Theoretically Robust Cross Sections for Neutrino Oscillation Experiments

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Outline

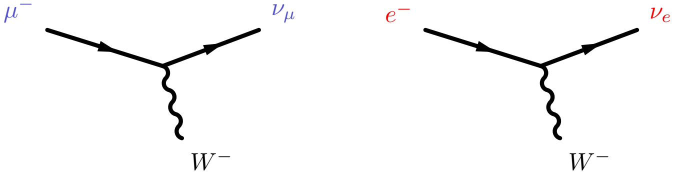
- Neutrino Oscillation Introduction
- Neutrino Cross Sections
- Quasielastic Scattering from Experiment
  - Constraints from Deuterium Scattering
- LQCD Introduction
- LQCD Survey of $F_A(Q^2)$
  - Summary of $F_A(Q^2)$ Calculations
  - T2K/DUNE Implications
- Future Prospects

Note: all references in online slides are hyperlinked

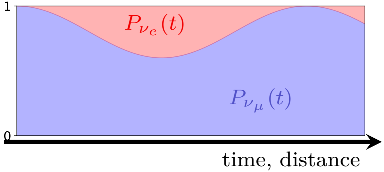
Neutrino Oscillation

Neutrino Oscillation (in a slide)

Neutrino flavor defined by charged lepton produced under weak interaction

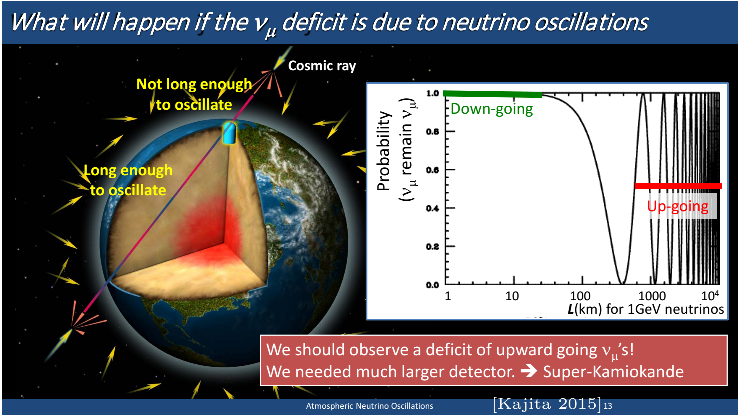


ν flavor changes spontaneously during near light-speed propagation: $\nu_\mu \rightarrow \nu_e$



Want to understand the mechanics governing this oscillation

Discovery of Neutrino Oscillation



What will happen if the ν_μ deficit is due to neutrino oscillations

We should observe a deficit of upward going ν_μ's!
We needed much larger detector. → Super-Kamiokande

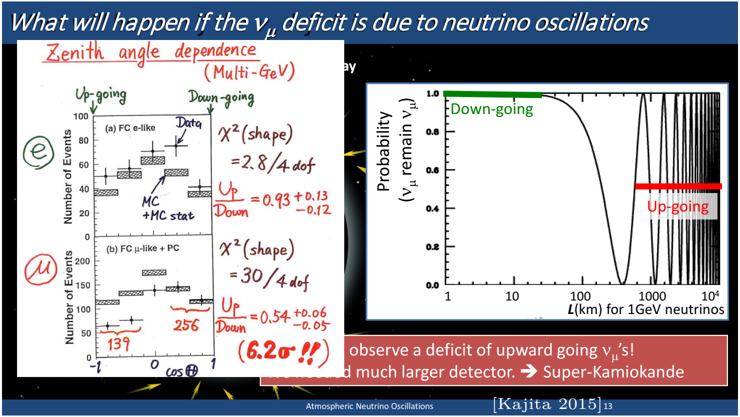
Atmospheric Neutrino Oscillations 13

[Kajita 2015]

Difference between upward vs downward ν at Super-Kamiokande (1998)
$\Longrightarrow$ Neutrinos have mass and oscillate!

2015 Nobel prize to Arthur McDonald, Takaaki Kajita

Discovery of Neutrino Oscillation



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Atmospheric Neutrino Oscillations

[Kajita 2015] 13

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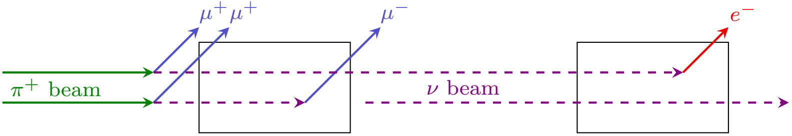
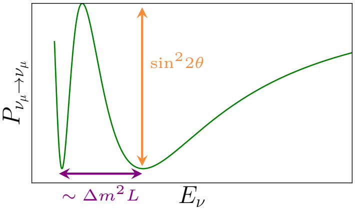
Neutrino Oscillation (with math)



$$\underbrace{|\nu_\ell\rangle}_{\text{flavor eigenstate}} = \sum_i U^*_{\ell i} \underbrace{|\nu_i\rangle}_{\textit{mass eigenstate}} \qquad |\nu_i\rangle \to e^{-iE_i t}|\nu_i\rangle$$

2 flavor model:

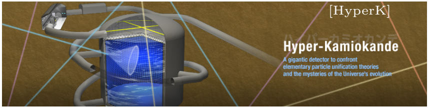
$$P_{\nu_\mu\to\nu_e} = \sin^2 2\theta \sin^2\left(1.27\frac{\Delta m^2[\text{eV}^2]\,L[\text{km}]}{E_\nu[\text{GeV}]}\right)$$



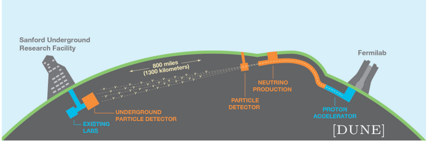
$$\left.\begin{array}{l}\textit{Mass eigenstates} \text{ – propagation} \\ \text{flavor eigenstates – interaction}\end{array}\right\} \text{Not the same}$$

Oscillation probability is function of L/E_ν at fixed L

Neutrino Physics Goals



[HyperK]



[DUNE]

Flagship long baseline experiments to measure neutrino oscillation

DUNE: USA, HyperK: Japan

Seek to answer fundamental questions about neutrinos:

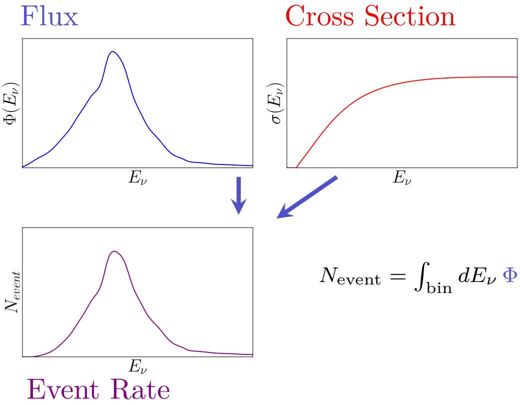
- mass ordering ($\Delta m^2_{32} > 0$?)
- octant ($\sin^2\theta_{23} = 0.5$?)
- CP violation ($\delta_{\mathrm{CP}} =$?)
- PMNS unitarity?
- 3 ν flavors?
- precision constraints

Measurements of solar, supernova ν

Data collection starts 2028–2029 $\implies$ need support from theory!

Neutrino Cross Sections

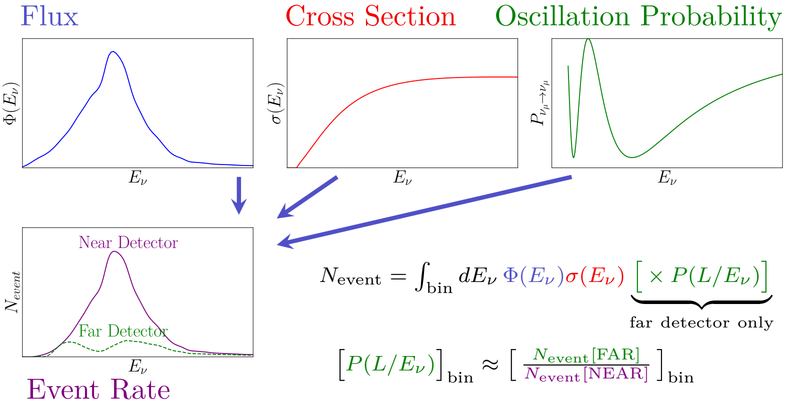
Measuring Oscillation Probability



$$N_{\text{event}} = \int_{\text{bin}} dE_\nu \, \Phi(E_\nu)\sigma(E_\nu)$$

Event rate from convolution over E_ν. Broad flux & distribution of event E_ν

Measuring Oscillation Probability



Flux

Cross Section

Oscillation Probability

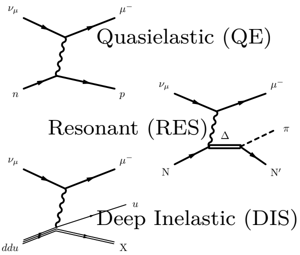
Event Rate

$$N_{\text{event}} = \int_{\text{bin}} dE_\nu\, \Phi(E_\nu)\sigma(E_\nu)\, \underbrace{\left[\times P(L/E_\nu)\right]}_{\text{far detector only}}$$

$$\left[P(L/E_\nu)\right]_{\text{bin}} \approx \left[\frac{N_{\text{event}}[\text{FAR}]}{N_{\text{event}}[\text{NEAR}]}\right]_{\text{bin}}$$

Event rate from convolution over E_ν. Broad flux & distribution of event E_ν far/near $\implies$ oscillation probability, but picture too simplified...

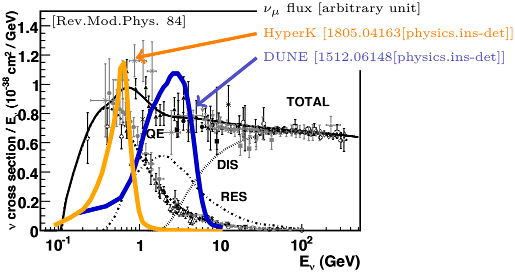
Neutrino Cross Sections



Quasielastic (QE)

Resonant (RES)

Deep Inelastic (DIS)



[Rev.Mod.Phys. 84]

ν_μ flux [arbitrary unit]

HyperK [1805.04163[physics.ins-det]]

DUNE [1512.06148[physics.ins-det]]

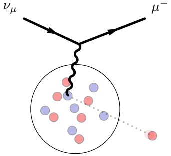
E_ν range spans several kinematic regimes

Different interaction channels contributing to event rates

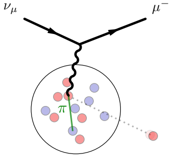
Event rate predictions need precise, theoretically robust cross sections
for multiple event topologies

Neutrino Event Topologies

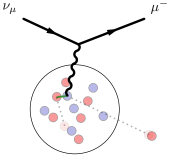
Quasielastic



π production



Multinucleon

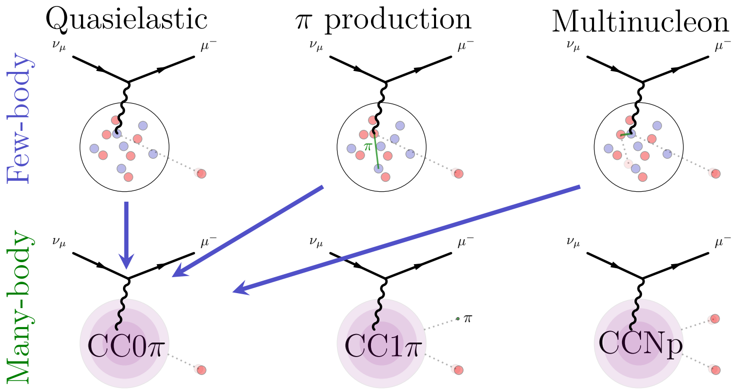


Many allowed interaction channels, reinteractions within nucleus

Particle kinematics change in nuclear medium

Only particles that escape are detectable

Neutrino Event Topologies



Many allowed interaction channels, reinteractions within nucleus

Particle kinematics change in nuclear medium

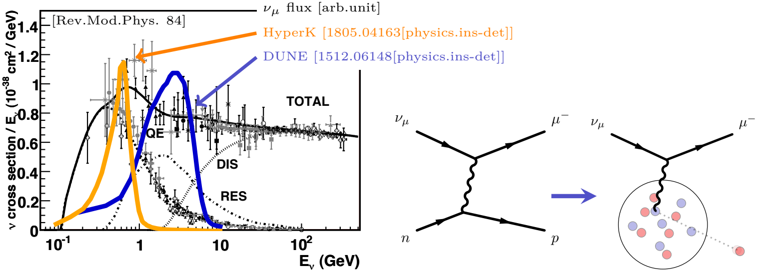
Only particles that escape are detectable

Mismatch between *nucleon* amplitudes & *nuclear* cross sections...

$\Longrightarrow$ Event-by-event E_ν measurements are not possible

$\Longrightarrow$ Reconstruct E_ν distributions from measured event rates

Neutrino Oscillation and Quasielastic



ν_μ flux [arb.unit]

HyperK [1805.04163[physics.ins-det]]

DUNE [1512.06148[physics.ins-det]]

Compute *nucleon* amplitudes, ingredients for *nuclear* models

Quasielastic is lowest E_ν, simplest $\implies$ most important

Question:

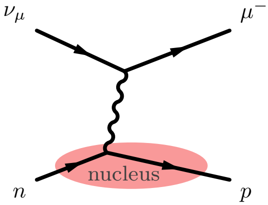
How well do we know nucleon quasielastic cross section from elementary target sources?

- Deuterium scattering
- Lattice QCD

QE Experimental Constraints

Quasielastic Form Factors

Quasielastic (QE) scattering assumes quasi-free nucleon inside nucleus



$$\mathcal{M}_{\text{nucleon}} = \langle \ell | \mathcal{J}^\mu | \nu_\ell \rangle \langle N' | \mathcal{J}_\mu | N \rangle$$

$$\begin{aligned} &\langle N'(p') | (V-A)_\mu(q) | N(p) \rangle \\ &= \bar{u}(p') \Big[\quad \gamma_\mu F_1(q^2) \quad + \tfrac{i}{2M_N} \sigma_{\mu\nu} q^\nu F_2(q^2) \\ &\qquad + \gamma_\mu \gamma_5 F_A(q^2) \quad + \tfrac{1}{2M_N} q_\mu \gamma_5 F_P(q^2) \quad \Big] u(p) \end{aligned}$$

- F_1, F_2: constrained by eN scattering
- F_P: subleading in cross section,
  $\propto F_A$ from pion pole dominance constraint

Axial form factor F_A is leading contribution to nucleon cross section uncertainty

Form Factor Parameterizations

Most common in experimental literature: dipole ansatz —

$$F_A(Q^2) = g_A \left(1 + \frac{Q^2}{m_A^2}\right)^{-2}$$

- Overconstrained by both experimental and LQCD data (revisit later)
- Inconsistent with QCD, requirements from unitarity bounds
- Motivated by $Q^2 \to \infty$ limit, data restricted to low Q^2

Model independent alternative: z expansion [Phys.Rev.D 84 (2011)] —

$$F_A(z) = \sum_{k=0}^{\infty} a_k z^k \qquad z(Q^2; t_0, t_{\text{cut}}) = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}} - t_0}} \qquad t_{\text{cut}} \leq (3M_\pi)^2$$

- Rapidly converging expansion
- Controlled procedure for introducing new parameters

Deuterium Constraints on F_A

- Outdated bubble chamber experiments:
  - Total $O(10^3)$ ν_μQE events
  - Digitized event distributions only
  - Unknown corrections to data
  - Deficient deuterium correction

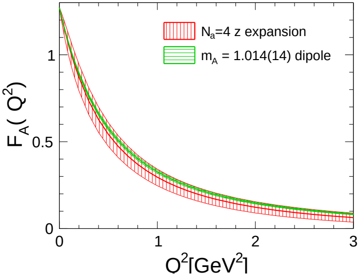
- Dipole overconstrained by data
  underestimated uncertainty $\times O(10)$

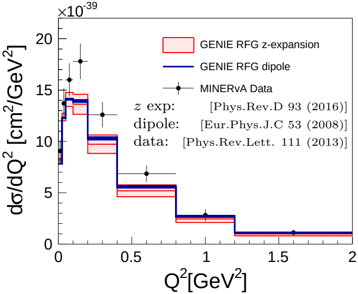
- Prediction discrepancies could be from
  nucleon and/or nuclear origins

Coming soon:

MINERνA $\bar{\nu}_\mu p \to \mu^+ n$ dataset
& updated form factor fits

See [Cai thesis (2021)]





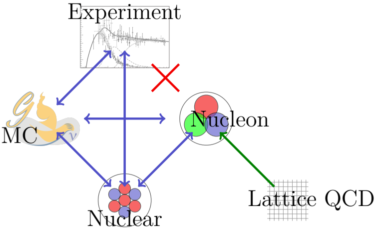
LQCD as Disruptive Technology

How can we improve precision?

Ideal: Modern high stats ν-D_2 scattering bubble chamber experiment

$\implies$ LQCD as a alternative/complement to experiment,
especially with experimentally inaccessible quantities

- ✓ No nuclear effects
- ✓ Realistic uncertainty estimates
- ✓ Systematically improvable
- ✓ Computers are (relatively) inexpensive



Build from the ground up:

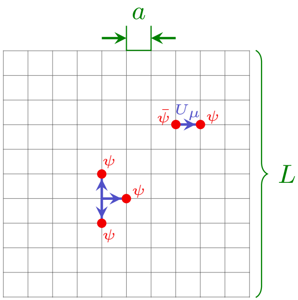
Nucleon amplitudes from first principles

Robust uncertainty quantification

Well motivated theory inputs to nuclear models/EFTs

Matrix Elements from LQCD

Lattice QCD Formalism



Numerical evaluation of path integral

Quark, gluon DOFs —

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \, \mathcal{D}U \, \exp(-S) \, \mathcal{O}_\psi \, [U]$$

Parameters: $am_{(u,d),\mathrm{bare}}$
$am_{s,\mathrm{bare}}$
$\beta = 6/g^2_{\mathrm{bare}}$

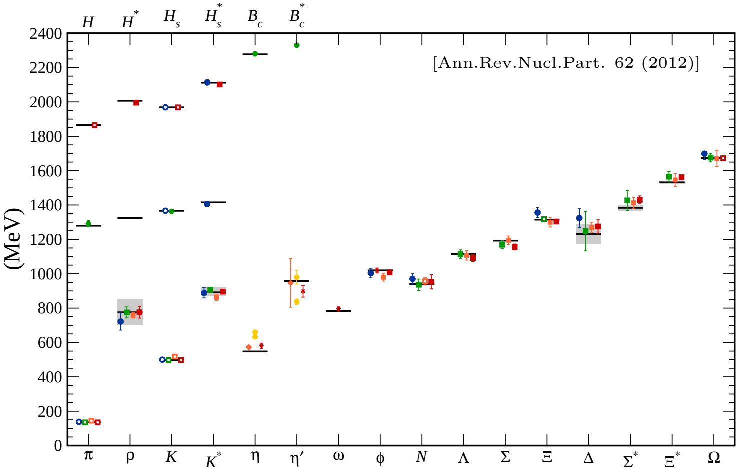
Matching: e.g. $\frac{M_\pi}{M_\Omega}$, $\frac{M_K}{M_\Omega}$, M_Ω
1 per parameter

Results — first principles predictions from QCD,
gluons to all orders

"Complete" error budget $\Longrightarrow$ extrapolation in a, L, M_π guided by EFT, FVχPT

- $a \to 0$ (continuum limit)
- $L \to \infty$ (infinite volume limit)
- $M_\pi \to M_\pi^{\mathrm{phys}}$ (chiral limit)

Successes of Lattice QCD



Open symbol: input Closed symbol: (pre/post)diction Line: experiment

- Excellent agreement across board
- Widely used in flavor physics

LQCD $g_A(Q^2 = 0)$

g_A is benchmark for
nucleon matrix elements in LQCD

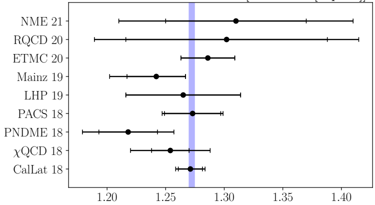
Status circa 2018 summarized by
USQCD white paper
[Eur.Phys.J.A 55 (2019)]

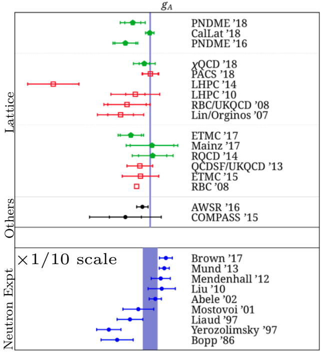
See also: FLAG review
[Eur.Phys.J.C 80 (2020)]

Historically g_A low compared to expt
excited states (+other...)

Lots of activity since 2018,
consistent agreement with PDG
full error budgets available

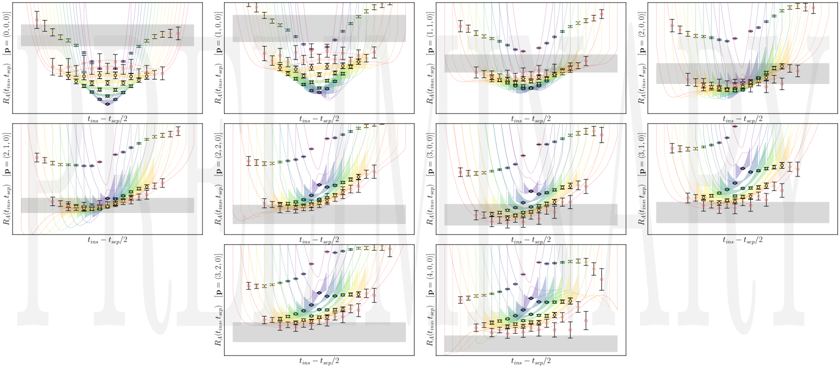
See refs. in: [2103.05599[hep-lat]]





[Eur.Phys.J.A 55 (2019)]

LQCD $F_A(Q^2)$

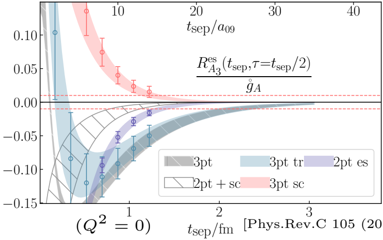


Lots of time separations, lots of momenta $\implies$ fit simultaneously

Low Q^2 $\implies$ fit curves (color) far from posterior (gray)

Curvature $\implies$ excited state contamination

Excited States



$(Q^2 = 0)$

[Phys.Rev.C 105 (2022)]

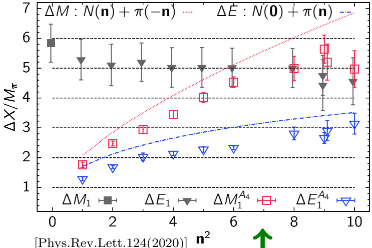
Compare fit to correlator data ratio

Remnant contamination dominated by “transition” states ($m \to n$, violet)

Statistically significant until 2 fm typical data $\lesssim$ 1 fm

Excited states still present in practically achievable large time limit

$(Q^2 \geq 0)$



[Phys.Rev.Lett.124(2020)]

↑

NOTE: expect only approx
agreement between data/curves

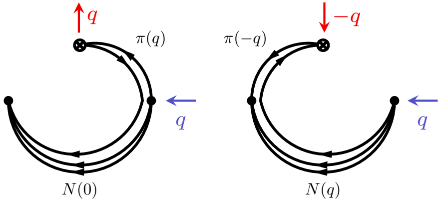
NME collab:

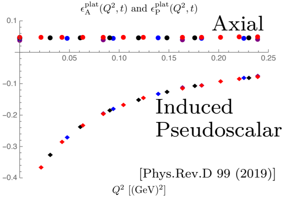
Q^2 contamination from $N \to N\pi$

Dominant contribution agrees with χPT expectation

$N\pi$ is important for $F_A(Q^2)$

Excited States - χPT and $N\pi$





[Phys.Rev.D 99 (2019)]

Contamination primarily from enhanced $N\pi$, mostly from induced pseudoscalar

Correlator fits without axial current not sensitive to $N\pi$

$\Longrightarrow$ need simultaneous fits including axial matrix elements

[Phys.Rev.C 105 (2022)] [Phys.Rev.D 105 (2022)]

Alternate fit strategies to remove $N\pi$ (are they comparable?):

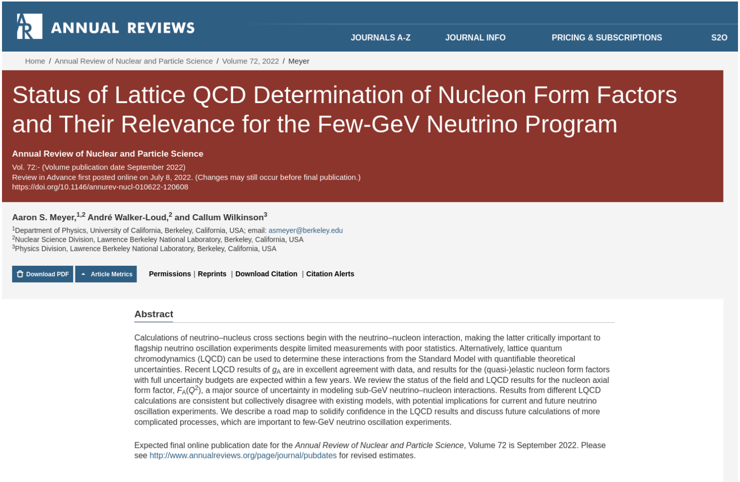
- Kinematic constraints ($F_P = 0$)
- include $\mathcal{A}_4$ (strong $N\pi$ coupling)
- explicit $N\pi$ operators

Prediction from χPT: [Phys.Rev.D 99 (2019)]

First demonstration by NME: [Phys.Rev.Lett. 124 (2020)]

χPT-inspired fit methods for fitting form factor data
[Phys.Rev.D 105 (2022)] [JHEP 05 (2020) 126]

LQCD Survey and Implications



Status of Lattice QCD Determination of Nucleon Form Factors and Their Relevance for the Few-GeV Neutrino Program

Annual Review of Nuclear and Particle Science
Vol. 72:- (Volume publication date September 2022)
Review in Advance first posted online on July 8, 2022. (Changes may still occur before final publication.)
https://doi.org/10.1146/annurev-nucl-010622-120608

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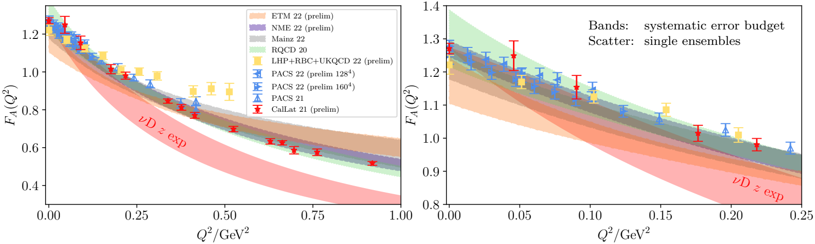
Abstract

Calculations of neutrino–nucleus cross sections begin with the neutrino–nucleon interaction, making the latter critically important to flagship neutrino oscillation experiments despite limited measurements with poor statistics. Alternatively, lattice quantum chromodynamics (LQCD) can be used to determine these interactions from the Standard Model with quantifiable theoretical uncertainties. Recent LQCD results of g_A are in excellent agreement with data, and results for the (quasi-)elastic nucleon form factors with full uncertainty budgets are expected within a few years. We review the status of the field and LQCD results for the nucleon axial form factor, $F_A(Q^2)$, a major source of uncertainty in modeling sub-GeV neutrino–nucleon interactions. Results from different LQCD calculations are consistent but collectively disagree with existing models, with potential implications for current and future neutrino oscillation experiments. We describe a road map to solidify confidence in the LQCD results and discuss future calculations of more complicated processes, which are important to few-GeV neutrino oscillation experiments.

Expected final online publication date for the *Annual Review of Nuclear and Particle Science*, Volume 72 is September 2022. Please see http://www.annualreviews.org/page/journal/pubdates for revised estimates.

Nucleon Axial Form Factor

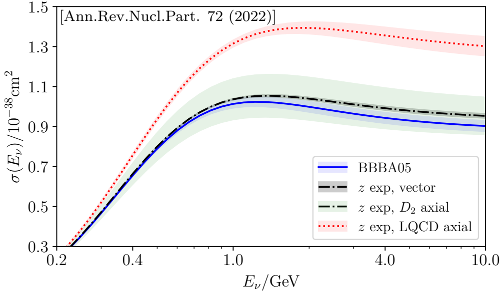


LQCD results maturing:

- Many results, all physical M_π: *independent data & different methods*
- Small systematic effects observed (expectation: largest at $Q^2 \to 0$)
- Nontrivial consistency checks from PCAC

Evidence of slow Q^2 falloff, situation unlikely to change drastically

Free Nucleon Cross Section



- LQCD prefers 30-40% enhancement of ν_μ CCQE cross section
- recent Monte Carlo tunes require 20% enhancement of QE

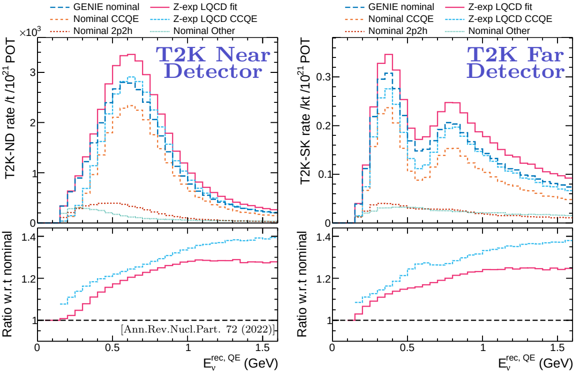
  [Phys.Rev.D 105 (2022)] [2206.11050 [hep-ph]]

  similar trend with continuum Schwinger function methods

  [Phys.Rev.D 105 (2022)] [2206.12518 [hep-ph]]
- With improved precision, sensitive to vector FF tension (black vs blue)

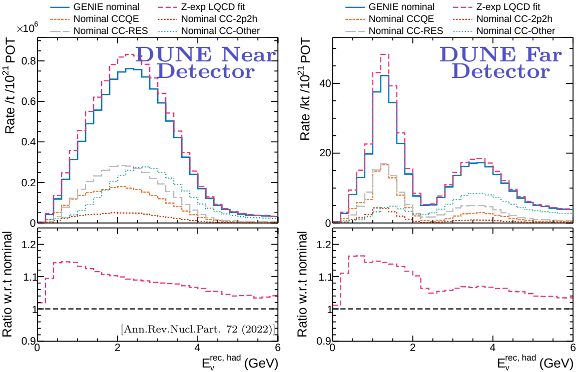
  [Phys.Rev.D 102 (2020)] vs [Nucl.Phys.B Proc.Suppl. 159 (2006)]

T2K Implications



- Dashed dark blue (GENIE nominal) vs solid magenta (z exp LQCD fit)
- QE enhancements produce 10-20% event rate enhancement, E_ν-dependent
- Monte Carlo tuning makes more detailed comparisons complicated
  $\Longrightarrow$ All channels are adjusted to compensate for QE changes
- cross section changes at ND $\neq$ *effective* cross section changes at FD:
  insufficient CCQE model freedom → bias in FD prediction

DUNE Implications

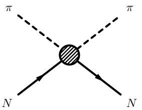


- Solid dark blue (GENIE nominal) vs dashed magenta (z exp LQCD fit)
- QE enhancements produce 10-20% event rate enhancement, E_ν-dependent
- Monte Carlo tuning makes more detailed comparisons complicated
  $\implies$ All channels are adjusted to compensate for QE changes
- cross section changes at ND $\neq$ *effective* cross section changes at FD:
  insufficient CCQE model freedom $\rightarrow$ bias in FD prediction

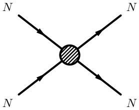
Future Directions

LQCD Target Calculations

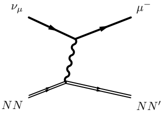
$N\pi$ Scattering



NN Scattering

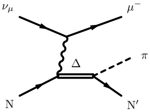


NN Quasielastic

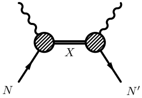


(incomplete list!)

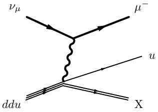
Resonant $N\pi$



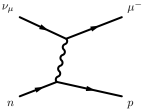
Four-point Inclusive

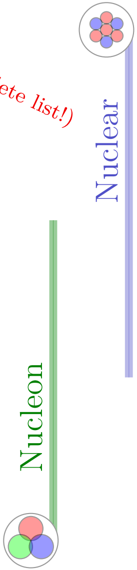


Deep Inelastic

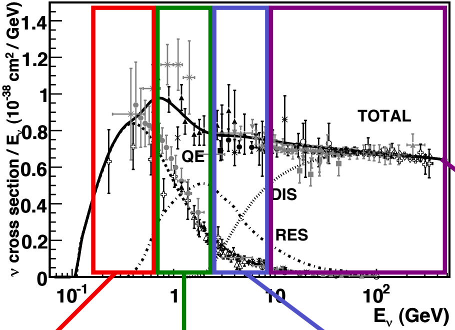


Quasielastic





Energy Regimes



Deep Inelastic Scattering

–Axial quasi/pseudo PDF

Quasielastic

–Nucleon Form Factors

–Full Error Budgets

“Shallow Inelastic Scattering” (SIS)

–Hadronic Tensor

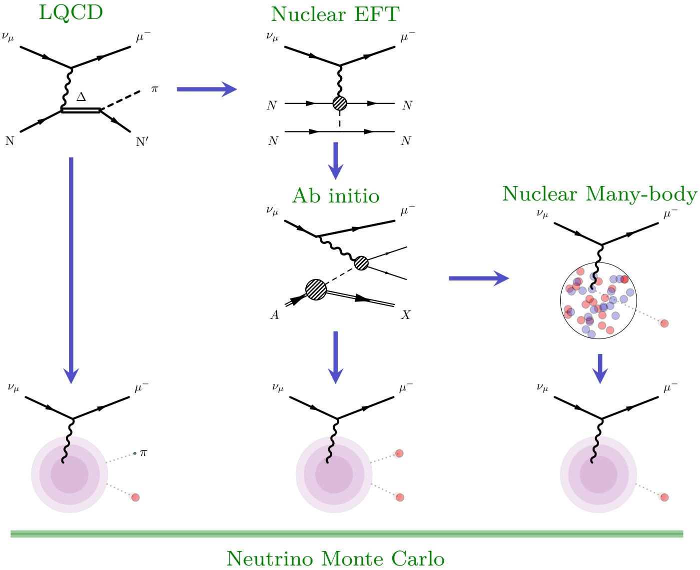
–Four Point Functions

$N \to \Delta, N \to N^*$

–Transition Matrix Elements

–Multiparticle Operators

Roadmap To Nuclear



LQCD

Nuclear EFT

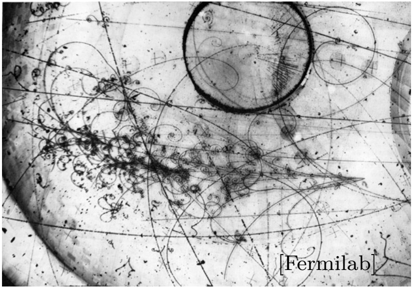
Ab initio

Nuclear Many-body

Neutrino Monte Carlo

Concluding Remarks

Outlook



[Fermilab]

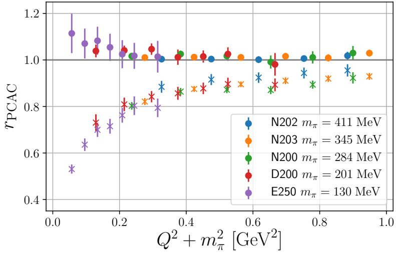
- *Nucleon* form factor uncertainty significantly underestimated
- LQCD is a proxy for missing experimental data
- Mounting evidence that ν QE cross section underestimated
  $\implies$ Attention needed to avoid biased results
- Unfilled niche: need support for neutrino experimental program
- Nucleon-pion effects are the next frontier...
  - Transition form factors
  - Low-energy constants for meson exchange
  - Pion production

Thank you for your attention!

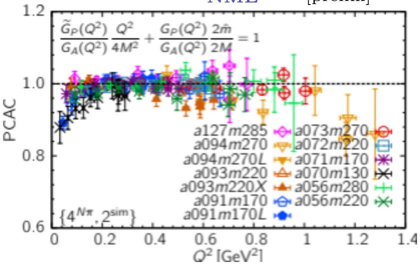
Backup

PCAC Checks

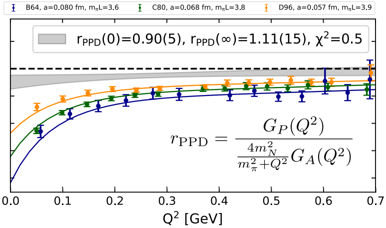
RQCD [JHEP 05 (2020)]



NME [prelim]

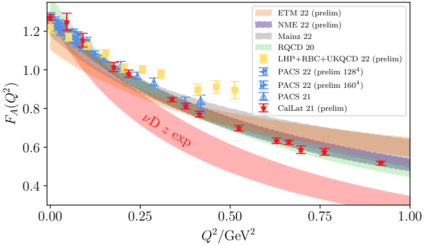


ETMC [prelim]

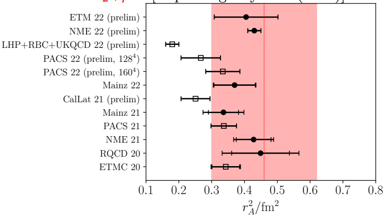


- Relation btw F_A, F_P, $\tilde F_P$ via PCAC
- Contamination in F_A and $\tilde F_P$, F_P very different [Phys.Rev.D 99 (2019)]
  $\Longrightarrow$ nontrivial consistency check

Axial Radius (r_A^2)



Filled circle: full error budget
Open square: incomplete
D$_2$+μH: [Rept.Prog.Phys. 81 (2018)]

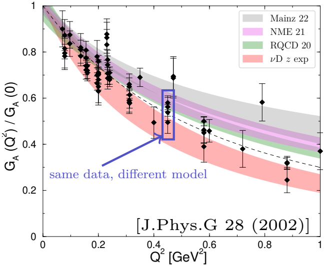


Radius related to slope: $r_A^2 = -\frac{6}{g_A}\frac{dF_A}{dQ^2}\Big|_{Q^2=0}$

Good agreement with r_A^2 from experiment, poor agreement with large Q^2
Fixing radius to agree at large Q^2 would bring radius down to $r_A^2 \sim 0.25$ fm^2

$\Longrightarrow$ Incompatible with dipole ansatz

Electro Pion Production



same data, different model

[J.Phys.G 28 (2002)]

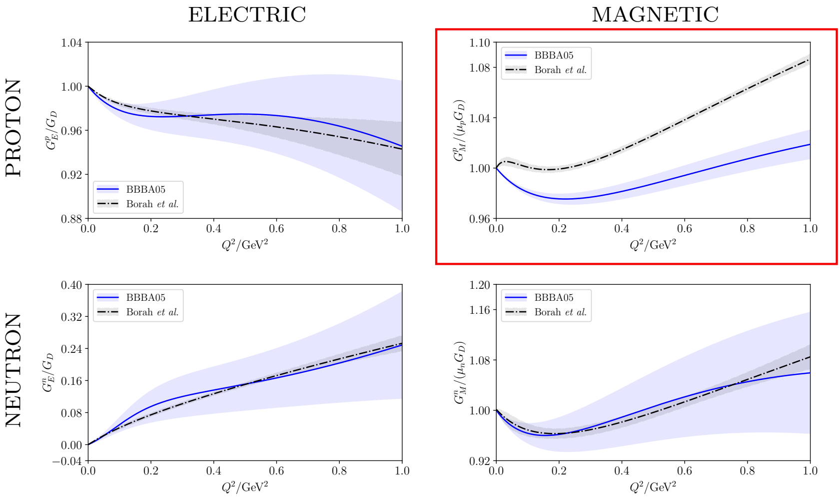
- Large model uncertainty, not included in world averages
- Valid only in $M_\pi \to 0$, $q \to 0$ limits
- Expansion to $O(M_\pi^2, Q^2)$:
  - restricted Q^2 validity
  - lacks shape freedom in Q^2
- Predates Heavy Baryon χPT, no systematic power counting

Modern experiments do not report $F_A(Q^2) \implies$ averages out of date

Possible argument for comparing to r_A^2 from low Q^2; high Q^2 untrustworthy

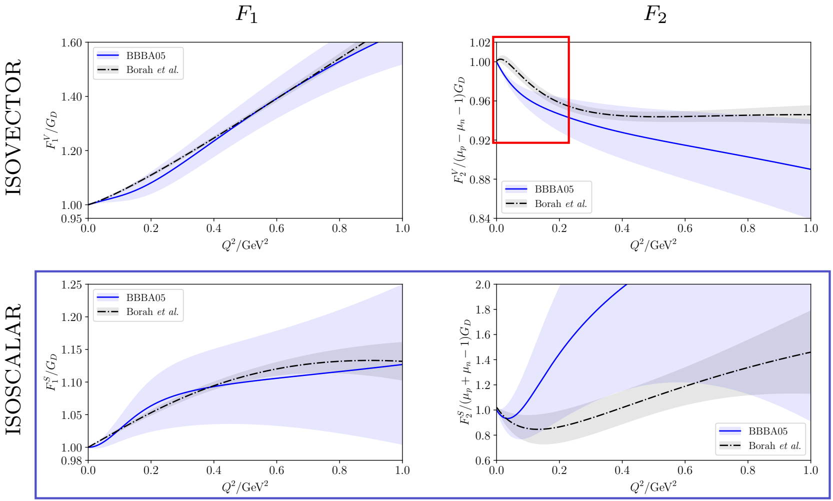
Effort needed to update prediction from photo/electro pion production

Vector Form Factors - Proton/Neutron



Large tension in proton magnetic form factor

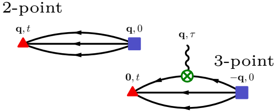
Vector Form Factors - Isospin Symmetric



Uncertain slope of F_2^V

Large uncertainty on isoscalar form factors

LQCD Computation Anatomy



Correction functions in euclidean time:

$\implies e^{-E_n t}$ decay of excited state contribs

2-point function

$$\langle \blacktriangle(t) \blacksquare(0) \rangle = \sum_n \langle 0|\blacktriangle|n\rangle\langle n|\blacksquare|0\rangle e^{-E_n t}$$

3-point function

$$\langle \blacktriangle(t) \otimes(\tau) \blacksquare(0) \rangle = \sum_{mn} \langle 0|\blacktriangle|n\rangle\langle n|\otimes|m\rangle\langle m|\blacksquare|0\rangle e^{-E_n(t-\tau)-E_m\tau}$$

Extract masses from 2-point, matrix elements from 3-point

Complications:

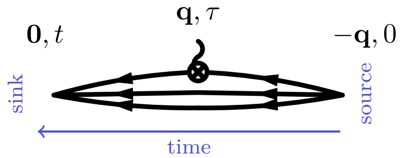
- exponentially degrading signal/noise with t
- $n > 0$ contaminations from excited states

Use many source/sink operators ($\blacksquare$, $\blacktriangle$) to suppress excited states:

$$C_{ij}(t) = \sum_n z_{i,n} z^\dagger_{j,n} e^{-E_n t}$$

$$\implies v^T C(t) v \approx e^{-E_0 t} \quad \text{when} \quad \sum_i v^T_i z_{i,n} \approx \delta_{0,n}$$

Setup



$$\mathcal{R}_{\mathcal{A}_z}(t,\tau,\mathbf{q}) = \frac{C^{3\text{pt}}_{\mathcal{A}_z}(t,\tau,\mathbf{q})}{\sqrt{C^{2\text{pt}}(t-\tau,\mathbf{0})C^{2\text{pt}}(\tau,\mathbf{q})}} \sqrt{\frac{C^{2\text{pt}}(\tau,\mathbf{0})}{C^{2\text{pt}}(t,\mathbf{0})} \frac{C^{2\text{pt}}(t-\tau,\mathbf{q})}{C^{2\text{pt}}(t,\mathbf{q})}}$$

$$\xrightarrow[t-\tau,\tau\to\infty]{} \frac{1}{\sqrt{2E_{\mathbf{q}}(E_{\mathbf{q}}+M)}} \left[-\frac{q_z^2}{2M} \mathring{\tilde{g}}_P(Q^2) + (E_{\mathbf{q}}+M)\mathring{g}_A(Q^2) \right]$$

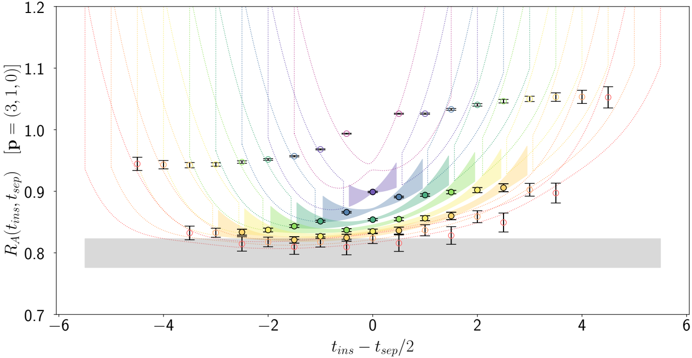
$Q^2 = |\mathbf{q}|^2 - (E_{\mathbf{q}} - M)^2$

$\mathcal{A}_z$ w/ $q_z = 0 \implies \mathcal{R}_{\mathcal{A}_z}(t,\tau,\mathbf{q}) \to \sqrt{\frac{E_{\mathbf{q}}+M}{2E_{\mathbf{q}}}}\mathring{g}_A(Q^2)$

$\implies$ No induced pseudoscalar

$\implies$ Simplified analysis of $\mathring{g}_A(Q^2)$

Correlation Function Ratio

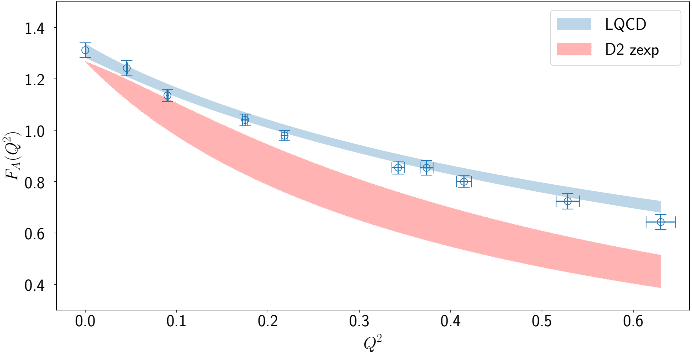


- Horizontal: source-insertion time, centered about midpoint
- Vertical: correlator ratio ∼ axial matrix element
- Color: source-sink separation time
- Colored bands: fit range
- Gray band: $\mathring{g}_A$ posterior value

Curvature $\implies$ excited state contamination

Left-right asymmetry $\implies$ different momentum, different spectrum

Axial Form Factor Fit

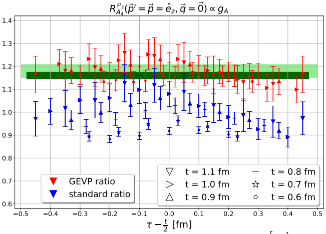


Trend of high-Q^2 enhancement seen in other LQCD results

2–4% LQCD uncertainty vs 10% uncertainty on D_2 result

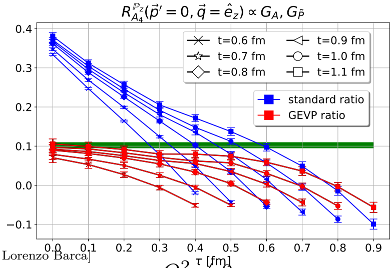
Single-ensemble result — could be systematically biased
(comparing to other LQCD results $\implies$ bias is likely small)

Axial FF - $N\pi$ Interpolating Operators



$Q^2 = 0$

[priv.comm. Lorenzo Barca]



$Q^2 \neq 0$

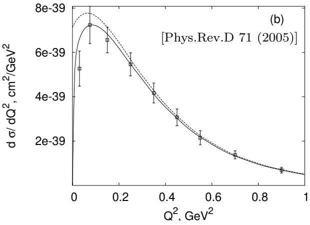
Address primary source of excited state contamination: $N\pi$

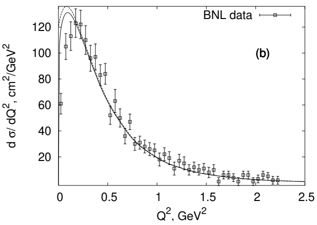
2×2 operator basis, explicit 3- and 5-quark interpolating operators

Significantly flatter ratios, simplified analysis

Will analysis with only 3-quark operators be consistent?

Resonance Production - $N \to \Delta$


[Phys.Rev.D 71 (2005)]



$N \to \Delta$ transition form factors are poorly known, but needed

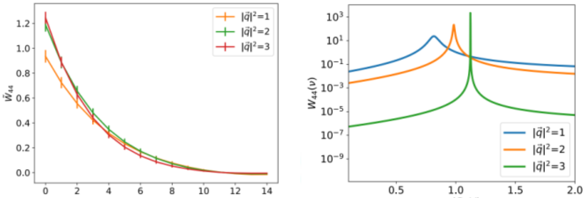
1π production cross section known to 30% [Phys.Rev.C 88 (2013)]
DUNE error budget anticipates $\lesssim$10% precision [2002.03005 [hep-ex]]

Completely unconstrained axial form factors in other $J^P = 3/2^-$ channels
$\implies$ 100% uncertainties from $V - A$, $A - A$ interference terms

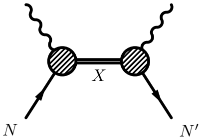
[Phys.Rev.D 74 (2006)]

Previous work by ETM: [Phys.Rev.D 83 (2011)] [Phys.Rev.Lett. 98 (2007)]

Resonance Production - $N \to N^*$



[priv.comm. Keh-Fei Liu]

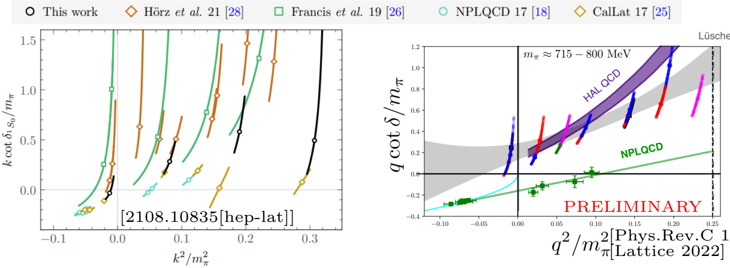


See also: [Phys.Rev.D 101 (2020)]

Four point function with $\langle \mathcal{O}(0)\mathcal{J}_4(-q)\mathcal{J}_4(q)\bar{\mathcal{O}}(0)\rangle$, $M_\pi \sim 370$ MeV
Removed elastic contribution $\implies$ resonant response (strong overlap with Roper)

Hadronic tensor methods for addressing SIS (1.4 GeV $\leq W \leq$ 2.0 GeV)
Large $N\pi$, $N\pi\pi$ contributions; strong interferences between resonant/nonresonant
Currently no practical $Q^2 \neq 0$ data in this region [S.Nakamura - NuSTEC S&DIS]

NN Scattering: D_2 Binding



[2108.10835[hep-lat]]

[Phys.Rev.C 103 (2021)]
[Lattice 2022]

Controversy over whether deuteron (un)bound at unphysically heavy M_π

Lüscher method (bound) vs potential method (unbound)

Bound: $\lim_{q^2 \to 0} q \cot \delta < 0$ Unbound: $\lim_{q^2 \to 0} q \cot \delta > 0$

Recent updates:

- updated Lüscher calculations → unbound
- both methods on same ensemble, agree on unbound deuteron
- large q^2 departure of potential from Lüscher

Still claims of bound deuteron; to be continued...