BEAM BREAKUP STUDIES FOR THE C$^3$ LINEAR COLLIDER

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Contents

- Initiatives utilizing distributed coupling linacs and our experience with such structures
- Sources of wakefields in RF linacs and BBU effects
- Modeling of the wakefield interaction: the code MILES
- BBU studies for the TeV-scale working point of the C$^3$ linear collider
- Investigation of phase advances alternative to the $\pi$-mode: SRWFs in $3\pi/4$ mode distributed coupling linac
Our Experience with Dist. Coupling Linacs

A novel NCRF concept for high gradient acceleration:

- Distributed coupling linac (DCL) scheme
- Optimized cavity topology
- Cryogenic operation of Cu structures (e.g. 45-77 K)


UC-XFEL

DARPA-GRIT ICS Source

C³: a “Cool Copper Collider”


E. Nanni et al. C³ demonstration research and development plan, 03 2022.
Sources of Wakefields in Linacs

- Field generated by the beam, **diffracted** by the surrounding environment and acting back on the beam
- Responsible for **energy spread** and **transverse deflection**

**Modes in resonant cavities**

- Longitudinal monopole and transverse dipole **higher order modes** (HOMs) in the accelerating cavities
- Responsible for the **long-range wakefield** (LRWF) interaction ($\omega_n = \omega_0 \sqrt{1 - (2Q)^{-2}}$ and $\alpha = \omega_0 / 2Q$)

$$
\begin{align*}
\Delta p &= \int F(r, r_0, t)|_{z=v_{r_0}t} \, dt \\
w_{\parallel} &= -\frac{c}{qq_0} \Delta p_{\parallel} \\
w_{\perp} &= \frac{c}{qq_0} \Delta p_{\perp}
\end{align*}
$$

- Field generated by the beam, diffracted by the surrounding environment and acting back on the beam
- Responsible for **energy spread** and **transverse deflection**

**Periodic accelerating structures**

- Longitudinal monopole and transverse dipole impedance from diffraction theory
- Responsible for the **short-range wakefield** (SRWF) interaction in **periodic accelerating structures**

$$
\begin{align*}
w_{\parallel}(s) &= \frac{Z_0 c}{\pi \alpha^2} \exp \left(-\sqrt{\frac{s}{s_0}}\right) \\
w_{\perp}(s) &= \frac{4Z_0 c \xi_1}{\pi \alpha^4} \left[1 - \left(1 + \sqrt{\frac{s}{s_1}}\right) \exp \left(-\sqrt{\frac{s}{s_1}}\right)\right]
\end{align*}
$$


Beam Breakup Effects

**Long-range transverse interaction**

\[ w_\perp(\tau) = \frac{\omega_0 R_\perp}{Q} e^{-\alpha \tau} \sin(\omega_n \tau) \]

- Off-axis bunches
- Transverse deflection
- Excitation of dipole HOMs

**Instability**: the amplitude of the center of mass oscillation grows

**Short-range transverse interaction**

\[ w_\perp(s) = \frac{4Z_0 cs_1}{\pi a^4} \left( 1 - \left( 1 + \sqrt{\frac{s}{s_1}} \right) \exp \left( -\sqrt{\frac{s}{s_1}} \right) \right) \]

**Phase space dilution**: the correlation between the planes is a cause of emittance growth
The Code MILES

Dedicated tracking code
• Motivation to investigate wakefields effects in linacs in presence of non-negligible space charge endangering the nominal beam quality
• Use of simple semi-analytical models: flexible and time-efficient tool (factors ∼1/30–1/10) with acceptable reduction of the accuracy
• Main applications: investigation of misalignment effects and design of possible correction schemes

MILES = Modeling Instabilities in Linacs with Ellipsoidal Space charge


Modeling overview and comparison with other codes

<table>
<thead>
<tr>
<th></th>
<th>MILES approach</th>
<th>Commonly utilized approaches</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space Charge</td>
<td>Equivalent ellipsoid or Multi-slice superposition</td>
<td>Particle in cell (PIC)</td>
</tr>
<tr>
<td>Wakefields</td>
<td>Algebraic formalism</td>
<td>Convolution integral or FFT equivalent</td>
</tr>
</tbody>
</table>
Long-range BBU in the C³ Linac

- BBU studies for the TeV scale working point of the C³ linear collider
- Center of mass oscillations in multi-bunch operation: sequence of point-like, structureless macro-particles carrying the whole bunch charge

C³ working point at TeV-scale
+ knowledge of dipole HOMs parameters (ω₀, Q, R⊥) for 5 < ω₀/2π < 20 GHz

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of bunches</td>
<td>75</td>
</tr>
<tr>
<td>Bunch charge</td>
<td>1 nC</td>
</tr>
<tr>
<td>Bunch separation</td>
<td>3.3 ns</td>
</tr>
<tr>
<td>Injection energy</td>
<td>10 GeV</td>
</tr>
<tr>
<td>Energy at the IP</td>
<td>1 TeV</td>
</tr>
<tr>
<td>Accelerating gradient</td>
<td>117 MeV/m</td>
</tr>
<tr>
<td>Avg betatron function</td>
<td>4 m</td>
</tr>
<tr>
<td>Linac length</td>
<td>≥ 12 km</td>
</tr>
</tbody>
</table>

Injection error:
- Linac sections are perfectly aligned (ideal)
- Injection with a transverse offset Δx for all bunches

Random alignment errors:
- On-axis injection
- Linac sections exhibit gaussian transverse offsets with std dev σΔx

Amplitude of the transverse oscillation of the bunches at the interaction point
Long-range BBU in the C$^3$ Linac (2)

Detuning of the HOMs

- **Unstable** motion if the ratio $f_{HOM}/f_b$ is close to an integer (see Mosnier)
- Mitigation through frequency **spread** (or *detuning*)
- Seven cavity-designs from small **variations** of the original geometry
- HOMs exhibit different, *i.e.* random, frequencies through subsequent linac sections

Amplitude of the transverse oscillation for each bunch in the rf-pulse at the interaction point normalized to the first bunch’s amplitude

Average on 50 iterations

- $\sigma_{\Delta x} = 100 \ \mu m$ - unif. freq. spread
- $\sigma_{\Delta x} = 100 \ \mu m$ - gauss freq. spread

Worst case scenario from previous plot:
Random misalignments with $\sigma_{\Delta x} = 100 \ \mu m$


$3\pi/4$-mode performance

Phase advances alternative to the conventional $\pi$-mode
135 degree phase advanced distributed coupling structure

• If one includes the period as an optimization parameter then the optimal structure with the minimal surface magnetic field, and consequently the highest shunt impedance occurs at ~132° phase advance; a natural choice is a 135° (i.e. $3\pi/4$ radians) phase advance.

• Every $4^{th}$ cavity has a $\pi$ phase shift (modulo $2\pi$), hence 4 manifolds are needed to feed the structure.

• Every two manifolds need to be fed with a $\pi/2$ phase shift; naturally fed by a hybrid that isolates the forward signal from the reflected signal.

Cell-to-cell phase advance:

$$\phi = \frac{\omega}{c} p$$

SRWF in the $3\pi/4$ Structures

Change of the geometrical dimensions implies changes in the wakefields

\[
\begin{align*}
  w_{\parallel}(s) &= \frac{Z_{0}c}{\pi \alpha^2} \exp \left( -\sqrt{\frac{s}{s_0}} \right) \\
  w_{\perp}(s) &= \frac{4Z_{0}c s_1}{\pi \alpha^4} \left( 1 - \left( 1 + \sqrt{\frac{s}{s_1}} \right) \exp \left( -\sqrt{\frac{s}{s_1}} \right) \right)
\end{align*}
\]

\[
\begin{align*}
  s_0 &\approx 0.41a^{1.8}g^{1.6}/p^{2.4} \\
  s_1 &\approx 0.169a^{1.79}g^{0.38}/p^{1.17}
\end{align*}
\]


<table>
<thead>
<tr>
<th>Iris $a$ [mm]</th>
<th>Gap $g$ [cm]</th>
<th>Period $p$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1, 2, 3$</td>
<td>$1.496, 1.534, 1.501$</td>
<td>$(3\lambda/8)$</td>
</tr>
<tr>
<td>$2$</td>
<td>$2$</td>
<td>$(\lambda/2)$</td>
</tr>
</tbody>
</table>

**Longitudinal Monopole SR wake-function**

**Transverse dipole SR wake-function**
Mitigation by Trajectory Steering

**Reference case**

- UC-XFEL scenario (100 pC, 0.5 mm rms beam, ~100 nm emittance at injection)
- 2 linac sections: $6.9 \text{ MeV} < \gamma mc^2 < 150 \text{ MeV}$ with the 1st section is 100 um off-axis
- Demonstration for the worst case scenario of 1 mm iris (two correctors upstream each linac section)
Conclusions and Outlook

• **Cryo-cooled distributed coupling linacs** are expected to play a crucial role for advanced and diverse **applications** (e⁻ e⁺ colliders, FELs, ICS, FLASH RT,...)
• The development of **MILES** for an efficient description of **BBU** (and other) **effects**
• BBU effects and their mitigation in the **C³ linear collider**
• Possible benefits in the operation with **3π/4** phase advance structures

**Ongoing work on long-range BBU**

• **Electromagnetic simulation**: eigenmode analysis of the structure finding frequencies, quality factors and shunt impedances for monopole and dipole
• **BBU studies with the 3π/4 structure**: multi-bunch simulations with MILES to estimate the center-of-mass oscillations as well as multi-bunch with intra-beam effects (SC + SRWF) for simultaneous description of short-range and long range wakefields

Thank you for your attention
(Questions are welcome)

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Additional Material

International Workshop on Future Linear Collider – May 15-19 2023
The Code MILES

Dedicated tracking code

- Motivation to investigate wakefields effects in linacs in presence of non-negligible space charge endangering the nominal beam quality
- Use of simple semi-analytical models: flexible and time-efficient tool (factors ~1/30-1/10) with acceptable reduction of the accuracy
- Main applications: investigation of misalignment effects and design of possible correction schemes

MILES = Modeling Instabilities in Linacs with Ellipsoidal Space charge

Comparison with other codes utilized in accelerator physics

- SPACE CHARGE: Particle in cell, PIC, approach (GPT, ASTRA, PARMELA, TSTEP...)
- WAKEFIELDS: Convolution integral with the beam slices, often solved with FFT algorithms (ASTRA, ELEGANT, LUCRETIA...)
- SC + WF: Complexity and CPU times increase and, thus, it is not a trivial feature (ASTRA, IMPACT-Z, OCELOT, HOMDYN)
The Code MILES

Dedicated tracking code

- Motivation to investigate **wakefields effects** in linacs in presence of non-negligible **space charge** endangering the nominal beam quality
- Use of simple **semi-analytical models**: flexible and time-efficient tool (**factors ∼1/30-1/10**) with acceptable reduction of the accuracy
- Main applications: investigation of misalignment effects and design of possible correction schemes

**MILES** = **M**odeling **I**nstabilities in **L**inacs with **E**llipsoidal **S**pace charge

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**Modeling overview and comparison with other codes**

- **SPACE CHARGE**: Equivalent ellipsoid or Multi-slice superposition instead of **PIC approaches** (GPT, ASTRA, PARMELA, TSTEP...)
- **WAKEFIELDS**: Algebraic formalism valid for short and long-range interaction instead of **convolution integral** with the beam slices (ASTRA, ELEGANT, LUCRETIA...)
- **SC + WF**: Complexity and CPU times increase and, thus, it is not a trivial feature (ASTRA, IMPACT-Z, OCELOT, HOMDYN)
Modeling Space Charge Forces

**Equivalent ellipsoid method**

- Ellipsoidal beams arise from the **blowout** process
- An **equivalent** uniform ellipsoid is associated to the actual beam distribution
- Self-induced **forces** are known analytically
- Forces are **linear**: self consistent model
  \[
  \Phi'(x, y, z') = \frac{3Q}{16\pi\epsilon_0} \int_0^{\infty} \left( 1 - \frac{x^2}{a^2 + t} - \frac{y^2}{b^2 + t} - \frac{z'^2}{c^2 + t} \right) \frac{dt}{\sqrt{\varphi(t)}}
  \equiv D_0 - A_0 x^2 - B_0 y^2 - C_0 z'^2
  \]

- Ellipsoidal beams arise from the blowout process
- An equivalent uniform ellipsoid is associated to the actual beam distribution
- Self-induced forces are known analytically
- Forces are linear: self consistent model

**Advantages**
- Very fast (~ 30 times than 3D PIC)
- Successfully reproduces rms size \((\sigma_x, \sigma_y, \sigma_z)\) and energy spread \((\sigma_E)\)

**Limitations**
- Absence of correlation prevents the observation of emittance dynamics
- Loss of axial symmetry in presence of strong transverse wakefield

**Multi-slice superposition method**

- Divide the beam in **cylindrical slices** with individual size, energy and aspect ratio
- Field produced by each (uniform) slice is known analytically \((\partial / \partial \theta = 0, \partial E_z / \partial r = 0)\)
- Force on each particle: **superposition** of the fields produced by all the slices (generalization of “HOMDYN”)

\[
E_z(0, \zeta) = \frac{Q}{2\pi\epsilon_0 R} h(\zeta, A)
\]

\[
E_r(r, \zeta) = \frac{Q r}{2\pi\epsilon_0 R^2 L} g(\zeta, A)
\]

**Field form factors**

\[
h(\zeta, A) = |\zeta| - |1 - \zeta| + \sqrt{\zeta^2 + (1 - \zeta)^2} - \sqrt{\zeta^2 + \zeta^2}
\]

\[
g(\zeta, A) = \frac{1}{2} \left[ \frac{1 - \zeta}{\sqrt{A^2 + (1 - \zeta)^2}} + \frac{\zeta}{\sqrt{A^2 + \zeta^2}} \right]
\]

**Advantages/ Limitations**
- Correlation between the planes describes the emittance dynamics
- Allows description of more arbitrary geometries
- More time-consuming but still faster than PIC
Modeling Space Charge Forces (2)

**Equivalent ellipsoid method**
- 250 pC $e^-$ beam from the C-band **hybrid gun**
- Evolution in a **field-free (drift)** region with GPT (**solid** lines) and with the ellipsoidal model in MILES (**dashed** lines)

**Multi-slice superposition method**
- 250 pC $e^-$ beam produced by the hybrid gun and matched to its **invariant envelope** in a booster linac
- Emittance **dynamics**: **oscillations** in the drift and **compensation** in the linac

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**Transverse plane: $\sigma_x$, $\epsilon_{nx}$**

**Longitudinal plane: $\sigma_z$, $\sigma_E$**

**Relative error within 3% with ~1/6 execution time**

**Relative error within 4% (except emittance) with ~1/40 execution time**

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K. Floettmann, [https://www.desy.de/~mpy8o/](https://www.desy.de/~mpy8o/)

Modeling Wakefield Effects

- Interaction of a particle with a cavity resonant mode
- The change in momentum (energy/deflection) is described by the equivalent shunt circuit voltage

Evolution of the voltage state

\[\mathcal{T}(\cdot) = p + M(\cdot)\]

Perturbation:
Kick induced by the passing charges
\[p(\parallel) = \frac{q \omega_0 R_{\parallel}}{Q} \left( -\frac{1}{\omega_n/Q} \right)\]
\[p(\perp) = q \omega_n R_{\perp} - \frac{Q}{Q} x_i \begin{pmatrix} 0 \\ 1 \end{pmatrix}\]

Free-evolution:
Homogenous 2nd order differential equation
\[\ddot{V}(t) + 2\alpha \dot{V}(t) + \omega_0^2 V(t) = 0\]
\[\left( \begin{array}{c} V(t) \\ \dot{V}(t) \end{array} \right) \rightarrow \left( \begin{array}{c} V(t + \tau) \\ \dot{V}(t + \tau) \end{array} \right) = M(\tau) \left( \begin{array}{c} V(t) \\ \dot{V}(t) \end{array} \right)\]
\[M(\tau) = e^{-\alpha \tau} \begin{pmatrix} \cos \omega_n \tau + \frac{\alpha \omega_n}{\omega_n} \sin \omega_n \tau & \frac{1}{\omega_n} \sin \omega_n \tau \\ -\frac{\omega_n^2}{\omega_n} \sin \omega_n \tau & \cos \omega_n \tau - \frac{\alpha \omega_n}{\omega_n} \sin \omega_n \tau \end{pmatrix}\]

Recursive formula
- **State** after the \( n \) charges have passed
- \( N \) particles \( \Rightarrow \) \( N - 1 \) operations

\[\mathcal{T}_n \mathcal{T}_{n-1} \cdots \mathcal{T}_1(\cdot) = (p_n + M_n)(p_{n-1} + M_{n-1}) \cdots (p_1 + M_1)(\cdot)\]

\[\left( \begin{array}{c} V_{n-1} \\ \dot{V}_{n-1} \end{array} \right) = p_n + M(\tau_n - \tau_{n-1}) \left( \begin{array}{c} V_{n-1} \\ \dot{V}_{n-1} \end{array} \right)\]
Modeling Wakefield Effects (2)

Short-range WF interaction

- 5 MeV rectangular beam propagating in two linac sections with $\Delta x_1 = 100 \, \mu m$ and $\Delta x_2 = 0$
- Wakes from **diffraction theory** in periodic linacs

Long-range WF interaction

- 100 bunches injected off-axis in a linear collider
- Wakes from a **single HOM**

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**DARPA-like parameters**

Misaligned linac section: the beam propagates off axis exciting dipole wakes which cause emittance growth

Transverse bunch oscillations induced by a dipole HOM responsible for LRWF interaction

---

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of bunches</td>
<td>100</td>
</tr>
<tr>
<td>Bunch population</td>
<td>$2 \cdot 10^9$</td>
</tr>
<tr>
<td>Bunch separation</td>
<td>40 ns</td>
</tr>
<tr>
<td>Injection energy</td>
<td>10 GeV</td>
</tr>
<tr>
<td>Accelerating gradient</td>
<td>100 MeV/m</td>
</tr>
<tr>
<td>Avg betatron function</td>
<td>10 m</td>
</tr>
<tr>
<td>Linac length</td>
<td>4900 m</td>
</tr>
<tr>
<td>$f_0$ (HOM)</td>
<td>15.7 GHz</td>
</tr>
<tr>
<td>$Q$ (HOM)</td>
<td>100</td>
</tr>
<tr>
<td>$R_c/Q$ (HOM)</td>
<td>0.45 MQ/m²</td>
</tr>
</tbody>
</table>

Enhanced radial focusing in rf linacs

- High accelerating gradient: \( \sim 50 \text{ MeV/m} \) (room temp) and \( \sim 125 \text{ MeV/m} \) (cryo-cooled)
- 1st-order focusing: transitions from field-free to non-zero field regions (a)
- 2nd-order focusing: non-synchronous space harmonics in the accelerating cells (b)

\[
\begin{align*}
(a) \quad \begin{pmatrix} x' \\ x'' \end{pmatrix} & \mapsto \begin{pmatrix} 1 & 0 \\ \mp \frac{\nu}{2} & 1 \end{pmatrix} \begin{pmatrix} x' \\ x'' \end{pmatrix} \\
(b) \quad \begin{pmatrix} x' \\ x'' \end{pmatrix} & \mapsto \begin{pmatrix} \cos \left( \frac{\nu \ln \frac{2a}{\gamma} \gamma} {\gamma} \right) & \frac{1}{\gamma} \sin \left( \frac{\nu \ln \frac{2a}{\gamma} \gamma} {\gamma} \right) \\ -\frac{\nu}{2} & \cos \left( \frac{\nu \ln \frac{2a}{\gamma} \gamma} {\gamma} \right) \end{pmatrix} \begin{pmatrix} x' \\ x'' \end{pmatrix}
\end{align*}
\]

Note: \( \nu = \eta / 8 \cos(\Delta \phi) \) with \( \eta = 1.12 - 0.5 \cos(2\Delta \phi) \)


Strong wakefield interaction

- Small irises cause stronger wakefield interaction: C-band structures with \( \sim 2 \text{ mm iris radius} \) \((\sim 0.038 \cdot \lambda_{\text{rf}})\)
- Field diffracted in periodic accelerating structures (K. Bane’s theory) (c)
- HOMs parasitically excited in the linac (d)

\[
\begin{align*}
w_{||}(s) &= \frac{Z_{oc}}{\pi a^2} \exp \left( -\frac{s}{s_0} \right) \\
w_{\perp}(s) &= \frac{4Z_{oc}}{\pi a^4} \left( 1 - 1 + \frac{s}{s_1} \right) \exp \left( -\frac{s}{s_1} \right)
\end{align*}
\]


Space-charge sensitivity

- Beams produced by rf photoinjectors are in the 3-5 MeV energy range
- Highly space charge-dominated nearby the injection in linac boosters
- Nominal focusing anomalously strong in absence of SC forces (overfocusing)

The Initiatives of the Collaboration (2)

Innovative RF-concepts for high gradient acceleration:

• **Distributed coupling** scheme: cells receive the RF power individually from a guiding manifold structure \((a)\)

• **Optimized cavity topology** \((b)\) enhances the efficiency in acceleration of charged particles (cell irises not subjected to coupling constraints)

• **Cryogenic operation**: copper structures in the 45-77 K range support higher electromagnetic fields for a given breakdown rate \((c)\)

• Combination of such techniques enables also next-generation, very high field RF **photoinjectors** \((d)\)

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The Initiatives of the Collaboration (3)

UC-XFEL
- New paradigm for university-scale facilities (~ 40 m footprint, < 40M$ costs)
- Next generation rf gun: high peak field (240 MV/m at C-band*)
- Distributed coupling, cryogenic rf linac structures (up to 125 MV/m, C-band)
- Short-period (~ mm) cryogenic undulators

DARPA-GRIT ICS Source
- Compact γ-rays source: high flux, tunable, narrow bandwidth
- Novel electron source: C-band hybrid photo-injector
- Room temperature, distributed coupling rf linac structures (C-band)

C³: a “Cool Copper Collider”
- 250 GeV (extendible to TeV-scale) Higgs factory: high luminosity $e^-e^+$ linear collider
- Modularized C-band cryo-cooled, distributed coupling linac structures (~ 125 MV/m)
- Near future (~2030-2040) HEP frontiers

(*C-band = 5.712 GHz)
SRWF in the $3\pi/4$ Structures (2)

Reference case

- UC-XFEL scenario (100 pC, 0.5 mm rms beam, ~100 nm emittance at injection)
- 2 linac sections: $6.9 \text{ MeV} < \gamma mc^2 < 150 \text{ MeV}$ with the 1st section is 100 um off-axis

**Centroid trajectory**

- No wake
- $a = 1 \text{ mm}$
- $a = 2 \text{ mm}$
- $a = 3 \text{ mm}$

**Emittance growth**

- No wake
- $a = 1 \text{ mm}$
- $a = 2 \text{ mm}$
- $a = 3 \text{ mm}$
The “Cool Copper Collider” or C$^3$ in Numbers

**Working point**
- A total charge of 75 nC is delivered within a 250 ns long macro-pulse (1428 rf-bucket in C-band, i.e. 5.712 GHz)
- 75 electron bunches (1 nC each) with a separation of 19 rf periods (3.3 ns)
- The electrons are injected at 10 GeV and accelerated up 1 TeV with an average gradient of 117 MeV/m -exploiting distributed coupling and cryo-cooling - (i.e., overall length $\gtrsim 12$ km)
- Alternate gradient FODO lattice with $\langle \beta_x \rangle = 4$ m provides focusing in the transverse plane

**Higher order modes**
- CST Studio Suite: 20 HOMs (excluding degeneracy) in the range $5<f<20$ GHz
- 9 of them are dipole modes responsible for BBU effects
- Mitigation through frequency spread for each dipole mode (unstable motion if the ratio $f_{HOM}/f_b$ is close to an integer)
Multi-bunch beam dynamics

Basic modeling

• Bunches within the rf-pulse: sequence of rigid macro-particles with no internal structure
• Bunches perform transverse oscillations in presence of an external focusing optics (alternating gradient FODO lattice with average beta function $\beta_x$)
• Bunches are accelerated as they propagate within the linac (average accelerating gradient $\gamma'$)

If $\gamma' L_c / \gamma \ll 1$

$$
\begin{pmatrix} x \\ x' \end{pmatrix} \rightarrow \sqrt{\frac{\gamma}{\gamma + \gamma' L_c}} \begin{pmatrix} \cos \mu_x & \beta_x \sin \mu_x \\ -\frac{1}{\beta_x} \sin \mu_x & \cos \mu_x \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}
$$

with $\mu_x = L_c / \beta_x$.

Injection and Alignment Errors

Injection error: ● ♦
• Linac sections are perfectly aligned
• Injection of all bunches occurs with a transverse shift $\Delta x$

Random alignment errors: ▲ ■
• Injection occurs on-axis
• Linac sections are shifted transversely with gaussian offsets with std dev $\sigma_{\Delta x}$

**NO DETUNING**
• All cavities are equal
• Each mode exhibits the same frequency in all cavities

Amplitude of the transverse oscillation for each bunch in the rf-pulse at the interaction point
Frequency Spread of the HOMs

Worst case scenario from previous plot:
Random misalignments with $\sigma_{\Delta x} = 100 \ \mu m$

WITH DETUNING
• Variations are applied section-by-section
• Each HOM exhibits a different (random) frequency through the subsequent linac sections
• The range of variations for the resonant frequencies is established by CST data

Amplitude of the transverse oscillation for each bunch in the rf-pulse at the interaction point normalized to the first bunch’s amplitude
• List of 20 HOMs
• 9 of which are dipole modes -> BBU effects
• (Realistic) frequency spread for each dipole mode

Courtesy of D. Kim
Ongoing Work on Long-range BBU

To do list

• Electromagnetic simulations
  • CAD drawing of the $3\pi/4$ phase advance cell (input received from S. Tantawi et al.)
  • Eigenmode analysis of the structure finding frequencies, quality factors and shunt impedances for monopole and dipole
• BBU studies with the $3\pi/4$ structure
  • Multi-bunch simulations with MILES to estimate the center-of-mass oscillations
  • Investigate multi-bunch with intra-beam effects (SC + SRWF): simultaneous description of short-range and long range wakefields

Thank you for your attention
(Quesions are welcome)

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If one includes the **period** as an optimization parameter then the optimal structure with the minimal surface magnetic field, and consequently the highest shunt impedance occurs at ~132° phase advance; a natural choice is a 135° (i.e. $3\pi/4$) **phase advance**.

Every 4th cavity has a $\pi$ phase shift (modulo $2\pi$), hence 4 manifolds are needed to feed the structure.

Every two manifolds need to be fed with a $\pi/2$ phase shift; naturally fed by a hybrid that isolates the forward signal from the reflected signal.

**Cell-to-cell phase advance:**

\[
\phi = \frac{\omega}{c} p
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