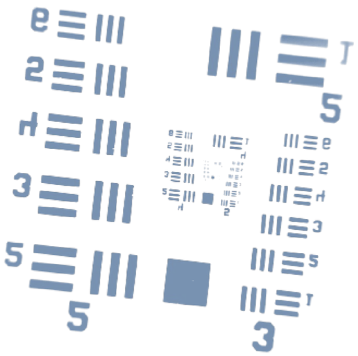
Spatial Frequency Domain Investigations of the Brighter-Fatter Effect

Dan Weatherill, Ian Shipsey,
Daniela Bortoletto, Richard Plackett

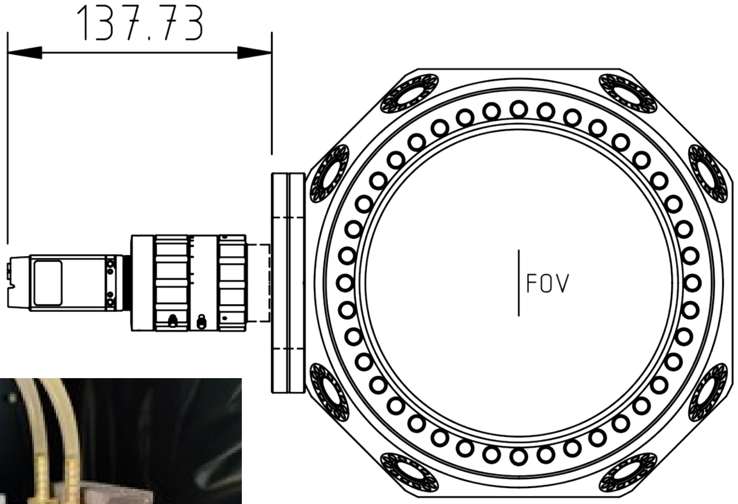
Oxford Physics Microstructure Detector Laboratory

Outline

- Intro to laser speckle & prior work
- Using laser speckle to measure MTF / PSF
- (**wonky**) – alternate method to calculate brighter-fatter correlations
- Using laser speckle to get detector gain
- Looking at evolution of correlations with integration time

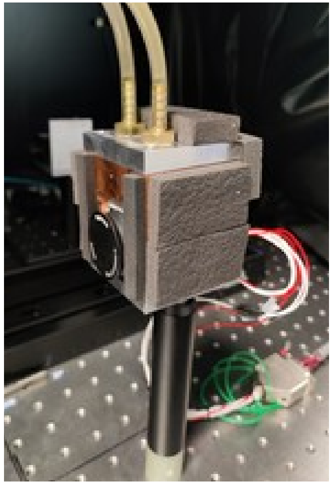


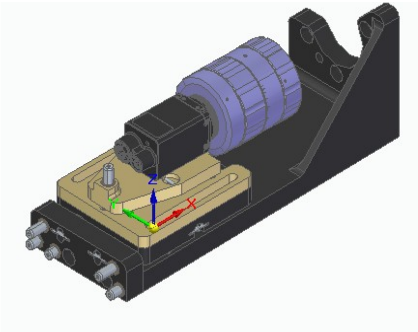
Motivations / Contexts



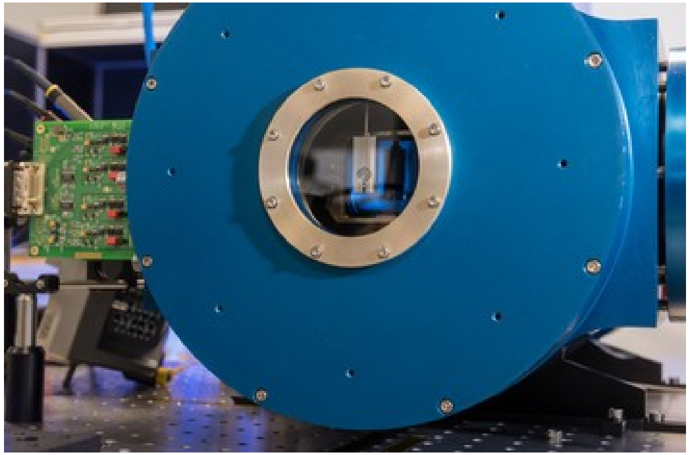
Laser speckle is a powerful way to measure sensor MTF.

Advantages for small pixel sizes (e.g. sony IMX541, left) – no refractive optics involved, no need to know and de-convolve those optics

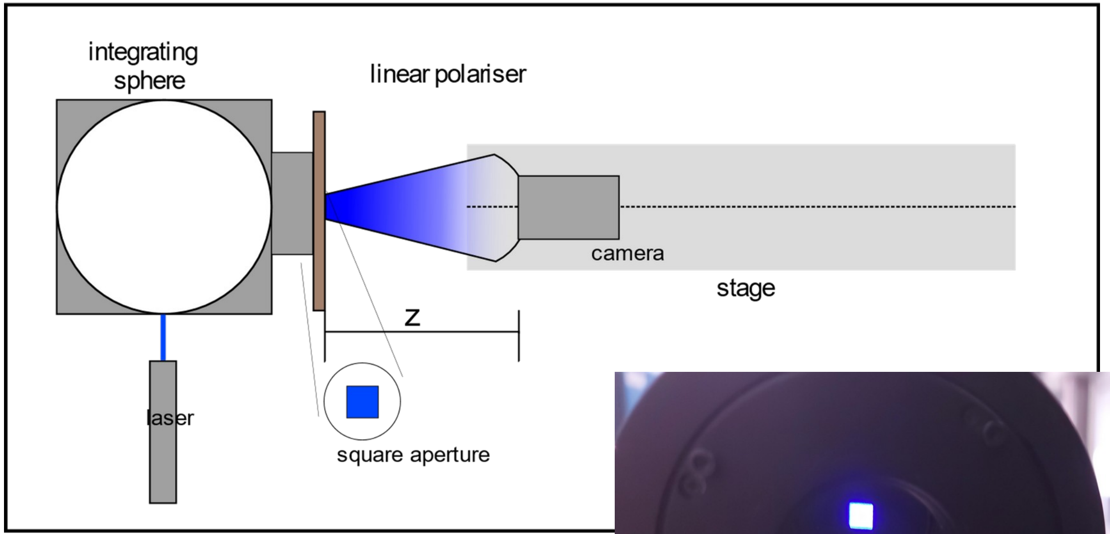




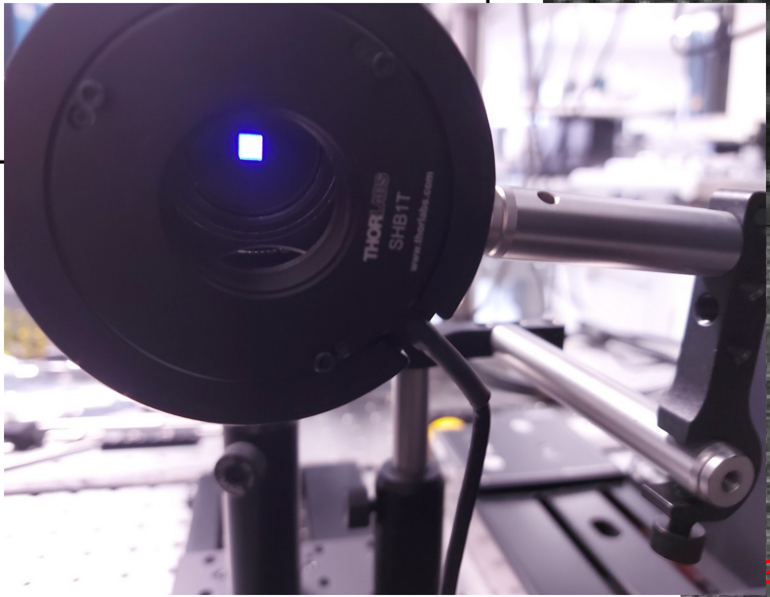
Larger pixels (e.g. 10um) – can illuminate large area with small device, has interesting interactions with brighter-fatter effect

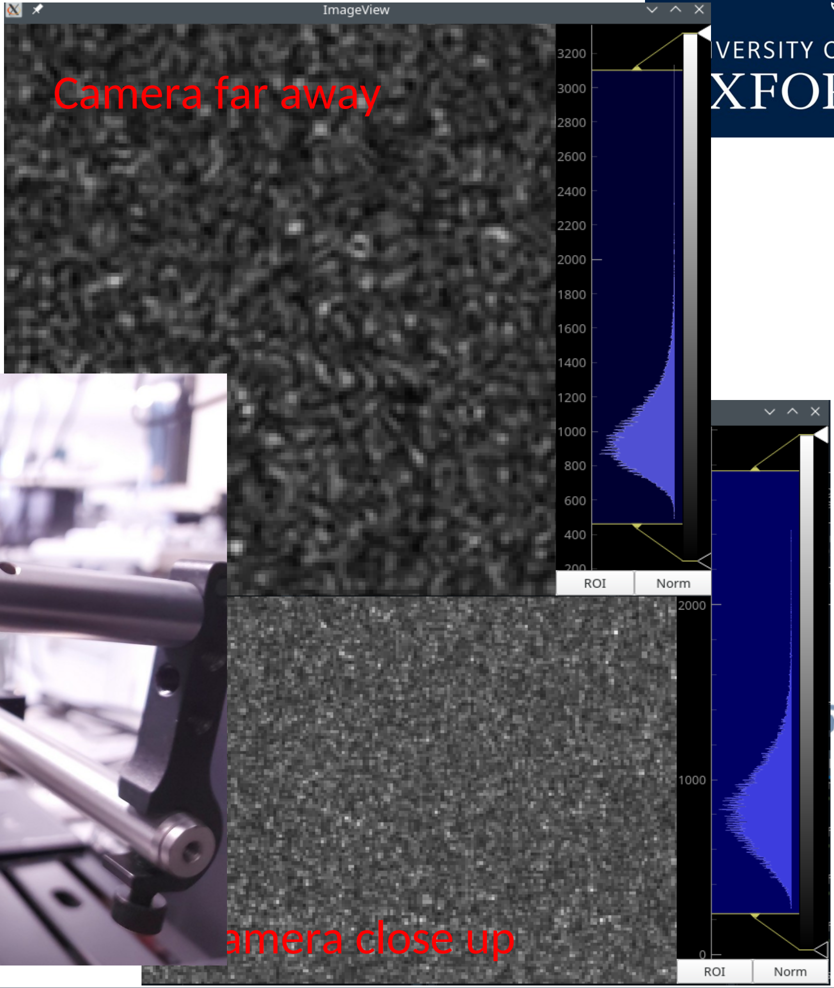


Background: Laser Speckle

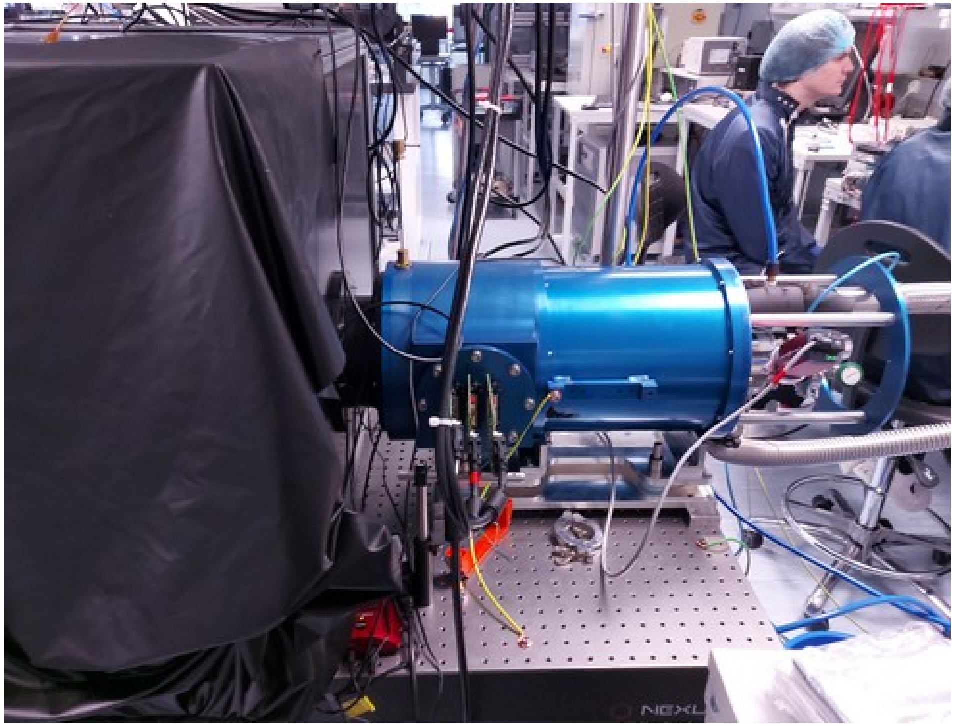


Shine a laser on a microscopically rough surface, pass through an aperture and a linear polariser. Resulting spatial noise pattern has **known** 1st and 2nd order statistics!

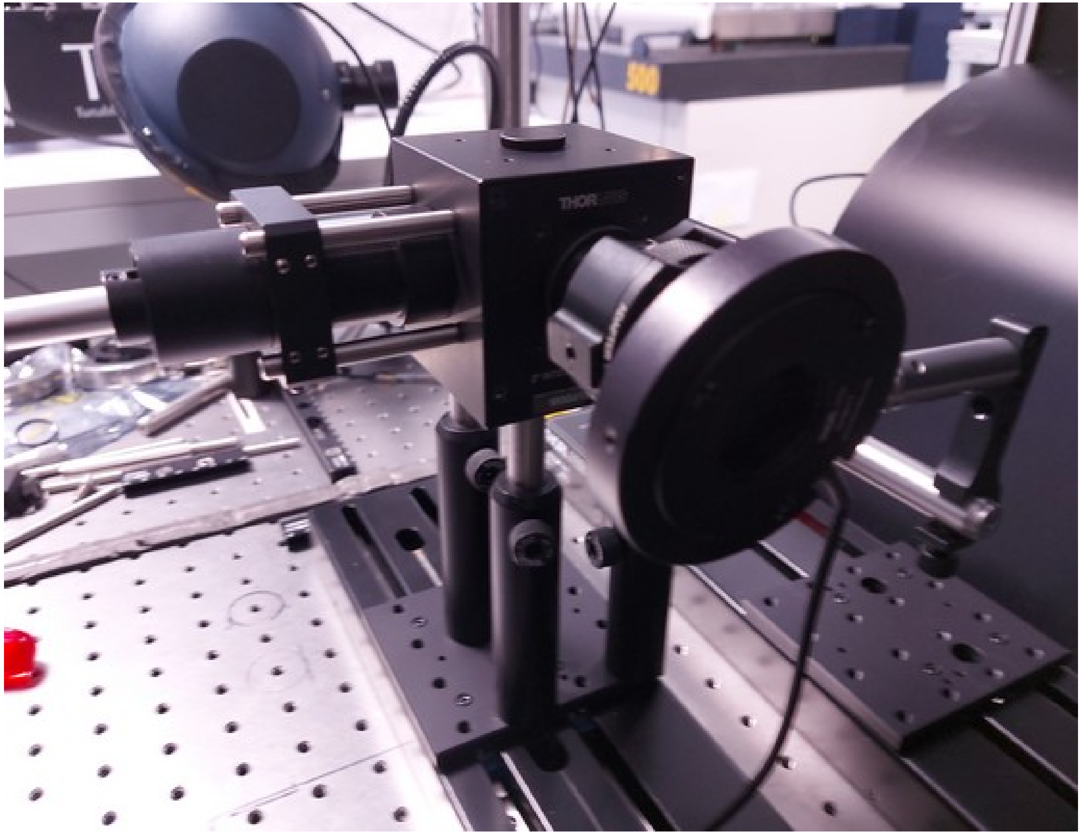




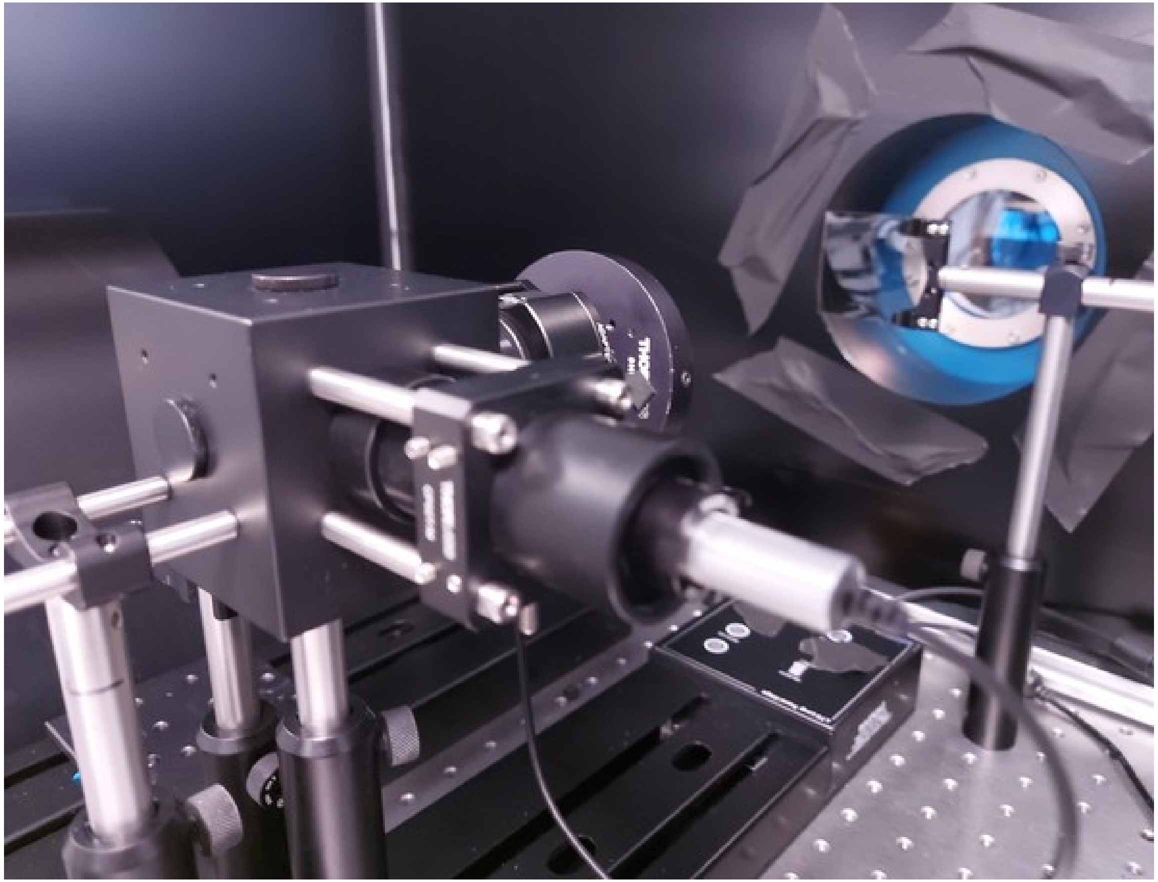
Hardware #1



N.b. in our setup for an te2v CCD250, we have the source moving rather than the detector



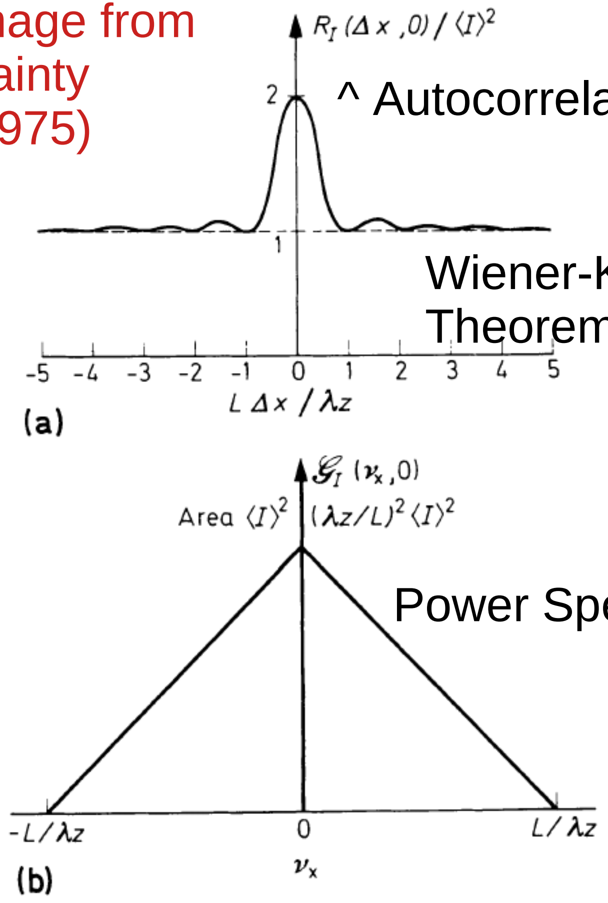
Hardware #2



Note the speckle pattern is **static** in time, provided you can get system vibrations low enough. In our case that lead to e.g. changing shutter mountings and other minor details.

Laser Speckle: Theory

Image from Dainty (1975)

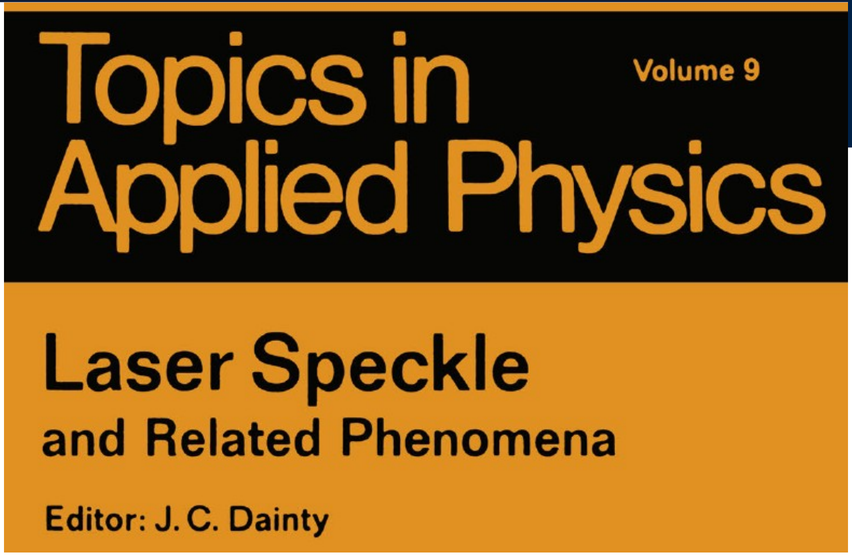


^ Autocorrelation function!

Wiener-Khintchin Theorem

Power Spectral Density (PSD)

Fig. 2.15a and b. Form of the (a) autocorrelation function and (b) power spectral density of speckle produced by a square scattering spot



$$R_I(\Delta x, \Delta y) = \langle I\rangle^2\left[1+\mathrm{sinc}^2\frac{L\Delta x}{\lambda z}\,\mathrm{sinc}^2\frac{L\Delta y}{\lambda z}\right],$$

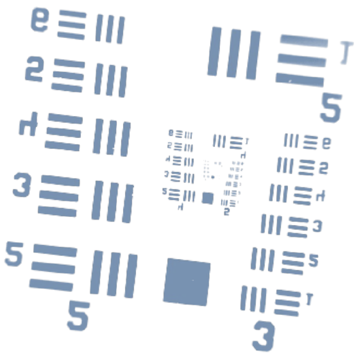
$$\mathcal{G}_I(\nu_X,\nu_Y)=\langle I\rangle^2\left[\delta(\nu_X,\nu_Y)+\left(\frac{\lambda z}{L}\right)^2\Lambda\left(\frac{\lambda z}{L}\nu_X\right)\Lambda\left(\frac{\lambda z}{L}\nu_Y\right)\right]$$

$$\Lambda(x)=1-|x| \quad \text{for} \quad |x|\le 1,$$

Key point

All of the party tricks that follow (and plenty more I haven’t had time to investigate yet!)   depend on this:

you don’t need to actually know the Fourier Transform, just the power spectrum!



Laser Speckle MTF Measurement History

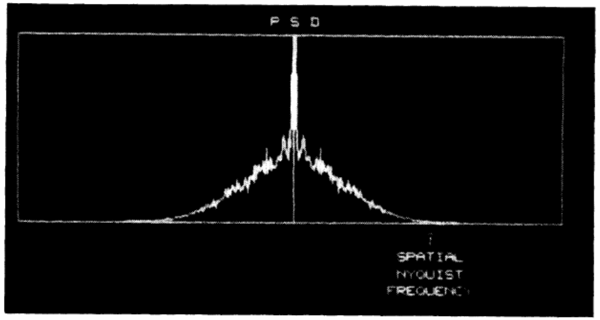
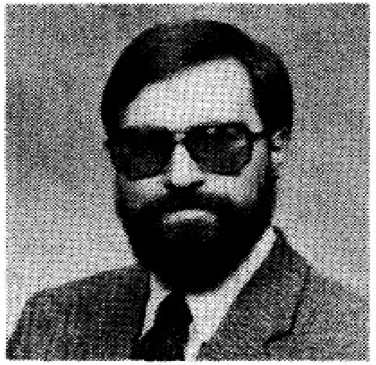


Fig. 6. PSD measured for speckle inside the shadow region. The arrow indicates the spatial Nyquist frequency of 21.5 cycles/mm.



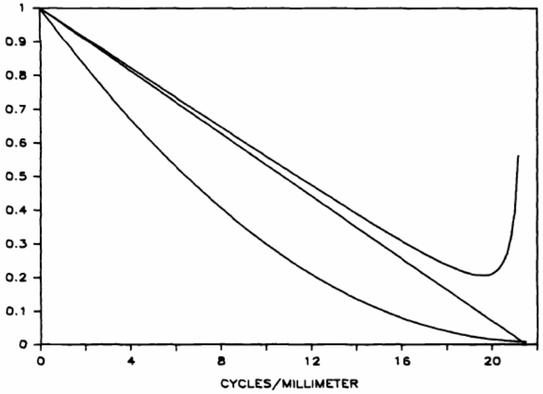


Fig. 8. Upper curve is the calculated MTF of the detector array. The middle curve is the input PSD of the speckle. The bottom curve is the polynomial fit to the normalized output PSD of the speckle from Fig. 7.

Images from Boreman et al, SPIE Op Eng 29, 1990 first demonstrated attempted MTF measurement via laser speckle and taking the power spectral density

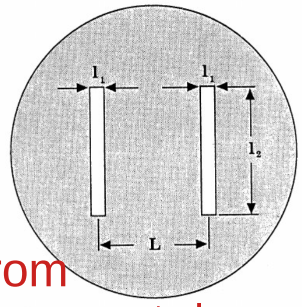


Figure 2. One-dimensional aperture.

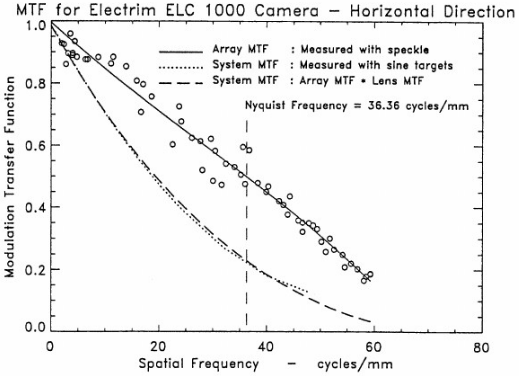


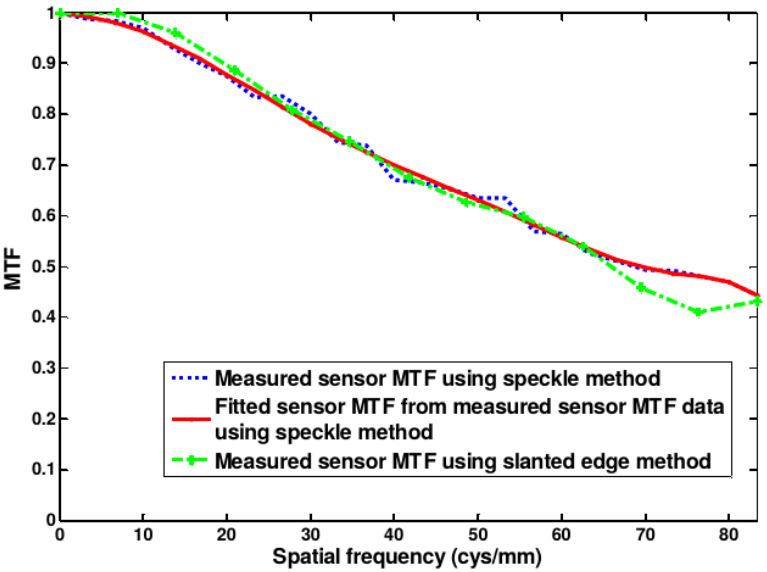
Figure 6. MTF of Electrim camera array using laser speckle and system MTF with sine targets vs system MTF as an array MTF times lens MTF.

Images from Sensiper, Boreman et al (1992)

Later, Sensiper, Boreman et al (1992) developed this method
Further using spaced slit aperture.

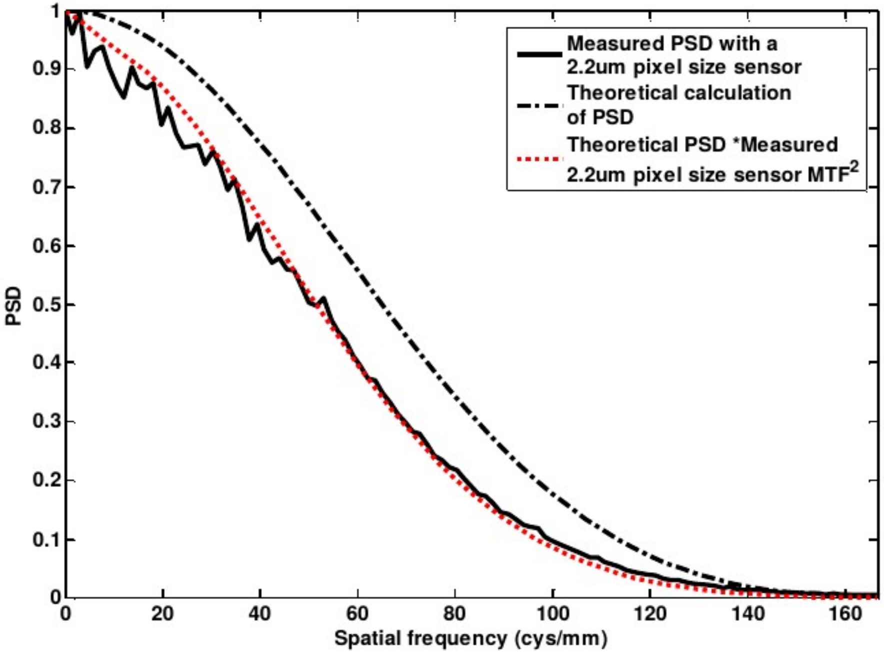
This has a known cutoff frequency, by moving the aperture along the optic axis, MTF can be reconstructed
Similar to our new method but with less statistical power

Past Work (2)



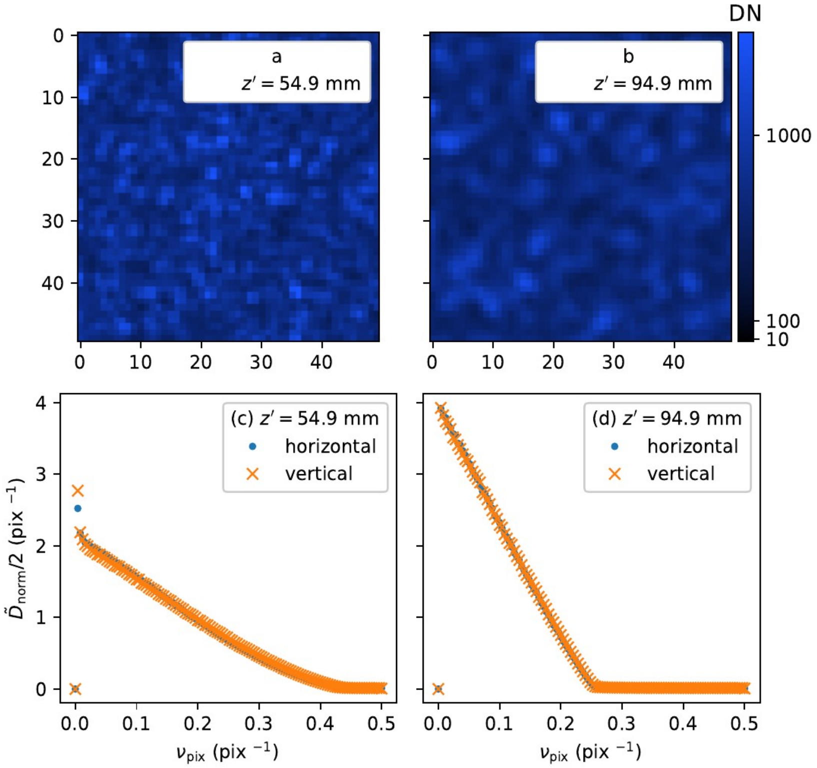
Chen et al (2008) did a **very** impressive measurement of speckle MTF and compared with slanted edge projection on a 2.2um pitch sensor.

However, to get this measurement beyond Nyquist they had to oversample the speckle by moving the detector laterally by half a pixel!!



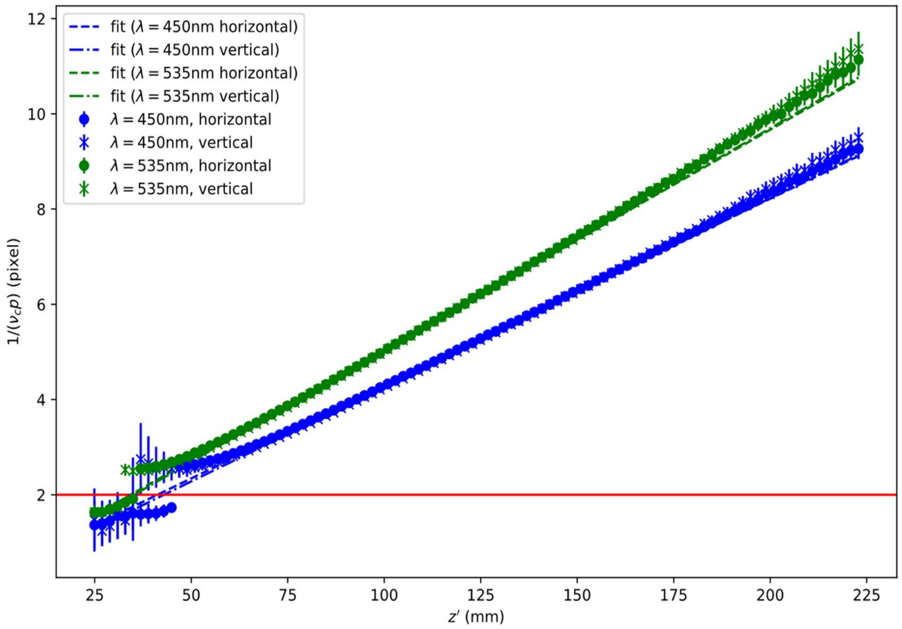
Chen et al method is highly sensitive to the accuracy of these lateral stage movements.
Also it requires an incredibly vibration stable speckle pattern (the exact same pattern must be imaged 4 times to get the oversampling)
It has the major advantage that only one projection distance is needed to reconstruct the full MTF
All above methods require pixel size to be known beforehand!

Laser Speckle MTF: Some improvements



- You **know** the shape of the PSD that entered the sensor (at least in the windowless case!), except for the **cutoff frequency**
- You can use the constraints of parseval's theorem to normalise the power spectral density, and from there, a highly constrained fit can determine the cutoff frequency.
- You can now use any type of deconvolution to remove the input PSD, and the result is the square of the MTF

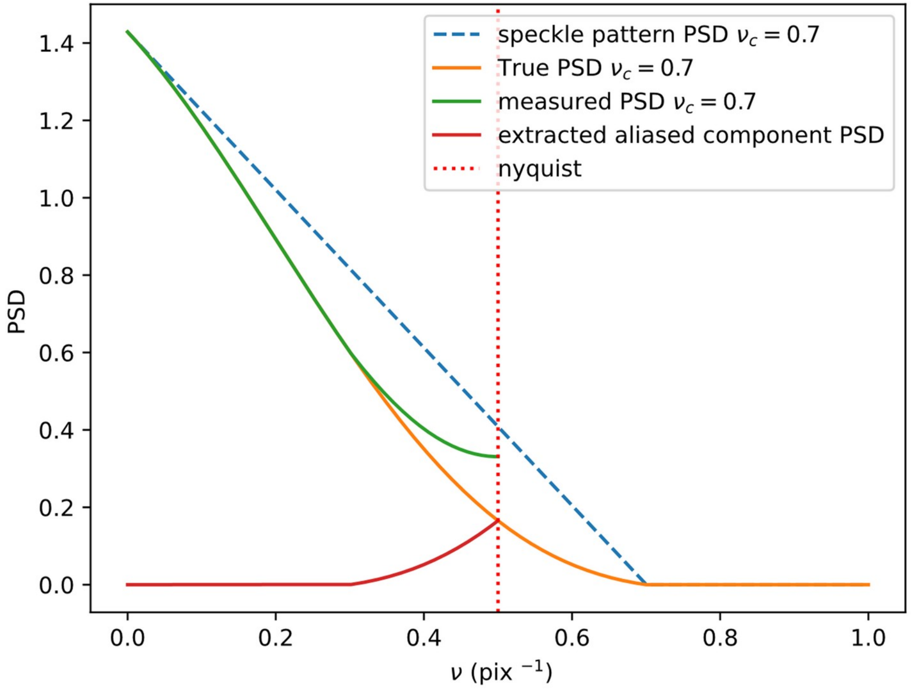
$$PSD_{\mathrm{meas}}(\nu) = MTF(\nu)^2 \times PSD_{\mathrm{speckle}}(\nu)$$

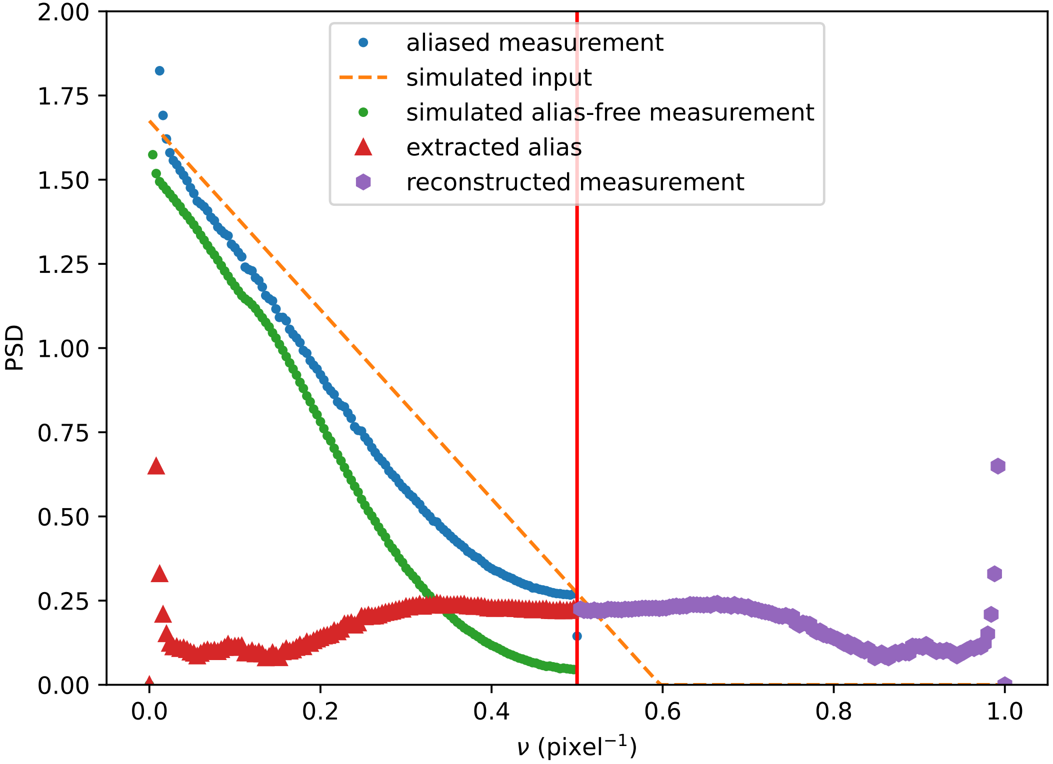


direction	λ (nm)	fitted gradient (m^{-1})	z_{offset} (mm)	p (μm)
horizontal	450	(39.2 ± 0.1)	(9.9 ± 0.4)	(2.80 ± 0.01)
vertical	450	(39.7 ± 0.1)	(7.5 ± 0.3)	(2.76 ± 0.02)
horizontal	535	(46.3 ± 0.2)	(9.0 ± 0.4)	(2.82 ± 0.01)
vertical	535	(46.7 ± 0.2)	(7.7 ± 0.3)	(2.79 ± 0.02)

- The theory says that the cutoff frequency depends reciprocally on distance, left shows fit to 1/cutoff freq vs distance.
- There are some experimental effects due to stray light and exposure time shot noise we need to take better account of (only basic implementation so far), but the fit looks pretty good
- As a by product, we get an estimate of the pixel size from this (conversion factor from mm to pixels!
- **(note nominal pixel size is 2.75um)**
- This calibration gives us “true” distance from our motor distance

Beyond the Nyquist Limit





Photon Transfer Curve

UNIVERSITY OF OXFORD

Image mean

Conventional PTC

Quadratic Fit

Relationship between BFE coefficients

Noise / offsets

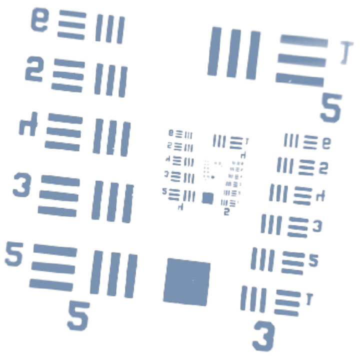
$$C_{ij} = \frac{\mu}{g}\left(\delta_{i0}\delta_{j0} + a_{ij}\mu g + \frac{2}{3}\left[\mathbf{a}*\mathbf{a}+\mathbf{a}\circ\mathbf{b}\right]_{ij}(\mu g)^2 + \frac{1}{6}\left[2\mathbf{a}*\mathbf{a}*\mathbf{a}+5\mathbf{a}*(\mathbf{a}\circ\mathbf{b})\right]_{ij}(\mu g)^3\right) + \frac{n_{ij}}{g^2}$$

Camera Gain

Measured Covariances

Next to leading order BFE relationships

Equation from work of Astier et al (2019) – who painstakingly worked out a peturbative expansion to the statistical photon transfer shape in the presence of redistributive correlations.

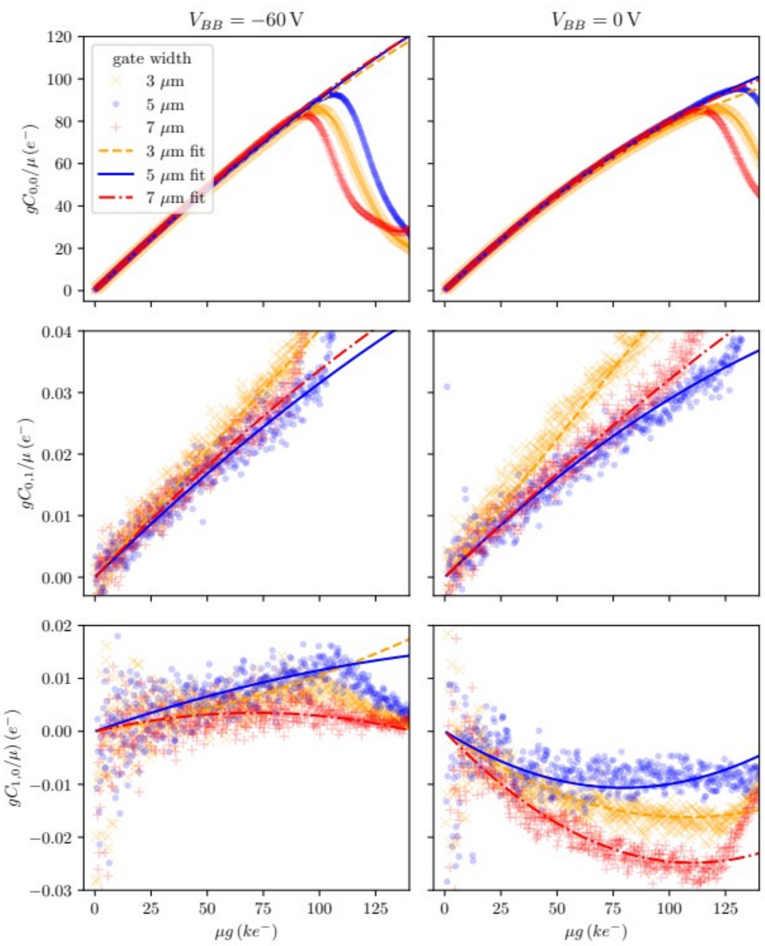


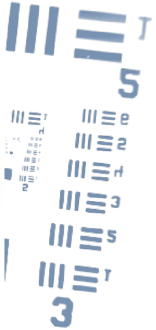
Covariance Measurements: Bartlett Method

We typically determine covariances/correlations from flat field data. And then fit Astier's equation (**right**, from Weatherill et al, 2020). Calculations usually done in frequency domain

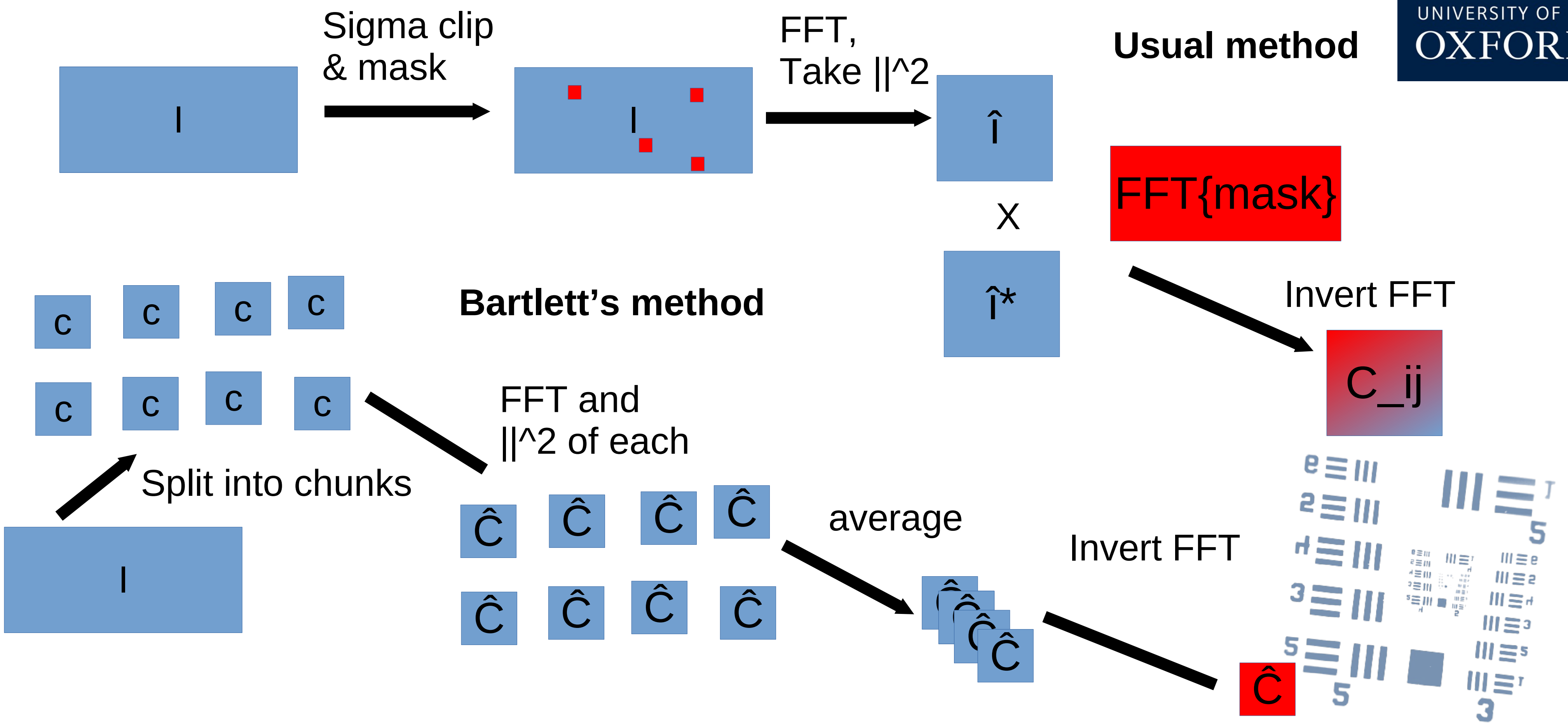
Unfortunately the correlations are quite sensitive to hot pixels, CTI defects etc etc. So, in Astier et al (2019) a masking procedure consisting of iterated sigma-clipping is described, which works well. We **cannot** sigma clip a speckle image, it would not work

However – there is an alternative! Note that we are interested in nearby correlations only, and that from Fourier theory, the consequence of using less data is not to reduce the resolution of correlations, but to reduce resolution of frequency bins...

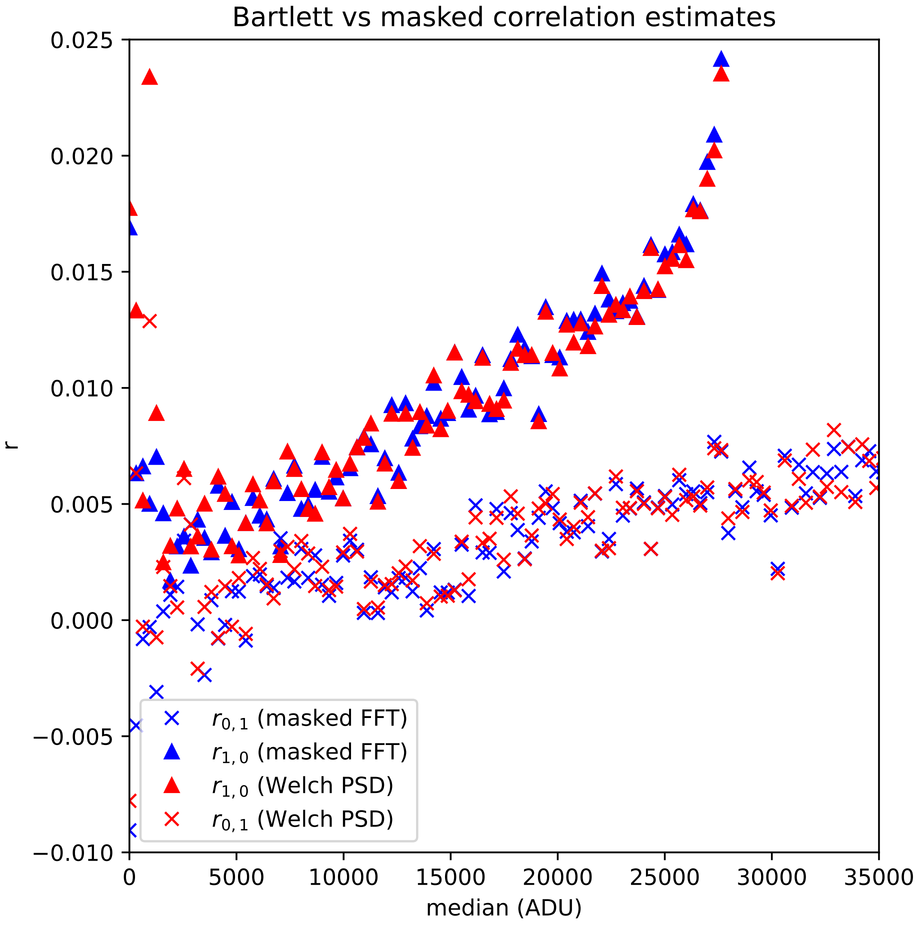


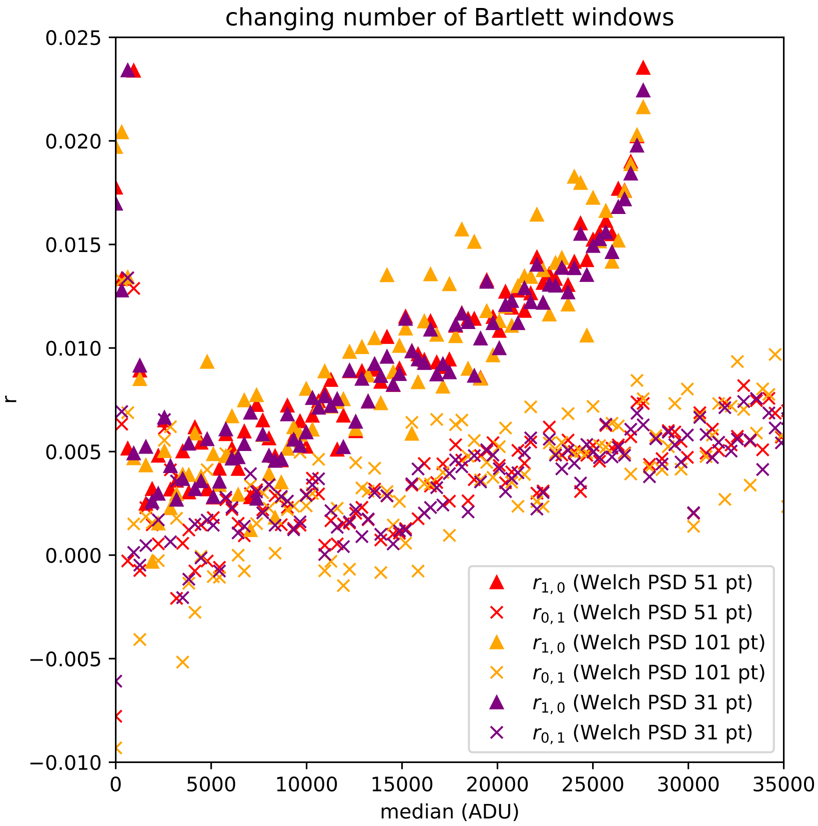


Covariance Measurements: Bartlett Method



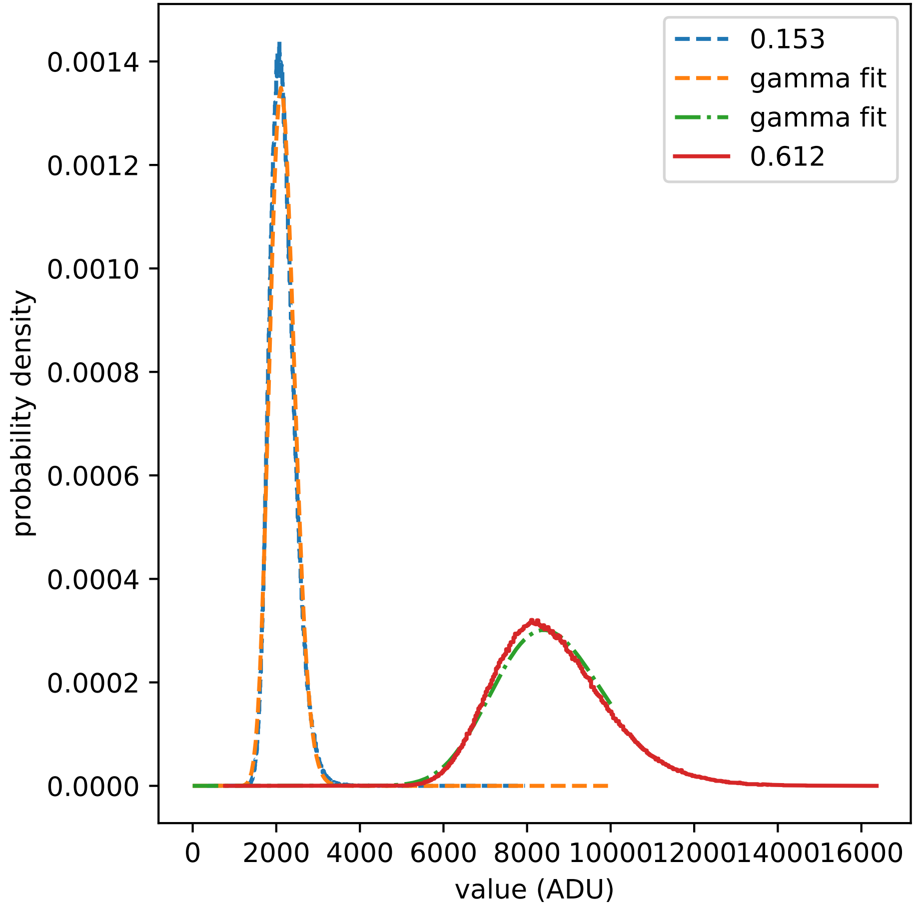
Covariance Measurement: Bartlett Method





Comparing usual masked FFT method with Bartlett method (without **masking**)

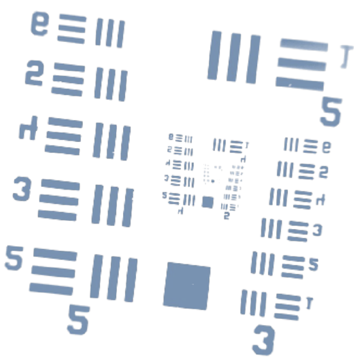
First order statistics – (approximately) gamma



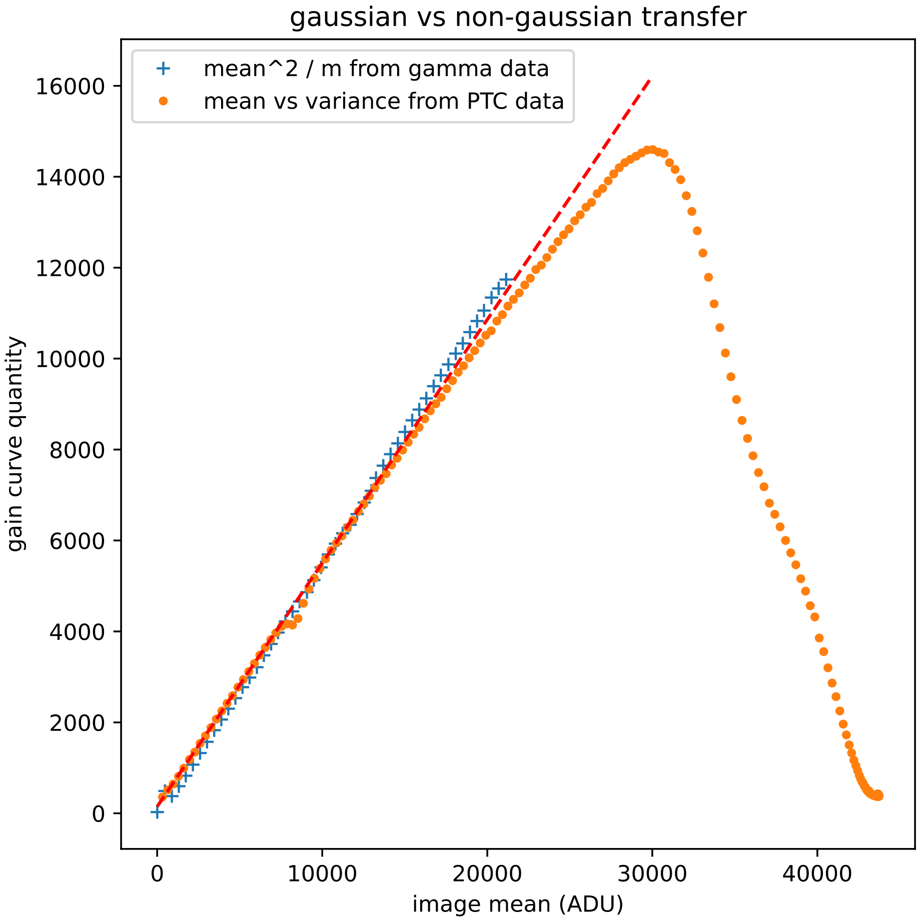
The distribution of the counts in an integrated speckle pattern are approximately gamma distributed.

Left – histograms from two exposure times at the same speckle projection distance, plus gamma fits.

NOTE these are not fits to the histograms, but MLE fits to a random sample of the data



Non-Gaussian Photon Transfer



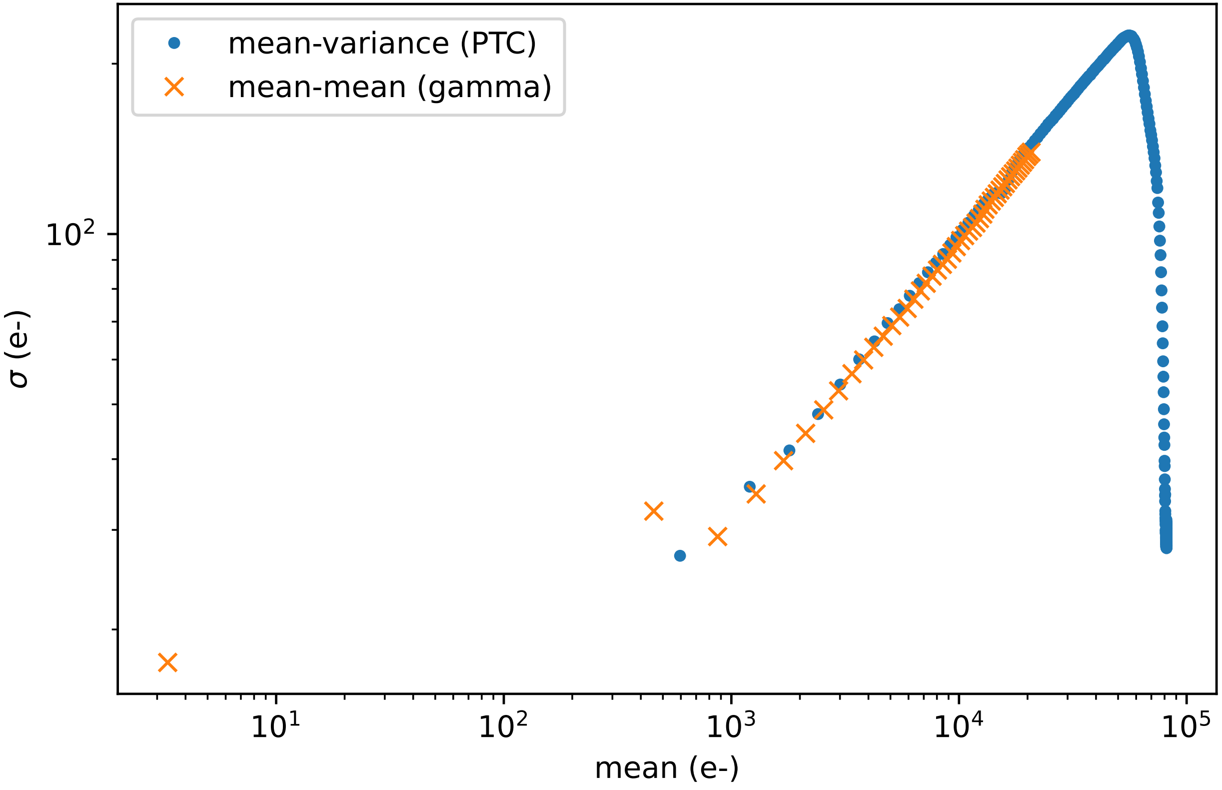
The particular gamma distribution for Laser speckle pattern has an extra Constraint. Its shape parameter is related to the scale parameter.

You can therefore make a similar curve to mean-Variance based on a combination of the fitted gamma values, i.e. a **gain curve**

$$p_{I_0}(I_0) \cong \begin{cases} \dfrac{\left(\dfrac{m}{\langle I \rangle}\right)^m I_0^{m-1} \exp\left(-m \dfrac{I_0}{\langle I \rangle}\right)}{\Gamma(m)} & I \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$

PTC from speckle vs flat fields

UNIVERSITY OF OXFORD

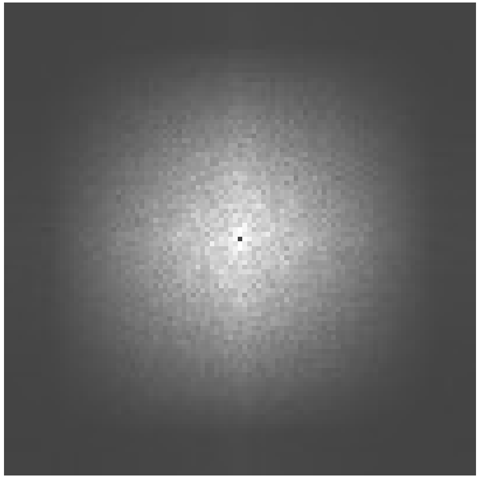


So far, we have not observed brighter fatter effect on this gain curve. We think for 2 reasons
1) one can average several frames together to beat down shot noise

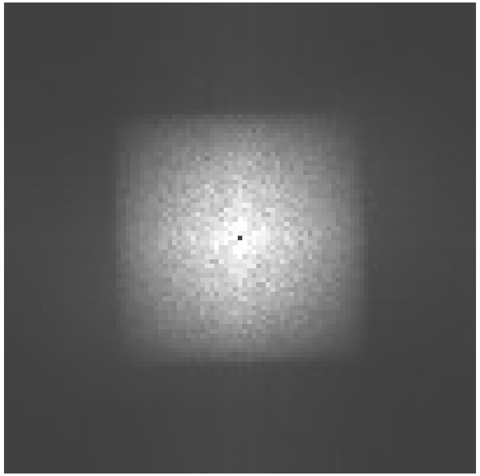
2) brighter-fatter effect can only very weakly affect the global shape property of the histogram

MTF vs signal...

- Can’t derive absolute MTF easily without doing full 2D fit of PSD. But we **can** look at differences with changing light level and cutoff frequency
- Note that the criticism sometimes made of flat field measurements is that brighter-fatter may depend on contrast – **speckle patterns sample over contrast ratios too!**
- Philosophical question – is brighter-fatter fundamentally a filtering process (multiply PSD) or a noise process (add to PSD), or both?

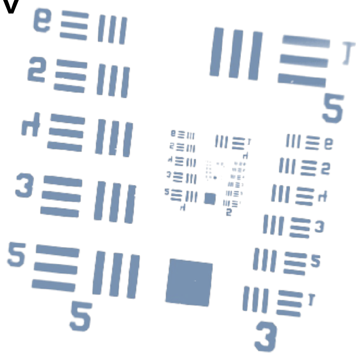


VBB = 0V



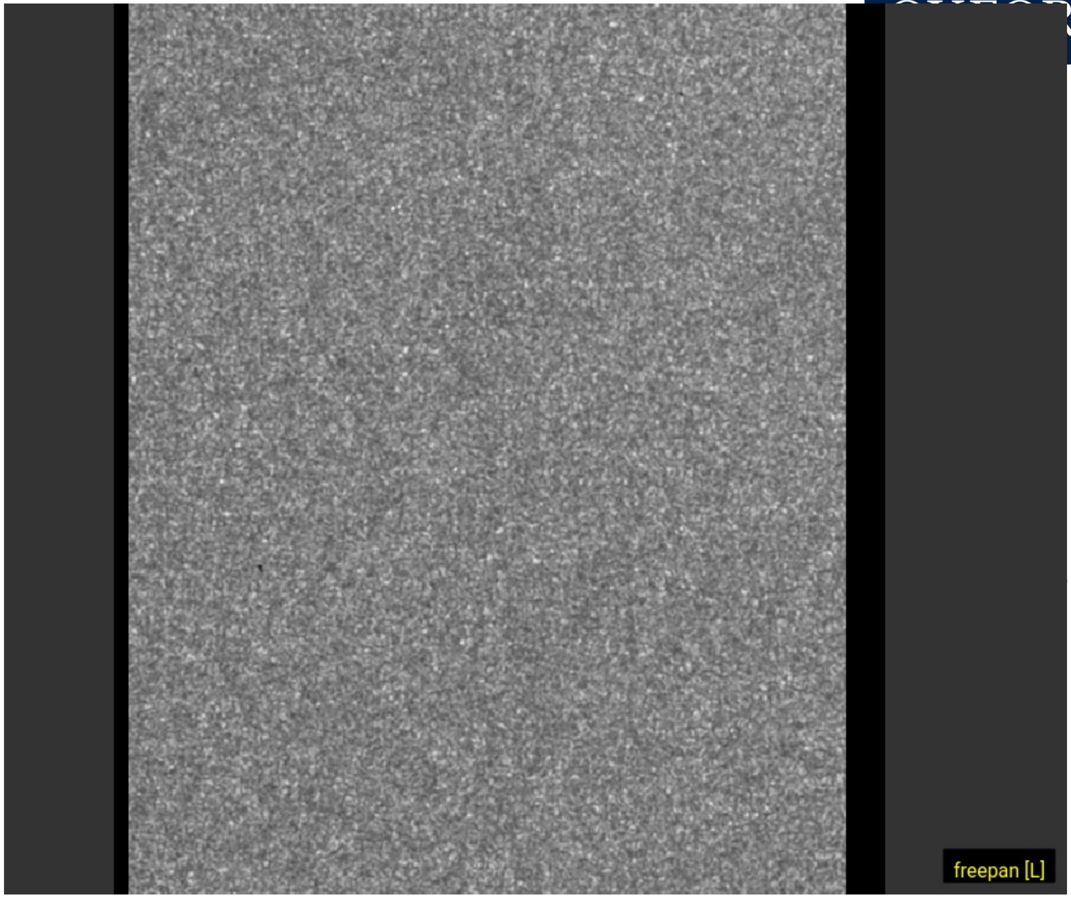
VBB = -60V

See proceeding for details!

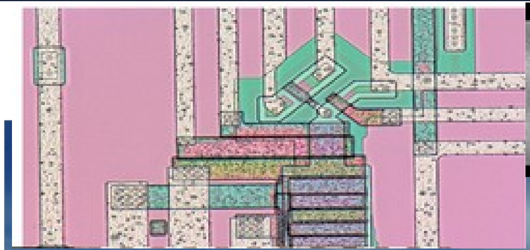


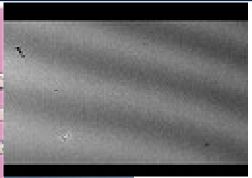
Summary

- Laser speckle has some interesting possibilities for detector calibration (known for a long time)
- We introduced some improvements to a previous MTF measurement method
- Detector gain can be obtained from first-order statistics fits to speckle patterns
- Hardware requirements may be favourable in some cases
- Working towards accurately obtaining brighter-fatter a_ij from speckle PTC data

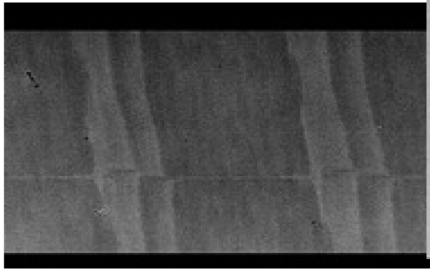


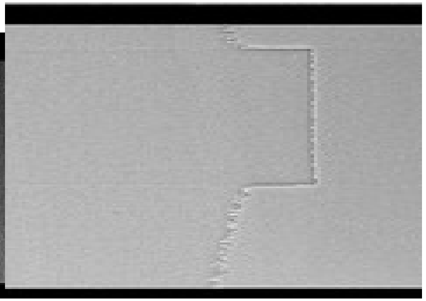
Thanks





Daniel.weatherill@physics.ox.ac.uk





Aknowledgements:

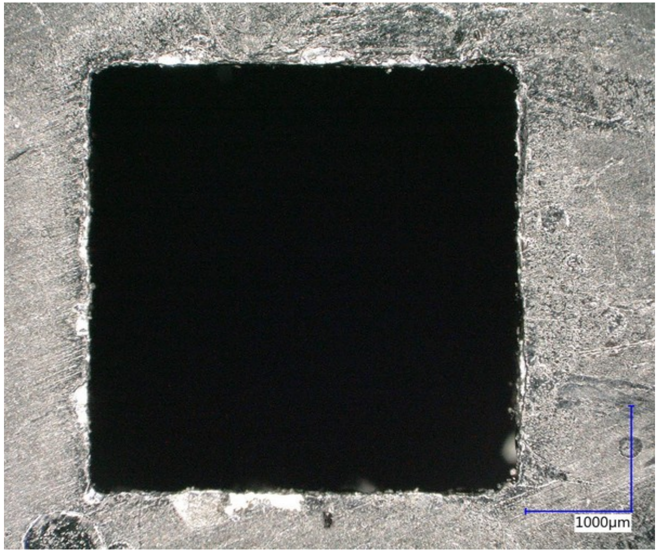
Thanks to all the OPMD group members for their continued assistance & support

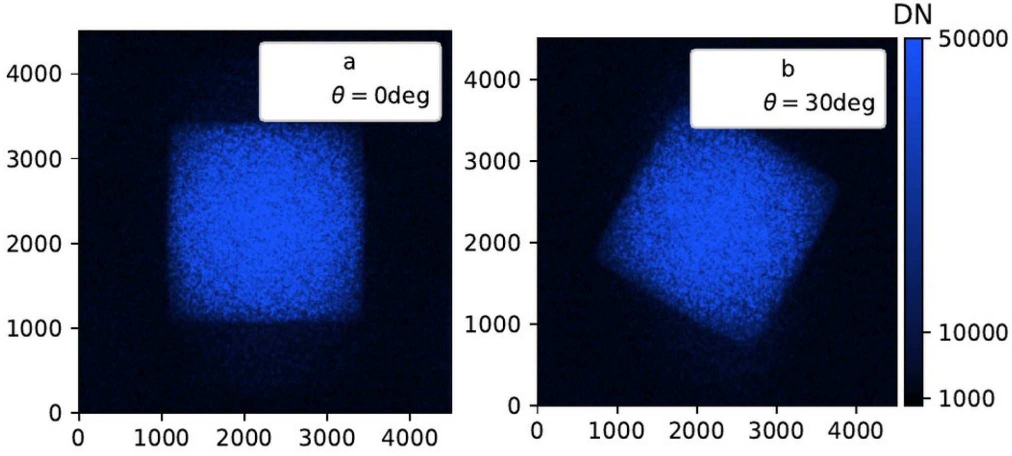
travel support provided by STFC for UK participation in LSST through grant ST/S006206/1

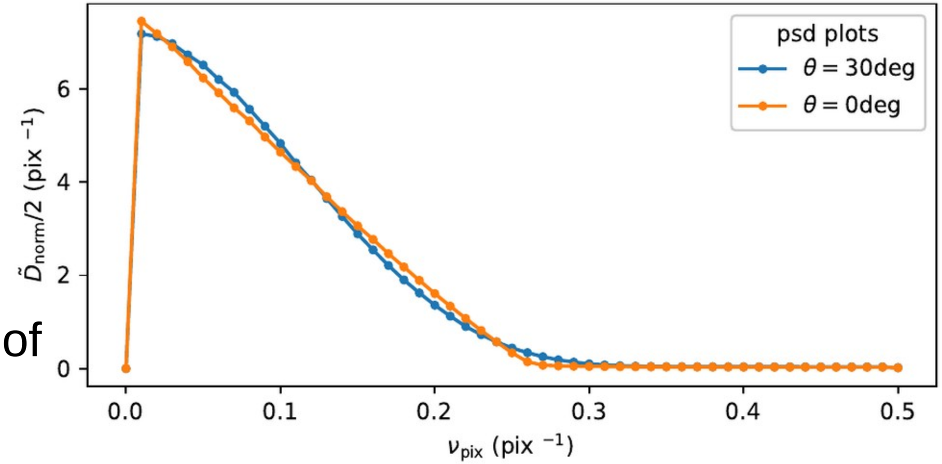
Science and Technology Facilities Council

LSST:UK Consortium

Backup – aperture shape / orientation

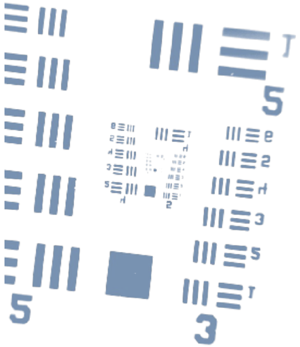






1D welch PSD is affected
By rotation of the aperture

Top row of plots are magnitude of
2D fourier transform of images



Backup – CTI correction

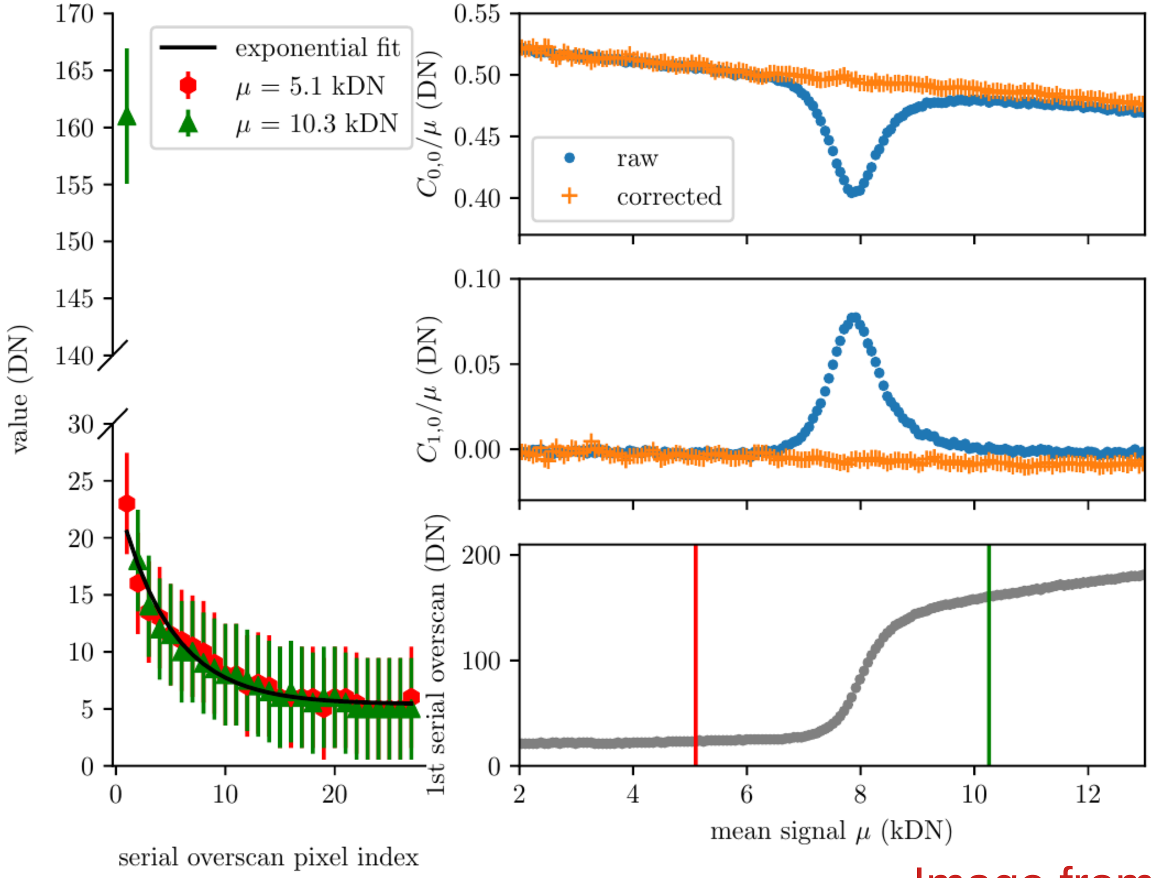


Image from Weatherill et al (2020)

Backup – tearing correction

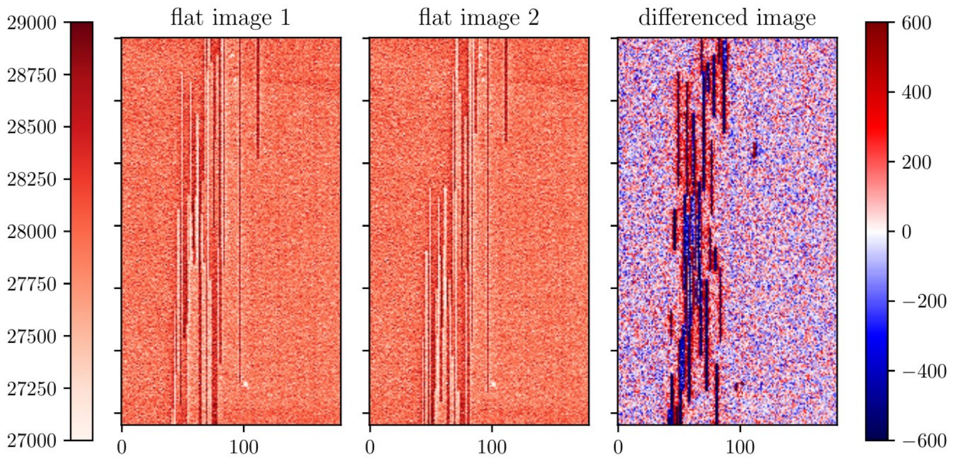


Figure 2. Illustration of the phenomenon of "tearing". Left: a region of one CCD channel after overscan correction and bias subtraction. Image tearing in the parallel direction (vertical axis on this plot) is clearly seen. Right: the same region of the resulting differenced image. Small time varying differences in the tearing patterns result in incomplete cancellation at the edges of the pattern.

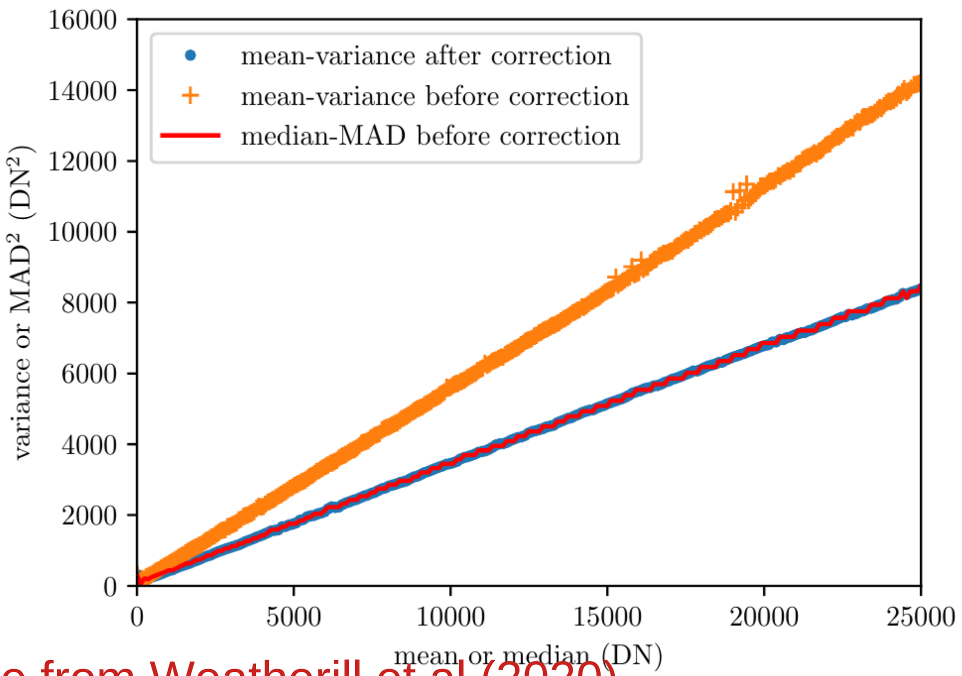


Image from Weatherill et al (2020)

Backup – Linearity Correction

Image from Weatherill et al (2020)