Beam Loading Effects in the tracking code RF-Track

International Workshop on Linear Colliders 2023 – Accelerator: Beam Dynamics Session

Javier Olivares Herrador (javier.olivares.herrador@cern.ch), Andrea Latina

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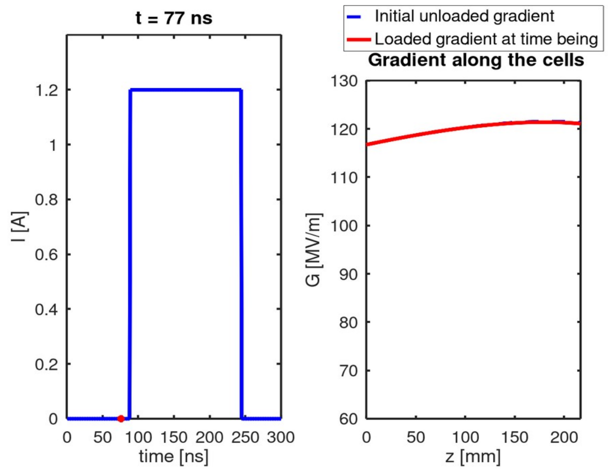
Beam Loading Effect

- **What**: **Reduction** of available accelerating **gradient**
- **Origin**: Beam – **Cavity** interaction
- **Consequences**: **Transient** response
  - Different energy loss from bunch to bunch
- **Motivation**: **High I, Compact** accelerating structures



> Theoretical analysis of beam loading effect based on CLIC's main linac [1]

[1] A. Grudiev, A.Lunin, V. Yakovlev. *Analytical solutions for transient and stead state beam loading in arbitrary travelling wave accelerating structures.* Phys. Rev. Special topics **14**, 052001 (2011)

Outline

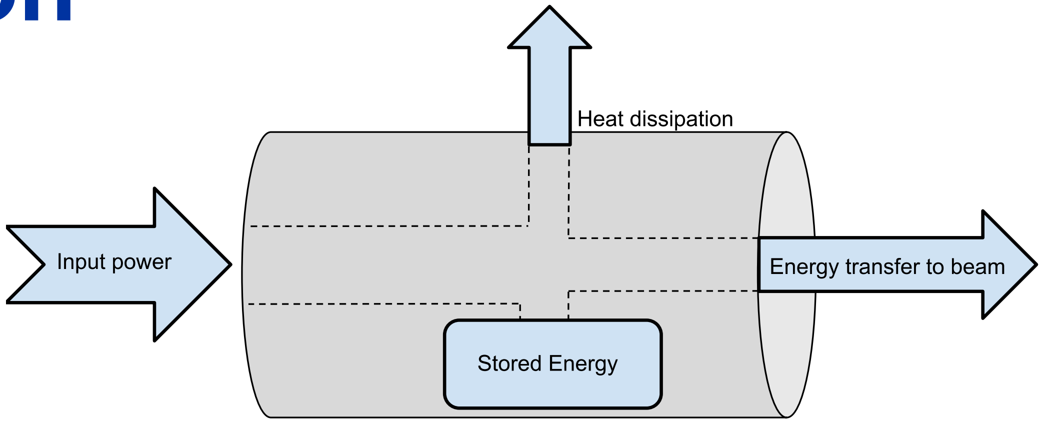
- **PART I**: Introduce **power-diffusive model** for Beam Loading Effect
  - Figures of merit
- **PART II**: Implementation into **RF-Track**
- **PART III**: Results
  - Reproducibility of **BL fields**
  - **Long** and **Short range** tracking results
  - **Transient** BL in **photoinjector guns**
  - Start-to-end BL simulations

PART I: Power-Diffusive Model

I. Energy Conservation

- Poynting Theorem

$$-\underbrace{\frac{\partial u(\vec{r},t)}{\partial t}}_{\text{Stored EM energy density variation}} = \underbrace{\vec{\nabla}\cdot\vec{S}(\vec{r},t)}_{\text{Power Flow \& Loss}} + \underbrace{\vec{E}(\vec{r},t)\cdot\vec{J}(\vec{r},t)}_{\text{Field-Beam Interaction}}$$



> Energy balance schematics for an accelerating structure

- Figures of merit:
  - Group velocity $\quad v_g = \frac{P_{\text{flow}}}{w}\,[\text{m/s}]$
  - Quality factor $\quad Q = \omega_{\text{RF}}\frac{w}{p_{\text{diss}}}$
  - Shunt impedance (p.u.l) $\quad r_e = \frac{G_{\text{eff}}^2}{p_{\text{diss}}}\,[\Omega/\text{m}]$

[2] Thomas P. Wangler. *RF linear accelerators*. Wiley-VCH 2008 (Amsterdam, Holland)

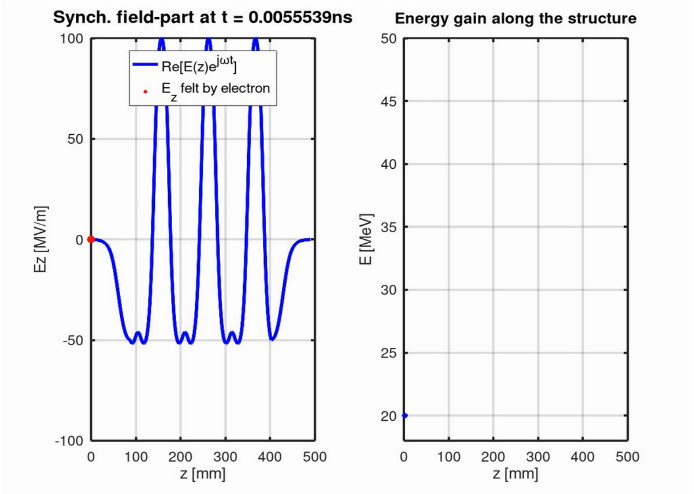
I. Gradient & Synchronization

- **Gradient**: Averaged E-field *affecting* the particle
- Time of flight:

$$t_q(z, t_0, \beta(z, t, t_0, \beta_0)) = t_0 + \int_0^z \frac{\mathrm{d}\zeta}{\beta(\zeta, t, t_0, \beta_0)c}$$

- Effective E-field:

$$E_z|_{\mathrm{eff}}(z, t, t_0, \beta) = \mathrm{Re}\left[\tilde{E}_z(z,t)e^{j\omega t_q(z,t_0,\beta(z,t,t_0,\beta_0))}\right]$$



> Electron synchronization with the on-axis electric field of an S-band accelerating cavity with 9 cells and peak E-field of 100 MV/m

I. Gradient & Synchronization

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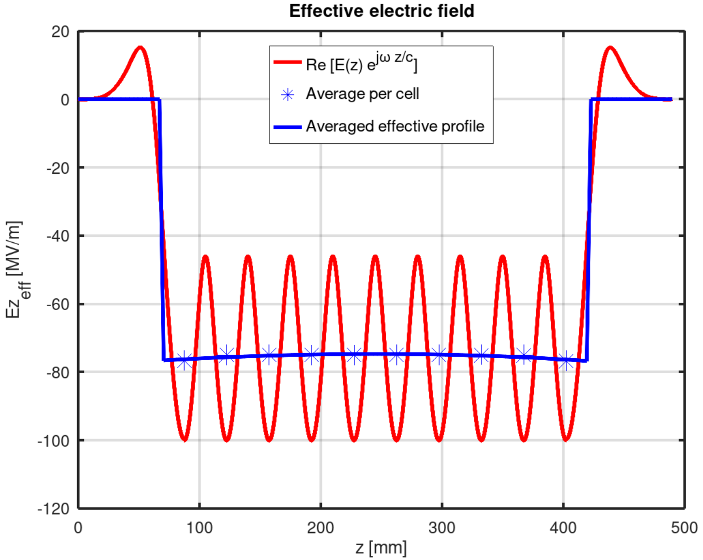
- Effective Gradient

$$G_{\text{eff}}(z_k, t, t_0, \beta) = \frac{1}{L}\int_{z_k}^{z_k+L} E_z|_{\text{eff}}(z, t, t_0, \beta)\ dz$$



> Effective electric field affecting the electron and its average over the cells

I. Power-Diffusion PDE

- From Poynting: Equation in terms of Gradient:

$$-\frac{\partial G_{\text{eff}}}{\partial t} = v_g \frac{\partial G_{\text{eff}}}{\partial z} + \left(-\frac{v_g Q}{r_{\text{eff}}} \frac{\partial (r_{\text{eff}}/Q)}{\partial z} + \frac{\omega}{Q} + \frac{\partial v_g}{\partial z} \right) \frac{G_{\text{eff}}}{2} + \underbrace{\frac{\omega r_{\text{eff}} \tilde{I}}{2Q}}_{\text{Beam Loading term!}}$$

Some features:

- **Paraxial** approximation
- From z_k to z → Cubic interpolation, continuity.
- **Beam Loading term**: **Decelerating** gradient dependent on **Intensity**.
- Assumes **causality**!
- Matches [1] for the TW ultrarelativistic case

PART II: RF-Track Implementation

II. RF-Track

- About **RF-Track** [3]:
  - Beam tracking in field maps/analytic structures including **space-charge** effects, **wakefields**, …
  - Multiple species (arbitrary q and m)
  - **Parallel C++**, interface with user via **Octave** or **Python**

- **Beam Loading** in RF-Track:
  - Self-consistent module
  - Additional decelerating kick (F_{BL})
    - Attached to Drift spaces, Analytic TW & SW structures, field maps

[3] A. Latina. *RF-Track Reference Manual*. CERN, Geneva, Switzerland, June 2020 DOI: 10.5281/zenodo.3887085

II. RF-Track – BL algorithm

- **INPUT**: $\omega,\ E_z,\ P,\ Q,\ v_g,\ \phi_{\mathrm{ad}},\ \texttt{BEAM}$
- **PHASE 1**: **Preparation**
  - 1.1) Structure characterization
    - From $Q,\ v_g$ → **Cubic interpolate** values from 0 to L_{total}
    - From $E_z,\ \omega,\ \phi_{\mathrm{ad}},\ P$ → **Integrate** and get $G(z, t=0)\ \forall z \in [0, L_{\mathrm{total}}]$
      → **Integrate** and get r_{eff}
  - 1.2) PDE solving
    - **Finite difference** method → Get $G(z,t) \forall t \in [0, t^*]$
- **PHASE 2**: **Tracking + BL**
  - 2.1) Perform RF-Track tracking
  - 2.2) Add F_{BL} kick $(z_{\mathrm{part}}, t_{\mathrm{part}}) \rightarrow F_{\mathrm{BL}} = -q\left(1 - \frac{G(z_{\mathrm{part}}, t_{\mathrm{part}})}{G(z_{\mathrm{part}}, 0)}\right) E_z(z_{\mathrm{part}}, t_{\mathrm{part}})$

II. RF-Track – BL Example

- Example in **Octave**: BL + TW field map

```
% Import RF-Track
RF_Track;

% Define bunch
B0 = Bunch6d([X XP Y YP t P MASS Q N]);

% Define RF structure
TWS  = RF_FieldMap_1d_CINT(Ez, hz, L, freq, -1);

% BL effect
BL_steady = BeamLoading(TWS, Pmap, VG, Qfactor, phaseadvance, -1 , particles_bunch, fb);
BL_trans  = BeamLoading(TWS, Pmap, VG, Qfactor, phaseadvance, -1 , particles_bunch, fb, Nbunches);

% Append BL effect to TWS
TWS.add_collective_effect(BL_trans);

% Tracking Lattice
L = Lattice();
L.append(TWS);

% Perform tracking
B1 = L.track(B0);

% Retrieve information and manipulate
M1 = B1.get_phase_space();
...
```

PART III: Results

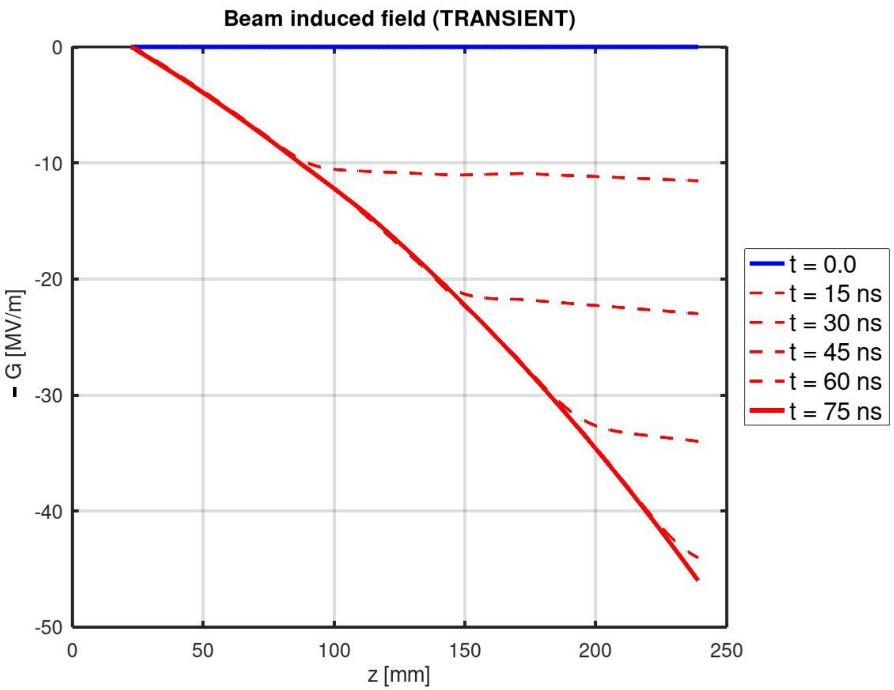
- BL field map
- BL Tracking (Long and Short Range)
- BL in Injectors
- Start-to-end BL simulations

III. BL field map (CLIC AS)

- CLIC main Linac **Accelerating Structure**
  - BL decelerating field

Magnitude	Units	Value
f_{RF}	GHz	$12,0$
ϕ_{ad}	rad	$2\pi/3$
N_{cells}		$26+1$
v_g (1^{st}, middle, last cell)	$\%c$	$(1,65,\ 1,20,\ 0,83)$
Q (1^{st}, middle, last cell)		$(5,54,\ 5,64,\ 5,74)\cdot 10^3$
t_{fill}	ns	$66,7$
P_{in}	MW	61,3
f_{inj}	GHz	$2,00$
$\langle I \rangle$	A	$1,00$
N_{bunches}		312

> CLIC main Linac Accelerating Structure Parameters [4]

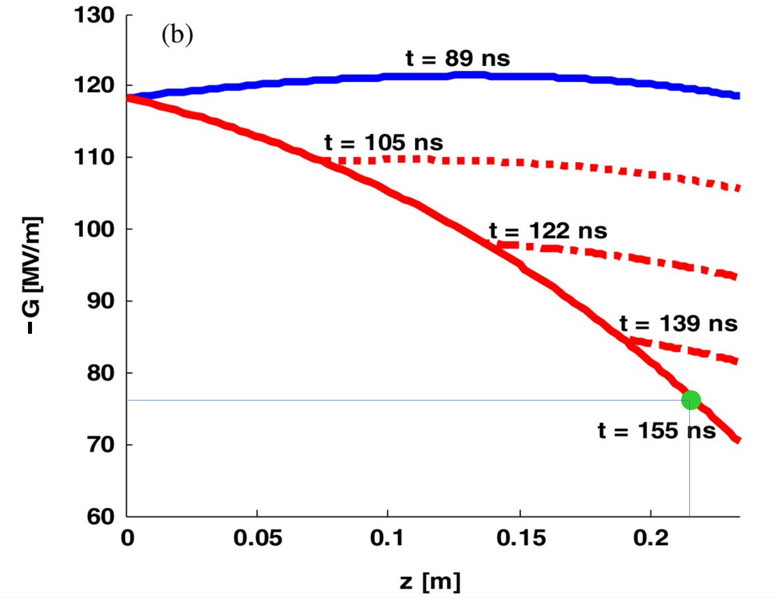


> Beam Induced Decelerating force for CLIC AS.

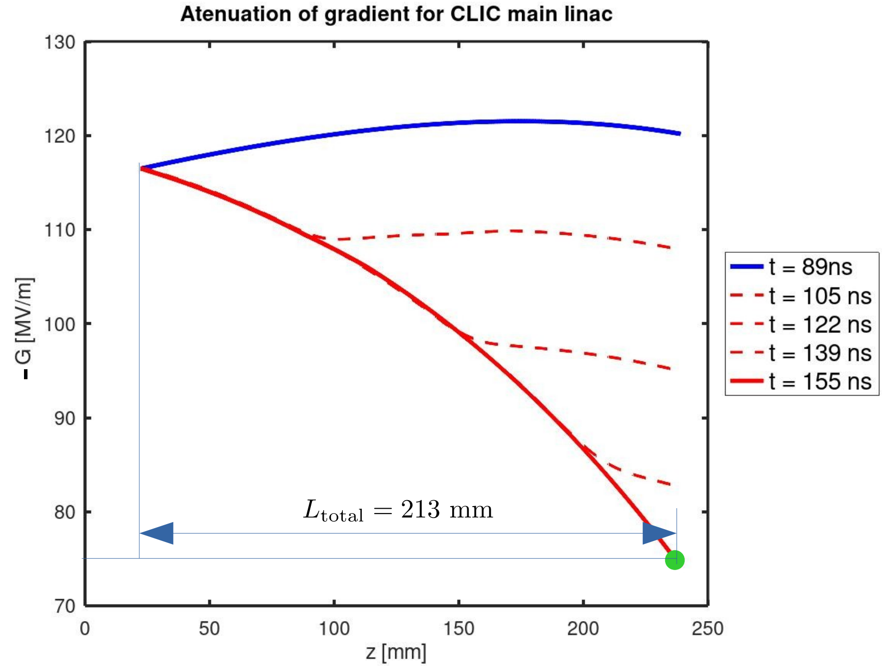
[4] *A Multi-TeV linear collider based on CLIC technology: CLIC Conceptual Design Report*, edited by M. Aicheler, P. Burrows, M. Draper, T. Garvey, P. Lebrun, K. Peach, N. Phinney, H. Schmickler, D. Schulte and N. Toge, CERN-2012-007

III. BL field map (CLIC AS)

- CLIC main Linac **Accelerating Structure**
  - Superposition to initial gradient (blue) → Total effect upon particles to track



> Theoretical calculation of gradient reduction in CLIC AS [1].



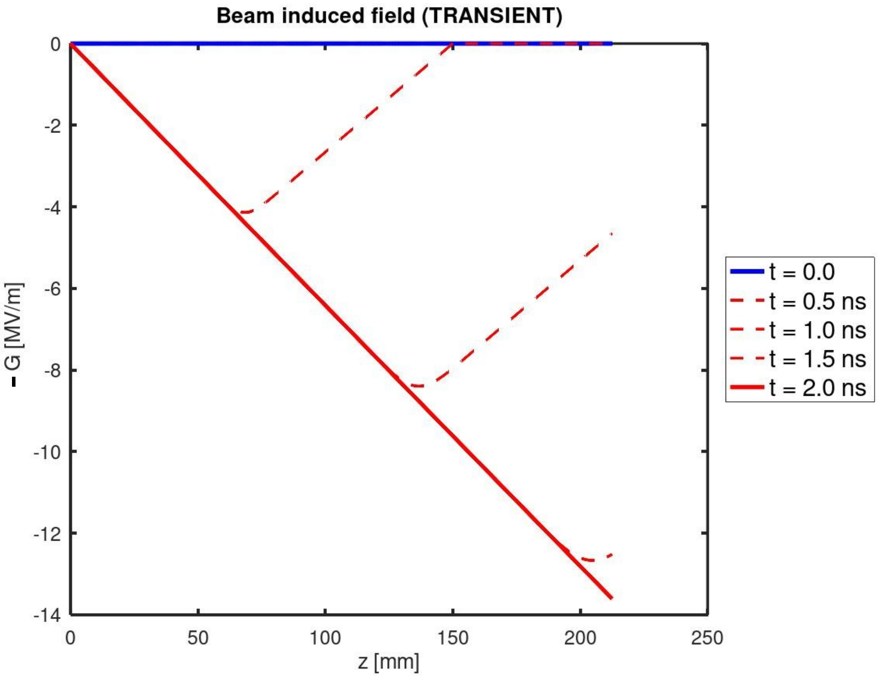
> RF-Track numerical calculation of the gradient reduction in CLIC AS.

III. BL field map (CLIC PETS)

- CLIC Power Extraction and Transfer Structures (**PETS**)
  - **Passive** structures → **Deceleration**

Magnitude	Units	Value
ϕ_{ad}	rad	$\pi/2$
N_{cells}		$34,0$
v_g	$\%c$	$45,3$
Q		$7,20 \cdot 10^3$
r/Q	$\mathrm{k}\Omega/\mathrm{m}$	$2,70$
t_{fill}	ns	$1,67$
f_{inj}	GHz	$12,0$
$\langle I \rangle$	A	101
N_{bunches}		$2{,}93 \cdot 10^3$
σ_t	mm/c	1,00
E_{inj}	GeV	2,40

> PETS parameters [5]

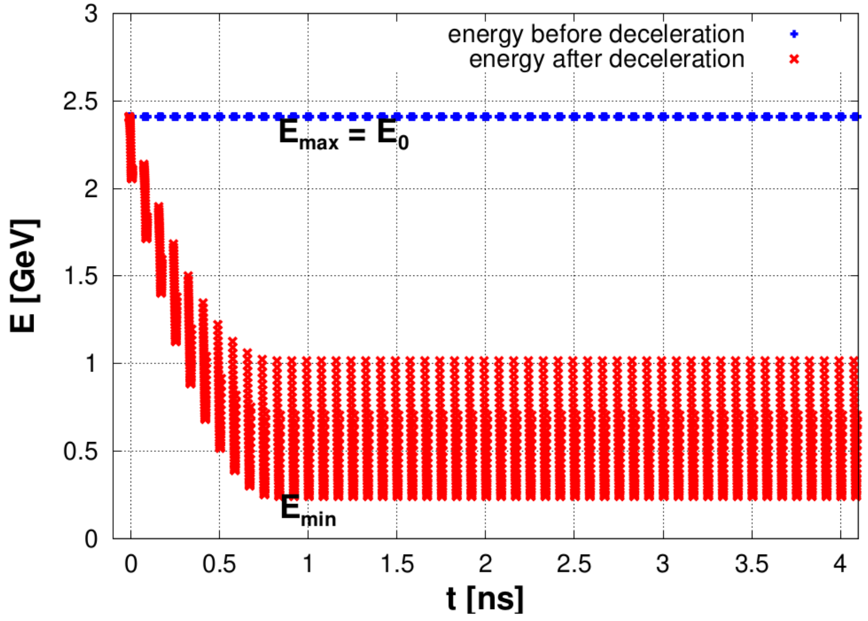


> RF-Track numerical calculation of the gradient reduction in CLIC PETS.

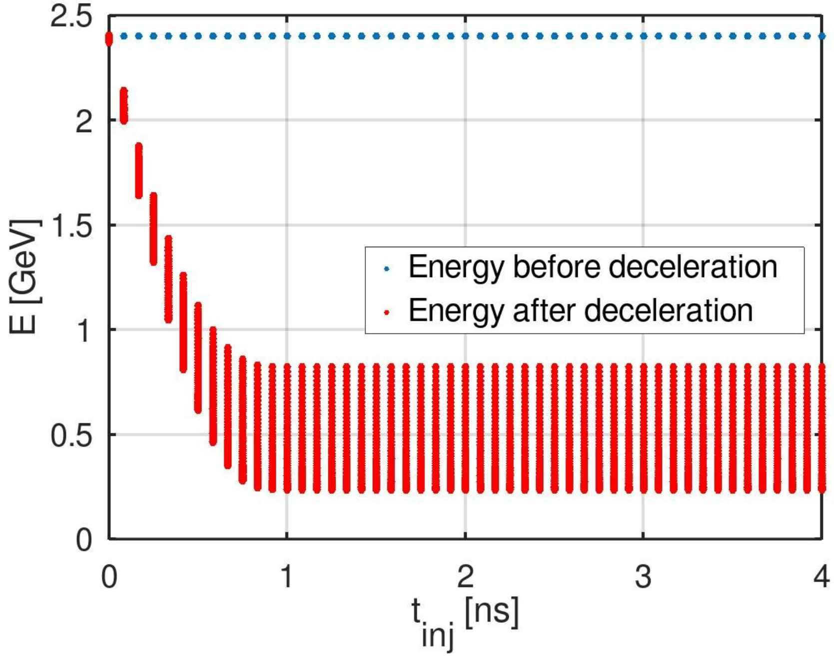
[5] Erik Adli (2009). *A Study of the Beam Physics in the CLIC Drive Beam Decelerator.* PhD Thesis.

III. BL tracking – Long Range

- CLIC Power Extraction and Transfer Structures (**PETS**)
  - Tracking of 50 bunches after **1492 PETS**



> PLACET train energy distribution after 1492 PETS [5].



> RF-Track train energy distribution after 1492 PETS.

III. BL tracking – Short Range

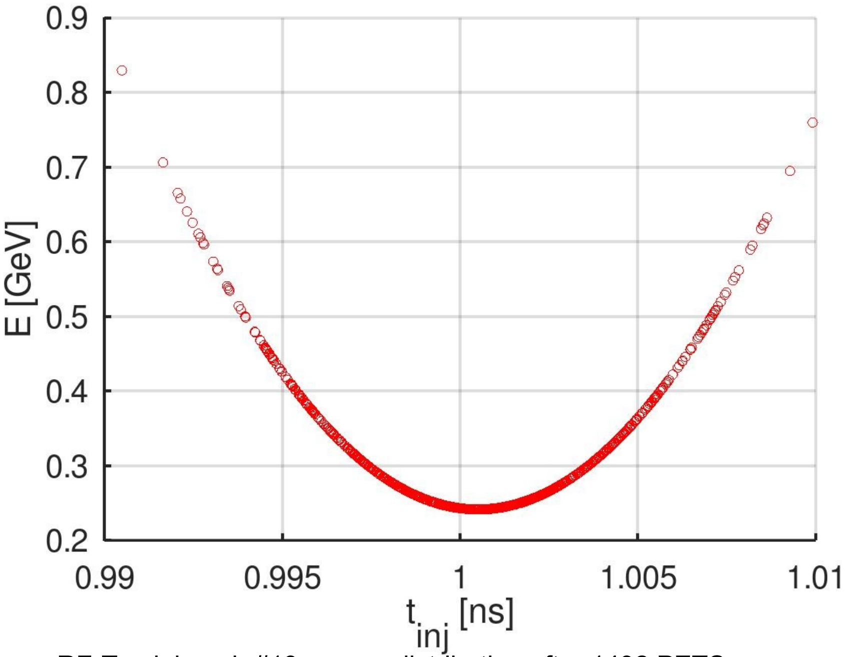
- CLIC Power Extraction and Transfer Structures (**PETS**)
  - Tracking of bunch #13 after **1492 PETS**

$$\eta = \frac{E_0 - E_{\min}}{E_0} \qquad \eta_{\text{PLACET}} = 90,0\%$$ [5]

Magnitude	Units	Value
E_{in}	GeV	2, 40
$E_{\min}$	MeV	242
η	%	89, 7
δ	%	0, 63

> Extraction efficiency parameters and comparison.

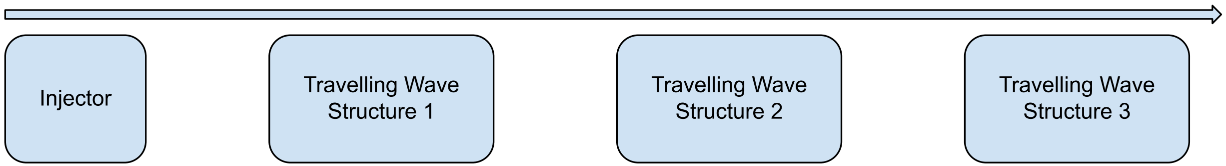
$$\delta = \frac{\eta_{\text{PLACET}} - \eta_{\text{RF-Track}}}{\eta_{\text{PLACET}}}$$



> RF-Track bunch #13 energy distribution after 1492 PETS.

III. CLEAR - BeamLine

- Accelerating structures at CLEAR:



Magnitude	Units	Value
f_{RF}	GHz	2.997
ϕ_{ad}	rad	π
L_{total}	m	0.175
Q		15773
r_{eff}/Q	Ω / m	3765
t_{fill}	ns	1492
E_z^{max}	MV/m	80.0

> Standing Wave Injector Parameters.

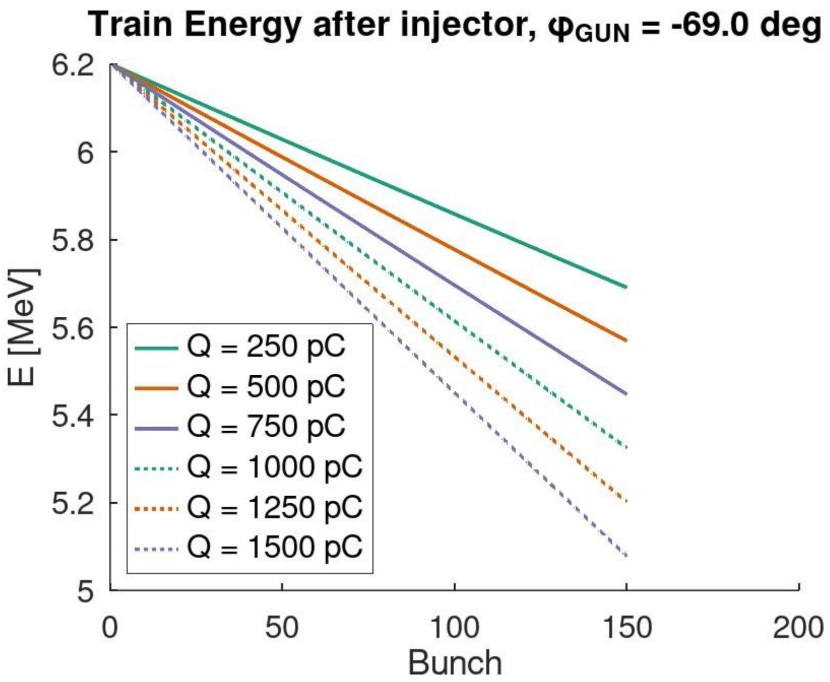
Magnitude	Units	Value
f_{RF}	GHz	2.997
ϕ_{ad}	rad	$2\pi/3$
L_{total}	m	4.5
$1/v_g$ (1^{st}, middle, last cell)	$1/c$	(46, 70, 133)
Q (1^{st}, middle, last cell)		(15300, 15210, 15130)
r_{eff}/Q (1^{st}, middle, last cell)	Ω/m	(4000, 4400, 4800)
t_{fill}	ns	1183
E_z^{max}	MV/m	15.0 – 20.0

> Travelling Wave Structures Parameters. [6] [7]

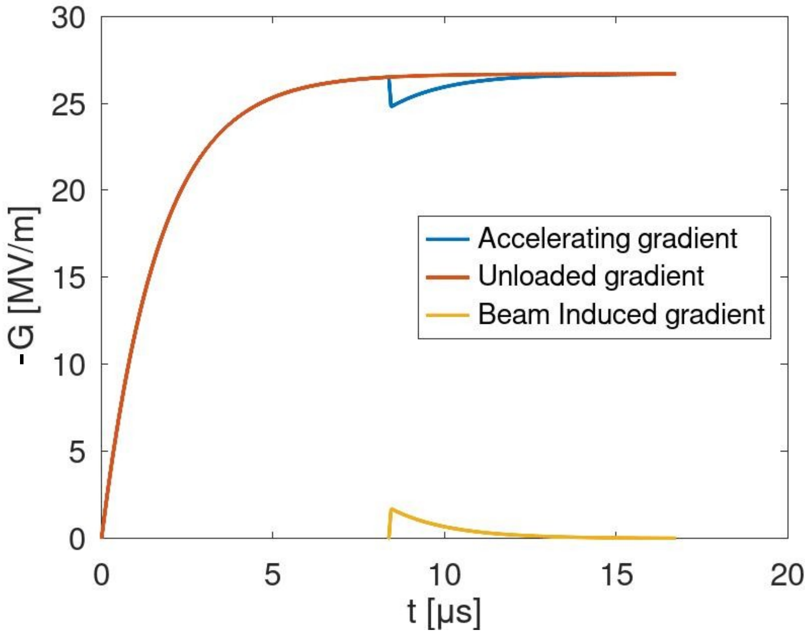
[6] CLEAR official site: https://clear.cern/content/beam-line-description
[7] LAL Report on LIL (Linac Injector of LEP)

III. BL in CLEAR Injector

- Train: 150 bunches; $f = f_{RF}/2$; Q_{bunch} = 250 pC – 1500 pC



> Energy profile for bunches with different charges after having travelled along the photoinjector



> Gun accelerating field evolution with time. Injection occurs at t = 8,37 us.

III. BL in CLEAR Injector

Another consequence: **Arrival time to TWS1**

- If all bunches traveled with same β , then the arrival time to TWS1 would be equally spaced.

$$t_k = \underbrace{\frac{4\pi k}{\omega}}_{\text{Injection time}} + \underbrace{\int_0^L \frac{\mathrm{d}z}{\beta(z)c}}_{\text{Flight time along gun}}; \; k = 0, .., N-1 \implies \Delta t_k = \frac{4\pi}{\omega}$$

- However, particles have different β because of **Gradient reduction** $\implies$ **Different Δt_k!**

III. BL in CLEAR Injector

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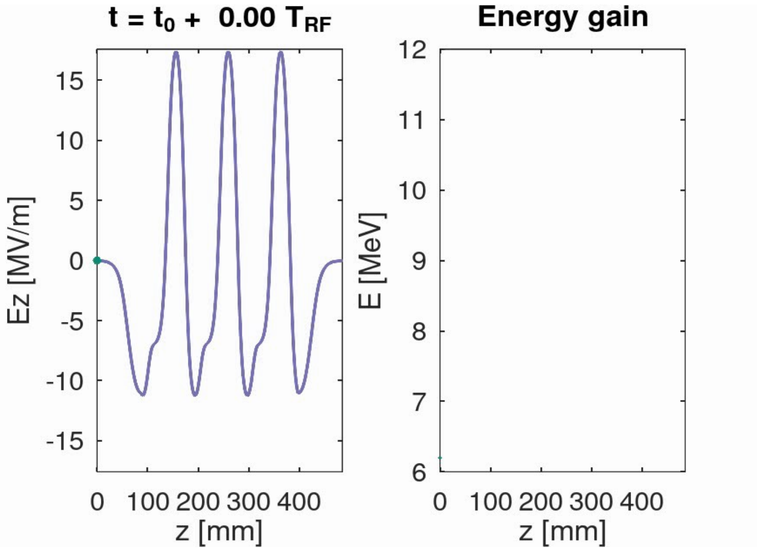
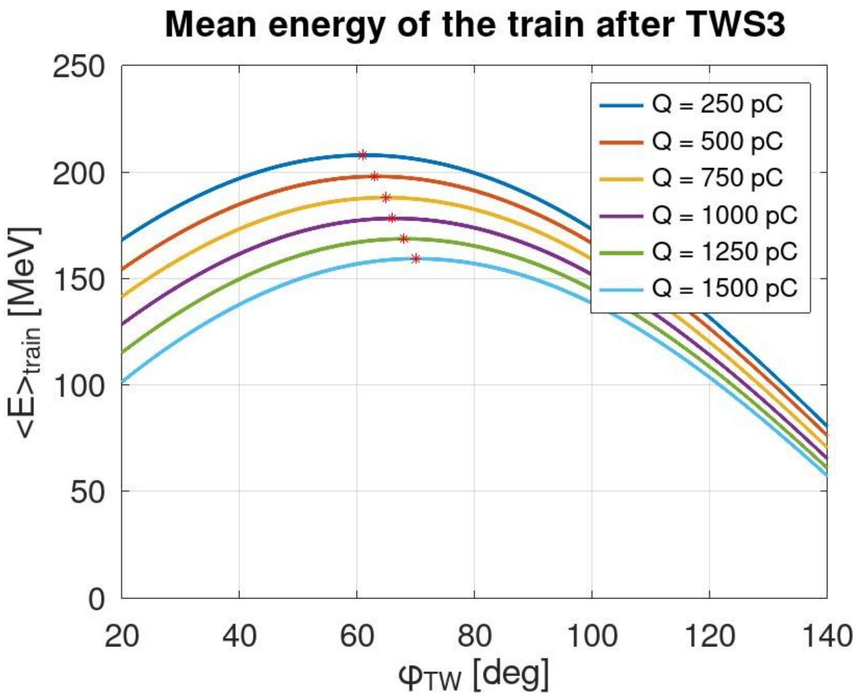


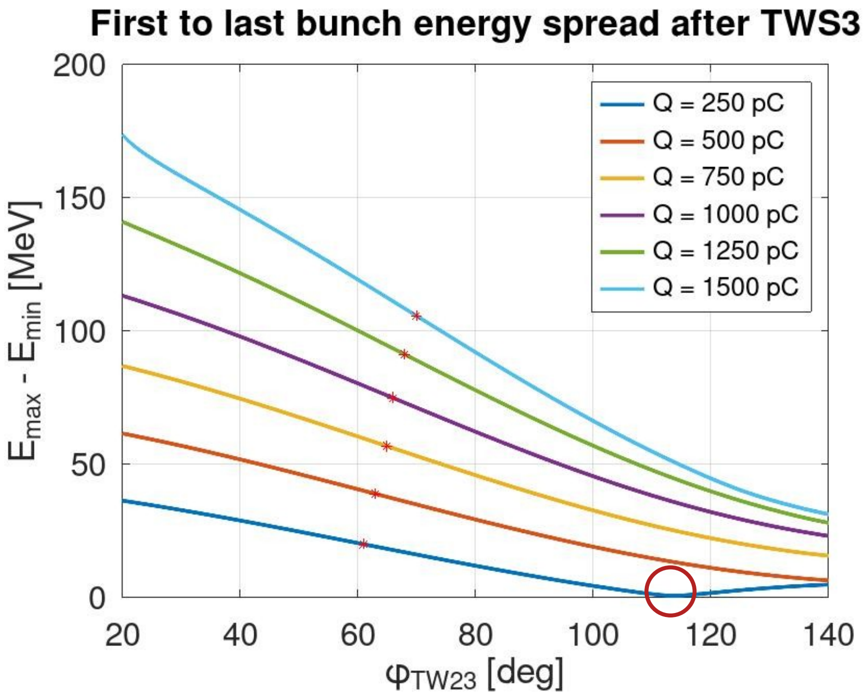
> Video showing the relevancy of synchronization for 2 bunches (macro-particles) injected on at off phase.

III. Start-to-end BL in CLEAR

- Consider the whole structure + Phase scan in $\phi_{\mathrm{TWS2}} = \phi_{\mathrm{TWS3}}$



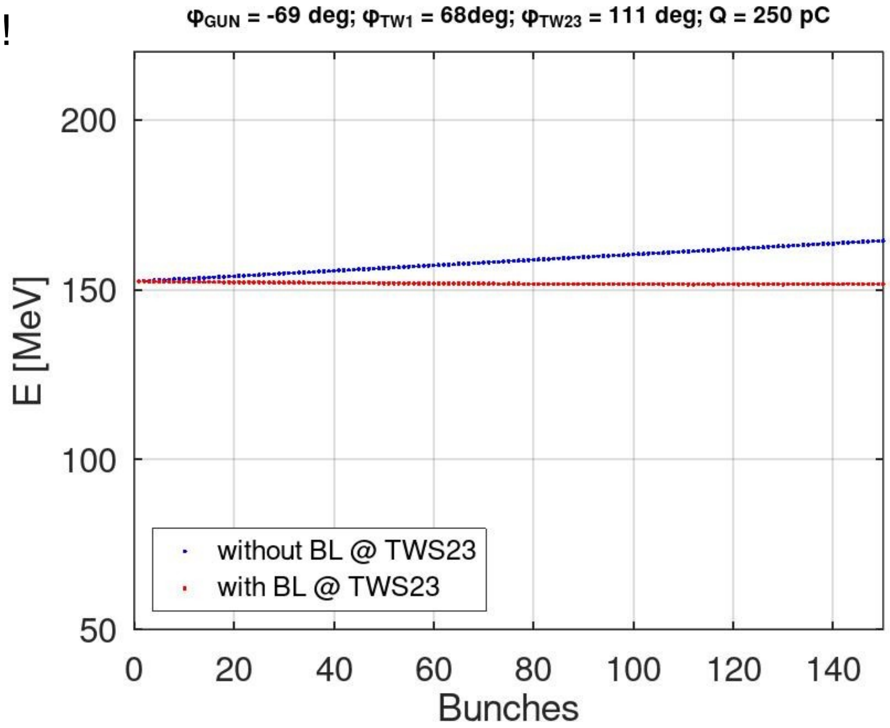
> Mean Energy of different trains dependency with the phase of TWS2-3.



> Energy range of different trains dependency with the phase of TWS2-3.

III. Start-to-end BL (Phase Compensation)

- **Beam Loading at GUN helps compensating overall Beam Loading**
  - Correct phase choice!



> Beam Loading compensated train.

Conclusions

- Gradient reduction due to beam-cavity interaction can be understood with the **Power-Diffusive model**
- The implementation of this model in **RF-Track** provides a **user-friendly**, **flexible** and **powerful** tool which allows:
  - To study gradient reduction in future linear colliders (in our case, CLIC)
  - To perform **long** and **short** range tracking considering **BL** effect
  - Study BL effects in **guns** and its **implications (unique?)**

- **Great agreement** has been found with previous BL studies
- Transient BL scenarios in gun injectors still require **experimental verification**

Acknowledgements

- **Supervision**, guidance and trust:
  - Andrea Latina (CERN, BE-ABP-LAF)
    also, RF-Track creator and main developer.
  - Nuria Fúster Martínez, Benito Gimeno, Daniel Esperante (UV – Spain, IFIC - CSIC)

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  - Alexej Grudiev (CERN, SY-RF-MKS)
  - Avni Aksoy (CERN, BE-ABP-LAF)
  - The CLIC Beam Dynamics team

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