Precision Flavor Theory

Wolfgang Altmannshofer
waltmann@ucsc.edu

UC SANTA CRUZ

SLAC Summer Institute 2024,
The Art of Precision: Calculations & Measurements

August 12 and 13, 2024

Outline of the Lectures

1. Introduction (today)
   - The CKM matrix (parametric input for precision predictions)
   - Wilson coefficients (perturbative physics)
   - Hadronic matrix elements (non-perturbative physics)

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3. B Decays (today/tomorrow)
   - $B \to D^{(*)} \ell \nu$ and $R_{D^{(*)}}$
   - $B_s \to \mu^+ \mu^-$
   - $B \to K \nu \bar{\nu}$
   - $B \to K^* \ell^+ \ell^-$ and $R_{K^{(*)}}$

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   - $B \to K^*\ell^+\ell^-$ and $R_{K^{(*)}}$
4. Kaon and Pion Decays (tomorrow)
   - Rare kaon decays $K \to \pi\nu\bar{\nu}$
   - Lepton universality in pion decays $\pi^+ \to e^+\nu$ vs. $\pi^+ \to \mu^+\nu$
   - Pion beta decay $\pi^+ \to \pi^0 e^+\nu$

Introduction

“Fishing Expeditions”

Promising Indirect Probes of New Physics

- Test bedrock assumptions of particle physics

  Lorentz invariance; CPT invariance; ...
  ($\Lambda \gtrsim M_{\text{Planck}} \sim 10^{19}$ GeV)

Probe more generic new physics

Reach to higher new physics scales

Promising Indirect Probes of New Physics

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  ($\Lambda \gtrsim M_{\text{Planck}} \sim 10^{19}$ GeV)

- Test (approximate) accidental symmetries of the SM

  Baryon Number: e.g. proton decay
  ($\Lambda \sim \Lambda_{\text{GUT}} \sim 10^{16}$ GeV)

  Lepton Number: e.g. neutrinoless double beta decay
  ($\Lambda \sim \Lambda_{\text{see-saw}} \sim 10^{12}$ GeV)

  Flavor: e.g. flavor changing neutral currents
  ($\Lambda \sim 10^3 - 10^8$ GeV)

  CP: e.g. electric dipole moments
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  CP: e.g. electric dipole moments
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- Test “ordinary” Standard Model processes

  Higgs precision program; Electroweak precision observables; muon anomalous magnetic moment; ...
  $(\Lambda \sim 10^3$ GeV)

Reach to higher new physics scales

Flavor in the Standard Model and Beyond

CC problem

Hierarchy problem

Vacuum stability?

Strong CP problem

$$\mathcal{L}_{\rm SM} \sim \Lambda^4 + \Lambda^2 H^2 + \lambda H^4$$

$$+ \bar{\Psi} \not{D} \Psi + (D_\mu H)^2 + (F_{\mu\nu})^2 + F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$+ Y H \bar{\Psi} \Psi$$

SM flavor puzzle

CC problem · Hierarchy problem · Vacuum stability?

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$$+ Y H \bar\Psi \Psi + \frac{1}{\Lambda}(LH)^2 + \frac{1}{\Lambda^2}\sum_i \mathcal{O}_i^{\rm dim6} + \ldots$$

SM flavor puzzle · Neutrino masses · Flavorful new physics?

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CC problem

Hierarchy problem

Vacuum stability?

Strong CP problem

SM flavor puzzle

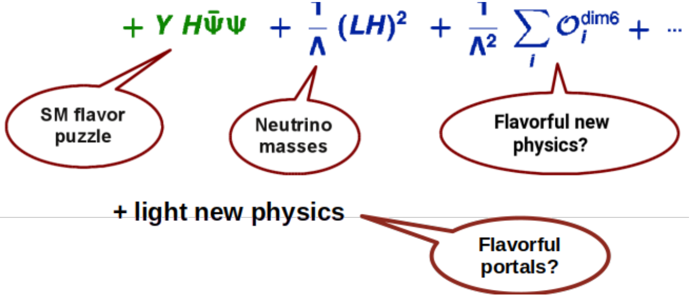
Neutrino masses

Flavorful new physics?

+ light new physics

Flavorful portals?

Two Basic Flavor Questions

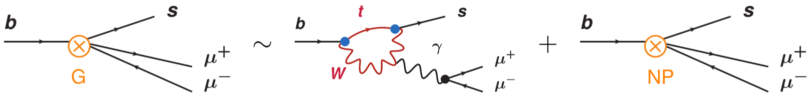


Q1: What is the origin of the hierarchical flavor structure of the SM?

Q2: Are there new sources of flavor violation beyond the SM?

Searching for New Physics with Flavor

Example: heavy new physics in rare B decays



$$G \sim \frac{1}{16\pi^2} \frac{g^4}{m_W^2} \frac{m_t^2}{m_W^2} V_{tb} V_{ts}^* + \frac{C_{\rm NP}}{\Lambda_{\rm NP}^2}$$

measure precisely

calculate precisely the SM contribution

get information on NP coupling and scale

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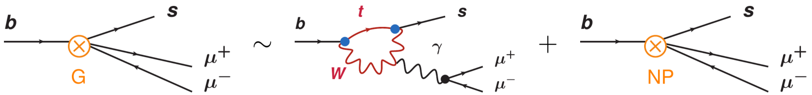

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measure precisely

calculate precisely the SM contribution

get information on NP coupling and scale

Mismatch between experiment and SM prediction
indicates new physics and provides a scale!

The Need for Precision

To maximize the sensitivity to new physics we need

- precision measurements of flavor observables
  $\rightarrow$ lectures by Jim
- precision theory prediction of the observables
  $\rightarrow$ these lectures

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precision theory predictions require

- high precision parametric input (in particular CKM)
- higher order perturbative calculations
- control over non-perturbative QCD uncertainties

The Weak Effective Hamiltonian

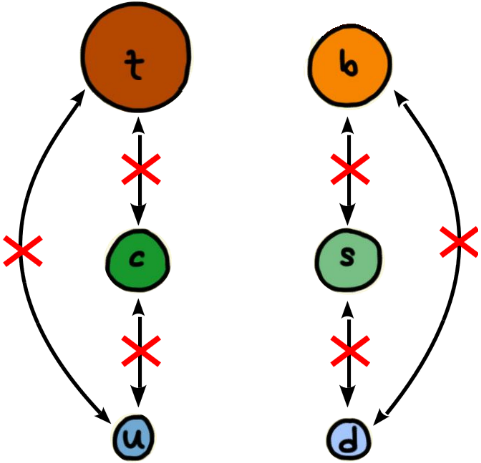
see e.g. Buras hep-ph/9806471 [hep-ph] for a review

Starting point for many theory predictions is the "weak effective Hamiltonian"

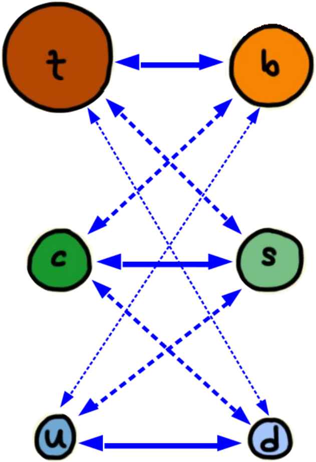
$$\langle f|\mathcal{H}_{\rm eff}|i\rangle = \frac{4G_F}{\sqrt{2}} \sum_k \lambda_{\rm CKM}^{(k)}\, C_k(\mu)\, \langle f|O_k(\mu)|i\rangle$$

- $\lambda_{\rm CKM}^{(k)}$ = combination of CKM matrix elements relevant for a given flavor changing process
- $C_k(\mu)$ = Wilson coefficients that encode the short distance physics (the weak interactions in the SM)
- $\langle f|O_k(\mu)|i\rangle$ = matrix elements of local operators made from light SM fields (light quarks, leptons, gluons, photon)
- Wilson coefficients and operator matrix elements depend on the renormalization scale μ

The CKM Matrix



no FCNCs at tree level



no FCNCs at tree level

transitions among the generations are mediated by the $W^{\pm}$ bosons and their relative strength is parametrized by the CKM matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

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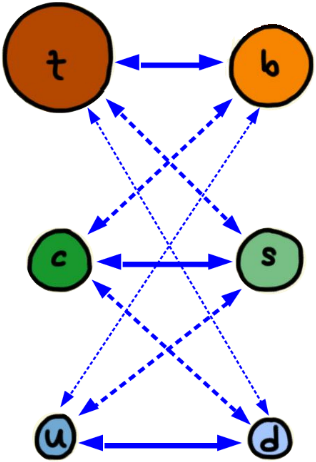

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CKM matrix is unitary and determined by 4 independent parameters

Standard Parametrization: product of 3 rotation matrices

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$s_{ij} = \sin(\theta_{ij}),\ c_{ij} = \cos(\theta_{ij})$$

Parametrization of the CKM Matrix

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$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

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(many equivalent parametrizations possible)

Parametrization of the CKM Matrix

Wolfenstein Parametrization: introduce the parameters λ, A, ρ, η

$$s_{12} = \lambda \quad , \quad s_{23} = A\lambda^2 \quad , \quad s_{13} e^{i\delta} = A\lambda^3(\rho + i\eta)$$

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measurements show that $\lambda \simeq 0.2 \ll 1$ is a good expansion parameter

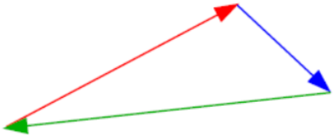
$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

Unitarity Triangles

The CKM matrix is unitary $\to$ relations between CKM elements

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

three complex numbers adding up to 0



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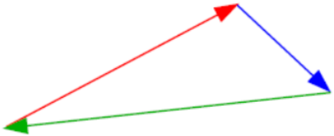
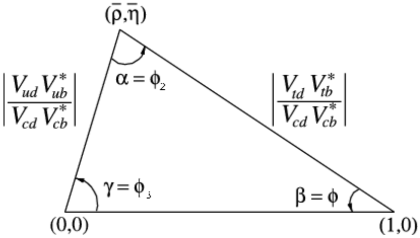




It is convenient to normalize one side to 1

$$\bar\rho + i\bar\eta = -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}$$

$$\bar\rho = \rho(1 + O(\lambda^2)) \ , \ \ \bar\eta = \eta(1 + O(\lambda^2))$$

global fits
of all data give
overall consistent
picture within
O(10%) uncertainties

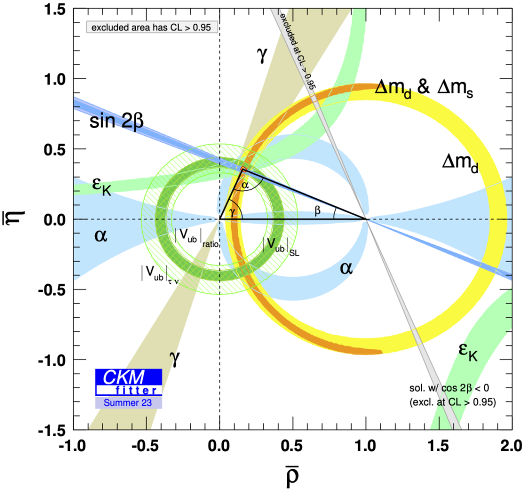
$$\lambda = 0.22498^{+0.00023}_{-0.00021}$$

$$A = 0.8215^{+0.0047}_{-0.0082}$$

$$\bar{\rho} = 0.1562^{+0.0112}_{-0.0040}$$

$$\bar{\eta} = 0.3551^{+0.0051}_{-0.0057}$$

http://ckmfitter.in2p3.fr/
http://www.utfit.org/



Alternative Approach

global CKM fits include many loop observables which might be affected by new physics

to avoid potential new physics contamination as much as possible, use 4 measurements based on tree level decays that are unlikely affected by new physics

$$V_{us} = 0.22431 \pm 0.00085 \;, \quad V_{cb} = (40.8 \pm 1.4) \times 10^{-3}$$

$$V_{ub} = (3.82 \pm 0.20) \times 10^{-3} \;, \quad \gamma = (65.9 \pm 3.5)^\circ$$

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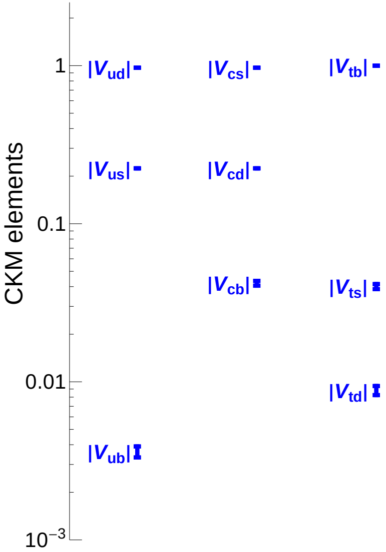
$$\begin{aligned}
&V_{ud} \simeq 1 - \frac{\lambda^2}{2} \,, && V_{us} \simeq \lambda \,, && V_{ub} \simeq |V_{ub}| e^{-i\gamma} \,, \\
&V_{cd} \simeq -\lambda \,, && V_{cs} \simeq 1 - \frac{\lambda^2}{2} \,, && V_{cb} = |V_{cb}| \,, \\
&V_{td} \simeq |V_{cb}|\lambda - |V_{ub}| e^{i\gamma} \left(1 - \frac{\lambda^2}{2}\right) \,, && V_{ts} \simeq -|V_{cb}| \left(1 - \frac{\lambda^2}{2}\right) - |V_{ub}|\lambda e^{i\gamma} \,, && V_{tb} \simeq 1 \,, \qquad (9)
\end{aligned}$$

(see e.g. WA, Lewis 2112.03437)

[I prefer this approach; I think it is more “robust” und transparent]

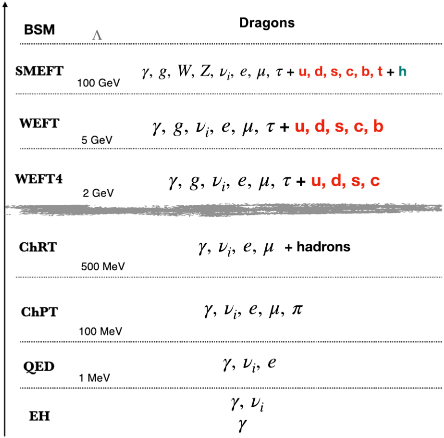
Quark Mixing Hierarchy



the measured CKM elements show a very hierarchical pattern

$$|V| \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad \lambda \simeq 0.2$$

- Flavor change comes from the weak scale $\mu_{\text{weak}} \sim 100$ GeV.
- But we observe flavor changing processes of hadrons at a low scale $\mu_{\text{had}} \sim 1$ GeV



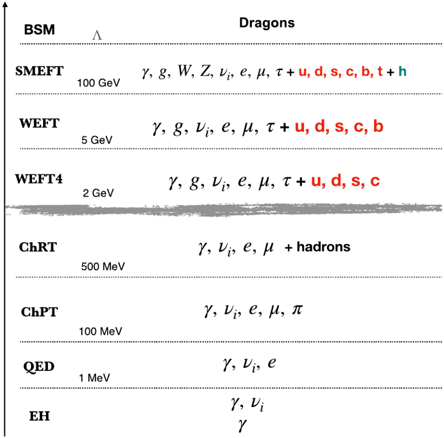
Falkowski Eur.Phys.J.C 83 (2023) 7, 656

(see lecture by Ilaria)

- Flavor change comes from the weak scale $\mu_{\text{weak}} \sim 100$ GeV.
- But we observe flavor changing processes of hadrons at a low scale $\mu_{\text{had}} \sim 1$ GeV
- Higher order loop corrections often come with large logs

$$\alpha_s \log\left(\frac{\mu^2_{\text{weak}}}{\mu^2_{\text{had}}}\right)$$

Can be O(1) corrections that need to be resummed.



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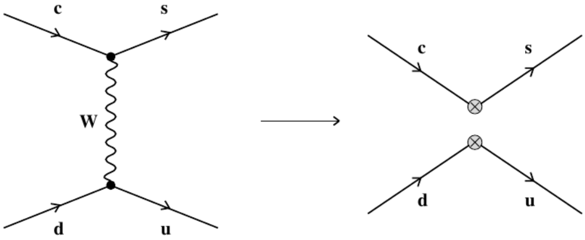
(see lecture by Ilaria)

Matching at Tree-Level

Buras hep-ph/9806471 [hep-ph]

Let's consider the effective Hamiltonian relevant for the decay $c \to s u \bar{d}$
(a simple example that illustrates many important features)

Integrating out the W boson at tree level gives one dim-6 operator and the corresponding Wilson coefficient



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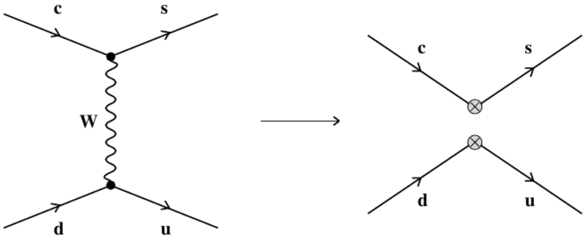
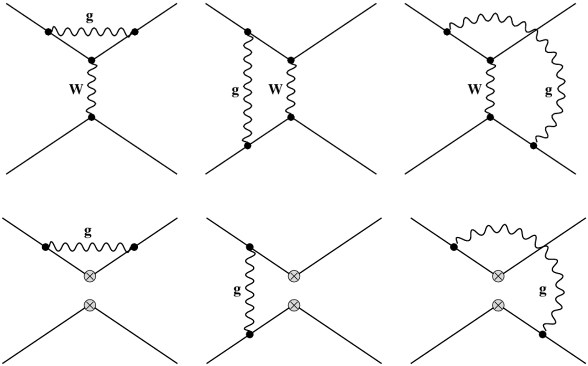


$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}\gamma_\mu P_L c)(\bar{u}\gamma^\mu P_L d) \ + \ \text{dim} \geq 8$$

Matching at 1-Loop

Buras hep-ph/9806471 [hep-ph]

What happens if we include 1-loop QCD corrections?



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We get two operators with different color structures

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cs}^* V_{ud} \Big(C_1 O_1 + C_2 O_2 \Big)$$

$$O_2 = (\bar{s}_\alpha \gamma_\mu P_L c_\alpha)(\bar{u}_\beta \gamma^\mu P_L d_\beta) \ , \quad C_2 = 1 + \frac{\alpha_s}{4\pi} \log\left(\frac{m_W^2}{\mu^2}\right)$$

$$O_1 = (\bar{s}_\alpha \gamma_\mu P_L c_\beta)(\bar{u}_\beta \gamma^\mu P_L d_\alpha) \ , \quad C_1 = -\frac{3\alpha_s}{4\pi} \log\left(\frac{m_W^2}{\mu^2}\right)$$

(α and β are color indices that are summed over)

RGE Running and Mixing

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- Including the higher order loops produces UV-divergencies that can be taken care of by renormalizing the Wilson coeffcients

$$C_i^{\text{bare}} = Z_{ij}^C C_j$$

- Need to introduce a matrix of renormalization constants, because loops with a Wilson coefficient C_i might produce divergencies that can only be absorbed by C_j

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- In the $\overline{\text{MS}}$ scheme one finds in our example

$$Z_{ij}^C = 1 - \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix}$$

RGE Running and Mixing

Buras hep-ph/9806471 [hep-ph]

- Determine the corresponding anomalous dimension matrix for the Wilson coefficients and determine their renormalization group running

$$\gamma = -2\alpha_s \frac{dZ^{(1)}}{d\alpha_s} = \frac{\alpha_s}{4\pi} \begin{pmatrix} -2 & 6 \\ 6 & -2 \end{pmatrix} = \frac{\alpha_s}{4\pi} \gamma_0$$

$$\vec{C}(\mu) = U(\mu, \mu_0) \vec{C}(\mu_0) \; , \quad U(\mu, \mu_0) = \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right)^{\frac{\gamma_0^T}{2\beta_0}}$$

- $\beta_0 = 23/3$ is the 1-loop coefficient of the QCD beta function with 5 active quark flavors

Connecting the High and Low Scales

$$\vec{C}(\mu) \cdot \langle f|\vec{O}(\mu)|i\rangle = \vec{C}(\mu_{\text{weak}}) \cdot U(\mu_{\text{weak}}, \mu_{\text{had}}) \cdot \langle f|\vec{O}(\mu_{\text{had}})|i\rangle$$

- Determine Wilson coefficients by matching at the weak scale.
- Run to the low scale using RGEs. This resumms the large logs.
- Combine the Wilson coefficients with hadronic matrix elements evaluated at the hadronic scale.

Dealing with Non-Perturbative QCD

1) "Cheat": Focus on observables that are vanishingly small in the Standard Model

   example: lepton flavor violating decays $B \to K\tau\mu$

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2) "Ratios": Design observables where hadronic physics (approximately) drops out

   example: lepton flavor universality ratios

$$\frac{\text{BR}(B \to K\mu\mu)}{\text{BR}(B \to Kee)} \,, \quad \frac{\text{BR}(\pi \to e\nu)}{\text{BR}(\pi \to \mu\nu)}$$

Dealing with Non-Perturbative QCD

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2) "Ratios": Design observables where hadronic physics (approximately) drops out

   example: lepton flavor universality ratios

$$\frac{\text{BR}(B \to K\mu\mu)}{\text{BR}(B \to Kee)} , \quad \frac{\text{BR}(\pi \to e\nu)}{\text{BR}(\pi \to \mu\nu)}$$

3) Parameterize the hadronic matrix elements and determine them e.g. with lattice QCD or data driven methods

   $\to$ see the discussion of hadronic contributions to $(g-2)_\mu$ by Martin and Aida

Parameterization of Hadronic Matrix Elements

examples of local matrix elements $\langle f|O(x)|i\rangle$

- decay constants

$$\langle 0|\bar{u}\gamma^\mu\gamma_5 d|\pi^+\rangle = if_\pi p_\pi^\mu$$

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- "Bag parameters" for meson mixing

$$\langle \bar{K}^0|(\bar{d}\gamma^\mu P_L s)(\bar{d}\gamma_\mu P_L s)|K^0\rangle = \frac{4}{3} B_K m_K f_K^2$$

Summary of the Introduction

Generic structure of a flavor changing amplitude:

$$\langle f|\mathcal{H}_{\text{eff}}|i\rangle = \frac{4G_F}{\sqrt{2}} \sum_k \lambda_{\text{CKM}}^{(k)} \, C_k(\mu) \, \langle f|O_k(\mu)|i\rangle$$

- CKM matrix elements (can be a limiting factor for precision)
- Wilson coefficients / short distance physics (in almost all cases under good perturbative control)
- hadronic matrix elements (can be a limiting factor for precision)

Neutral Meson Mixing

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There are 4 neutral meson anti-meson systems

$B_s - \bar{B}_s$ mixing $\quad b\bar{s} \leftrightarrow \bar{b}s$

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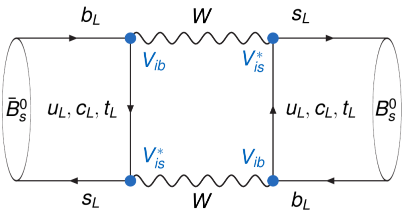
Meson mixing arises in the SM through “box-diagrams”



Time Evolution of Neutral Meson Systems

$$i\partial_t \begin{pmatrix} B(t) \\ \bar{B}(t) \end{pmatrix} = \left(\hat{M} + \frac{i}{2}\hat{\Gamma} \right) \begin{pmatrix} B(t) \\ \bar{B}(t) \end{pmatrix}$$

mass matrix $\hat{M} = \hat{M}^\dagger = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix}$, decay matrix $\hat{\Gamma} = \hat{\Gamma}^\dagger = \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}$

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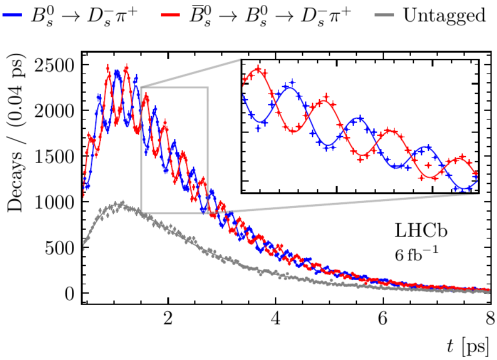
diagonalize the Hamiltonian

$$B_H = pB + q\bar{B} \;, \quad B_L = pB - q\bar{B} \;, \quad \left(\frac{q}{p}\right)^2 = \frac{2M_{12}^* - i\Gamma_{12}^*}{2M_{12} - i\Gamma_{12}}$$

$$\Delta M_s = M_s^H - M_s^L \simeq 2|M_{12}^s|$$
$$\Delta M_d = M_d^H - M_d^L \simeq 2|M_{12}^d|$$

Mixing Frequencies



$$\Gamma(B_s(t) \to D_s^- \pi^+) \sim e^{-\Gamma_s t}\left(\cosh(\frac{\Delta\Gamma_s t}{2}) + \cos(\Delta M_s t)\right)$$

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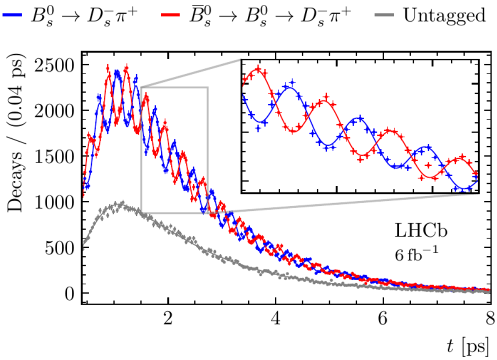

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$$\Delta M_s = (17.765 \pm 0.006)/ps \quad , \quad \Delta M_d = (0.5069 \pm 0.0019)/ps$$

(Heavy Flavor Averaging Group `hflav.web.cern.ch`)

SM Predictions for B-Meson Mixing

$$\Delta M_d^{\rm SM} = \frac{G_F^2 m_W^2}{6\pi^2} m_{B_d} |V_{td}^* V_{tb}|^2 S_0(m_t^2/m_W^2) \eta_B f_{B_d}^2 \hat{B}_{B_d} \ ,$$
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- S_0 is a loop function that depends on the top mass. It correponds to the Wilson coeffcient of a 4-fermion operator

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- hadronic matrix elements from lattice with $\sim 5\%$ uncertainty

$$f_{B_d}\sqrt{\hat{B}_{b_d}} = 210.6(5.5)~{\rm MeV} \qquad f_{B_s}\sqrt{\hat{B}_{B_s}} = 256.1(5.7)~{\rm MeV}$$

[see Flavour Lattice Averaging Group `flag.unibe.ch` for compilation of state-of-the-art lattice results relevant for flavor physics and the corresponding original lattice references.]

Probing New Physics with Meson Mixing

4 fermion contact interactions leading to kaon mixing

$$\frac{C_1}{\Lambda^2}(\bar{d}\gamma_\mu P_L s)(\bar{d}\gamma^\mu P_L s)$$

$$\frac{C_2}{\Lambda^2}(\bar{d}P_L s)(\bar{d}P_L s)$$

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$$\frac{C_4}{\Lambda^2}(\bar{d}P_L s)(\bar{d}P_R s)$$

$$\frac{C_5}{\Lambda^2}(\bar{d}_\alpha P_L s_\beta)(\bar{d}_\beta P_R s_\alpha)$$

(analogous for other meson systems)

[need hadronic matrix elements for all operators from lattice]

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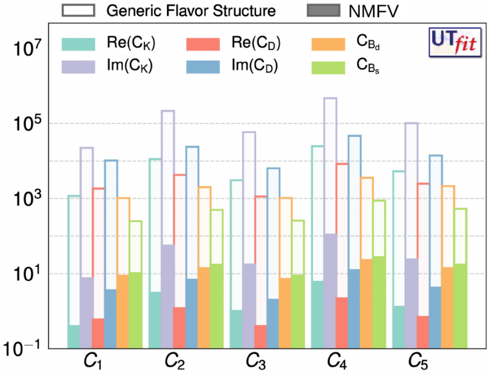
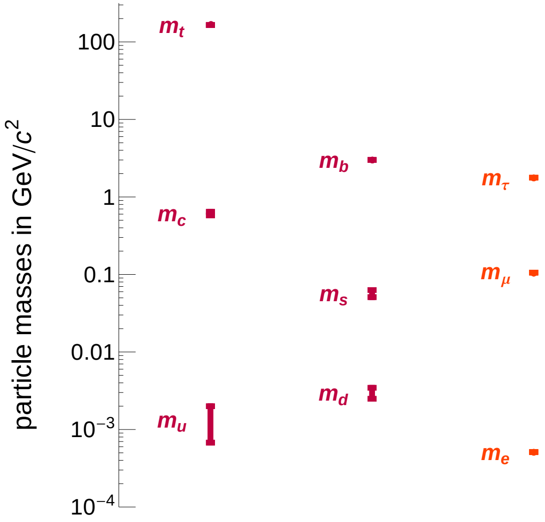


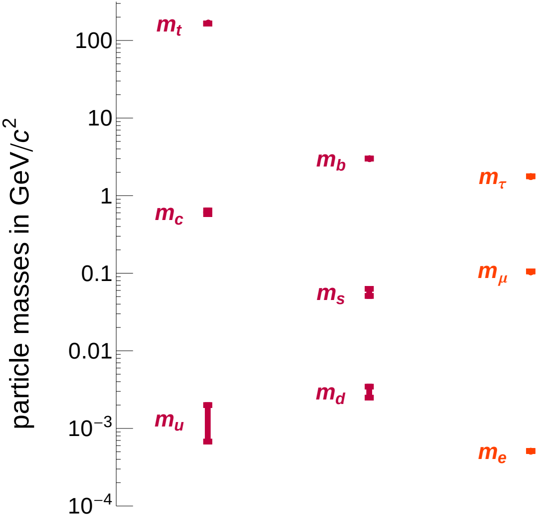
bounds on Λ in TeV assuming $|C_i| = 1$ or $|C_i| = \lambda_{\text{CKM}}^{\text{SM}}$

Decays of B Hadrons



$$\overline{m}_b(\overline{m}_b) = 4.18^{+0.03}_{-0.02}\ \text{GeV}$$

The b Quark



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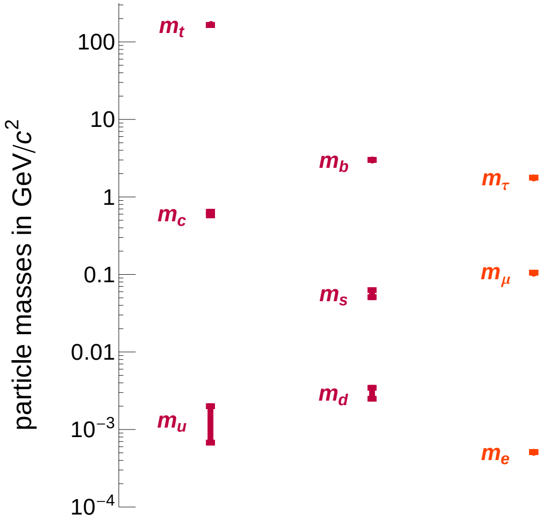

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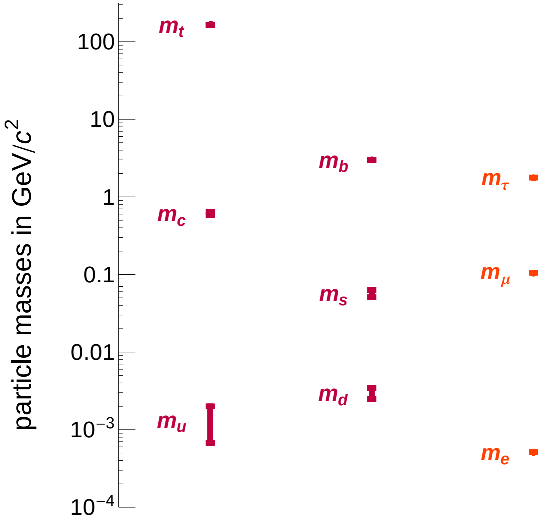

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- B baryons:
  Λ_b, ...
- exotics?

Lifetime

- Decay of b quarks proceeds through the weak interactions
- Exchange of a heavy virtual W boson

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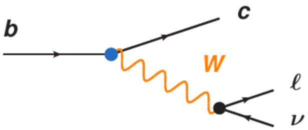
- Estimate the decay width

$$\Gamma(b \to c\ell\nu) \sim \frac{G_F^2}{192\pi^3} m_b^5 |V_{cb}|^2$$



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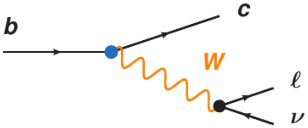
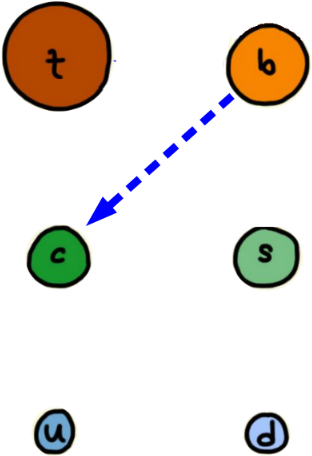


$$\Rightarrow \quad \tau = \frac{1}{\Gamma_{\text{tot}}} \sim \mathcal{O}(10^{-12} s)$$

- small decay width $\Rightarrow$ sizable lifetime
- high sensitivity to new physics effects

Charged Current Decays



- arise at tree level through W exchange

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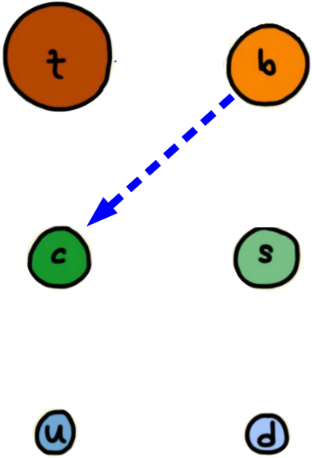

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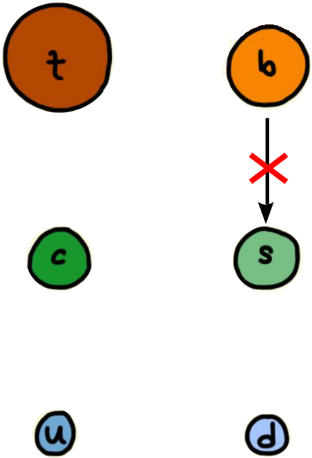
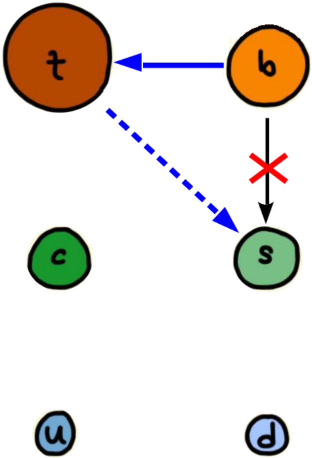
$$(\text{e.g. } B \to \pi\mu\nu)$$



- absent in the SM at tree level (GIM mechanism)

Flavor Changing Neutral Current Decays



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$$A(b \to s) \sim \frac{1}{16\pi^2} V_{ts}^* V_{tb} \sim 2.5 \times 10^{-4}$$

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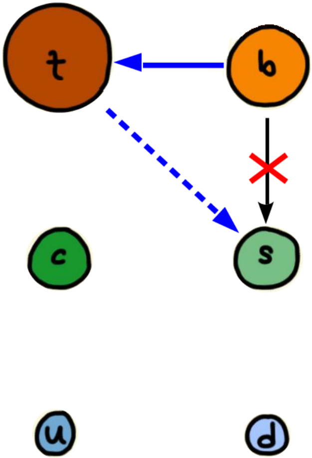

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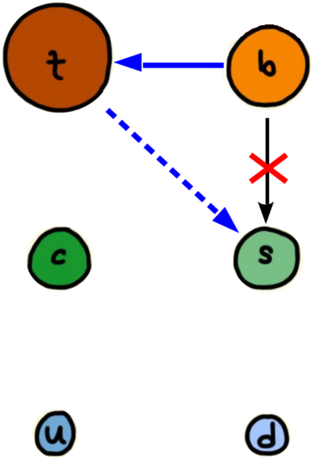

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"rare decays"

Classification of Charged Current Decays

- Semi-leptonic decay modes
  (both charged and neutral B mesons)

  exclusive: e.g. $B \to D\tau\nu$, $B \to D^*\mu\nu$, $B \to \pi e\nu$ ...
  inclusive: e.g. $B \to X_c\tau\nu$, $B \to X_c\mu\nu$, $B \to X_u e\nu$ ...

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- Purely hadronic decay modes
  (both charged and neutral B mesons)

  hundreds of possible final states

Classification ... ys

- ▶ Semi-leptonic
  (both charged
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- ▶ Purely leptonic
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Γ_{99}	$[K^+\pi^-]_D K^+\pi^-\pi^+$		
Γ_{100}	$[K^-\pi^+]_D K^+\pi^-\pi^+$		
Γ_{101}	$D_{CP(+1)} K^+\pi^-\pi^+$		
Γ_{102}	$\overline{D}^0 K^+ \overline{K}^0$		$(5.5 \pm 1.6) \times 10^{-4}$
Γ_{103}	$\overline{D}^0 K^+ \overline{K}^*(892)^0$		$(7.5 \pm 1.7) \times 10^{-4}$
Γ_{104}	$\overline{D}^0 \pi^+\pi^+\pi^-$		$(5.6 \pm 2.1) \times 10^{-3}$
Γ_{105}	$[K^-\pi^+]_D \pi^+\pi^-\pi^+$		
Γ_{106}	$\overline{D}^0 \pi^+\pi^+\pi^-$ nonresonant		$(5 \pm 4) \times 10^{-3}$
Γ_{107}	$\overline{D}^0 \pi^+ \rho^0$		$(4.2 \pm 3.0) \times 10^{-3}$
Γ_{108}	$\overline{D}^0 a_1(1260)^+$		$(4 \pm 4) \times 10^{-3}$
Γ_{109}	$\overline{D}^0 \omega\pi^+$		$(4.1 \pm 0.9) \times 10^{-3}$
Γ_{110}	$D^*(2010)^- \pi^+\pi^+$		$(1.35 \pm 0.22) \times 10^{-3}$
Γ_{111}	$D^*(2010)^- K^+\pi^+$		$(8.2 \pm 1.4) \times 10^{-5}$
Γ_{112}	$\overline{D}_1(2420)^0 \pi^+$, $\overline{D}_1^0 \to D^*(2010)^- \pi^+$		$(5.2 \pm 2.2) \times 10^{-4}$
Γ_{113}	$D^- \pi^+\pi^+$		$(1.07 \pm 0.05) \times 10^{-3}$
Γ_{114}	$D^- K^+\pi^+$		$(7.7 \pm 0.5) \times 10^{-5}$
Γ_{115}	$D_0^*(2300)^0 K^+$, $D_0^{*0} \to D^-\pi^+$		$(6.1 \pm 2.4) \times 10^{-6}$
Γ_{116}	$D_2^*(2460)^0 K^+$, $D_2^{*0} \to D^-\pi^+$		$(2.32 \pm 0.23) \times 10^{-5}$
Γ_{117}	$D_1^*(2760)^0 K^+$, $D_1^{*0} \to D^-\pi^+$		$(3.6 \pm 1.2) \times 10^{-6}$
Γ_{118}	$D^+ K^0$		$< 2.9 \times 10^{-6}$
Γ_{119}	$D^+ K^+ \pi^-$		$(5.6 \pm 1.1) \times 10^{-6}$
Γ_{120}	$D_2^*(2460)^0 K^+$, $D_2^{*0} \to D^+\pi^-$		$< 6.3 \times 10^{-7}$
Γ_{121}	$D^+ K^{*0}$		$< 4.9 \times 10^{-7}$
Γ_{122}	$D^+ \overline{K}^{*0}$		$< 1.4 \times 10^{-6}$
Γ_{123}	$\overline{D}^*(2007)^0 \pi^+$		$(4.90 \pm 0.17) \times 10^{-3}$
Γ_{124}	$\overline{D}^{*0}_{CP(+1)} \pi^+$	[4]	$(2.7 \pm 0.6) \times 10^{-3}$
Γ_{125}	$D^{*0}_{CP(-1)} \pi^+$	[4]	$(2.4 \pm 0.9) \times 10^{-3}$
Γ_{126}	$\overline{D}^*(2007)^0 \omega\pi^+$		$(4.5 \pm 1.2) \times 10^{-3}$
Γ_{127}	$\overline{D}^*(2007)^0 \rho^+$		$(9.8 \pm 1.7) \times 10^{-3}$
Γ_{128}	$\overline{D}^*(2007)^0 K^+$		$(3.97^{+0.31}_{-0.28}) \times 10^{-4}$
Γ_{129}	$\overline{D}^{*0}_{CP(+1)} K^+$	[4]	$(2.60 \pm 0.33) \times 10^{-4}$
Γ_{130}	$\overline{D}^{*0}_{CP(-1)} K^+$	[4]	$(2.19 \pm 0.30) \times 10^{-4}$
Γ_{131}	$D^*(2007)^0 K^+$		$(7.8 \pm 2.2) \times 10^{-6}$
Γ_{132}	$\overline{D}^*(2007)^0 K^*(892)^+$		$(8.1 \pm 1.4) \times 10^{-4}$
Γ_{133}	$\overline{D}^*(2007)^0 K^+ \overline{K}^0$		$< 1.06 \times 10^{-3}$
Γ_{134}	$\overline{D}^*(2007)^0 K^+ \overline{K}^*(892)^0$		$(1.5 \pm 0.4) \times 10^{-3}$
Γ_{135}	$\overline{D}^*(2007)^0 \pi^+\pi^+\pi^-$		$(1.03 \pm 0.12)\%$
Γ_{136}	$\overline{D}^*(2007)^0 a_1(1260)^+$		$(1.9 \pm 0.5)\%$
Γ_{137}	$\overline{D}^*(2007)^0 \pi^-\pi^+\pi^+\pi^0$		$(1.8 \pm 0.4)\%$
Γ_{138}	$\overline{D}^{*0}\, 3\,\pi^+ 2\,\pi^-$		$(5.7 \pm 1.2) \times 10^{-3}$
Γ_{139}	$D^*(2010)^+ \pi^0$		$< 3.6 \times 10^{-6}$
Γ_{140}	$D^*(2010)^+ K^0$		$< 9.0 \times 10^{-6}$
Γ_{141}	$D^*(2010)^- \pi^+\pi^+\pi^0$		$(1.5 \pm 0.7)\%$
Γ_{142}	$D^*(2010)^- \pi^+\pi^+\pi^+\pi^-$		$(2.6 \pm 0.4) \times 10^{-3}$

Classification of FCNC Decays (Rare Decays)

- Radiative decay modes
  (both charged and neutral B mesons)

  exclusive: e.g. $B \to K^* \gamma$, $B \to \rho\gamma$, ...
  inclusive: e.g. $B \to X_s \gamma$, ...

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- Purely leptonic decay modes
  (only neutral B mesons)

  e.g. $B_s \to \mu^+ \mu^-$, $B_d \to \tau^+ \tau^-$, ...

Application (0th order picture)

- Charged Current Decays:

  determination of CKM matrix elements

- Rare Decays:

  search for new physics

Application (0th order picture)

- Charged Current Decays:

  determination of CKM matrix elements

  (but can also be used to probe new physics, if the new physics is "strong" enough to compete with tree level W exchange)

- Rare Decays:

  search for new physics

  (but can also be used to determine CKM parameters, if one assumes that the decays are free of new physics)

Can’t discuss all the decay modes.

Will focus on a few examples in more detail:

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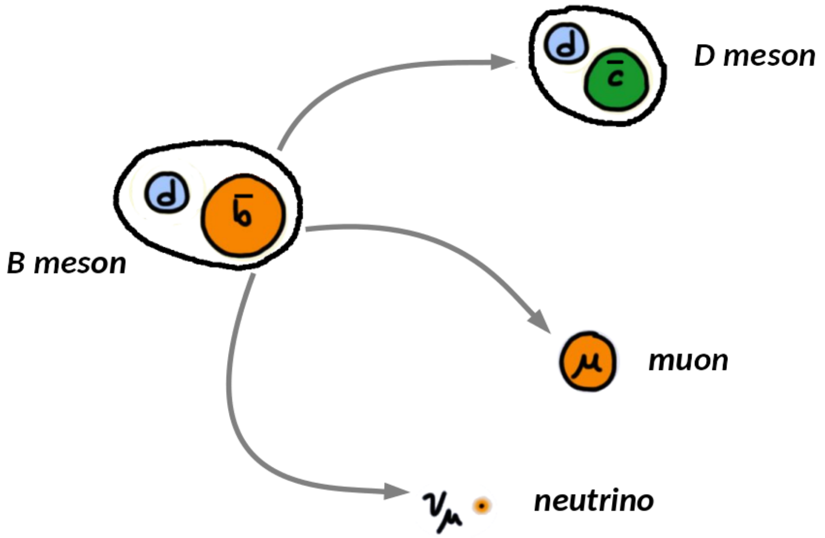
The $B \to D^{(*)}\ell\nu$ Decays



Effective Hamiltonian for $B \to D^{(*)}\ell\nu$ in the SM

- Charged current decays
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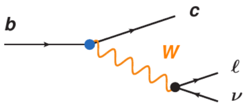

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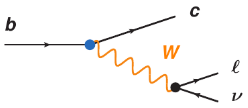

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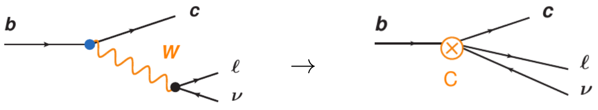

- characteristic energy scale of B decays: $\mathcal{O}(m_B)$
- characteristic energy scale of weak interactions: $\mathcal{O}(m_W) \gg \mathcal{O}(m_B)$
- decays can be described by an effective Hamiltonian ("integrate out the W boson")

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb}\, C\, (\bar{c}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu P_L \nu_\ell)$$

Wilson coefficient

4-fermion contact interaction

Hadronic Matrix Elements

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Parameterization in terms of form factors

$$\left\langle D\right| \bar{c}\gamma^\mu b \left|\bar{B}\right\rangle \equiv f_+(q^2)(p_B + p_D)^\mu + [f_0(q^2) - f_+(q^2)]\, \frac{m_B^2 - m_D^2}{q^2}\, q^\mu$$

$$\left\langle D^*\right| \bar{c}\gamma^\mu b \left|\bar{B}\right\rangle \equiv -ig(q^2)\, \epsilon^{\mu\nu\rho\sigma}\, \varepsilon^*_\nu\, (p_B + p_{D^*})_\rho\, q_\sigma\ ,$$

$$\left\langle D^*\right| \bar{c}\gamma^\mu\gamma^5 b \left|\bar{B}\right\rangle \equiv \varepsilon^{*\mu} f(q^2) + a_+(q^2)\, \varepsilon^* \cdot p_B\, (p_B + p_{D^*})^\mu + a_-(q^2)\, \varepsilon^* \cdot p_B\, q^\mu$$

$$\frac{\mathrm{d}\Gamma(\overline{B} \to D l \bar{\nu})}{\mathrm{d}w} = \frac{G_F^2 |V_{cb}|^2 \, \eta_{\mathrm{EW}}^2 \, m_B^5}{48\pi^3} \, (w^2 - 1)^{3/2} \, r_D^3 \, (1 + r_D)^2 \, \mathcal{G}(w)^2 \, ,$$

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if $\mathcal{G},\mathcal{F}$ are known, can use experimental data on the decay rates to determine the CKM element V_{cb}

Parameterization of the Form Factors

Boyd, Grinstein, Lebed hep-ph/9412324; Caprini, Lellouch, Neubert hep-ph/9712417; ...
... Flynn, Juttner, Tsang 2303.11285; Gubernari, Reboud, van Dyk, Virto 2305.06301

- One would like to work with a robust parameterization of the q^2 dependence of the form factors
- Use a conformal mapping to the variable z, and use analytic properties of the form factors to express them in a power series in z with coefficients bounded by unitarity

$$z = \frac{\sqrt{1+\omega} - \sqrt{2}}{\sqrt{1+\omega} + \sqrt{2}} , \quad f(z) = \frac{1}{P(z)\phi(z)} \sum_n a_n z^n , \quad \sum_n |a_n|^2 \leq 1$$

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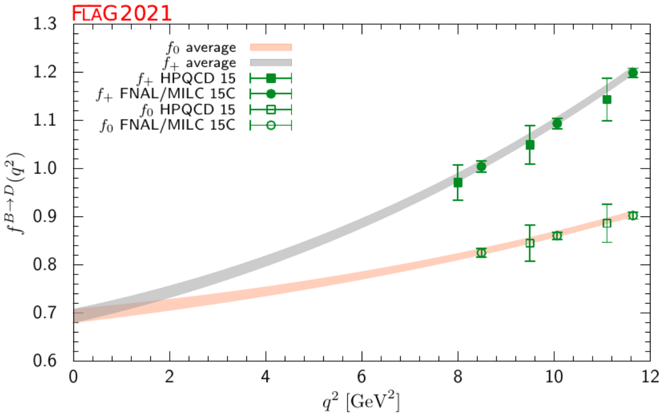
- For $B \to D$ the physical region corresponds to $0 < z \lesssim 0.064$.
- $P(z)$ = Blaschke factor that takes into account poles.
- $\phi(z)$ = outer function ensures unitarity bounds take a simple form.

(can also use HQET to constrain the form factor shapes)

Lattice Determination of the Form Factors



percent level uncertainty from lattice form factors translates into percent level uncertainty on V_{cb}

Lepton Flavor Universality Ratios

Take ratios of branching ratios with different leptons in the final state

$$R_{D^{(*)}} = \frac{BR(B \to D^{(*)} \tau \nu)}{BR(B \to D^{(*)} \ell \nu)}$$

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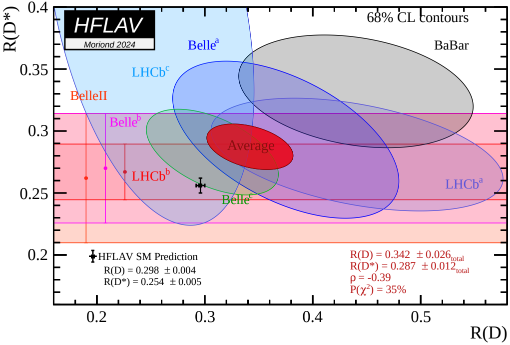
- LFU ratios do not depend on the CKM matrix elements
- Have reduced dependence on form factors
- can be predicted in the SM with high precision

$$R_D^{\rm SM} = 0.298 \pm 0.004 \quad , \qquad R_{D^*}^{\rm SM} = 0.254 \pm 0.005$$

[values adopted by HFLAV, based on many theory papers ...]

world average from the heavy flavor averaging group



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$\ell = \mu, e$ (BaBar/Belle)

$\ell = \mu$ (LHCb)

$$R_D^{\text{exp}} = 0.342 \pm 0.026 \ , \qquad R_{D^*}^{\text{exp}} = 0.287 \pm 0.012$$

combined discrepancy with the SM of 3.3σ

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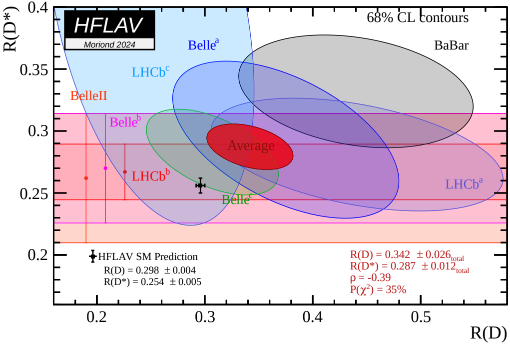

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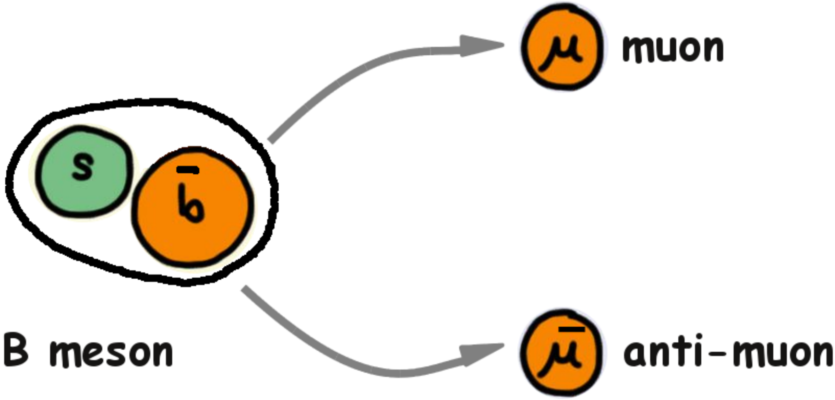
Hint for new physics? Belle II will clear this up soon.

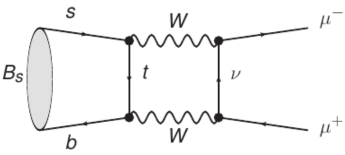
$B_s \to \mu^+\mu^-$

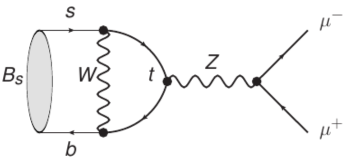
The $B_s \to \mu^+\mu^-$ Decay



SM Contribution

- Flavor changing neutral current process
- induced by Boxes and Z penguins





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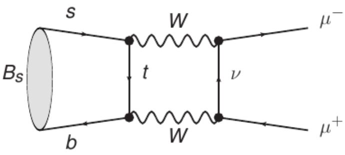
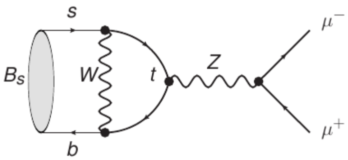


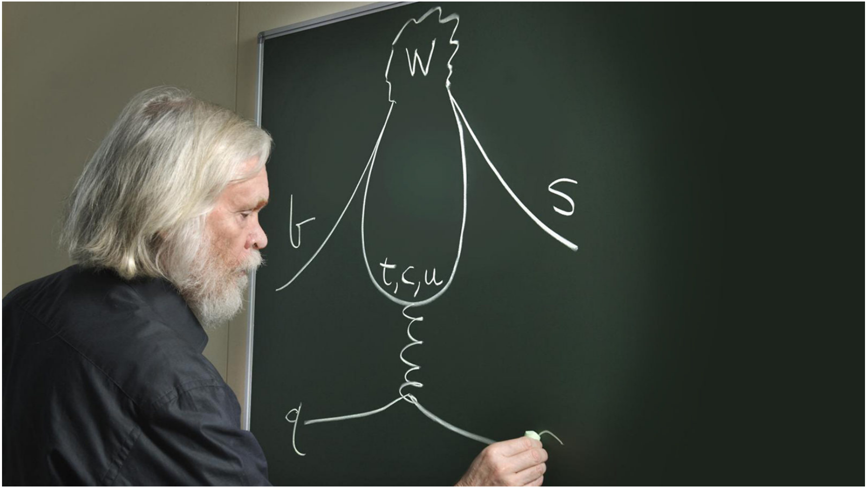


- helicity suppressed decay (similar to pion decay):

  B meson is spin 0, muons spin 1/2
  $\rightarrow$ one muon has to be left-handed, other one right-handed

  electroweak interactions only give muons of the same handedness
  $\rightarrow$ branching ratio is helicity suppressed by m_μ^2/m_B^2

Penguin Diagrams



https://www.symmetrymagazine.org/article/june-2013/the-march-of-the-penguin-diagrams

Effective Hamiltonian for $B_s \to \mu^+\mu^-$ in the SM

- Integrate out the top, W, Z, ... to arrive at an effective Hamiltonian that describes the $B_s \to \mu^+\mu^-$ decay

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \; C_{10} (\bar{s}\gamma_\alpha P_L b)(\bar{\mu}\gamma^\alpha \gamma_5 \mu)$$

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- s_W is the sine of the weak mixing angle
- Y_0 and Y_1 are loop functions that depend on $x_t = m_t^2 / m_W^2$
- known at NNLO in QCD and NLO in the electroweak interactions

The Hadronic Matrix Element

$$\langle \mu^+ \mu^- | \mathcal{H}_{\text{eff}} | B_s \rangle = -\frac{4 G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \, C_{10} \langle \mu^+ \mu^- | (\bar{\mu} \gamma^\alpha \gamma_5 \mu) | 0 \rangle \langle 0 | (\bar{s} \gamma_\alpha P_L b) | B_s \rangle$$

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- Hadronic matrix element is given by the B_s meson decay constant

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- Hadronic matrix element is given by the B_s meson decay constant

$$\langle 0 | (\bar s \gamma^\alpha b) | B_s \rangle = 0$$

$$\langle 0 | (\bar s \gamma^\alpha \gamma_5 b) | B_s \rangle = i f_{B_s} p_{B_s}^\alpha$$

- decay constants can be determined on the lattice

$$f_{B_s} = (230.3 \pm 1.3)\text{MeV} \quad , \qquad f_{B_d} = (190.0 \pm 1.3)\text{MeV} \quad \text{(FLAG)}$$

sub-percent precision!

Branching Ratio Prediction

$$BR(B_s \to \mu^+\mu^-)_{SM} = \tau_{B_s} \frac{G_F^2}{\pi} m_{B_s} f_{B_s}^2 m_\mu^2 \frac{\alpha^2}{16\pi^2} |V_{ts}^* V_{tb}|^2 \frac{Y(x_t)^2}{s_W^4} \frac{1}{1-y_s}$$

- decay constant: $f_{B_s}^2$
- Effect of lifetime difference of Bs and Bs-bar: $\frac{1}{1-y_s}$
- helicity suppression: m_μ^2
- loop suppression: $\frac{\alpha^2}{16\pi^2}$
- CKM suppression: $|V_{ts}^* V_{tb}|^2$

Branching Ratio Prediction

decay constant

Effect of lifetime difference of Bs and Bs-bar

$$BR(B_s \to \mu^+\mu^-)_{\rm SM} = \tau_{B_s} \frac{G_F^2}{\pi} m_{B_s} f_{B_s}^2 m_\mu^2 \frac{\alpha^2}{16\pi^2} |V_{ts}^* V_{tb}|^2 \frac{Y(x_t)^2}{s_W^4} \frac{1}{1-y_s}$$

helicity suppression

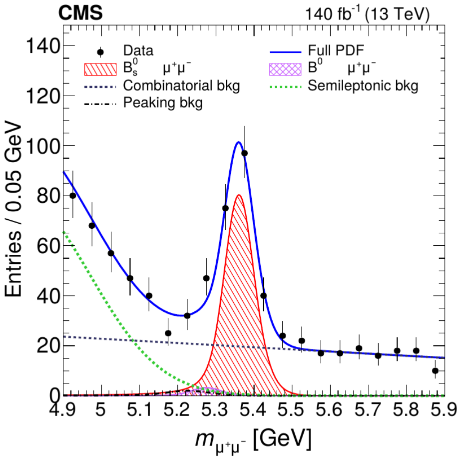
loop suppression

CKM suppression

$$BR(B_s \to \mu^+\mu^-)_{\rm SM} = (3.46 \pm 0.24) \times 10^{-9}$$ (using my preferred CKM input)

a truly rare decay!

Experimental status of $B_s \to \mu^+\mu^-$



$$BR(B_s \to \mu^+\mu^-)_{\text{exp}} = (3.34 \pm 0.27) \times 10^{-9}$$ (PDG average of ATLAS, CMS, LHCb)

In good agreement with SM prediction.

$b \to s\nu\bar{\nu}$

The $b \to s\nu\bar{\nu}$ Decays

- There are various hadronic versions of the decay

$$B \to K\nu\bar{\nu}\ , \quad B \to K^*\nu\bar{\nu}\ , \quad B_s \to \phi\nu\bar{\nu}\ , \quad \Lambda_b \to \Lambda\nu\bar{\nu}$$

- Similar story as we have seen before: integrate out W,Z,t and match onto an effective Hamiltonian. One finds a single operator in the Standard Model

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}\frac{\alpha}{4\pi}V_{ts}^* V_{tb} C_L (\bar{s}\gamma^\mu P_L b)(\bar{\nu}\gamma_\mu(1-\gamma_5)\nu)$$

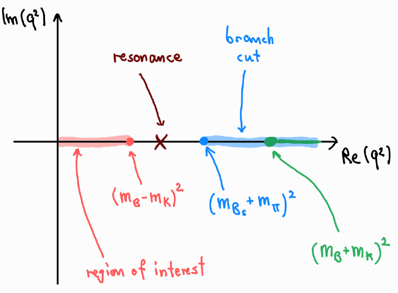
- Wilson coefficient is known at NNLO in QCD and NLO electro-weak

(Brod, Gorbahn, Stamou, 1009.0947, 2105.02868)

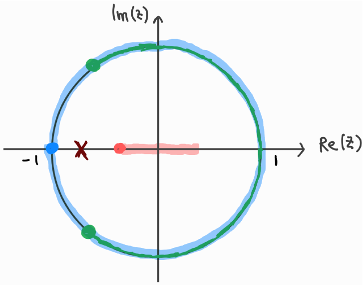
$$C_L^{\text{SM}} = -6.322 \pm 0.031\Big|_{m_t} \pm 0.074\Big|_{\text{QCD}} \pm 0.009\Big|_{\text{EW}}$$

$B \to K$ Form Factors

Form factors are parameterized similarly to $B \to D$:
polynomials in z with coefficients bounded by unitarity



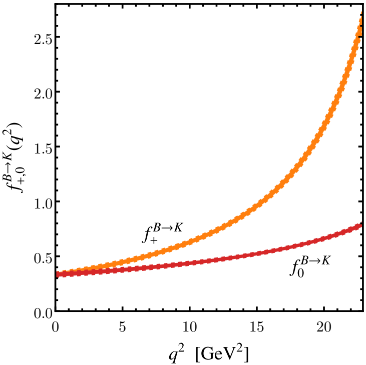
$\Rightarrow$

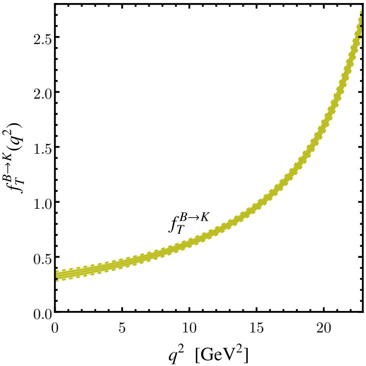


$$\mathcal{F}(q^2) = \frac{1}{\mathcal{B}_\mathcal{F}(z)\phi_\mathcal{F}(z)} \sum_k \alpha_k^\mathcal{F} p_k^\mathcal{F}(z) \ , \quad \sum_{\mathcal{F},k} |\alpha_k^\mathcal{F}|^2 < 1$$

$B \to K$ Form Factors from the Lattice

- Astonishing precision is achieved on the lattice
- Plots show 2σ error bands!





[plots based on HPQCD 2207.12468, Fermilab/MILC 1509.06235,
Gubernari, Reboud, van Dyk, Virto 2305.06301]

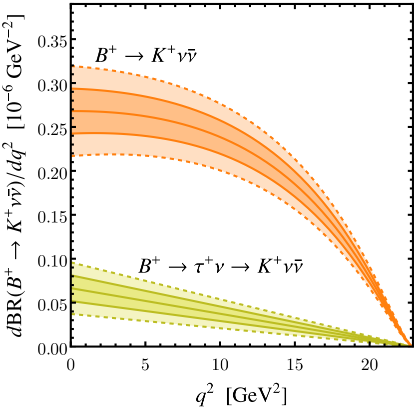
Standard Model Prediction for $B \to K\nu\bar{\nu}$

- SM branching ratio predicted with $\sim$ 8% precision

$$\text{BR}(B^+ \to K^+\nu\bar{\nu}) =$$

$$= (4.46 \pm 0.36) \times 10^{-6}$$

- For the charged B decays need also to take into account a "long-distance" contribution from $B^+ \to \tau^+\nu \to K^+\nu\bar{\nu}$



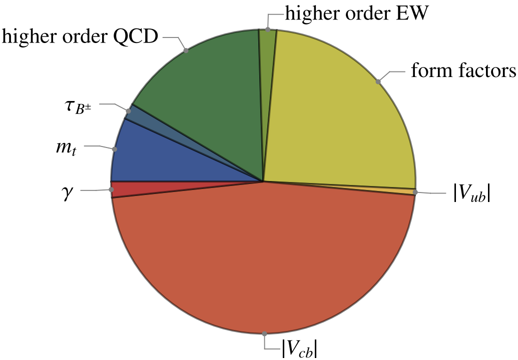
[work in progress with Gadam and Toner]

Error Budget

$\text{BR}(B^+ \to K^+ \nu\bar{\nu}) =$

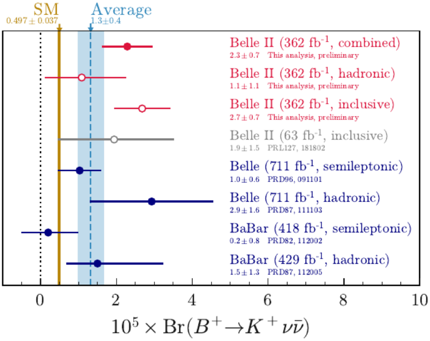
$= (4.46 \pm 0.36) \times 10^{-6}$

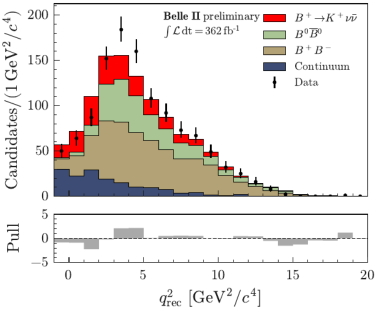
- Uncertainty is dominated by CKM input
- Uncertainties for $B \to K^* \nu\bar{\nu}$ and $B_s \to \phi\nu\bar{\nu}$ somewhat higher because of less precise form factors



[work in progress with Gadam and Toner]

Belle II 2311.14647



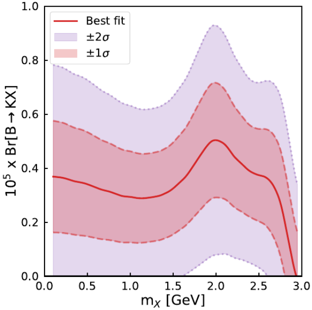


- Evidence for $B \to K\nu\bar{\nu}$ at 3.5σ above background and 2.7σ above the SM prediction.
- Excess of events is particularly pronounced around $q^2 \simeq 4$ GeV^2.

A Hint for Light New Physics?

- Instead of fitting the excess with a continuous 3-body spectrum from $B \to K\nu\bar{\nu}$ one gets a better fit with a new resonance $B \to KX$

WA, Crivellin, Haigh, Inguglia, Martin Camalich 2311.14629



A Hint for Light New Physics?

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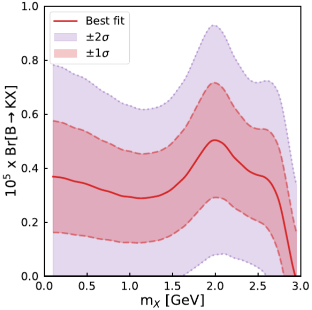
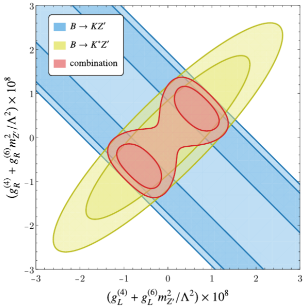




- Could be for example a Z' or ALP with mass around 2 GeV
- Constraints from $B \to K^*\nu\bar{\nu}$ narrow down couplings

see also Bause et al. 2309.00075; Allwicher et al. 2309.02246; Felkl et al. 2309.02940; McKeen et al. 2312.00982; Fridell et al. 2312.12507; Ho et al. 2401.10112; Gabrielli et al. 2402.05901; Hou et al 2402.19208; Bolton et al. 2403.13887; He et al 2403.12485; Marzocca et al 2404.06533; Eguren et al 2405.00108; Buras et al. 2405.06742; ...

SM Prediction for $\Lambda_b \to \Lambda \nu \bar{\nu}$

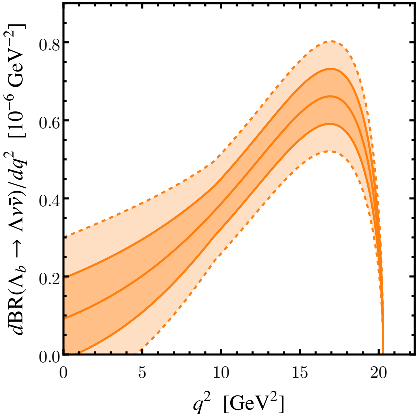
- SM branching ratio predicted with $\sim 15\%$ precision

$$\text{BR}(\Lambda_b \to \Lambda \nu \bar{\nu}) =$$

$$= (7.71 \pm 1.06) \times 10^{-6}$$

- Need FCC-ee/CEPC in Z-factory mode to access this decay experimentally

Amhis et al. 2309.11353

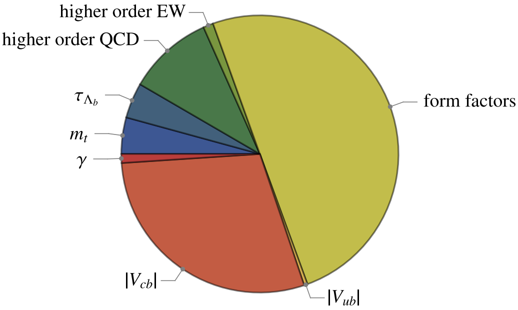


[work in progress with Gadam and Toner]

Error Budget

$$\text{BR}(\Lambda_b \to \Lambda \nu \bar{\nu}) =$$

$$= (7.71 \pm 1.06) \times 10^{-6}$$



[work in progress with Gadam and Toner]

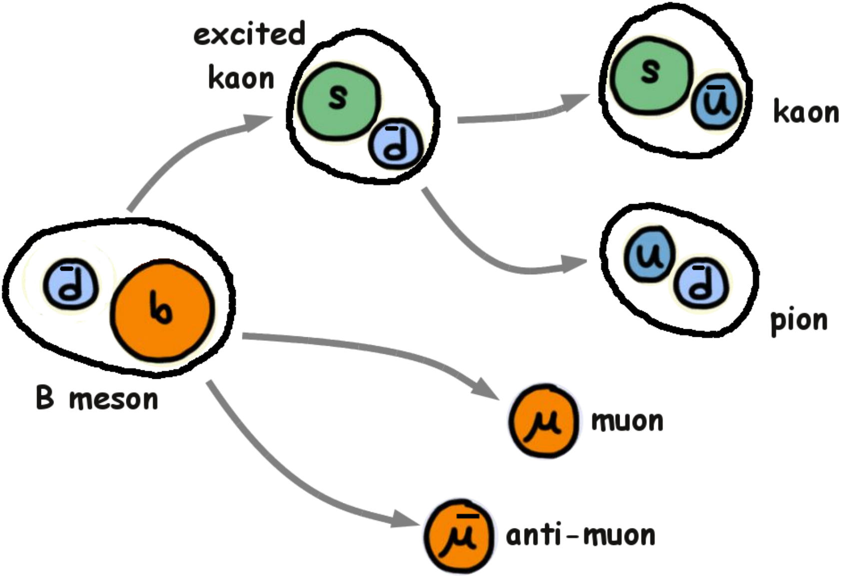
- Lattice calculations of $\Lambda_b \to \Lambda$ form factors are less established and currently have larger uncertainties

  Detmold, Meinel 1602.01399;

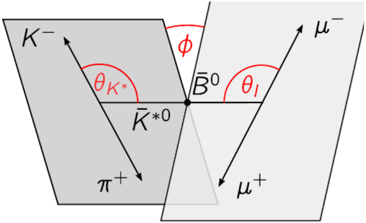
  Blake et al. 2205.06041

$B \to K^* \ell^+ \ell^-$ and $R_{K^{(*)}}$

The $B \to K^*(\to K\pi)\mu^+\mu^-$ Decay



The $B \to K^*(\to K\pi)\mu^+\mu^-$ Decay

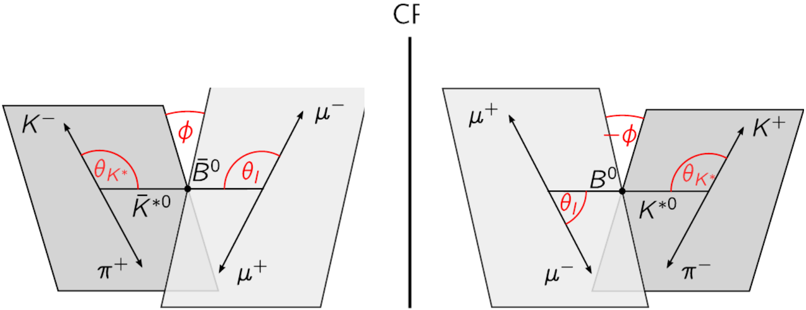


- kinematics described by 4 variables

  invariant mass squared of the two muons: q^2
  three angles: $0 < \theta_{K^*} < \pi$, $0 < \theta_\ell < \pi$, $-\pi < \phi < \pi$

  $\to$ many observables accessible from the angular distribution

The $B \to K^*(\to K\pi)\mu^+\mu^-$ Decay

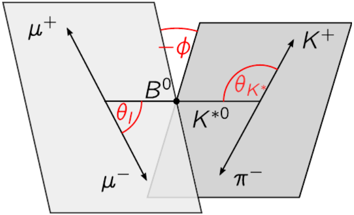


- self tagging:

  $K^+\pi^-$ final state for B^0
  $K^-\pi^+$ final state for $\bar{B}^0$

  $\to$ in principle easy access to CP asymmetries

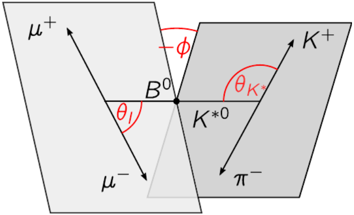
The $B \to K^* \mu^+ \mu^-$ Angular Decay Distribution



$$\frac{d^4\bar{\Gamma}}{dq^2\, d\cos\theta_\ell\, d\cos\theta_{K^*}\, d\phi} =$$

$$= \frac{9}{32\pi} \bar{I}(q^2, \theta_\ell, \theta_{K^*}, \phi)$$

The $B \to K^* \mu^+ \mu^-$ Angular Decay Distribution



$$\frac{d^4\bar{\Gamma}}{dq^2\, d\cos\theta_\ell\, d\cos\theta_{K^*}\, d\phi} = \\ = \frac{9}{32\pi} \bar{I}(q^2, \theta_\ell, \theta_{K^*}, \phi)$$

$$\begin{aligned}\bar{I}(q^2, \theta_\ell, \theta_{K^*}, \phi) = \\ = \bar{I}_1^s \sin^2\theta_{K^*} + \bar{I}_1^c \cos^2\theta_{K^*} + (\bar{I}_2^s \sin^2\theta_{K^*} + \bar{I}_2^c \cos^2\theta_{K^*}) \cos 2\theta_\ell \\ + \bar{I}_3 \sin^2\theta_{K^*} \sin^2\theta_\ell \cos 2\phi + \bar{I}_4 \sin 2\theta_{K^*} \sin 2\theta_\ell \cos\phi \\ - \bar{I}_5 \sin 2\theta_{K^*} \sin\theta_\ell \cos\phi \\ - (\bar{I}_6^s \sin^2\theta_{K^*} + \bar{I}_6^c \cos^2\theta_{K^*}) \cos\theta_\ell + \bar{I}_7 \sin 2\theta_{K^*} \sin\theta_\ell \sin\phi \\ - \bar{I}_8 \sin 2\theta_{K^*} \sin 2\theta_\ell \sin\phi - \bar{I}_9 \sin^2\theta_{K^*} \sin^2\theta_\ell \sin 2\phi\end{aligned}$$

The I's are moments of the angular distribution.

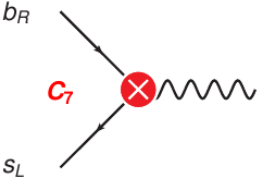
Effective Hamiltonian for $B \to K^* \ell^+ \ell^-$ in the SM

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_{i=7,9,10} C_i \mathcal{O}_i + \ldots$$

Effective Hamiltonian for $B \to K^* \ell^+ \ell^-$ in the SM

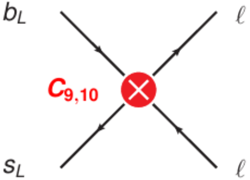
$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_{i=7,9,10} C_i \mathcal{O}_i \ + \ \ldots$$

magnetic dipole operators



$C_7 (\bar{s} \sigma_{\mu\nu} P_R b) F^{\mu\nu}$

semileptonic operators



$C_9 (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu \ell)$

$C_{10} (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu \gamma_5 \ell)$

$B \to K^*$ Form Factors

Hadronic matrix elements are parameterized in terms of form factors

$$\langle \bar{K}^*(k)|\bar{s}\gamma_\mu(1-\gamma_5)b|\bar{B}(p)\rangle = -i\epsilon^*_\mu(m_B+m_{K^*})A_1(q^2) + i(2p-q)_\mu(\epsilon^*\cdot q)\,\frac{A_2(q^2)}{m_B+m_{K^*}}$$
$$+\,iq_\mu(\epsilon^*\cdot q)\,\frac{2m_{K^*}}{q^2}\,\left[A_3(q^2)-A_0(q^2)\right] + \epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho k^\sigma\,\frac{2V(q^2)}{m_B+m_{K^*}}$$

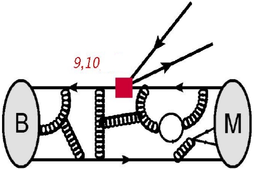
$$\langle \bar{K}^*(k)|\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|\bar{B}(p)\rangle = i\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho k^\sigma\,2T_1(q^2)$$
$$+\,T_2(q^2)\left[\epsilon^*_\mu(m_B^2-m_{K^*}^2)-(\epsilon^*\cdot q)\,(2p-q)_\mu\right] + T_3(q^2)(\epsilon^*\cdot q)\left[q_\mu - \frac{q^2}{m_B^2-m_{K^*}^2}\,(2p-q)_\mu\right]$$

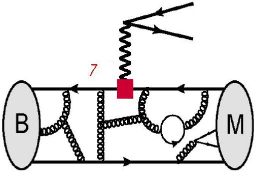
Predictions exist from lattice QCD and
other non-perturbative methods (light cone sum rules)

most recent fit to a z-parameterization by Gubernari, Reboud, van Dyk, Virto 2305.06301

Non-Local Effects

So far we discussed the local contributions

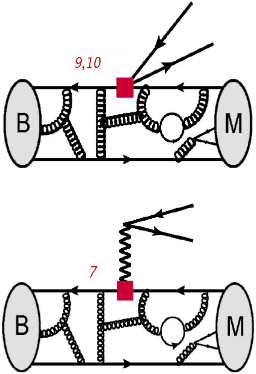


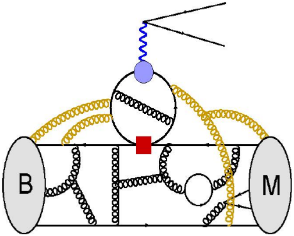


(illustrations by Danny van Dyk)

So far we discussed the local contributions

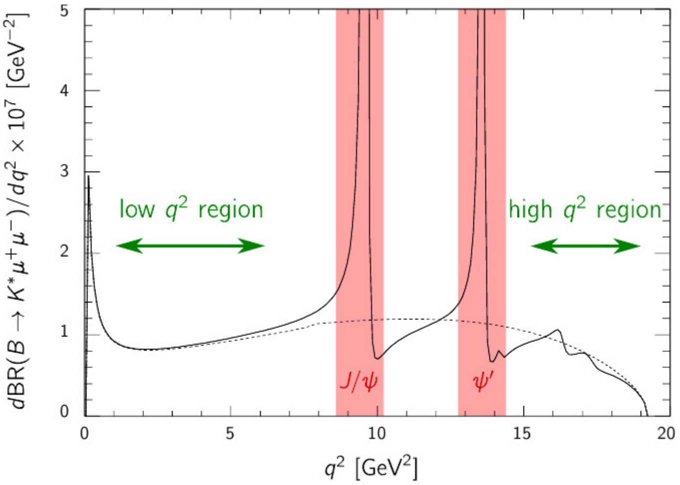
there are also non-local effects coming from 4-quark operators; often referred to a “charm loop” effects.



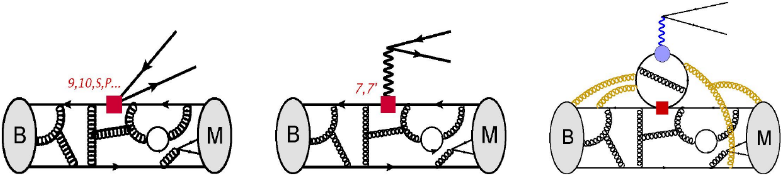


(illustrations by Danny van Dyk)

The q^2 Spectrum



$b \to s\ell\ell$ Amplitudes

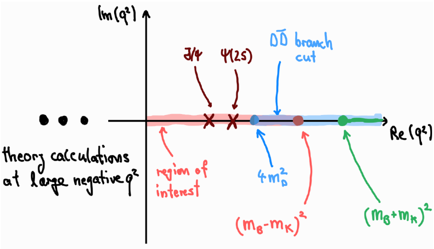


$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\} + \mathcal{O}(\alpha^2)$$

- Local (Form Factors) : $\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) | \, \bar{s}\, \Gamma_\lambda^{(T)}\, b \, | \bar{B}(k+q) \rangle$
- Non-Local : $\mathcal{H}_\lambda(q^2) = i \mathcal{P}_\mu^\lambda \int d^4x \, e^{iq\cdot x} \langle \bar{M}_\lambda(k) | T\{ j^\mu_{\rm em}(x), \mathcal{C}_i \mathcal{O}_i(0)\} | \bar{B}(q+k) \rangle$

(talk by Javier Virto at Flavour@TH workshop, CERN May 11, 2023)

Parameterization of the Charm Loop



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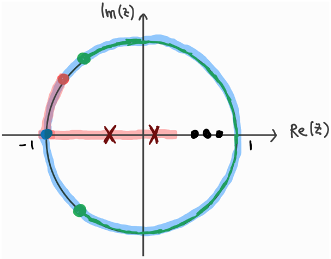


- Proposed parameterization analogous to the local form factors.
- Works for q^2 below the $D\bar{D}$ branch cut.

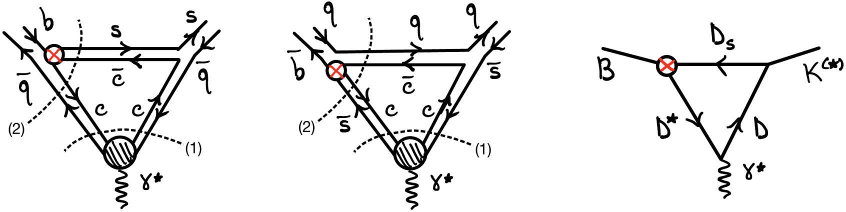
Bobeth, Chrzaszcz, van Dyk, Virto 1707.07305; Gubernari, van Dyk, Virto 2011.09813; Gubernari, Reboud, van Dyk, Virto 2206.03797

$$\mathcal{H}(q^2) = \frac{1}{\mathcal{B}_\mathcal{H}(z)\phi_\mathcal{H}(z)} \sum_k \beta_k^\mathcal{H} p_k^\mathcal{H}(z) \ , \quad \sum_{\mathcal{H},k} |\beta_k^\mathcal{H}|^2 < 1$$

Additional Charm Loop Effects?

- The charm loop also gives “triangle diagrams” involving e.g. intermediate $D_s \bar{D}$ states

  Ciuchini, Fedele, Franco, Paul, Silvestrini, Valli 2212.10516



- E.g. decay $B \to D_s D^*$ followed by rescattering $D_s D^* \to K^{(*)}\gamma^*$

- This gives anomalous thresholds that distort the analytic structure

  (Mutke, Hoferichter, Kubis 2406.14608)

- How disruptive is this to the proposed parameterization?

Lepton Flavor Universality Ratios

- Hadronic uncertainties drop out almost entirely in lepton flavor universality ratios

$$R_{K^*} = \frac{BR(B \to K^* \mu^+ \mu^-)}{BR(B \to K^* e^+ e^-)}$$

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$$R_{K^*} = \frac{BR(B \to K^* \mu^+ \mu^-)}{BR(B \to K^* e^+ e^-)}$$

- Analogously for the $B \to K \ell^+ \ell^-$ decays

$$R_K = \frac{BR(B \to K \mu^+ \mu^-)}{BR(B \to K e^+ e^-)}$$

Lepton Flavor Universality Ratios

- Hadronic uncertainties drop out almost entirely in lepton flavor universality ratios

$$R_{K^*} = \frac{BR(B \to K^* \mu^+ \mu^-)}{BR(B \to K^* e^+ e^-)}$$

- Analogously for the $B \to K \ell^+ \ell^-$ decays

$$R_K = \frac{BR(B \to K \mu^+ \mu^-)}{BR(B \to K e^+ e^-)}$$

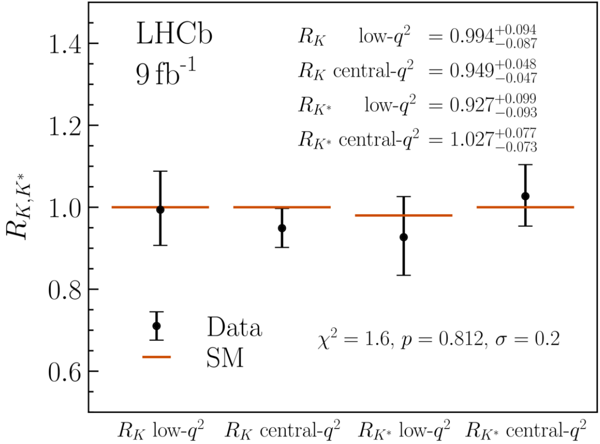
- Standard Model Predictions Bordone, Isidori, Pattori 1605.07633

$$R_K^{[1,6]} = 1.00 \pm 0.01 \ , \quad R_{K^*}^{[1.1,6]} = 1.00 \pm 0.01 \ , \quad R_{K^*}^{[0.045,1.1]} = 0.91 \pm 0.03$$

(The numbers in square brackets indicate the q^2 region)

Lepton Flavor Universality Tests in $b \to s\ell\ell$

LHCb 2212.09152, 2212.09153

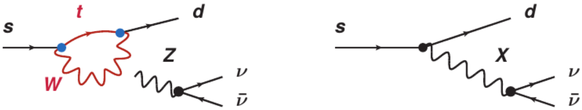


R_K and R_{K^*} are consistent with SM expectations at the $\sim 5\%$ level

Kaon and Pion Decays

Probing New Physics with Rare Kaon Decays

	Standard Model	generic New Physics
$s \to d$	$\sim \frac{1}{16\pi^2} \frac{G_F}{\sqrt{2}} V_{ts} V_{td}^* \simeq \frac{1}{(250\text{ TeV})^2}$	$\sim \frac{1}{M_X^2}$
$b \to d$	$\sim \frac{1}{16\pi^2} \frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \simeq \frac{1}{(50\text{ TeV})^2}$	$\sim \frac{1}{M_X^2}$
$b \to s$	$\sim \frac{1}{16\pi^2} \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \simeq \frac{1}{(20\text{ TeV})^2}$	$\sim \frac{1}{M_X^2}$



"the rarer the better"

$K \to \pi\nu\bar{\nu}$

- The $K \to \pi\nu\bar{\nu}$ decays are among the theoretically cleanest flavor changing neutral current processes.

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- Relevant hadronic matrix element can be extracted from data.

$$\text{BR}(K \to \pi\nu\bar{\nu}) = \frac{\text{BR}(K \to \pi\nu\bar{\nu})}{\text{BR}(K \to \pi\ell\nu)} \times \text{BR}(K \to \pi\ell\nu)$$

Want to predict this

Can be calculated with high precisions

Can be measured with high precisions

- Hadronic matrix element drops out in the ratio, up to iso-spin and QED corrections which are under good control. (Mescia, Smith 0705.2025)

$K^+ \to \pi^+\nu\bar{\nu}$ in the SM

Brod, Gorbahn, Stamou 2105.02868

$$\text{Br}\left(K^+ \to \pi^+\nu\bar{\nu}(\gamma)\right) = \kappa_+(1+\Delta_{\text{EM}})\left[\left(\frac{\text{Im}\lambda_t}{\lambda^5}X_t\right)^2 + \left(\frac{\text{Re}\lambda_c}{\lambda}\left(P_c + \delta P_{c,u}\right) + \frac{\text{Re}\lambda_t}{\lambda^5}X_t\right)^2\right].$$

- κ_+: prefactor that includes the hadronic matrix element extracted from $K \to \pi\ell\nu$ decays.

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- Δ_{EM}: known NLO QED corrections

Prediction and Error Budget

Brod, Gorbahn, Stamou 2105.02868

$$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu}) = 7.73(16)(25)(54) \times 10^{-11}\,.$$

- first uncertainty from perturbative physics, second from non-perturbative physics, third from input parameters.

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Brod, Gorbahn, Stamou 2105.02868

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- first uncertainty from perturbative physics, second from non-perturbative physics, third from input parameters.

$$10^{11} \times \mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu}) = 7.73 \pm 0.12_{X_t^{\mathrm{QCD}}} \pm 0.01_{X_t^{\mathrm{EW}}} \pm 0.11_{P_c} \pm 0.24_{\delta P_{cu}} \pm 0.04_{\kappa_+}$$
$$\pm 0.13_{\lambda} \pm 0.46_{A} \pm 0.18_{\bar{\rho}} \pm 0.03_{\bar{\eta}} \pm 0.05_{m_t} \pm 0.15_{m_c} \pm 0.05_{\alpha_s} .$$

- uncertainty is dominated by CKM; “intrinsic” theory uncertainty is only a few percent.

Experimental Situation

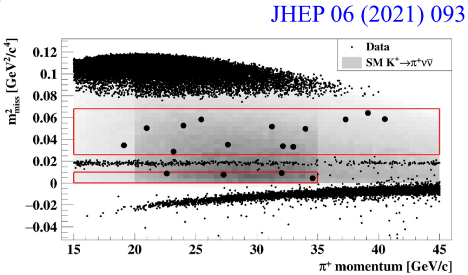
NA62 experiment has evidence for the decay

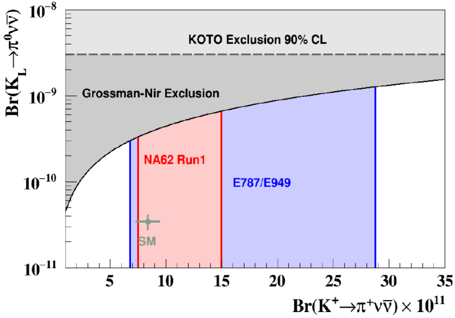
$$BR(K^+ \to \pi^+ \nu\bar{\nu}) =$$

$$= (10.6^{+4.0}_{-3.4} \pm 0.9) \times 10^{-11}$$

Expect 15% uncertainty with the full data set.

(Unfortunately no prospects for further improvement because of cancellation of the HIKE proposal)





$K_L \to \pi^0 \nu\bar{\nu}$ in the SM

Brod, Gorbahn, Stamou 2105.02868

$$\mathrm{Br}\left(K_L \to \pi^0 \nu\bar{\nu}\right) = \kappa_L r_{\epsilon_K} \left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X_t\right)^2$$

- Decay is CP violating and depends to an excellent approximation only on the top contribution

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- Decay is CP violating and depends to an excellent approximation only on the top contribution
- As in the case of the charged kaon decay, hadronic matrix elements can be obtained from data (with small isospin and QED corrections)

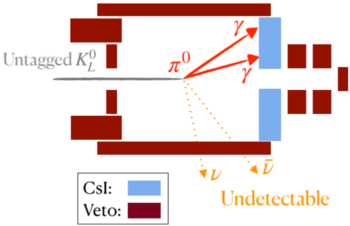
$$\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu}) = 2.59(6)(2)(28) \times 10^{-11} \, .$$

$$10^{11} \times \mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu}) = 2.59 \pm 0.06_{X_t^{\mathrm{QCD}}} \pm 0.01_{X_t^{\mathrm{EW}}} \pm 0.02_{\kappa_L} \pm 0.16_{\bar{\eta}} \pm 0.22_A \pm 0.04_\lambda \pm 0.02_{m_t} \, .$$

- Intrinsic theory uncertainty only few percent;
uncertainty from CKM input $\sim 10\%$

Experimental Situation

- The KOTO experiment at J-PARC is searching for the $K_L \to \pi^0 \nu \bar{\nu}$ decay
- Very challenging experiment!



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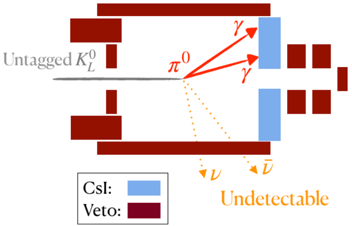

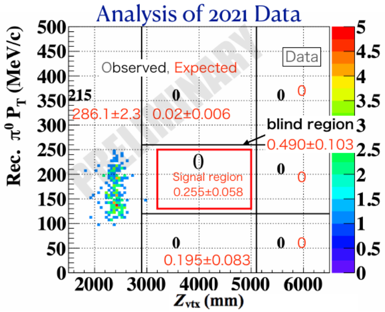

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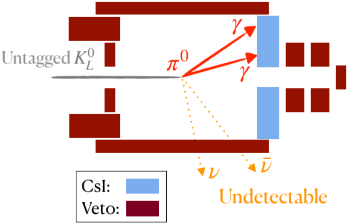
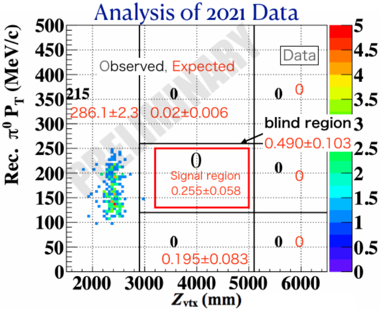
- KOTO can still improve by 1 order of magnitude
- KOTO II proposal to observe the decay at the SM rate





Pion Decays

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(+ additional photons or $e^+ e^-$ pairs)

Lepton Universality in Pion Decays

- $\pi^+ \to \ell^+\nu$ is the textbook example of a helicity suppressed decay

$$\Gamma(\pi^+ \to \ell^+\nu) \simeq \frac{G_F^2}{8\pi}|V_{ud}|^2 f_\pi^2 m_\pi m_\ell^2 \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2$$

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- Take electron to muon ratio to get rid of CKM factors and the pion decay constant

$$R_\pi = \frac{\text{BR}(\pi^+ \to e^+\nu)}{\text{BR}(\pi^+ \to \mu^+\nu)} = \frac{m_e^2}{m_\mu^2}\frac{(m_\pi^2 - m_e^2)^2}{(m_\pi^2 - m_\mu^2)^2}(1 + \Delta_{\text{rad}})$$

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- The by far largest uncertainty comes from higher order QED
- Leading effect for point like pions: $\Delta_{\text{rad}} = -\frac{3\alpha}{2\pi}\log\left(\frac{m_\mu^2}{m_e^2}\right) \simeq -3.7\%$

(Kinoshita '59)

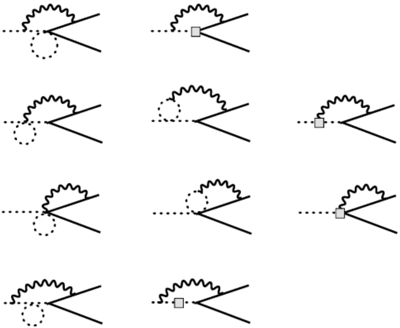
SM Prediction of R_π

Resum the logs, and include structure dependent QED corrections using chiral perturbation theory at 2-loops

Marciano, Sirlin ’93; Cirigliano, Rosell ’07

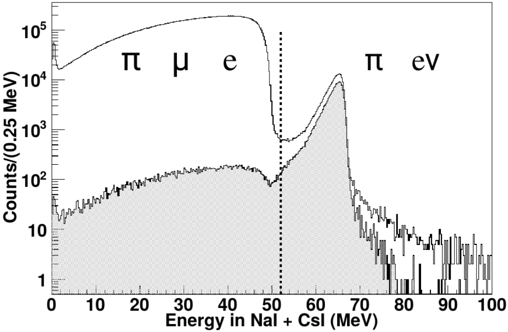
$R_\pi^{\rm SM} = 1.23524(15) \times 10^{-4}$

Probably the most precisely known hadronic observable!



Existing Measurement from PIENU

- look for mono-energetic positrons from the decay of stopped charged pions
- Compatible with the SM prediction, but 1 order of magnitude larger uncertainty



PIENU 1506.05845

$$R_\pi = (1.2344 \pm 0.0023 \pm 0.0019) \times 10^{-4}$$

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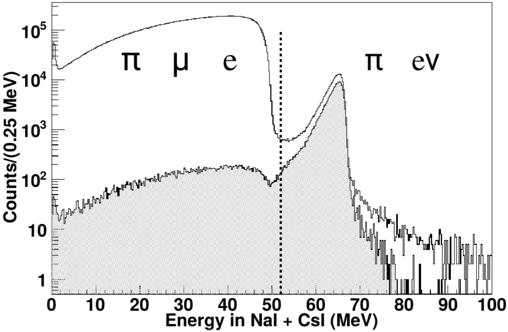

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$$R_\pi = (1.2344 \pm 0.0023 \pm 0.0019) \times 10^{-4}$$

PIENU result corresponds to a test of
$\mu - e$ universality of the weak interactions at the 10^{-3} level

The PIONEER Experiment

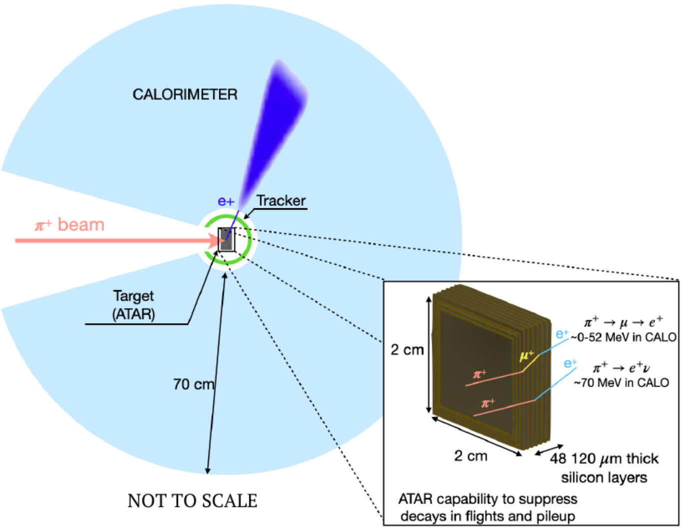
Goal is to match the theory uncertainty and thus test lepton universality of the weak interactions with an order of magnitude better precision

2203.01981

PSI Ring Cyclotron Proposal R-22-01.1
PIONEER: Studies of Rare Pion Decays

W. Altmannshofer,[1] H. Binney,[2] E. Blucher,[3] D. Bryman,[4,5] L. Caminada,[6] S. Chen,[7] V. Cirigliano,[8] S. Corrodi,[9] A. Crivellin,[6,10,11] S. Cuen-Rochin,[12] A. DiCanto,[13] L. Doria,[14] A. Gaponenko,[15] A. Garcia,[2] L. Gibbons,[16] C. Glaser,[17] M. Escobar Godoy,[1] D. Göldi,[18] S. Gori,[1] T. Gorringe,[19] D. Hertzog,[2] Z. Hodge,[2] M. Hoferichter,[20] S. Ito,[21] T. Iwamoto,[22] P. Kammel,[2] B. Kiburg,[15] K. Labe,[16] J. LaBounty,[2] U. Langenegger,[6] C. Malbrunot,[5] S.M. Mazza,[1] S. Mihara,[21] R. Mischke,[5] T. Mori,[22] J. Mott,[15] T. Numao,[5] W. Ootani,[22] J. Ott,[1] K. Pachal,[5] C. Polly,[15] D. Počanić,[17] X. Qian,[13] D. Ries,[23] R. Roehnelt,[2] B. Schumm,[1] P. Schwendimann,[2] A. Seiden,[1] A. Sher,[5] R. Shrock,[24] A. Soter,[18] T. Sullivan,[25] M. Tarka,[1] V. Tischenko,[13] A. Tricoli,[13] B. Velghe,[5] V. Wong,[5] E. Worcester,[13] M. Worcester,[26] and C. Zhang[13]

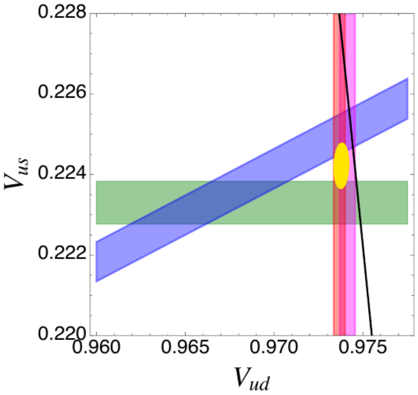
The PIONEER Experiment



Precision Test of First Row CKM Unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- $|V_{ub}|^2 \sim 10^{-5}$ and can be neglected
- current best determination of V_{ud} from nuclear beta decays and neutron decay
- V_{us}/V_{ud} from leptonic kaon and pion decays $K \to \mu\nu$ vs. $\pi \to \mu\nu$
- V_{us} from $K \to \pi\ell\nu$ decays
- combination gives a $2-3$ sigma deficit from unitarity



Cirigliano, Crivellin, MH, Moulson 2022

Pion Beta Decay

Pion beta decay could give the theoretically cleanest determination of V_{ud}

- Master formula Cirigliano, Knecht, Neufeld, Pichl 2003, Czarnecki, Marciano, Sirlin 2020, Feng et al. 2020

$$\Gamma(\pi^+ \to \pi^0 e^+ \nu_e(\gamma)) = \frac{G_F^2 |V_{ud}|^2 M_{\pi^\pm}^5 |f_+^\pi(0)|^2}{64\pi^3}(1 + \Delta_{RC}^{\pi\ell}) I_{\pi\ell}$$

$\hookrightarrow$ need branching fraction and pion life time from experiment

- (Theory) inputs
  - Phase space $I_{\pi\ell} = 7.3766(43) \times 10^{-8}$, uncertainty from $\Delta_\pi = M_{\pi^+} - M_{\pi^0}$
  - Form factor $f_+^\pi(0) = 1 - 7 \times 10^{-6}$

    $\hookrightarrow$ protected by $SU(2)$ Ademollo–Gatto theorem (Behrends–Sirlin)
  - Radiative corrections $\Delta_{RC}^{\pi\ell} = 0.0334(10)$ ChPT, Cirigliano et al., $\Delta_{RC}^{\pi\ell} = 0.0332(3)$ lattice QCD, Feng et al.

- Resulting V_{ud} extracted from PIBETA 2004

$$V_{ud}^{\pi,\text{ChPT}} = 0.97376(281)_{\text{BR}}(9)_{\tau_\pi}(47)_{\Delta_{RC}^{\pi\ell}}(28)_{I_{\pi\ell}}[287]_{\text{total}}$$

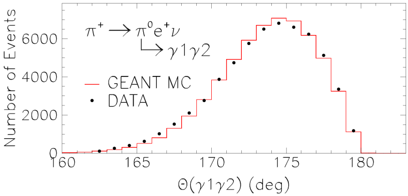
$$V_{ud}^{\pi,\text{lattice}} = 0.97386(281)_{\text{BR}}(9)_{\tau_\pi}(14)_{\Delta_{RC}^{\pi\ell}}(28)_{I_{\pi\ell}}[283]_{\text{total}}$$

Martin Hoferichter, seminar at UC Santa Cruz 8/9/24

PIONEER Phase II and Phase III

Experimental signature of beta decay of a stopped pion:

two (almost) back to back photons from the π^0 plus a very soft positron



PiBeta hep-ex/0312030

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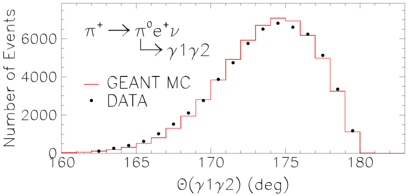

PiBeta hep-ex/0312030

- PiBeta experiment made a measurement with 10^{-3} precision

$$\mathrm{BR}(\pi^+ \to \pi^0 e^+ \nu) = 1.036(4)(5) \times 10^{-8}$$

- In phase II and III, PIONEER aims at measuring $\pi^+ \to \pi^0 e^+ \nu$ 1 order of magnitude more precisely than PiBeta and thus get a V_{ud} that rivals the determination from nuclear decays.

Tight Lines!