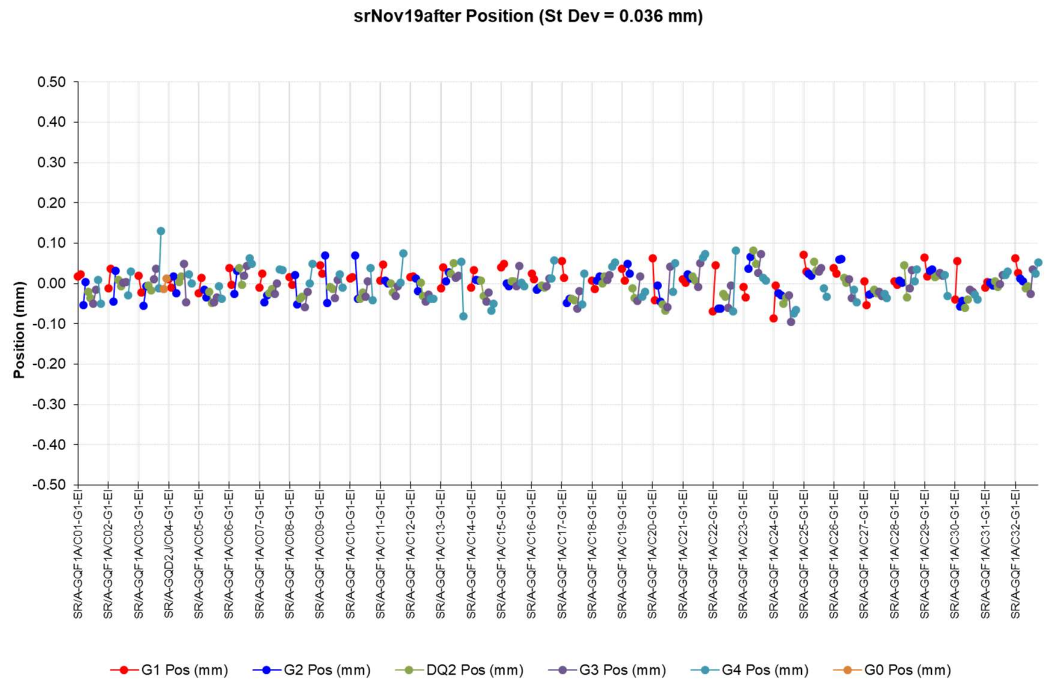


Figure 15 The final smoothing vertical position of the new EBS machine just before commissioning.

General combined uncertainty

Table 4 Final general uncertainty calculation

Ref	*Quantity*	*U(x) [μm]*	*U(y) [μm]*	*U(z) [μm]*	*U(z)† [μm]*
B_{final1}	Fiducialisation	23	24	29	41
B_{final2}	Assembly	126	29	32	33
B_{final3}	Transport and bakeout	42	17	20	20
B_{final4}	Tunnel alignment	6	6	19	19
B_{final5}	Girder smoothing	219	39	36	36
†Low gradient quadrupoles, octupoles and sextupoles use shims for the vertical alignment. This means the girder rectitude must be also included.					

The combined uncertainties for the Type B contributions are:

$$u_x = \sqrt{23^2 + 126^2 + 42^2 + 4^2 + 219^2} = 257\ \mu\text{m}$$

$$u_y = \sqrt{24^2 + 29^2 + 17^2 + 6^2 + 39^2} = 57\ \mu\text{m}$$

$$u_z = \sqrt{29^2 + 32^2 + 20^2 + 19^2 + 36^2} = 63\ \mu\text{m}$$

$$u_{z\_shimmed} = \sqrt{41^2 + 33^2 + 20^2 + 19^2 + 36^2} = 69\ \mu\text{m}$$

This general calculation applies the same uncertainty to all magnets which poses two significant issues. First, the uncertainties related to the tunnel alignment B_{final4} and the girder smoothing B_{final5} are different for different magnets. This variation is clearly illustrated in Figure 16. The uncertainty for a given magnet is impacted by its location and the method of measurement. Due to some obstructed lines of sight, some magnets are observed more frequently than others making them more geometrically defined, or stable. While the impact of the tunnel alignment is minimal, the differences in smoothing across girders are substantial.

The disposition of the magnets and girders in one typical cell, of 32 cells, in the ESRF EBS SR is shown in Figure 17. The following tables (Table 5 to Table 8) give uncertainty calculations for magnet families considering variations in tunnel alignment B_{final4} and smoothing B_{final5}. It is important to note that the values for fiducialisation, assembly and transport and bakeout remain unchanged, so they are combined into one contribution $B_{final123}$ i.e. B_{final1}, B_{final2} and B_{final3} from Table 4.

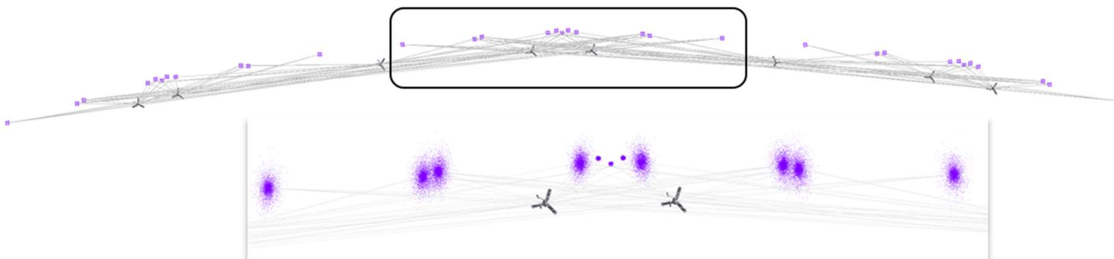


Figure 16 Measurement scheme in a typical cell, over 3 adjacent cells, of the EBS SR. The lower image is a zoom of a SA measurement simulation using the survey measurements.

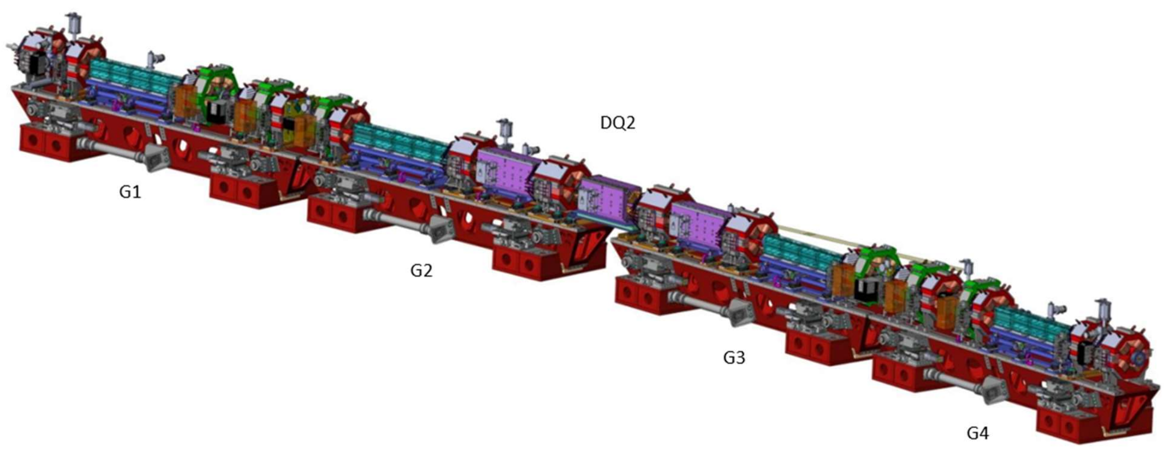


Figure 17 The disposition of magnets and girders of one cell in the tunnel

Table 5 Final uncertainty for the high gradient quadrupoles and combined function dipole Type B contributions. Note these magnets are not shimmed.

Ref	*Quantity*	*U(x) [µm]*	*U(y) [µm]*	*U(z) [µm]*
$B_{final123}$	B_{final1}, B_{final2} and B_{final3} Table 4	135	41	47
B_{final4}	Tunnel alignment	4	4	13
B_{final5}	Girder smoothing	219	24	30

The combined uncertainties for the high gradient quadrupoles and combined function dipoles Type B contributions are:

$$u_x = \sqrt{135^2 + 4^2 + 219^2} = 257\ \mu\text{m}$$

$$u_y = \sqrt{41^2 + 4^2 + 24^2} = 48\ \mu\text{m}$$

$$u_z = \sqrt{47^2 + 13^2 + 30^2} = 58\ \mu\text{m}$$

Table 6 Final uncertainty for the sextupole Type B contributions. Note these magnets are shimmed.

Ref	*Quantity*	*U(x) [μm]*	*U(y) [μm]*	*U(z) [μm]*
$B_{final123}$	B_{final1}, B_{final2} and B_{final3} Table 4	135	42	56
B_{final4}	Tunnel alignment	4	7	11
B_{final5}	Girder smoothing	219	41	36

The combined uncertainties for the sextupole Type B contributions are:

$$u_x = \sqrt{135^2 + 4^2 + 219^2} = 257\ \mu m$$

$$u_y = \sqrt{42^2 + 7^2 + 41^2} = 58\ \mu m$$

$$u_z = \sqrt{56^2 + 11^2 + 36^2} = 68\ \mu m$$

Table 7 Final uncertainty for the medium gradient quadrupole Type B contributions. Note these magnets are shimmed.

Ref	*Quantity*	*U(x) [μm]*	*U(y) [μm]*	*U(z) [μm]*
$B_{final123}$	B_{final1}, B_{final2} and B_{final3} Table 4	135	42	56
B_{final4}	Tunnel alignment	6	6	11
B_{final5}	Girder smoothing	219	53	42

The combined uncertainties for the medium gradient quadrupole Type B contributions are:

$$u_x = \sqrt{135^2 + 6^2 + 219^2} = 257\ \mu m$$

$$u_y = \sqrt{42^2 + 6^2 + 53^2} = 67\ \mu m$$

$$u_z = \sqrt{56^2 + 11^2 + 42^2} = 71\ \mu m$$

Table 8 Final uncertainty for the octupole Type B contributions. Note these magnets are shimmed.

Ref	*Quantity*	*U(x) [μm]*	*U(y) [μm]*	*U(z) [μm]*
$B_{final123}$	B_{final1}, B_{final2} and B_{final3} Table 4	135	42	56
B_{final4}	Tunnel alignment	4	7	11
B_{final5}	Girder smoothing	219	43	31

The combined uncertainties for the Type B contributions for the octupole Type B contributions are:

$$u_x = \sqrt{135^2 + 4^2 + 219^2} = 257\ \mu m$$

$$u_y = \sqrt{42^2 + 7^2 + 43^2} = 61\ \mu m$$

$$u_z = \sqrt{56^2 + 11^2 + 31^2} = 65\ \mu m$$

Table 9 Uncertainty summary for magnets in the EBS SR. The cell is symmetric and moving up the table in reverse order from DQ2C to QF1A gives the uncertainties for the magnets on the G3 and G4 girders.

Magnet	*Magnet type*	*Nominal* $U(y)$ *[μm]*	*Measured* $U(y)$ *[μm]*	*Nominal* $U(z)$ *[μm]*	*Measured* $U(z)$ *[μm]*	
QF1A	Med. Grad. Quad.	100	67	85	71	↑
QD2A	Med. Grad. Quad.	100	67	85	71	.
QD3A	Med. Grad. Quad.	100	67	85	71	.
SD1A	Sextupole	70	58	*50*	*68*	G1
QF4A	Med. Grad. Quad.	100	67	85	71	
SF2A	Sextupole	70	58	*50*	*68*	.
QF4B	Med. Grad. Quad.	100	67	85	71	.
OF1B	Octupole	100	61	100	65	↓
SD1B	Sextupole	70	58	*50*	*68*	↑
QD3A	Med. Grad. Quad.	100	67	85	71	.
QF6B	High Grad. Quad.	60	48	60	58	G2
DQ1B	Dipole-Quadrupole	60	48	60	58	.
QF8B	High Grad. Quad.	60	48	60	58	↓
DQ2C	Dipole-Quadrupole	60	48	60	58	DQ2
Symmetric moving back up the table for in the order DQ2 exit, G3 and G4 girders …						

Uncertainty summary

Summarising the previous sections, the final uncertainty for the magnets in the EBS SR are given in Table 9. Note this table gives the uncertainties of the magnets in the first half of the cell i.e. G1, G2 and the DQ2. The cell is symmetric and moving up the table in reverse order from DQ2C to QF1A gives the uncertainties for the magnets on the G3 and G4 girders. See also Figure 17 for the magnet layout.

Note the uncertainty $U(x) = 257$ μm along the beam for all of the magnets is well within the nominal tolerance of $U(x) = 500$ μm. $U(z) = 68$ μm for the sextupoles are above the nominal uncertainty of $U(x) = 50$ μm.

Part, but not all of the problem in $U(z)$ for the sextupoles, is the shimming. Apart from $U(z)$ for the sextupoles all of the other magnet tolerances have been achieved.

DISCUSSION

"It is seen that the alignment of the storage ring ranges from 22 to 42 μm in the horizontal plane and from 22 to 54 μm in the vertical plane." (Raimondi P., 2021) This was the estimate for the EBS SR alignment uncertainty based on orbit measurements, and therefore mainly quadrupole alignment uncertainties, determined during the machine commissioning. It is an independent and direct assessment of the alignment uncertainty. It is also more optimistic than what we have calculated in Table 9.

Recall, the values in Table 9 are our best estimate of the alignment uncertainties based on the information we have, the techniques and instruments we use. The values in Table 9 are not a direct measurement of the alignment uncertainties and their effects on the electron beam in the machine.

By using the Guide to the Expression of Uncertainty in Measurement (GUM) (BIPM, 2008), we are following a particularly rigorous framework for the determination of uncertainty in measurement. An argument can be made that certain contributions are repeated or superfluous. One example might be the *overall uncertainty* in the assembly uncertainty calculation in Table 3. Other contributions, like the *wire block reference* and *wire position determination* repeatability in the fiducialisation uncertainty calculation in Table 2 are determined using data taken over more than a year. It is possible that taking data over shorter time periods may be a more appropriate approach with a smaller uncertainty contribution.

It can be argued that the magnets that are not at the ends of the girders, because they are not measured during the final alignment, are not affected by the tunnel alignment B_{final4} and the girder smoothing B_{final5} in Table 4 to Table 9. We know that the magnets did not move appreciably with respect to one another on the girders[‡‡]. However, we prefer to argue that even though they are not measured, their uncertainty is affected like the other magnets at the ends of the girders because they were moved.

There are several contributions like this that could be reconsidered or removed altogether. Naturally, this would reduce the overall uncertainty. Indeed, we could in principle modify the uncertainty calculation in a way that it could agree with almost anything within reason.

This approach can be dangerous, so we will not change our uncertainty estimate in Table 9, at least for the time being. Furthermore, removing or not including contributions can always invite *what about ...* questions.

A more appropriate approach is to discuss among experts what is important and should be considered in an uncertainty calculation. A universal consensus among experts would be ideal.

Finally, it would be beneficial to explore why there is a difference between the orbit determination of the alignment uncertainty and the uncertainty calculation in Table 9. This could provide insights into important, and less important uncertainty contributions to consider.

CONCLUSION

The only way to know whether tolerances have been achieved or not is to make measurements. However, all measurements have some degree of uncertainty that arise from a variety of sources.

It seems there is not a common way to express uncertainty in measurement in the accelerator alignment community; a way to know how well something is aligned with respect to where it should be; a way to know whether or not tolerances have been achieved.

At the ESRF we decided to use the approach proposed in the GUM. We have presented a detailed proposal for an uncertainty calculation for the alignment of magnets in the ESRF EBS SR based on this approach. This calculation considers all major sources of error: fiducialisation, assembly, transportation and installation; and combines each of these contributions into a final statement of uncertainty.

The advantage of this is that it provides an easy way to compare what is done in different accelerators, at least those using a similar approach. An uncertainty calculation like the one made in this paper provides an easily interpretable baseline for everyone impacted by the results of accelerator alignment. It provides clarity that decision makers need to evaluate if something is or it isn't achievable. Ultimately this approach could even lead to improvement in accelerator alignment.

The key point is that we are not interested in the smallest possible estimate, but rather the most correct estimate of accelerator alignment uncertainty.

[‡‡] All of the magnets were measured on the girders in 2022. The coordinates from these girder surveys were adjusted on the initial assembly coordinates giving $U(x) = 37$ µm, $U(y) = 27$ µm, $U(z) = 19$ µm. These results can be directly compared to those in the section on bakeout and the entry B_{final3} in Table 4. The values are nearly unchanged over the 2½ year period between November 2019 and summer 2022 indicating the magnets don't move much with respect to each other on the girders.

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