

BEAM BREAKUP STUDIES FOR THE C³ LINEAR COLLIDER

Speaker: F. Bosco^{1,2,3*}

Co-authors: M. Behtouei⁴, O. Camacho¹, M. Carillo^{1,2,3}, E. Chiadroni^{2,4},
L. Faillace⁴, L. Ficcadenti³, D. Francescone^{2,3}, A. Fukasawa¹, A. Giribono⁴, L. Giuliano^{2,3},
G. Lawler¹, Z. Li⁵, N. Majernik^{5,1}, M. Migliorati^{2,3}, A. Mostacci^{2,3}, L. Palumbo^{2,3},
J. Rosenzweig^{1,2}, G. J. Silvi^{2,3}, B. Spataro⁴, S. Tantawi⁵

¹UCLA, Los Angeles, 90095 CA, USA

²La Sapienza University of Rome, 00161 Rome, Italy

³INFN-Sez. Roma1, 00161 Rome, Italy

⁴INFN-LNF, 00044 Frascati, Italy

⁵SLAC, Menlo Park, 94025, CA, USA

UCLA

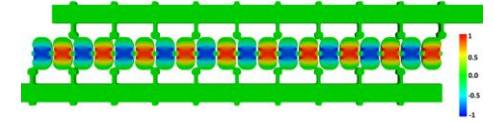


* fbosco@physics.ucla.edu

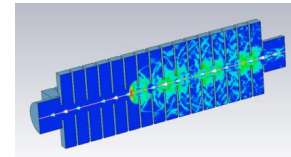
This work is supported by DARPA under Contract N.HR001120C0072, by DOE Contract DE-SC0009914 and DE-SC0020409, by the National Science Foundation Grant N.PHY-1549132 and by INFN through the project ARYA.

Contents

- Initiatives utilizing **distributed coupling linacs** and our **experience** with such structures



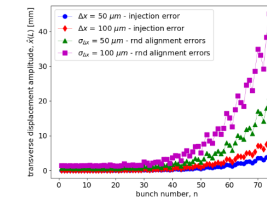
- Sources of **wakefields** in RF linacs and **BBU effects**



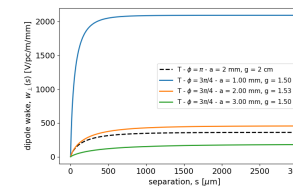
- **Modeling** of the wakefield interaction: the code **MILES**



- BBU studies for the TeV-scale working point of the **C³ linear collider**



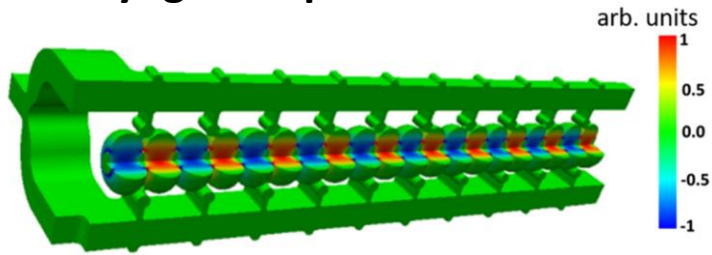
- Investigation of **phase advances** alternative to the π -mode: SRWFs in $3\pi/4$ mode distributed coupling linac



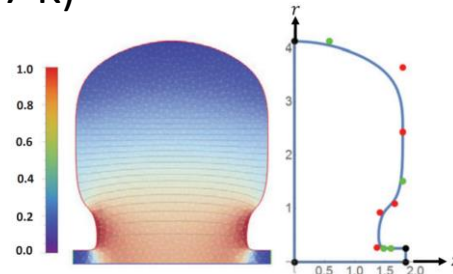
Our Experience with Dist. Coupling Linacs

A novel NCRF concept for high gradient acceleration:

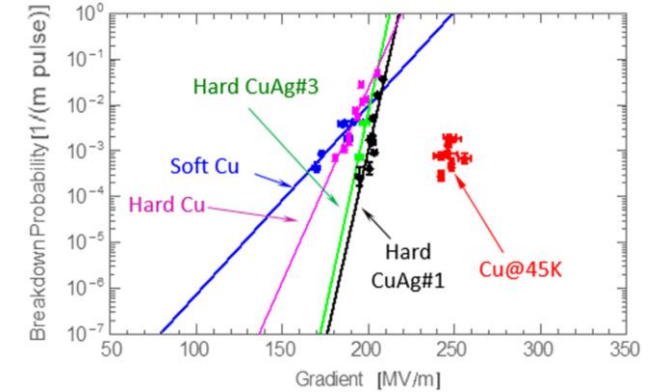
- **Distributed coupling linac (DCL) scheme**
- **Optimized cavity topology**
- **Cryogenic operation of Cu structures (e.g. 45-77 K)**



S. Tantawi, M. Nasr, Z. Li, C. Limborg, and P. Borchard, "Design and demonstration of a distributed-coupling linear accelerator structure," *Phys. Rev. Accel. Beams*, vol. 23, p. 092001, Sep 2020.

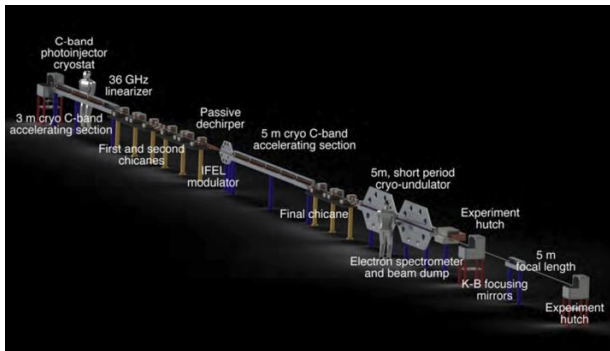


M. Nasr and S. Tantawi. New Geometrical-Optimization Approach using Splines for Enhanced Accelerator Cavities' Performance. In *9th International Particle Accelerator Conference*, 6 2018.



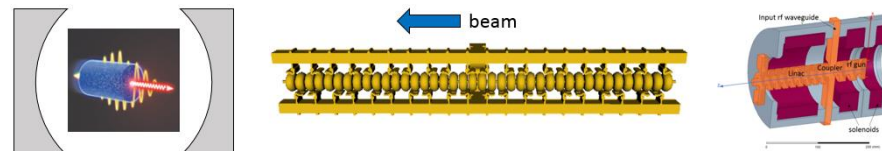
A. D. Cahill, J. B. Rosenzweig, V. A. Dolgashev, S. G. Tantawi, and S. Weathersby, "High gradient experiments with X-band cryogenic copper accelerating cavities," *Phys. Rev. Accel. Beams*, vol. 21, p. 102002, Oct 2018.

UC-XFEL



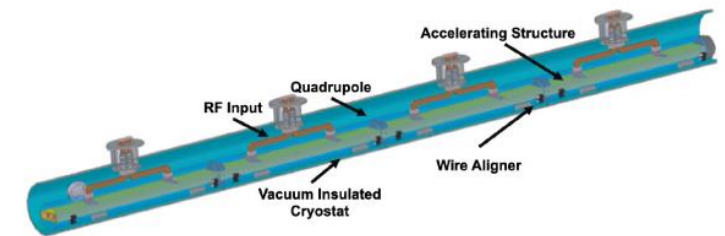
J. B. Rosenzweig *et al.*, "An ultra-compact X-ray free-electron laser," *New Journal of Physics*, vol. 22, p. 093067, sep 2020.

DARPA-GRIT ICS Source



L. Faillace *et al.* Start-to-End Beam-Dynamics Simulations of a Compact C-Band Electron Beam Source for High Spectral Brilliance Applications. In *Proc. IPAC'22*, number 13 in International Particle Accelerator Conference, pages 687–690. JACoW Publishing, Geneva, Switzerland, 07 2022.

C³: a "Cool Copper Collider"



E. Nanni *et al.* C³ demonstration research and development plan, 03 2022.

Sources of Wakefields in Linacs

- Field generated by the beam, **diffracted** by the surrounding environment and acting back on the beam
- Responsible for **energy spread** and **transverse deflection**

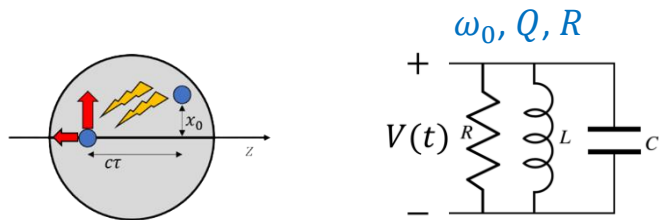
$$\Delta \mathbf{p} = \int \mathbf{F}(\mathbf{r}, \mathbf{r}_0, t)|_{z=vt-s} dt$$

$$w_{\parallel} = -\frac{c}{qq_0} \Delta p_z$$

$$w_{\perp} = \frac{c}{qq_0} \Delta \mathbf{p}_{\perp}$$

Modes in resonant cavities

- Longitudinal monopole and transverse dipole **higher order modes** (HOMs) in the accelerating cavities
- Responsible for the **long-range wakefield** (LRWF) interaction ($\omega_n = \omega_0 \sqrt{1 - (2Q)^{-2}}$ and $\alpha = \omega_0/2Q$)



$$w_{\parallel}(\tau) = \frac{\omega_0 R_{\parallel}}{Q} e^{-\alpha\tau} \left(\cos(\omega_n\tau) - \frac{\alpha}{\omega_n} \sin(\omega_n\tau) \right)$$

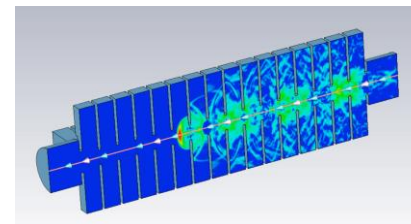
$$w_{\perp}(\tau) = \frac{\omega_0 R_{\perp}}{Q} e^{-\alpha\tau} \sin(\omega_n\tau)$$

A. W. Chao, *Physics of collective beam instabilities in high energy accelerators*. New York, USA: Wiley, 1993.

L. Palumbo, V. G. Vaccaro, and M. Zobov, Wake fields and impedance, CAS Accelerator School, Geneva, Switzerland, Rep. CERN 95-06, 1995.

Periodic accelerating structures

- Longitudinal monopole and transverse dipole impedance from **diffraction theory**
- Responsible for the **short-range wakefield** (SRWF) interaction in *periodic accelerating structures*

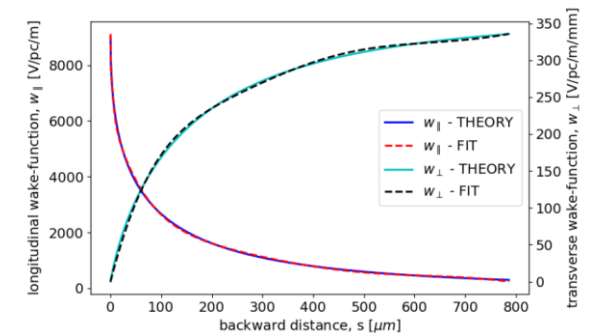


$$w_{\parallel}(s) = \frac{Z_0 c}{\pi a^2} \exp\left(-\sqrt{\frac{s}{s_0}}\right)$$

$$w_{\perp}(s) = \frac{4Z_0 c s_1}{\pi a^4} \left(1 - \left(1 + \sqrt{\frac{s}{s_1}} \right) \exp\left(-\sqrt{\frac{s}{s_1}}\right) \right)$$

J. D. Lawson, "Radiation from a ring charge passing through a resonator", Rutherford High Energy Laboratory, Chilton, England, Rep. RHEL/M144, 1968.

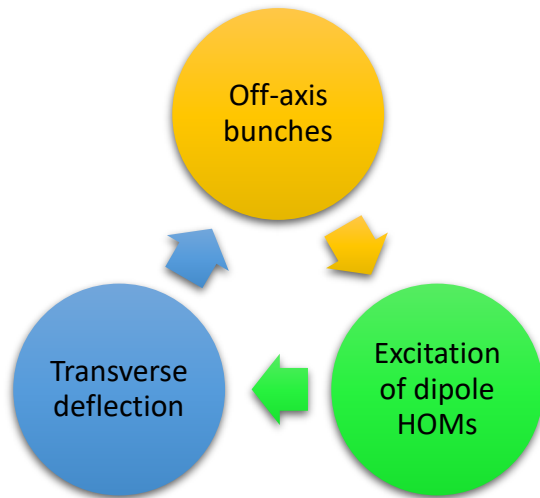
K. Bane, "Short-range dipole wakefields in accelerating structures for the NLC", SLAC, Menlo Park, USA, Rep. SLAC-PUB-9663, Mar. 2003.



Beam Breakup Effects

Long-range transverse interaction

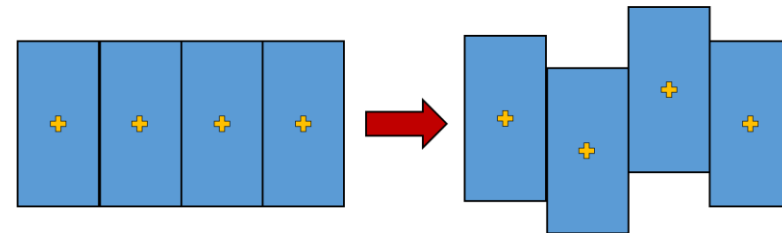
$$w_{\perp}(\tau) = \frac{\omega_0 R_{\perp}}{Q} e^{-\alpha\tau} \sin(\omega_n\tau)$$



Instability: the amplitude of the center of mass oscillation grows

Short-range transverse interaction

$$w_{\perp}(s) = \frac{4Z_0 c s_1}{\pi a^4} \left(1 - \left(1 + \sqrt{\frac{s}{s_1}} \right) \exp\left(-\sqrt{\frac{s}{s_1}}\right) \right)$$



Phase space dilution: the correlation between the planes is a cause of emittance growth

The Code MILES

Dedicated tracking code

- Motivation to investigate **wakefields effects** in linacs in presence of non-negligible **space charge** endangering the nominal beam quality
- Use of simple **semi-analytical models**: *flexible* and *time-efficient* tool (**factors ~1/30-1/10**) with acceptable reduction of the accuracy
- Main applications: investigation of misalignment effects and design of possible **correction** schemes



Miles Davis (May 26, 1926 – September 28, 1991)

MILES = Modeling Instabilities in Linacs with Ellipsoidal Space charge

F. Bosco *et al.*, “Modeling Short Range Wakefield Effects in a High Gradient Linac,” in *Proc. IPAC’21*, no. 12 in International Particle Accelerator Conference, JACoW Publishing, Geneva, Switzerland, 08 2021.

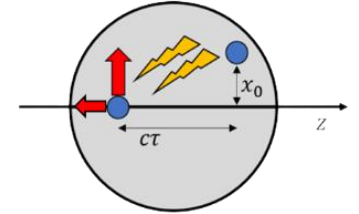
F. Bosco, O. Camacho, M. Carillo, E. Chiadroni, L. Faillace, A. Fukasawa, A. Giribono, L. Giuliano, N. Najernik, A. Mostacci, L. Palumbo, B. Spataro, C. Vaccarezza, J. B. Rosenzweig, and M. Migliorati. A fast tracking code for evaluating collective effects in linear accelerators, 2022.

Modeling overview and comparison with other codes

	MILES approach	Commonly utilized approaches
Space Charge	Equivalentelellipsoid or Multi-slice superposition	Particle in cell (PIC)
Wakefields	Algebraic formalism	Convolution integral or FFT equivalent

Long-range BBU in the C³ Linac

- BBU studies for the TeV scale working point of the C³ linear collider
- Center of mass **oscillations** in multi-bunch operation: sequence of point-like, structureless macro-particles carrying the whole bunch charge



K. L. Bane *et al.*, “An advanced NCRF linac concept for a high energy e⁺e⁻ linear collider,” 2018.

C³ working point at TeV-scale

+ knowledge of dipole HOMs parameters (ω_0, Q, R_{\perp}) for $5 < \omega_0/2\pi < 20$ GHz

Parameter	Value
Number of bunches	75
Bunch charge	1 nC
Bunch separation	~ 3.3 ns
Injection energy	10 GeV
Energy at the IP	1 TeV
Accelerating gradient	117 MeV/m
Avg betatron function	4 m
Linac length	≳ 12 km

Emilio Nanni *et al.* C³ demonstration research and development plan, 03 2022.

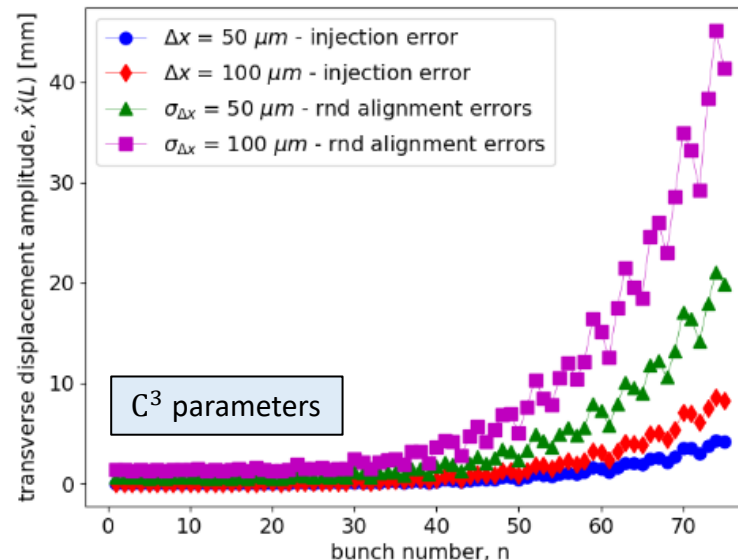
D. Kim *et al.*, Design Study of HOM Coupler for the C-band Accelerating Structure, doi:10.18429/JACoW-IPAC2022-TUPOMS057

Injection error: ●◆

- Linac sections are perfectly aligned (ideal)
- Injection with a transverse offset Δx for all bunches

Random alignment errors: ▲■

- On-axis injection
- Linac sections exhibit gaussian transverse offsets with std dev $\sigma_{\Delta x}$



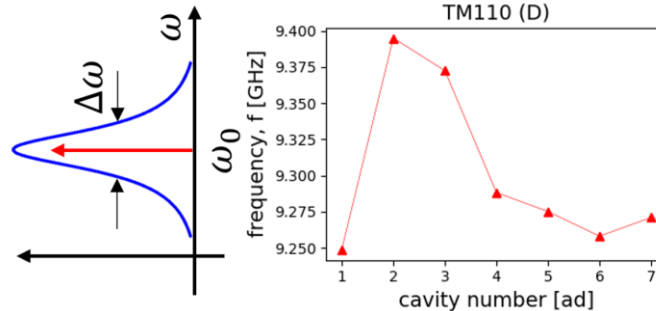
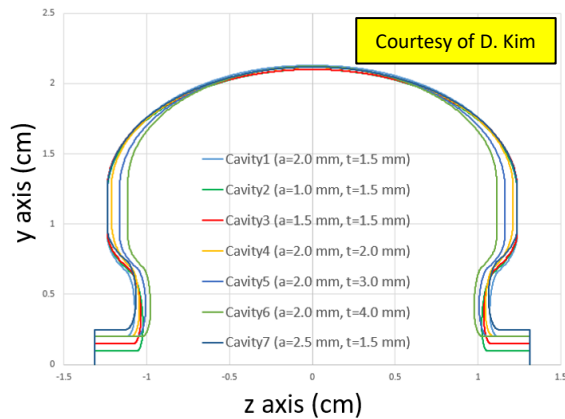
Amplitude of the transverse oscillation of the bunches at the interaction point

F. Bosco *et al.* “Modeling and Mitigation of Long-Range Wakefields for Advanced Linear Colliders”, 13th Int. Particle Accelerator Conf. (IPAC’22), Bangkok, Thailand, June 2022, paper WEPOMS045,

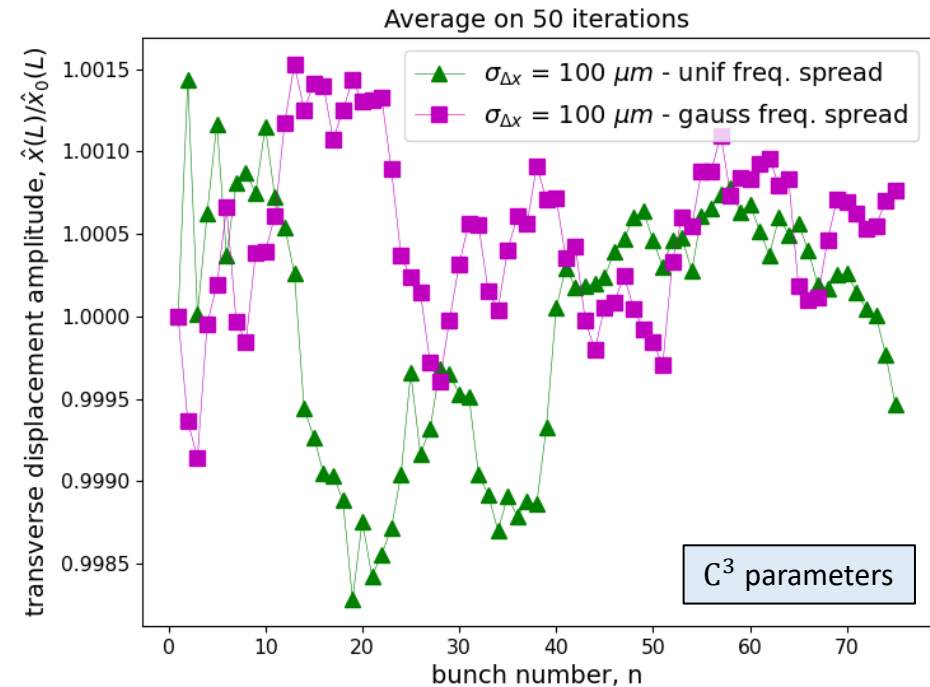
Long-range BBU in the C³ Linac (2)

Detuning of the HOMs

- **Unstable** motion if the ratio f_{HOM}/f_b is close to an integer (see Mosnier)
- Mitigation through frequency **spread** (or *detuning*)
- Seven cavity-designs from small **variations** of the original geometry
- HOMs exhibit different, *i.e.* **random**, frequencies through subsequent linac sections



Worst case scenario from previous plot:
Random misalignments with $\sigma_{\Delta x} = 100 \mu\text{m}$



Amplitude of the transverse oscillation for each bunch in the rf-pulse at the interaction point normalized to the first bunch's amplitude

Alban Mosnier. Instabilities in linacs. In *CERN Accelerator School: Course on Advanced Accelerator Physics (CAS)*, pages 0459–514, 12 1993.

K. L. F. Bane and R. L. Gluckstern "The transverse wakefield of a detuned X-band accelerator structure", *Particle Accelerators*, Vol. 42, 1993.

K. A. Thompson, C. Adolphsen and K. L. F. Bane "Multi-bunch beam break-up in detuned structures", SLAC-PUB-6153, 1993.

$3\pi/4$ -mode performance

Phase advances alternative to the conventional π -mode

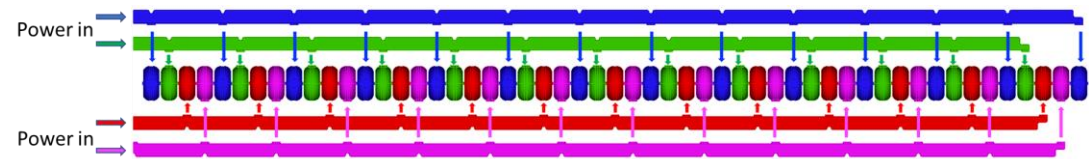
135 degree phase advanced distributed coupling structure

Courtesy of S. Tantawi et. al

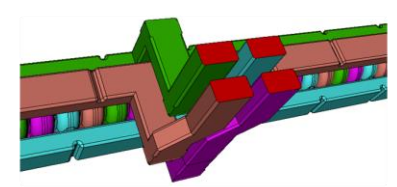
- If one includes the **period** as an optimization parameter then the optimal structure with the minimal surface magnetic field, and consequently the highest shunt impedance occurs at $\sim 132^\circ$ phase advance; a natural choice is a 135° (i.e. $3\pi/4$ radians) **phase advance**.
- Every 4^{th} cavity has a π phase shift (modulo 2π), hence 4 manifolds are needed to feed the structure.
- Every *two* manifolds need to be fed with a $\pi/2$ phase shift; naturally fed by a hybrid that isolates the forward signal from the reflected signal.

Cell-to-cell phase advance:

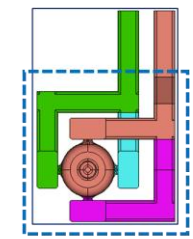
$$\phi = \frac{\omega}{c} p$$



4 waveguide feed schematics



4 waveguide feed layout



Shumail, Li, Tantawi

S. Tantawi, Z. Li, M. Shumail, C. Nantista, M. Oriunno, G. Bowden, B. Loo, E. Snively, "Cryogenically Cooled 135 Degree Distributed-Coupling Linear Accelerator and Its Application to Very High Electron Energy Radiation Therapy," to be published.

SRWF in the $3\pi/4$ Structures

Change of the geometrical dimensions implies changes in the wakefields

$$w_{\parallel}(s) = \frac{Z_0 c}{\pi a^2} \exp\left(-\sqrt{\frac{s}{s_0}}\right)$$

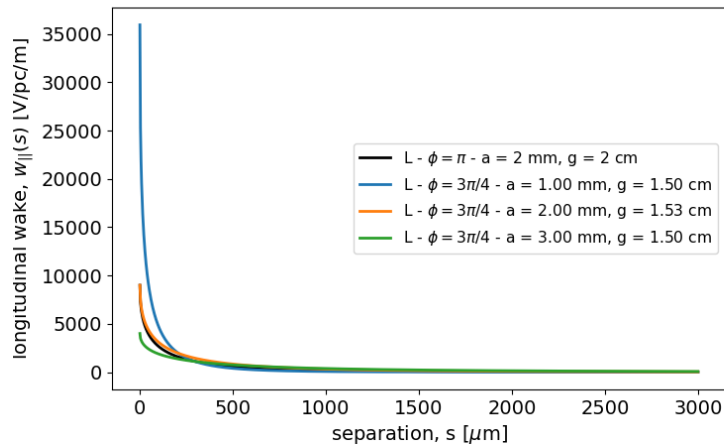
$$w_{\perp}(s) = \frac{4Z_0 c s_1}{\pi a^4} \left(1 - \left(1 + \sqrt{\frac{s}{s_1}}\right) \exp\left(-\sqrt{\frac{s}{s_1}}\right)\right)$$

$$s_0 \approx 0.41 a^{1.8} g^{1.6} / p^{2.4}$$

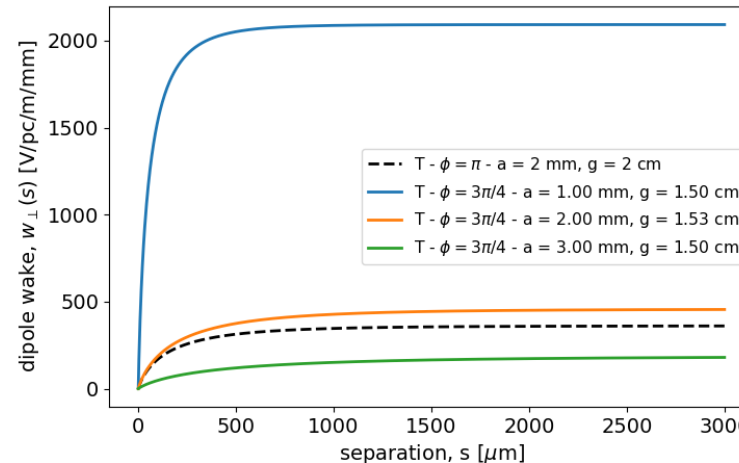
$$s_1 \approx 0.169 a^{1.79} g^{0.38} / p^{1.17}$$

K. Bane, "Short-range dipole wakefields in accelerating structures for the NLC", SLAC, Menlo Park, USA, Rep. SLAC-PUB-9663, Mar. 2003. doi:10.2172/812954

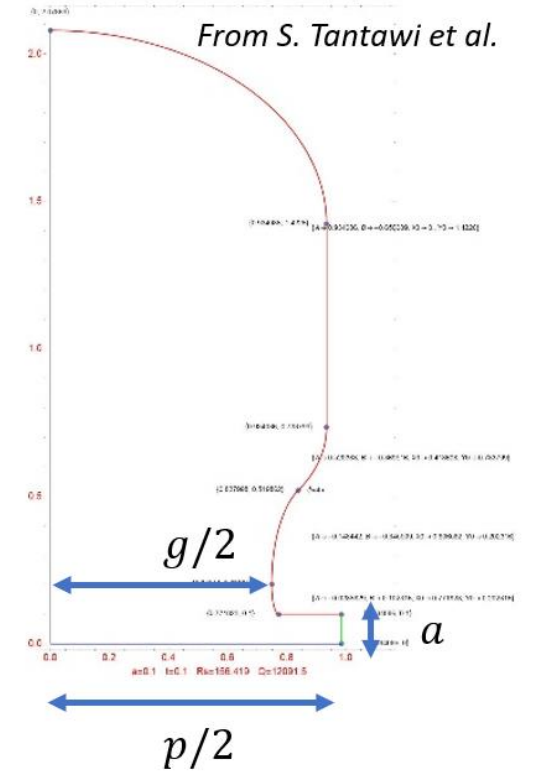
$\phi = 3\pi/4$	$\phi = \pi$ (original)
Iris a [mm] = 1,2,3	Iris a [mm] = 2
Gap g [cm] = 1.496, 1.534, 1.501	Gap g [cm] = 2
Period p [mm] = 19.682 ($3\lambda/8$)	Period p [mm] = 26.242 ($\lambda/2$)



Longitudinal Monopole SR wake-function



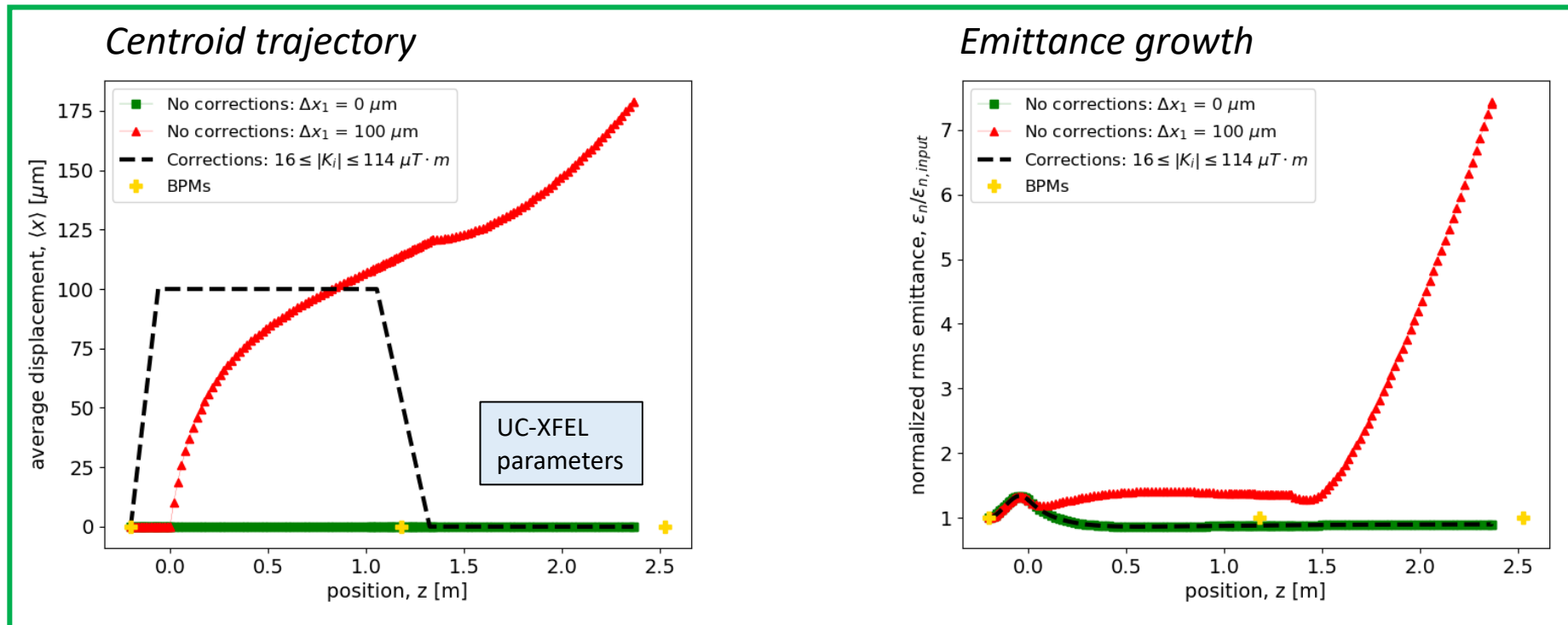
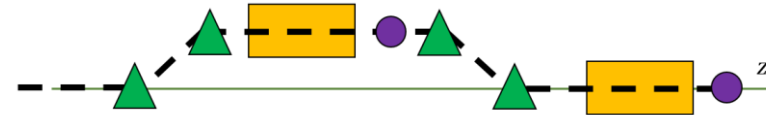
Transverse dipole SR wake-function



Mitigation by Trajectory Steering

Reference case

- UC-XFEL scenario (100 pC, 0.5 mm rms beam, ~ 100 nm emittance at injection)
- 2 linac sections: $6.9 \text{ MeV} < \gamma mc^2 < 150 \text{ MeV}$ with the 1st section is **100 μm** off-axis
- Demonstration for the worst case scenario of **1 mm iris** (two correctors upstream each linac section)



Conclusions and Outlook

- **Cryo-cooled distributed coupling linacs** are expected to play a crucial role for advanced and diverse **applications** (e^-e^+ colliders, FELs, ICS, FLASH RT,...)
- The development of **MILES** for an efficient description of **BBU** (and other) **effects**
- BBU effects and their mitigation in the **C³ linear collider**
- Possible benefits in the operation with **$3\pi/4$** phase advance structures

Ongoing work on long-range BBU

- **Electromagnetic simulation:** eigenmode analysis of the structure finding frequencies, quality factors and shunt impedances for monopole and dipole
- **BBU studies with the $3\pi/4$ structure:** multi-bunch simulations with MILES to estimate the center-of-mass oscillations as well as multi-bunch with intra-beam effects (SC + SRWF) for simultaneous description of short-range and long range wakefields



Thank you for your attention
(Questions are welcome)

Additional Material

International Workshop on Future Linear Collider – May 15-19 2023

The Code MILES

Dedicated tracking code

- Motivation to investigate **wakefields effects** in linacs in presence of non-negligible **space charge** endangering the nominal beam quality
- Use of simple **semi-analytical models**: *flexible* and *time-efficient* tool (**factors $\sim 1/30-1/10$**) with acceptable reduction of the accuracy
- Main applications: investigation of misalignment effects and design of possible **correction** schemes



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Comparison with other codes utilized in accelerator physics

- **SPACE CHARGE**: Particle in cell, PIC, approach (GPT, ASTRA, PARMELA, TSTEP...)
- **WAKEFIELDS**: Convolution integral with the beam slices, often solved with FFT algorithms (ASTRA, ELEGANT, LUCRETIA...)
- **SC + WF**: Complexity and CPU times increase and, thus, it is not a trivial feature (ASTRA, IMPACT-Z, OCELOT, HOMDYN)

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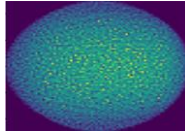
F. Bosco, O. Camacho, M. Carillo, E. Chiadroni, L. Faillace, A. Fukasawa, A. Giribono, L. Giuliano, N. Najernik, A. Mostacci, L. Palumbo, B. Spataro, C. Vaccarezza, J. B. Rosenzweig, and M. Migliorati. A fast tracking code for evaluating collective effects in linear accelerators, 2022.

Modeling overview and comparison with other codes

- **SPACE CHARGE**: **Equivalent ellipsoid** or **Multi-slice superposition** instead of **PIC approaches** (GPT, ASTRA, PARMELA, TSTEP...)
- **WAKEFIELDS**: **Algebraic formalism** valid for short and long-range interaction instead of **convolution integral** with the beam slices (ASTRA, ELEGANT, LUCRETIA...)
- **SC + WF**: Complexity and CPU times increase and, thus, it is not a trivial feature (ASTRA, IMPACT-Z, OCELOT, HOMDYN)

Modeling Space Charge Forces

Equivalent ellipsoid method



- Ellipsoidal beams arise from the **blowout** process
- An **equivalent** uniform ellipsoid is associated to the actual beam distribution
- Self-induced **forces** are known analytically
- Forces are **linear**: self consistent model

$$\Phi'(x, y, z') = \frac{3Q}{16\pi\epsilon_0} \int_0^{+\infty} \left(1 - \frac{x^2}{a^2 + t} - \frac{y^2}{b^2 + t} - \frac{z'^2}{c'^2 + t} \right) \frac{dt}{\sqrt{\varphi(t)}}$$

$$\equiv D_0 - A_0x^2 - B_0y^2 - C_0z'^2$$

O. Kellogg, *Foundation of Potential Theory*. Dover, 1953.

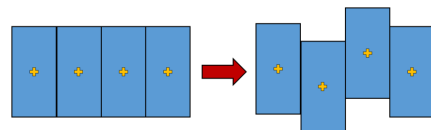
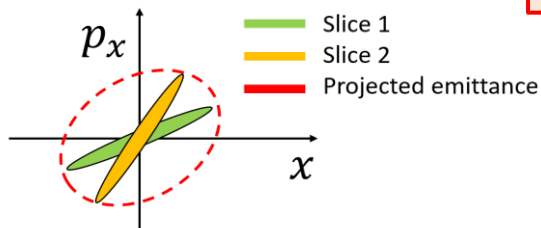
Luca Serafini. The short bunch blow-out regime in rf photoinjectors. *AIP Conference Proceedings*, 413(1):321-334, 1997. doi: 10.1063/1.54425. URL <https://aip.scitation.org/doi/abs/10.1063/1.54425>.

Advantages

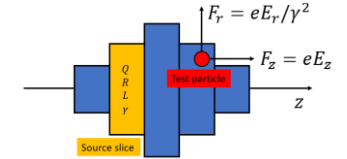
- Very **fast** (~ 30 times than 3D PIC)
- Successfully reproduces rms size ($\sigma_x, \sigma_y, \sigma_z$) and energy spread (σ_E)

Limitations

- Absence of *correlation* prevents the observation of emittance dynamics
- Loss of *axial symmetry* in presence of strong transverse wakefield



Multi-slice superposition method



- Divide the beam in **cylindrical slices** with individual size, energy and aspect ratio
- Field produced by each (uniform) slice is known analytically ($\partial/\partial\theta = 0, \partial E_z/\partial r = 0$)
- Force on each particle: **superposition** of the fields produced by all the slices (generalization of "HOMDYN")

M. Ferrario, L. Serafini, and F. Tazzioli. Multi-bunch dynamics in accelerating structures including interaction with higher order modes. In *Proceedings of International Conference on Particle Accelerators*, pages 3279-3281 vol.5, 1993. doi: 10.1109/PAC.1993.309625.

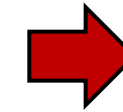
$$E_z(0, \zeta) = \frac{Q}{2\pi\epsilon_0 R^2} h(\zeta, A)$$

$$E_r(r, \zeta) = \frac{Qr}{2\pi\epsilon_0 R^2 L} g(\zeta, A)$$

Field form factors

$$h(\zeta, A) = |\zeta| - |1 - \zeta| + \sqrt{A^2 + (1 - \zeta)^2} - \sqrt{A^2 + \zeta^2}$$

$$g(\zeta, A) = \frac{1}{2} \left[\frac{1 - \zeta}{\sqrt{A^2 + (1 - \zeta)^2}} + \frac{\zeta}{\sqrt{A^2 + \zeta^2}} \right]$$



$$F_x = q \sum_s E_x(x_s, \zeta_s) / \gamma_s^2$$

$$F_y = q \sum_s E_y(y_s, \zeta_s) / \gamma_s^2$$

$$F_z = q \sum_s E_z(0, \zeta_s)$$

Advantages/Limitations

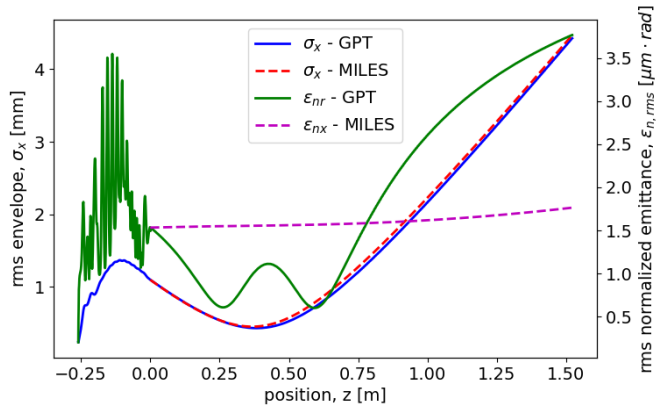
- Correlation between the planes describes the emittance dynamics
- Allows description of more arbitrary geometries
- More time-consuming but still faster than PIC

Modeling Space Charge Forces (2)

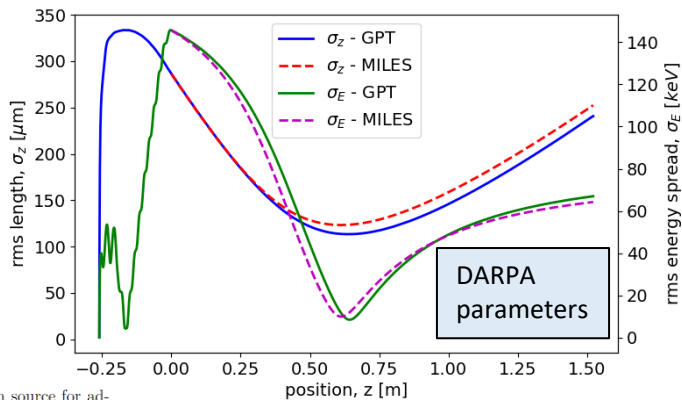
Equivalent ellipsoid method

- 250 pC e^- beam from the C-band **hybrid gun**
- Evolution in a **field-free (drift)** region with GPT (*solid lines*) and with the ellipsoidal model in MILES (*dashed lines*)

Transverse plane: σ_x, ϵ_{nx}



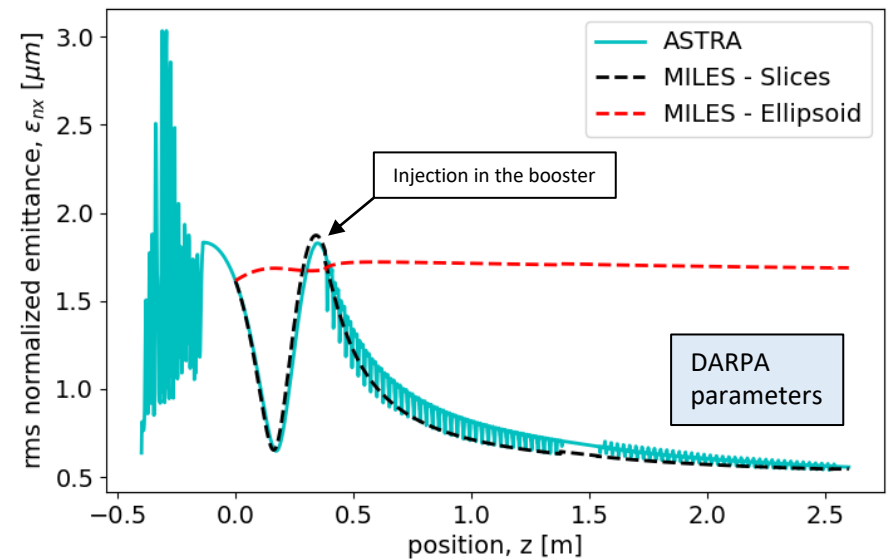
Longitudinal plane: σ_z, σ_E



Relative error within 4% (except emittance) with $\sim 1/40$ execution time

Multi-slice superposition method

- 250 pC e^- beam produced by the hybrid gun and matched to its **invariant envelope** in a booster linac
- Emittance **dynamics**: *oscillations* in the drift and *compensation* in the linac



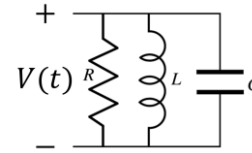
Relative error within 3% with $\sim 1/6$ execution time

K. Floettmann, <https://www.desy.de/~mpyflo/>

L. Serafini and J. B. Rosenzweig, "Envelope analysis of intense relativistic quasilaminar beams in rf photoinjectors: A theory of emittance compensation," *Phys. Rev. E*, vol. 55, pp. 7565-7590, June 1997.

Modeling Wakefield Effects

- Interaction of a particle with a cavity resonant **mode**
- The change in momentum (energy/deflection) is described by the equivalent shunt circuit **voltage**



$$c\Delta p_z = c \int F_z dt = -qq_0 w_{\parallel}(\tau) \doteq -qV_{\parallel}(\tau)$$

$$c\Delta \mathbf{p}_{\perp} = c \int \mathbf{F}_{\perp} dt = qq_0 \rho_0 w_{\perp}(\tau) \doteq q\mathbf{V}_{\perp}(\tau)$$

Evolution of the voltage state

$$\begin{pmatrix} V(t) \\ \dot{V}(t) \end{pmatrix} \xrightarrow{\mathcal{T}(\cdot)} \mathbf{p} + M(\cdot)$$

Perturbation:
Kick induced by the passing charges

$$\mathbf{p}_i^{(\parallel)} = q \frac{\omega_0 R_{\parallel}}{Q} \begin{pmatrix} 1 \\ -\omega_0/Q \end{pmatrix}$$

$$\mathbf{p}_i^{(\perp)} = q\omega_n \frac{\omega_0 R_{\perp}}{Q} x_i \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Free-evolution:
Homogenous 2nd order differential equation

$$\ddot{V}(t) + 2\alpha\dot{V}(t) + \omega_0^2 V(t) = 0$$

$$\begin{pmatrix} V(t) \\ \dot{V}(t) \end{pmatrix} \mapsto \begin{pmatrix} V(t+\tau) \\ \dot{V}(t+\tau) \end{pmatrix} = M(\tau) \begin{pmatrix} V(t) \\ \dot{V}(t) \end{pmatrix}$$

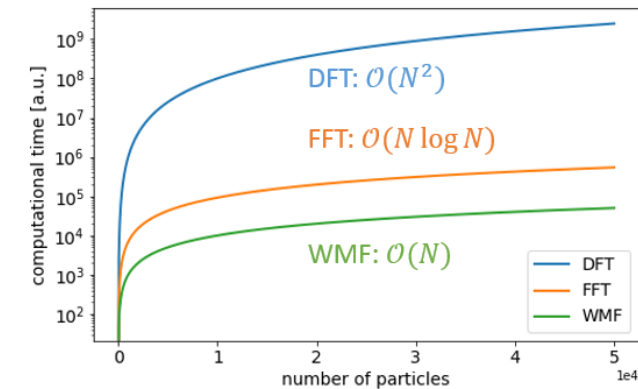
$$M(\tau) = e^{-\alpha\tau} \begin{pmatrix} \cos \omega_n \tau + \frac{\alpha}{\omega_n} \sin \omega_n \tau & \frac{1}{\omega_n} \sin \omega_n \tau \\ -\frac{\omega_0^2}{\omega_n} \sin \omega_n \tau & \cos \omega_n \tau - \frac{\alpha}{\omega_n} \sin \omega_n \tau \end{pmatrix}$$

Recursive formula

- **State** after the n charges have passed
- N particles $\Rightarrow N - 1$ **operations**

$$\mathcal{T}_n \mathcal{T}_{n-1} \cdots \mathcal{T}_1(\cdot) = (\mathbf{p}_n + M_n)(\mathbf{p}_{n-1} + M_{n-1}) \cdots (\mathbf{p}_1 + M_1)(\cdot)$$

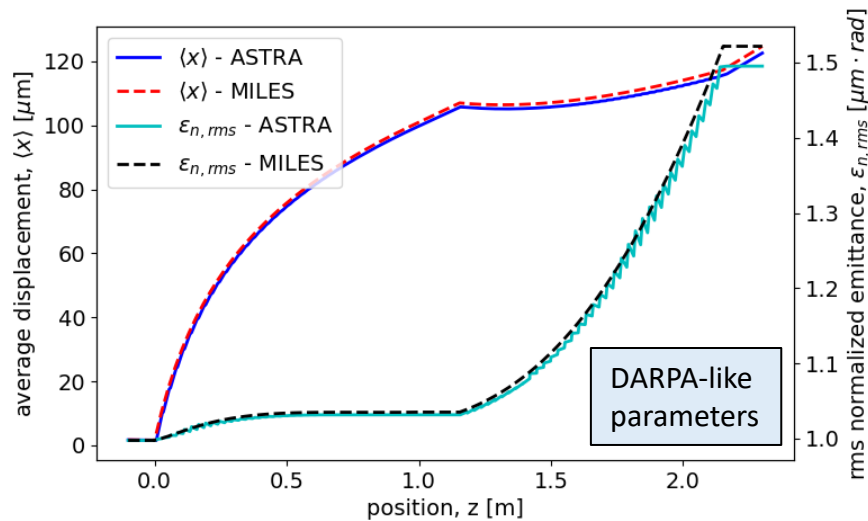
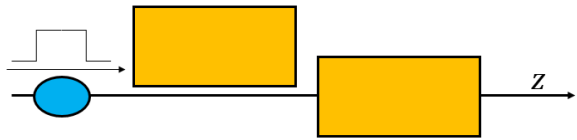
$$\begin{pmatrix} V_n \\ \dot{V}_n \end{pmatrix} = \mathbf{p}_n + M(\tau_n - \tau_{n-1}) \begin{pmatrix} V_{n-1} \\ \dot{V}_{n-1} \end{pmatrix}$$



Modeling Wakefield Effects (2)

Short-range WF interaction

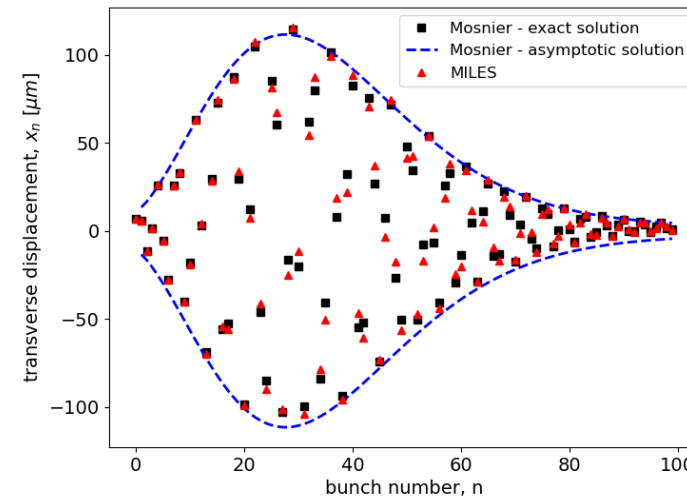
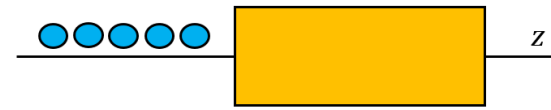
- 5 MeV rectangular beam propagating in two linac sections with $\Delta x_1 = 100 \mu\text{m}$ and $\Delta x_2 = 0$
- Wakes from **diffraction theory** in periodic linacs



Misaligned linac section: the beam propagates off axis exciting dipole wakes which cause emittance growth

Long-range WF interaction

- 100 bunches injected off-axis in a linear collider
- Wakes from a **single HOM**



Transverse bunch oscillations induced by a dipole HOM responsible for LRWF interaction

Parameter	Value
Number of bunches	100
Bunch population	$2 \cdot 10^9$
Bunch separation	40 ns
Injection energy	10 GeV
Accelerating gradient	100 MeV/m
Avg betatron function	10 m
Linac length	4900 m
f_0 (HOM)	15.7 GHz
Q (HOM)	100
R_{\perp}/Q (HOM)	$0.45 \text{ M}\Omega/\text{m}^2$

A. Mosnier, "Instabilities in Linacs", CAS Accelerator School, Rhodes, 1993.

Motivation: Unique Beam Physics

Enhanced radial focusing in rf linacs

- High accelerating **gradient**: ~ 50 MeV/m (room temp) and ~ 125 MeV/m (cryo-cooled)
- 1st - order focusing: transitions from field-free to non-zero field regions **(a)**
- 2nd - order focusing: non-synchronous *space harmonics* in the accelerating cells **(b)**

(a)
$$\begin{pmatrix} x \\ x' \end{pmatrix} \mapsto \begin{pmatrix} 1 & 0 \\ \mp \frac{\gamma'}{2\gamma} & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}$$

(b)
$$\begin{pmatrix} x \\ x' \end{pmatrix} \mapsto \begin{pmatrix} \cos\left(\nu \ln \frac{\gamma_2}{\gamma_1}\right) & \frac{\gamma_1}{\nu \gamma'} \sin\left(\nu \ln \frac{\gamma_2}{\gamma_1}\right) \\ -\frac{\nu \gamma'}{\gamma_2} \sin\left(\nu \ln \frac{\gamma_2}{\gamma_1}\right) & \frac{\gamma_1}{\gamma_2} \cos\left(\nu \ln \frac{\gamma_2}{\gamma_1}\right) \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}$$

Note: $\nu = \eta/8 \cos(\Delta\phi)$ with $\eta = 1.12 - 0.5 \cos(2\Delta\phi)$

J. Rosenzweig and L. Serafini, *Transverse particle motion in radio-frequency linear accelerators*, Physical Review E 49, 1599 (1994).

S. C. Hartman and J. B. Rosenzweig, "Ponderomotive focusing in axisymmetric rf linacs," *Phys. Rev. E*, vol. 47, pp. 2031–2037, Mar 1993.

River R. Robles, Obed Camacho, Atsushi Fukasawa, Nathan Majernik, and James B. Rosenzweig, Versatile, high brightness, cryogenic photoinjector electron source. *Phys. Rev. Accel. Beams*, 24:063401, Jun 2021. doi: 10.1103/PhysRevAccelBeams.24.063401. URL

Strong wakefield interaction

- Small irises cause stronger **wakefield interaction**: C-band structures with ~ 2 mm iris radius ($\sim 0.038 \cdot \lambda_{\text{rf}}$)
- Field diffracted in *periodic accelerating structures* (K. Bane's theory) **(c)**
- HOMs parasitically excited in the linac **(d)**

(c)
$$w_{\parallel}(s) = \frac{Z_0 c}{\pi a^2} \exp\left(-\sqrt{\frac{s}{s_0}}\right)$$
 SRWF

$$w_{\perp}(s) = \frac{4Z_0 c s_1}{\pi a^4} \left(1 - \left(1 + \sqrt{\frac{s}{s_1}}\right) \exp\left(-\sqrt{\frac{s}{s_1}}\right)\right)$$

J. D. Lawson, "Radiation from a ring charge passing through a resonator", Rutherford High Energy Laboratory, Chilton, England, Rep. RHEL/M144, 1968.

K. Bane, "Short-range dipole wakefields in accelerating structures for the NLC", SLAC, Menlo Park, USA, Rep. SLAC-PUB-9663, Mar. 2003. doi: 10.2172/812954

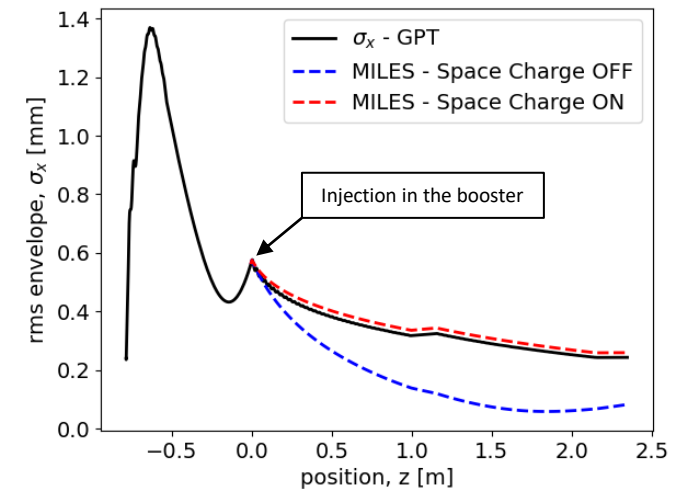
(d)
$$w_{\parallel}(\tau) = \frac{\omega_0 R_{\parallel}}{Q} e^{-\alpha\tau} \left(\cos(\omega_n\tau) - \frac{\alpha}{\omega_n} \sin(\omega_n\tau)\right)$$
 LRWF

$$w_{\perp}(\tau) = \frac{\omega_0 R_{\perp}}{Q} e^{-\alpha\tau} \sin(\omega_n\tau)$$

L. Palumbo, V. G. Vaccaro, and M. Zobov, Wake fields and impedance, CAS Accelerator School, Geneva, Switzerland, Rep. CERN 95-06, 1995.

Space-charge sensitivity

- Beams produced by rf photoinjectors are in the **3-5 MeV** energy range
- Highly **space charge**-dominated nearby the injection in linac boosters
- Nominal focusing anomalously strong in absence of SC forces (**overfocusing**)

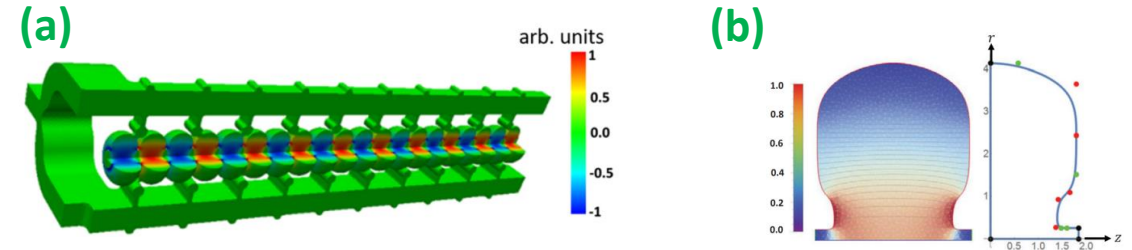


The Initiatives of the Collaboration (2)

Innovative RF-concepts for high gradient acceleration:

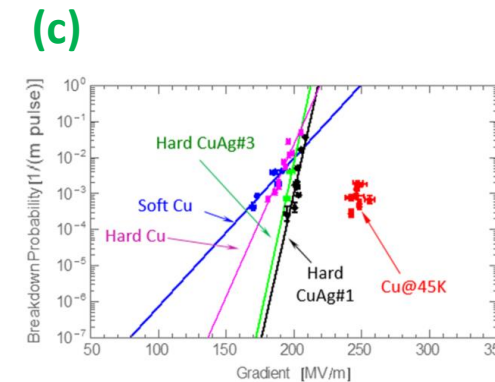
- **Distributed coupling** scheme: cells receive the RF power individually from a guiding manifold structure **(a)**
- Optimized **cavity topology** **(b)** enhances the efficiency in acceleration of charged particles (cell irises not subjected to coupling constraints)
- **Cryogenic operation**: copper structures in the 45-77 K range support higher electromagnetic fields for a given breakdown rate **(c)**
- Combination of such techniques enables also next-generation, very high field RF **photoinjectors** **(d)**

J. B. Rosenzweig, A. Cahill, V. Dolgashev, *et al.*, "Next generation high brightness electron beams from ultrahigh field cryogenic rf photocathode sources," *Phys. Rev. Accel. Beams*, vol. 22, p. 023403, Feb 2019.



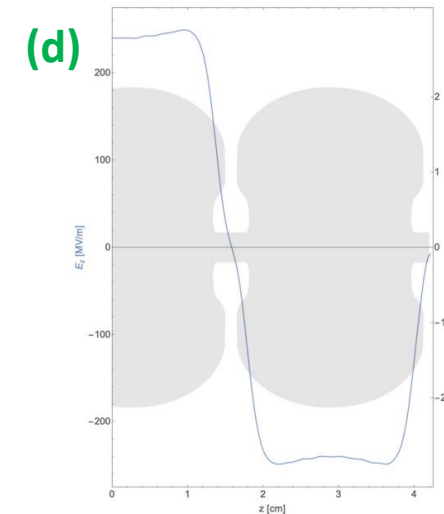
S. Tantawi, M. Nasr, Z. Li, C. Limborg, and P. Borchard, "Design and demonstration of a distributed-coupling linear accelerator structure," *Phys. Rev. Accel. Beams*, vol. 23, p. 092001, Sep 2020.

M. Nasr and S. Tantawi. New Geometrical-Optimization Approach using Splines for Enhanced Accelerator Cavities' Performance. In *9th International Particle Accelerator Conference*, 6 2018.



A. D. Cahill, J. B. Rosenzweig, V. A. Dolgashev, S. G. Tantawi, and S. Weathersby, "High gradient experiments with X-band cryogenic copper accelerating cavities," *Phys. Rev. Accel. Beams*, vol. 21, p. 102002, Oct 2018.

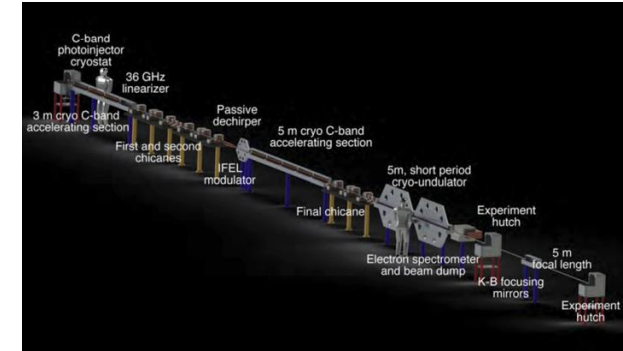
R. R. Robles, O. Camacho, A. Fukasawa, N. Majernik, and J. B. Rosenzweig. Versatile, high brightness, cryogenic photoinjector electron source. *Phys. Rev. Accel. Beams*, 24:063401, Jun 2021.



The Initiatives of the Collaboration (3)

UC-XFEL

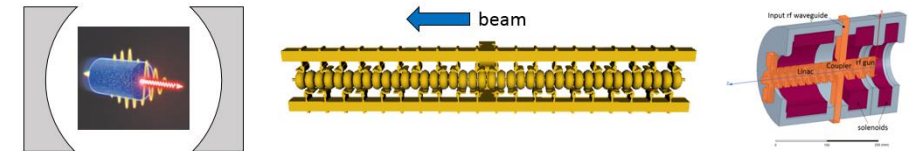
- New **paradigm** for university-scale facilities (~ 40 m footprint, < 40 M\$ costs)
- Next generation **rf gun**: high peak field (240 MV/m at C-band*)
- Distributed coupling, cryogenic **rf linac** structures (up to 125 MV/m, C-band)
- Short-period (\sim mm) cryogenic **undulators**



J. B. Rosenzweig *et al.*, “An ultra-compact X-ray free-electron laser,” *New Journal of Physics*, vol. 22, p. 093067, sep 2020.

DARPA-GRIT ICS Source

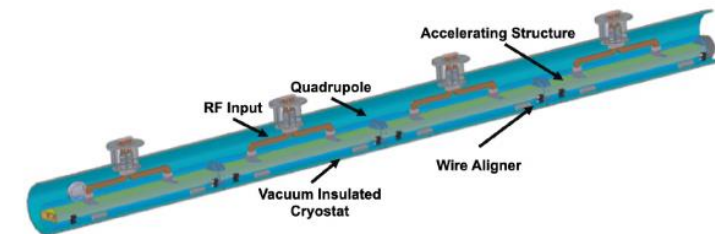
- Compact **γ -rays source**: high flux, tunable, narrow bandwidth
- Novel electron source: C-band **hybrid** photo-injector
- Room temperature, distributed coupling **rf linac** structures (C-band)



L. Faillace *et al.* Start-to-End Beam-Dynamics Simulations of a Compact C-Band Electron Beam Source for High Spectral Brilliance Applications. In *Proc. IPAC'22*, number 13 in International Particle Accelerator Conference, pages 687–690. JACoW Publishing, Geneva, Switzerland, 07 2022.

C³: a “Cool Copper Collider”

- 250 GeV (extendible to TeV-scale) **Higgs factory**: high luminosity e^-e^+ linear collider
- **Modularized** C-band cryo-cooled, distributed coupling linac structures (~ 125 MV/m)
- Near future ($\sim 2030-2040$) **HEP** frontiers



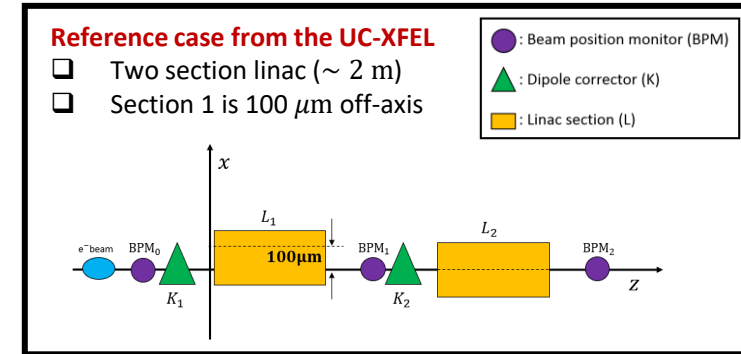
E. Nanni *et al.* C³ demonstration research and development plan, 03 2022. 23

(*C-band = 5.712 GHz)

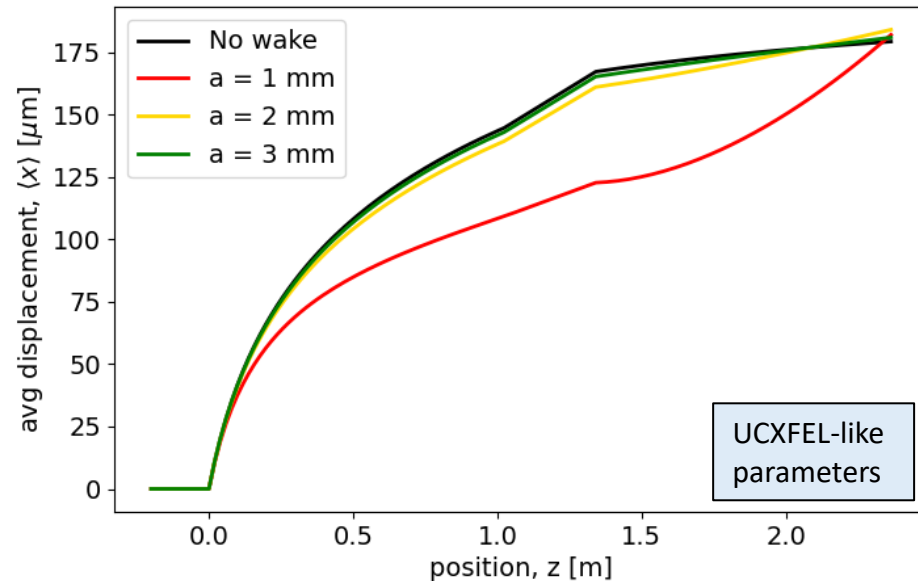
SRWF in the $3\pi/4$ Structures (2)

Reference case

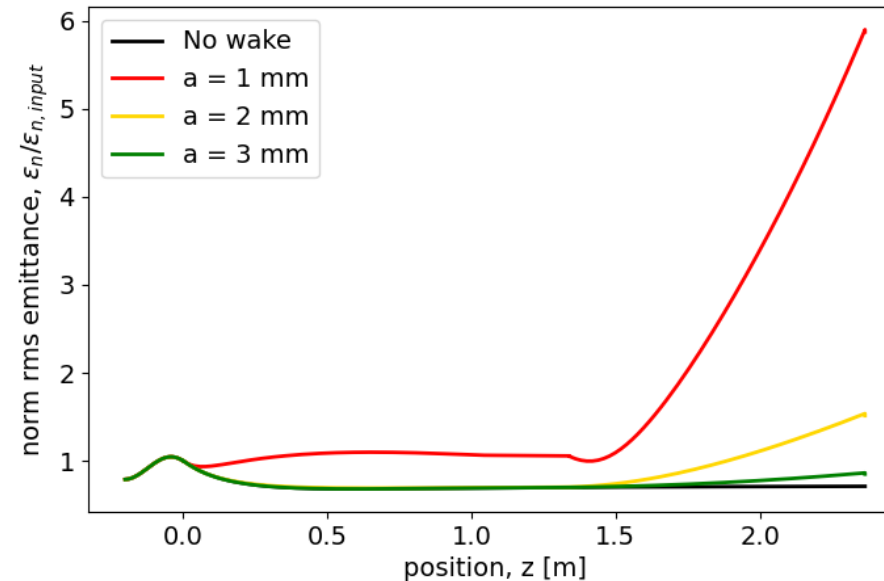
- UC-XFEL scenario (100 pC, 0.5 mm rms beam, ~ 100 nm emittance at injection)
- 2 linac sections: $6.9 \text{ MeV} < \gamma mc^2 < 150 \text{ MeV}$ with the 1st section is **100 μm** off-axis



Centroid trajectory



Emittance growth



The “Cool Copper Collider” or C³ in Numbers

Working point

- A total **charge** of 75 nC is delivered within a 250 ns long macro-pulse (1428 rf-bucket in C-band, *i.e.* 5.712 GHz)
- 75 electron bunches (1 nC each) with a **separation** of 19 rf periods (3.3 ns)
- The electrons are injected at 10 GeV and **accelerated** up 1 TeV with an average gradient of 117 MeV/m -exploiting *distributed coupling* and *cryo-cooling* - (*i.e.*, overall length ≥ 12 km)
- Alternate gradient FODO lattice with $\langle \beta_x \rangle = 4$ m provides **focusing** in the transverse plane

Higher order modes

- CST Studio Suite: 20 **HOMs** (excluding degeneracy) in the range $5 < f < 20$ GHz
- 9 of them are **dipole** modes responsible for BBU effects
- Mitigation through frequency **spread** for each dipole mode (unstable motion if the ratio f_{HOM}/f_b is close to an integer)

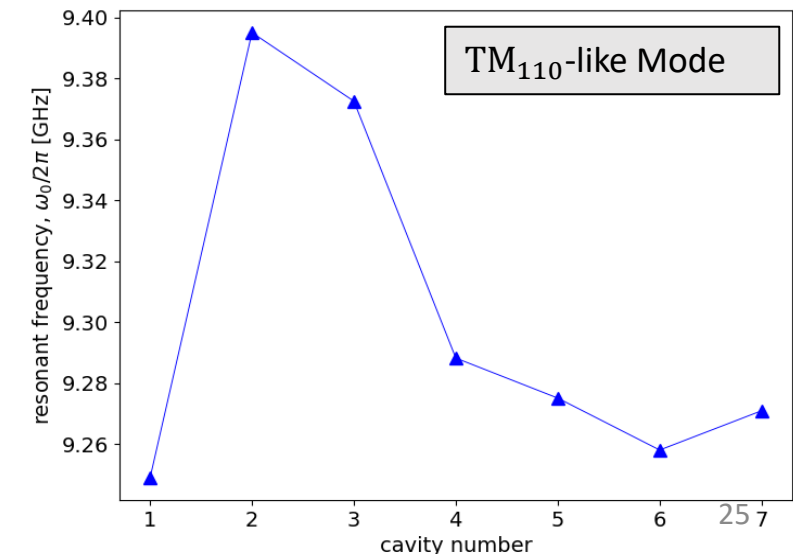
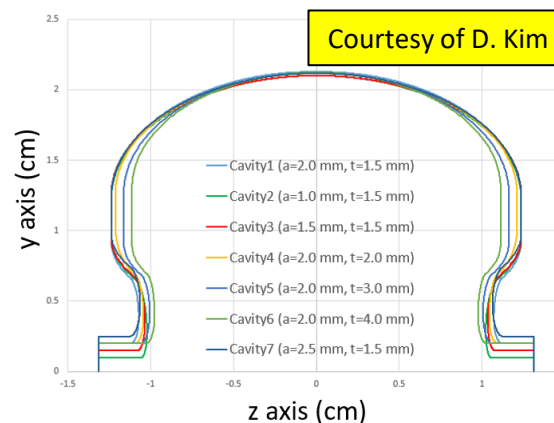
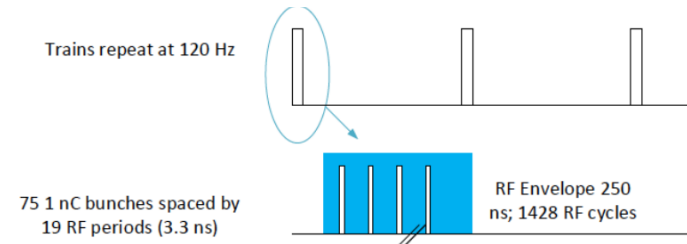
An Advanced NCRF Linac Concept for a High Energy e⁺e⁻ Linear Collider

K. L. Bane, T. L. Barklow, M. Breidenbach, C. P. Burkhart, E. A. Fauve, A. R. Gold, V. Heloim, Z. Li, E. A. Nanni, M. Nasr, M. Oriunno, E. Paterson, M. E. Peskin, T. O. Raubenheimer and S. G. Tantawi

SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, CA 94025

S. Tantawi, M. Nasr, Z. Li, C. Limborg, and P. Borchard, “Design and demonstration of a distributed-coupling linear accelerator structure”, *Physical Review Accelerators and Beams*, vol. 23, no. 9, Sep. 2020. doi:10.1103/PhysRevAccelBeams.23.092001

A. D. Cahill, J. B. Rosenzweig, V. A. Dolgashev, Z. Li, S. G. Tantawi and S. Weathersby, “RF losses in a high gradient cryogenic copper cavity”, *Phys. Rev. Accel. Beams*, vol. 21 n. 6, Jun. 2018. doi:10.1103/PhysRevAccelBeams.21.061301



Multi-bunch beam dynamics

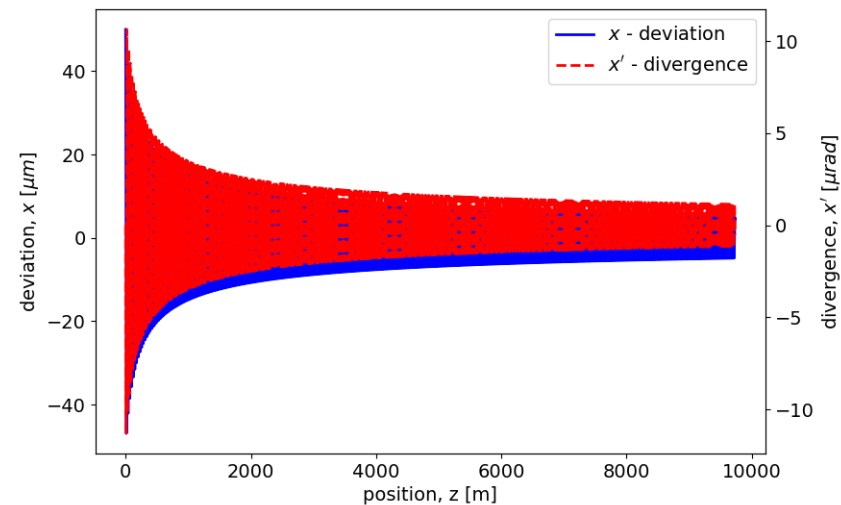
Basic modeling

- Bunches within the rf-pulse: sequence of rigid **macro-particles** with no internal structure
- Bunches perform transverse oscillations in presence of an external **focusing optics** (alternating gradient FODO lattice with average beta function β_x)
- Bunches are **accelerated** as they propagate within the linac (average accelerating gradient γ')

If $\gamma' L_c / \gamma \ll 1$

$$\begin{pmatrix} x \\ x' \end{pmatrix} \mapsto \sqrt{\frac{\gamma}{\gamma + \gamma' L_c}} \begin{pmatrix} \cos \mu_x & \beta_x \sin \mu_x \\ -\frac{1}{\beta_x} \sin \mu_x & \cos \mu_x \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}$$

with $\mu_x = L_c / \beta_x$.



M. Reiser, *Theory and Design of Charged Particle Beams*.
John Wiley & Sons, New York, 1994.

Adiabatic damping of the transverse oscillations in absence of collective effects

Injection and Alignment Errors

Injection error: ● ◆

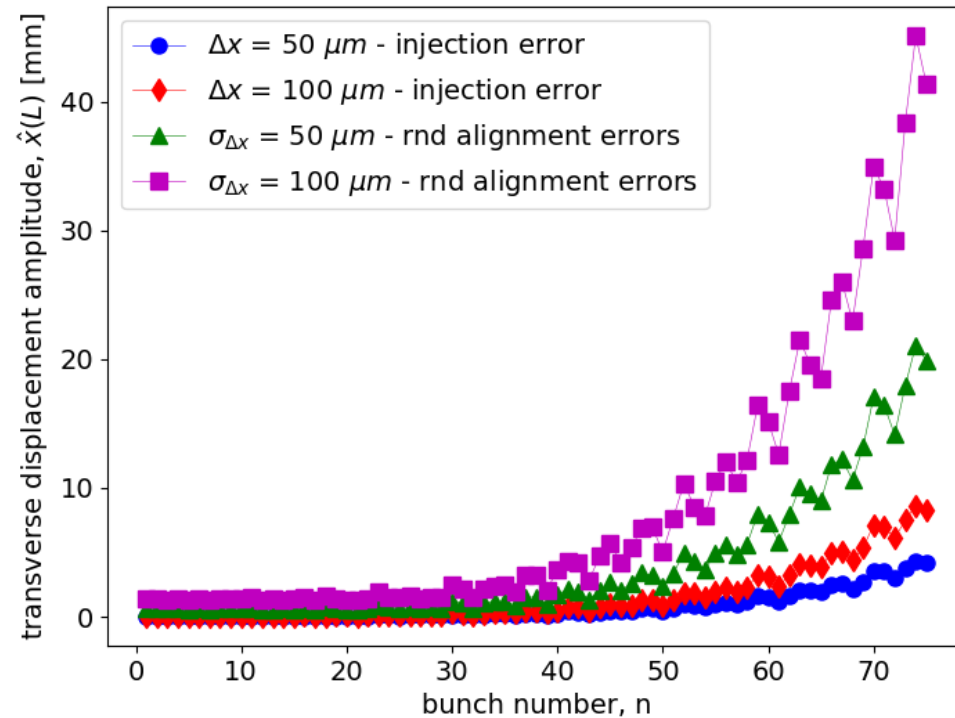
- Linac sections are perfectly aligned
- Injection of all bunches occurs with a transverse shift Δx

Random alignment errors: ▲ ■

- Injection occurs on-axis
- Linac sections are shifted transversely with gaussian offsets with std dev $\sigma_{\Delta x}$

NO DETUNING

- All cavities are equal
- Each mode exhibits the same frequency in all cavities



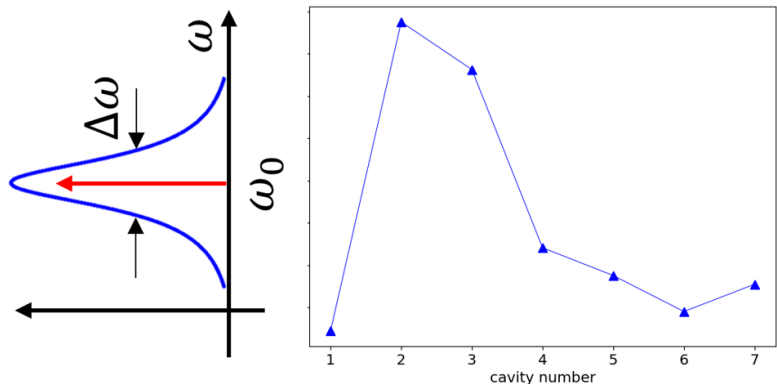
Amplitude of the transverse oscillation for each bunch in the rf-pulse at the interaction point

Frequency Spread of the HOMs

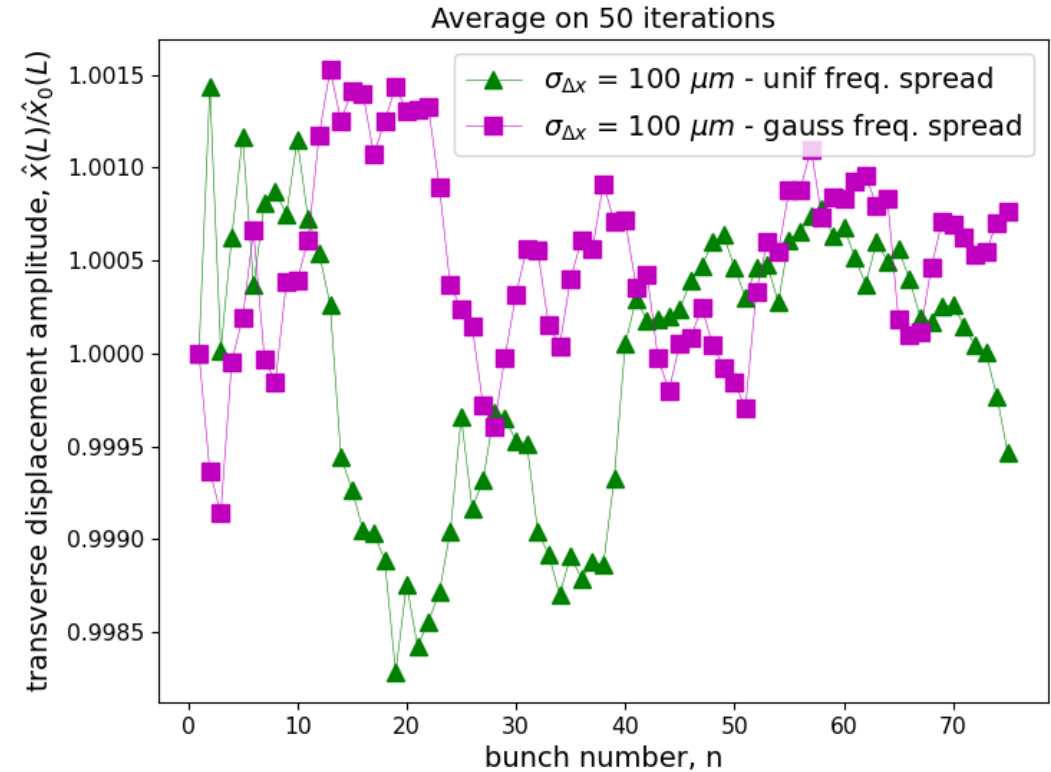
Worst case scenario from previous plot:
Random misalignments with $\sigma_{\Delta x} = 100 \mu\text{m}$

WITH DETUNING

- Variations are applied section-by-section
- Each HOM exhibits a different (random) frequency through the subsequent linac sections
- The range of variations for the resonant frequencies is established by CST data



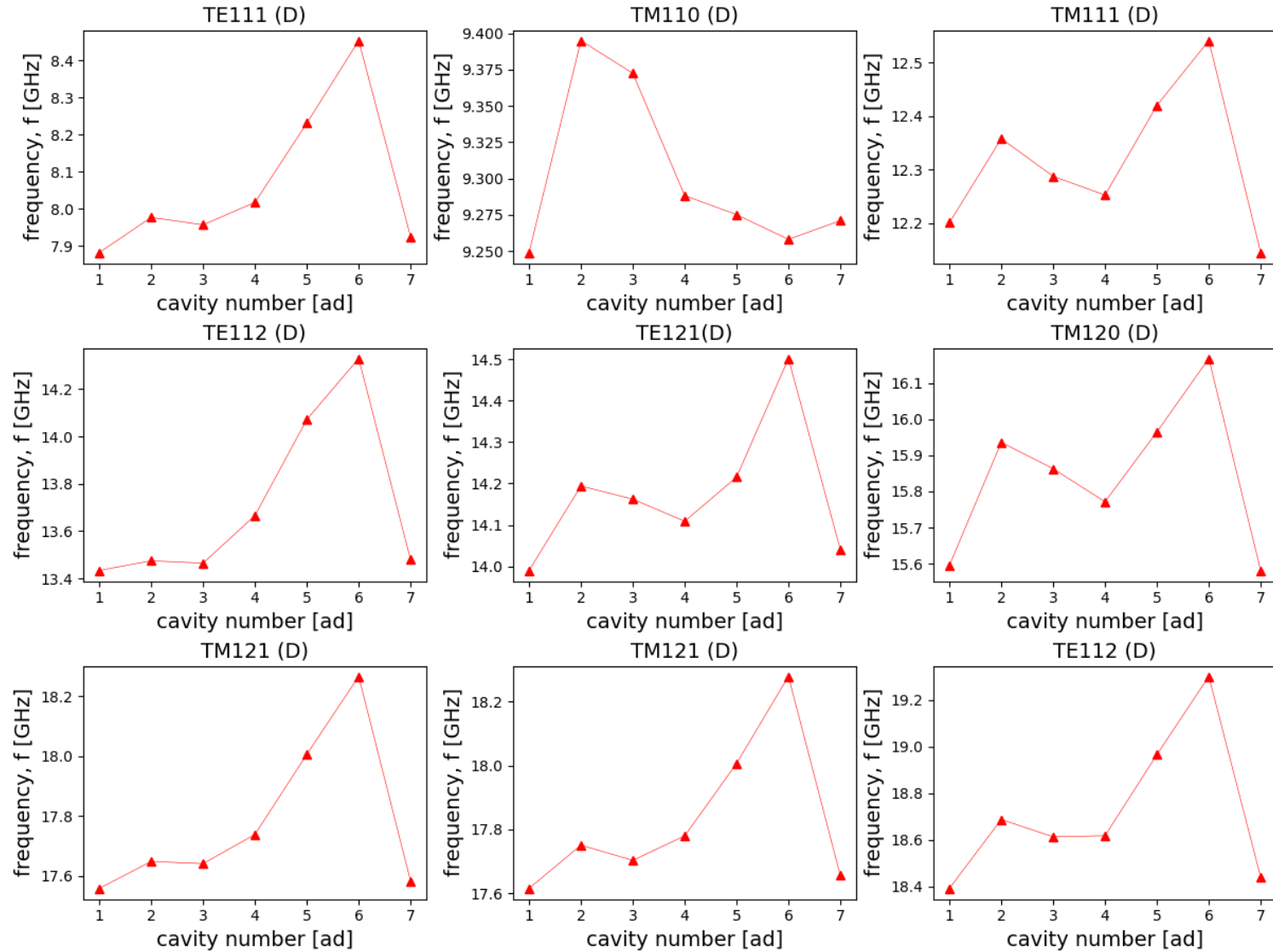
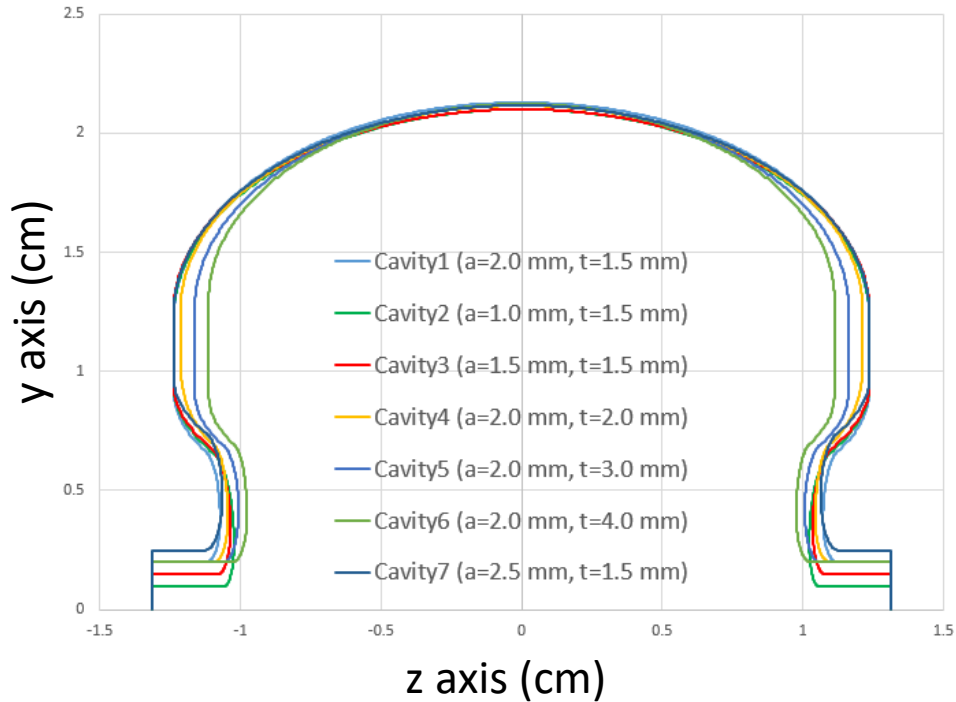
K. L. F. Bane and R. L. Gluckstern "The transverse wakefield of a detuned X-band accelerator structure", *Particle Accelerators*, Vol. 42, 1993.
K. A. Thompson, C. Adolphsen and K. L. F. Bane "Multi-bunch beam break-up in detuned structures", SLAC-PUB-6153, 1993.



Amplitude of the transverse oscillation for each bunch in the rf-pulse at the interaction point normalized to the first bunch's amplitude

SLAC 7 Cavities

Courtesy of D. Kim



- List of 20 HOMs
- 9 of which are dipole modes -> BBU effects
- (Realistic) frequency spread for each dipole mode

Ongoing Work on Long-range BBU

To do list

- **Electromagnetic simulations**
 - CAD drawing of the $3\pi/4$ phase advance cell (input received from S. Tantawi *et al.*)
 - Eigenmode analysis of the structure finding frequencies, quality factors and shunt impedances for monopole and dipole
- **BBU studies with the $3\pi/4$ structure**
 - Multi-bunch simulations with MILES to estimate the center-of-mass oscillations
 - Investigate multi-bunch with intra-beam effects (SC + SRWF): simultaneous description of short-range and long range wakefields

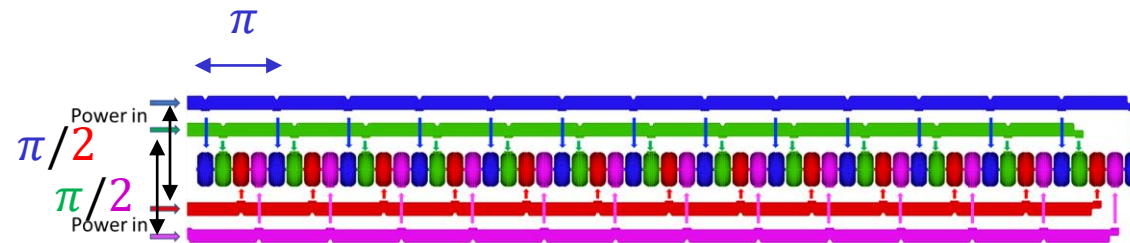


Thank you for your attention
(Questions are welcome)

135 degree phase advanced distributed coupling structure

Courtesy of S. Tantawi *et. al*

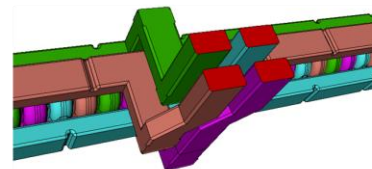
- If one includes the **period** as an optimization parameter then the optimal structure with the minimal surface magnetic field, and consequently the highest shunt impedance occurs at $\sim 132^\circ$ phase advance; a natural choice is a 135° (*i.e.* $3\pi/4$) **phase advance**.
- Every 4^{th} cavity has a π phase shift (modulo 2π), hence 4 manifolds are needed to feed the structure.
- Every *two* manifolds need to be fed with a $\pi/2$ phase shift; naturally fed by a hybrid that isolates the forward signal from the reflected signal.



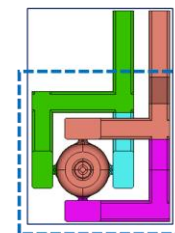
Cell-to-cell phase advance:

$$\phi = \frac{\omega}{c} p$$

4 waveguide feed schematics



4 waveguide feed layout



Shumail, Li, Tantawi