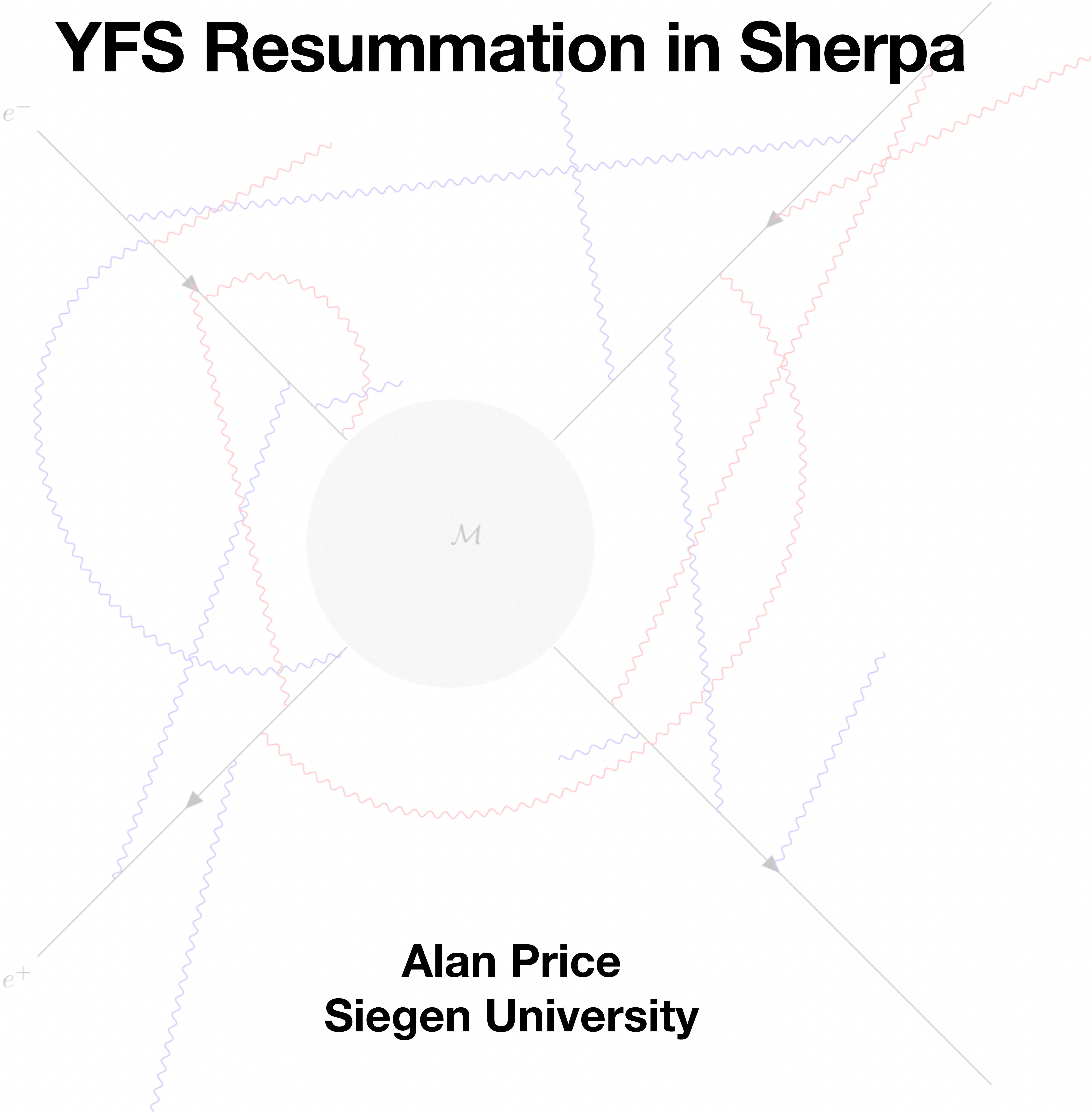
# YFS Resummation in Sherpa



**Alan Price**
**Siegen University**

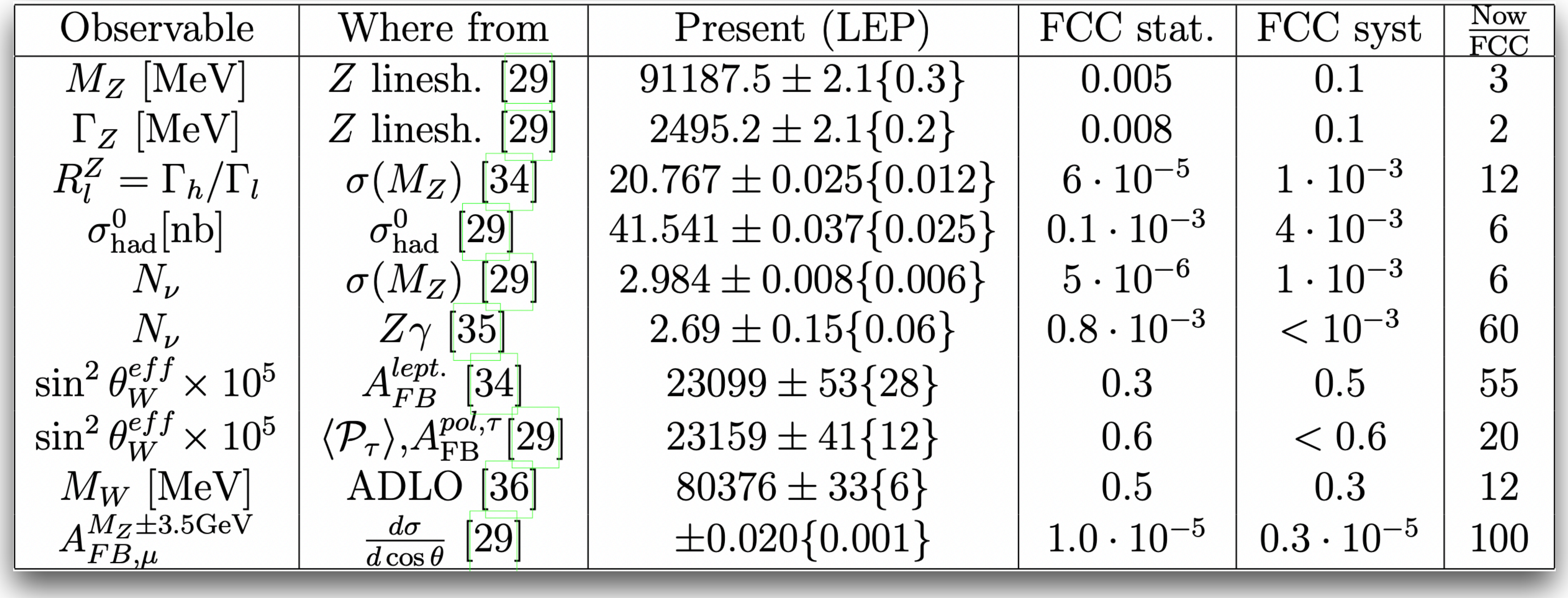
MARIE CURIE ACTIONS

# Overview

- Motivation
- YFS Brief Recap
- Automation of Perturbative Corrections
- Implementation in SHERPA
- Future Plans

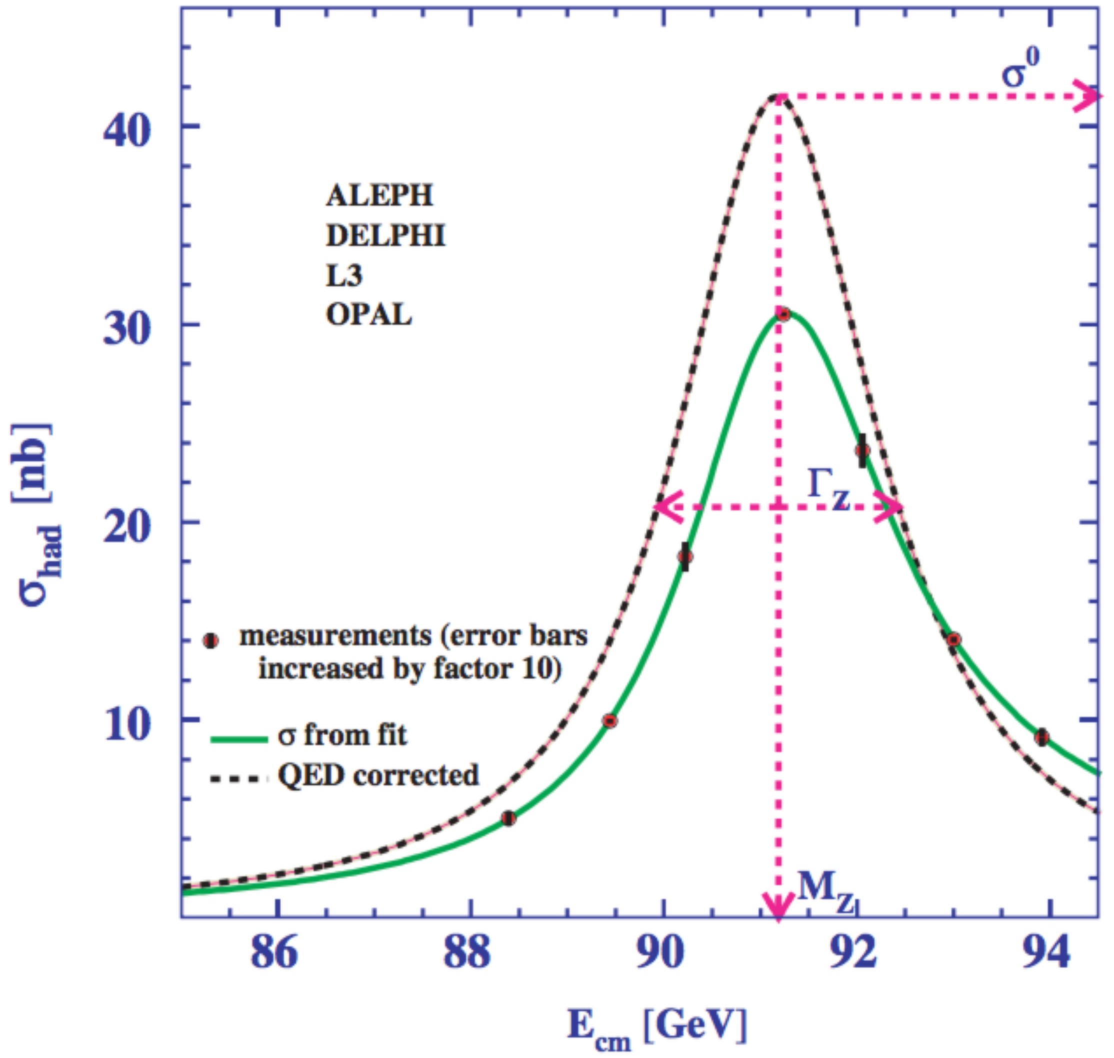
# Motivation

- Factor 5-200 reduction of experimental error
- QED effects of 0.1% could be included in LEP error budget
- Future colliders will deliver full LEP Statistics in minutes



| Observable | Where from | Present (LEP) | FCC stat. | FCC syst | $\frac{\text{Now}}{\text{FCC}}$ |
|---|---|---|---|---|---|
| $M_Z$ [MeV] | $Z$ linesh. [29] | $91187.5 \pm 2.1\{0.3\}$ | 0.005 | 0.1 | 3 |
| $\Gamma_Z$ [MeV] | $Z$ linesh. [29] | $2495.2 \pm 2.1\{0.2\}$ | 0.008 | 0.1 | 2 |
| $R_l^Z = \Gamma_h/\Gamma_l$ | $\sigma(M_Z)$ [34] | $20.767 \pm 0.025\{0.012\}$ | $6 \cdot 10^{-5}$ | $1 \cdot 10^{-3}$ | 12 |
| $\sigma^0_{\mathrm{had}}$[nb] | $\sigma^0_{\mathrm{had}}$ [29] | $41.541 \pm 0.037\{0.025\}$ | $0.1 \cdot 10^{-3}$ | $4 \cdot 10^{-3}$ | 6 |
| $N_\nu$ | $\sigma(M_Z)$ [29] | $2.984 \pm 0.008\{0.006\}$ | $5 \cdot 10^{-6}$ | $1 \cdot 10^{-3}$ | 6 |
| $N_\nu$ | $Z\gamma$ [35] | $2.69 \pm 0.15\{0.06\}$ | $0.8 \cdot 10^{-3}$ | $< 10^{-3}$ | 60 |
| $\sin^2\theta_W^{eff} \times 10^5$ | $A_{FB}^{lept.}$ [34] | $23099 \pm 53\{28\}$ | 0.3 | 0.5 | 55 |
| $\sin^2\theta_W^{eff} \times 10^5$ | $\langle\mathcal{P}_\tau\rangle, A_{\mathrm{FB}}^{pol,\tau}$ [29] | $23159 \pm 41\{12\}$ | 0.6 | $< 0.6$ | 20 |
| $M_W$ [MeV] | ADLO [36] | $80376 \pm 33\{6\}$ | 0.5 | 0.3 | 12 |
| $A_{FB,\mu}^{M_Z \pm 3.5\mathrm{GeV}}$ | $\frac{d\sigma}{d\cos\theta}$ [29] | $\pm 0.020\{0.001\}$ | $1.0 \cdot 10^{-5}$ | $0.3 \cdot 10^{-5}$ | 100 |

*S.Jadach and M.Skrzypek, Eur. Phys. J.C 79, no.9, 756 (2019)*



$$d\sigma(L,\hat{L}) = \alpha^k \sum_n^{\infty} \alpha^n \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \hat{\sigma}_{n,i,j} L^i \hat{L}^j$$

$$\hat{L} = \log\left(\frac{Q^2}{E_\gamma^2}\right) \qquad L = \log\left(\frac{Q^2}{m_e^2}\right)$$

**Yennie-Frautschi-Surra (YFS) resums all soft Logs**

# YFS a Brief Review

- Yennie-Frautschi-Suura allows us to resum **soft logs to infinite order**
- Provides a systematic method to include **perturbative corrections**
- The multi-photon phasespace is treated exactly ⇒ Explicit Photons

$$d\sigma = \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!} d\Phi_Q \left[ \prod_{i=1}^{n_\gamma} d\Phi_i^\gamma \, S(k_i) \, \Theta(k_i, \Omega) \right] \left( \tilde{\beta}_0 + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1(k_j)}{S(k_j)} + \sum_{\substack{j,k=1 \\ j<k}}^{n_\gamma} \frac{\tilde{\beta}_2(k_j, k_k)}{S(k_j)S(k_k)} + \cdots \right),$$

# YFS a Brief Review

$$Y(\Omega) = 2\alpha \sum_{i<j} \left( \mathscr{R}e\, B(p_i, p_j) + \tilde{B}(p_i, p_j, \Omega) \right)$$

$$S(k) = \sum_{i,j} \frac{\alpha}{4\pi^2} Z_i Z_j \theta_i \theta_j \left( \frac{p_i}{p_i \cdot k} - \frac{p_j}{p_j \cdot k} \right)^2$$

$$B(p_i, p_j) = -\frac{i}{8\pi^3} Z_i Z_j \theta_i \theta_j \int \frac{d^4k}{k^2} \left( \frac{2p_i\theta_i - k}{k^2 - 2(k \cdot p_i)\theta_i} + \frac{2p_j\theta_j + k}{k^2 + 2(k \cdot p_j)\theta_j} \right)^2$$

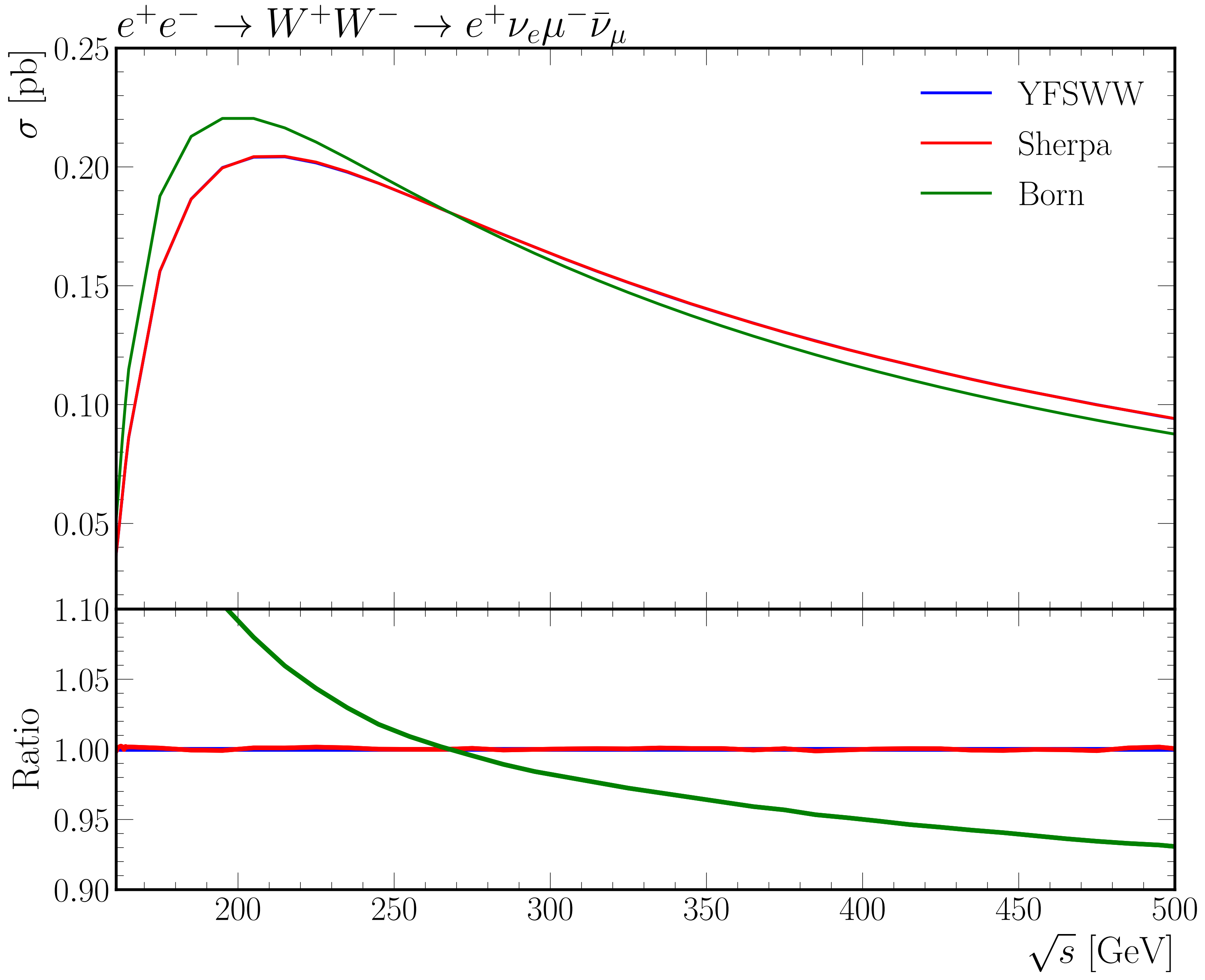
$$\tilde{B}(p_i, p_j, \Omega) = \frac{1}{4\pi^2} Z_i Z_j \theta_i \theta_j \int d^4k\, \delta(k^2) \big(1 - \Theta(k, \Omega)\big) \left( \frac{p_i}{(p_i \cdot k)} - \frac{p_j}{(p_j \cdot k)} \right)^2$$

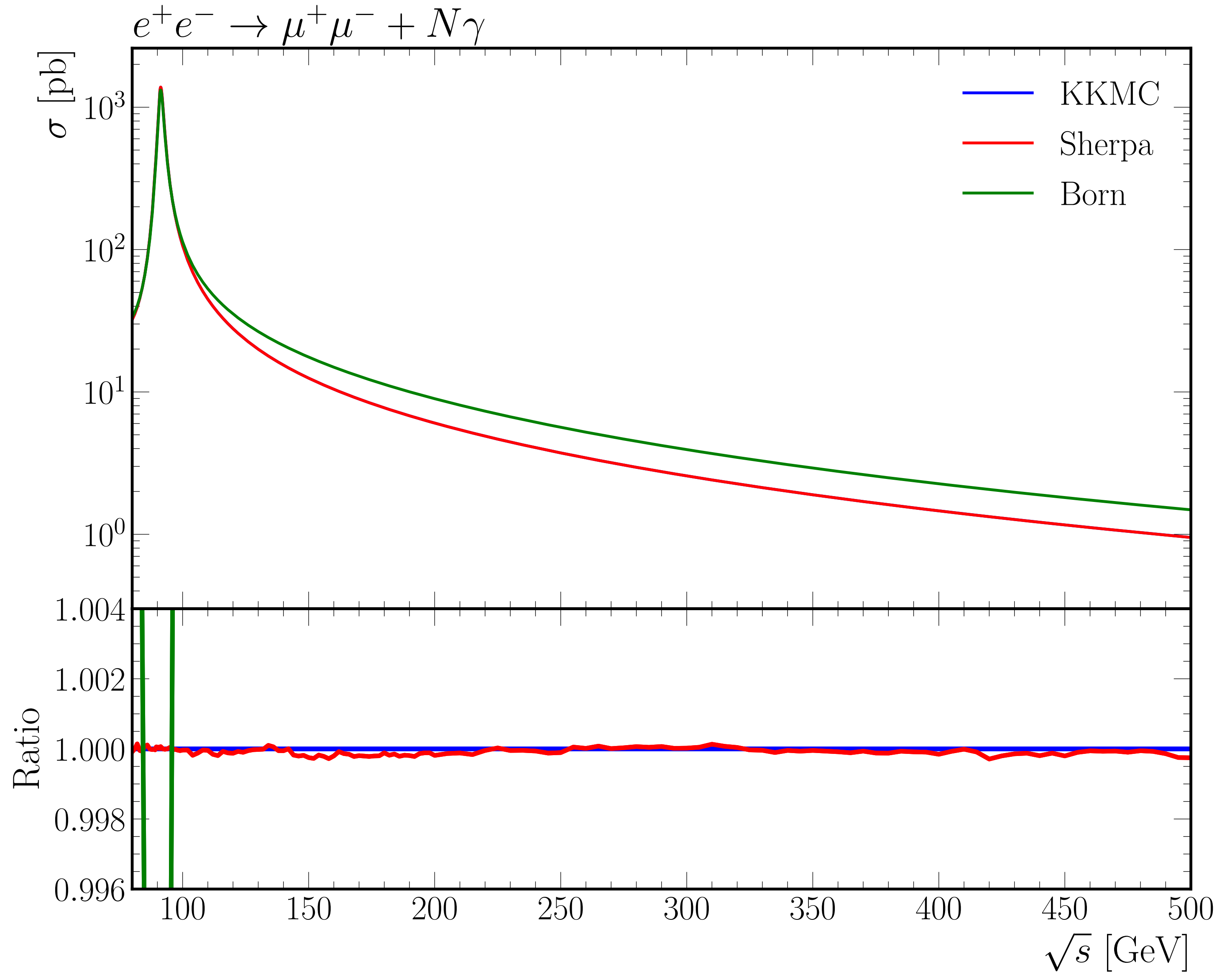
$$d\sigma = \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!} d\Phi_Q \left[ \prod_{i=1}^{n_\gamma} d\Phi_i^\gamma S(k_i) \Theta(k_i, \Omega) \right] \left( \tilde{\beta}_0 + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1(k_j)}{S(k_j)} + \sum_{\substack{j,k=1 \\ j<k}}^{n_\gamma} \frac{\tilde{\beta}_2(k_j, k_k)}{S(k_j)S(k_k)} + \cdots \right),$$

IR finite residues,  will return to later

# YFS Validation

- Detailed and dedicated checks against all current codes
  - KKMC $e^+e^- \rightarrow f\bar{f}$ Comput.Phys.Commun. 130 (2000) 260-325
  - YFSWW $e^+e^- \rightarrow W^+W^-$ Comput.Phys.Commun. 140 (2001) 475-512
- KKMC C++ can be interfaced with Sherpa ⇒ Point-by-Point comparison
- More detail see [SciPost Phys. 13 (2022) 2, 026]





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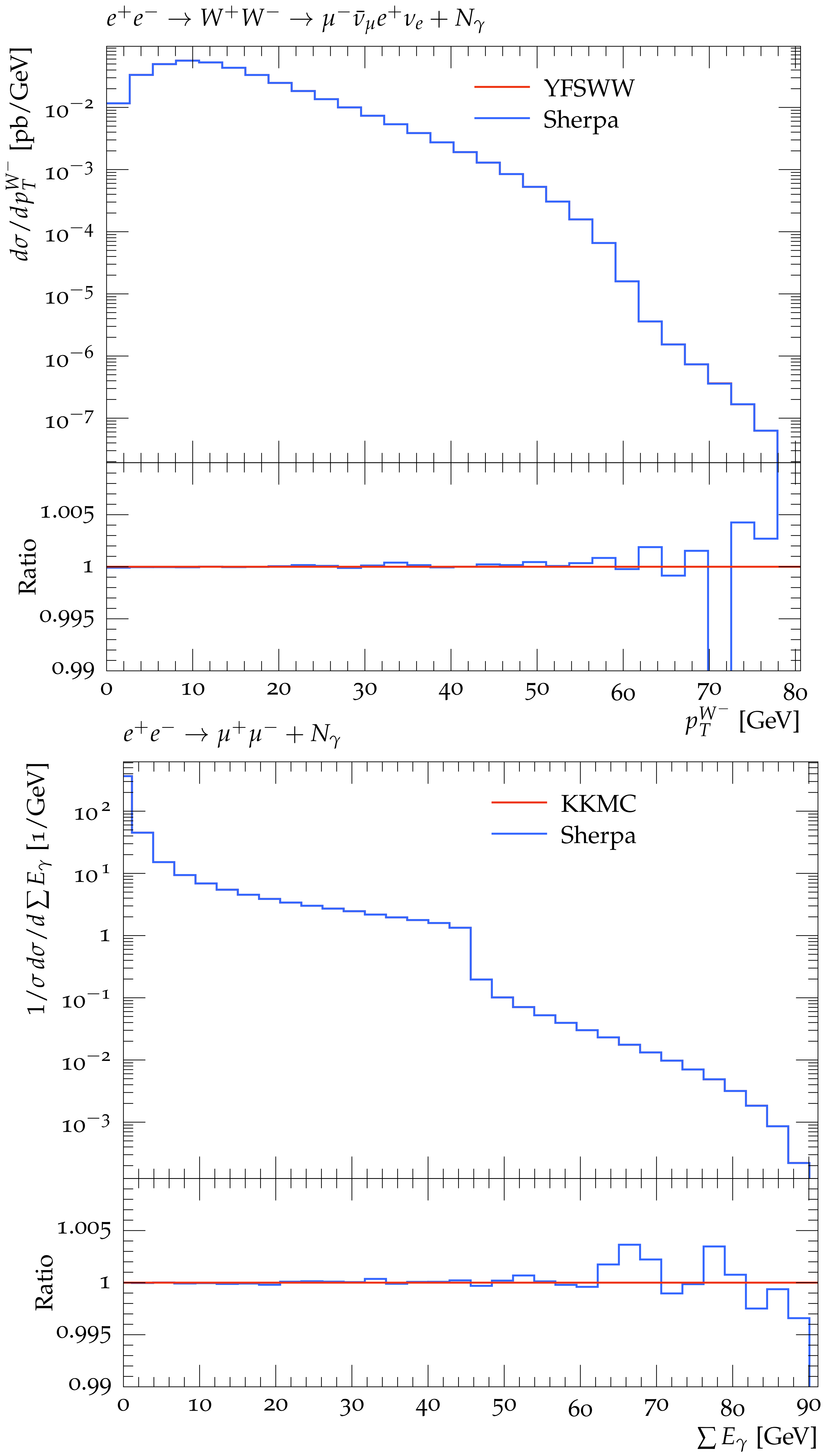
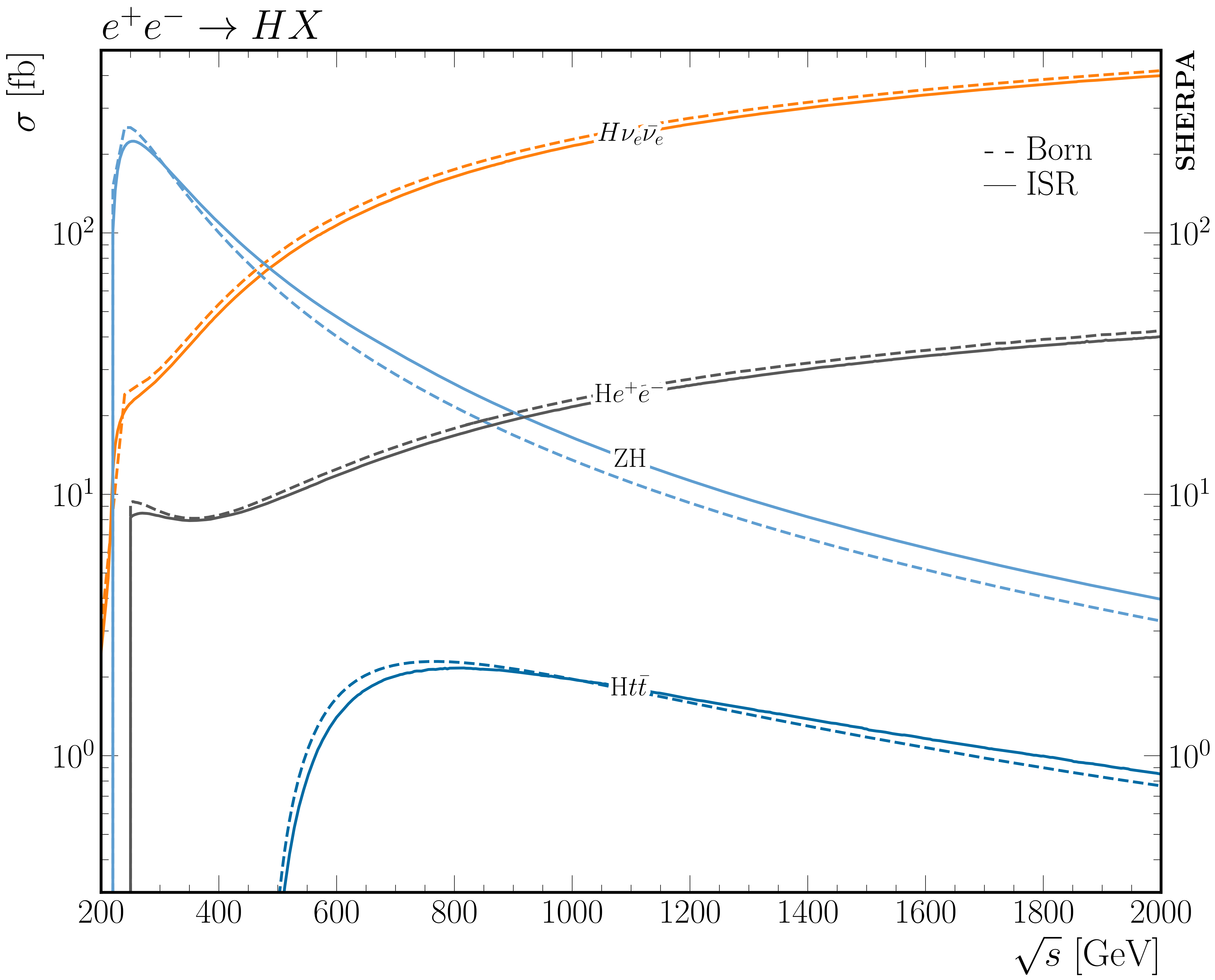


# YFS Caveats

- General Rule of thumb: If Sherpa can do the born e+e- process, we can also apply YFS.
- **All leptons must be massive**
- Should not be applied to colored partons
- Automated for both initial and final states
- Photons will appear in the event record
  - Can be disabled



# Perturbative Corrections

YFS@NLO

$$\tilde{\beta}_0 + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1(k_j)}{S(k_j)} + \sum_{\substack{j,k=1\\ j<k}}^{n_\gamma} \frac{\tilde{\beta}_2(k_j, k_k)}{S(k_j)S(k_k)} + \cdots ,$$

**YFS provides a method to matching the resummation to N….NLO calculations**

# Perturbative Corrections

$$\left(\frac{1}{2(2\pi)^3}\right)^{n_\gamma}\left|\sum_{\bar{n}_\gamma=0}^{\infty} M_{n_\gamma}^{\bar{n}_\gamma+\frac{1}{2}n_\gamma}\right|^2 = \tilde{\beta}_0\prod_{i=1}^{n_\gamma}\left[\tilde{S}(k_i)\right] + \sum_{i=1}^{n_\gamma}\left[\frac{\tilde{\beta}_1(k_i)}{\tilde{S}(k_i)}\right]\prod_{j=1}^{n_\gamma}\left[\tilde{S}(k_j)\right] + \sum_{\substack{j,k=1\\ j<k}}^{n_\gamma}\left[\frac{\tilde{\beta}_2(k_i,k_j)}{\tilde{S}(k_i)\tilde{S}(k_j)}\right]\prod_{l=1}^{n_\gamma}\left[\tilde{S}(k_l)\right]$$

Notation: The IR finite correction for

$n_\gamma$ and $\bar{n}_\gamma$ is given as $\tilde{\beta}_{n_\gamma}^{\bar{n}_\gamma+n_\gamma}$

$$\tilde{\beta}_{n_\gamma} = \sum_{\bar{n}_\gamma=0}^{\infty}\tilde{\beta}_{n_\gamma}^{\bar{n}_\gamma+n_\gamma}$$

Each $\tilde{\beta}$ is by itself **IR finite**

IR divergences **are not** canceled à la Bloch-Nordsiek! Loop calculators take note

# One-Loop Corrections

$$\tilde{\beta}^1_0\left(\Phi_n\right) = \mathscr{V}\left(\Phi_n\right) - \sum_{ij}\mathscr{D}_{ij}\left(\Phi_{ij}\right)$$

- ❖Full One Loop EW contribution
    - ❖Contains IR divergent terms
- ❖Need a loop generator that can include all lepton masses!
- ❖Currently only Recola can provide this
    - ❖Bonus: Allows Massive Reg
- ❖Provided to YFS using Sherpa interface
- ❖All or nothing. Cannot separate ISR/FSR

- ❖ One Loop Subtraction term calculated using Mass Reg
- ❖ Fully automated within YFS module
- ❖ Constructed from <u>all dipoles</u>
- ❖ Really should be limited to leptonic final states only
    - ❖ Works for massive quarks but should not be combined with QCD resummation

# One Loop Example

IR Cancellation in action

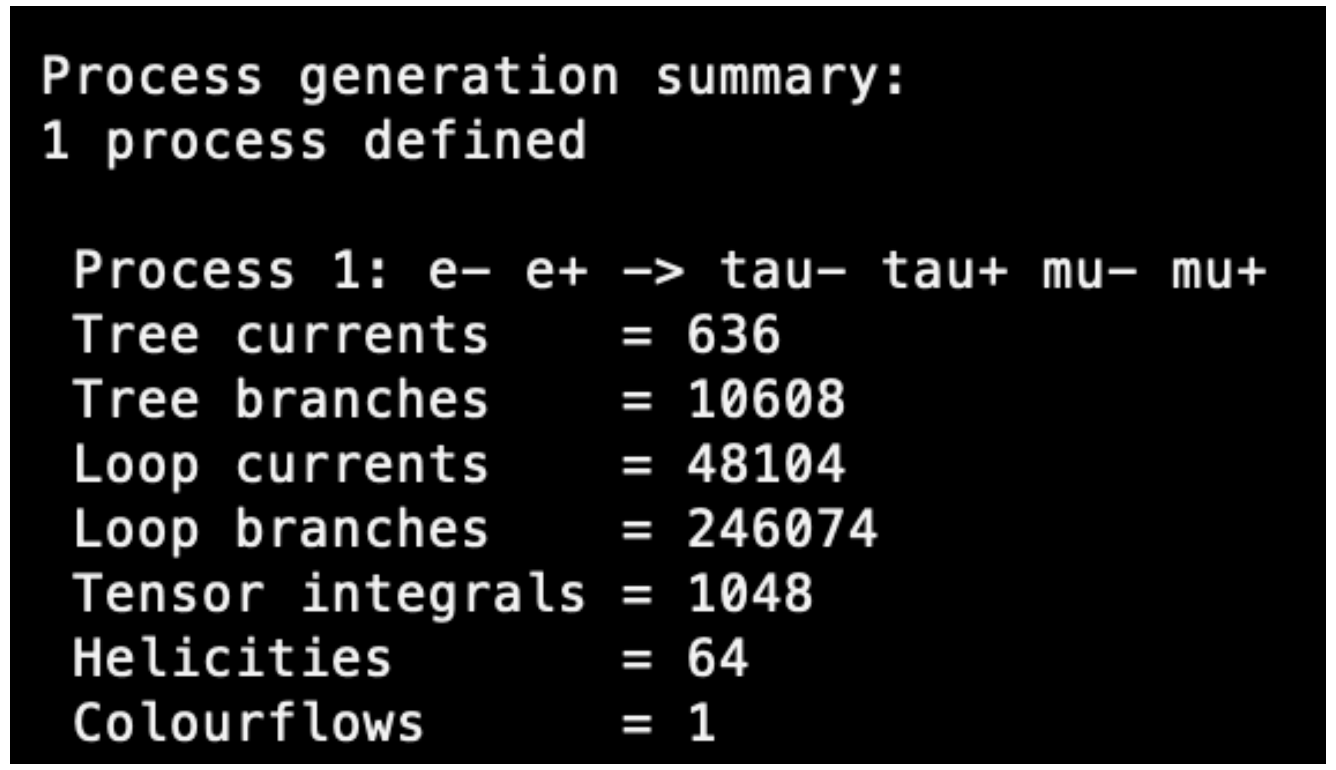
**$2 \to 4$ Process**

15 dipoles:

1 Initial

6 Final

8 Initial-Final



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Process generation summary:
1 process defined

 Process 1: e- e+ -> tau- tau+ mu- mu+
 Tree currents     = 636
 Tree branches     = 10608
 Loop currents     = 48104
 Loop branches     = 246074
 Tensor integrals = 1048
 Helicities        = 64
 Colourflows       = 1
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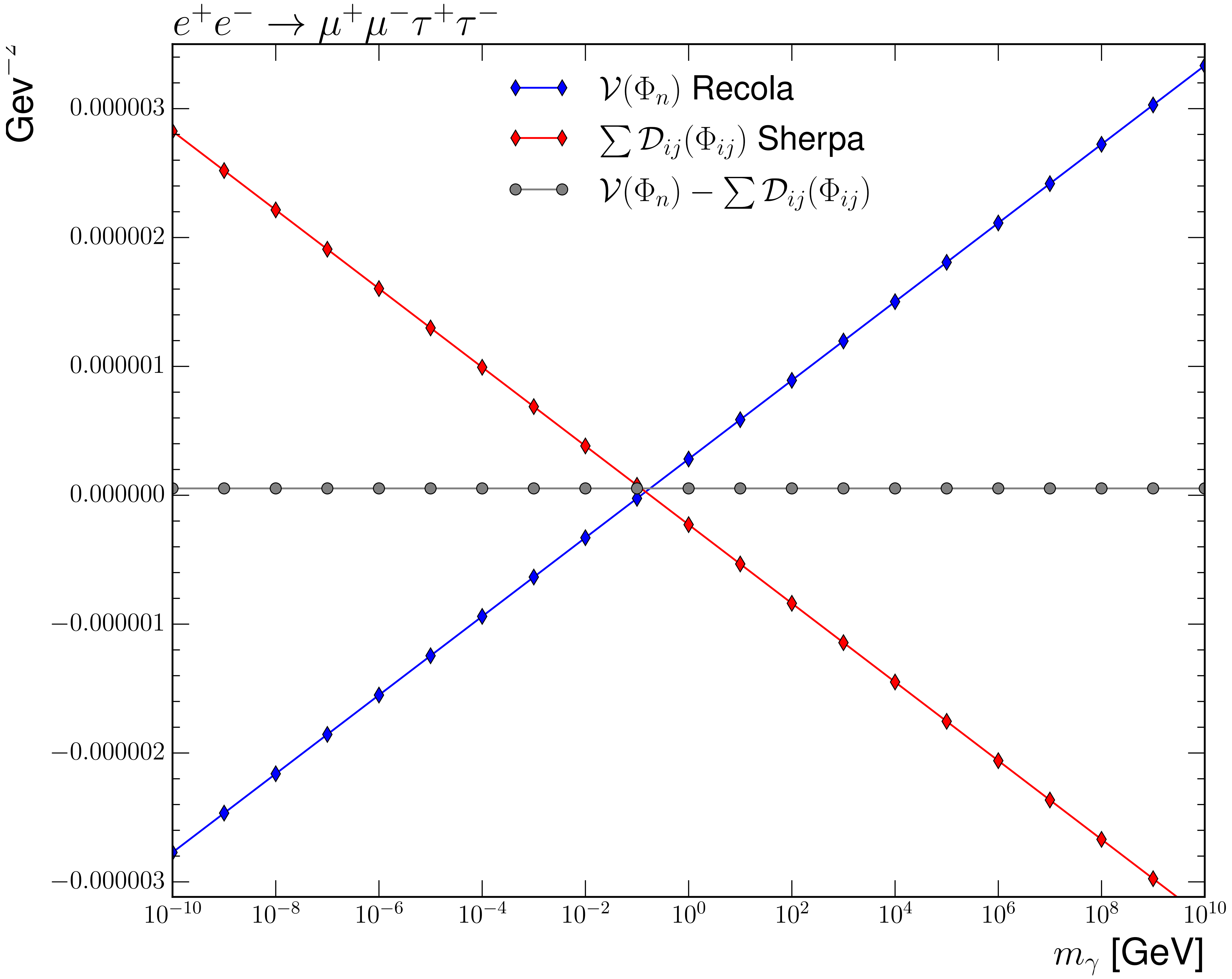


$$\log(m_\gamma^2) \to \frac{\Gamma(1+\epsilon)}{\epsilon}(4\pi\mu^2)^\epsilon$$

# Real Corrections

$$\tilde{\beta}_1^1\left(\Phi_{n+1}\right) = \mathscr{R}\left(\Phi_{n+1}\right) - \tilde{\beta}_0^0\left(\Phi_n\right)\sum_{ij}\tilde{S}_{ij}(k)$$

❖Real photon correction to born process

❖In Sherpa, can be taken from AMEGIC or COMIX ME generators

❖ Subtraction term calculate from the eikonals of all dipoles

❖ Automated within YFS

# Real-Virtual Corrections

$$\tilde{\beta}_1^2\left(\Phi_{n+1}\right) = \mathscr{R}\mathscr{V}\left(\Phi_{n+1};k\right) - \tilde{\beta}_0^1\left(\Phi_n\right)\sum_{ij}\tilde{S}_{ij}(k)$$

- ❖Real photon correction to born process
- ❖Calculated with Recola

- ❖ Subtraction term calculate from the eikonals of all dipoles
- ❖ Automated within YFS

# Real-Real Corrections

$$\tilde{\beta}_2^2\left(\Phi_{n+2}\right) = \mathscr{R}\mathscr{R}\left(\Phi_{n+2}\right) - \sum_{ij} \tilde{\beta}_1^1(\Phi_{n+1}; k_1)\tilde{S}_{ij}(k_2)$$

$$- \sum_{ij} \tilde{\beta}_1^1(\Phi_{n+1}; k_2)\tilde{S}(k_1)$$

$$-\tilde{\beta}_0^0(\Phi_n) \sum_{ij} \tilde{S}_{ij}(k_1)\tilde{S}_{ij}(k_2)$$

- ❖Real-Real corrections are generated with SHERPA’s ME generators
- ❖Subtraction, while complicated, also automated

# Virtual-Virtual Corrections

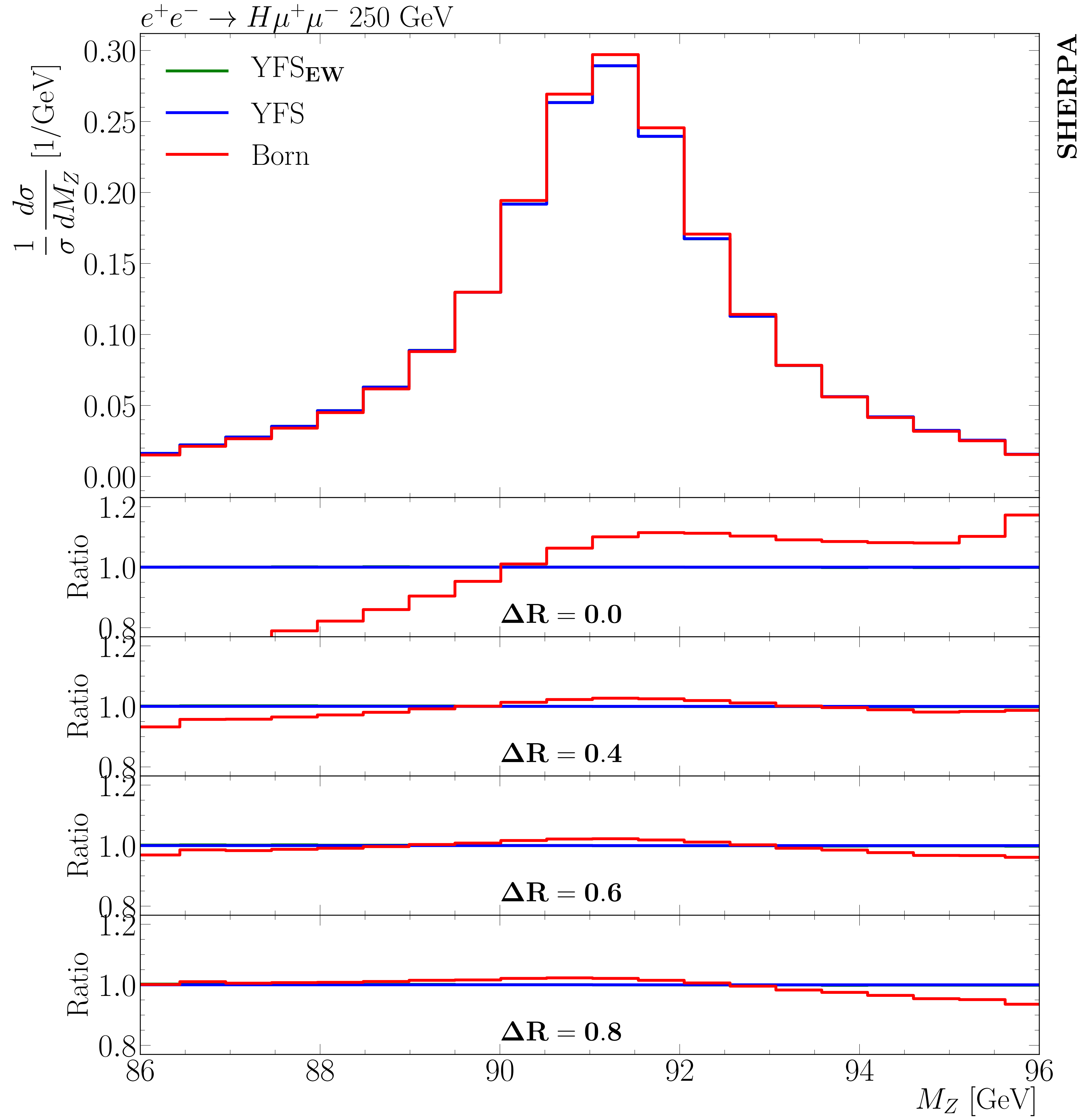
$$\tilde{\beta}_0^2\left(\Phi_n\right) = \mathscr{V}\mathscr{V}\left(\Phi_n\right) - \sum_{ij} \mathscr{D}_{ij}\left(\Phi_{ij}\right)$$

- ❖Virtual-Virtual corrections limiting factor
- ❖Subtraction, while complicated, also automated

# HZ at NLO

| $e^+e^- \to$ | Scheme | LO | YFS | $\mathrm{YFS}_{\mathbf{EW}}$ | $\delta_{\mathbf{EW}}$ |
|---|---|---|---|---|---|
| $HZ$ | $G_\mu$ | 240.280(2) | 213.80(6) | 207.48(6) | -13.65% |
| | $\alpha(M_Z^2)$ | 253.002(2) | 223.29(7) | 202.98(6) | -19.77% |
| | $\delta_{G_\mu}^{\alpha(M_Z^2)}$ | -5.03% | -4.25% | 2.22% | |
| $H\mu^+\mu^-$ | $G_\mu$ | 7.8554(4) | 6.911(2) | 6.666(2) | -15.13% |
| | $\alpha(M_Z^2)$ | 8.4875(5) | 7.401(3) | 6.444(2) | -24.07% |
| | $\delta_{G_\mu}^{\alpha(M_Z^2)}$ | -7.45% | -6.62% | 3.45% | |
| $H\tau^+\tau^-$ | $G_\mu$ | 7.8376(5) | 6.933(2) | 6.696(2) | -14.56% |
| | $\alpha(M_Z^2)$ | 8.4682(5) | 7.429(3) | 6.485(2) | -23.41% |
| | $\delta_{G_\mu}^{\alpha(M_Z^2)}$ | -7.45% | -6.67% | 3.26% | |
| $H\nu_\mu\bar{\nu}_\mu$ | $G_\mu$ | 15.5300(1) | 13.808(4) | 13.501(5) | -13.06% |
| | $\alpha(M_Z^2)$ | 16.7796(7) | 14.804(5) | 13.132(4) | -21.74% |
| | $\delta_{G_\mu}^{\alpha(M_Z^2)}$ | -7.45% | -6.73% | 2.81% | |
| $H\nu_\tau\bar{\nu}_\tau$ | $G_\mu$ | 15.5300(1) | 13.808(4) | 13.501(5) | -13.06% |
| | $\alpha(M_Z^2)$ | 16.7796(7) | 14.804(5) | 13.132(4) | -21.74% |
| | $\delta_{G_\mu}^{\alpha(M_Z^2)}$ | -7.45% | -6.73% | 2.81% | |

Total cross-section for leptonically decaying ZH at 250 GeV. The results are quoted in fb



# NNLO Corrections with GRIFFIN

GRIFFIN: A C++ library for EW radiative corrections [2211.16272](https://arxiv.org/abs/2211.16272)

Developed by A. Freitas and L.Chen

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           Lisong Chen and Ayres Freitas
          https://arxiv.org/abs/2211.16272
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GRIFFIN=**G**auge-**R**esonance-**I**n-**F**our-**F**ermion-**IN**teraction

- ❖ Modern version of LEP era EW tools such as DIZET (used by KKMC)
- ❖ Only for $f\bar{f} \rightarrow f'\bar{f}'$
- ❖NNLO accurate a Z pole
- ❖ Modular nature simple to interface to Sherpa

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Framework for $f\bar{f} \to Z^*/\gamma* \to f'\bar{f}'$:

- Laurent expansion about Z-pole + regular matrix element off-resonance

$$M_{ij} = M_{ij}^{\text{exp},s_0} + M_{ij}^{\text{noexp}} - M_{ij}^{\text{exp},M_Z^2},$$

@NLO

avoid double counting

$$M_{ij}^{\text{exp},s_0} = \frac{R_{ij}}{s - s_0} + S_{ij} + (s - s_0)S'_{ij} + \ldots \qquad s_0 \equiv M_Z^2 - iM_Z\Gamma_Z$$

@NNLO @NLO

Stuart '91; Veltman '94

From **A.Freitas**

How does GRIFFIN Treat IR divergent terms?

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avoid double counting

$$M_{ij}^{\text{exp},s_0} = \frac{R_{ij}}{s - s_0} + S_{ij} + (s - s_0)S'_{ij} + \ldots \qquad s_0 \equiv M_Z^2 - iM_Z\Gamma_Z$$

@NNLO @NLO

Stuart '91; Veltman '94

From **A.Freitas**

With YFS inspired Subtraction!

$$\gamma\gamma\ \text{box:} \qquad B_{\text{VV}(1)} = B^{\text{tot}}_{\text{VV}(1)} - S^{(0)}_{\text{VV}}\,\frac{\alpha}{\pi} Q_e Q_f\, f_{\text{IR}}(m_\gamma, t, u),$$

$$\gamma Z\ \text{box:} \qquad B_{\gamma\text{Z},ij(1)} = B^{\text{tot}}_{\gamma\text{Z},ij(1)} - \frac{R^{(0)}_{ij}}{s - s_0}\,\frac{\alpha}{\pi} Q_e Q_f\,[f_{\text{IR}}(m_\gamma, t, u) + \delta_G(s, t, u)],$$

$$f_{\text{IR}}(m_\gamma, t, u) = \ln\!\left(\frac{1 - c_\theta}{1 + c_\theta}\right)\left[\ln\!\left(\frac{2m_\gamma^2}{s\sqrt{1 - c_\theta^2}}\right) + \frac{1}{2}\right],$$

$$\delta_G(s, t, u) = -2\ln\!\left(\frac{1 - c_\theta}{1 + c_\theta}\right)\,\ln\!\left(\frac{s_0 - s}{s_0}\right).$$

# NNLO Corrections with GRIFFIN

GRIFFIN: A C++ library for EW radiative corrections 2211.16272

Developed by A. Freitas and L.Chen

Test Process $e^+e^- \rightarrow \mu^+\mu^-$ at 91.2GeV

| Born | YFS | YFS+Recola | YFS+GRIFFIN |
|---|---|---|---|
| 2114.5 pb | 1463.09 pb | 1494.7(8) pb | 1497.5(7) pb |

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Ongoing validation effort

GRIFFIN=**G**auge-**R**esonance-**I**n-**F**our-**F**ermion-**IN**teraction

Griffin Integration time: **30s** with 8 cores

Recola Integration time: **4mins** with 8 cores

# Sherpa for Linear Colliders

## Beam Polarization

- Historically has always been present in Sherpa
  - Need to be resurrected
- Validated against Madgraph with multiple processes
- Automatically included in YFS
- Sophisticated treatment also available for intermediate particles **Hoppe, Siegert, Schönherr**

| $P_{e^-}, P_{e^+}$ | **0,0** | **-0.8,03** |
|---|---|---|
| Madgraph | 0.206 pb | 0.03 pb |
| Sherpa | 0.206 pb | 0.03 pb |

$e^+e^- \to \mu^+\mu^- jj$ at 181 GeV

## Beamstrahlung

- Implementation of simple double Gaussian BES
- More sophisticated spectra handle under developments
  - Historically present

# Conclusion

- YFS ISR that can be applied to **any** $e^+e^-$ process
- For purely leptonic processes we have a new NLO framework
  - Virtual EW and real corrections fully automated and matched!
  - Approaching NNLO Precision
- Sherpa committed to being a $e^+e^-$ generator: One dedicated postdoc
  - Dedicate Sherpa3 lepton release coming soon
    - All of this talk and more