Evaluating accelerator, detector, and physics limitations on $\mathrm{e^+e^-}$ collider center-of-mass energy determination using dileptons

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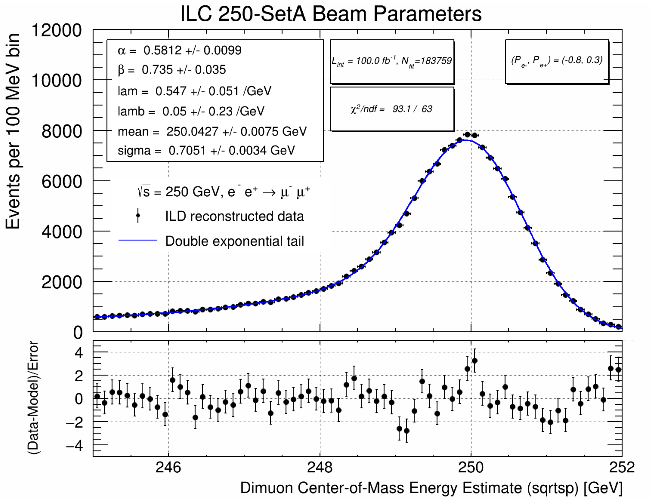
- See "Center-of-mass energy determination using $\mathrm{e^+e^-} \rightarrow \mu^+\mu^-(\gamma)$ events at future $\mathrm{e^+e^-}$ colliders" (2209.03281) with Brendon Madison for in-depth overview.
- We use a muon **momenta** based estimator, $\sqrt{s}_p$, to measure precisely the absolute center-of-mass energy scale of actual **collisions** without assuming ISR is collinear.
- Needs great momentum resolution and exquisite control of tracker momentum scale.
- Uses all dimuon events. Can work well at all $\sqrt{s}$ and especially for $\sqrt{s} \approx M_{\mathrm{Z}}$.
- Relevant to linear and circular $\mathrm{e^+e^-}$ colliders: $\mathrm{C^3}$, CLIC, ILC, ReLiC, FCC-ee.
- Also applies to Bhabhas, $\mathrm{e^+e^-} \rightarrow \mathrm{e^+e^-}(\gamma)$.

Silver Quality Dimuon PFOs (After BS)

Example $\sqrt{s}_p$ (detector-level) distribution with ILD full simulation.



Peak width 1.69 ± 0.01 wider than $\sqrt{s}_p$ (gen). Leads to **2 ppm** stat. precision @ ILC250.

$\sqrt{s}$ Sensitivity Estimates for ILC at $\sqrt{s} = 250$ GeV

Stat. uncertainties in ppm on $\sqrt{s}$ for $\mu^+\mu^-$ channel with BES, BS, ILD detector

$\mathcal{L}_{\text{int}}$ [ab^{-1}]	$\mathcal{P}_-$, $\mathcal{P}_+$ [%]	ε [%]	Gold	Silver	Bronze	All categories
2.0	$0, 0$	69.3	5.1	2.4	6.1	2.1
0.9	$-80, +30$	70.4	6.4	3.1	7.7	2.6
0.9	$+80, -30$	68.0	7.5	3.4	8.7	2.9
0.1	$-80, -30$	70.1	25	12	30	10
0.1	$+80, +30$	68.3	28	13	33	11
2.0	Combined	-	4.7	2.2	5.6	**1.9**

Fractional errors on μ parameter (mode of peak) for 6-parameter double exponential tail function fit with all 5 shape parameters fixed to their best-fit values. The $\mathrm{e}^+\mathrm{e}^-$ channel can and should also be used. Much larger statistics from t-channel enhanced Bhabhas (also at wide angle!).

Bottom-line

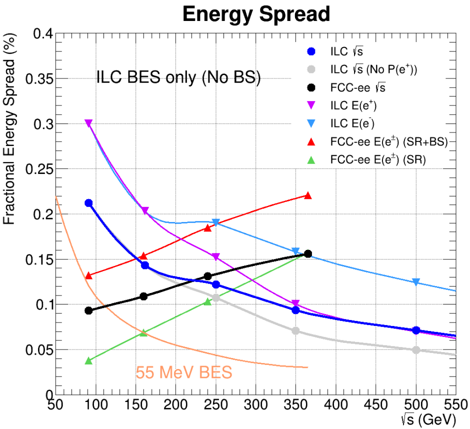
Statistical uncertainty at $\sqrt{s} = 250$ GeV of 2 ppm with momentum-based estimator. This far exceeds the 25 ppm stat. uncertainty (Hinze 2005 $\mu^+\mu^-$ channel only) of the Zγ angles-based estimator used at LEP2.

Overview of Newer Material in this Talk

- Energy spread considerations
- Detector resolution
- Working on generator-level studies of **radiative correction effects** on the $\sqrt{s}_p$ observable (using in particular KKMCee) including fast simulation estimates of some detector effects (mainly momentum resolution and acceptance). Issues with incorporating BES, BS and polarization with KKMCee. Seem to now have reasonable BES (after fixes).
- Developments related to **modeling the ILC luminosity spectrum**. Current event generators (WHIZARD, KKMCee, SHERPA) do not include these. In parallel, Brendon, has been working on GP2X (GUINEA-PIG (GP) to X: brute-force merging of GP with event generators). See his Tuesday talk.
- A new method for measuring **single colliding beam energy** reported already in late Fall 2022. It uses the same setup as in 1909.12245 focused on beam energy spread measurements, where one does assume collinear ISR. But it also leverages measurements of momentum not just of angles.
- We also have been looking into **Bhabha** event rates with BHWIDE. Very encouraging also at wide-angle for high $\sqrt{s}$.

Energy Spread

The energy spread is one of the fundamental limitations to how well one can measure the center-of-mass energy. It affects the "luminosity spectrum" and induces a variable longitudinal boost. Approx. Gaussian for both FCC-ee and ILC.

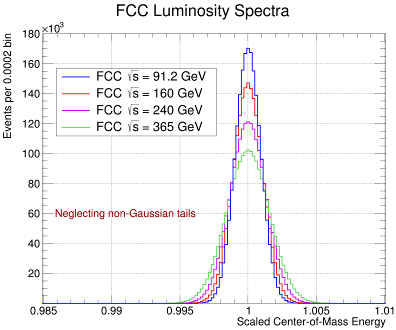


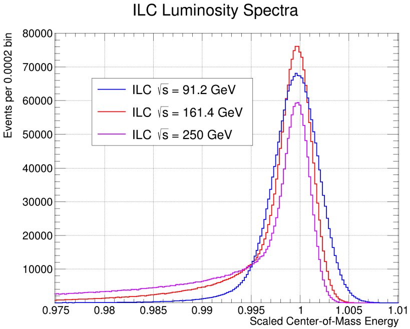
- Circular.
  $\sigma_E/E \sim E$
- Linear.
  $\sigma_E/E \sim 1/E$
- RDP feasible in lower left region with $\sigma_E \lessapprox 55$ MeV for circular.

Characterizing the energy spread is also a goal.

Luminosity Spectrum

2M events in each sample





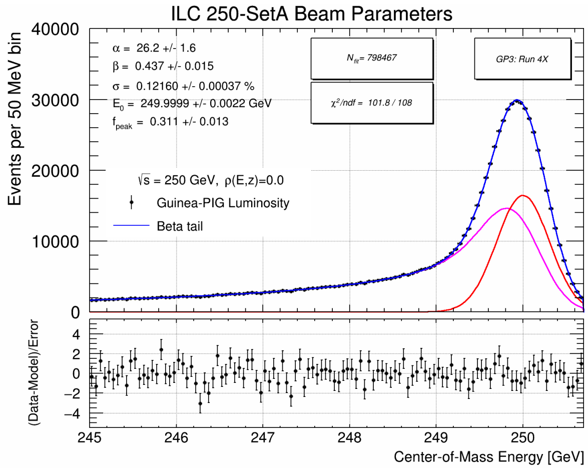
Just the BES shown previously.

GUINEA-PIG beamstrahlung simulation convolved with above BES.

Characterizing the peak position is the main goal for determining the center-of-mass energy.

Center-of-Mass Energy (After Beamstrahlung)

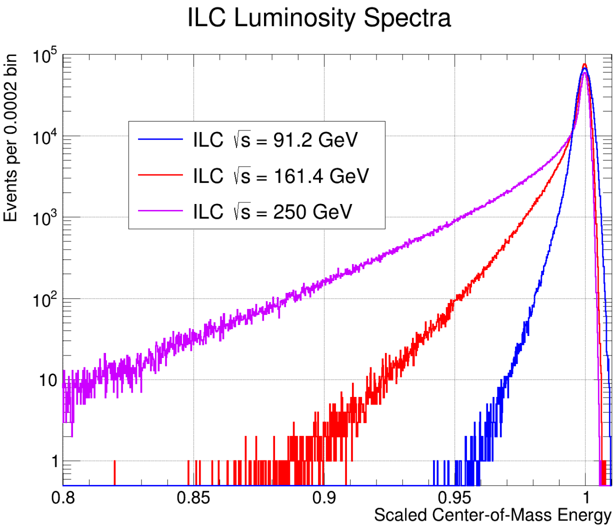
See ECFA Mini-workshop talk for more details.



$$\sigma/\sqrt{s} = 0.1216 \pm 0.0004\% \text{ (cf } 0.1217\% \text{ in TDR (} 0.190\% \oplus 0.152\%)/2)$$

Negligible bias now with single Beta function in the convolution.

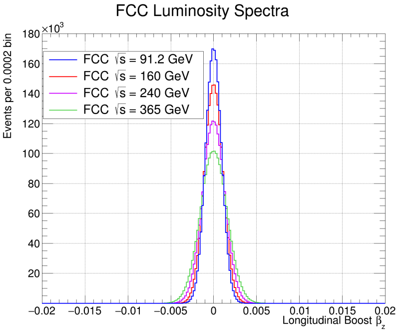
Luminosity Spectrum II

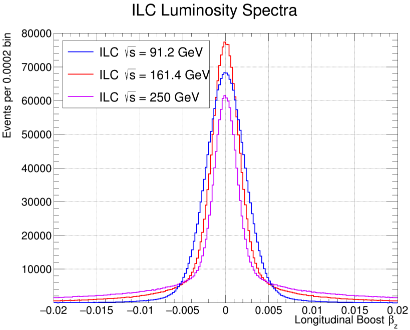


Beamstrahlung tail of some importance at 250 GeV, less important at 161 GeV, and of little importance at 91 GeV. ISR plays a similar role for all colliders.

$$\beta_z = (E_- - E_+)/(E_- + E_+)$$

2M events in each sample

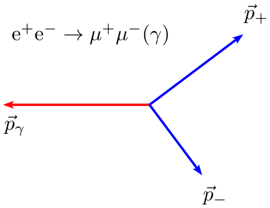




Just the BES shown previously.

GUINEA-PIG beamstrahlung simulation convolved with above BES.

$\sqrt{s}_p$ Method in a Nutshell



Measure $\sqrt{s}_p$ using,
$(|\vec{p}_+|, |\vec{p}_-|, |\vec{p}_+ + \vec{p}_-|)$

Assuming,

- **Equal** beam energies, E_b
- The lab **is** the CM frame, $(\sqrt{s} = 2\,E_\mathrm{b},\ \sum \vec{p}_i = 0)$
- The system recoiling against the dimuon is **massless**

$$\sqrt{s} = \sqrt{s}_p \equiv E_+ + E_- + |\vec{p}_+ + \vec{p}_-|$$

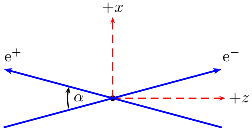
$$\sqrt{s}_p = \sqrt{p_+^2 + m_\mu^2} + \sqrt{p_-^2 + m_\mu^2} + |\vec{p}_+ + \vec{p}_-|$$

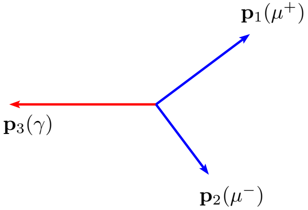
An estimate of $\sqrt{s}$ using only the (precisely measurable) muon momenta

- No assumption on the photon direction.
- With ILD detector at ILC - expect 0.14% momentum resolution for typical 71 GeV muons in Zγ events at $\sqrt{s} = 250$ GeV. Event $\sqrt{s}$ to $\approx 0.1\%$.
- Detector-level studies are with full simulation and reconstruction.

Essentials Explained

General case. 3 nuisance parameters: **crossing angle**, α, **recoil mass**, M_3, **event collision energy asymmetry**, $(E_{\mathrm{b}}^{-} - E_{\mathrm{b}}^{+})/(E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}) = \overline{\Delta E_{\mathrm{b}}}/E_{\mathrm{ave}}$.





1. $\sqrt{s} = E_1^* + E_2^* + E_3^* = E_{12}^* + E_3^*$
2. $\sqrt{s} = E_{12}^* + \sqrt{(p_{12}^*)^2 + M_3^2}$ (general M_3)
3. $\sqrt{s} = E_{12}^* + |\mathbf{p}_{12}^*|$ (assuming $M_3 = 0$)

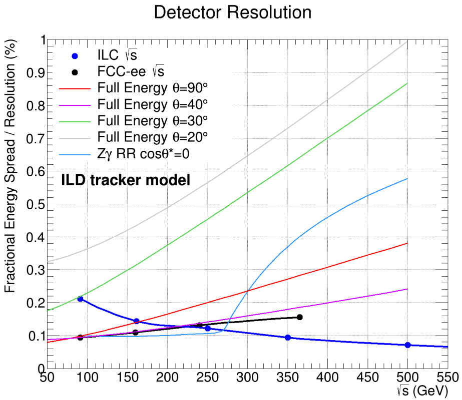
We have the measured dimuon 4-vector in the detector frame $(E_{12}, \mathbf{p}_{12})$. Need to apply the appropriate boost from lab back to the CM frame to obtain $(E_{12}^*, \mathbf{p}_{12}^*)$. The boost velocity (in the horizontal x-z plane) is

$$\boldsymbol{\beta} = (\beta_x, \beta_y, \beta_z) = \left(\sin(\alpha/2), 0, \frac{\overline{\Delta E_{\mathrm{b}}}}{E_{\mathrm{ave}}}\cos(\alpha/2)\right)$$

$\beta_x = 0.007/0.015$ (ILC/FCC-ee). β_z depends on the collision energy asymmetry.

Is Your Detector Resolution Commensurate with BES?

Superimpose on previous BES curves the expected resolution on the event center-of-mass energy estimator for both $2 \rightarrow 2$ events where the mass is measured, and highly radiative $Z\gamma$ events using $\sqrt{s}_p$ (muon momentum based). Neglects crossing-angle.



- Uses ILD at ILC for illustration.
- RR events. Great below 300 GeV.
- High $\sqrt{s}$ - more challenging.
- ILD 3.5T well suited to ILC Z.

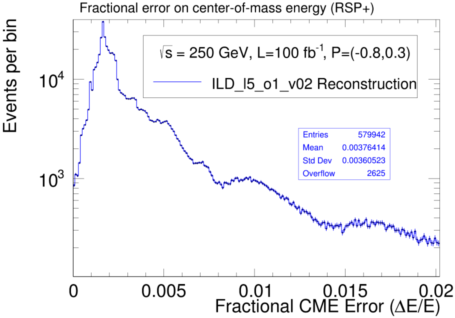
Additional much higher statistics from Bhabhas will help a lot at higher $\sqrt{s}$, but forward polar angles less powerful.

ILD Study: Event Selection Requirements

Currently rather simple.

Use latest full ILD simulation/reconstruction at 250 GeV.

- Require exactly two identified muons
- Opposite sign pair
- Require uncertainty on estimated $\sqrt{s}_p$ of the event of less than 0.8% of $\sqrt{s}_{\mathrm{nom}}$ based on propagating track-based error matrices
- Categorize reconstruction quality as **gold** ($<$0.15%), **silver** ([0.15, 0.3]%), **bronze** ([0.3, 0.8]%)
- Require the two muons pass a vertex fit with p-value > 1 %



Selection efficiencies for (80%/30%) beam polarizations:

- $\varepsilon_{-+} = 70.4 \pm 0.1$ %
- $\varepsilon_{+-} = 68.0 \pm 0.1$ %
- $\varepsilon_{--} = 70.1 \pm 0.1$ %
- $\varepsilon_{++} = 68.3 \pm 0.1$ %

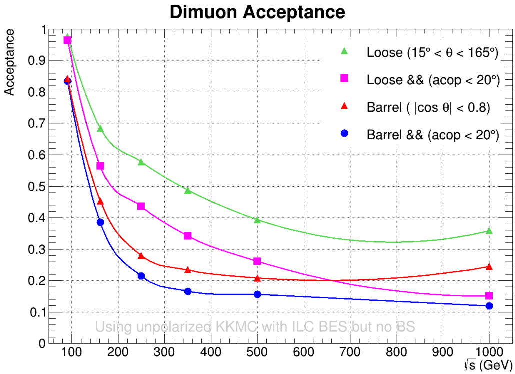
Backgrounds not yet studied in detail, ($\tau^+\tau^-$ is small:0.15%, of no import for the $\sqrt{s}$ peak region).

Center-of-Mass Energy Dependence of $\sqrt{s}$ Measurements

Found 1.9 ppm (stat. uncertainty) for ILC at 250 GeV.

Relevant Factors

1. Cross-section
2. Acceptance
3. Momentum Resolution
4. Calorimetric Energy Resolution
5. Integrated Luminosity
6. Intrinsic Beam Energy Spread
7. Beamstrahlung



- Factors 4, 5, and 6 **improve** with higher $\sqrt{s}$ for ILC
- Factors 1, 2, 3, and 7 **degrade** with higher $\sqrt{s}$ for ILC

Given current limitations in full start-to-end modeling of BS, polarization, physics (RCs), detector effects, integration of Bhabhas, and consistent estimation procedure do not yet have a full quantitative assessment for all $\sqrt{s}$ beyond essentially that presented in 2209.03281 focused on $\sqrt{s} = 250$ GeV.

Evaluating Physics Limitations I (WIP)

Method: physical precision

Use weighted KKMCee (4.32) generator to re-weight the estimators at generator level ($\sqrt{s}_p$, inferred E_- and E_+) to alternative physics levels. Use $\frac{1}{2}(x' - x)$.

(Coherent) exclusive exponentiation. $\mathcal{O}(\alpha^2)$ CEEX (**CEEX2**) is the standard one.

- $\mathcal{O}(\alpha)$ CEEX (CEEX1)
- $\mathcal{O}(\alpha^3)$ EEX (EEX3)
- $\mathcal{O}(\alpha^2)$ CEEX but no ISR/FSR interference (CNIF2)

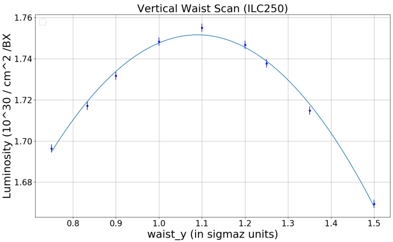
$\sqrt{s}_p$ physical precision estimates (ppm). (CEEX1, EEX3, CNIF2) cf **CEEX2**

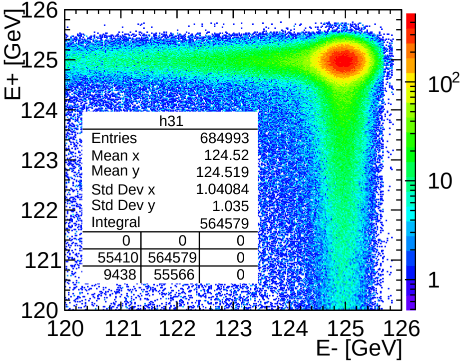
Use barrel acceptance with 20° acoplanarity cut (blue curve).

$\sqrt{s}$	CEEX1	EEX3	CNIF2
91 GeV	-0.9	0.4	0.02
161 GeV	-0.2	-10	-11
250 GeV	-0.3	-11	-11
350 GeV	-0.6	-10	-10
500 GeV	-0.5	-9	-9
1000 GeV	-0.6	-8	-8

- Use change in mean of $(\sqrt{s}_p - \sqrt{s})/\sqrt{s_{\mathrm{nom}}}$ in $\pm$ 0.5% range.
- 10^7 unpolarized weighted events per $\sqrt{s}$ with ILC BES but no BS
- Z is different
- Prospects now for ppm precision

- New GUINEA-PIG simulations done to model ILC luminosity spectrum paying attention to adequate stochasticity of large statistics E_{b} distributions.
- Did GP vertical waist scan. Choose $1.1\sigma_z$. Find $\mathcal{L}$ consistent with ILC.
- How to model the bivariate (E_+, E_-) distribution with BS-induced correlations?





Use the **copula** approach to factorize the problem:

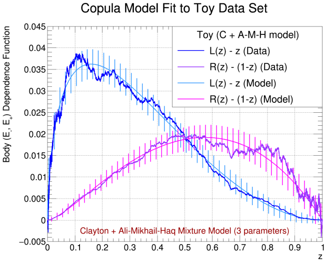
1. modeling the marginal (1-d) distributions of E_+ and E_- (pre-BES)
2. modeling the dependence distribution between E_+ and E_- (pre-BES)

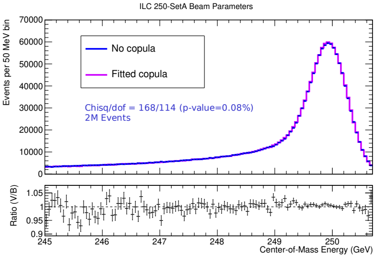
followed by Gaussian BES smearing of the beam energies

What is a bivariate copula?

- It is the link between the full 2-d probability distribution and the two 1-d marginal distributions.
- Specifically the copula is the 2-d cdf for which the marginal pdfs $\sim \mathrm{Un}(0,1)$.
- Thus we can split the luminosity spectrum modeling problem in two.

In practice, especially at low $\sqrt{s}$, the dependence effects are rather **small**.





Chosen 3-parameter copula model fits the empirical copula cdf (p-value=98%) of the ILC250 GP "body" data (parametric bootstrap Cramer-von-Mises G-o-F test). Models 91, 161 GeV data too. See CopulaGenerator github for a stochastic implementation of an 18 parameter model. Use separate double Beta distributions for arm and body marginals. **More details in backup**. Welcome collaboration to implement in event generators.

Collision Beam Energies using $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-(\gamma)$

Infer the e^- and e^+ beam energies from the muons alone under the *assumption* of one collinear **undetected** ISR photon. (E, p_z) conservation equations:

$$E_- + E_+ = E_1 + E_2 + |p_\gamma^z|/\cos(\alpha/2) \quad (1)$$

$$(E_- - E_+)\cos(\alpha/2) = p_1^z + p_2^z + p_\gamma^z \quad (2)$$

Solve for E_- and E_+,

$$E_- = \frac{1}{2}\left[(E_1 + E_2) + \frac{(p_1^z + p_2^z)}{\cos(\alpha/2)} + \frac{(|p_\gamma^z| + p_\gamma^z)}{\cos(\alpha/2)}\right] \quad (3)$$

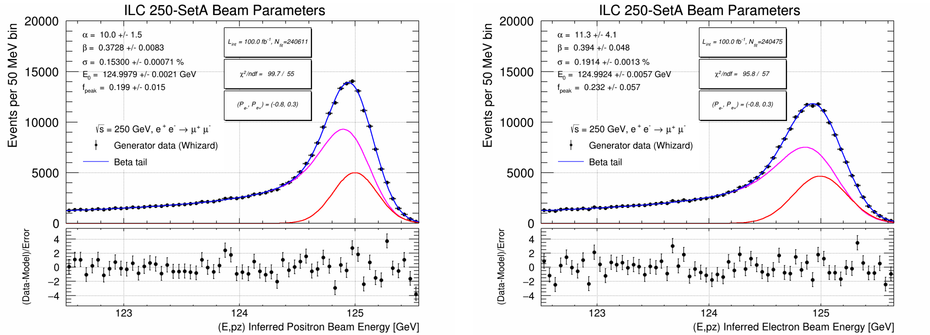
$$E_+ = \frac{1}{2}\left[(E_1 + E_2) - \frac{(p_1^z + p_2^z)}{\cos(\alpha/2)} + \frac{(|p_\gamma^z| - p_\gamma^z)}{\cos(\alpha/2)}\right] \quad (4)$$

- If $p_\gamma^z \leq 0$, there is NO p_γ^z induced error for the muons-only E_- equation[0]
- If $p_\gamma^z \geq 0$, there is NO p_γ^z induced error for the muons-only E_+ equation[0]

Exact for one of the beams with one collinear ISR photon present! But really wrong for the other beam - especially for $Z\gamma$ events ($E_\pm$ error is $\frac{-|p_\gamma^z|}{\cos(\alpha/2)}$).

[0]Obtained by neglecting the **unmeasured** red p_γ^z dependent terms.

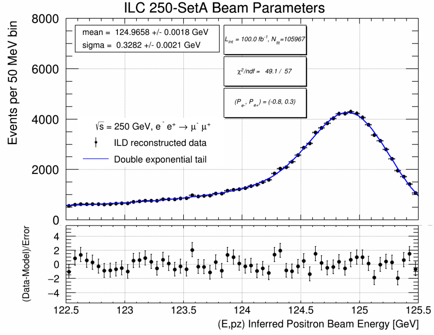
Collision Beam Energy (Generator level)

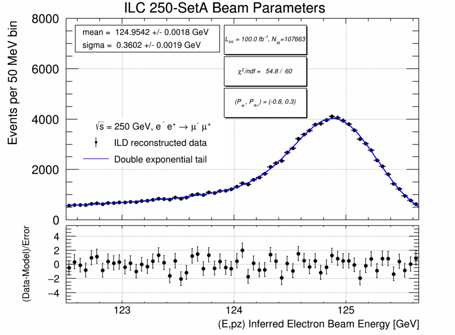


Generator-level rms of peak very similar to intrinsic expectation from beam energy spread alone of 0.152% (e^+) and 0.190% (e^-).

Collision Beam Energy (Reconstructed)

Here for "silver" quality dimuons.





- Also relevant for luminosity spectrum extraction. (Note. T. Barklow also discussed these estimators in the past).
- Precision degraded by detector resolution as expected, but can still resolve well the differences.
- Likely complementary to $\sqrt{s_p}$ approach. Although the advantage of a more direct single beam measurement is diluted statistically by the wrong hemisphere feature.

Statistical Precision Estimates

Statistical precision estimates were given in the preprint for $\sqrt{s_p}$. The following table summarizes an initial comparison of the $\sqrt{s_p}$ estimator and the (E, p_z) inferred beam energy.

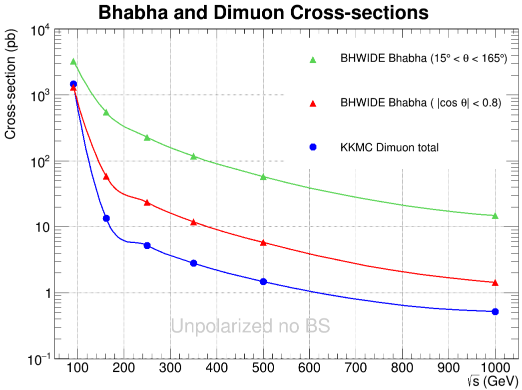
Observable	$\sqrt{s}$	E_{b}^{-}	E_{b}^{+}	$2\sqrt{\hat{E_{\mathrm{b}}^{-}}\hat{E_{\mathrm{b}}^{+}}}\cos(\alpha/2)$
WHIZARD generator level	3.8	3.2	2.6	2.1
Generator level estimator	5.8	6.4	5.3	4.1
Detector level estimator	7.8	12.5	12.2	8.7

Table E.5: Statistical precision estimates on the absolute scale of the center-of-mass energy, $\sqrt{s}$, and the electron and positron beam energies (E_{b}^{-} and E_{b}^{+}) in parts per million (ppm) for 100 fb^{-1} of $\mathrm{e^+e^-} \to \mu^+\mu^-(\gamma)$ events with $P(\mathrm{e}^-) = -0.8$ and $P(\mathrm{e}^+) = +0.3$ at $\sqrt{s} = 250$ GeV for ILC. The statistical energy scale uncertainties are the result of fits to the relevant distributions with the shape parameters fixed to their best-fit values, but the one scale-related location parameter floating. The $\sqrt{s_p}$ estimator is used for the $\sqrt{s}$ estimate, and the (E, p_z)-inferred colliding beam energy estimator is used for E_{b}^{-} and E_{b}^{+}. The WHIZARD generator level values represent fits to the true distributions of $\sqrt{s}$, E_{b}^{-}, and E_{b}^{+} differing only from the pure luminosity spectrum equivalent by the convolution with the cross-section. The generator level estimator includes effects from ISR and FSR that invalidate the assumptions. Detector level estimates use ILD full simulation with dimuons classed in the gold, silver, and bronze resolution categories and includes acceptance, efficiency and resolution effects. The last column gives the propagated uncertainty on the center-of-mass energy scale using the single beam energy scale estimates neglecting potential correlations.

- Beam energy distributions benefit from higher peak/tail ratio at generator level (higher probability of no significant beamstrahlung).
- Final $E_{\mathrm{b}}^{\pm}$ uncertainty estimates interesting, but higher than $\sqrt{s_p}$ for $\sqrt{s}$.

Bhabhas

Wait: there is more!



- Barrel Bhabha cross-section at $\sqrt{s} = 250$ GeV of 23.5 pb exceeds the accepted dimuon barrel cross-section of 1.46 pb by a **FACTOR of 16!**
- For ILD these Bhabha events are in the pristine momentum resolution regime. So should contribute a factor of up to 4 improvement in measurement precision (although of course electrons are not muons...).

Summary of Progress

Progress

- New high precision method for momentum-scale using especially $\mathrm{K^0_S}$ and Λ. Promises 2.5 ppm uncertainty per 10M hadronic Zs.
- More detailed investigation of dimuons for $\sqrt{s}$ and dL/d$\sqrt{s}$ reconstruction. Capable of 2 ppm stat. uncertainty for ILC at $\sqrt{s} = 250$ GeV and 2 ppm for every $1\mathrm{fb}^{-1}$ of the standard 100 fb^{-1} ILC run at the Z. QED theoretical uncertainties appear under control at 1 ppm level (KKMC study).
- Baseline ILC250 can make precision measurements at the Z and at the WW threshold. Use the actual colliding beams for center-of-mass energy measurement. Opens up capabilities for high precision A_{LR}, M_{W}, M_{Z}, Γ_{Z}.
- New ideas on luminosity spectrum modeling and on colliding beam energy measurements.

Conclusions

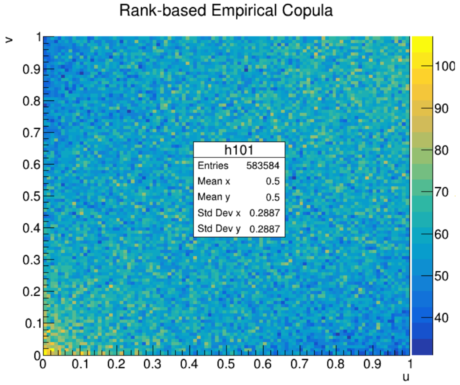
- Tracking detectors designed for ILC have the potential to measure beam energy related quantities with precision similar to the intrinsic energy spread using dimuon events (and also especially wide-angle Bhabha events).
- The $\sqrt{s}_p$ technique enables a high precision electroweak measurement program for ILC taking advantage of absolute CoM energy scale knowledge.

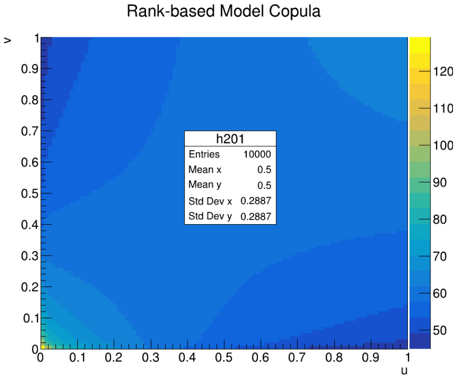
Backup Slides

Acknowledgments

Thanks to Paolo Azzurri, Tim Barklow, Mikael Berggren, Alain Blondel, Patrick Janot, Jenny List, Brendon Madison, Thorsten Ohl, Alan Price, Alberto Ruiz, Andre Sailer, Daniel Schulte, Zbigniew Was, Glen White, Guy Wilkinson, and Kaoru Yokoya for useful comments.

Plot (u, v) with rank-based E_- as u and rank-based E_+ as v from "body" (both beams lose non-negligible energy) GUINEA-PIG events. (No BES applied in GP).

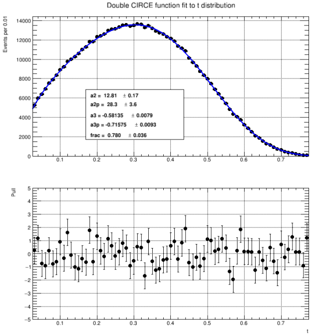




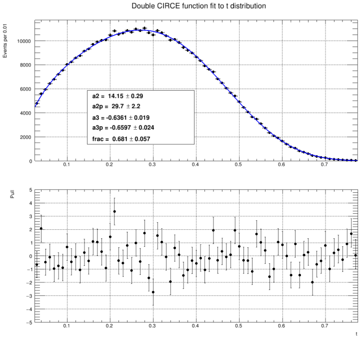
Find mild dependence structure. With most visible action in lower left corner when both beams lose a lot of energy. Here $\chi^2 = 9488$ for 9798 dof (p-value=98.7%).

A "double Beta function" is needed to fully describe these "deconvolved" distributions at the 1M event level. Use $\eta = 4$ here. Cut at $x \leq 1 - 4 \times 10^{-7}$.

$$t \equiv (1 - x)^{1/\eta}$$



Body (χ^2/dof = 58.8/70)



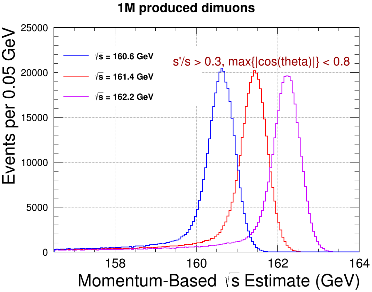
Arms (χ^2/dof = 89.6/70)

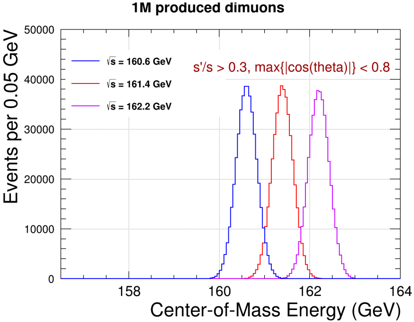
Fits with 1M events for ILC250 are fine! 2-d event populations: 24.55% (peak), 29.80% (body), 45.65% (arms).

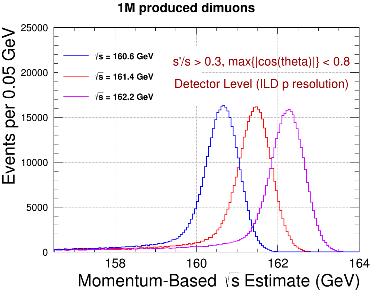
Center-of-Mass Energy near WW Threshold

Study with $e^+e^- \to \mu^+\mu^-(\gamma)$

- Use KKMCee with energy spread of 0.203% (ILC-like)
- No beamstrahlung for now
- Tail from radiative effects
- 44.7% of events pass muon cuts
- Plots: $\Delta(\sqrt{s})/\sqrt{s} = 5.0$ ppm stat.

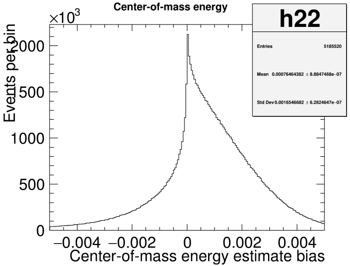


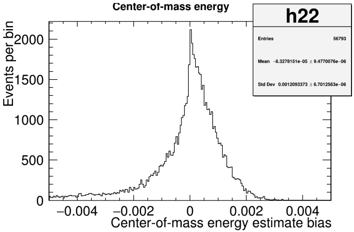


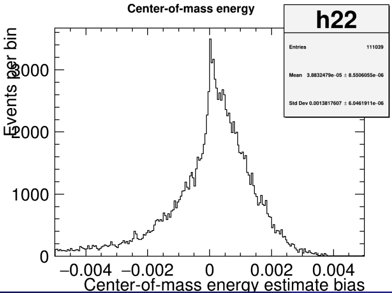


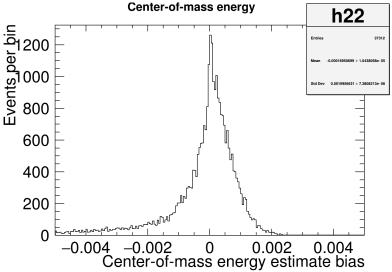
$\sqrt{s}_p$ Fractional Bias

KKMCee study. ILC BES. No BS. No polarization. Gen level.
91 (Top Left), 250 (Top Right), 350 (Bottom Left), 500 (Bottom Right)



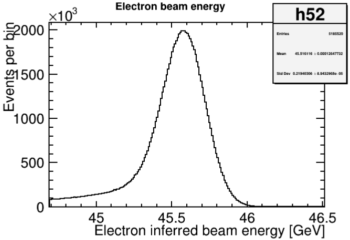


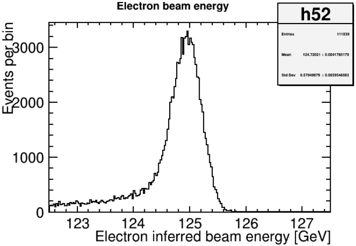


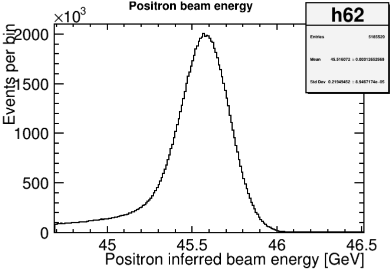


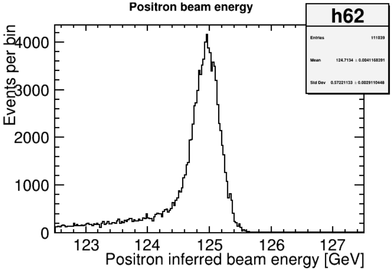
Inferred beam energies

KKMCee study. ILC BES. No BS. No polarization. Gen level.
91 (Top), 250 (Bottom)



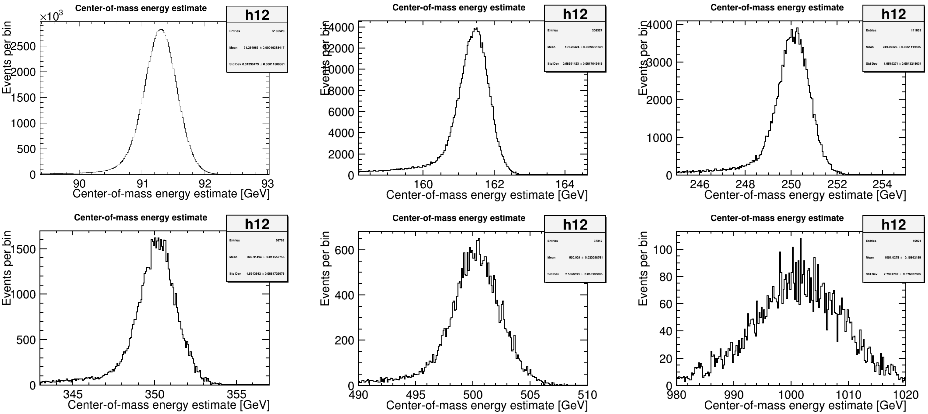






$\sqrt{s_p}$ at detector level with dimuons

- Plots from KKMCee study. ILC BES, but no BS nor polarization for now.
- Use barrel + acoplanarity acceptance. Normalized to 100 fb^{-1}.
- Each plot covers $\pm 2\%$.



Evaluating Physics Limitations II (WIP)

Method: physical precision

Use weighted KKMCee (4.32) generator to re-weight the estimators at generator level ($\sqrt{s}_p$, inferred E_- and E_+) to alternative physics levels. Use $\frac{1}{2}(x' - x)$.

(Coherent) exclusive exponentiation. $\mathcal{O}(\alpha^2)$ CEEX (**CEEX2**) is the standard one. See acceptance slide for acceptance cuts.

$\sqrt{s}_p$ physical precision estimates (ppm). (CEEX1, EEX3, CNIF2) cf **CEEX2**

Loose acceptance

$\sqrt{s}$	CEEX1	EEX3	CNIF2
91 GeV	-1.0	0.4	0.1
161 GeV	-1.6	-17	-16
250 GeV	-5.4	-14	-13
350 GeV	-12	-14	-13
500 GeV	-25	-10	-10
1000 GeV	-69	-1.3	-9

Barrel (no acop cut)

$\sqrt{s}$	CEEX1	EEX3	CNIF2
91 GeV	-1.0	0.4	0.08
161 GeV	-2.8	-10	-9
250 GeV	-11	-10	-8
350 GeV	-21	-8	-7
500 GeV	-35	-4	-4
1000 GeV	-72	11	4

Evaluating Physics Limitations III (WIP)

Method: physical precision

Use weighted KKMCee (4.32) generator to re-weight the estimators at generator level ($\sqrt{s}_p$, inferred E_- and E_+) to alternative physics levels. Use $\frac{1}{2}(x' - x)$.

(Coherent) exclusive exponentiation. $\mathcal{O}(\alpha^2)$ CEEX (**CEEX2**) is the standard one. See acceptance slide for acceptance cuts.

Inferred E_- (left) and E_+ (right) physical precision estimates (ppm)

Barrel + acop acceptance. Use mean in ± 2 % range around E_{nom}.

$\sqrt{s}$	CEEX1	EEX3	CNIF2
91 GeV	-0.8	-0.2	-0.2
161 GeV	-3.1	-37	-36
250 GeV	-1.8	-50	-50
350 GeV	-1.7	-47	-46
500 GeV	-1.5	-41	-41
1000 GeV	-1.8	-5	-5

$\sqrt{s}$	CEEX1	EEX3	CNIF2
91 GeV	-0.8	-0.3	-0.3
161 GeV	-3.3	-44	-42
250 GeV	-2.0	-41	-41
350 GeV	-1.7	-44	-44
500 GeV	-1.8	-36	-36
1000 GeV	-1.7	-43	-43

Few ppm for CEEX2 cf CEEX1.

Outlook and Work Plans

Lots of opportunities to improve this:

1. Constrained kinematic fits. For example one can test the consistency with the pure 2-body hypothesis of $e^+e^- \to \mu^+\mu^-$ while fitting for the two unmeasured parameters of E_{ave} and $\overline{\Delta E_b}$, and also perform fits with the $e^+e^- \to \mu^+\mu^-\gamma$ hypothesis.
2. Extend the techniques to the $e^+e^- \to e^+e^-$ channel.
3. Exploit fully events with detected photons.
4. Implement complete end-to-end measurement scheme and understand how best to use different kinematic regimes and correct/mitigate observed biases.
5. Characterize better the intrinsic limitations associated with beam energy spread, beamstrahlung, ISR, FSR, backgrounds, and detector acceptance and resolution. This includes studies with more specialized physics event generators such as KKMCee [29].
6. Tracker momentum scale studies using $J/\psi \to \mu^+\mu^-$, $K_S^0 \to \pi^+\pi^-$, $\Lambda^0 \to p\pi^-$. We have some preliminary results [30] further applying the technique advocated in [31] based on the Armenteros-Podolanski [32] reconstruction technique. A more novel aspect is that one can aspire to simultaneously improve the measurements of the K_S^0 and Λ masses and the momentum scale given that the masses of their decay products are very well known.
7. Understand the relative merit of dimuons for luminosity spectrum determination compared with Bhabhas and integrate both techniques in a global analysis.
8. Characterize further the scope for measuring accelerator parameters such as the crossing angle and beamstrahlung-induced correlations including the observed dependence of the beam energy spectrum on the longitudinal collision vertex. The latter has been shown to be easily measurable with vertex fits in $e^+e^- \to \mu^+\mu^-(\gamma)$ events.