Thermodynamics of oscillating neutrinos

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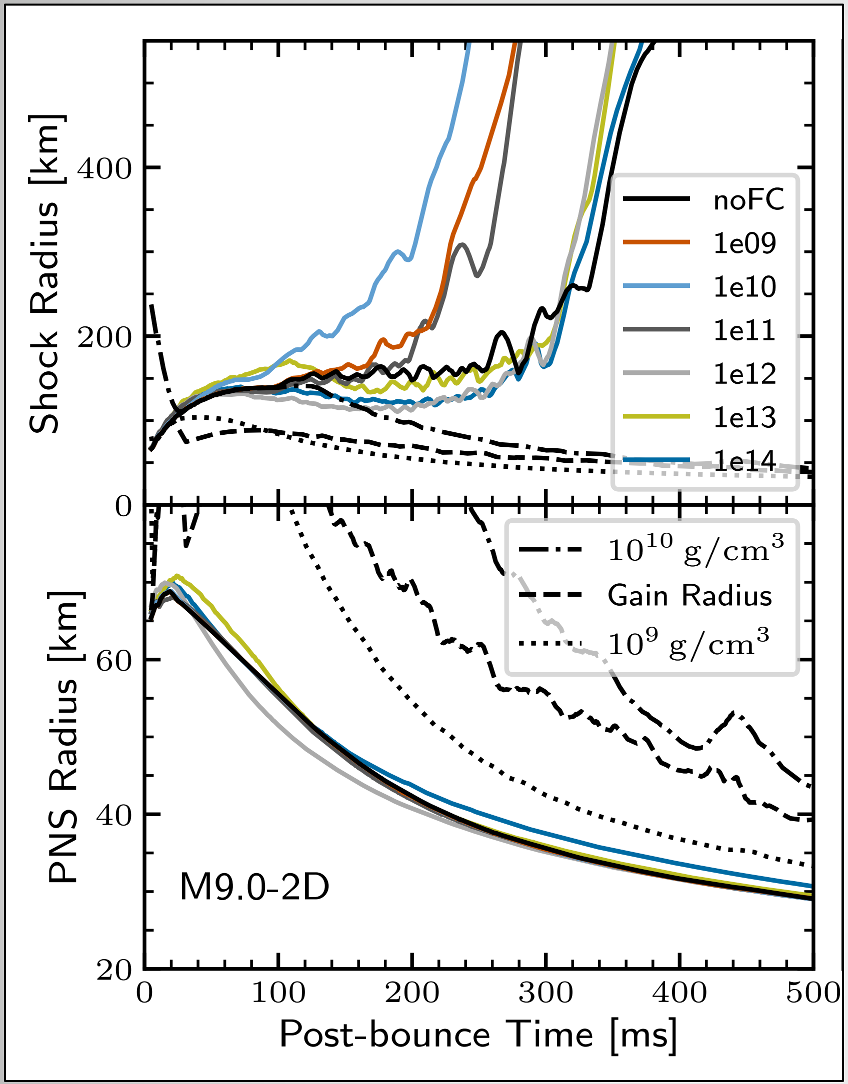

Ehring, Abbar, Janka, Raffelt, & Tamborra, 2305.11207

Flavor mixing is estimated to significantly alter CCSN heating rates—and thus the explosion dynamics.

Also see:
Ehring et al., PRD (2023)
Nagakura, PRL (2023)

Neutrino mass raises fundamental questions for particle physics.

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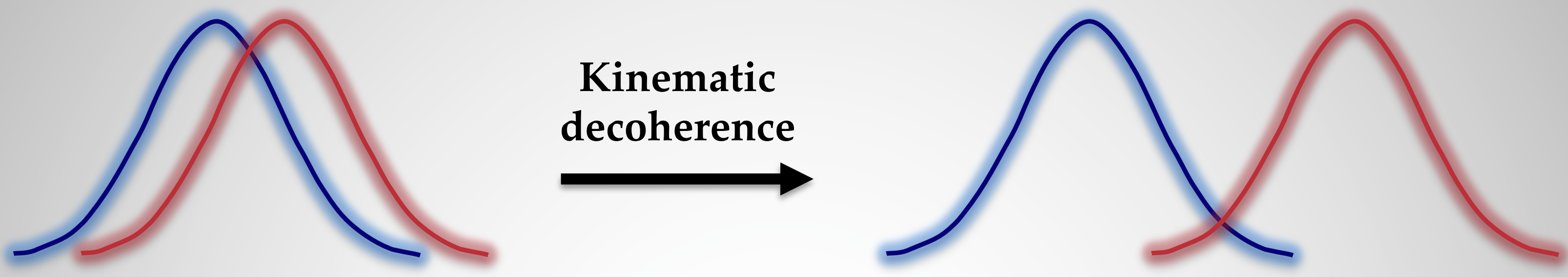
and statistical

Every neutrino is a superposition of wave packets:



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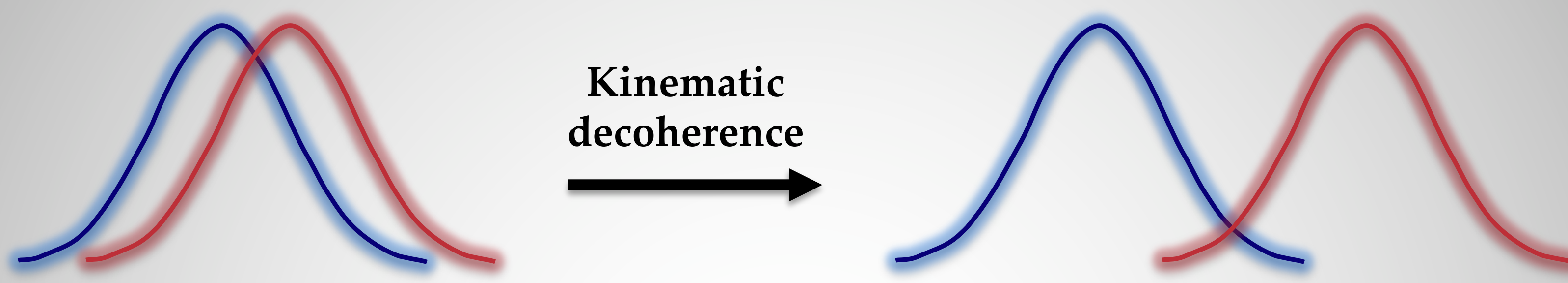

There's a term for superposed Gaussians…

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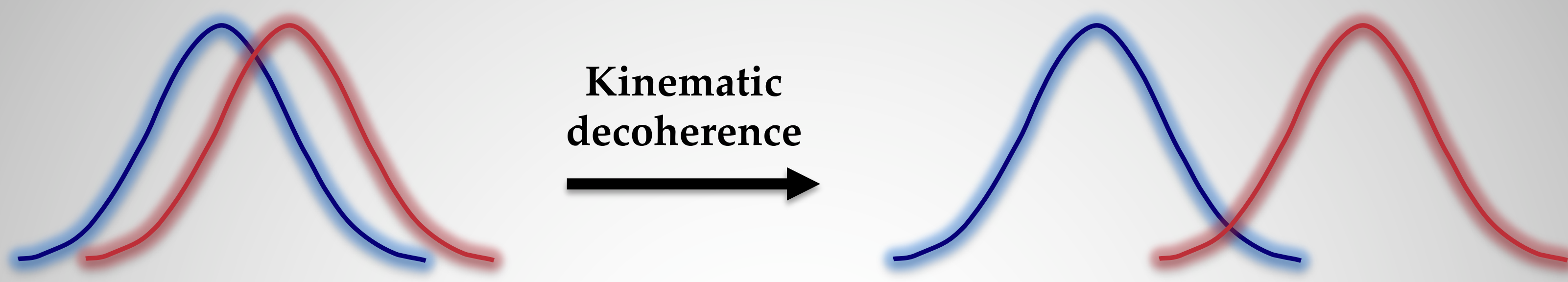
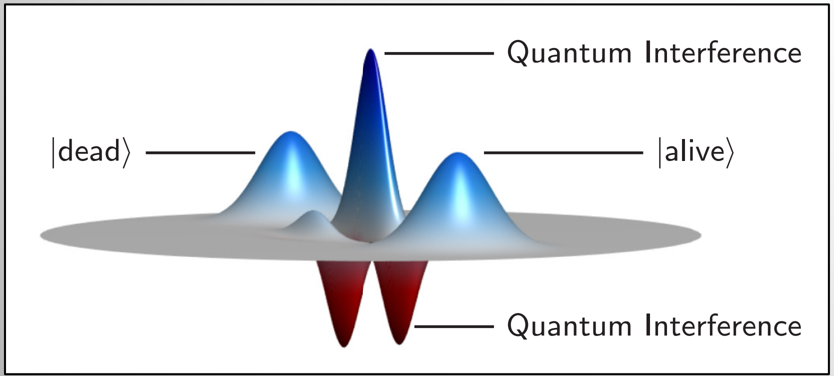


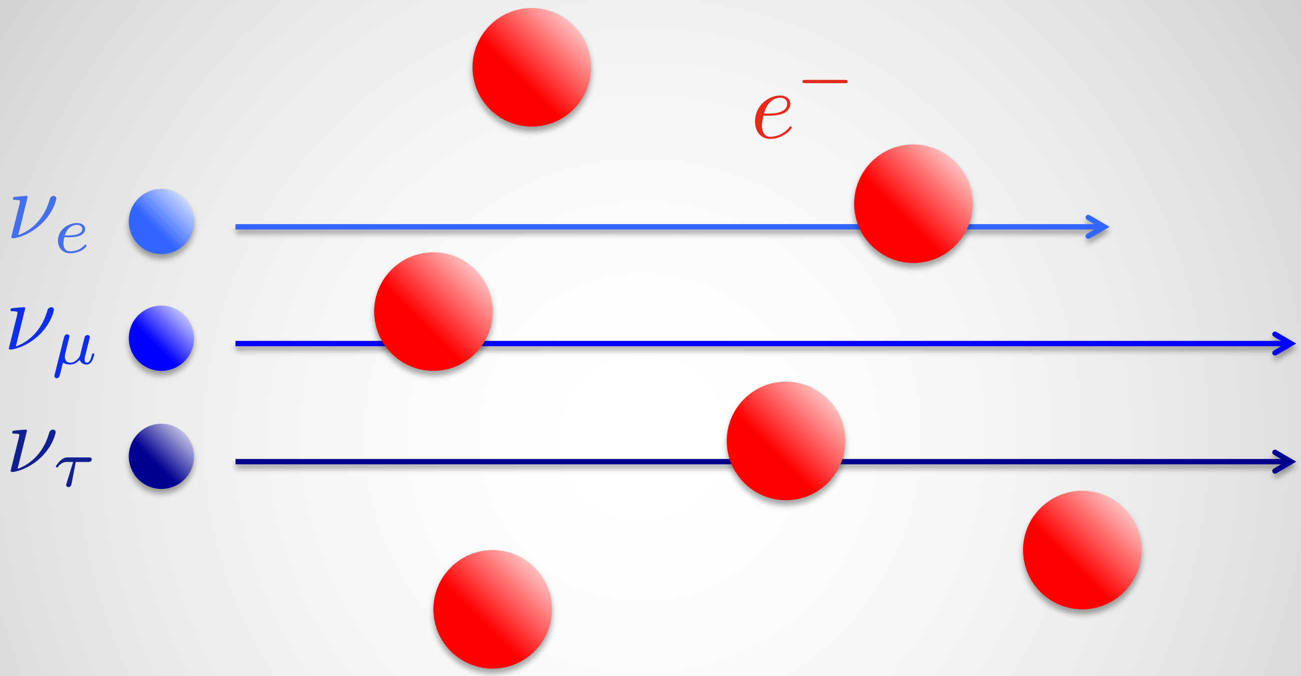
There's a term for superposed Gaussians…

Schrödinger cat states

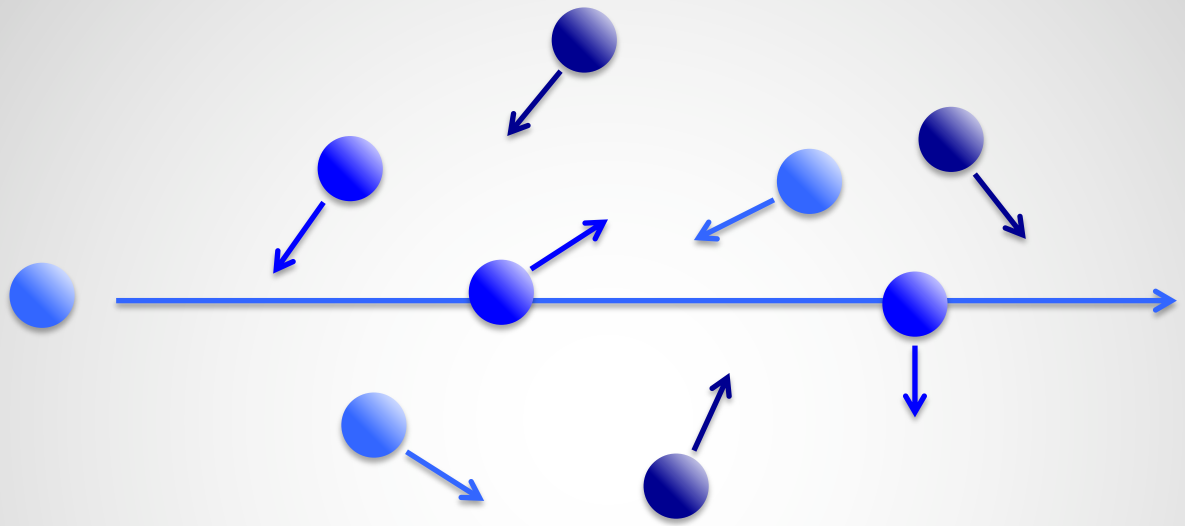


Rundle, Mills, Tilma, Samson, Everitt, PRA 2017

S. Haroche via *Science et Vie Junior*

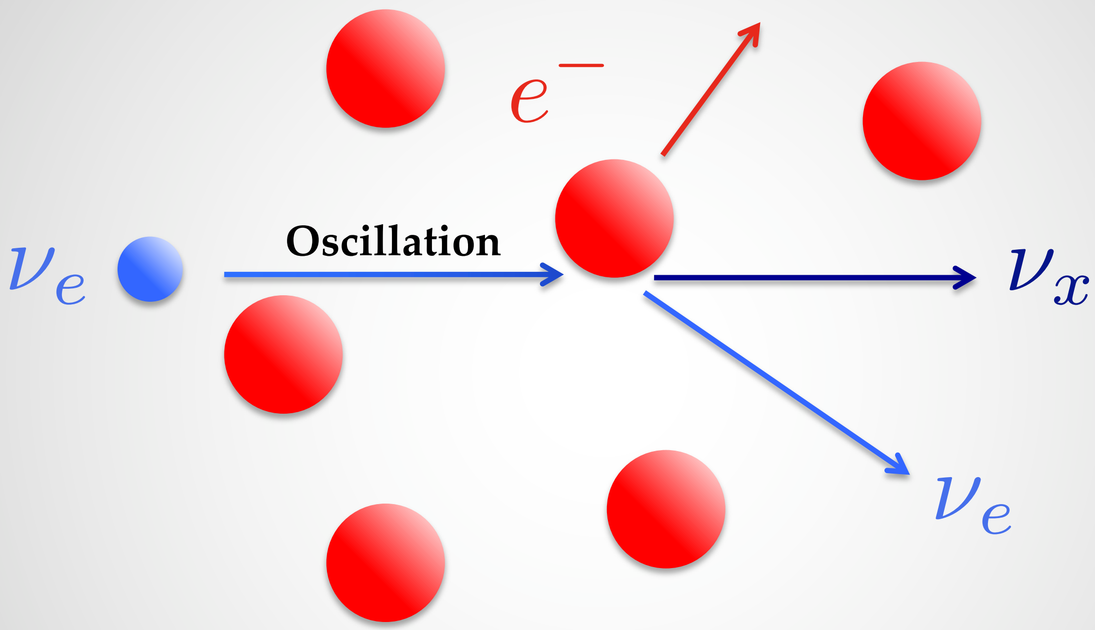


When neutrinos **forward scatter** on background particles,
they acquire in-medium effective masses.



Neutrinos contribute to their own background. As a result, forward scattering changes oscillations in a nonlinear way.

Collective oscillations



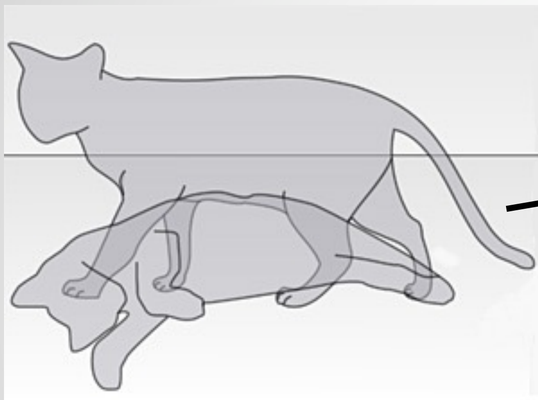
Absorption and momentum-changing scattering
cause **collisional decoherence**.

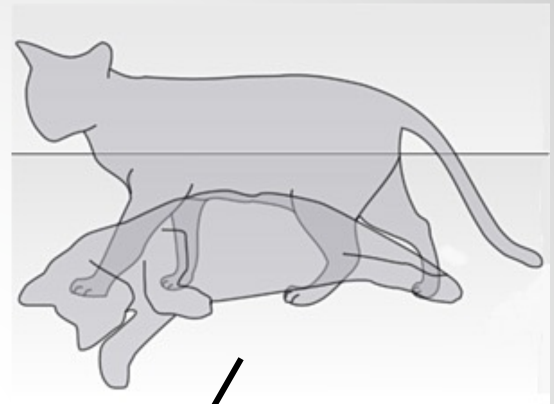
clowder *noun*

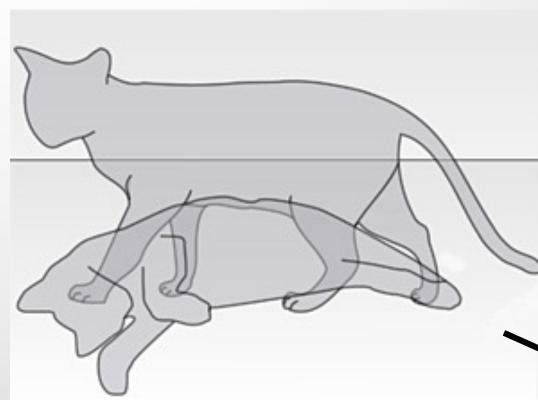
: a group of cats

Merriam-Webster

Schrödinger's clowder







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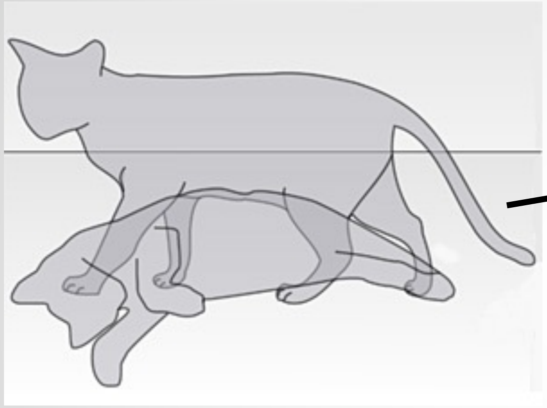
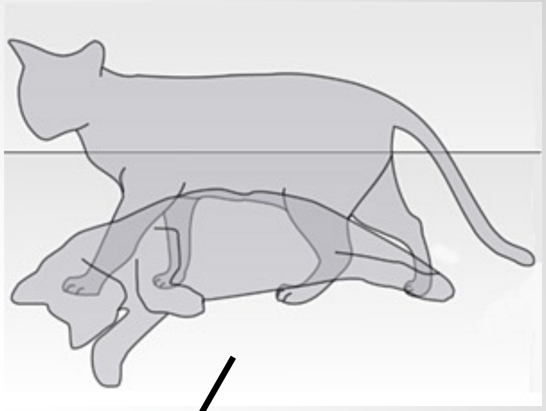
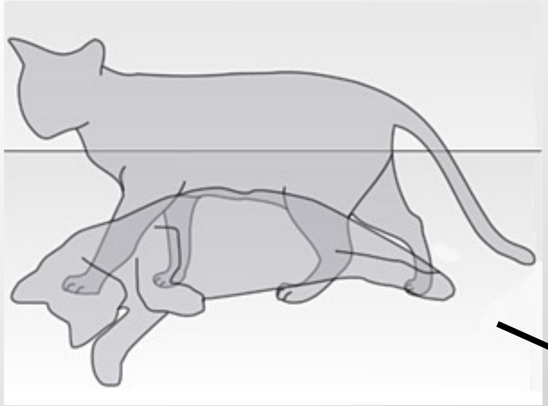






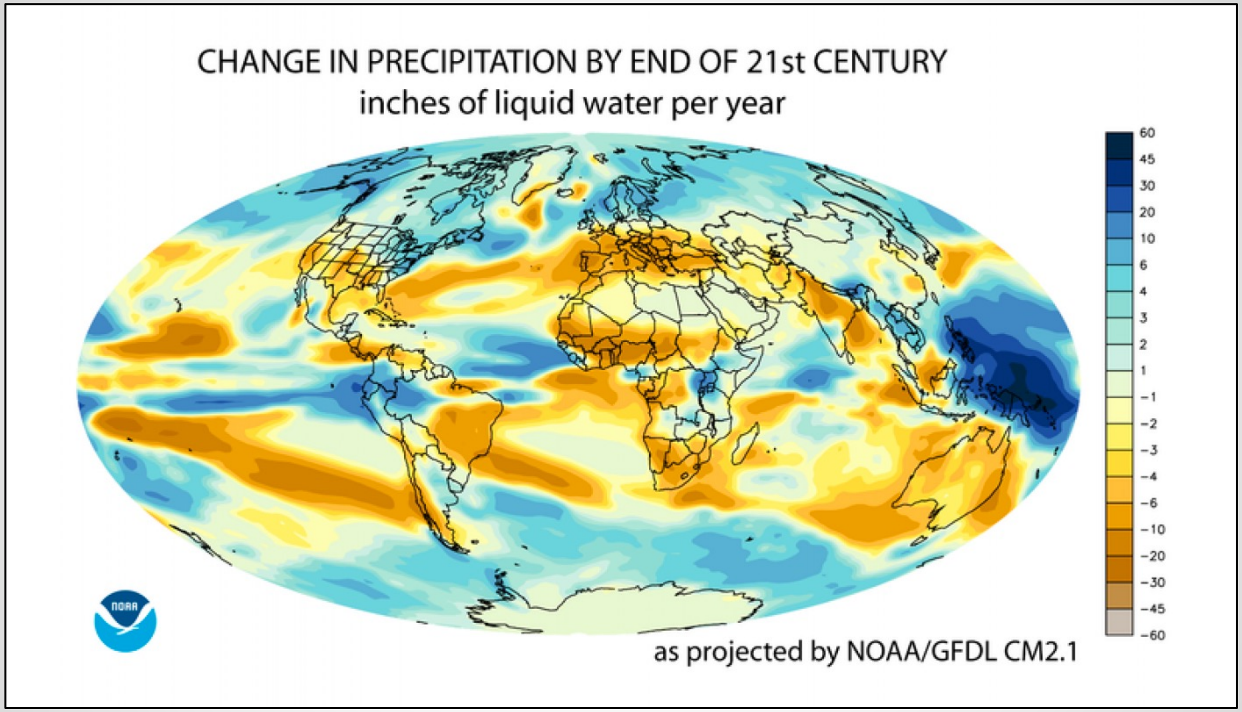
Statistical mechanics of superpositions of particles

Quantum kinetic equation for density matrix $\rho(t, \boldsymbol{r}, \boldsymbol{p})$:

$$\underbrace{i\left(\partial_t + \hat{\mathbf{p}} \cdot \partial_{\mathbf{r}}\right)\rho}_{\textbf{Particle advection}} = \underbrace{[H, \rho]}_{\substack{\textbf{Flavor}\\ \textbf{mixing}}} + \underbrace{iC}_{\textbf{Collisions}}$$

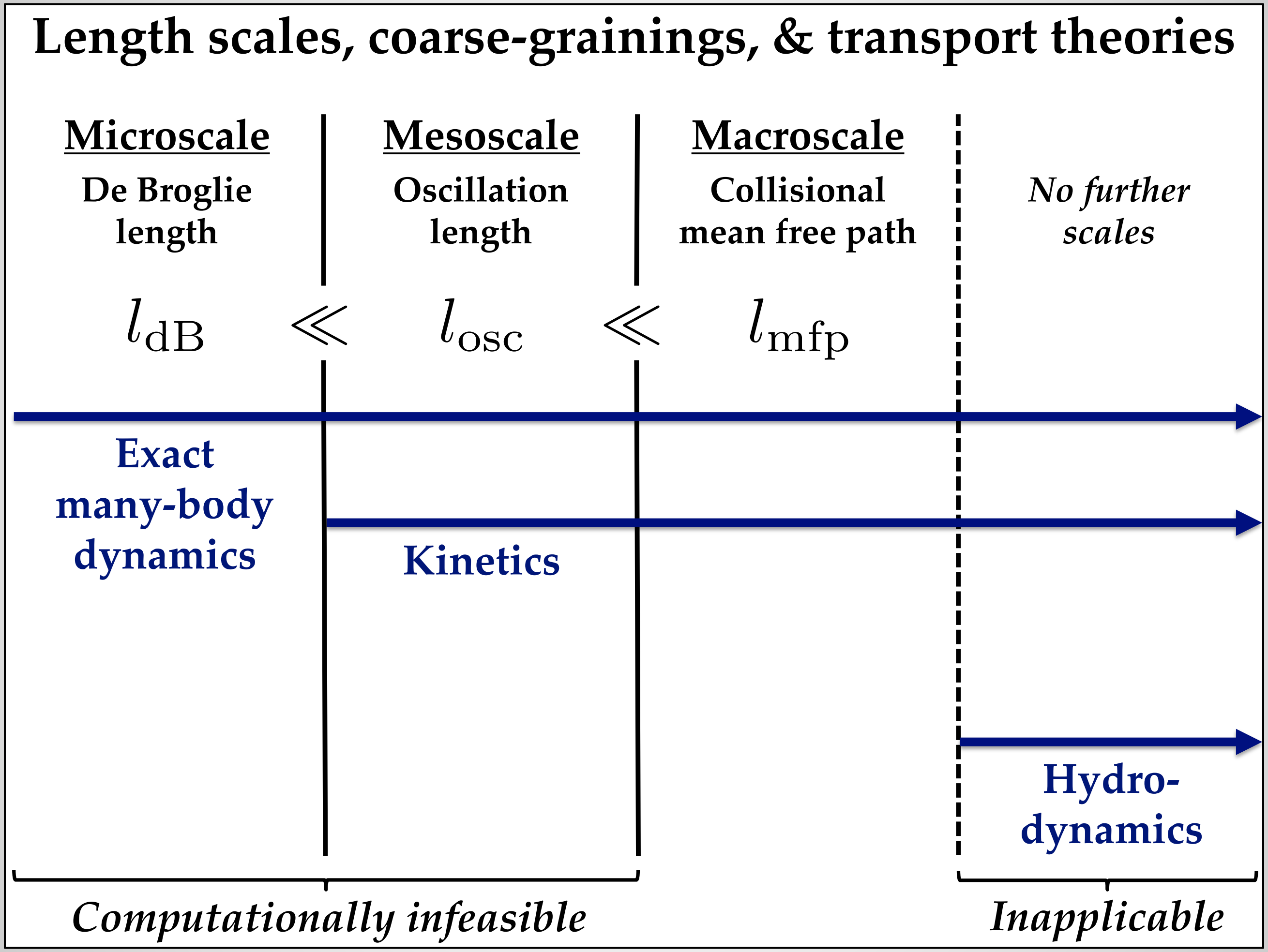
Dolgov, SJNP (1981); Stodolsky, PRD (1987); Nötzold & Raffelt, NPB (1988); Pantaleone, PLB (1992); Sigl & Raffelt, NPB (1993); Raffelt, Sigl, & Stodolsky, PRL (1993); Raffelt & Sigl, AP (1993); Loreti & Balantekin, PRD (1994); Yamada, PRD (2000); Friedland & Lunardini, PRD (2003); Strack & Burrows, PRD (2005); Cardall, PRD (2008); Volpe, Väänänen, & Espinoza, PRD (2013); Vlasenko, Fuller, & Cirigliano, PRD (2014); Kartavtsev, Raffelt, & Vogel, PRD (2015); Stirner, Sigl, & Raffelt, JCAP (2018); Nagakura, PRD (2022); **Johns**, 2305.04916; plus many others

Compare to climate modeling…

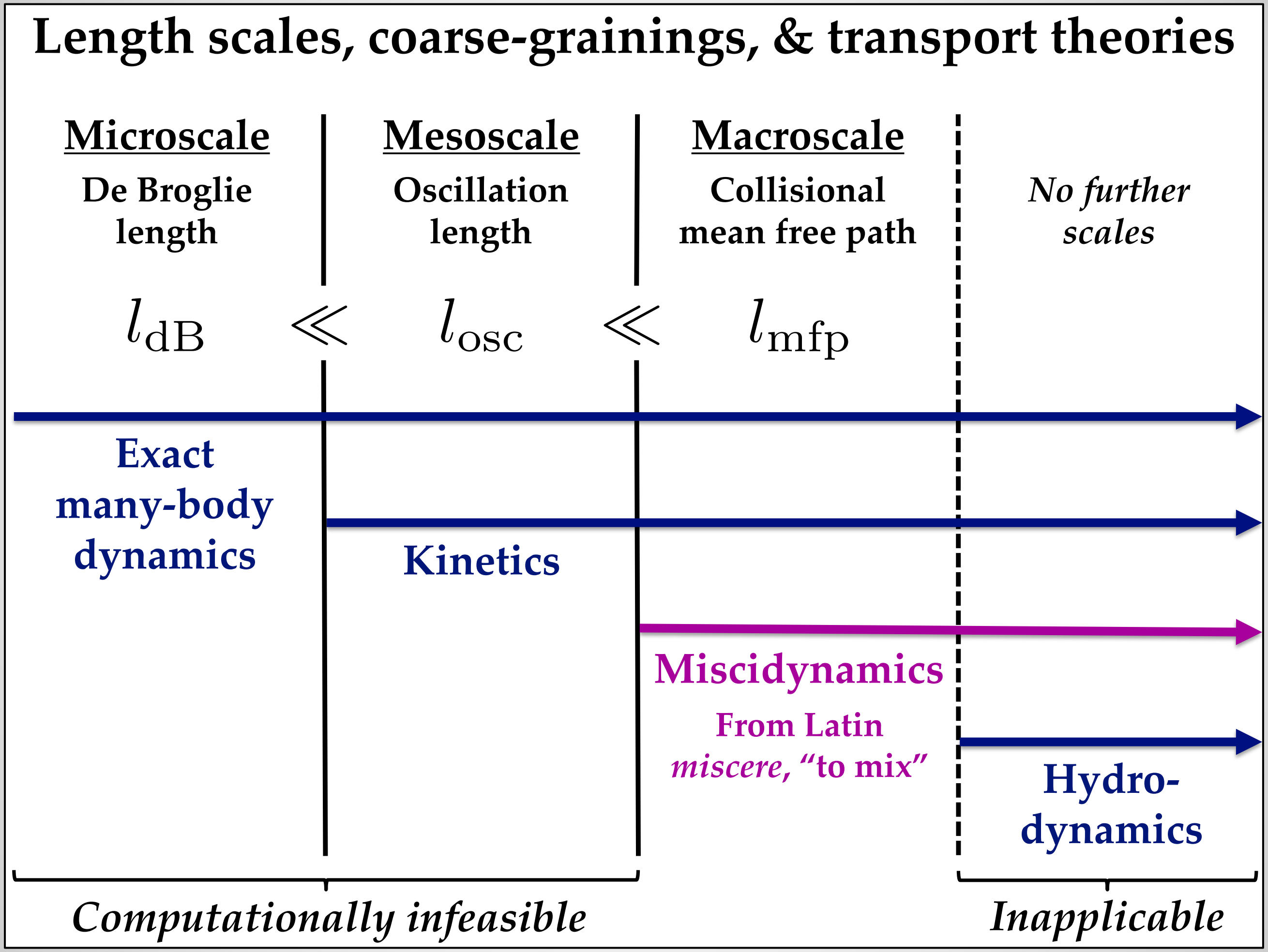


NOAA Geophysical Fluid Dynamics Laboratory

Without hydrodynamics, this field would not have gotten far.



Johns, 2306.14982



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Varieties of equilibration

Systems relax through information loss. Where does the information go?

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  Classical: *H*-theorem in Boltzmann gases.
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- **Many-body correlations**

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  Quantum: Thermalization in ultracold atomic gases.

- **Small scales**

  Classical: Collisionless relaxation in galaxies & plasmas. Turbulence.
  Quantum: Mixing equilibration.

To formulate the thermodynamics of oscillating neutrinos, we need to define an appropriate entropy:

$$S = -\int d\mathbf{p}\ \mathrm{Tr}\left[\overline{\rho_{\mathbf{p}}} \log \overline{\rho_{\mathbf{p}}} + (1 - \overline{\rho_{\mathbf{p}}}) \log (1 - \overline{\rho_{\mathbf{p}}})\right]$$

Spatially coarse-grained density matrix

Johns, 2306.14982

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Spatially coarse-grained density matrix

<u>Second law of thermodynamics</u>

S is maximal at equilibrium, subject to conservation laws.

We then appeal to two physical principles: **scale separation** & **ergodicity**.

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Equilibrium distribution of collisionless neutrino matter:

$$\rho_{\mathbf{p}}^{\mathrm{eq}} = \frac{1}{e^{\beta\left(H_{\mathbf{p}}^{\mathrm{eq}} - \mu_{\mathbf{p}}\right)} + 1}$$

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<u>Third law of thermodynamics</u>

The unique ($S = 0$) ground state:

$$\left(\rho^{\text{eq}}_{\mathbf{p}}\right)_{IJ} \xrightarrow{T \to 0} \begin{cases} \delta_{IJ} & \left(H^{\text{eq}}_{\mathbf{p}}\right)_{IJ} \leq \mu_{\mathbf{p}} \\ 0 & \left(H^{\text{eq}}_{\mathbf{p}}\right)_{IJ} > \mu_{\mathbf{p}} \end{cases}$$

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First law of thermodynamics

$$\Delta U = W + Q$$

with W and Q appropriately defined.

$$\Delta U = \overbrace{\frac{1}{N_f} H_0 \Delta P_0 + \frac{1}{2}\vec{H} \cdot \Delta|\vec{P}|\hat{P}}^{\equiv Q^{\mathrm{env}}} + \overbrace{\frac{1}{2}|\vec{H}||\vec{P}|\Delta\left(\hat{H}\cdot\hat{P}\right)}^{\equiv Q^{\mathrm{kin}}}$$

$$+ \underbrace{\frac{1}{N_f}\Delta H_0 P_0 + \frac{1}{2}\Delta|\vec{H}||\vec{P}|\hat{H}\cdot\hat{P}}_{\equiv W}$$

$$\rho = \frac{1}{N_f} P_0 + \frac{1}{2}\vec{P}\cdot\vec{\Lambda}$$

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From here it's easy to show that quantum-adiabatic effects (MSW, spectral swaps, MNR) are **adiabatic processes in the thermodynamic sense**.

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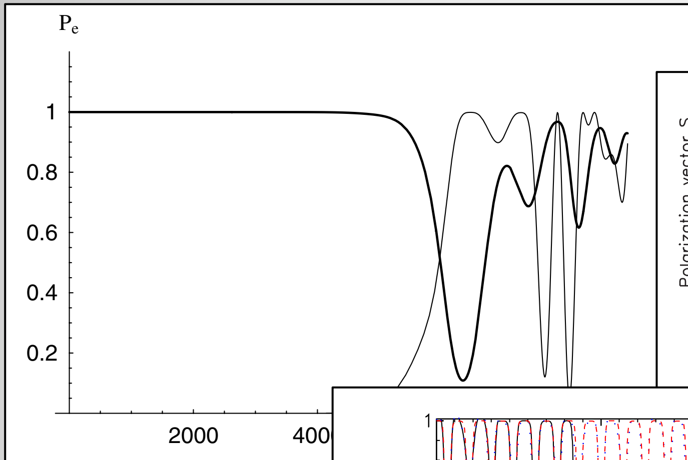
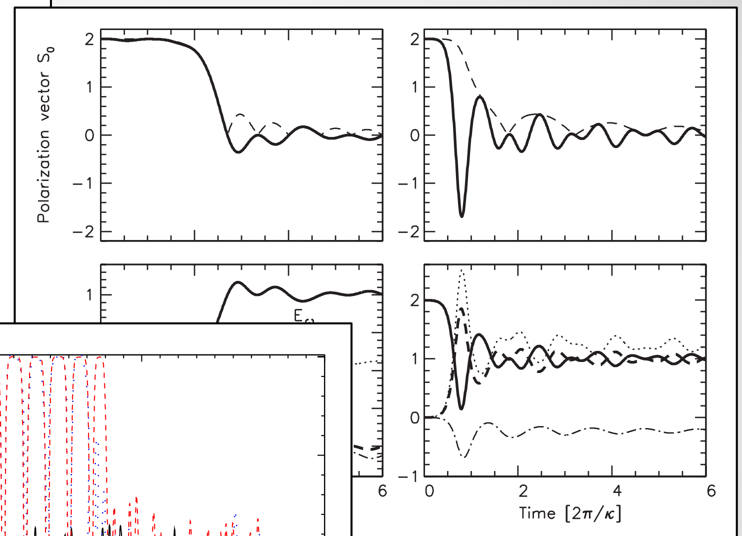
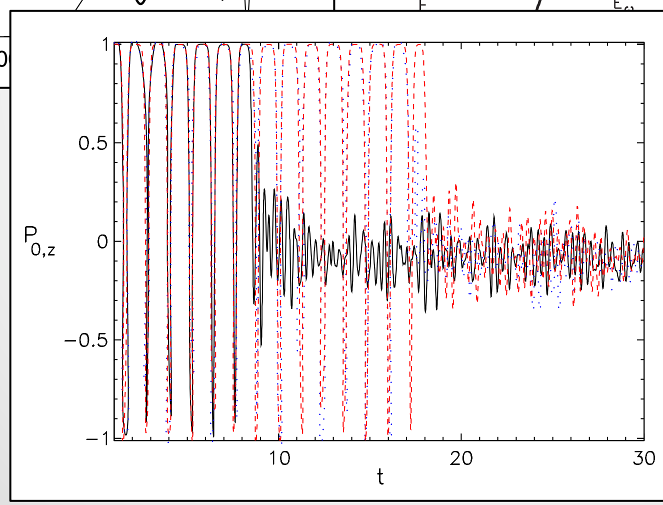
Quasi-steady states are the numerically observed outcomes of collisionless instabilities:

Sawyer, PRD (2005)



Raffelt & Sigl, PRD (2007)





Mangano, Mirizzi, & Saviano, PRD (2014)

+ many more papers
on fast instabilities

Bhattacharyya & Dasgupta, PRD (2020)
Richers, Willcox, & Ford, PRD (2021)
Nagakura & Zaizen, PRL (2022) & others

Thermodynamics predicts the mean asymptotic distributions.

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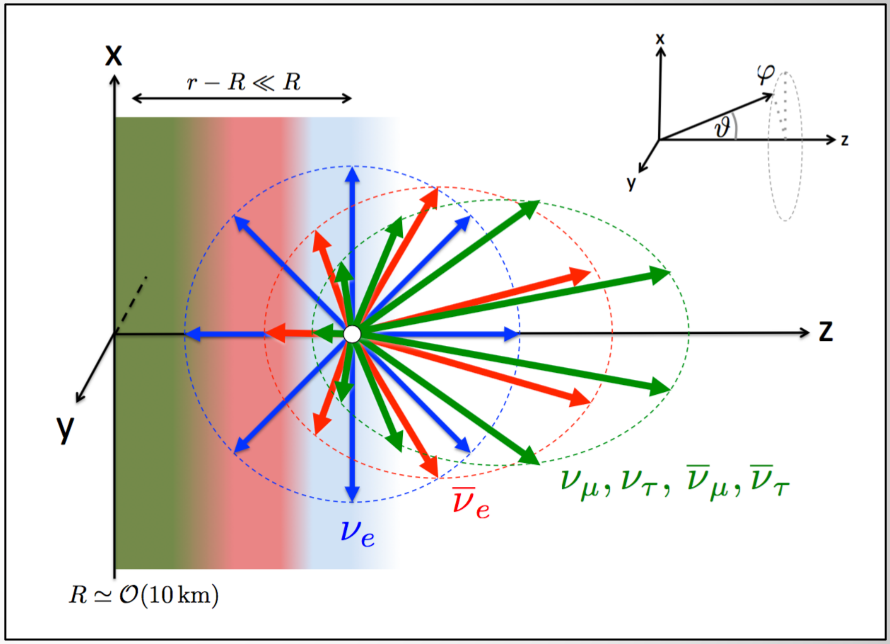
Thermodynamics also explains the association between fast instabilities and **angular crossings**.



Sawyer, PRL (2016); Chakraborty, Hansen, Izaguirre, & Raffelt, JCAP (2016); Dasgupta, Mirizzi, & Sen, JCAP (2017) [*figure*];
Izaguirre, Raffelt, & Tamborra, PRL (2017); Capozzi, Dasgupta, Lisi, Marrone, & Mirizzi, PRD (2017);
Abbar & Duan, PRD (2018); Capozzi, Dasgupta, Mirizzi, Sen, & Sigl, PRL (2019); Martin, Yi, & Duan, PLB (2020);
Johns, Nagakura, Fuller, & Burrows, PRD (2020a); **Johns** & Nagakura, PRD (2021); Nagakura, Burrows, **Johns**, & Fuller, PRD (2021);
Morinaga, PRD (2022); Dasgupta, PRL (2022); & many others

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Non-collective neutrino oscillations

Work | **The MSW effect**

Wolfenstein, PRD (1978)
Mikheyev & Smirnov, SJNP (1985)
Bethe, PRL (1986)

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Heat (internal)

Kinematic decoherence

Nussinov, PLB (1976)
Kayser, PRD (1981)
Kiers, Nussinov, & Weiss, PRD (1996)

Johns, 2306.14982

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Heat (internal)	**Kinematic decoherence** Nussinov, PLB (1976) Kayser, PRD (1981) Kiers, Nussinov, & Weiss, PRD (1996)
Heat (external)	**Collisional decoherence** Harris & Stodolsky, PLB (1978) Thomson, PRA (1992) Raffelt, Sigl, & Stodolsky, PRL (1993)

Johns, 2306.14982

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Work **MSW-like effects** **Spectral swaps. Matter–neutrino resonances.**

Duan, Fuller, Carlson, & Qian, PRD (2006)
Raffelt & Smirnov, PRD (2007)
Malkus, Friedland, & McLaughlin, 1403.5797

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Heat (internal)

Collisionless instabilities **Slow instabilities. Fast instabilities.**

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Johns, PRL (2023)
Xiong, **Johns**, Wu, & Duan, 2212.03750
Liu, Zaizen, & Yamada, PRD (2023)

Johns, 2306.14982

Local mixing equilibrium

$$\rho \longrightarrow \rho^{\mathrm{eq}}$$

The miscidynamic equation

$$i\left(\partial_t + \hat{\mathbf{p}} \cdot \partial_{\mathbf{x}}\right)\rho^{\mathrm{eq}} = iC^{\mathrm{eq}}$$

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What changes need to be made to current simulations?

(1) Distribution functions $\longrightarrow$ Density matrices.

(2) Add off-diagonals to collision terms.

(3) Re-equilibrate ρ after each step.

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Main idea #2. We've outlined the thermodynamic theory of oscillating neutrinos. The primary equilibration mechanism is collisionless phase-space transfer.

Main idea #3. A transport theory—miscidynamics—follows from the assumption of local equilibrium. It might enable the accurate incorporation of neutrino mixing into simulations.