First results of beam-beam simulations with WarpX

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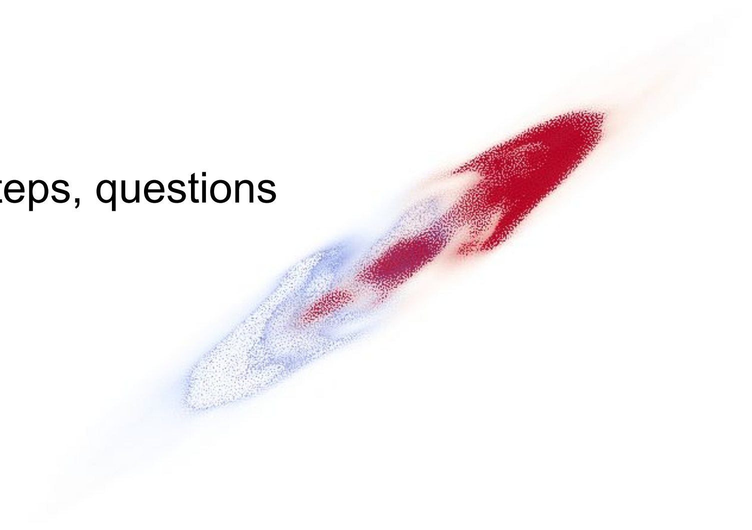
Outline

1. WarpX improvements for beam-beam collisions
2. Test case 1: ultra-tight beams
3. Test case 2: ILC @ 250 GeV
4. Modeling efforts
5. Conclusions, next steps, questions



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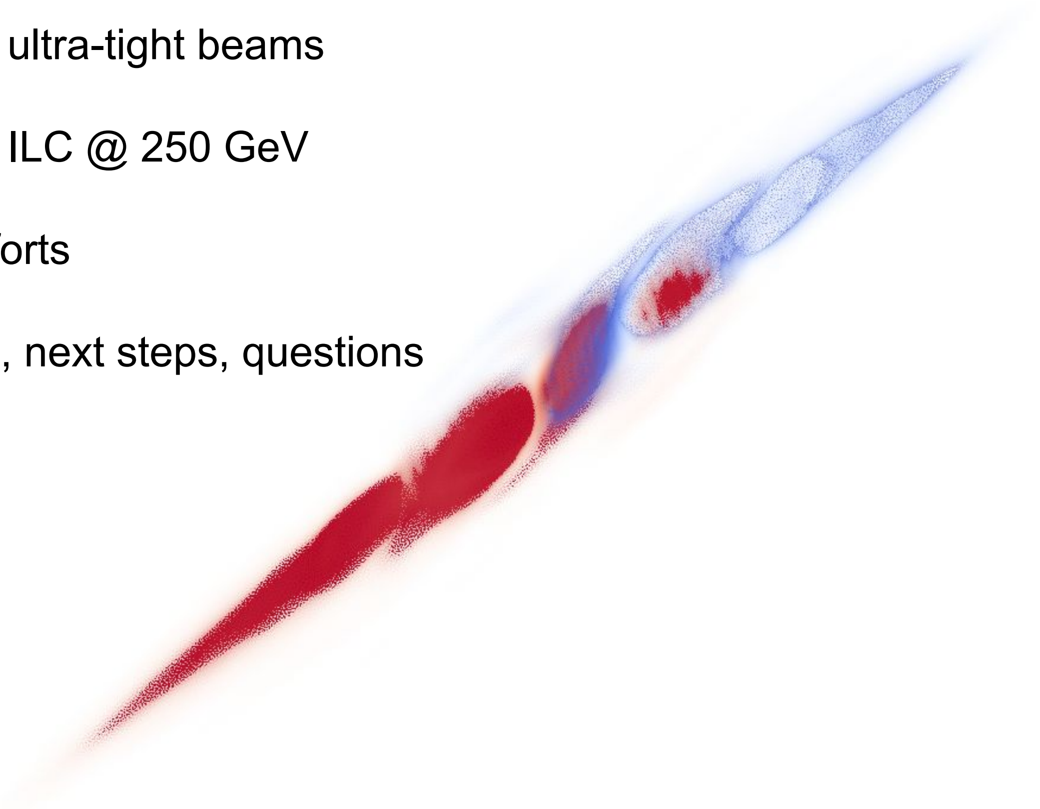

Summary of WarpX capabilities

https://ecp-warpx.github.io/

PIC code:

- electrostatic & electromagnetic
- massively-parallel
- portable & multi-platform
- lots of diagnostics
- broad & active community
- applications: laser-plasma acceleration, astrophysics, nuclear fusion, low-temperature plasmas, etc.

physics relevant to beam-beam collisions:

- electrostatic solver in the beams' rest frame
  - iterative
  - based on integrated Green functions (WIP)
- QED physics
  - synchrotron emission of photons
  - coherent pair generation or nonlinear BW

see Remi and Jean-Luc's talk!

Work so far on simulations @ IP

- added and adding some relevant features to the code (in progress!)
  - luminosity diagnostic
  - 2 electrostatic solvers
  - incoherent pair creation via linear Breit-Wheeler process
- benchmarks with other PIC codes under ultra-tight parameters
  - Osiris
  - VLPL
  - GUINEA-PIG
- comparisons between GUINEA-PIG vs. WarpX under ILC @ 250 GeV
- performance comparisons with GUINEA-PIG
- exploration of different electron-positron colliders (not shown here)
- tentative simplified model to get some key quantities (e.g. energy spread)
- …visualizations!

Special acknowledgements

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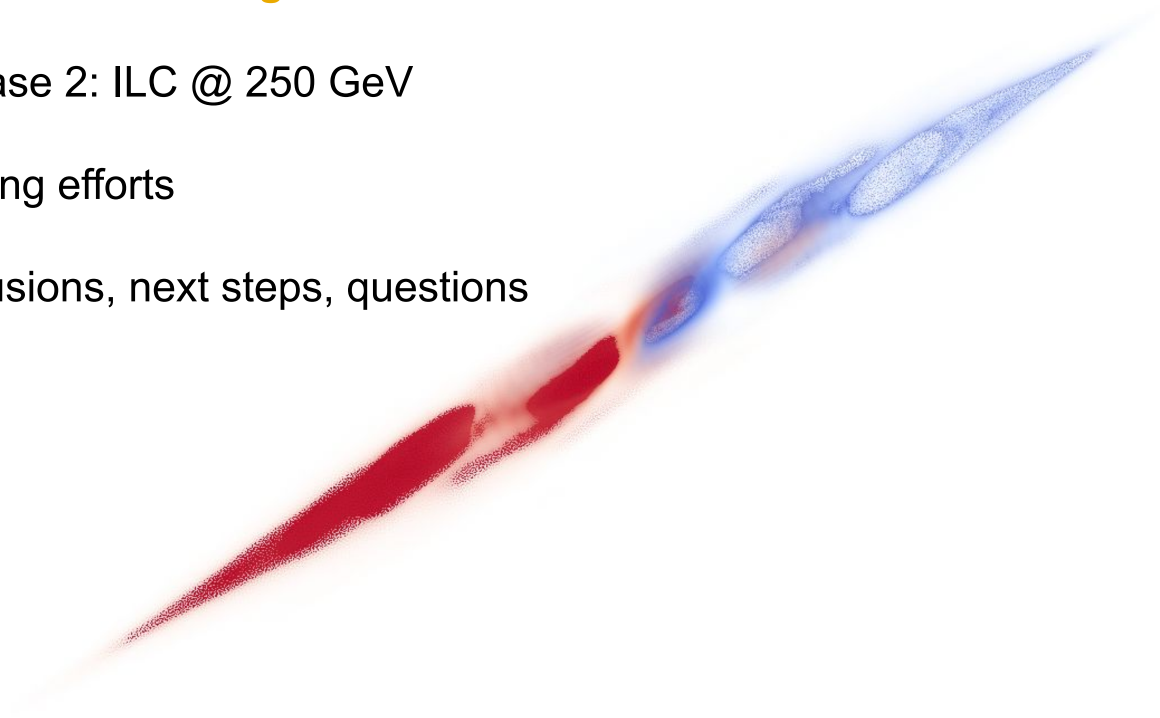

Test case 1: ultra-tight beams

- main parameters: E_{COM} = 250 GeV | N = 8.7 * 10^8 | $\sigma_z = \sigma_x = \sigma_y$ = 10 nm
- significant generation of coherent pairs
- low disruption D = 0.001
- spherical beams → “convenient” for “traditional” PIC simulations since the domain is not stretched (as in flat beams)

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Yakimenko et al. PRL 2019

Prospect of Studying Nonperturbative QED with Beam-Beam Collisions

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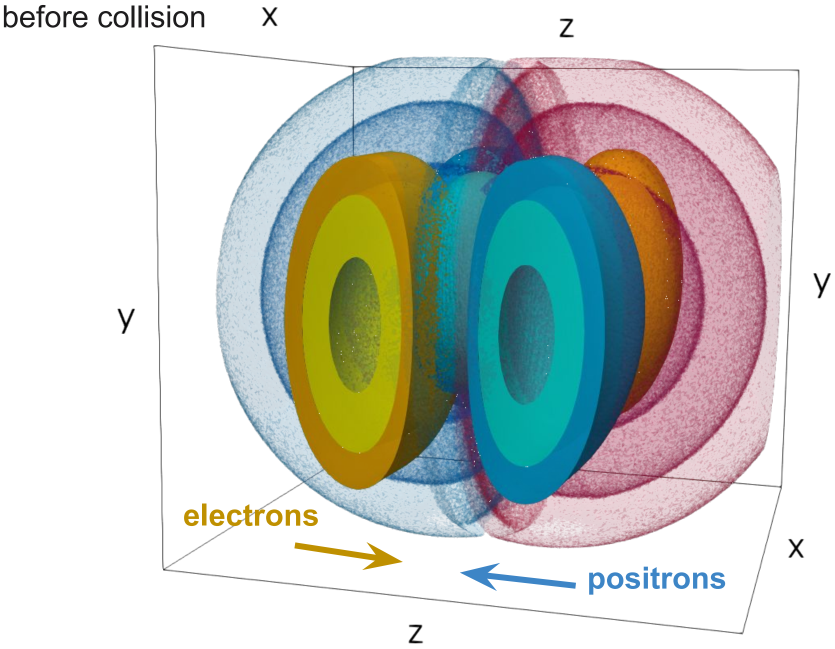
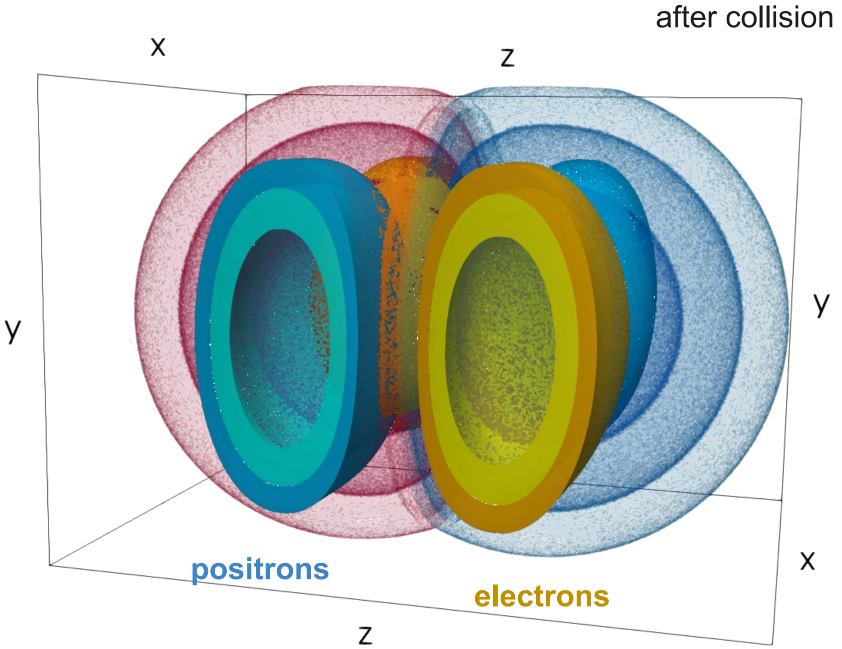
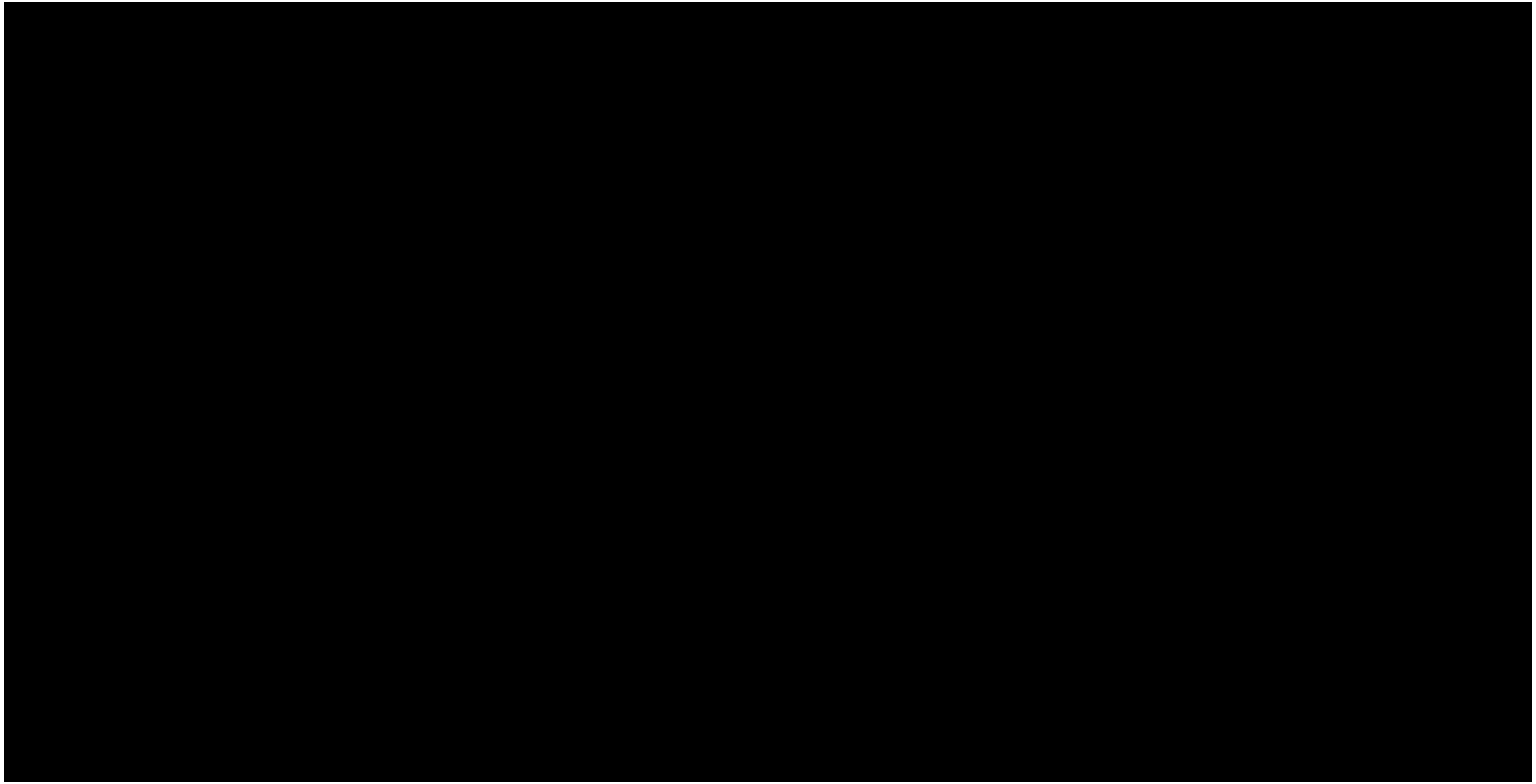




WarpX agrees well with GUINEA-PIG

time evolution of the ***integrated density***

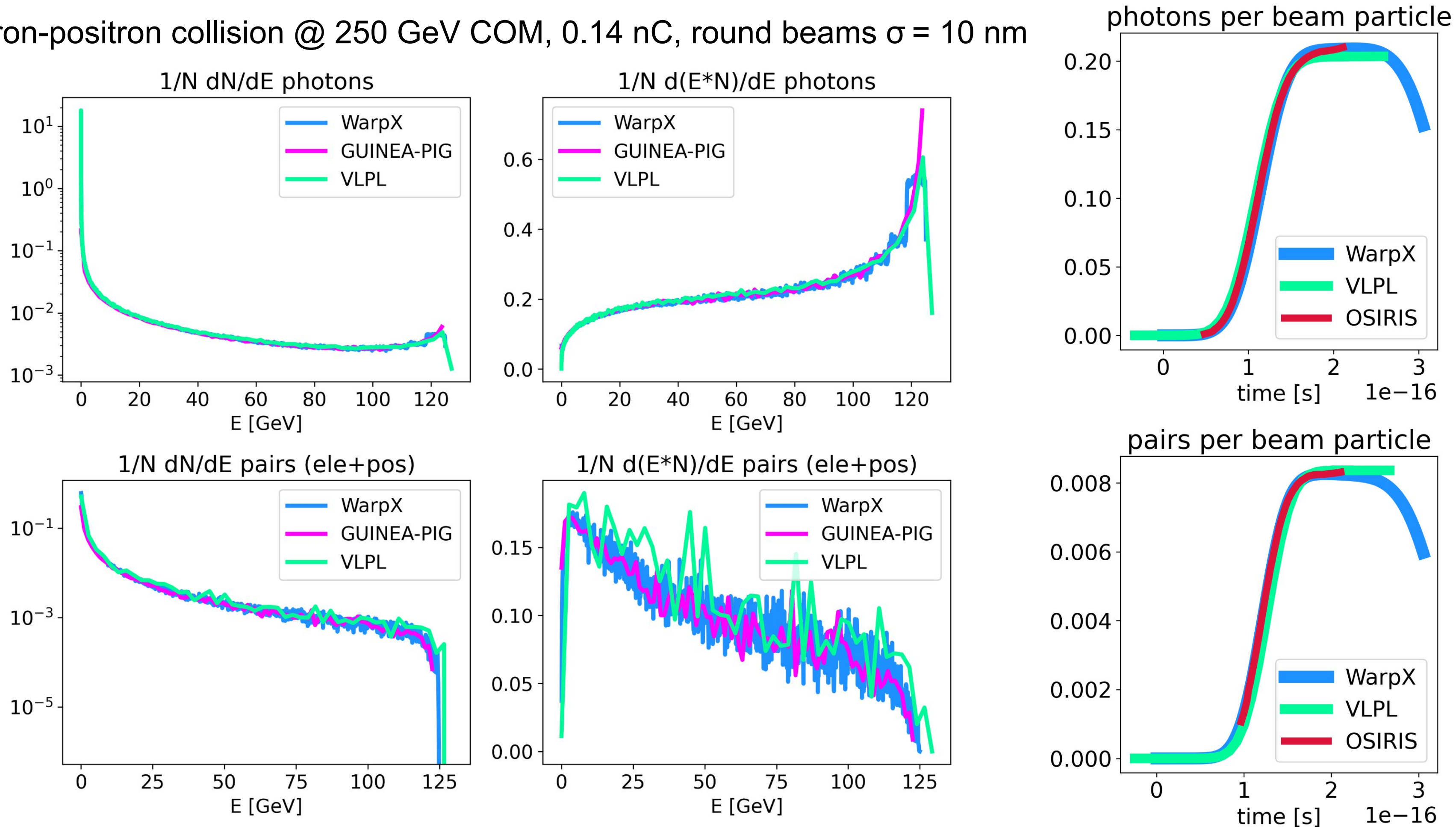
(z,x)
plane



(z,y)
plane

Different PIC codes with different implementations all agree

electron-positron collision @ 250 GeV COM, 0.14 nC, round beams σ = 10 nm

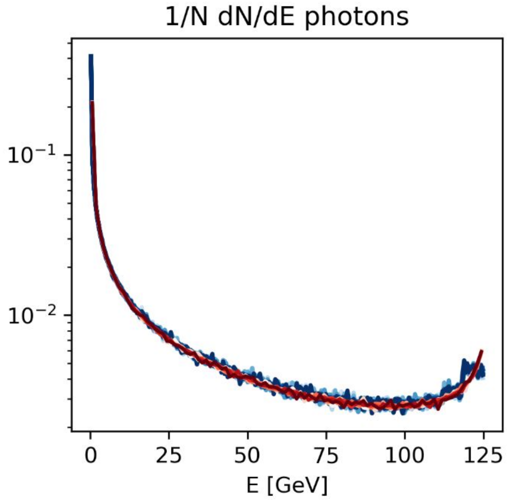


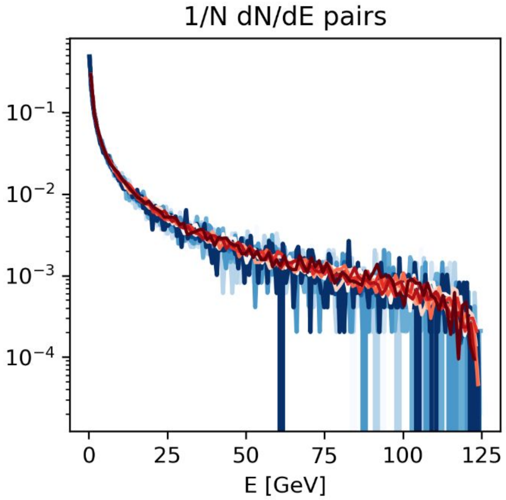
WarpX can be faster than GUINEA-PIG

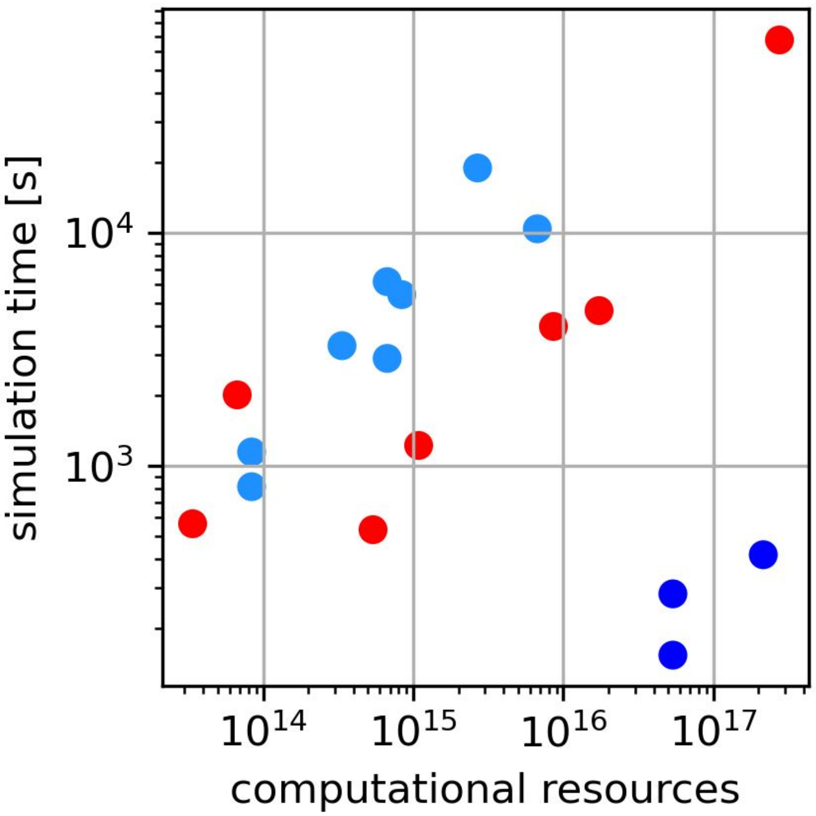
- GUINEA-PIG
- WarpX on 1 CPU
- WarpX on GPUs

as fast as or slower* than GUINEA-PIG in serial runs
…but much, much faster in parallel runs

* WarpX uses an iterative solver here!







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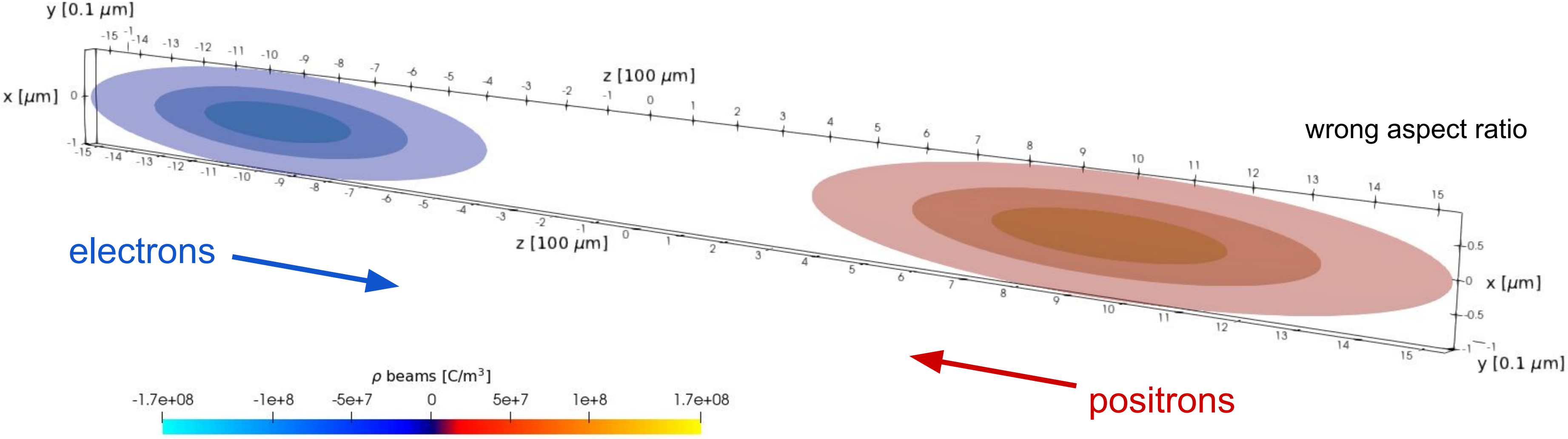
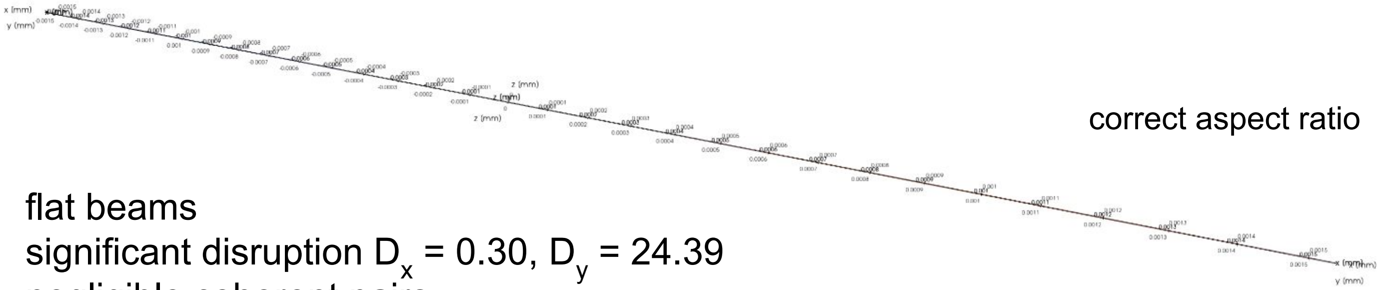
Test case 2: ILC 250 GeV COM



main parameters

- E_{COM} = 250 GeV
- N = 2 10^{10}
- σ_z = 300 μm
- σ_x = 516 nm
- σ_y = 7.7 nm



- flat beams
- significant disruption D_x = 0.30, D_y = 24.39
- negligible coherent pairs

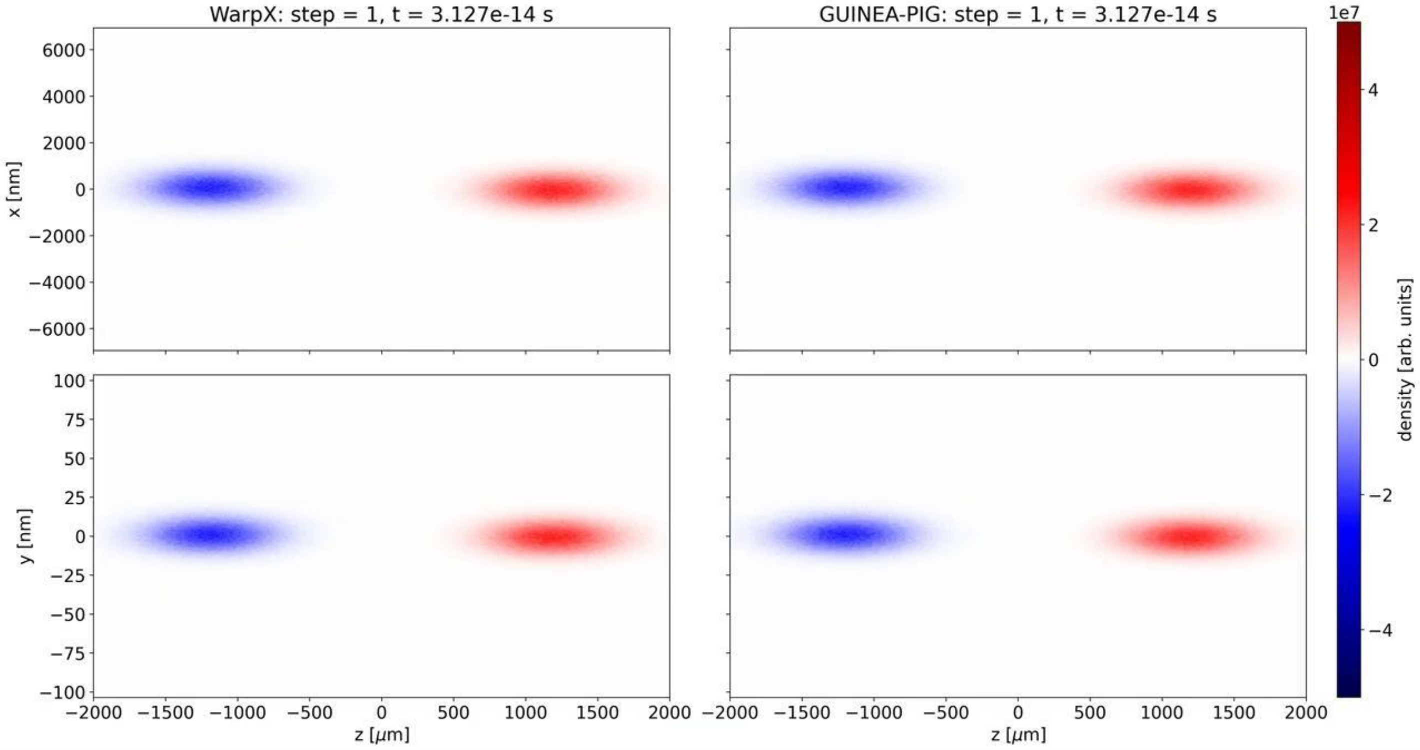
[ILC @ Snowmass 21]

WarpX agrees well with GUINEA-PIG

time evolution of the ***integrated density***
shifted beams to mitigate the discrepancies due to the kink or head-tail instability

(z,x) plane

(z,y) plane

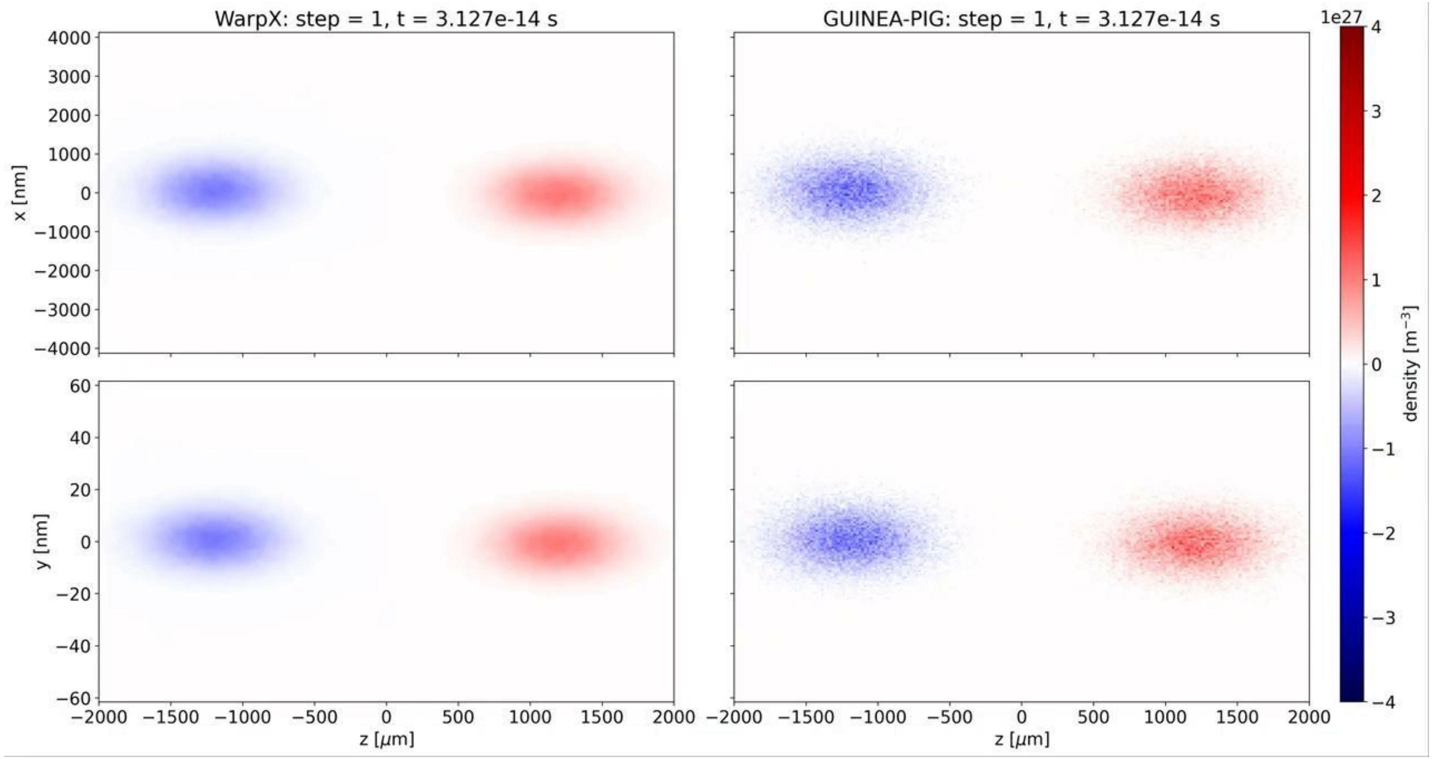


WarpX agrees well with GUINEA-PIG

time evolution of the ***density slices***
shifted beams to mitigate the discrepancies due to the kink or head-tail instability

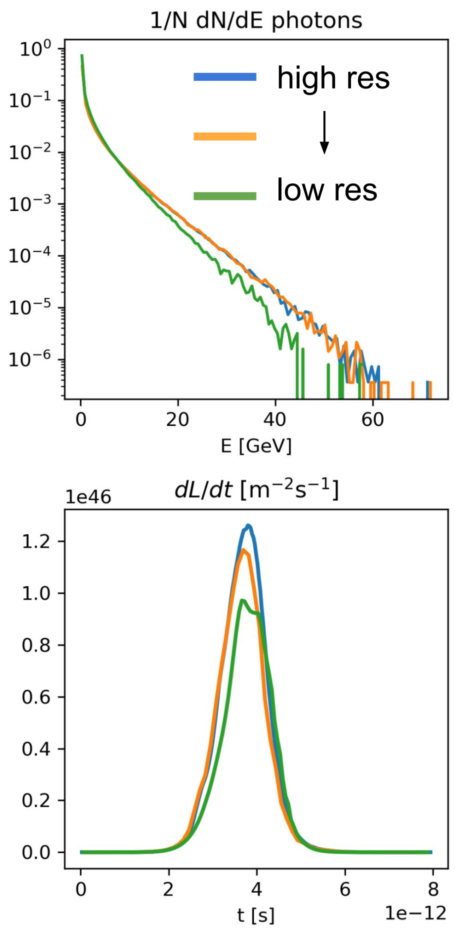
(z,x) plane

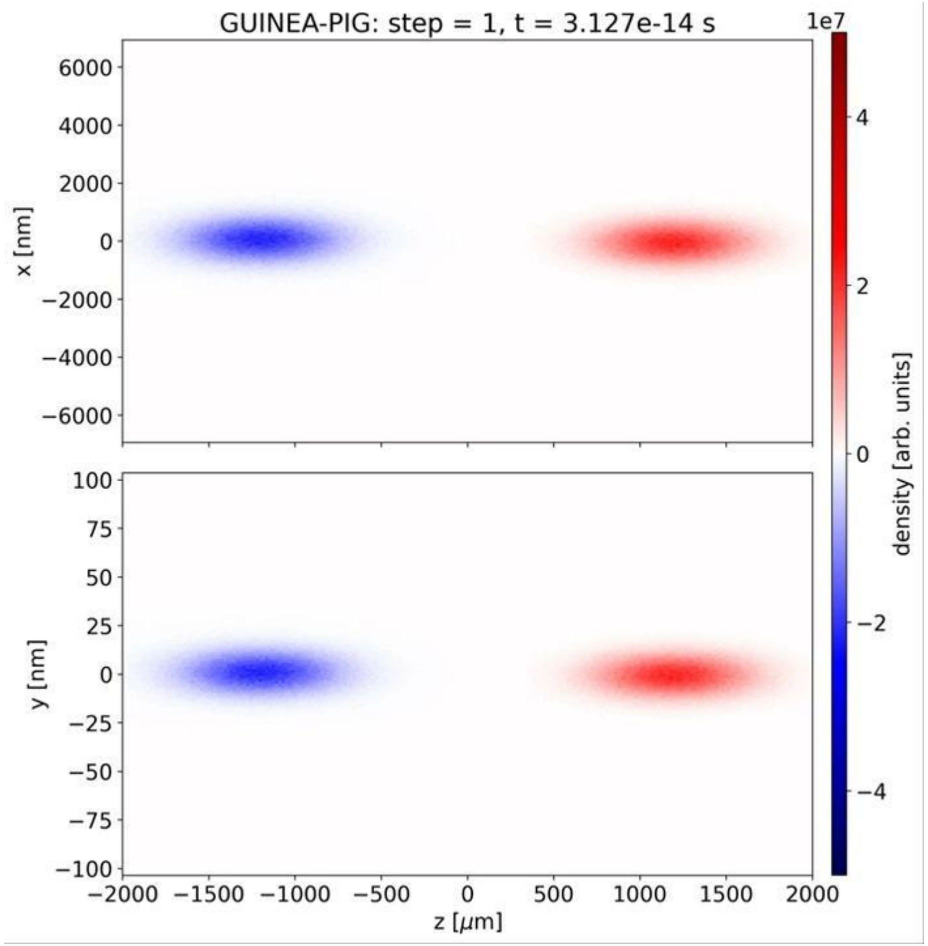
(z,y) plane



High enough resolution is needed to capture the details

low resolution:
→ the structures that form in the beams are blurred
→ luminosity might be wrong



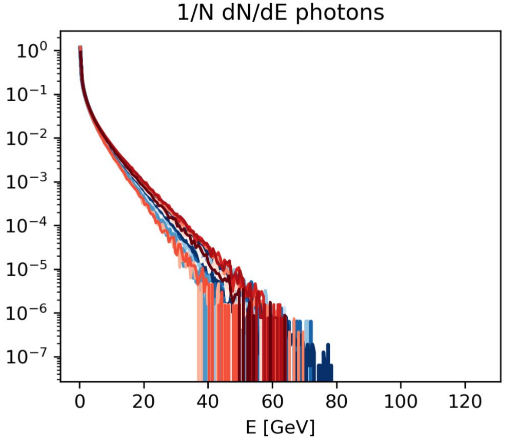


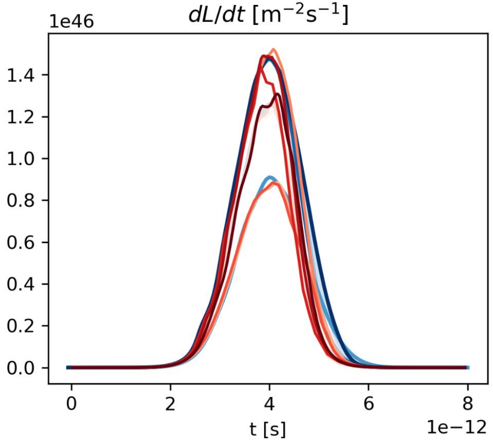
WarpX can be efficient also with flat beams

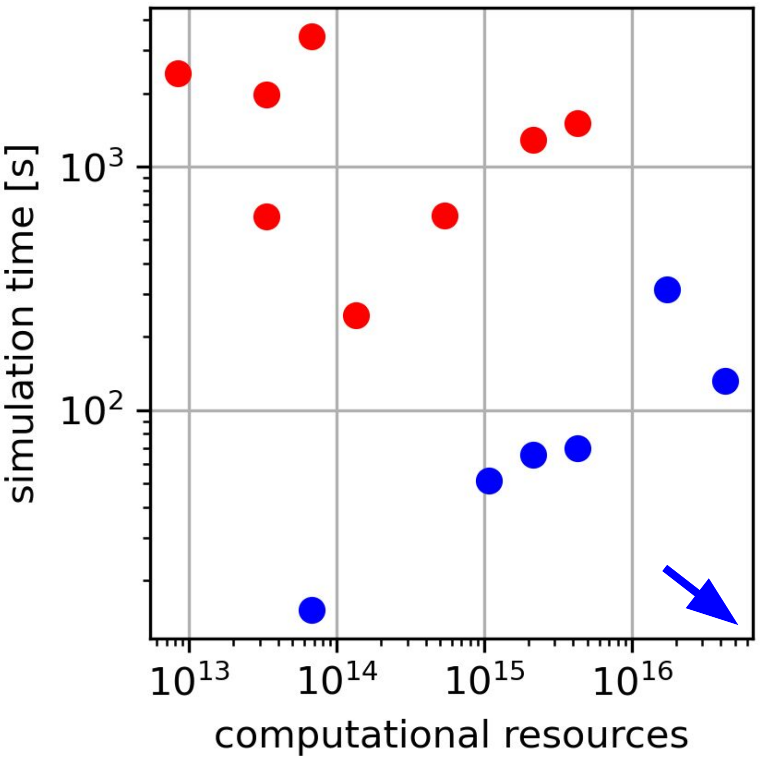
not completely fair comparison…
- GUINEA-PIG → 1 CPU
- WarpX → 1 CPU + 1GPU

…but shows that with WarpX we can be orders of magnitude faster using orders of magnitude more computational resources

all simulations together: again, resolution matters







WarpX enables complex 2D visualizations

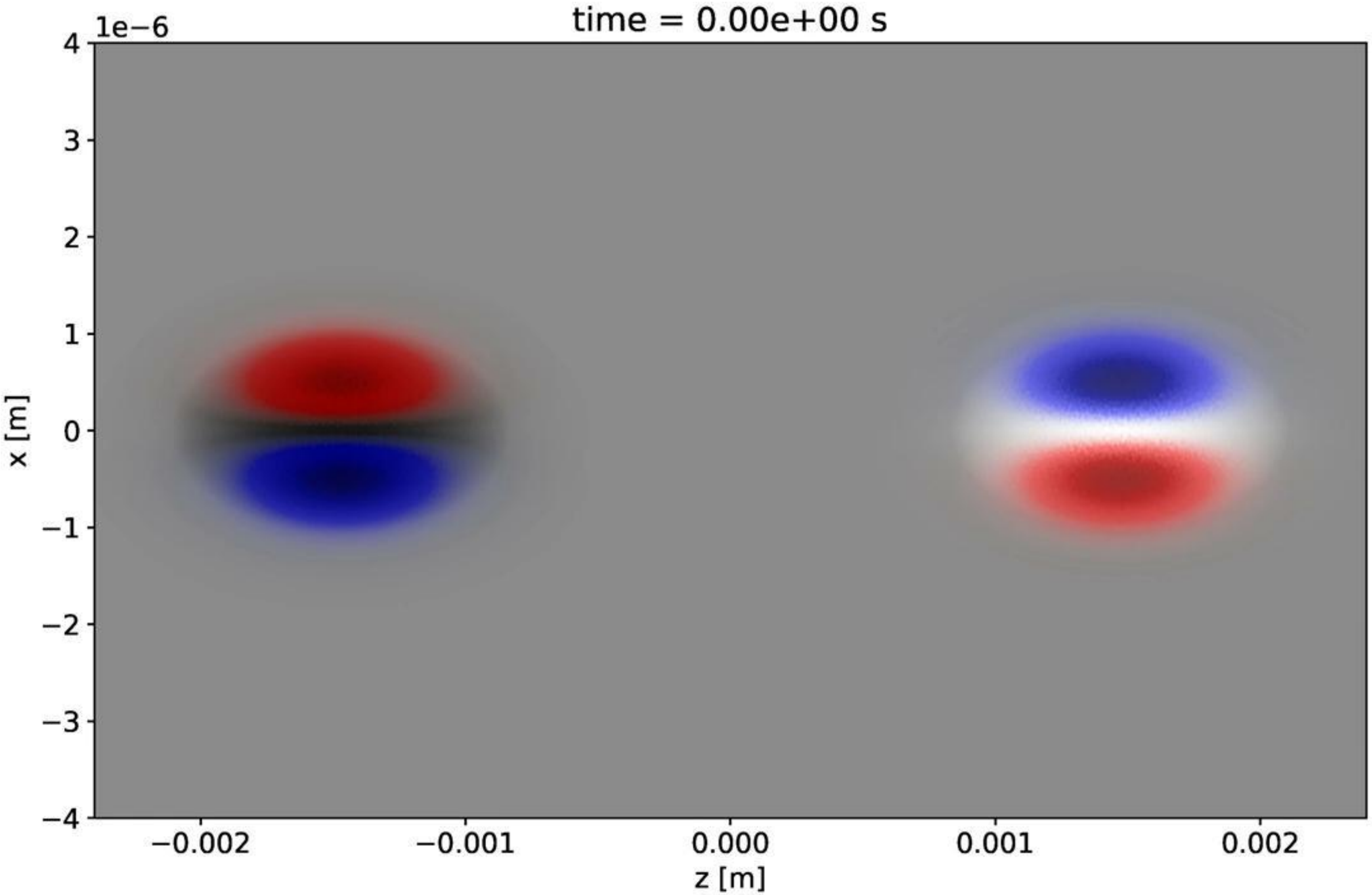
field E_x

beams ρ

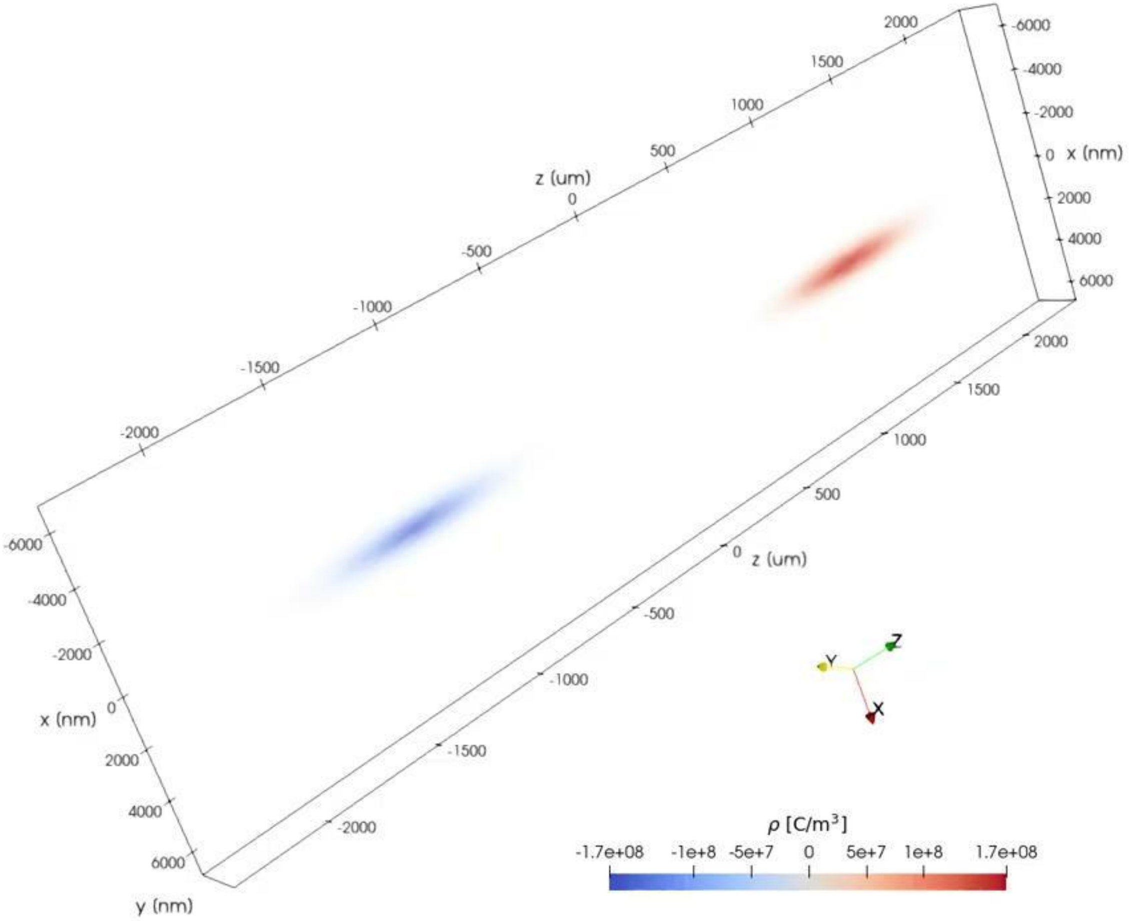
photons

WARNING: this is just an example of what we can visualize and how!

very wrong aspect ratio!



WarpX enables complex 3D visualizations



very wrong
aspect ratio!

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A simple model to capture the energy spread

momentum equation

From Niel PRE 2018

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{u} \times \mathbf{B}) - mc^2 S(\chi)\frac{\mathbf{u}}{c\mathbf{u}^2} + mc^2\sqrt{R(\chi,\gamma)}\frac{dW}{dt}\frac{\mathbf{u}}{c\mathbf{u}^2}$$

deterministic

random (ok for $\chi<1$)

dW is a Wiener process of variance dt

$$\mathbf{p} = mc\gamma\mathbf{u} = m\gamma\mathbf{v}$$

main HPs

- self-force is neglected
- ultrarelativistic longitudinal dynamics
- beams are Gaussian
- EM fields are transverse
- equipartition between EM forces

From Samsonov NJP 2021

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{u} \times \mathbf{B}) - \frac{P(\chi)}{c}\mathbf{u}$$

position equation

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\boxed{\begin{cases} \dfrac{d\gamma}{dt} = -S(\chi) + \sqrt{R(\chi,\gamma)}\dfrac{dW}{dt} \\ \dfrac{d\mathbf{p}_\perp}{dt} = -2e\mathbf{E} \\ \dfrac{d\mathbf{x}_\perp}{dt} = \mathbf{v}_\perp \end{cases} \qquad \begin{array}{c} \chi = 2\gamma|\mathbf{E}|/E_S \\ \dfrac{dN_\gamma}{dt} = \dfrac{2\alpha m_e c^2}{3\hbar\gamma}G(\chi) \\ \dfrac{dN_\pm}{dt} = \dfrac{\alpha m_e c^2}{\hbar\gamma_\gamma}T(\chi_\gamma) \end{array}}$$

Samsonov NJP 2021
Niel PRE 2018
Fedeli NJP 2022

Some auxiliary functions (just for reference)

terms in the longitudinal equation of motion

Niel PRE 2018

$$S(\chi) = \int_0^{+\infty} d\gamma_\gamma \gamma_\gamma w_\chi(\gamma,\gamma_\gamma) = \frac{2}{3}\frac{\alpha^2}{\tau_e}\chi^2 g(\chi), \quad (39)$$

$$R(\chi,\gamma) = \int_0^{+\infty} d\gamma_\gamma \gamma_\gamma^2 w_\chi(\gamma,\gamma_\gamma) = \frac{2}{3}\frac{\alpha^2}{\tau_e}\gamma h(\chi), \quad (40)$$

$$g(\chi) = \int_0^{+\infty} d\chi_\gamma \frac{G(\chi,\chi_\gamma)}{\chi^3}$$
$$= \frac{9\sqrt{3}}{8\pi}\int_0^{+\infty} d\nu \left[\frac{2\nu^2 \mathrm{K}_{5/3}(\nu)}{(2+3\nu\chi)^2} + \frac{4\nu(3\nu\chi)^2}{(2+3\nu\chi)^4}\mathrm{K}_{2/3}(\nu)\right]. \quad (25)$$

$$h(\chi) = \frac{9\sqrt{3}}{4\pi}\int_0^{+\infty} d\nu \left[\frac{2\chi^3\nu^3}{(2+3\nu\chi)^3}\mathrm{K}_{5/3}(\nu) + \frac{54\chi^5\nu^4}{(2+3\nu\chi)^5}\mathrm{K}_{2/3}(\nu)\right]. \quad (41)$$

to compute the photon emission rate

Fedeli NJP 2022

$$G(\chi) = \int_0^\chi \frac{S(\chi,u/\chi)}{u}\mathrm{d}u = \int_0^1 \frac{S(\chi,\xi)}{\xi}\mathrm{d}\xi = \int_0^1 \frac{\sqrt{3}}{2\pi}\left[\left(\int_{Y(\chi,\xi)}^{\infty} K_{5/3}(s)\mathrm{d}s\right) + \frac{\xi^2}{1-\xi}K_{2/3}(Y(\chi,\xi))\right]\mathrm{d}\xi$$

to compute the NLBW pair generation rate

$$T(\chi_\gamma) = \int_0^{\chi_\gamma} F(\chi_\gamma,u)\mathrm{d}u = \frac{1}{\pi\sqrt{3}\chi_\gamma^2}\int_0^{\chi_\gamma}\left[\int_{X(\chi_\gamma,u)}^{+\infty}\sqrt{s}K_{1/3}\left(\frac{2}{3}s^{3/2}\right)\mathrm{d}s\right.$$
$$\left. - \left(2-\chi_\gamma X^{3/2}(\chi_\gamma,u)\right)K_{2/3}\left(\frac{2}{3}X^{3/2}(\chi_\gamma,u)\right)\right]\mathrm{d}u.$$

Some auxiliary functions (just for reference)

electric field for round beams

$$E_r(r,z) = \frac{1}{2\pi\varepsilon_0}\frac{q_0}{\sqrt{2\pi}\sigma_z\sigma_r^2 r} e^{-z^2/2\sigma_z^2}\int_0^r e^{-r'^2/2\sigma_r^2} r'\mathrm{d}r' = \frac{1}{2\pi\varepsilon_0}\frac{q_0}{\sqrt{2\pi}\sigma_z} e^{-z^2/2\sigma_z^2}\left[\frac{1-e^{-r^2/2\sigma_r^2}}{r}\right]$$

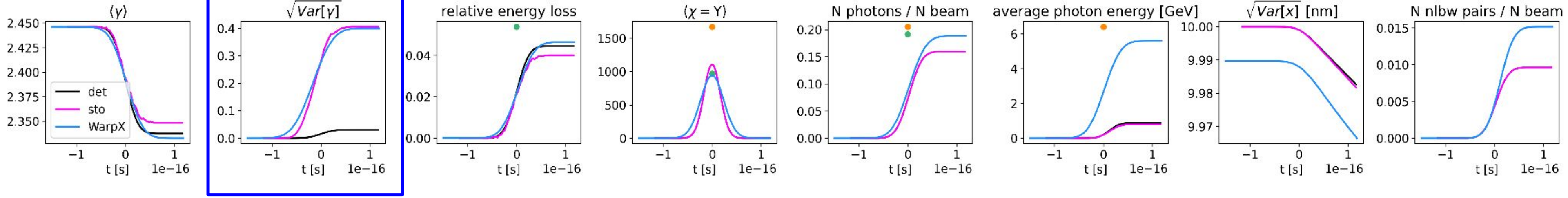
electric field for flat beams

$$E_x = \frac{Q}{2\varepsilon_o\sqrt{2\pi(\sigma_x^2-\sigma_y^2)}}\,\mathrm{Im}\left[w\left(\frac{x+iy}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right) - e^{\left[-\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}\right]} w\left(\frac{x\frac{\sigma_y}{\sigma_x}+iy\frac{\sigma_x}{\sigma_y}}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right)\right]$$

$$E_y = \frac{Q}{2\varepsilon_o\sqrt{2\pi(\sigma_x^2-\sigma_y^2)}}\,\mathrm{Re}\left[w\left(\frac{x+iy}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right) - e^{\left[-\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}\right]} w\left(\frac{x\frac{\sigma_y}{\sigma_x}+iy\frac{\sigma_x}{\sigma_y}}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right)\right]$$

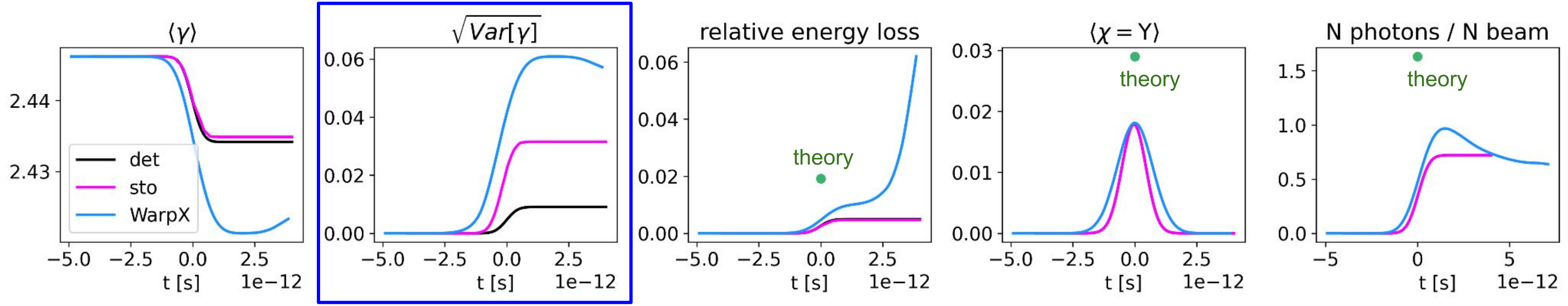
The model can predict some results for the two test cases

ultra-tight: E_{COM} = 125 GeV, σ_x = 10 nm, σ_y = 10 nm, σ_z = 10 nm, N = 8.7 * 10^8



ILC: E_{COM} = 250 GeV, σ_x = 516 nm, σ_y = 7.7 nm, σ_z = 300 μm, N = 2^{10}

the simulation results are not the latest!



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WarpX as a new generation beam-beam code

Two main takeaways:

1. WarpX agrees well with GUINEA-PIG under very different parameters
2. WarpX can be much faster than GUINEA-PIG

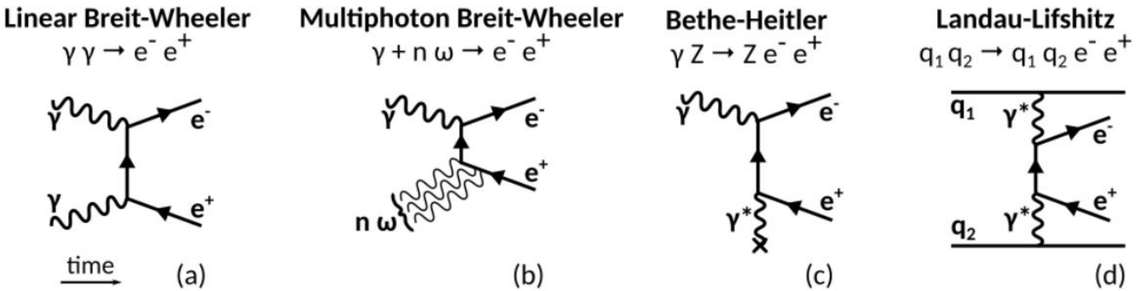
…plus:

- self-consistent → pairs contribute to the fields
- adaptable to the desired scenario
- parallel computing, GPUs → many particles, high resolution
- wide variety diagnostics
- active & broad community → maintenance
- cool visualizations ;)

Next steps in beam-beam investigations

- electrostatic relativistic Poisson solvers
  - finalize the implementation of the Poisson solver based on the integrated Green functions
  - implementation of a 2D version similar to the slice approach used in GUINEA-PIG, e.g.
  - a new intern will contribute

- add missing QED physics: Bethe-Heitler & Landau-Lifshitz (incoherent pair generation)
  - using a binary collision approach, like in [Martinez PoP 2019]
  - different from GUINEA-PIG and CAIN where they use a [virtual photon approximation]
  - should also help to benchmark linear Breit-Wheeler module



- expand the test cases: laser-plasma, ILC @ other energies, HALFH, others?
- comparisons with CAIN?

Thank you for your time!