Modeling Center-of-Mass Energy Precision using Dimuons and Bhabhas at ILC

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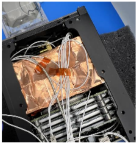
Contents

Background: Who am I?

- Native of Kansas
- PhD student at Univ. Kansas
- Currently work with Graham Wilson on next generation colliders
- Focus on ILC so far
- Previously worked on Astroparticle physics and Aerospace Engineering



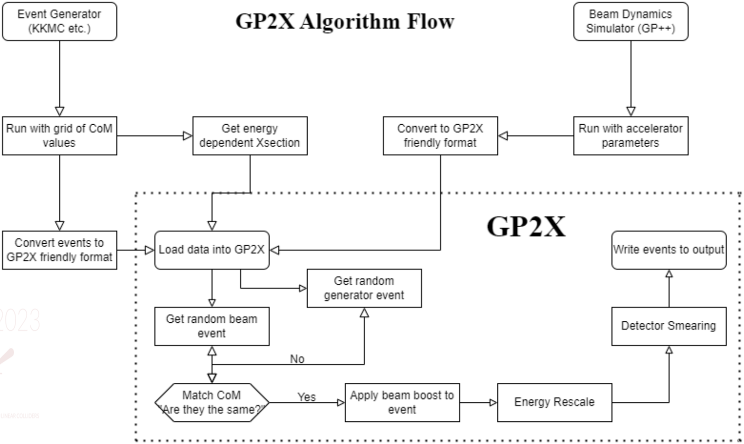
Broad Outline

- **What am I doing today?**
- Work towards precision $\sqrt{s}$ and beam energy measurements – More detail[1]
- **Why?**
- Need to measure energies, particles precisely as well as monitor beams and luminosity
- **How?**
- Dimuons in tracker and Bhabhas in ECAL
- **How? Pt. 2**
- Writing software and doing analysis (GP2X)[2]

AI Artist's depiction of $ee \to \mu\mu(\gamma)$

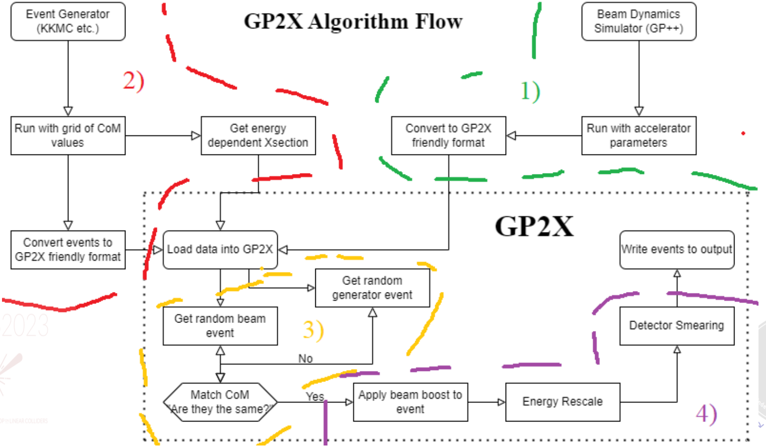
Outline of GP2X

- GP2X (GuineaPig++ to X)
- X is an event generator and GuineaPig++[3] does beam collision simulation
- Written in ROOT (C++ in future?)
- Consider the Algorithm Diagram:



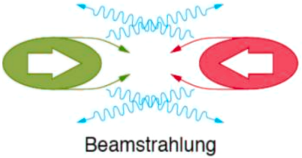
Outline of GP2X

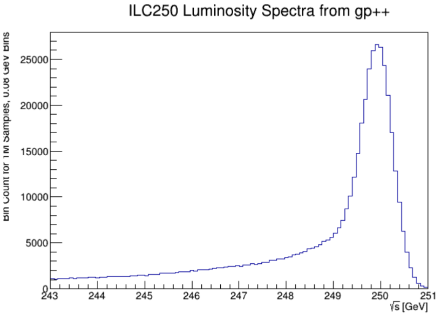
- GP2X (GuineaPig++ to X)
- X is an event generator and GuineaPig++[3] does beam collision simulation
- Written in ROOT (C++ in future?)
- Cut this into sections!



GP2X 1) GuineaPig++

- Use to simulate events for the "initial state" i.e. "ee" after beamstrahlung etc.
- Beam Energy Spread (BES) OFF , handled by GP2X





- **How?**
- Give GuineaPig an input card and then run GuineaPig numerous times on computing cluster
-
- Take output **lumi.ee.out** file and convert to format GP2X can use

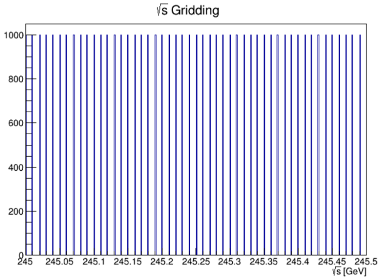
- Note: $\sqrt{s}$ spectra!

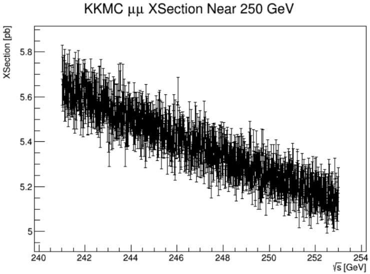
Output

```
125.000000 125.000000 500.819733 1.806819 -57.376547 1213 -7.70116e-05 -7.75947e-05 -0.000141939 1.13057e-05 0 0 0 0 0 0 451149
125.000000 124.848885 -406.269989 0.567856 -39.200738 1726 0.000189155 -3.39702e-05 0.000263655 5.53292e-05 0 0 0 0 0 0 1476664
119.642281 125.000000 -231.475555 -0.875431 -257.309109 3142 0.000747022 2.73545e-05 0.000536495 -2.4685e-05 0 0 0 0 0 0 4835672
124.631737 124.676758 -51.434143 -1.227208 86.730359 2188 0.000167454 5.25277e-05 5.38255e-05 -4.53217e-05 0 0 0 0 0 0 2652315
123.881699 121.094826 8.253984 2.962857 -147.835844 3051 -0.000198026 1.48113e-05 -0.000433987 -5.38493e-06 0 0 0 0 0 0 4701593
125.000000 108.386497 -8.205165 1.589622 205.139047 3282 -6.52207e-05 3.66908e-06 0.000749865 4.83599e-05 0 0 0 0 0 0 4995272
124.973595 125.000000 -1.554769 -3.063253 126.066984 1998 0.000171935 -1.12584e-05 2.01508e-05 -4.2083e-05 0 0 0 0 0 0 2149479
116.623161 122.506683 -530.349731 -2.478820 -17.218301 3306 0.000705048 -6.13798e-05 0.000646323 4.97031e-05 0 0 0 0 0 0 5023828
```

GP2X 2) , Event Generator

- Use to simulate events for the final state e.g. $\mu\mu(\gamma)$
- Do with $\sqrt{s}$ grid to get a sample with all possible $\sqrt{s}$
- Important: All events done with 5 degree (θ) acceptance
- Generators used so far:
  - KKMC (LHE)
  - WHIZARD3 (LHE)
  - BHWIDE (LHE)
- Output LHE so LHE to ROOT TTree converter needed to use with GP2X!
- Important: Get $\sigma(\sqrt{s})$, will need later

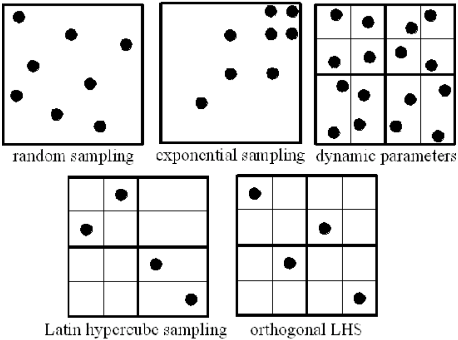




GP2X 3) , Pairing Initial and Final

- Have: Initial database and final database
- Want: Convolved database
- Need: Pair initial and final state by $\sqrt{s}$
- Randomly from both database?
- Really slow! $\approx \frac{100k}{week}$, $\mathcal{O}(a^2N^2)$, $a \approx 4$ here
- Use fitting and $\sqrt{s}$ grid to construct Fitted Orthogonal Sampling (FOS) to randomly sample both at once
- Much faster , $\approx \frac{300k}{hr}$, $\mathcal{O}(aN)$

Overview of types of multidimensional random sampling



GP2X 4) , Do physics (and then write)

- Have: Paired database
- Want: Convolved database
- Need: Boost final state, Gaussian BES, rescaling, fold in Xsection and detector effects
- Energy Rescale $\rightarrow$ 2nd Order NR Method

$$\sum_{i=1}^{N} E_i' = \sum_{i=1}^{N} \sqrt{K^2 p_i^2 + m_i^2} = \sqrt{s'}$$

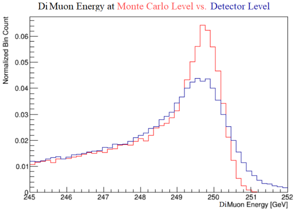
$$K_1 = \frac{\sqrt{s'} - \sum_{i=1}^{N} \frac{m_i^2}{E_i}}{\sqrt{s} - \sum_{i=1}^{N} \frac{m_i^2}{E_i}}$$

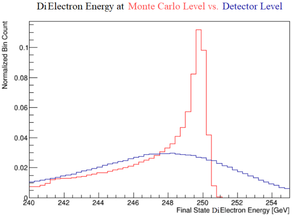
$$K_{n+1} = K_n - f(K_n)/f'(K_n)$$

$$f(K) = \sum_{i=1}^{N} \sqrt{K^2 p_i^2 + m_i^2} - \sqrt{s'} = 0$$

$$f'(K) = K \sum_{i=1}^{N} \frac{p_i^2}{\sqrt{K^2 p_i^2 + m_i^2}}$$

- Xsection $\rightarrow$ fit event generator $\sigma(\sqrt{s})$ and then reweight
- Detector $\rightarrow$ Smear using parametric model of ILD tracker or 18% ERes ECAL

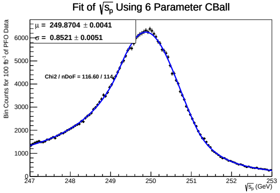




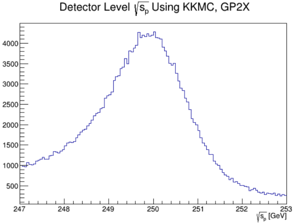
Center of Mass From DiMuons

- Why: DiMuons can be measured well in tracker, kinematically simple
- Issue: Beamstrahlung, ISR, FSR complicate things
- First Step: Derive solution that works in limit of massless 3rd body, $\mu\mu(\gamma)$
- Extensive work here [1] finds: $\sqrt{s_p} = E_{\mu\mu} + \sqrt{E_{\mu\mu}^2 - M_{\mu\mu}^2}$
- Where $\sqrt{s_p}$ is momentum based estimate of $\sqrt{s}$

Example from iLCSoft fitted using crystal ball function

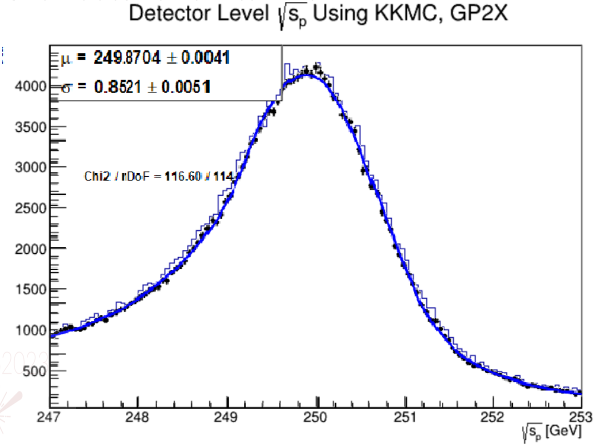


Or from GP2X ... Look similar!



Center of Mass From DiMuons

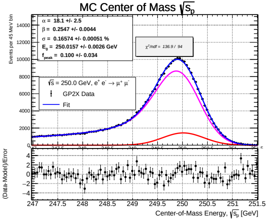
- Overlay them ... very similar
- Check that iLCSoft and GP2X agree in future
- Issue: Look at CBall fit mean , it is many σ off of 250!
- CBall doesn't fit mean well?



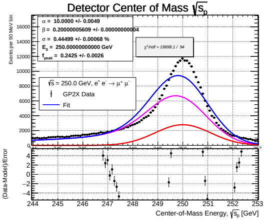
A Better Fit?

- Want: Fit that can separate (and fit) peak well
- How: Convolution Fits
- Today: Show beta convolved fit
- Following example of CIRCE [4] use beta distribution convolved with Gaussian:
- $F(x, \alpha, \beta, \sigma) = \int x^{\beta-1}(1-x)^{\alpha-1}G(x-1, \sigma)dx$
- With $x = \frac{\sqrt{s}}{E_0}$ and fraction of Gaussian, f_{peak}, have 5 parameters
- 1 less parameter than before!

Fit from GP2X $\sqrt{s_p}$ at MC level



At detector level ... (WIP)

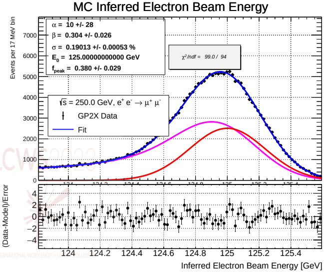


What about Beam Energies?

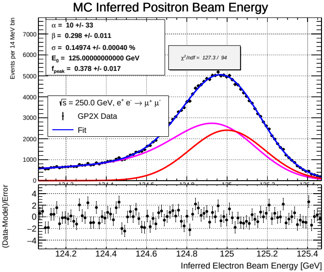
- Why: Monitor beam
- How: Derive relationship in limit of colinear photon
- Result:

$$E_i^- = \frac{1}{2}(E_{\mu\mu} + p^z_{\mu\mu}) \ ,$$
$$E_i^+ = \frac{1}{2}(E_{\mu\mu} - p^z_{\mu\mu})$$

- Can fit with fixed energy at design values

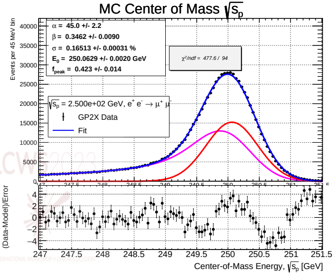


- χ^2 of both supports the fixed hypothesis
- σ also matches design BES for ILC250
- Potential issue: uncertainty on α

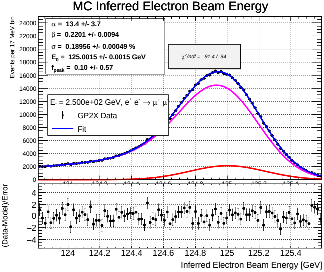


Fitting Bhabhas at MC Level

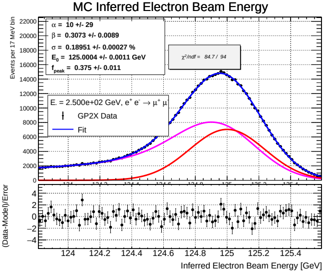
- Why: Xsection ...
  Bhabha ≈ 2200 pb while
  DiMuon ≈ 5 pb
- Issue: Bhabha prefers small angles, ECAL measured
- Use BHWIDE, WHIZARD event generators
- BHWIDE+GP2X $\sqrt{s_p}$



BHWIDE+GP2X Electron beam energy

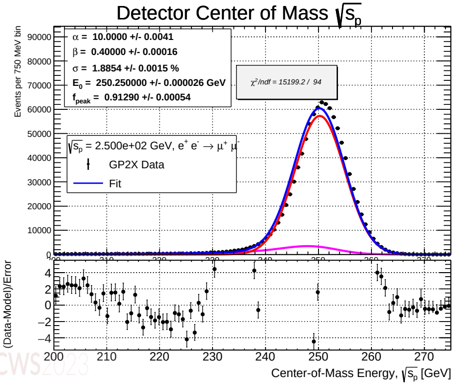


WHIZARD+GP2X Electron beam energy



Fitting Bhabhas at Detector Level

- As with DiMuons the detector level fitting is WIP
- WHIZARD+GP2X



- Need:
- Determine Bhabhas that can be measured well in tracker
- Incorporate detector into fitting

Fits Summarized in Table

Results using ILC250 and ILD at MC level and 1M Samples				
Metric	E_0 at 100 fb^{-1} (2 ab^{-1}) [GeV]	Fit, True σ [GeV]	XSection (pb)	Pol. (e^- , e^+)
KKMC $\sqrt{s_p}$	250.016 $\pm$ 0.003 (0.0007)	0.414 , 0.303	5.28 $\pm$ 0.05	No
KKMC E_i^-	124.988 $\pm$ 0.004 (0.0009)	0.237 , 0.238	—	—
KKMC E_i^+	124.982 $\pm$ 0.003 (0.0007)	0.192 , 0.188	—	—
BHWIDE $\sqrt{s_p}$	250.063 $\pm$ 0.0001 (0.00003)	0.413 , 0.303	2313 $\pm$ 15	No
BHWIDE E_i^-	125.002 $\pm$ 0.0001 (0.00002)	0.237 , 0.238	—	—
BHWIDE E_i^+	124.999 $\pm$ 0.0001 (0.00002)	0.188 , 0.188	—	—
WHIZARD $\mu\mu$ $\sqrt{s_p}$	250.093 $\pm$ 0.007 (0.002)	0.415 , 0.303	5.91 $\pm$ 0.04	(-0.8,+0.3)
WHIZARD $\mu\mu$ E_i^-	125.012 $\pm$ 0.001 (0.0003)	0.235 , 0.238	—	—
WHIZARD $\mu\mu$ E_i^+	124.998 $\pm$ 0.003 (0.0006)	0.188 , 0.188	—	—
WHIZARD ee $\sqrt{s_p}$	250.087 $\pm$ 0.0001 (0.00003)	0.407 , 0.303	2270 $\pm$ 20	(-0.8,+0.3)
WHIZARD ee E_i^-	125.001 $\pm$ 0.00007 (0.00001)	0.235, 0.238	—	—
WHIZARD ee E_i^+	124.999 $\pm$ 0.0001 (0.00004)	0.188 , 0.188	—	—

BES is fit well in all beam metrics

Spread in $\sqrt{s_p}$ does not match simple Gaussian spread of $E^+ + E^-$

Precision in E_0 maybe "too good" , need work on validation

Need work on precision calibration for tracker and ECAL

Future Work

- Integration with event generators
- Incorporate detector in fits
- Model ISR in fit so it can be monitored
- Investigate other $\sqrt{s}$
- Work on precision tracker and ECAL calibration

Acknowledgement

Thanks to Graham Wilson for advice and discussions.
Go to Graham's talk on Thursday!
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References

[1] Brendon Madison and Graham W. Wilson. "Center-of-mass energy determination using e+e → +() events at future e+e colliders". In: Snowmass 2021. Sept. 2022. arXiv: 2209.03281 [hep-ex].
[2] Brendon Madison. Brendonmadison/GP2X: Guinea pig to X , X being a difermion event generator . takes initial beam dynamics from Guineapig and uses them to boost the final state difermions according to their center of mass energies. url: https://github.com/BrendonMadison/GP2X.
[3] D. Schulte. "Beam-beam simulations with Guinea-Pig". In: eConf C980914 (1998). Ed. by Kwok Ko and Robert D. Ryne, pp. 127–131.
[4] Thorsten Ohl. "circe Version 1.02: beam spectra for simulating linear collider physics". In: Computer Physics Communications 101.3 (1997), pp. 269–288. issn: 0010-4655. doi: https://doi.org/10.1016/S0010-4655(96)00167-1. url: https://www.sciencedirect.com/science/article/pii/ S0010465596001671.