Beam Loading Effects in the tracking code RF-Track

International Workshop on Linear Colliders 2023 – Accelerator: Beam Dynamics Session

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Beam Loading Effect

- **What**: Reduction of available accelerating gradient
- **Origin**: Beam – Cavity interaction
- **Consequences**: Transient response
  - Different energy loss from bunch to bunch
- **Motivation**: High I, Compact accelerating structures


> Theoretical analysis of beam loading effect based on CLIC’s main linac [1]
Outline

- **PART I**: Introduce *power-diffusive model* for Beam Loading Effect
  - Figures of merit
- **PART II**: Implementation into *RF-Track*
- **PART III**: Results
  - Reproducibility of *BL fields*
  - **Long** and **Short range** tracking results
  - **Transient** BL in *photoinjector guns*
  - Start-to-end BL simulations
PART I: Power-Diffusive Model
I. Energy Conservation

- **Poynting Theorem**

\[- \frac{\partial u(\vec{r}, t)}{\partial t} = \nabla \cdot \vec{S}(\vec{r}, t) + \vec{E}(\vec{r}, t) \cdot \vec{J}(\vec{r}, t)\]

- **Figures of merit:**
  - Group velocity
  - Quality factor
  - Shunt impedance (p.u.l)

\[v_g = \frac{P_{\text{flow}}}{w} \quad [\text{m/s}]\]

\[Q = \frac{\omega_{\text{RF}}}{P_{\text{diss}}} \cdot \frac{w}{P_{\text{diss}}}\]

\[r_e = \frac{G_{\text{eff}}^2}{P_{\text{diss}}} \quad [\Omega/\text{m}]\]

> Energy balance schematics for an accelerating structure

I. Gradient & Synchronization

- **Gradient**: Averaged E-field *affecting* the particle

- Time of flight:
  \[ t_q(z, t_0, \beta(\zeta, t, t_0, \beta_0)) = t_0 + \int_0^z \frac{d\zeta}{\beta(\zeta, t, t_0, \beta_0)c} \]

- Effective E-field:
  \[ E_{z|\text{eff}}(z, t, t_0, \beta) = \Re \left( \tilde{E}_z(z, t)e^{j\omega t_q(z, t_0, \beta(z, t_0, \beta_0))} \right) \]

> Electron synchronization with the on-axis electric field of an S-band accelerating cavity with 9 cells and peak E-field of 100 MV/m
I. Gradient & Synchronization

- **Gradient**: Averaged E-field affecting the particle

- Time of flight:
  \[ t_q(z, t_0, \beta(z, t, t_0, \beta_0)) = t_0 + \int_0^z \frac{d\zeta}{\beta(\zeta, t, t_0, \beta_0)c} \]

- Effective E-field:
  \[ E_{z|\text{eff}}(z, t, t_0, \beta) = \text{Re} \left[ \bar{E}_z(z, t)e^{j\omega t_q(z, t_0, \beta(z, t, t_0, \beta_0))} \right] \]

- Effective Gradient
  \[ G_{\text{eff}}(z_k, t_0, \beta) = \frac{1}{L} \int_{z_k}^{z_k + L} E_{z|\text{eff}}(z, t, t_0, \beta) \, dz \]

> Effective electric field affecting the electron and its average over the cells
I. Power-Diffusion PDE

- From Poynting: Equation in terms of Gradient:

\[-\frac{\partial G_{\text{eff}}}{\partial t} = v_g \frac{\partial G_{\text{eff}}}{\partial z} + \left( -\frac{v_g Q}{r_{\text{eff}}} \frac{\partial (r_{\text{eff}}/Q)}{\partial z} + \frac{\omega}{Q} + \frac{\partial v_g}{\partial z} \right) \frac{G_{\text{eff}}}{2} + \frac{\omega r_{\text{eff}} \tilde{I}}{2Q}\]

Some features:

- **Paraxial** approximation
- From \(z_k\) to \(z\) → Cubic interpolation, continuity.
- **Beam Loading term**: **Decelerating** gradient dependent on **Intensity**.
- Assumes **causality**!
- Matches [1] for the TW ultrarelativistic case

Beam Loading term!
PART II: RF-Track Implementation
II. RF-Track

• About RF-Track [3]:
  – Beam tracking in field maps/analytic structures including space-charge effects, wakefields, ...
  – Multiple species (arbitrary $q$ and $m$)
  – Parallel C++, interface with user via Octave or Python

• Beam Loading in RF-Track:
  – Self-consistent module
  – Additional decelerating kick ($F_{BL}$)
    • Attached to Drift spaces, Analytic TW & SW structures, field maps

II. RF-Track – BL algorithm

- **INPUT**: $\omega, E_z, P, Q, v_g, \phi_{ad}, \text{BEAM}$

- **PHASE 1: Preparation**
  - 1.1) Structure characterization
    - From $Q, v_g$ → **Cubic interpolate** values from 0 to $L_{total}$
    - From $E_z, \omega, \phi_{ad}, P$ → **Integrate** and get $G(z, t = 0) \forall z \in [0, L_{total}]$
    - Integrate and get $r_{eff}$
  - 1.2) PDE solving
    - **Finite difference** method → Get $G(z, t) \forall t \in [0, t^*]$

- **PHASE 2: Tracking + BL**
  - 2.1) Perform RF-Track tracking
  - 2.2) Add $F_{BL}$ kick
    $$(z_{part}, t_{part}) \rightarrow F_{BL} = -q \left(1 - \frac{G(z_{part}, t_{part})}{G(z_{part}, 0)}\right) E_z(z_{part}, t_{part})$$
II. RF-Track – BL Example

- Example in Octave: BL + TW field map

```octave
% Import RF-Track
RF_Track;

% Define bunch
B0 = Bunch6d([X XP YYP t P MASS Q N]);

% Define RF structure
TWS = RF_FieldMap_1d_CINT(Ez, hz, L, freq, -1);

% BL effect
BL_steady = BeamLoading(TWS, Pmap, VG, Qfactor, phaseadvance, -1, particles_bunch, fb);
BL_trans = BeamLoading(TWS, Pmap, VG, Qfactor, phaseadvance, -1, particles_bunch, fb, Nbunches);

% Append BL effect to TWS
TWS.add_collective_effect(BL_trans);

% Tracking Lattice
L = Lattice();
L.append(TWS);

% Perform tracking
B1 = L.track(B0);

% Retrieve information and manipulate
M1 = B1.get_phase_space();```

PART III: Results

- BL field map
- BL Tracking (Long and Short Range)
- BL in Injectors
- Start-to-end BL simulations
III. BL field map (CLIC AS)

- CLIC main Linac **Accelerating Structure**

  - BL decelerating field

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{RF}$</td>
<td>GHz</td>
<td>12.0</td>
</tr>
<tr>
<td>$\phi_{ad}$</td>
<td>rad</td>
<td>$2\pi/3$</td>
</tr>
<tr>
<td>$N_{cells}$</td>
<td></td>
<td>26 + 1</td>
</tr>
<tr>
<td>$v_g$ (1\textsuperscript{st}, middle, last cell)</td>
<td>%c</td>
<td>(1, 65, 1, 20, 0, 83)</td>
</tr>
<tr>
<td>$Q$ (1\textsuperscript{st}, middle, last cell)</td>
<td></td>
<td>(5, 54, 5, 64, 5, 74) · 10\textsuperscript{3}</td>
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<tr>
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<td>ns</td>
<td>66, 7</td>
</tr>
<tr>
<td>$P_{\text{in}}$</td>
<td>MW</td>
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<tr>
<td>$f_{\text{inj}}$</td>
<td>GHz</td>
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<tr>
<td>$\langle f \rangle$</td>
<td>A</td>
<td>1.00</td>
</tr>
<tr>
<td>$N_{\text{bunches}}$</td>
<td></td>
<td>312</td>
</tr>
</tbody>
</table>

> CLIC main Linac Accelerating Structure Parameters [4]

> Beam Induced Decelerating force for CLIC AS.

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III. BL field map (CLIC AS)

- CLIC main Linac **Accelerating Structure**
  - Superposition to initial gradient (blue) → Total effect upon particles to track

> Theoretical calculation of gradient reduction in CLIC AS [1].

> RF-Track numerical calculation of the gradient reduction in CLIC AS.
III. BL field map (CLIC PETS)

- CLIC Power Extraction and Transfer Structures (PETS)
  - Passive structures → Deceleration

<table>
<thead>
<tr>
<th>Magnitude</th>
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<th>Value</th>
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<tr>
<td>$\varphi_{ad}$</td>
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<td>kΩ/m</td>
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<td>GHz</td>
<td>12, 0</td>
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<tr>
<td>$\langle I \rangle$</td>
<td>A</td>
<td>101</td>
</tr>
<tr>
<td>$N_{bunches}$</td>
<td></td>
<td>2,93 $\cdot 10^3$</td>
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<tr>
<td>$\sigma_t$</td>
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<tr>
<td>$E_{inj}$</td>
<td>GeV</td>
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</table>

> PETS parameters [5]

> RF-Track numerical calculation of the gradient reduction in CLIC PETS.

III. BL tracking – Long Range

- CLIC Power Extraction and Transfer Structures (PETS)
  - Tracking of 50 bunches after 1492 PETS

> PLACET train energy distribution after 1492 PETS [5].

> RF-Track train energy distribution after 1492 PETS.
III. BL tracking – Short Range

- CLIC Power Extraction and Transfer Structures (PETS)
  - Tracking of bunch #13 after 1492 PETS

\[ \eta = \frac{E_0 - E_{\text{min}}}{E_0} \quad \eta_{\text{PLACET}} = 90.0\% \text{ [5]} \]

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>Units</th>
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<tbody>
<tr>
<td>( E_{\text{in}} )</td>
<td>GeV</td>
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</tr>
<tr>
<td>( E_{\text{min}} )</td>
<td>MeV</td>
<td>242</td>
</tr>
<tr>
<td>( \eta )</td>
<td>%</td>
<td>89.7</td>
</tr>
<tr>
<td>( \delta )</td>
<td>%</td>
<td>0.63</td>
</tr>
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</table>

> Extraction efficiency parameters and comparison.

\[ \delta = \frac{\eta_{\text{PLACET}} - \eta_{\text{RF-Track}}}{\eta_{\text{PLACET}}} \]

> RF-Track bunch #13 energy distribution after 1492 PETS.
### III. CLEAR - BeamLine

- Accelerating structures at CLEAR:

#### Travelling Wave Structure 1

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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<td>$f_{RF}$</td>
<td>GHz</td>
<td>2.997</td>
</tr>
<tr>
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<td>rad</td>
<td>$\pi$</td>
</tr>
<tr>
<td>$L_{total}$</td>
<td>m</td>
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<tr>
<td>$Q$</td>
<td></td>
<td>15773</td>
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<tr>
<td>$r_{eff}/Q$</td>
<td>$\Omega / m$</td>
<td>3765</td>
</tr>
<tr>
<td>$t_{\text{fill}}$</td>
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<td>1492</td>
</tr>
<tr>
<td>$E_{z}^{\text{max}}$</td>
<td>MV/m</td>
<td>80.0</td>
</tr>
</tbody>
</table>

> Standing Wave Injector Parameters.

#### Travelling Wave Structure 2

<table>
<thead>
<tr>
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<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{RF}$</td>
<td>GHz</td>
<td>2.997</td>
</tr>
<tr>
<td>$\phi_{ad}$</td>
<td>rad</td>
<td>$2\pi/3$</td>
</tr>
<tr>
<td>$L_{total}$</td>
<td>m</td>
<td>4.5</td>
</tr>
<tr>
<td>$1/v_a$ ($1^{st}$, middle, last cell)</td>
<td></td>
<td>$1/c$ (46, 70, 133)</td>
</tr>
<tr>
<td>$Q$ ($1^{st}$, middle, last cell)</td>
<td></td>
<td>(15300, 15210, 15130)</td>
</tr>
<tr>
<td>$r_{eff}/Q$ ($1^{st}$, middle, last cell)</td>
<td>$\Omega / m$</td>
<td>(4000, 4400, 4800)</td>
</tr>
<tr>
<td>$t_{\text{fill}}$</td>
<td>ns</td>
<td>1183</td>
</tr>
<tr>
<td>$E_{z}^{\text{max}}$</td>
<td>MV/m</td>
<td>15.0 – 20.0</td>
</tr>
</tbody>
</table>

> Travelling Wave Structures Parameters. [6] [7]

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[6] CLEAR official site: https://clear.cern/content/beam-line-description
III. BL in CLEAR Injector

- Train: 150 bunches; \( f = \frac{f_{RF}}{2} \); \( Q_{bunch} = 250 \text{ pC} \text{ – 1500 pC} \)

> Energy profile for bunches with different charges after having travelled along the photoinjector

> Gun accelerating field evolution with time. Injection occurs at \( t = 8.37 \mu s \).
III. BL in CLEAR Injector

Another consequence: **Arrival time to TWS1**

- If all bunches traveled with same $\beta$, then the arrival time to TWS1 would be equally spaced.

$$ t_k = \frac{4\pi k}{\omega} + \int_0^L \frac{dz}{\beta(z)c}; \quad k = 0, \ldots, N - 1 \implies \Delta t_k = \frac{4\pi}{\omega} $$

Injection time Flight time along gun

- However, particles have different $\beta$ because of **Gradient reduction** $\implies$ Different $\Delta t_k$!
III. BL in CLEAR Injector

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Injection time Flight time along gun

- However, particles have different $\beta$ because of Gradient reduction $\Rightarrow$ Different $\Delta t_k$!

> Video showing the relevancy of synchronization for 2 bunches (macro-particles) injected on at off phase.
III. Start-to-end BL in CLEAR

- Consider the whole structure + Phase scan in $\phi_{TWS2} = \phi_{TWS3}$

> Mean Energy of different trains dependency with the phase of TWS2-3.

> Energy range of different trains dependency with the phase of TWS2-3.
III. Start-to-end BL (Phase Compensation)

- Beam Loading at GUN helps compensating overall Beam Loading
  - Correct phase choice!

\[ \Phi_{\text{GUN}} = -69 \, \text{deg}; \Phi_{\text{TW1}} = 68 \, \text{deg}; \Phi_{\text{TW23}} = 111 \, \text{deg}; Q = 250 \, \text{pC} \]
Conclusions

- Gradient reduction due to beam-cavity interaction can be understood with the **Power-Diffusive model**
- The implementation of this model in **RF-Track** provides a **user-friendly, flexible and powerful** tool which allows:
  - To study gradient reduction in future linear colliders (in our case, CLIC)
  - To perform **long** and **short** range tracking considering **BL** effect
  - Study BL effects in **guns** and its **implications (unique?)**

- **Great agreement** has been found with previous BL studies
- Transient BL scenarios in gun injectors still require **experimental verification**
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