Generative Model Applications 3: Liquid Argon TPCs

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The NSF Institute for Artificial Intelligence and Fundamental Interactions





ML Reconstruction

Last week I talked primarily about Reconstruction for LArTPCs where

Deep neural networks having success in identifying key physical quantities for studying neutrino interactions





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Challenges

This talk will touch on early work to eventually address other

challenges through generative models ation that we can from our data, precisely and accurately?

Simulation/Modeling: can we translate physics to observables faster? Can we better use data-driven methods?

Inference: are we testing our models against data as best as we can while accounting properly for and mitigating our model uncertainties?

Operations: are we saving the right events? Is the experiment running optimally? Can we detect and make decisions faster?

Fast ML on FPGAs to implement special rare process triggers

frai.2022.855184



Simulation/Modeling: can we translate physics to observables faster? Can we better use data-driven methods?

Like in other frontiers, generating simulated data is a major bottleneck of analyses

- Example: Individual events can take 5-10 mins/event+ for MicroBooNE TPC with simulated cosmics + neutrino interaction
- Data driven methods are used to get better estimate: in MicroBooNE and SBN experiments which are on surface, cosmic background data is collected and used in simulated data by adding neutrino interaction – but cannot save enough of these events due to processing and storage constraints



Simulations for LArTPCs





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- Approach to improve **optical data** modeling using machine learning
- Early work to use generative models to reproduce **LArTPC wire plane images**





time





Optical Data

Liquid argon (and other cryogenic noble liquids) are excellent scintillators: 40k scintillation photons produced per MeV of deposited energy

Provides **measurement of time** when particles(s) traverse the detector

Useful for reconstruction: pattern of light helps with **same-time clustering** for ionization 3D spacepoints

There is some Particle ID information as well between **high** (proton, nuclei) vs **low** (electron, muon) density ionizing particles



Matching pattern of sensor position and total charge to cluster spacepoints



Optical Simulation Challenges

- CPU photon transport simulation for O(GeV) events is <u>prohibitively expensive</u>: Requires transporting O(10M) photons!
- Current standard practice is to use an approximate model
 - Generate table of output vs. position for voxelized data
 - Analytic function for mean sensor output vs. position [EPJC 81: 349 (2021)]
- Unknown parameters
 - Reflection/Transmission for 128 nm (VUV) scintillation photons not known for many materials/surfaces
 - Still some uncertainties in response of the wavelength shifter (TPB: converts 128 nm to detectable 420 nm light)



One direction: GPU photon sim



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Neural Network Estimate

visint

1 neuro

dense layer

vistmp

 $n \times m$ neuros outer product

 Neural network learning function to map 3D position and photons emitted at that location to expected signal in the array of optical sensors



Figure 2. Photon detection systems in ProtoDUNE-like (left) and DUNE-like geometries (right).

• Trained on point-source full photon transport simulations distributed throughout the detector

Wei Mu et al 2022 Mach. Learn.: Sci. Technol. 3 015033



pos_x

1 neuro

input layer

pos

1 neuro

input layer

pos_z 1 neuro

input laver

viscol

n neuros

dense layer

visrow

m neuros

dense laver.

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VISar

 $n \times m$ neuros

multiply layer

Visfull

output laver

Neural Network Estimate

- Good reproduction of predicted simulation
- Simple Neural network operations produces large speed-up compared to geant4: 20-50x





Outline

- Approach to improve **optical data** modeling using machine learning
- Early work to use generative models to reproduce **LArTPC wire plane images**











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Regions of ionization past induction wires and collect on last collection wires.

Induces current on wires intersecting at (y,z)





Recording wire signals over time, detector produces image-like data





wire number

Example of data event in MicroBooNE. View of same event for each projection.

Color scale indicates amount of ionization electrons seen on wire at given time



LArTPC data

- Patches of LArTPC images
- Globally Sparse: most pixels are empty
 - Non-zero pixels ~3-5%
- Locally Dense: patches have specific, thin shapes
 - Tracks: lines
 - Showers: branching lines
 - The pixel values along the lines contain info on particle type





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Generative Model

- Define latent vector, \mathcal{Z} , whose distribution is one we know how to sample, e.g. $z \sim \mathcal{N}(0,1)$
- Find a map from ${\mathcal Z}$ to ${\mathcal X}$, $f_{ heta}(z)$, so that $p(x)=p(f_{ heta}(z))$



How to Determine f

- Use a neural net for $f_{ heta}(z)$ and train on data to map into data region
- Need something to tell you if good or bad spot: hard since region where images located not defined
- Learn a function to tell you if good or bad: Discriminator



DCGANs

Not gotten very good results with GANS

(Curious to know about any projects with GANs using IF sample – don't hesitate to let me know your experience: good or bad!)

Typical result training collapses: discriminator, too good, too early. Generator produces unconving images



Generated Samples using DCGAN[<u>arxiv:1511.06434</u>]





For LArTPC Images

- Is sparse nature of LArTPCs an issue?
- Smaller space to find?



Adding Noise

- Training collapses when generated and real images very different
- Discriminator becomes too good
- Add noise to real images, so initial overlap with generated sample









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Adding noise helps a little – but not very good







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Pixel-CNN

Auto-regressive approach train to predict sequence in smaller space

$$p(\mathbf{x}) = p(x_1, x_2, ..., x_{n^2})$$

Bayes Theorem:

$$p(\mathbf{x}) = \prod_{i=1}^{n^2} p(x_i | x_1, \dots, x_{i-1})$$

A sequential model!



Use a CNN (with masking) to learn these distributions



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25

VQ-VAE for LArTPCs

- Use Vector-Quantized VAE to describe image as sequence of codes
- Patches in image mapped to 1 of 256 embedding vectors
- Embedding vectors learned during Autoencoder training
- Decoder learned as part of AE training
- Does not require that we train a discriminator



(1) Encoder maps image patch to latent space

(2) Encoded patch is mapped to the nearest, learned embedding vector (3) Encoded, quantized image composed of embedding vectors and is passed to decoder



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Pixel-CNN Results

- Image quality better than GAN
- Seeing wider variety of patterns
- Not seeing as many long lines or showers
- More quantitative
 measures latter



(a) Generated images



(b) Training images

arXiv:2204.02496



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Diffusion Models

- Change our generative picture a little
- Choose our latent space to be same size as X
- Associate map between **X** and **Z** as some process that runs for time, t=0 to t=T so that $p_T(x) = \mathcal{N}(0, I)$





SDEs

- The process can be modeled as a stochastic differential equation (SDE)
- Diffusion process defined by drift and brownian motion
- Can define a reverse process back to data. Requires estimating the score function: related to gradient of p(x). In practice, this is a neural net.





SDE

- Diffusion Process SDE
- Choice: Variance Preserving SDE

<u>Y. Song et al.</u> arXiv:2011.13456 (2020)

$$\begin{array}{ll} \begin{array}{ll} \text{Change in pixel} \\ \text{values at t} \end{array} \mathrm{d} \vec{\boldsymbol{X}}_t = -\frac{1}{2} \beta(t) \vec{\boldsymbol{X}}_t \mathrm{d} t + \sqrt{\beta(t)} \mathrm{d} \vec{\boldsymbol{W}}_t. \\ \\ \text{Drift} & \begin{array}{ll} \text{Brownian Motion} \\ \text{Provides change in random directions} \end{array} \end{array}$$



Note: For us, equilibrium not uniform, but the Normal distribution



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SDE: Forward Process

• The distribution after some time, *t*, using SDE

$$p(\vec{\boldsymbol{X}}_t | \vec{\boldsymbol{X}}_0) = \mathcal{N}(\vec{\boldsymbol{X}}_0 e^{-\frac{1}{2}\int_0^t \beta(s) ds}, \boldsymbol{I} - \boldsymbol{I} e^{-\int_0^t \beta(s) ds})$$

Starting with original image at **t=0**, \vec{X}_0 , gives the distribution of pixels at later time, **t**.



At time *t=T*, configure parameters such that

$$ec{X}_T \sim \mathcal{N}(0,I)$$







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Reverse process SDE, with time parameter *s goes from 0 to T* and *s=T-t*, is

$$\begin{split} \text{score function} \\ \mathrm{d}\dot{\boldsymbol{X}}_s &= [-\mathbf{f}(\dot{\boldsymbol{X}}_s,s) + g^2(s) \nabla \log p(\dot{\boldsymbol{X}}_s)] \mathrm{d}s + g(s) \mathrm{d}\boldsymbol{W}_s \\ \text{Estimate with neural net: } \boldsymbol{s}_{\theta}(\vec{\boldsymbol{X}}_t,t) \end{split}$$

Output is change for every pixel value: U-Net Architecture



B. Anderson. Stoch. Proc. and their App., 12 (3):313-326 (1982)



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Empirical loss follows from a previous slide's: $p(\vec{X}_t | \vec{X}_0) = \mathcal{N}(\vec{X}_0 e^{-\frac{1}{2}\int_0^t \beta(s)ds}, I - Ie^{-\int_0^t \beta(s)ds})$

$$\mathcal{L}(\boldsymbol{\theta}) = \frac{1}{N} \sum_{i}^{N} ||\boldsymbol{s}_{\boldsymbol{\theta}}(\vec{\boldsymbol{X}}_{t^{i}}^{i}, t^{i}) - \frac{-(\vec{\boldsymbol{X}}_{t^{i}}^{i} - \vec{\boldsymbol{\mu}}_{t^{i}}^{i})}{\vec{\boldsymbol{\sigma}^{2}}_{t^{i}}^{i}}||_{2}^{2}$$

Where we can sample a batch with *N* images $ec{X}_t$

Each image is associated with a different time, t, sampled uniformly from (0,T]

After training **s**, we can solve Reverse SDE to sample \vec{X}_T

B. Anderson. Stoch. Proc. and their App., 12 (3):313-326 (1982)



Sampling

Recall: variance changes with time. Largest at time of sampling, near zero when at end of reverse process. Thus we are using Annealed Langevin Dynamics.

Large variance at beginning helps produce images from different modes



Sampling (reverse process) time direction



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Score-matching Diffusion Model for LArTPCs

Trained a VPSDE using the Y. Song repo

Dataset consisted of 64x64 pixel crops of images from the PiLArNet open dataset Note: not full particle trajectories.









Z. Imani **Tufts Physics**























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S. Aeron **Tufts ECE**



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Score-matching Diffusion Model for LArTPCs

Which set, left or right, is training images – which are generated?



Image fidelity enough to be indistinguishable?



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With exception of one person, accuracy for small sample of scanners statistically consistent with random guessing



Want to try yourself?

100 examples to judge can be found <u>here</u>.

(Don't forget to do the practice quiz to help train yourself.)

arXiv:2307.13687 (2023)



Evaluating Quality

Need additional measures to quantify beyond visual inspection:

- High dimensional comparisons between images
- Comparison of other neural network outputs
- Physics-analysis motivated quantities



Evaluation: High-Dim. Metrics Directly on Images

• Each comparison is done between the training and generated images and the validation and generated images





Evaluation: Comparing SSNet Behavior

- Another approach: compare outputs of a neural network
- Given prominent role of track/shower pixel labeling in neutrino reconstruction, we use SSNet [Phys. Rev. D 99, 092001]



Version of SSNet trained on the training data.

SSNet Weights for those who want to compare: <u>zenodo</u>



Evaluation: Comparing SSNet Behavior

- labeling frequencies and output scores
- For SBDM, matches well after 50 epochs of training
- VQ-VAE generated images also compared – SBDM clear improvement, matching visual judgement





Evaluation: SSNet-FID

- For natural images, a standard metric is the FID or Frechet Inception Distance
- It compares the feature map of the last layer of the ImageNet-trained Inception v3 classifier
- SSNet being a U-Net, using that layer was not tractable, so we looked at layers near the bottleneck





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Evaluation: SSNet-FID

- SSNet-FID comparing train and test sets each to the generated image set
- Generated images produced for several epochs
- Plateau around 50 epochs, in agreement with metrics



Epochs	10	20	30	40	50	60	100	150	300
Training	121.63	18.96	10.2	9.15	8.94	8.99	8.95	8.75	8.79
Validation	121.38	18.31	10.2	9.21	9.09	8.87	8.94	8.71	8.84

 $SSNet-FID_{N=10K}$ for future comparisons



Evaluation: Reco-inspired Quantities

- Ultimately, threshold for "good enough" means no bias introduced into reconstruction quantities and physical observables
- Our dataset are only crops: so full reconstruction of momenta not possible yet
- Use algorithms related to energy calculation
 - Track length of track-like pixels (judged by SSNet) proxy for range
 - Number of shower pixels in an image proxy for calorimetry





Evaluation: Reco-inspired Quantities

- Track length and width derived from 1st and 2nd PCA axes
- Similar but statistically different not perfect, close enough?
- Track width least similar: generated tracks somewhat too wide (Jitter? Scattering? Delta-rays?)





Checking for Mode Collapse

- Do not want generated images to be small variations on training set examples
- Used Earth Mover's distance (Wasserstein-1 distance) to find closest neighbors
- Match trajectory direction
- Shower patterns varied
- Tracks have variation in high-energy deposits (expected from Bethe-Block distribution)
- Similar EMD among generated and training images





SBDM Next Steps

- Proof-of-principle that despite LArTPC sparse structure, generative models seem possible if using Diffusion Models
- Establish metrics to compare future work by others nascent effort so lot's of easy things to try
 - E.g. How do Flow networks perform?
- All methods and image sets used will be posted on zenodo (coming soon: watch for updates to <u>arXiv:2307.13687 (2023)</u>
- All metrics tell a consistent story: converge around 50 epochs and plateau, matching (subjective) visual quality trends



SBDM Next Steps

- Next biggest step is generation of individual particles conditioned on momentum
- Require bigger image sizes: 128x128 or 512x512
- Neutrino interaction can be composed with individual particles



Uses: Image repair/Gap Filling



Past efforts identity right regions, but do not produce realistic looking track segments

Missing sensor-signals either from induction dynamics or non-responsive wires is a major cause of downstream reconstruction errors

Limited use (e.g. no generation of particles entirely in a gap) can help a lot to:

- Reduce broken tracks
- Reduce False 3D spacepoints
- Connect tracks in uninstrumented regions between detector modules





Uses: Cosmic Background Augmentation

- In order to estimate backgrounds and mis-reconstruction rates due to cosmic background, surface LArTPCs use data taken when beam is off
- Simulated neutrino are overlaid into image to make simulation samples
- Not nearly enough samples can be recorded due to storage
- Can we generate large enough background crop (512x512 or 1024x1024)?
- Must generate 3-plane consistent
 image: potential challenge





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Uses: Iterative final state comparisons (speculative)

- Reconstruction makes one-pass and extracts final state particles from image of neutrino interaction
- Use generative models to generate expected image of propose final states and use comparison to inform errors and *action*:
 - E.g. adjust reco vertex, recluster particle, retrieve portion of raw image for review, instigate image repair
 - Moving from the ML into **AI** for Neutrino Physics
- Near-term useful comparisons possible
 - Compare photon-induced shower (BG) versus small proton+electron shower (signal) using matching images
 - Probing classifier robustness by modifications of particles in images





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Other Data-Driven Possibilities

- Elements of Neutrino Interaction Generators do not have good theoretical models yet, e.g. hadronic output
- Neutrino oscillations have near and far detector data to constrain models of flux and cross section and to extrapolate far detector neutrino flux
 - Remove reliance of cross section or flux models through data representations learned by training generative models





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Conclusions

- Early days of generative modeling for Neutrino Physics
- Despite sparse LArTPC images, at least diffusion models produce quality images both visually and in the reproduction of various metrics
- Opens doors to approach several challenges in new ways









Definitions

The Wasserstein Distance

Let $\alpha \in \mathcal{M}^1_+(\mathcal{X})$ and $\beta \in \mathcal{M}^1_+(\mathcal{Y})$,

$$W_{c}(\alpha,\beta) = \min_{\pi \in \Pi(\alpha,\beta)} \int_{\mathcal{X} \times \mathcal{Y}} c(x,y) d\pi(x,y) \qquad (\mathcal{P})$$

For $c(x, y) = ||x - y||_2^p$, $W_c(\alpha, \beta)^{1/p}$ is the Wasserstein distance.



https://audeg.github.io/talks/talkAIP.pdf



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Definitions: Frechet Inception Distance

$$d_F(\mathcal{N}(\mu,\Sigma),\mathcal{N}(\mu',\Sigma'))^2 = \|\mu-\mu'\|_2^2 + \mathrm{tr}\left(\Sigma+\Sigma'-2igl(\Sigma^{rac{1}{2}}\cdot\Sigma'\cdot\Sigma^{rac{1}{2}}igr)^{rac{1}{2}}igr)^{rac{1}{2}}
ight)$$

From Wikipedia





Flash Matched Cluster





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frai.2022.855184



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Like in other frontiers, generating simulated data is a major bottleneck of analyses

- Example: Individual events can take 5-10 mins/event+ for MicroBooNE TPC with simulated cosmics + neutrino interaction
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Simulations - Generative Models

Score-based Generative Modeling shows promise for generating LArTPC-like images



Model generation as the reverse of a diffusion process bringing data images to noise



Simulations - Generative Models

Which set, left or right, is training images – which are generated?



More details in next Wed. talk: "Generative Model Applications 3"



Simulations - Data Driven/Differentiable

Differentiable (surrogate) simulations save time by allowing a way to possible reweight MC events rather than generating additional samples with variation in detector physics parameters

Also enables simulation-based inference for a number of exciting applictions



https://arxiv.org/abs/2211.01505



Accelerating Development: Are we providing enough tools to the community to enable new ideas and new contributors?



LArTPC Neutrino Interaction (Simulation) Dataset

MicroBooNE has released some LArTPC simulations: cosmic data overlaid with simulated neutrino interaction





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LArTPC Neutrino Interaction (Simulation) Dataset

Another public dataset of 3D voxels along with simple 2D projections



https://arxiv.org/abs/2006.01993

● osf.io/vruzp/ 👬 OSF**HOME 🗸** LArTPC 2D/3D - Simulation - Particle Se.. t Metadata Analytics Files Wiki Registrations Particle Imaging in Liquid Argon (PILArNet) / I ArTPC 2D/3D - Simulation - Particle Segmentation & Clustering Contributors: DeepLearnPhysics Date created: 2018-12-04 06:43 PM | Last Updated: 2020-07-02 01:31 PM Identifier: DOI 10.17605/OSF.IO/VRUZP Category: 🛢 Data Description: This sub-project is organized by DeepLearnPhysics (www.deeplearnphysics.org), and is a part of a bigger project to share public imaging detector. It is particularly aimed for developing pixel-level particle classification technique for pixel-type (=3D readout) LArTPC. License: CC-By Attribution 4.0 International

https://osf.io/vruzp



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- ML techniques have already impacted the physics program of several neutrino experiments
- Many develops across experiments should provide further impact
- Developments have been somewhat focused on reconstruction
- But there are still other research challenges that ML might help to advance
- Cross pollination between experiments and frontiers will surely accelerate progress



BACK UP SLIDES



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Sampling with Reverse Process

With the choices for:

 $\beta(t)$

 $t_i = T + (i - M)\Delta t$ $\Delta t = -(T - \epsilon)/(M - 1)$ $\epsilon = 1.0 \times 10^{-3}$ Point is not to get too into the weeds, but can integrate

$$\vec{\boldsymbol{\mu}}_{t_{i-1}} = \vec{\boldsymbol{X}}_{t_i} + \left[-\frac{1}{2}\beta(t_i)\vec{\boldsymbol{X}}_{t_i} - \beta(t_i)\boldsymbol{s}_{\boldsymbol{\theta}}(\vec{\boldsymbol{X}}_{t_i}, t_i)\right]\Delta t$$
$$\vec{\boldsymbol{X}}_{t_{i-1}} = \vec{\boldsymbol{\mu}}_{t_{i-1}} + z\sqrt{-\beta(t_i)\Delta t},$$

 $z = \mathcal{N}(0, I)$ Sampled Brownian motion direction



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With the choices for:

$$\beta(t)$$

$$t_i = T + (i - M)\Delta t$$
$$\Delta t = -(T - \epsilon)/(M - 1)$$
$$\epsilon = 1.0 \times 10^{-3}$$

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$$\vec{\boldsymbol{X}}_{t_{i-1}} = \vec{\boldsymbol{\mu}}_{t_{i-1}} + z\sqrt{-\beta(t_i)\Delta t},$$



Transport Problem

• Comparison of two sampled distributions



Fig. 1. Illustration of the setup. We need to transport the iron ore from the mines to the factories, which is the transportation problem.

Find P_{ii} that minimizes total cost $d = \min \sum P_{i,j} C_{i,j}$ $i\in\{A,B\}$ $j \in \{X,Y,Z\}$

 $P_{i,j} \ge 0$ $P_{i,j}$ $= r_i$ $P_{i,j}$ $= c_i$


Wasserstein vs. Sinkhorn Cost Optimization

Sinkhorn distance relaxes constrain on P_{ii} and imposes term that









To get around how "thin" the image space might be, we trained an autoencoder to learn embedding for images into a lower dimensional space

Train Generator to map from **Z** to embedding space **H**

Used Optimal Transport to measure how close or far generated examples are from real ones. [AE-OT]

Training examples



Generated examples





LArTPC Primer



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Recording wire signals over time, detector produces image-like data



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Capture 3 projection images with wire planes

Can solver inverse problem to recover 3D energy deposits

lonization signals on wires coincident in time provide info for (Y,Z) position

X position given by time delay from light signal





wire number

Example of data event in MicroBooNE. View of same event for each projection.

Color scale indicates amount of ionization electrons seen on wire at given time



Flavor determined from finding partner lepton (muon,electron) produced in interaction Neutrino energy inferred from momenta of resulting particles





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Neutrino Oscillation Analysis



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Neutrino oscillations occur because

- neutrinos have mass
- and flavor states are mixture of mass states

Neutrinos <u>created/interact</u> in flavor states They <u>propagate</u> in their mass states

Don't have to line up!



Neutrino Oscillation: 2-flavor example

Neutrino oscillations occur because

- neutrinos have mass
- and flavor states are mixture of mass states





Neutrino Oscillation: 2-flavor example

Let's start with a neutrino created in flavor state $| u_{\mu} angle$



Want to know state at time, *t*, so we need to apply the propagator, U(t), on the $|\nu_{\mu}\rangle$ in the mass basis

$$U(t)U_{PMNS}^{-1}|\nu_{\mu}\rangle = -\sin\theta e^{-iE_{1}t}|\nu_{1}\rangle + \cos\theta e^{-iE_{2}t}|\nu_{2}\rangle$$
$$E = \sqrt{p^{2} + m^{2}}$$

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$$E = \sqrt{p^{2} + m^{2}}$$

what we have are two states oscillating at slightly two frequencies due to the slightly different masses



with a slight shift in frequency, you get periods where two waves in phase for some time, and out of phase for others

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$$U(t)U_{PMNS}^{-1}|\nu_{\mu}\rangle = -\sin\theta e^{-iE_{1}t}|\nu_{1}\rangle + \cos\theta e^{-iE_{2}t}|\nu_{2}\rangle$$
$$E = \sqrt{p^{2} + m^{2}}$$

what we have are two states oscillating at slightly two frequencies due to the different masses

Classical analogue is the beat-frequency phenomenon



Neutrino Oscillation: 2-flavor example

Probability of transition from flavor ν_{μ} to ν_{e} :

$$P(\nu_{\mu} \to \nu_{e}) = |\langle \nu_{e} | U_{PNMS} U(t) U_{PNMS}^{-1} | \nu_{\mu} \rangle|^{2}$$
$$= \sin^{2} 2\theta \sin^{2} \left(([1.27 \text{ GeV } \text{km}^{-1}] \Delta m^{2} \frac{L}{E}) \right) \text{ osc}$$

Key signature is oscillatory prob function of L/E







Neutrino Oscillation: 2-flavor example

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Key signature is oscillatory prob function of L/E





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Neutrino nucleon Interactions

Never scattering on a free quark.

Dominant interactions at u_{μ} at $E_{\nu} \sim 1 \text{ GeV}$ (typical flavor and energy for accelerator ν)



Cherenkov

Other experiments analyze the pattern of Cherenkov Radiation to infer particle momenta and type.

Spatial arrange of optical sensors not grid-like





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