Precision Calculations in QCD and EWK Interactions at Hadron Colliders

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QUIZ: Getting to know the room

★ Please raise your hands!
QUIZ: Getting to know the room

★ Please raise your hands!

➡️ Who is currently a PhD student?
QUIZ: Getting to know the room

★ Please raise your hands!

➡️ Who is currently a PhD student?

➡️ Who already has a PhD?
QUIZ: Getting to know the room

★ Please raise your hands!

➡ Who is currently a PhD student?

➡ Who already has a PhD?

➡ Who has already finished a PostDoc?
QUIZ: Getting to know the room

★ Please raise your hands!

➡ Who is currently a PhD student?

➡ Who already has a PhD?

➡ Who has already finished a PostDoc?

➡ Who is staff member?
QUIZ: Getting to know the room

★ Let’s divide the room…!

→ Who is a theorist?
QUIZ: Getting to know the room

★ Let’s divide the room…!

→ Who is a theorist?

→ Who is an experimentalist?
QUIZ: Getting to know the room

★ Let’s divide the room…!

➔ Who is a theorist?

➔ Who is an experimentalist?

➔ Who is non-binary?
QUIZ: Getting to know the room

★ Let’s divide the room…!

→ **Who is a theorist?**

→ **Who is an experimentalist?**

→ **Who is non-binary?**

**phenomenologist**
Combined results: the excess

4th July 2012
Did we need theory to observe the Higgs resonance?

![Graph showing the expected and observed four-lepton invariant mass distribution for the selected Higgs boson candidates with a constrained Z boson mass, shown for an integrated luminosity of 36.1 fb⁻¹ and at \( p_s = 13 \) TeV assuming the SM Higgs boson signal with a mass \( m_H = 125.09 \) GeV.](image)

![Graph showing the expected and observed numbers of signal and background events in the four-lepton decay channels for an integrated luminosity of 36.1 fb⁻¹ and at \( p_s = 13 \) TeV, assuming the SM Higgs boson signal with a mass \( m_H = 125.09 \) GeV. The second column shows the expected number of signal events for the full mass range while the subsequent columns correspond to the mass range of \( 118 < m_4 l < 129 \) GeV. In addition to the ZZ\(^*\) background, the contribution of other backgrounds is shown, comprising the data-driven estimate from Table 4 and the simulation-based estimate of contributions from rare triboson and t\(\bar{t}\)V processes. Statistical and systematic uncertainties are added in quadrature.](image)
Did we need theory to observe the Higgs resonance?

...no! (not really)
Do we need theory to measure Higgs couplings?
Do we need theory to measure Higgs couplings?
Yes, absolutely!
Do we need theory to find a New-Physics resonance?
Do we need theory to find a New-Physics resonance?

No!
Do we need theory to find NP as a small deviation?

---

**Indirect searches**

- Possibility that new states exist (just) beyond the energy reach of the LHC
- We may still observe indirect effects of such particles in the kinematic tails of distributions, e.g., LEP limits on ~ TeV Z'
- Intrinsically small effects that require precise theoretical control on signal and background predictions

**Framework**: SM effective field theory (SMEFT)

- Theoretically consistent, 'model independent' approach to deviations of interactions between SM fields

**Effective Field Theory (EFT)**

\[
\mathcal{L} = \mathcal{L}_{SM} + \sum_i c_i \bar{\psi} \psi \phi_i + O(\frac{\Lambda^4}{\Lambda_{UV}^4})
\]
Do we need theory to find NP as a small deviation? Yes, absolutely!

\[ E > E_{\text{LHC}} \]

**Diagram:**
- **SM:** Standard Model
- **EFT:** Effective Field Theory
- **M \sim \Lambda:** Mass scale relation
- **indirect searches**
How do we get here?
Outline

Lecture 1
★ Fixed-order calculations
  • QCD basics (Lagrangian, Feynman rules, strong coupling)
  • LHC Factorization/Master Formula (PDFs, partonic cross section)
  • Matrix Elements (tree-level, loops)
  • NLO QCD (methods, slicing vs. subtraction vs. analytic)
  • NNLO QCD (methods, timeline)
  • EW corrections (NLO, Sudakov logarithms, mixed QCD-EW corrections?)

Lecture 2
★ Monte Carlo Event Generation & Resummation
  • Resummation
  • Parton Shower Generators (formalism, hadronization, MPI)
  • NLO+PS Matching (MC@NLO, Powheg, merging)
  • NNLO+PS Matching (MiNNLO, Geneva)
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Useful literature

★ Introductory level (QCD lecture notes from CERN schools)
  • Peter Skands, arXiv:1207.2389
  • Gavin Sakam, arXiv:1011.5131

★ Books on QCD
  • "The Black Book of Quantum Chromodynamics: A Primer for the LHC Era", J. Campbell, J. Houston, F. Krauss, Oxford, 2018
Imagine...

...LHC records enough statistics...

...to observe an excess in a Higgs distribution

LHC data

prediction
Imagine…

…LHC records enough statistics…

...to observe an excess in a Higgs distribution

New Physics discovered!

➤ point-like Higgs-gluon interaction see e.g. [Grazzini, Ilnicka, Spira, MW '16]

➤ new heavy particle running in loop

LHC discovers new particle

Likely another Nobel prize in particle physics
Now Imagine…

…the theory error was five times larger

WE MISSED DISCOVERING NEW PHYSICS
make sure there are only two LHC scenarios:
1. establish SM for accessible energy scales at LHC
2. find deviation pattern that hints to BSM Physics
more precise predictions translate into higher discovery reach almost "for free"
The QED Lagrangian

$$\mathcal{L}_{\text{QED}} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Maxwell}} + \mathcal{L}_{\text{int}}$$

$$= \bar{\psi} (i\slashed{\partial} - m) \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - e \bar{\psi} \gamma^\mu A_\mu \psi$$

$\psi$ dirac fermion fields with mass $m$

$A_\mu^a$ electromagnetic photon gauge fields

$F^{a}_{\mu\nu}$ photon field strength tensor $F^{a}_{\mu\nu} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a$
The QED Lagrangian

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\[ = \bar{\psi} \left( i \slashed{\partial} - m \right) \psi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - e \bar{\psi} \gamma^\mu A_\mu \psi \]

**Feynman rules:**

- In:
  - \( \psi \rightarrow p \) = \( u(p) \)
  - \( \bar{\psi} \rightarrow p \) = \( \bar{v}(p) \)
  - \( A_\mu \rightarrow \) = \( \epsilon_\mu \)

- Out:
  - \( p \psi \rightarrow \) = \( \bar{u}(p) \)
  - \( p \bar{\psi} \rightarrow v(p) \)
  - \( A_\mu \rightarrow \) = \( \epsilon^*_\mu \)

**Examples:**

- Feynman diagram for electron-positron scattering:
  - \( \psi \rightarrow e^{-} \bar{\psi} \)
  - \( \psi \rightarrow e^{+} \bar{\psi} \)

- Feynman rules for photon emission:
  - \( A_\mu \rightarrow \) = \( \epsilon^*_\mu \)

- Momentum conservation in QED:
  - \( p \rightarrow \bar{p} \) = \( \frac{i (\not{p} + m)}{p^2 - m^2} \)
The QCD Lagrangian

\[ \mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Yang–Mills}} + \mathcal{L}_{\text{int}} \]

\[ = \bar{\psi}_i (i\not{\partial} - m) \psi_i - \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^{a} - g_s \bar{\psi}_i \gamma^\mu A_{\mu}^a t_{ij}^a \psi_j \]

- \( \psi_i \) quark fields with colour charge index \( i \) and mass \( m \) \( \rightarrow \) quarks come in 3 colours \( \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \)
- \( A_{\mu}^a \) gluon gauge fields \( a = 1, \ldots, 8 \)
- \( F_{\mu\nu}^a \) gluon field strength tensor \( F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f_{abc} A_\mu^b A_\nu^c \) with \( \text{SU}(3) \) structure constants \( f_{abc} \)
- \( t_{ij}^a \) \( \text{SU}(3) \) colour matrices (generators of the \( \text{SU}(3) \) gauge group; representation: Gell-Mann matrices)
The QCD Lagrangian

\[ \mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Yang–Mills}} + \mathcal{L}_{\text{int}} \]

\[ = \bar{\psi}_i (i\not\!D - m) \psi_i - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu}_a - g_s \bar{\psi}_i \gamma^\mu A^a_\mu t^a_{ij} \psi_j \]

Feynman rules:

in: \( A^a_\mu \) = \( e^a_\mu \)

out: \( A^a_\mu \) = \( e^a_\mu \)

\[ \alpha, i \rightarrow \beta, j \]

\[ k, m \]

\[ = \left( \frac{i}{k - m} \right)_{\alpha\beta} \delta_{ij} \]

\[ a, \mu \]

\[ b, \nu \]

\[ = \left( \frac{-ig_{\mu\nu}}{k^2} \right) \delta^{ab} \]

\[ a, \mu \]

\[ b, \nu \]

\[ c, \rho \]

\[ \gamma^{\mu} \]

\[ g_s f^{abc} \left[ g^{\mu\nu} (k - p)^\rho + g^\nu_{\rho} (p - q)^\mu + g^\mu_{\rho} (q - k)^\nu \right] \]

\[ = -ig_s^2 \left[ f^{abcde} (g^{\mu\rho} g_{\nu\sigma} - g^{\mu\sigma} g_{\nu\rho}) + f^{abce} f^{\rho\mu}(g^{\nu\sigma} - g^{\mu\sigma}) + f^{ade} f^{\rho\mu}(g^{\nu\sigma} - g^{\mu\sigma}) \right] \]

\[ = i g_s \gamma^\mu t^a \]
The strong coupling constant

★ The SM is a renormalizable gauge theory

→ couplings (and masses) need to be renormalized (because of UV divergences)

→ theory does not predict value of $\alpha$, but the dependence on scale

Renormalization group equation (RGE):

$$\frac{d\alpha(\mu^2)}{d \ln(\mu^2)} = \beta(\alpha(\mu^2)) = \beta_0 \alpha^2 + \beta_1 \alpha^3 + \beta_2 \alpha^4 + \cdots$$
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perturbative ($\alpha \ll 1$) solution at one-loop:

$$\alpha(\mu^2) = \frac{\alpha(\mu_0^2)}{1 - \alpha(\mu_0^2) \beta_0 \ln(\mu^2/\mu_0^2)}$$

Asymptotic freedom

\[ S_{\beta_0} = S_{\mu_0} + O(\beta_0^3 \beta_0^2) \]

At one-loop order we have

$$\frac{d\beta(\mu^2)}{d \ln(\mu^2)} = \beta(\alpha(\mu^2)) = \beta_0 \alpha^2 + \beta_1 \alpha^3 + \beta_2 \alpha^4 + \cdots$$

$$\beta(\alpha(\mu^2)) = \frac{\alpha(\mu_0^2)}{1 - \alpha(\mu_0^2) \beta_0 \ln(\mu^2/\mu_0^2)}$$

The behaviour crucially depends on the sign of the coefficient $\beta_0$

In QED the dependence of the coupling on the scale has a simple physical interpretation

$$\beta_0 > 0$$

$$\beta_0 < 0$$
The strong coupling constant

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$$\alpha(\mu^2) = \frac{\alpha(\mu_0^2)}{1 - \alpha(\mu_0^2) \beta_0 \ln(\mu^2/\mu_0^2)}$$

QED: $\beta_0 > 0$

The behaviour crucially depends on the sign of the coefficient $\beta_0$.

In QED the dependence of the coupling on the scale has a simple physical interpretation.

- $\beta_0 < 0$: decrease with distance
- $\beta_0 > 0$: increase with energy
The strong coupling constant

★ The SM is a renormalizable gauge theory

→ couplings (and masses) need to be renormalized (because of UV divergences)

→ theory does not predict value of \( \alpha \), but the dependence on scale

Renormalization group equation (RGE):

\[
\frac{d\alpha(\mu^2)}{d \ln(\mu^2)} = \beta(\alpha(\mu^2)) = \beta_0 \alpha^2 + \beta_1 \alpha^3 + \beta_2 \alpha^4 + \cdots
\]

perturbative \( (\alpha \ll 1) \) solution at one-loop:

\[
\alpha(\mu^2) = \frac{\alpha(\mu_0^2)}{1 - \alpha(\mu_0^2) \beta_0 \ln(\mu^2/\mu_0^2)}
\]

QED: \( \beta_0 > 0 \)

QCD: \( \beta_0 < 0 \) (due to gluon self interaction)
We can actually measure asymptotic freedom

\[ \alpha_s(Q^2) \]

\[ QCD \; \alpha_s(M_Z) = 0.1181 \pm 0.0011 \]

- $\tau$ decays (N$^3$LO)
- DIS jets (NLO)
- Heavy Quarkonia (NLO)
- $e^+e^-$ jets & shapes (res. NNLO)
- e.w. precision fits (N$^3$LO)
- $p\bar{p} \rightarrow$ jets (NLO)
- $p\bar{p} \rightarrow$ tt (NNLO)

Nobel prize in 2004
Gross, Politzer, Wilczek
We can actually measure asymptotic freedom

\[\alpha_s(Q^2)\]

Asymptotic freedom

\[\alpha_s(Q^2) \rightarrow \text{perturbation theory valid for high-energy collisions } (Q \gg \Lambda_{QCD} \approx 0.2 \text{ GeV})\]
How to make predictions for proton-proton collisions
LHC event
LHC event

Hadrons

Charged Leptons (e, μ) / Photons

proton

proton
LHC event
LHC event

Hard Process

proton

proton

Marius Wiesemann  (MPP Munich)  Precision Calculations in QCD and EWK Interactions at Hadron Colliders  August 5, 2024
LHC event

Hadronization

Parton Shower

Hard Process

proton

proton
LHC event

Hadronization

Parton Shower

Hard Process

Underlying Event
$\sigma_{\text{had}} =$

**LHC Master Formula**

**Hard Process**
\[ \sigma_{\text{had}} = f_i(x_1, \mu_F) f_j(x_2, \mu_F) \]

Parton Distribution Functions (PDFs):
- probability to find a certain a parton (here: gluon) with momentum fraction \( x_i \) inside the proton
- long distance (non-perturbative)
\[ \sigma_{\text{had}} = f_i(x_1, \mu_F) f_j(x_2, \mu_F) \times \sigma_{ij}(x_1 P_1, x_2 P_2, \mu_F) \]

**LHC Master Formula**

**Hard Process**

- **Proton**
- **Proton**

partonic cross section:
- transition scattering amplitude (probability) for the partonic process \( ij \rightarrow X \) (to produce some final state \( X \))
- short distance (perturbative)
LHC Master Formula

\[ \sigma_{\text{had}} = \sum_{ij} \int dx_1 \, dx_2 \, f_i(x_1, \mu_F) \, f_j(x_2, \mu_F) \times \sigma_{ij}(x_1 P_1, x_2 P_2, \mu_F) \]
LHC Master Formula

\[ \sigma_{\text{had}} = \sum_{ij} \int dx_1 dx_2 f_i(x_1, \mu_F) f_j(x_2, \mu_F) \times \sigma_{ij}(x_1 P_1, x_2 P_2, \mu_F) + \mathcal{O}(\Lambda^2/Q^2) \]

factorization holds up to additional non-perturbative corrections (higher "twist")
Parton Distribution Functions (PDFs)

\[ \sigma_{\text{had}} = \sum_{ij} \int dx_1 \, dx_2 \left( f_i(x_1, \mu_F) \, f_j(x_2, \mu_F) \right) \times \sigma_{ij}(x_1 P_1, x_2 P_2, \mu_F) + \mathcal{O}(\Lambda^2/Q^2) \]

- universal distributions containing long-distance structure of hadrons
- scale dependence via DGLAP evolution (Dokshitzer, Gribov, Lipatov, Altarelli, Parisi):

\[ \mu^2 \frac{\partial}{\partial \mu^2} f_i(x, \mu^2) = \frac{\alpha_S(\mu^2)}{2\pi} \sum_j \int_1^1 dy \, f_j(y, \mu^2) P_{ij}(x/y, \alpha_S(\mu^2)) \]

- \( f_i(x, \mu_0^2) \) determined from:
  - lattice QCD (in principle)
  - fits to data (in practice)
  - e.g. MSTW, MMHT, CTEQ, HERA, ABM, NNPDF, ...
  - photon PDF calculated

[Manohar, Nason, Salam, Zanderighi, '17]
Partonic Cross Section

\[ \sigma_{\text{had}} = \sum_{ij} \int dx_1 \, dx_2 \, f_i(x_1, \mu_F) \, f_j(x_2, \mu_F) \times \sigma_{ij}(x_1 P_1, x_2 P_2, \mu_F) + \mathcal{O}(\Lambda^2/Q^2) \]

\[ \sigma_{ij} \sim \sigma_{\text{LO}} \cdot (1 + \alpha + \alpha^2 + \ldots) \]

LO \sim \mathcal{O}(100\%)

NLO \sim \mathcal{O}(10\%)

NNLO \sim \mathcal{O}(1\%)

Uncertainties:

\( \alpha \sim 0.118 \)

Hard Process
Partonic Cross Section

\[ \sigma_{\text{had}} = \sum_{ij} \int dx_1 \, dx_2 \, f_i(x_1, \mu_F) f_j(x_2, \mu_F) \times \sigma_{ij}(x_1 p_1, x_2 p_2, \mu_F) + \mathcal{O}(\Lambda^2 / Q^2) \]

\[ \sigma_{ij} = \frac{1}{2s} \int \left[ \frac{1}{(2\pi)^3 2E_i} \prod_{i=1}^{n} \right] \left[ (2\pi)^4 \delta^4 \left( \sum_{i=1}^{n} q_i^\mu - (p_1 + p_2)^\mu \right) \right] \left| M_{ij}(p_1, p_2, q_i) \right|^2 \]

[flux factor] [phase-space integral - \( \Phi_n \)] [squared matrix element]
Partonic Cross Section

\[
\sigma_{\text{had}} = \sum_{ij} \int dx_1 \, dx_2 \, f_i(x_1, \mu_F) \, f_j(x_2, \mu_F) \times \sigma_{ij}(x_1 P_1, x_2 P_2, \mu_F) + \mathcal{O}(\Lambda^2/Q^2)
\]

\[
\sigma_{ij} = \frac{1}{2s} \int \left\{ \left[ \prod_{i=1}^{n} \frac{d^3 \vec{q}_i}{(2\pi)^3 2E_i} \right] \left[ (2\pi)^4 \delta^4 \left( \sum_{i=1}^{n} q_i^\mu - (p_1 + p_2)^\mu \right) \right] |M_{ij}(p_1, p_2, q_i)|^2 \right\}
\]

[flux factor] [phase-space integral - \(\Phi_n\)] [squared matrix element]

LO

\( + \) + ... \( 2 \)

to illustrate the concepts, we don’t care what the particles are — just draw lines
Importance of QCD corrections (example WZ)

NNLO crucial for accurate description of data

[Grassini, Kallweit, Rathlev, MW '16]

NNLO

NLO
Higher-order corrections

\[ d\sigma = d\sigma^{(0)} + \alpha d\sigma^{(1)} + \alpha^2 d\sigma^{(2)} + \mathcal{O}(\alpha^3) \]

Two (complicated) main problems to solve:

1. **evaluate (loop) amplitudes**
   (ingredients of calculation, difficulty \( \sim e^{\text{loops}} \), understood at 1-loop, various 2-loop results, very few 3-loop results)

2. **combination of different (singular) ingredients**
   (final cross section prediction, difficulty \( \sim e^{\text{order}} \), understood up to NNLO, very few N3LO results)
Calculation of Scattering Amplitudes

★ Tree-level/LO calculation is relatively simple
Calculation of Scattering Amplitudes

★ Tree-level/LO calculation is relatively simple

1. draw all Feynman diagrams (same external states, same coupling order)

\[ q\bar{q} \rightarrow \gamma^{*}g : \]

\[ p_{1} \quad k, e_{g} \]
\[ p_{2} \quad q, e_{\gamma} \]

\[ + \]

\[ p_{1} \quad k, e_{g} \]
\[ p_{2} \quad q, e_{\gamma} \]

\[ s = (p_{1} + p_{2})^{2} \]
\[ t = (p_{1} - q)^{2} \]
\[ u = (p_{2} - q)^{2} \]
\[ p_{1} + p_{2} = q + k \]
Calculation of Scattering Amplitudes

★ Tree-level/LO calculation is relatively simple

1. draw all Feynman diagrams (same external states, same coupling order)
2. plug in Feynman rules and add expressions of all diagrams to obtain amplitude

\[
q\bar{q} \rightarrow \gamma^* g : \\
\begin{align*}
p_1 & \quad k, e_g \\
p_2 & \quad q, e_\gamma
\end{align*}
\]

\[
M_{q\bar{q} \rightarrow \gamma^* g} = u_\gamma^c(p_1) \left( -ig_s t_a^{a_{bc}} e_8 \right) \frac{i}{p_1 - k_1} \left( -ie_\gamma e_\gamma^* \right) \bar{v}_c^a(p_2) + u_\gamma^c(p_1) \left( -ie_\gamma e_\gamma^* \right) \frac{i}{p_2 - k_1} \left( -ig_s t_a^{a_{bc}} e_8 \right) \bar{v}_c^a(p_2)
\]

in: \[
\psi \rightarrow p = u(p) \\
\bar{\psi} \rightarrow p = \bar{v}(p)
\]

out: \[
A_\mu = e_\mu^* \\
A_\mu = e_\mu^*
\]
Calculation of Scattering Amplitudes

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1. draw all Feynman diagrams (same external states, same coupling order)
2. plug in Feynman rules and add expressions of all diagrams to obtain amplitude
3. square amplitude & average/sum over incoming/outgoing spins and colours

\[ q\bar{q} \rightarrow \gamma^* g : \]
\[ p_1 \]
\[ p_2 \]
\[ \bar{k}, e_g \]
\[ q, e_\gamma \]
\[ q, e_\gamma \]
\[ \sum \]
\[ \sum \]
\[ \sum \]
\[ \sum \]
\[ p_1 \]
\[ p_2 \]
\[ k, e_g \]
\[ k, e_g \]
\[ (p_1 - k) \]
\[ (p_2 - k) \]
\[ u_s(p_1)( - ig_s t^a c \cdot k_\gamma) \frac{i}{(p_1 - k)} ( - i e_q e_\gamma^* ) \bar{v}_c^{\gamma'}(p_2) + u_s(p_1)( - i e_q e_\gamma^* ) \frac{i}{(p_2 - k)} ( - ig_s t^a c \cdot k_\gamma) \bar{v}_c^{\gamma'}(p_2) \]

\[ |M_{q\bar{q} \rightarrow \gamma^* g}|^2 = \frac{1}{4} \sum_{s s'} \frac{1}{9} \sum_{c c'} \sum_{e_\gamma e_\gamma'} \sum_a |M_{q\bar{q} \rightarrow \gamma^* g}|^2 \]

- spin & colour avg. (incoming)
- spin & colour sum (outgoing)
Calculation of Scattering Amplitudes

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1. draw all Feynman diagrams (same external states, same coupling order)
2. plug in Feynman rules and add expressions of all diagrams to obtain amplitude
3. square amplitude & average/sum over incoming/outgoing spins and colours
4. perform some algebra and do simplifications

\[ q\bar{q} \rightarrow \gamma^* g : \]

\[ M_{q\bar{q} \rightarrow \gamma^* g} = u_c^e(p_1) \left( -ig_s t^{a}_{cc'} \varepsilon_g \right) \frac{i}{p_1 - k_1} \left( -ie_q \varepsilon^* \right) \bar{\nu}_c^e(p_2) + u_c^e(p_1) \left( -ie_q \varepsilon^* \right) \frac{i}{p_2 - k_1} \left( -ig_s t^{a}_{cc'} \varepsilon_g \right) \bar{\nu}_c^e(p_2) \]

\[ \left| M_{q\bar{q} \rightarrow \gamma^* g} \right|^2 = \left( \sum_{ss'} \sum_{cc'} \sum_{\varepsilon^*_i \varepsilon_j} \sum_a \right) \left| M_{q\bar{q} \rightarrow \gamma^* g} \right|^2 = \frac{32\pi^2 \alpha_s e_q^2}{3} C_F \left( \frac{u}{t} + \frac{t}{u} + \frac{2s(s-u-t)}{ut} \right) \]

\[ s = (p_1 + p_2)^2 \]
\[ t = (p_1 - q)^2 \]
\[ u = (p_2 - q)^2 \]
\[ p_1 + p_2 = q + k \]
Calculation of Scattering Amplitudes

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3. square amplitude & average/sum over incoming/outgoing spins and colours
4. perform some algebra and do simplifications
5. multiply flux factor, sum initial states, multiply PDFs and integrate over phase+parton fractions

\[
\sigma_{ij} = \frac{1}{2s} \int \left[ \prod_{i=1}^{n} \frac{d^3q_i}{(2\pi)^3 2E_i} \right] \left[ (2\pi)^4 \delta^4 \left( \sum_{i=1}^{n} q_i^\mu - (p_1 + p_2)^\mu \right) \right] |M_{ij}(p_1, p_2, q_i)|^2 
\]

\[
\sigma_{\text{had}} = \sum_{ij} \int dx_1 \ dx_2 \ f_i(x_1, \mu_F) \ f_j(x_2, \mu_F) \times \sigma_{ij}(x_1 P_1, x_2 P_2, \mu_F) + \mathcal{O}(\Lambda^2/Q^2)
\]
Calculation of Scattering Amplitudes

★ Tree-level/LO calculation is relatively simple

1. draw all Feynman diagrams (same external states, same coupling order)
2. plug in Feynman rules and add expressions of all diagrams to obtain amplitude
3. square amplitude & average/sum over incoming/outgoing spins and colours
4. perform some algebra and do simplifications
5. multiply flux factor, sum initial states, multiply PDFs and integrate over phase+parton fractions

- (automated) tools for 1.-4.: FeynArts/Qgraf, Mathematica/Form etc.
- note: number of diagrams grows factorially. e.g. $gg \rightarrow n$ gluons:

<table>
<thead>
<tr>
<th>$n$ diag.</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<td></td>
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<td>220</td>
<td>2485</td>
<td>34300</td>
<td>559405</td>
<td>10525900</td>
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Calculation of Scattering Amplitudes

★ Loop amplitudes are much more difficult (although 1-loop is a solved issue by now):
  • integration over momenta of virtual particles in loops
  • leads to UV divergences that require renormalization
  • leads to IR divergences (expressed through regulator)

\[ M_{1\text{-loop}} = \int \frac{d^4 q}{(2\pi)^4} \cdots \]
Calculation of Scattering Amplitudes

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  • integration over momenta of virtual particles in loops
  • leads to UV divergences that require renormalization
  • leads to IR divergences (expressed through regulator)

★ Dimensional regularization is the standard procedure used today

\[ \int \frac{d^4 l}{(2\pi)^4} \to \mu^{2\epsilon} \int \frac{d^d l}{(2\pi)^d}, \quad d = 4 - 2\epsilon < 4 \]

• leads poles in $1/\epsilon^n$, e.g.

\[ \int_0^1 \frac{dx}{x} \to \int_0^1 \frac{dx}{x^{1-\epsilon}} = \frac{1}{\epsilon} \]

• every (squared) 1-loop amplitude in dim.-reg. has the following form:

\[ 2 \text{Re} \langle M_{1\text{-loop}} | M_0 \rangle = \frac{A}{\epsilon^2} + \frac{B}{\epsilon} + C \]
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   \[ 2 \Re \langle M_{1\text{-loop}} | M_0 \rangle = \frac{A}{\epsilon^2} + \frac{B}{\epsilon} + C \]

→ but it remains very difficult to actually solve the loop integrals and arrive at this form
Calculation of Scattering Amplitudes

★ Breakthrough in 1-loop calculations

\[
\begin{align*}
\quad & = \sum_i d_i + \sum_i c_i + \sum_i b_i + \sum_i a_i
\end{align*}
\]

• all integrals in 1-loop amplitudes can be reduced to a basis of master integrals (box, triangle, bubble, tadpole)

• the coefficients can be extracted through an algorithmic reduction approach

\textit{Ossola, Pittau, Papadopolous '06}
Calculation of Scattering Amplitudes

★ Breakthrough in 1-loop calculations

\[ \text{all integrals in 1-loop amplitudes can be reduced to a basis of master integrals (box, triangle, bubble, tadpole)} \]

\[ \text{the coefficients can be extracted through an algorithmic reduction approach} \]

★ At 2-loop things are much more cumbersome

\[ \text{no general basis of master integrals for all processes (yet/ever?)} \]

\[ \text{reduction to master integrals substantially more involved & numerically demanding} \]
INGREDIENTS FOR A CALCULATION (generic 2→2 process)

LO

Tree

2→2

2

...slide borrowed from Gavin Salam
INGREDIENTS FOR A CALCULATION (generic 2→2 process)

LO
2→2
Tree

NLO
2→3
Tree

1-loop
2→2

+ complex conj.
...slide borrowed from Gavin Salam
INGREDIENTS FOR A CALCULATION (generic 2→2 process)

NNLO

Tree
2→4

1-loop
2→3

2-loop
2→2

1-loop
2→2

2-loop
2→2

×

+ complex conj.

...slide borrowed from Gavin Salam
How to do a NLO calculation
NLO Calculation: The Issue

$$\sigma_{\text{LO}} = \int_{\Phi_B} \text{d}\sigma^B.$$
NLO Calculation: The Issue

\[ \sigma_{NLO} = \int_{\Phi_{B}} d\sigma^{B} + \int_{\Phi_{B+1}} d\sigma^{R} + \int_{\Phi_{B}} d\sigma^{V} \]

LO

\((pp\rightarrow WZ)\)

\(\rightarrow d\sigma^{B}\)

NLO

\((pp\rightarrow WZ)\)

\(\rightarrow d\sigma^{R}\)

\(\rightarrow d\sigma^{V}\)
NLO Calculation: The Issue

\[ \sigma_{NLO} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} d\sigma^R + \int_{\Phi_B} d\sigma^V \]

LO
(pp→WZ)
\[ \rightarrow d\sigma^B \]

NLO
(pp→WZ)
\[ \rightarrow d\sigma^R \]
\[ \rightarrow d\sigma^V \]

1-loop virtual amplitude:
\[ d\sigma^V \sim \left( \frac{A_0}{\epsilon^2} + \frac{B_0}{\epsilon} + C_0 \right) \]
NLO Calculation: The Issue

$$\sigma_{\text{NLO}} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} d\sigma^R + \int_{\Phi_B} d\sigma^V$$

real-emission amplitude finite, but integrand (propagators) become singular during phase-space integration. After phase-space integration:

LO (pp→WZ)

NLO (pp→WZ)

I-loop virtual amplitude:

$$d\sigma^V \sim \left( \frac{A_0}{\epsilon^2} + \frac{B_0}{\epsilon} + C_0 \right)$$

$$d\sigma^V \sim \left( \frac{A}{\epsilon^2} + \frac{B}{\epsilon} + C \right)$$
NLO Calculation: The Issue

\[ \sigma_{\text{NLO}} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} d\sigma^R + \int_{\Phi_B} d\sigma^V = C + D \]

real-emission amplitude finite, but integrand (propagators) become singular during phase-space integration. After phase-space integration:

\[ \int_{\Phi_{B+1}} d\sigma^R \sim \left( \frac{A}{e^2} + \frac{B}{e} + D \right) \]

1-loop virtual amplitude:

\[ d\sigma^V \sim \left( \frac{A_0}{e^2} + \frac{B_0}{e} + C_0 \right) \]

LO

(pp → WZ)

\[ \rightarrow d\sigma^B \]

NLO

(pp → WZ)

\[ \rightarrow d\sigma^R \]

\[ \rightarrow d\sigma^V \]
\[ f(z) \text{ is some function with finite limit for } z \to 0 \]

**LOCAL SUBTRACTION**

\[
\sigma = c \cdot f(0) + \int_0^1 dz \left[ \frac{f(z)}{z} - \frac{f(0)}{z} \right]
\]

- **Virtual & Counterterm:**
  - May need (tough) analytic calculation

- **Real Part:**
  - MC integration is finite even without cut

**“SLICING”**

\[
\sigma = \left( c - \ln \frac{1}{\text{cut}} \right) \cdot f(0) + \int_{\text{cut}}^1 dz \frac{f(z)}{z}
\]

- **Virtual & Counterterm:**
  - Get from soft-collinear resummation

- **Real Part:**
  - Use MC integration (cut has to be small)

---

**NNLO approaches**

- **Sector decomposition**
  - Anastasiou, Melnikov, Petriello; Binoth, Heinrich

- **Antenna subtraction**
  - Kosower; Gehrmann, Gehrmann-de Ridder, Glover

- **Stripper**
  - Czakon

- **Sector-improved residue**
  - Boughezal, Melnikov, Petriello

- **CoLorFul subtraction**
  - Del Duca, Somogyi, Trocsanyi

- **Projection-to-Born**
  - Cacciari, Dreyer, Karlberg, Salam, Zanderighi

- **qT subtraction**
  - Catani, Grazzini

- **N-jettiness subtraction**
  - Boughezal, Focke, Liu, Petriello; Gaunt, Stahlhofen, Tackmann, Walsh

...slide borrowed from Gavin Salam
Our calculation is performed in the complex-mass scheme \[21\], and besides resonances, it includes all contributions at one-loop amplitudes for tree amplitudes for $Z/W$ with closed fermion loops (see Figure 3 (a)); all other contributions that would enter a complete process at LO, we only include diagrams \[68\] to deal with exceptional phase-space points.

The intermediate vertex $\nu_\ell$ initiates contributions. The boson can be replaced by an odd-bilocal quark-gluon (gg) approximation is applied. Our implementation can deal with any combination of leptonic flavours, though.

In the SF channel, each diagram is duplicated according to the two possible assignments of the three charged leptons—one opposite-sign, same-flavour (OSSF) lepton pair, and another charged lepton with charge opposite to the lepton in the OSSF pair along with the other two leptons. These OSSF lepton pairs are later referred to as same-flavour (SF) leptons.

In the DF channel, the usual experimental requirement of a mass window $m_W^\text{cut} - 3\sigma < m_W < m_W^\text{cut} + 3\sigma$ is applied. Our approximation includes all contributions at NLO through subtraction.

The one-loop amplitudes for $Z/W$ production are achieved by charge conjugation. The leptonic final states are generated, as shown in Figure 1 for the (b) channel.

### NLO through subtraction

\[
\sigma_{NLO} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} d\sigma^R + \int_{\Phi_B} d\sigma^V \quad \text{sum finite}
\]

- **LO**
  - (pp→WZ)

- **NLO**
  - (pp→WZ)

Arrow $\rightarrow d\sigma^B$ and $\rightarrow d\sigma^R$ denote the contributions at LO and NLO, respectively. Arrow $\rightarrow d\sigma^V$ denotes the contributions at VBF.
NLO through subtraction

\[
\sigma_{NLO} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} d\sigma^R + \int_{\Phi_B} d\sigma^V \\
= \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} (d\sigma^R - d\sigma^S) + \int_{\Phi_B} \left( d\sigma^V + \int_1 d\sigma^S \right) \epsilon = 0
\]

d\sigma^S: subtraction term

→ Dipole [Catani, Seymour '96]
→ FKS [Frixione, Kunszt, Signer '96]
→ Antenna [Gehrmann et al. '05]
→ ...

LO
(pp→WZ)

NLO
(pp→WZ)
Subtraction terms?

\[
\sigma_{\text{NLO}} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} d\sigma^R + \int_{\Phi_B} d\sigma^V \\
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\]

✦ use factorization properties of squared amplitudes
✦ singularities appear when final-state parton soft or colliner
✦ singularity structure of amplitudes universal and known

schematically:
Subtraction terms?

\[ \sigma_{\text{NLO}} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} d\sigma^R + \int_{\Phi_B} d\sigma^V \]

\[ = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} (d\sigma^R - d\sigma^S) + \int_{\Phi_B} \left( d\sigma^V + \int_1 d\sigma^S \right) \epsilon = 0 \]

✦ use factorization properties of squared amplitudes
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✦ singularity structure of amplitudes universal and known

splitting parton

schematically:
Subtraction terms?

\[ \sigma_{NLO} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} d\sigma^R + \int_{\Phi_B} d\sigma^V \]
\[ = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} (d\sigma^R - d\sigma^S) + \int_{\Phi_B} \left( d\sigma^V + \int_1 d\sigma^S \right) \epsilon = 0 \]

- use factorization properties of squared amplitudes
- singularities appear when final-state parton soft or colliner
- singularity structure of amplitudes universal and known

schematically:

\[ \begin{array}{c}
\text{splitting parton getting soft or colliner} \\
\end{array} \]

\[ F(\leftarrow \rightarrow) \otimes \]

\[ 2 \]

\[ 2 \]
Subtraction terms?

\[
\sigma_{\text{NLO}} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} d\sigma^R + \int_{\Phi_B} d\sigma^V \\
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\]

✦ use factorization properties of squared amplitudes
✦ singularities appear when final-state parton soft or colliner
✦ singularity structure of amplitudes universal and known

schematically:

\[
\begin{array}{ccc}
\text{J}^a \text{ (soft limit)} \text{ or splitting function } P_{ij} \text{ (collinear limit)}
\end{array}
\]
Subtraction terms?

\[
\sigma_{NLO} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} d\sigma^R + \int_{\Phi_B} d\sigma^V
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\]

schematically:

eikonal factor \( J^a \) (soft limit) or splitting function \( P_{ij} \) (collinear limit)
Subtraction terms?

\[ \sigma_{\text{NLO}} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} d\sigma^R + \int_{\Phi_B} d\sigma^V \]

\[ = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} (d\sigma^R - d\sigma^S) + \int_{\Phi_B} (d\sigma^V + \int_1 d\sigma^S) \]

finite

\[ \sim (\frac{A_0}{\epsilon^2} + \frac{B_0}{\epsilon} + C_0) \]

\[ \sim (-\frac{A_0}{\epsilon^2} - \frac{B_0}{\epsilon} + D_0) \]

\[ = C_0 + D_0 \]

eikonal factor \( J^a \) (soft limit) or splitting function \( P_{ij} \) (collinear limit)

schematically:
Subtraction terms?

\[ \sigma_{\text{NLO}} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} d\sigma^R + \int_{\Phi_B} d\sigma^V \]

\[ = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} (d\sigma^R - d\sigma^S) + \int_{\Phi_B} \left( d\sigma^V + \int_1^1 d\sigma^S \right) \epsilon = 0 \]

\[ d\sigma^S: \text{subtraction term} \]

→ Dipole [Catani, Seymour '96] \( \Rightarrow \) combines soft & collinear limit in dipole function

→ FKS [Frixione, Kunszt, Signer '96] \( \Rightarrow \) partitions phase space into soft, coll. & soft+coll.

→ Antenna [Gehrmann et al. '05] \( \Rightarrow \) like dipole, but 1 Antenna \( \simeq \) 1/2 Dipole

Schematically:

Eikonal factor \( J^a \) (soft limit) or splitting function \( P_{ij} \) (collinear limit)
Automation

★ Automation at LO (tree-level amplitudes & phase space) understood for very long time
Automation

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• Automation of 1-loop amplitudes (OPP, OpenLoops, …)

  MadLoop   OpenLoops   Gosam    Recola    Helac-NLO   BlackHat   NJet   NLOX   ...

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  MadGraph5_aMC@NLO   Munich/Matrix   Powheg    Sherpa    Herwig++   WHIZARD    ...

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<table>
<thead>
<tr>
<th>Process</th>
<th>Syntax</th>
<th>LO 13 TeV</th>
<th>NLO 13 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pp \rightarrow H$ (HEFT)</td>
<td>$pp &gt; h$</td>
<td>$1.593 \pm 0.003 \cdot 10^1$</td>
<td>$3.261 \pm 0.010 \cdot 10^1$</td>
</tr>
<tr>
<td>$pp \rightarrow Hj$ (HEFT)</td>
<td>$pp &gt; hj$</td>
<td>$8.367 \pm 0.003 \cdot 10^1$</td>
<td>$1.422 \pm 0.006 \cdot 10^1$</td>
</tr>
<tr>
<td>$pp \rightarrow Hjj$ (HEFT)</td>
<td>$pp &gt; hjj$</td>
<td>$3.020 \pm 0.002 \cdot 10^1$</td>
<td>$5.124 \pm 0.020 \cdot 10^1$</td>
</tr>
<tr>
<td>$pp \rightarrow Hjj$ (VBF)</td>
<td>$pp &gt; hjj \ s \ s \ w^+ w^- z$</td>
<td>$1.987 \pm 0.002 \cdot 10^0$</td>
<td>$1.900 \pm 0.006 \cdot 10^0$</td>
</tr>
<tr>
<td>$pp \rightarrow Hjj$ (VBF)</td>
<td>$pp &gt; hjj \ s \ s \ w^+ w^- z$</td>
<td>$2.824 \pm 0.005 \cdot 10^1$</td>
<td>$3.085 \pm 0.010 \cdot 10^1$</td>
</tr>
<tr>
<td>$pp \rightarrow HW^\pm$</td>
<td>$pp &gt; hw^\pm m$</td>
<td>$1.195 \pm 0.002 \cdot 10^1$</td>
<td>$1.419 \pm 0.005 \cdot 10^1$</td>
</tr>
<tr>
<td>$pp \rightarrow HW^\pm j$</td>
<td>$pp &gt; hw^\pm m j$</td>
<td>$4.018 \pm 0.003 \cdot 10^1$</td>
<td>$4.842 \pm 0.017 \cdot 10^1$</td>
</tr>
<tr>
<td>$pp \rightarrow HW^\pm jj$</td>
<td>$pp &gt; hw^\pm m j j$</td>
<td>$1.198 \pm 0.016 \cdot 10^1$</td>
<td>$1.574 \pm 0.014 \cdot 10^1$</td>
</tr>
<tr>
<td>$pp \rightarrow HZ$</td>
<td>$pp &gt; h z$</td>
<td>$6.468 \pm 0.008 \cdot 10^1$</td>
<td>$7.674 \pm 0.027 \cdot 10^1$</td>
</tr>
<tr>
<td>$pp \rightarrow HZ j$</td>
<td>$pp &gt; h z j$</td>
<td>$2.225 \pm 0.001 \cdot 10^1$</td>
<td>$2.667 \pm 0.010 \cdot 10^1$</td>
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<tr>
<td>$pp \rightarrow HZ jj$</td>
<td>$pp &gt; h z j j$</td>
<td>$7.202 \pm 0.012 \cdot 10^2$</td>
<td>$8.753 \pm 0.037 \cdot 10^2$</td>
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<tr>
<td>$pp \rightarrow HW^+W^-$ (4f)</td>
<td>$pp &gt; hw^+ w^-$</td>
<td>$8.325 \pm 0.139 \cdot 10^3$</td>
<td>$1.065 \pm 0.003 \cdot 10^3$</td>
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<tr>
<td>$pp \rightarrow HW^\pm\gamma$</td>
<td>$pp &gt; hw^\pm m a$</td>
<td>$2.518 \pm 0.006 \cdot 10^3$</td>
<td>$3.309 \pm 0.011 \cdot 10^3$</td>
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<tr>
<td>$pp \rightarrow HZW^\pm$</td>
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<td>$3.763 \pm 0.007 \cdot 10^3$</td>
<td>$5.292 \pm 0.015 \cdot 10^3$</td>
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<tr>
<td>$pp \rightarrow HZZ$</td>
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<td>$2.093 \pm 0.003 \cdot 10^3$</td>
<td>$2.538 \pm 0.007 \cdot 10^3$</td>
</tr>
<tr>
<td>$pp \rightarrow Htt$</td>
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<td>$3.579 \pm 0.003 \cdot 10^1$</td>
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</tr>
<tr>
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<td>$4.994 \pm 0.005 \cdot 10^2$</td>
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<tr>
<td>$pp \rightarrow Hbb$ (4f)</td>
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<td>$6.085 \pm 0.026 \cdot 10^1$</td>
</tr>
<tr>
<td>$pp \rightarrow Htt^\sim j$</td>
<td>$pp &gt; h t t^\sim j$</td>
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<td>$3.244 \pm 0.025 \cdot 10^1$</td>
</tr>
<tr>
<td>$pp \rightarrow Hbb j$ (4f)</td>
<td>$pp &gt; h b b^\sim j$</td>
<td>$7.367 \pm 0.002 \cdot 10^2$</td>
<td>$9.034 \pm 0.032 \cdot 10^2$</td>
</tr>
</tbody>
</table>

MadGraph5_aMC@NLO: sample from 172 processes

...slide borrowed from Massimiliano Grazzini
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→ NLO has become the minimal standard now in (most) LHC analyses
Automation at LO (tree-level amplitudes & phase space) understood for very long time

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NNLO(+PS) only as process libraries in (public) codes
How to do a NNLO calculation
NNLO local subtraction

LO
(pp→WZ)

(\bar{d} u) \rightarrow d\sigma^B

NLO
(pp→WZ)

(\bar{d} d) \rightarrow d\sigma^V

(\bar{d} u) \rightarrow d\sigma^R

NNLO
(pp→WZ)

(\bar{d} d) \rightarrow d\sigma^{VV}

(\bar{d} u) \rightarrow d\sigma^{RV}

(\bar{d} u) \rightarrow d\sigma^{RR}
NNLO local subtraction

\[ \sigma_{\text{NNLO}} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} d\sigma^R + \int_{\Phi_B} d\sigma^V + \int_{\Phi_{B+2}} d\sigma^{RR} + \int_{\Phi_{B+1}} d\sigma^{RV} + \int_{\Phi_B} d\sigma^{VV} \]

LO
(pp→WZ)
\[ \rightarrow d\sigma^B \]

NLO
(pp→WZ)
\[ \rightarrow d\sigma^V \]
\[ \rightarrow d\sigma^R \]

NNLO
(pp→WZ)
\[ \rightarrow d\sigma^{VV} \]
\[ \rightarrow d\sigma^{RV} \]
\[ \rightarrow d\sigma^{RR} \]
NNLO local subtraction

$$\sigma_{\text{NLO}} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} (d\sigma^R - d\sigma^S) + \int_{\Phi_B} \left( d\sigma^V + \int_1 d\sigma^S \right)$$
NNLO local subtraction

\[ \sigma_{\text{NNLO}} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} (d\sigma^R - d\sigma^S) + \int_{\Phi_B} \left( d\sigma^V + \int_1 d\sigma^S \right) \]

\[ + \int_{\Phi_{B+2}} (d\sigma^{RR} - d\sigma^{S21} - d\sigma^{S22}) + \int_{\Phi_{B+1}} \left( d\sigma^{RV} - d\sigma^S + \int_1 d\sigma^{S21} \right) + \int_{\Phi_B} \left( d\sigma^{VV} + \int_1 d\sigma^S + \int_2 d\sigma^{S22} \right) \]
NNLO local subtraction

\[
\sigma_{\text{NNLO}} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} (d\sigma^R - d\sigma^S) + \int_{\Phi_B} \left( d\sigma^V + \int_1 d\sigma^S \right) \\
+ \int_{\Phi_{B+2}} (d\sigma^{RR} - d\sigma^{S_{21}} - d\sigma^{S_{22}}) + \int_{\Phi_{B+1}} \left( d\sigma^{RV} - d\sigma^{S_1} + \int_1 d\sigma^{S_{21}} \right) + \int_{\Phi_B} \left( d\sigma^{VV} + \int_1 d\sigma^{S_1} + \int_2 d\sigma^{S_{22}} \right)
\]

1 unresolved (soft/colliner) & 1 resolved emission (NLO-like) \\
2 unresolved (soft/colliner) emissions
NNLO local subtraction

\[ \sigma_{\text{NNLO}} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} (d\sigma^R - d\sigma^S) + \int_{\Phi_B} \left( d\sigma^V + \int_1 d\sigma^S \right) \]

\[ + \int_{\Phi_{B+2}} (d\sigma^{RR} - d\sigma^{S_{21}} - d\sigma^{S_{22}}) + \int_{\Phi_{B+1}} \left( d\sigma^{RV} - d\sigma^S_1 + \int_1 d\sigma^{S_{21}} \right) + \int_{\Phi_B} \left( d\sigma^{VV} + \int_1 d\sigma^S_1 + \int_2 d\sigma^{S_{22}} \right) \]

1 unresolved (soft/colliner) & 1 resolved emission (NLO-like)

2 unresolved (soft/colliner) emissions

cancels \(1/\epsilon^n\) poles of RV

\[ \text{LO INGREDIENTS FOR A CALCULATION (generic } 2 \rightarrow 2 \text{ process)} \]
\[ \sigma_{\text{NNLO}} = \int_{\Phi_B} \sigma_B + \int_{\Phi_{B+1}} (\sigma^R - \sigma^S) + \int_{\Phi_B} \left( \sigma^V + \int_1 \sigma^S \right) \\
+ \int_{\Phi_{B+2}} (\sigma^{RR} - \sigma^{S_{21}} - \sigma^{S_{22}}) + \int_{\Phi_{B+1}} \left( \sigma^{RV} - \sigma^{S_1} + \int_1 \sigma^{S_{21}} \right) + \int_{\Phi_B} \left( \sigma^{VV} + \int_1 \sigma^{S_1} + \int_2 \sigma^{S_{22}} \right) \]

1 unresolved (soft/colliner) & 1 resolved emission (NLO-like)  
2 unresolved (soft/colliner) emissions  
subtracts one unresolved emission (NLO-like)  
cancels $1/\epsilon^n$ poles of RV  

LLL
\[ \sigma_{\text{NNLO}} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} (d\sigma^R - d\sigma^S) + \int_{\Phi_B} \left( d\sigma^V + \int_1 d\sigma^S \right) \]

\[ + \int_{\Phi_{B+2}} (d\sigma^{RR} - d\sigma^{S_{21}} - d\sigma^{S_{22}}) + \int_{\Phi_{B+1}} (d\sigma^{RV} - d\sigma^{S_1}) + \int_1 d\sigma^{S_{21}} \]

\[ + \int_{\Phi_B} \left( d\sigma^{VV} + \int_1 d\sigma^S + \int_2 d\sigma^{S_{22}} \right) \]

1 unresolved (soft/colliner) & 1 resolved emission (NLO-like)

2 unresolved (soft/colliner) emissions

subtracts one unresolved emission (NLO-like)

cancels $1/\epsilon^n$ poles of RV

sum has to cancel all $1/\epsilon^n$ poles of $VV$
\[ \sigma_{\text{NNLO}} = \int_{\Phi_B} d\sigma^B + \int_{\Phi_{B+1}} (d\sigma^R - d\sigma^S) + \int_{\Phi_B} \left( d\sigma^V + \int_1 d\sigma^S \right) \]

\[ + \int_{\Phi_{B+2}} (d\sigma^{RR} - d\sigma^{S21} - d\sigma^{S22}) + \int_{\Phi_{B+1}} \left( d\sigma^{RV} - d\sigma^S \right) + \int_1 d\sigma^S \]

1 unresolved (soft/colliner) & 1 resolved emission (NLO-like)

2 unresolved (soft/colliner) emissions

subtracts one unresolved emission (NLO-like)

cancels \(1/\epsilon^n\) poles of RV

sum has to cancel all \(1/\epsilon^n\) poles of VV

Antenna subtraction
Kosower; Gehrmann, Gehrmann-de Ridder, Glover

Stripper
Czakon

Sector-improved residue subtraction
Boughezal, Melnikov, Petriello

CoLorFul subtraction
Del Duca, Somogyi, Troscanyi

Local analytic sector subtraction
Bertolotti, Magna, Maina, Pelliccioli, Ratti, Signorile-Signorile, Torrielli, Uccirati
NNLO through X+jet at NLO + Slicing
NNLO through X+jet at NLO + Slicing

\[
\sigma_{\text{NLO}}^{X+\text{jet}} = \int_{\Phi_{\text{RV}}} d\sigma^{\text{RV}} + \int_{\Phi_{\text{RV}+1}} (d\sigma^{\text{RR}} - d\sigma^{\text{S}}) + \int_{\Phi_{\text{RV}}} \left( d\sigma^{\text{RV}} + \int_1 d\sigma^{\text{S}} \right)
\]

\(d\sigma^{\text{S}}\): subtraction term
  → Dipole [Catani, Seymour '96]
  → FKS [Frixione, Kunszt, Signer '96]
  → Antenna [Gehrmann et al. '05]
  → ...

LO
(pp→WZ)

NLO
(pp→WZ+jet)

NNLO
(pp→WZ+jet)
NNLO through X+jet at NLO + Slicing

\[
\sigma_{\text{NLO}}^{X+\text{jet}} = \left[ \int_{\Phi_{RV}} \, d\sigma^{RV} + \int_{\Phi_{RV+1}} \, (d\sigma^{RR} - d\sigma^{S}) + \int_{\Phi_{RV}} \, (d\sigma^{RV} + \int_{1} \, d\sigma^{S}) \right] \frac{q_T}{Q} \equiv r > r_{\text{cut}}
\]

\[
\rightarrow \left[ A \cdot \log^4 (r_{\text{cut}}) + B \cdot \log^3 (r_{\text{cut}}) + C \cdot \log^2 (r_{\text{cut}}) + D \cdot \log (r_{\text{cut}}) \right] \otimes d\sigma^B
\]
NNLO through X+jet at NLO + Slicing

\[
\sigma_{NLO}^{X+jet} = \left[ \int_{\Phi_{RV}} d\sigma^{RV} + \int_{\Phi_{RV+1}} (d\sigma^{RR} - d\sigma^{S}) + \int_{\Phi_{RV}} \left( d\sigma^{RV} + \int_{1} d\sigma^{S} \right) \right] \frac{q_T}{Q} \equiv r > r_{cut}
\]

\[
r_{cut} \ll 1 \rightarrow \left[ A \cdot \log^4(r_{cut}) + B \cdot \log^3(r_{cut}) + C \cdot \log^2(r_{cut}) + D \cdot \log(r_{cut}) \right] \otimes d\sigma^{B}
= \int_{r > r_{cut}} \left[ d\sigma^{(res)} \right]_{f.o.} \equiv \Sigma_{NNLO}(r_{cut}) \otimes d\sigma^{B}
\]

[Collins, Soper, Sterman '85]
[Bozzi, Catani, de Florian, Grazzini '06]
NNLO through $X$+jet at NLO + Slicing

\[
\sigma_{NLO}^{X+jet} = \left[ \int_{\Phi_{RV}} d\sigma_{RV} + \int_{\Phi_{RV+1}} (d\sigma_{RR} - d\sigma_{S}) + \int_{\Phi_{RV}} (d\sigma_{RV} + \int_1 d\sigma_{S}) \right] \frac{q_r^+}{q_r^-} \equiv r > r_{cut}
\]

\[
\int_{r > r_{cut}} \left[ A \cdot \log^4(r_{cut}) + B \cdot \log^3(r_{cut}) + C \cdot \log^2(r_{cut}) + D \cdot \log(r_{cut}) \right] \otimes d\sigma^B
\]

\[
\int_{r > r_{cut}} \left[ d\sigma^{(res)} \right] \text{f.o.} \equiv \Sigma_{NNLO}(r_{cut}) \otimes d\sigma^B
\]

\[
d\sigma_{NNLO}^X = \left| d\sigma_{NLO}^{X+jet} \right|_{r > r_{cut}} - \Sigma_{NNLO}(r_{cut}) \otimes d\sigma^B
\]
NNLO through X+jet at NLO + Slicing

\[
d\sigma_{\text{NNLO}}^X = \left [ \left. d\sigma_{\text{NLO}}^{X+\text{jet}} \right|_{r > r_{\text{cut}}} - \Sigma_{\text{NNLO}}(r_{\text{cut}}) \otimes d\sigma^B \right ] + \]

**q_T subtraction**

[Catani, Grazzini '07]

**LO**

\[
\begin{align*}
\bar{d} & \rightarrow d \rightarrow Z/\gamma \rightarrow l^+ l^- \\
& \rightarrow d\sigma^B
\end{align*}
\]

**NLO**

\[
\begin{align*}
\bar{d} & \rightarrow d \rightarrow W^+ \rightarrow l^+ \nu_l \\
& \rightarrow d\sigma^V
\end{align*}
\]

**NNLO**

\[
\begin{align*}
\bar{d} & \rightarrow d \rightarrow W^+ \rightarrow l^+ \nu_l \\
& \rightarrow d\sigma^{VV}
\end{align*}
\]

**LO**

\[
\begin{align*}
\bar{d} & \rightarrow d \rightarrow Z/\gamma \rightarrow l^+ l^- \\
& \rightarrow d\sigma^R
\end{align*}
\]

**NLO**

\[
\begin{align*}
\bar{d} & \rightarrow d \rightarrow W^+ \rightarrow l^+ \nu_l \\
& \rightarrow d\sigma^{RV}
\end{align*}
\]

**NNLO**

\[
\begin{align*}
\bar{d} & \rightarrow d \rightarrow W^+ \rightarrow l^+ \nu_l \\
& \rightarrow d\sigma^{RR}
\end{align*}
\]
NNLO through $X$+jet at NLO + Slicing

\[ d\sigma_{\text{NNLO}}^X = \left[ d\sigma_{\text{NLO}}^{X+\text{jet}} \bigg|_{r > r_{\text{cut}}} - \Sigma_{\text{NNLO}}(r_{\text{cut}}) \otimes d\sigma^B \right] + \mathcal{H}_{\text{NNLO}} \otimes d\sigma^B \]

$q_T$ subtraction

[Catani, Grazzini '07]
$r_{\text{cut}} \to 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]

automatically computed in every single MATRIX NNLO run

\[
\sigma / \sigma_{\text{NNLO}} - 1\% = \left[ d\sigma_{\text{NNLO}}^{X} \bigg|_{r > r_{\text{cut}}} - \Sigma_{\text{NNLO}}(r_{\text{cut}}) \otimes d\sigma^{B} \right] + \mathcal{H}_{\text{NNLO}} \otimes d\sigma^{B}
\]

\[
d\sigma_{\text{NNLO}}^{X} = d\sigma_{\text{NLO}}^{X+\text{jet}} \bigg|_{r > r_{\text{cut}}} - \Sigma_{\text{NNLO}}(r_{\text{cut}}) \otimes d\sigma^{B}
\]
$r_{\text{cut}} \rightarrow 0$ extrapolation in MATRIX

[Grassini, Kallweit, MW '17]

simple quadratic fit ($A \cdot r_{\text{cut}}^2 + B \cdot r_{\text{cut}} + C$) to extrapolate to $r_{\text{cut}} = 0$

\[
\frac{\sigma}{\sigma_{\text{NNLO}}} - 1\% \quad pp \rightarrow Z \oplus 13 \text{ TeV}
\]

\[
d\sigma^X_{\text{NNLO}} = \left[ \frac{d\sigma^{X+\text{jet}}_{\text{NLO}}}{r > r_{\text{cut}}} - \sum_{\text{NNLO}} (r_{\text{cut}}) \otimes d\sigma^B \right] + \mathcal{H}_{\text{NNLO}} \otimes d\sigma^B
\]
$r_{\text{cut}} \rightarrow 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]

\[
d\sigma^X_{\text{NNLO}} = \left[ d\sigma^{X+\text{jet}}_{\text{NLO}} \big|_{r > r_{\text{cut}}} - \sum_{\text{NNLO}} (r_{\text{cut}}) \otimes d\sigma^B \right] + \mathcal{H}_{\text{NNLO}} \otimes d\sigma^B
\]
$r_{\text{cut}} \to 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]

\[ \frac{\sigma}{\sigma_{\text{NNLO}}} - 1\% \]

Drell-Yan

\[ pp \to Z @ 13 \text{ TeV} \]

\[ d\sigma^X_{\text{NNLO}} = \left[ d\sigma^{X+\text{jet}}_{\text{NLO}} \bigg| r > r_{\text{cut}} \right] - \Sigma_{\text{NNLO}}(r_{\text{cut}}) \otimes d\sigma^B + \mathcal{H}_{\text{NNLO}} \otimes d\sigma^B \]

[Hamberg, van Neerven, Matsuura '91]
$r_{\text{cut}} \to 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]
Explosion of NNLO results

...slide borrowed from Gavin Salam
Remarkable progress in the development of methods to perform NNLO computations!
Example #1: R-ratio

\[
\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \alpha_s(\sqrt{s_{e^+e^-}})
\]

\[
= R_0 \left(1 + 0.32\alpha_s + 0.14\alpha_s^2 - 0.47\alpha_s^3 - 0.59316\alpha_s^4 + \cdots\right)
\]

\[\text{Baikov et al., 1206.1288}\]

(numbers for $\gamma$-exchange only)

This is one of the few quantities calculated to N4LO

Good convergence of the series at every order
(at least for $\alpha_s(M_Z) = 0.118$)

...slide borrowed from Gavin Salam
Example #2: Higgs production

\[ \sigma(pp \rightarrow H) = (961 \text{ pb}) \times (\alpha_s^2 + 10.4\alpha_s^3 + 38\alpha_s^4 + 48\alpha_s^5 + \cdots) \]

\[ \alpha_s \equiv \alpha_s(M_H/2) \]

\[ \sqrt{s_{pp}} = 13 \text{ TeV} \]

Anastasiou et al., 1602.00695 (ggF, hEFT)

\[ pp \rightarrow H \text{ (via gluon fusion) is one of only few hadron-collider processes known at N3LO} \]

The series does not converge well
(explanations for why are only moderately convincing)

...slide borrowed from Gavin Salam
Example #2: Higgs production

Convention: "theory uncertainty" (i.e. from missing higher orders) is estimated by change of cross section when varying $\mu$ in range $1/2 \rightarrow 2$ around central value.

Higgs cross section (EFT)

$\sigma(pp \rightarrow H)$ [pb]

$\mu_0 = m_H/2$

measurement

[ATLAS-CONF-2020-027]

prediction

$\mathcal{E}_T$, NLO, pp 13 TeV, PDF4LHC15, $\mu_F = \mu_R = \mu_0$. 

$\mathcal{E}_T$, NLO, pp 13 TeV, PDF4LHC15, $\mu_F = \mu_R = \mu_0$. 

$\mathcal{E}_T$, NLO, pp 13 TeV, PDF4LHC15, $\mu_F = \mu_R = \mu_0$. 

$\mathcal{E}_T$, NLO, pp 13 TeV, PDF4LHC15, $\mu_F = \mu_R = \mu_0$. 

$\mathcal{E}_T$, NLO, pp 13 TeV, PDF4LHC15, $\mu_F = \mu_R = \mu_0$. 

$\mathcal{E}_T$, NLO, pp 13 TeV, PDF4LHC15, $\mu_F = \mu_R = \mu_0$. 

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$\mathcal{E}_T$, NLO, pp 13 TeV, PDF4LHC15, $\mu_F = \mu_R = \mu_0$. 

$\mathcal{E}_T$, NLO, pp 13 TeV, PDF4LHC15, $\mu_F = \mu_R = \mu_0$. 

$\mathcal{E}_T$, NLO, pp 13 TeV, PDF4LHC15, $\mu_F = \mu_R = \mu_0$. 

$\mathcal{E}_T$, NLO, pp 13 TeV, PDF4LHC15, $\mu_F = \mu_R = \mu_0$. 

$\mathcal{E}_T$, NLO, pp 13 TeV, PDF4LHC15, $\mu_F = \mu_R = \mu_0$. 

$\mathcal{E}_T$, NLO, pp 13 TeV, PDF4LHC15, $\mu_F = \mu_R = \mu_0$. 

$\mathcal{E}_T$, NLO, pp 13 TeV, PDF4LHC15, $\mu_
Example #2: Higgs production

Convention: “theory uncertainty” (i.e. from missing higher orders) is estimated by change of cross section when varying $\mu$ in range $1/2 \rightarrow 2$ around central value.

Scale dependence as the “THEORY UNCERTAINTY”

Here, only the renorm. scale $\mu$ has been varied. In real life you need to change renorm. and factorisation scales.

Higgs cross section (EFT)

$\sigma(pp \rightarrow H)$ [pb]

$\mu_0 = m_H/2$

measurement [ATLAS-CONF-2020-027]

LO prediction

$\sigma(pp \rightarrow H)$ [pb]
Example #2: Higgs production

Higgs cross section (EFT)

\( \sigma(pp \rightarrow H) \) [pb]

- **LO**
- **NLO**

\( \mu_0 = m_H/2 \)

Scale dependence as the "THEORY UNCERTAINTY"

Convention: "theory uncertainty" (i.e. from missing higher orders) is estimated by change of cross section when varying \( \mu \) in range \( 1/2 \rightarrow 2 \) around central value.
Example #2: Higgs production

Higgs cross section (EFT)

\[ \sigma(pp \rightarrow H) \text{ [pb]} \]

- **LO**
- **NLO**
- **NNLO**

\( \mu_0 = m_H/2 \)

Scale dependence as the "THEORY UNCERTAINTY"
Example #2: Higgs production

Convention: “theory uncertainty” (i.e. from missing higher orders) is estimated by change of cross section when varying $\mu$ in range $1/2 \rightarrow 2$ around central value.

Higgs cross section (EFT)

$\sigma(pp \rightarrow H)$ [pb]

$\mu_0 = m_H/2$

[Anastasiou et al. '15], [Mistlberger '18]

see Bernhards's lecture
Convention: “theory uncertainty” (i.e. from missing higher orders) is estimated by change of cross section when varying $\mu$ in range $1/2 \rightarrow 2$ around central value.

Here, only the renorm. scale $\mu$ has been varied. In real life you need to change renorm. and factorisation scales.

...slide borrowed from Gavin Salam
For many processes NNLO scale band is $\pm 2\%$. But only in 3/17 cases is NNLO (central) within NLO scale band...

...slide borrowed from Gavin Salam
NNLO frontier 2 $\rightarrow$ 3 processes

- **massless/one mass (full 2-loop):**
  - $pp \rightarrow \gamma \gamma \gamma$ [Chawdhry, Czakon, Mitov, Poncelet '19], [Kallweit, Sotnikov, MW '20]
  - $pp \rightarrow \gamma \gamma + \text{jet}$ [Chawdhry, Czakon, Mitov, Poncelet '21]
  - $pp \rightarrow 3\text{-jet}$ [Czakon, Mitov, Poncelet '21]
  - $pp \rightarrow \text{bbW} (m_b=0)$ [Hartano, Poncelet, Popescu, Zoia '22]
  - $pp \rightarrow \gamma + 2\text{-jet}$ [Badger, Czakon, Hartano et al. '23]

- **massive (with approximated 2-loop):**
  - $pp \rightarrow \text{ttH} \ (\text{soft approx.})$ [Catani, Devoto, Grazzini et al. '22]
  - $pp \rightarrow \text{bbW} \ (\text{small m}_b)$ [Buonocore, Devoto, Grazzini et al. '23]
  - $pp \rightarrow \text{ttW} \ (\text{both})$ [Buonocore, Devoto, Kallweit et al. '22]
Example #3

- massless/one mass (full 2-loop):
  - $pp \rightarrow \gamma\gamma\gamma$  [Chawdhry, Czakon, Mitov, Poncelet '19], [Kallweit, Sotnikov, MW '20]
  - $pp \rightarrow \gamma\gamma + \text{jet}$  [Chawdhry, Czakon, Mitov, Poncelet '21]
  - $pp \rightarrow 3\text{-jet}$  [Czakon, Mitov, Poncelet '21]

- massive (with approximated 2-loop):
  - $pp \rightarrow ttH$ (soft approx.)  [Catani, Devoto, Grazzini et al. '22]
  - $pp \rightarrow bbW$ (small $m_b$)  [Buonocore, Devoto, Grazzini et al. '23]
  - $pp \rightarrow ttW$ (both)  [Buonocore, Devoto, Kallweit et al. '22]

First 2→3 process at NNLO QCD

$\bar{q} \rightarrow \gamma \rightarrow \gamma \gamma$

+ two-loop five-point function  [Abreu, Page, Pascual, Sotnikov '20]
Example #4

- massless/one mass (full 2-loop):
  - $pp \to \gamma\gamma\gamma$
    [Chawdhry, Czakon, Mitov, Poncelet '19],
    [Kallweit, Sotnikov, MW '20]
  - $pp \to \gamma\gamma + \text{jet}$
    [Chawdhry, Czakon, Mitov, Poncelet '21]
  - $pp \to 3\text{-jet}$
    [Czakon, Mitov, Poncelet '21]

"Tour de force in Quantum Chromodynamics"

LH '17 wishlist

\[
\begin{array}{c|c}
  pp \to 3\text{jets} & \text{NLO}_{\text{QCD}} \\
  \text{(N}^2\text{LO}_{\text{QCD}}) & \\
\end{array}
\]
Example #5

- massless/one mass (full 2-loop):
  - \( pp \rightarrow \gamma \gamma \gamma \) [Chawdhry, Czakon, Mitov, Poncelet '19], [Kallweit, Sotnikov, MW '20]
  - \( pp \rightarrow \gamma \gamma + \text{jet} \) [Chawdhry, Czakon, Mitov, Poncelet '21]
  - \( pp \rightarrow 3\text{-jet} \) [Czakon, Mitov, Poncelet '21]
  - \( pp \rightarrow \bar{b}bW \) (\( m_b = 0 \)) [Hartano, Poncelet, Popescu, Zoia '22]
  - \( pp \rightarrow \gamma + 2\text{-jet} \) [Badger, Czakon, Hartano et '23]

- massive (with approximated 2-loop):
  - \( pp \rightarrow \bar{t}tH \) (soft approx.) [Catani, Devoto, Grazzini et al. '22]
  - \( pp \rightarrow \bar{b}bW \) (small \( m_b \)) [Buonocore, Devoto, Grazzini et al. '23]
  - \( pp \rightarrow \bar{t}tW \) (both) [Buonocore, Devoto, Kallweit et al. '22]

+ two approximations for two-loop

1. \( W \) assumed to be soft and factorizing
2. tops assumed to have small mass

small-mass expansion [Mitov, Moch '06]

\[
2\text{Re}\left(R^{(0)}|R^{(2)}\right) = \sum_{i=1}^{4} \kappa_{i} \log^{i}(m_{t}/\mu_{R}) + 2\text{Re}(R^{(0)}|R^{(2)}) + \mathcal{O}(m_{b}/\mu)
\]

massive amplitude massless amplitude [Badger, Hartano, Krys, Zoia '21]

validation at 1-loop
Example #5

- massless/one mass (full 2-loop):
  - $pp \rightarrow \gamma \gamma \gamma$
    [Chawdhry, Czakon, Mitov, Poncelet '19],
    [Kallweit, Sotnikov, MW '20]
  - $pp \rightarrow \gamma \gamma + \text{jet}$
    [Chawdhry, Czakon, Mitov, Poncelet '21]
  - $pp \rightarrow 3$-jet
    [Czakon, Mitov, Poncelet '21]
  - $pp \rightarrow \bar{b}bW$ ($m_b=0$)
    [Hartano, Poncelet, Popescu, Zoia '22]
  - $pp \rightarrow \gamma + 2$-jet
    [Badger, Czakon, Hartano et al. '23]

- massive (with approximated 2-loop):
  - $pp \rightarrow ttH$ (soft approx.)
    [Catani, Devoto, Grazzini et al. '22]
  - $pp \rightarrow bbW$ (small $m_b$)
    [Buonocore, Devoto, Grazzini et al. '23]
  - $pp \rightarrow ttW$ (both)
    [Buonocore, Devoto, Kallweit et al. '22]
N³LO QCD frontier 2 → 1 processes

- inclusive N³LO calculations:
  - $pp \rightarrow H$ [Anastasiou et al. '15], [Mistlberger '18]
  - $pp \rightarrow Z/W$ [Duhr, Dulat, Mistlberger '20 '20]
  - $pp \rightarrow Hjj$ (VBF) [Dreyer, Karlberg '16]
  - $pp \rightarrow HHjj$ (VBF) [Dreyer, Karlberg '18]

- differential N³LO calculations:
  - $pp \rightarrow H$ [Cieri, Chen, Gehrmann, Glover, Huss '18], [Dulat, Mistlberger, Pelloni '18], [Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni '21], [Billis, Dehnadi, Ebert, Michel, Tackmann '21]
  - $pp \rightarrow \ell\ell$ [Chen, Gehrmann, Glover, Huss, Yang, Zhu '21], [Camarda, Cieri, Ferrera '21], [Neumann, Campbell '22]
  - $pp \rightarrow \ell\nu$ [Chen, Gehrmann, Glover, Huss, Yang, Zhu '21], [Neumann, Campbell '23]
  - $H \rightarrow bb$ [Mondini, Schiavi, Willams '19]
Example #6

- inclusive $N^3$LO calculations:
  - $pp \to H$
  - $pp \to Z/W$
  - $pp \to Hjj$ (VBF)
  - $pp \to HHjj$ (VBF)

- differential $N^3$LO calculations:
  - $pp \to H$
  - $pp \to \ell \ell$ [Chen, Gehrmann, Glover, Huss, Yang, Zhu '21]

$$
\sigma_{N^3LO}^Z = \left[ \sigma_{N^3LO}^{Z+\text{jet}} \bigg|_{r > r_{\text{cut}}} - \Sigma_{N^3LO}(r_{\text{cut}}) \otimes d\sigma^B \right] + \mathcal{H}_{N^3LO} \otimes d\sigma^B
$$

- $N^3$LO for $Z$+jet via Antenna subtraction
- $N^3$LO via $q_T$ slicing
- **inclusive $N^3$LO calculations:**
  - $pp \rightarrow H$
  - $pp \rightarrow Z/W$
  - $pp \rightarrow Hjj$ (VBF)
  - $pp \rightarrow HHjj$ (VBF)

- **differential $N^3$LO calculations:**
  - $pp \rightarrow H$
  - $pp \rightarrow \ell\ell$
  - $pp \rightarrow \ell\nu$ [Neumann, Campbell '23]
  - $H \rightarrow bb$

**Figure 5**

$W^+$ 5.02 TeV

<table>
<thead>
<tr>
<th>$W^+$ σ [pb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2300</td>
</tr>
<tr>
<td>2200</td>
</tr>
<tr>
<td>2100</td>
</tr>
<tr>
<td>2000</td>
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</tbody>
</table>

Uncertainties

- ATLAS data
- NLO
- NNLO
- $N^3$LO

- **Uncertainties**
  - Scale + 0.5% num.
  - MSHT20 PDF

- **NNLO for $W$+jet via 1-jettiness slicing**
- **$N^3$LO via $q_T$ slicing**
EW corrections

★ EW corrections just like (abelian version of) QCD corrections, and yet different…

NLO QCD

NLO EW

EW corrections just like (abelian version of) QCD corrections, and yet different…

W/Z

ɣ

ɣ

W/Z
EW corrections

★ EW corrections just like (abelian version of) QCD corrections, and yet different…

NLO QCD

NLO EW

cancellation of IR singularities
EW corrections

★ EW corrections just like (abelian version of) QCD corrections, and yet different…

NLO QCD

NLO EW

cancellation of IR singularities

IR singularities regulated by $m_{Z/W}$

→ separately finite

→ real Z’s/W’s can be measured
EW corrections

★ EW corrections just like (abelian version of) QCD corrections, and yet different…

NLO QCD

NLO EW

cancellation of IR singularities

IR singularities regulated by $m_{Z/W}$

$\rightarrow$ separately finite

$\rightarrow$ real Z’s/W’s can be measured

$\rightarrow$ large EW Sudakov logs:

$$\alpha^n \log^k \left( \frac{s}{m_{Z/W}^2} \right), \quad k \leq 2n$$
Example #8

[Grazzini, Kallweit, Lindert, Pozzorini, MW '19]

\[ pp \rightarrow \ell^- \ell^+ \nu \bar{\nu} \ell \quad \text{LHC } \sqrt{s} = 13 \text{ TeV} \]

- \( H_T^{\text{jet}} \) < 0.2 \( H_T^{\text{lep}} \)

- \( \frac{d\sigma}{dp_{T,v_2}} \) [fb/GeV]
- \( \frac{d\sigma}{dp_{T,v_2}} - 1\% \)

**NLO EW effect**
EW corrections

[Grazzini, Kallweit, Lindert, Pozzorini, MW '19]

\[ pp \rightarrow \ell^- \ell^+ \nu \bar{\nu}_\ell \ \text{LHC} \ \sqrt{s} = 13 \text{ TeV} \]

\[ W^+W^- \]

\[ H_T^{\text{jet}} < 0.2 \ H_T^{\text{lep}} \]

Precision Calculations in QCD and EWK Interactions at Hadron Colliders

August 5, 2024

Marius Wiesemann (MPP Munich)
Summary so far

★ High energy colliders allow us to probe fundamental interactions among elementary particle in a controlled environment at very short distances, but it requires that SM Physics has to be described with:

★ physical observables that can be reliably calculated and measured at the same time

★ accurate+precise predictions (and measurements)
  -- very difficult & highly advanced technology
Summary so far

Theory predictions reached an accuracy considered impossible some years ago:

<table>
<thead>
<tr>
<th>Order</th>
<th>Details</th>
</tr>
</thead>
</table>
| LO    | fully automated  
Edge: 10-12 particles in the final state |
| NLO   | fully automated  
Edge: 4-6 particles in the final state |
| NNLO  | dedicated calculations, few public codes  
essentially all $2 \rightarrow 2$ reactions, several $2 \rightarrow 3$ recently |
| $N^3$LO | first few calculations  
only $2 \rightarrow 1$ reactions so far, but differential recently |
Many Theory Aspects NOT Talked About

★ Resummation and Event Generation
(highly sophisticated methods to describe soft physics in collider processes; high-accuracy analytic resummation; improving shower accuracy; matching to NNLO; ...)

★ How to do loop calculations in detail
(five-point functions for $2 \rightarrow 3$ processes currently being solved; four-point functions with internal masses for $2 \rightarrow 2$ processes; ...)

★ PDF determination from data

★ Extraction of SM parameters (couplings, masses, ...)

★ ...
Many Theory Aspects NOT Talked About

★ Resummation and Event Generation \( \rightarrow \) tomorrow in lecture 2
(highly sophisticated methods to describe soft physics in collider processes; high-accuracy analytic resummation; improving shower accuracy; matching to NNLO; ...)

★ How to do loop calculations in detail
(five-point functions for \( 2 \rightarrow 3 \) processes currently being solved; four-point functions with internal masses for \( 2 \rightarrow 2 \) processes; ...)

★ PDF determination from data

★ Extraction of SM parameters (couplings, masses, ...)

Thank you very much for your attention!
Extra Slides
Figure 2: Dependence of the NNLO cross sections on the r_{cut} → 0 extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]
$r_{\text{cut}} \to 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]