Measuring the tau polarisation at the ILC

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Motivation

The aim of this study

The reconstruction of tau spin orientation ("Polarimeter") in order to measure polarisation to investigate new physics.

Two tools are available at ILC to measure the chirality of such new interactions.

- At the ILC, forward-backward asymmetry $A_{FB} = \frac{3}{4} A_e \cdot A_f$ can be measured

  Thanks to ILC's polarised beams, $A_e$ can be measured $\Rightarrow A_f$ can be extracted from $A_{FB}$

- We can also directly measure $A_{\tau}$ by using tau polarisation $P(\tau)$

$$\frac{dP(\tau)}{d \cos \theta} = \frac{3}{8} A_{\tau}(1 + \cos^2 \theta) + \frac{3}{4} \left(\frac{A_e - P_e}{1 - A_e P_e}\right) \cos \theta$$
Polarimeter

Reconstruction of tau polarisation $P(\tau)$ depends on tau decay mode.

only look at $\tau \rightarrow \pi \nu$ (BR $\sim$ 10 %) in this talk

$\tau \rightarrow \rho \nu$ (BR $\sim$ 26 %)

Polarimeter vectors of $\tau \rightarrow \pi \nu$ in $\tau$ rest frame

$$h(\tau^\pm \rightarrow \pi^\pm \nu) = \frac{p_\nu}{|p_\nu|}$$

Polarimeter vectors of $\tau \rightarrow \rho \nu$ in $\tau$ rest frame

$$h(\tau^\pm \rightarrow \pi^\pm \pi^0 \nu) = 2 \left( q \cdot p_\nu \right) q - m_q^2 p_\nu$$

“Polarimeter”

The cosine of the angle this polarimeter vector makes to the tau flight direction

Where $q = p_{\pi^\pm} - p_{\pi^0}$

$p_\nu, p_{\pi^\pm}, p_{\pi^0}$ the 3-momenta of the neutrinos, charged pions, neutral pions
Previous study

Extract polarimeter without using neutrino information

"Approximate" polarimeters based only on the momenta of visible tau decay products

"Optimal" polarimeters including the neutrino component

In this talk: reconstruct neutrino momentum $\rightarrow$ optimal polarimeters

mean statistical error on tau polarisation

\[ E_{CM} = 500 \text{ GeV}, \ L = 1.6 \text{ ab}^{-1} \]

Do not Use neutrino

Use neutrino

0.40 %

0.30 %
Simulation setup

- ILD mc-2020 $e^+e^- \rightarrow \tau^+\tau^-$ signal event sample with 100% beam polarisations
- The decay of the polarised tau was done using TAUOLA.
- MC truth information was used.
\[ \tau \text{ reconstruction method} \]

Assume

- Two taus are produced along the beam line (interaction point has \( x = y = 0 \)),
- Two taus are back-to-back in x-y plane,
  - any ISR photons have negligible \( p_T \)
- Charged particle travels approximately in a straight line near IP.
**τ reconstruction method**

Assume

- Primary interaction occurs along the beam line (interaction point has $x = y = 0$),
- Two taus are back-to-back in x-y plane,
- Charged particle travels approximately in a straight line near IP.

- Two tau momenta lie in a plane containing z-axis, at some azimuthal angle $\phi$
The intersection between plane and trajectory: the decay points of $\tau$

For a plane with azimuthal angle $\phi$,
the intersection of trajectories with this plane can be calculated.
\( \tau \) reconstruction method

\( \tau - \tau \) production plane

then choice of \( z_{IP} \) gives direction of tau momenta

\( (\phi, z_{IP}) : \text{unknown} \)

How can we choose \( \phi, z_{IP} \) ?
We choose the values of $z$ and $\phi$ which result in neutrino masses closest to zero.

**τ reconstruction method**

- **Unknown**
  - neutrino 3-momentum $\times 2$
  - ISR momentum
  - $z_{IP}$

- **Constraints**
  - 4-momentum conservation
  - tau mass $\times 2$
  - Decay point on trajectory $\times 2$

For choice of $z_{IP}$, $\phi$
we can calculate tau 4-momenta $P_\tau$

the invariant mass of the missing (neutrino) momentum for each tau can be calculated

$$P_\nu = P_\tau - P_{vis}$$

assume 1 ISR photon collinear with beam

expected impact parameter resolution $\sim$ few $\mu$m

IP

tau decay length $\sim$ few mm

equipped trajectory

neutrino

Constraints

$\sigma_{d0}$

We choose the values of $z$ and $\phi$ which result in neutrino masses closest to zero.
\[ \tau \text{ reconstruction method} \]

We have tried another method

\[
\vec{p} : \text{unit vector} \\
\alpha : \text{real number}
\]

\[
(\phi, z_{IP}) \rightarrow (\alpha, z_{IP})
\]
We have tried another method

\[ \alpha \vec{p} \]

\[ \alpha_2 \vec{p} \]

\[ z_{IP} \]

\[ \vec{p} : \text{unit vector} \]

\[ \alpha : \text{real number} \]

\[ \sigma_{d_0} \]

\[ \text{tau decay length} \sim \text{few mm} \]

\[ \alpha_2 \text{ can be calculated by imposing back-to-back-ness in the x-y projection} \]
τ reconstruction method

Two methods to find solutions

(φ, z_{IP}) : unknown

We have combined them
Find solutions

We choose the values of \( z \) and \( \phi \) which result in neutrino masses closest to zero:

\[
|m_{\nu_1}^2|, \quad |m_{\nu_2}^2|, \quad \sum_i |m_{\nu_i}^2|
\]

Example event with 1 solution:

\[ \sum_i |m_{\nu_i}^2| \]

\( \phi \) [rad] \hspace{2cm} m_{\nu_1} \text{ [GeV]} \hspace{2cm} z \text{ [mm]}

\( \Delta \) : “the identified solutions”

Find local minima in \( \sum_i |m_{\nu_i}^2| \).
Find solutions

We choose the values of \( z \) and \( \phi \) which result in neutrino masses closest to zero.

Example event with 2 solutions: 

\[
| m_{\nu_1}^2 | \quad ... | m_{\nu_2}^2 | \quad \sum_i | m_{\nu_i}^2 |
\]

\( \phi \) [rad] \quad \Delta : “the identified solutions”

\( m_{\nu_1} \) [GeV] \quad find local minima in \( \sum_i | m_{\nu_i}^2 | \)

\( z \) [mm]
Impact parameter method efficiency is > 90% for events with $m_{\tau\tau} \sim 250$ GeV
Polarimeter using reconstructed $\nu$ is in reasonable agreement with MC one.
We have up to 20 possible solutions per event

Some entries per event

$\tau^\pm \rightarrow \pi^\pm \nu$

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Polarimeter distributions for $\tau^\pm \rightarrow \pi^\pm \nu$ decays in events with $m_{\tau\tau} > 240$ GeV

=> we cannot trust the statistical errors from simple fit

Use Jackknife method
The basic idea is to calculate the estimator (e.g. tau polarisation) by sequentially deleting a single event polarimeter from the sample. The estimator is recomputed until there are $n$ estimates for a sample size of $n$.

Variation of $n$ estimates gives

$$
\sigma_{\text{jackknife}} = \sqrt{\frac{n - 1}{n} \sum_{i=1}^{n} (\hat{P}_i - \hat{\mu})^2}
$$

arXiv:1606.00497
Jackknife method

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Extract tau polarisation from log likelihood fit
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Variation of \( n \) estimates gives

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extract tau polarisation from log likelihood fit

arXiv:1606.00497
Method Calibration

To check the bias, an artificial polarisation was created by changing the ratio of $N_R$ and $N_L$ to calculate $P_{fit}$

$$P_{in} = \frac{N_R - N_L}{N_R + N_L}$$

$N_R$: right-handed tau  
$N_L$: left-handed tau

At least 1 tau $\rightarrow \pi/\rho$

$m_{\tau\tau} > 240$ GeV

- Bias due to the presence of wrong solution was found
- There seems have no $\cos \theta_{\tau}$ dependence
Summary

- Full reconstruction of $e^+e^- \rightarrow \tau^+\tau^-$ using impact parameter was investigated.
- New method to find solutions was implemented and method efficiency was improved.
  
  For events with both $m_{\tau\tau} \sim 91$ GeV and $\sim 250$ GeV, new method efficiency is $> 90\%$
- Polarimeters were reconstructed in the $\tau \rightarrow \pi\nu$ and $\tau \rightarrow \rho\nu$ decay modes and reasonable agreement between MC truth polarimeter and the one from new method were found.
- Jackknife method was used to estimate tau polarisation errors.

Future plan

- Investigate search for new physics by using the tau polarisation.
w/o reset

w/reset
const int NN = 11;            // different polarisation

for ( int i = 0; i < NN; i++ ) {
    TString samp;
    samp += i;
    _htot[i] = new TH1F( "htot" + samp, "htot" + samp, 100, -1, 1 );
}

for ( int icos = 0; icos < ncos; icos++ ) {
    for ( int pwant = 0; pwant < NN; pwant++ ) {
        for ( int itau = 0; itau < 2; itau++ ) {
            if ( newmeth_polarimeter[0][itau] >= -999 ) {
                for ( int isol = 0; isol < newmeth_nsol; isol++ ) {
                    _htot[pwant]->Fill( newmeth_polarimeter[isol][itau], solweight );
                }
            }
        }
    }
} // end itau loop

} // end loop icos

After FSR

on-shell Z is produced

Significant FSR

\( m_{\tau\tau} < m_Z \)
Method Calibration

MC linked PFO: which MC particles produced the hits included in this reconstructed particle.

At least 1 tau $\rightarrow \pi/\rho$

$m_{\tau\tau} > 240$ GeV

Artificial input polarisation

$N = 5000$

PFO-MC

Errors from Jackknife method