Modeling Center-of-Mass Energy Precision using Dimuons and Bhabhas at ILC

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Background: Who am I?

- Native of Kansas
- PhD student at Univ. Kansas
- Currently work with Graham Wilson on next generation colliders
- Focus on ILC so far
- Previously worked on Astroparticle physics and Aerospace Engineering
Broad Outline

- What am I doing today?
  - Work towards precision $\sqrt{s}$ and beam energy measurements – More detail[1]
- Why?
  - Need to measure energies, particles precisely as well as monitor beams and luminosity
- How?
  - Dimuons in tracker and Bhabhas in ECAL
- How? Pt. 2
  - Writing software and doing analysis (GP2X)[2]

AI Artist’s depiction of $ee \rightarrow \mu\mu(\gamma)$
Outline of GP2X

- **GP2X** (GuineaPig++ to X)
- X is an event generator and GuineaPig++[3] does beam collision simulation
- Written in ROOT (C++ in future?)
- Consider the Algorithm Diagram:

![Algorithm Diagram](image-url)
Outline of GP2X

- **GP2X** (GuineaPig++ to X)
- X is an event generator and GuineaPig++[3] does beam collision simulation
- Written in ROOT (C++ in future?)
- Cut this into sections!
GP2X 1) GuineaPig++

- Use to simulate events for the “initial state” i.e. “ee” after beamstrahlung etc.

- Beam Energy Spread (BES) OFF, handled by GP2X

**How?**

- Give GuineaPig an input card and then run GuineaPig numerous times on computing cluster

- Take output **lumi.ee.out** file and convert to format GP2X can use

**Note:** $\sqrt{s}$ spectra!
GP2X 2) Event Generator

- Use to simulate events for the final state e.g. $\mu\mu(\gamma)$
- Do with $\sqrt{s}$ grid to get a sample with all possible $\sqrt{s}$
- Important: All events done with 5 degree ($\theta$) acceptance
- Generators used so far:
  - KKMC (LHE)
  - WHIZARD3 (LHE)
  - BHWIDE (LHE)
- Output LHE so LHE to ROOT TTree converter needed to use with GP2X!
- Important: Get $\sigma(\sqrt{s})$, will need later
GP2X 3, Pairing Initial and Final

- Have: Initial database and final database
- Want: Convolved database
- Need: Pair initial and final state by $\sqrt{s}$
- Randomly from both database?
- Really slow! $\approx \frac{100k}{\text{week}}$, $O(a^2N^2)$, $a \approx 4$ here
- Use fitting and $\sqrt{s}$ grid to construct Fitted Orthogonal Sampling (FOS) to randomly sample both at once
- Much faster, $\approx \frac{300k}{\text{hr}}$, $O(aN)$

Overview of types of multidimensional random sampling

- random sampling
- exponential sampling
- dynamic parameters
- Latin hypercube sampling
- orthogonal LHS
GP2X 4) , Do physics (and then write)

- Have: Paired database
- Want: Convolved database
- Need: Boost final state, Gaussian BES, rescaling, fold in Xsection and detector effects
- Energy Rescale → 2nd Order NR Method

\[ \sum_{i=1}^{N} E_i' = \sum_{i=1}^{N} \sqrt{K^2 p_i^2 + m_i^2} = \sqrt{s'} \]

\[ K_1 = \frac{\sqrt{s'} - \sum_{i=1}^{N} \frac{m_i^2}{E_i}}{\sqrt{s} - \sum_{i=1}^{N} \frac{m_i^2}{E_i}} \]

\[ K_{n+1} = K_n - f(K_n) / f'(K_n) \]

\[ f(K) = \sum_{i=1}^{N} \sqrt{K^2 p_i^2 + m_i^2} - \sqrt{s'} = 0 \]

\[ f'(K) = K \sum_{i=1}^{N} \frac{p_i^2}{\sqrt{K^2 p_i^2 + m_i^2}} \]

- Xsection → fit event generator \( \sigma(\sqrt{s}) \) and then reweight
- Detector → Smear using parametric model of ILD tracker or 18% ERes ECAL
Center of Mass From DiMuons

- **Why**: DiMuons can be measured well in tracker, kinematically simple
- **Issue**: Beamstrahlung, ISR, FSR complicate things
- **First Step**: Derive solution that works in limit of massless 3rd body, $\mu\mu(\gamma)$
- **Extensive work here [1]** finds: $\sqrt{s_p} = E_{\mu\mu} + \sqrt{E_{\mu\mu}^2 - M_{\mu\mu}^2}$
- Where $\sqrt{s_p}$ is momentum based estimate of $\sqrt{s}$

Example from iLCSoft fitted using crystal ball function

Or from GP2X ... Look similar!
Center of Mass From DiMuons

- Overlay them ... very similar
- Check that iLCSoft and GP2X agree in future
- Issue: Look at CBall fit mean, it is many $\sigma$ off of 250!
- CBall doesn’t fit mean well?

Detector Level $\sqrt{s_p}$ Using KKMC, GP2X
A Better Fit?

- Want: Fit that can separate (and fit) peak well
- How: Convolution Fits
- Today: Show beta convolved fit
- Following example of CIRCE [4]
  - use beta distribution convolved with Gaussian:
    \[ F(x, \alpha, \beta, \sigma) = \int x^{\beta-1} (1 - x)^{\alpha-1} G(x - 1, \sigma) \, dx \]
  - With \( x = \frac{\sqrt{s}}{E_0} \) and fraction of Gaussian, \( f_{peak} \), have 5 parameters
  - 1 less parameter than before!

Fit from GP2X \( \sqrt{s_p} \) at MC level

At detector level ... (WIP)
What about Beam Energies?

- Why: Monitor beam
- How: Derive relationship in limit of colinear photon
- Result:
  \[ E_i^- = \frac{1}{2}(E_{\mu\mu} + p_{\mu\mu}^z) \]
  \[ E_i^+ = \frac{1}{2}(E_{\mu\mu} - p_{\mu\mu}^z) \]
- Can fit with fixed energy at design values

\[ \chi^2 \] of both supports the fixed hypothesis

- \( \sigma \) also matches design BES for ILC250
- Potential issue: uncertainty on \( \alpha \)
Fitting Bhabhas at MC Level

- Why: Xsection ...
  Bhabha $\approx 2200$ pb while DiMuon $\approx 5$ pb
- Issue: Bhabha prefers small angles, ECAL measured
- Use BHWIDE, WHIZARD event generators
- BHWIDE+GP2X $\sqrt{s_p}$

**WHIZARD+GP2X Electron beam energy**

<table>
<thead>
<tr>
<th>$E$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\sigma$</th>
<th>$E_{\text{true}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3750</td>
<td>0.3073 +/- 0.0089</td>
<td>0.18951 +/- 0.00027 %</td>
<td>125.0004 +/- 0.0011</td>
<td>125.004 +/- 0.011</td>
</tr>
</tbody>
</table>

**BHWIDE+GP2X Electron beam energy**

<table>
<thead>
<tr>
<th>$E$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\sigma$</th>
<th>$E_{\text{true}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1000</td>
<td>0.2201 +/- 0.0094</td>
<td>0.16513 +/- 0.00031 %</td>
<td>125.0015 +/- 0.0015</td>
<td>125.015 +/- 0.015</td>
</tr>
</tbody>
</table>

$\chi^2/\nu = 84.7 / 94$

$E = 2.500e+02$ GeV, $e^- e^+ \rightarrow \mu^- \mu^+$

$\chi^2/\nu = 91.4 / 94$

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$\chi^2/\nu = 91.4 / 94$
Fitting Bhabhas at Detector Level

- As with DiMuons the detector level fitting is WIP
- WHIZARD+GP2X

Need:
- Determine Bhabhas that can be measured well in tracker
- Incorporate detector into fitting

![Graph showing detector center of mass and fit parameters.](image)
BES is fit well in all beam metrics

Spread in $\sqrt{s_p}$ does not match simple Gaussian spread of $E^+ + E^-$

Precision in $E_0$ maybe “too good”, need work on validation

Need work on precision calibration for tracker and ECAL
Future Work

- Integration with event generators
- Incorporate detector in fits
- Model ISR in fit so it can be monitored
- Investigate other $\sqrt{s}$
- Work on precision tracker and ECAL calibration
Acknowledgement

Thanks to Graham Wilson for advice and discussions. Go to Graham’s talk on Thursday!
This work was performed at the HPC facilities operated by the Center for Research Computing at the University of Kansas supported in part through the National Science Foundation MRI Award 2117449.
References

[2] Brendon Madison. Brendonmadison/GP2X: Guinea pig to $X$, $X$ being a differmion event generator. takes initial beam dynamics from Guineapig and uses them to boost the final state differmions according to their center of mass energies. url: https://github.com/BrendonMadison/GP2X.