

Collective Neutrino Oscillations

Huaiyu Duan



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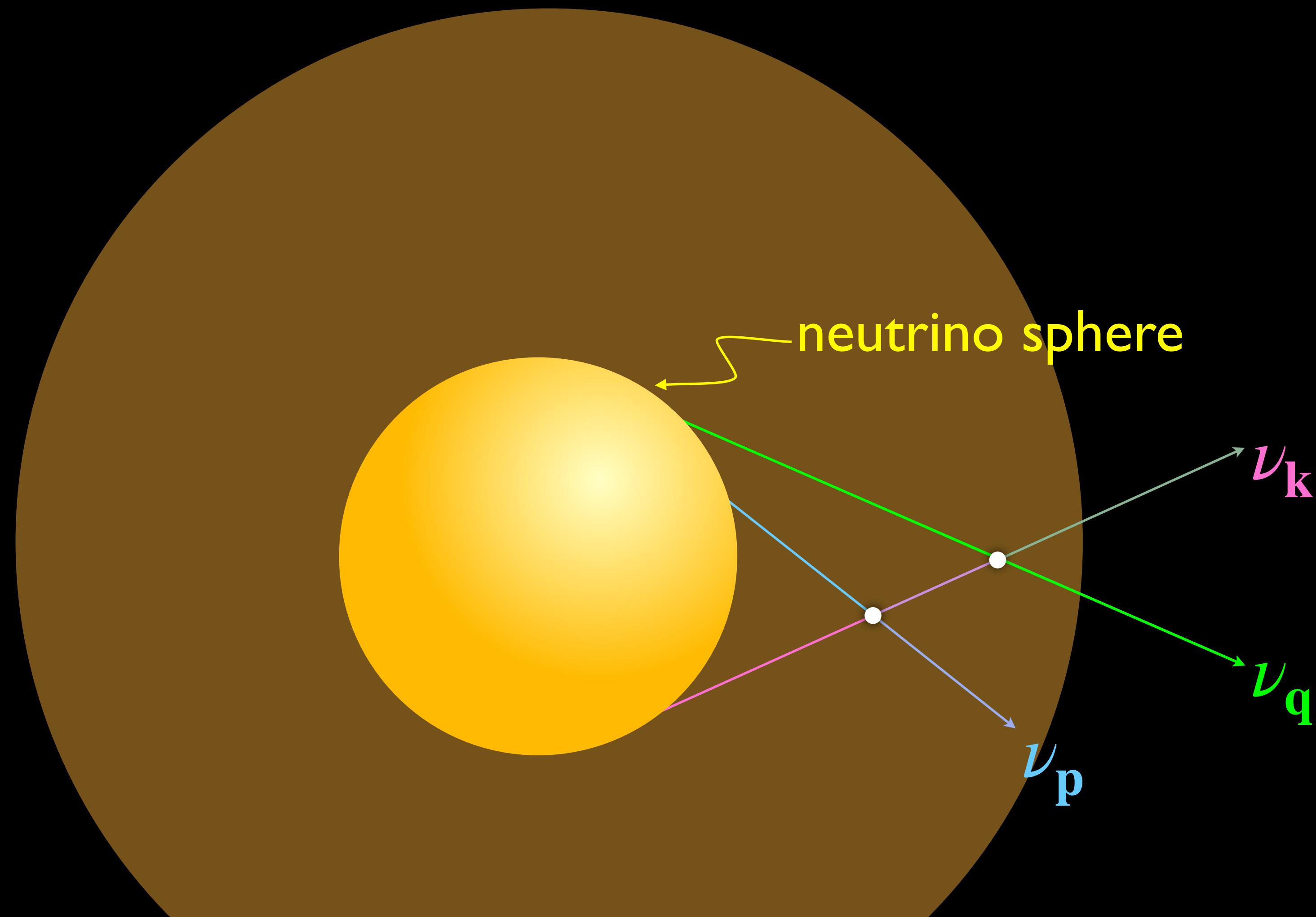


Outline

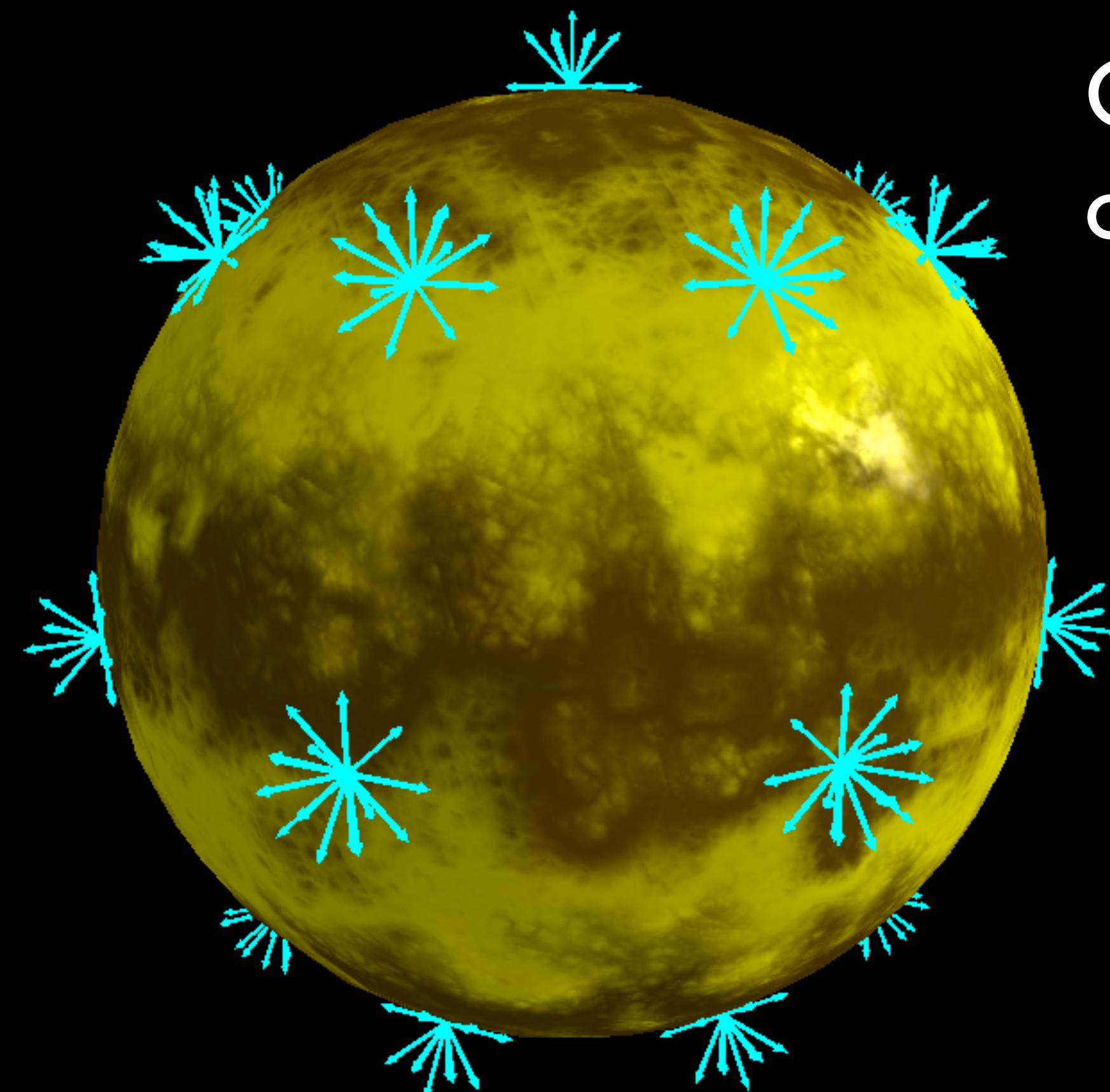
- Introduction to collective neutrino oscillations
- Flavor instabilities & fast flavor conversion
- Effects of collisions on collective oscillations
- “Integrating” collective oscillations in SN simulations
- Summary

Introduction

Dense Neutrino Gas



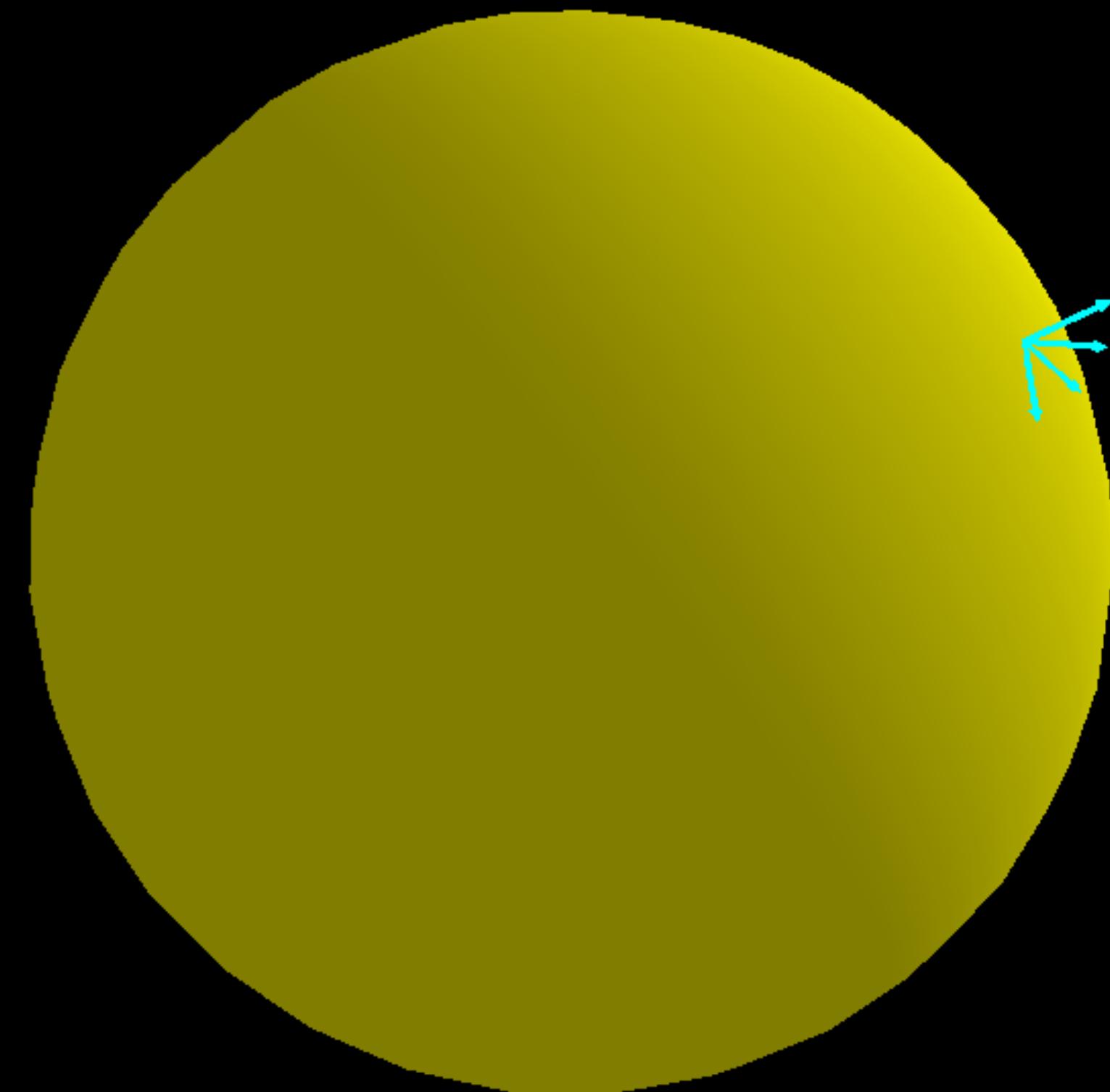
7D Problem



Coherent forward scattering
outside neutrino sphere

$$\rho_{\mathbf{p}}(t, \mathbf{r})$$

Bulb Model



Azimuthal symmetry around
any radial direction

$$\rho_{E,\vartheta}(r)$$

Flavor Transport

Equation of motion

$$(\partial_t + \hat{\mathbf{v}} \cdot \nabla) \rho = -i[\mathcal{H}, \rho] + \mathcal{C} \xleftarrow{\text{Collision}}$$

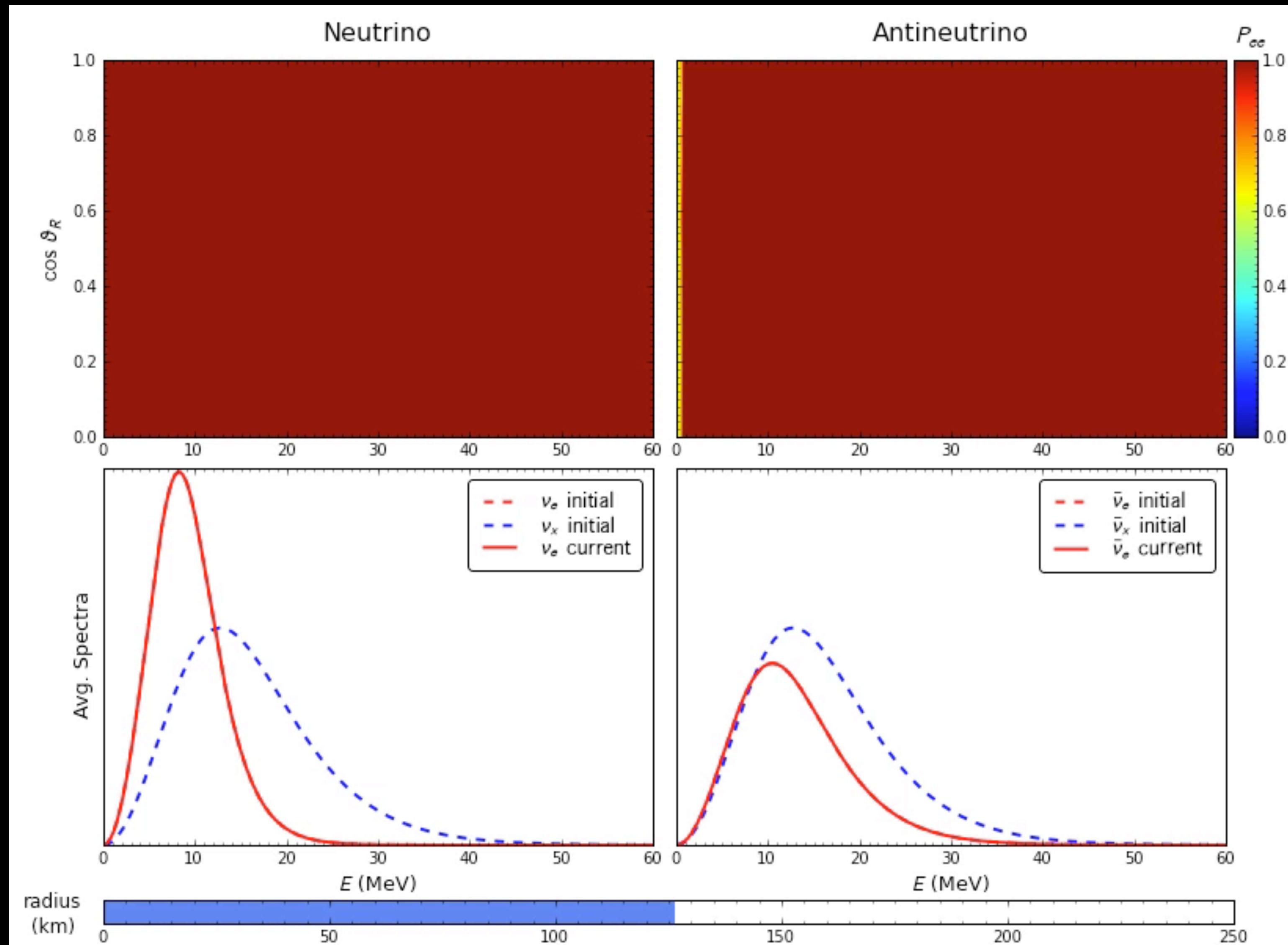
$$\mathcal{H} = \frac{M^2}{2E} + \sqrt{2}G_F \text{diag}[n_e, 0, 0] + \mathcal{H}_{\nu\nu}$$

mass matrix $\xrightarrow{\quad}$ electron density \downarrow
neutrino energy $\xleftarrow{\quad}$ $\nu\text{-}\nu$ forward scattering
 (self-coupling) \uparrow

$$\mathcal{H}_{\nu\nu} = \sqrt{2}G_F \int d^3\mathbf{p}' (1 - \hat{\mathbf{v}} \cdot \hat{\mathbf{v}}') (\rho_{\mathbf{p}'} - \bar{\rho}_{\mathbf{p}'})$$

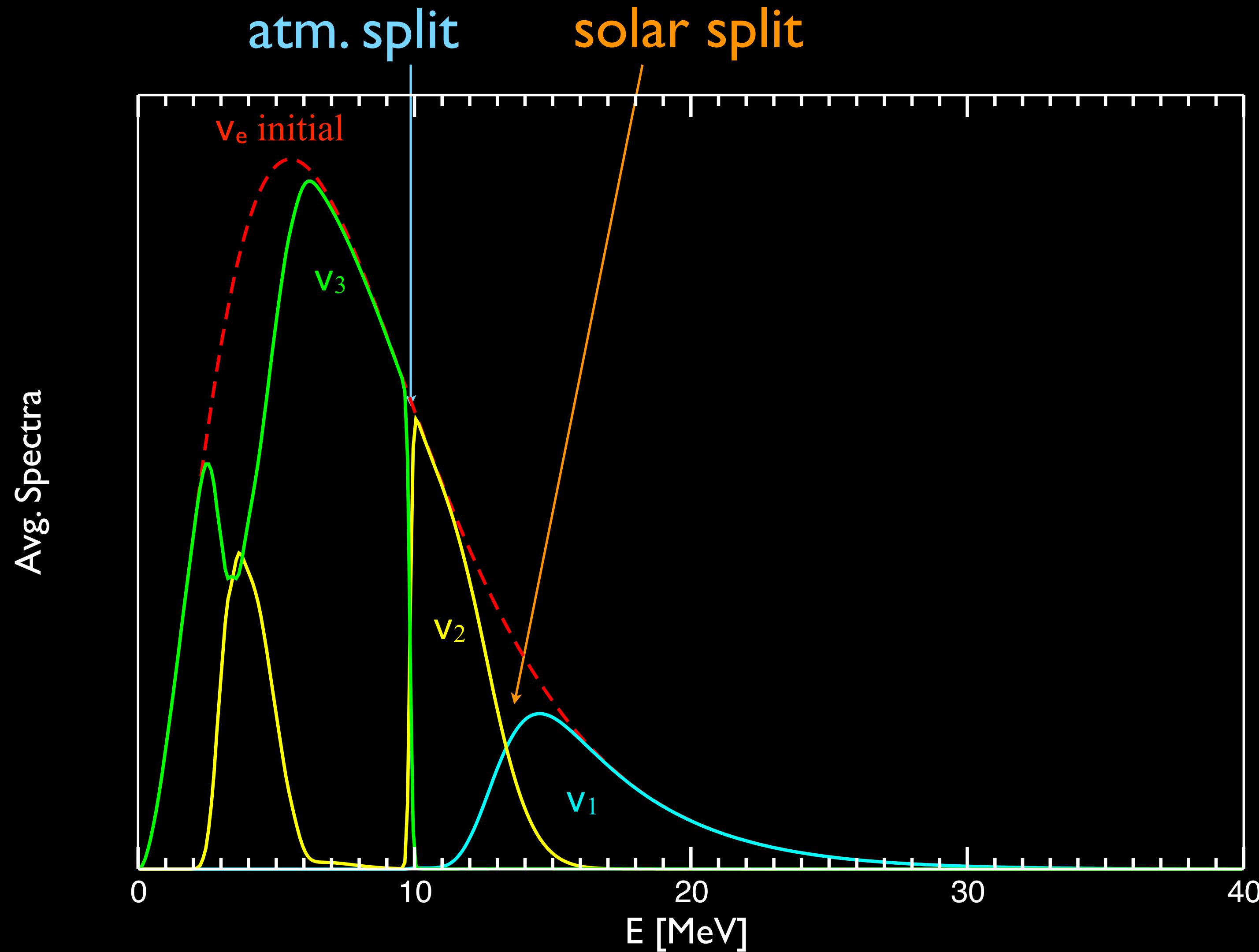
$$\langle L_{\nu_e} \rangle = 4.1 \text{ foe}, \langle L_{\bar{\nu}_e} \rangle = 4.3 \text{ foe}, \langle L_{\nu_x, \bar{\nu}_x} \rangle = 7.9 \text{ foe}$$

$$\langle E_{\nu_e} \rangle = 9.4 \text{ MeV}, \langle E_{\bar{\nu}_e} \rangle = 13.0 \text{ MeV}, \langle E_{\nu_x, \bar{\nu}_x} \rangle = 15.8 \text{ MeV}$$



Spectral Splits

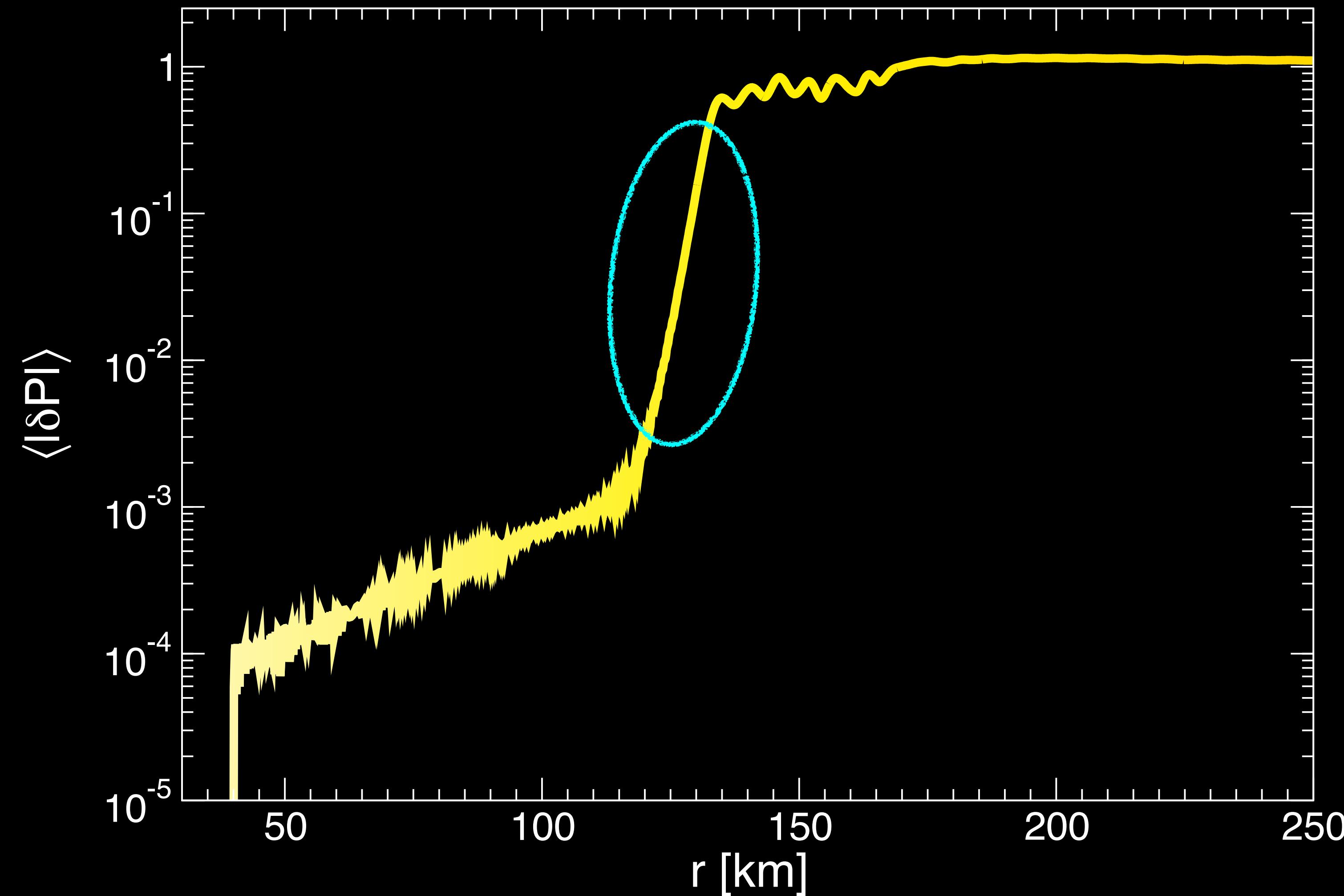
Neutronization burst



HD+ (2007), Cherry+ (2010)

Flavor Instabilities and Fast Flavor Conversion

Exponential Growth



HD & Friedland (2010)

Linear Stability Analysis

Homogeneous and isotropic gas

Electron flavor neutrinos and antineutrinos initially

$$\rho \propto \begin{bmatrix} 1 & S \\ S^* & 0 \end{bmatrix} \quad \bar{\rho} \propto \begin{bmatrix} 1 & \bar{S} \\ \bar{S}^* & 0 \end{bmatrix} \quad |S| \ll 1$$

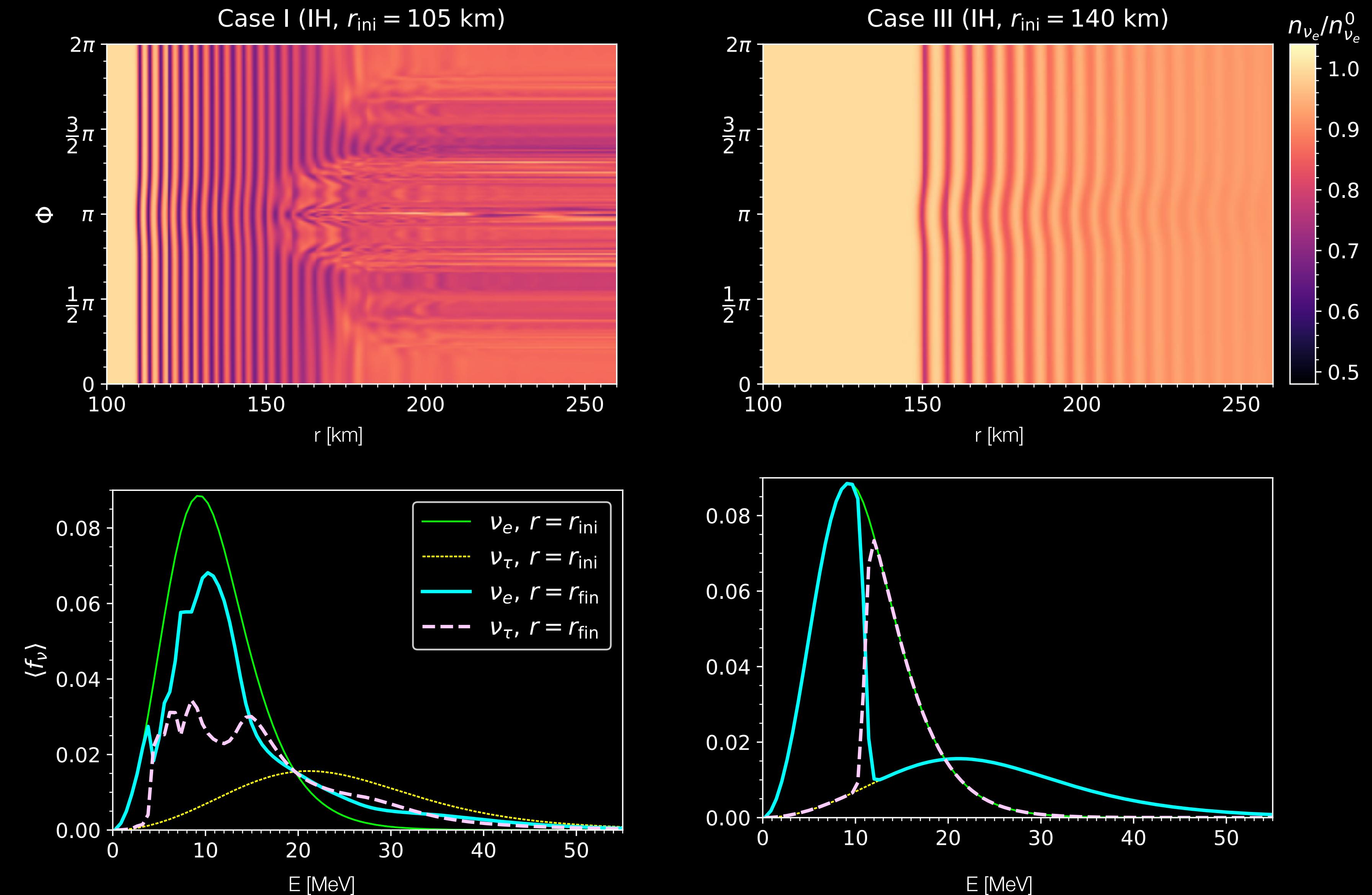
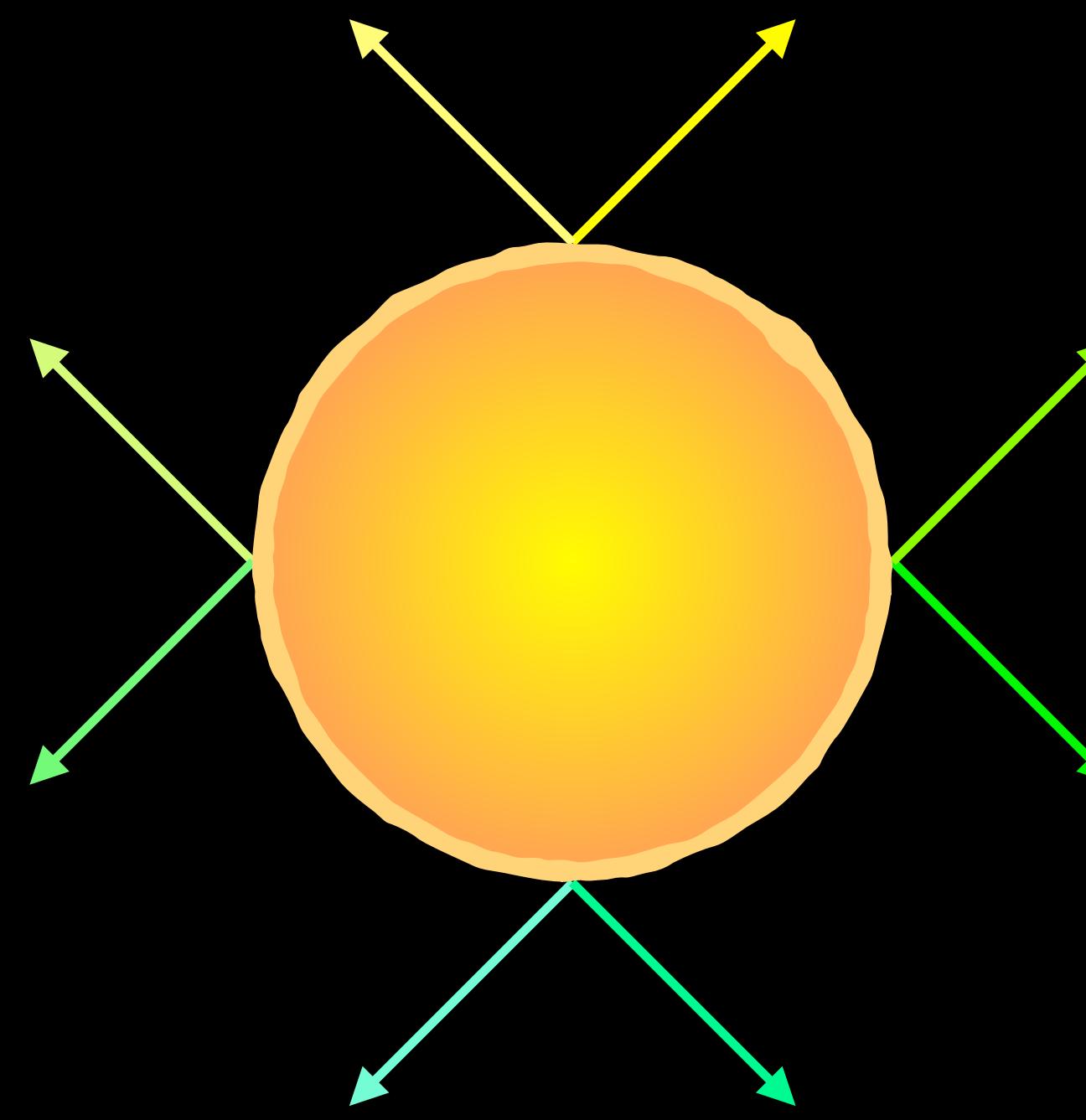
$$i \begin{bmatrix} \dot{S} \\ \dot{\bar{S}} \end{bmatrix} \approx \begin{bmatrix} -\omega - \cancel{\alpha}\mu & \cancel{\alpha}\mu \\ -\cancel{\nu} & \omega + \cancel{\mu} \end{bmatrix} \begin{bmatrix} S \\ \bar{S} \end{bmatrix}$$

$\omega = \Delta m^2/2E$
 $\alpha = n_{\bar{\nu}}/n_{\nu}$
 ~~$\mu \propto n_{\bar{\nu}}$~~

- Normal modes \rightarrow Collective oscillations ($S, \bar{S} \sim e^{-i\Omega t}$)
- $\text{Im}(\Omega) > 0 \rightarrow$ Flavor instabilities

Breaking the Symmetries

Ring model



Collective Oscillation Wave

Dynamic models

$$S_{\mathbf{p}}(t, \mathbf{r}) \propto e^{-i(\Omega t - \mathbf{K} \cdot \mathbf{r})}$$

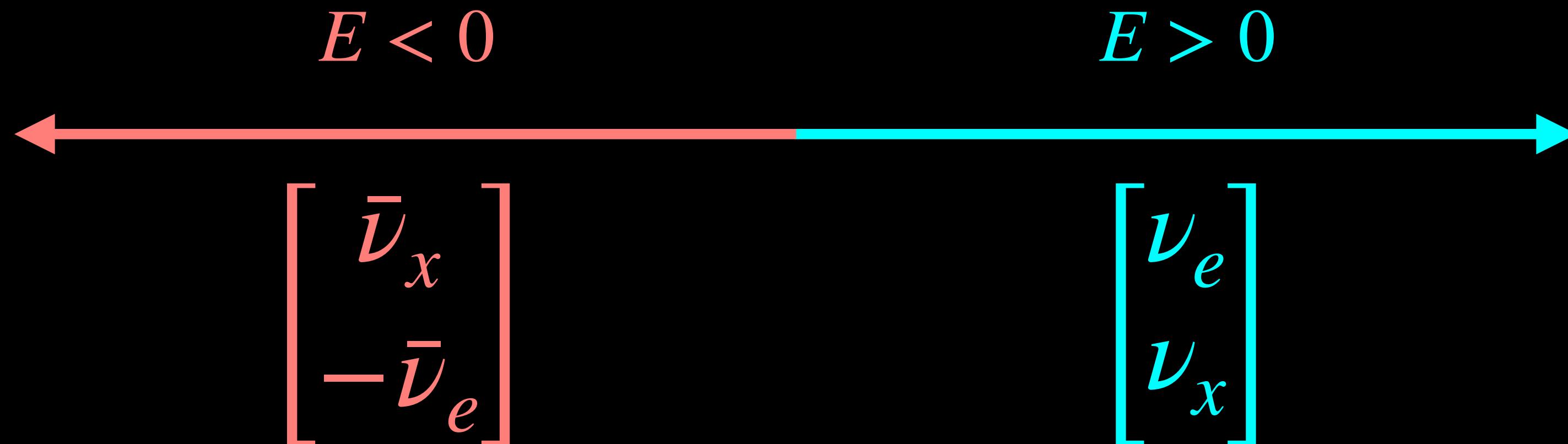
Duan+ (2008)
Izaguirre+ (2016)

- Collective flavor oscillations are the collective wave modes in the neutrino gas with the dispersion relation $\Omega(\mathbf{K})$.
- $\text{Im}(\Omega) > 0 \rightarrow$ Flavor instabilities.
- **Slow** oscillations occur on the distance scale of **1 km** ($\sim 10 \text{ MeV}/\Delta m_{\text{atm}}^2$).
- **Fast** oscillations can occur on the distance scale of **1 cm** ($\sim 1/G_F n_\nu$), independent of the neutrino energies (Sawyer, 2016).

Neutrino Flavor-Spin and Distribution

Aka flavor isospin and ELN distribution

$$G(E, \hat{v}) = \begin{cases} f_{\nu_e} - f_{\bar{\nu}_x} & \text{if } E > 0 \\ f_{\bar{\nu}_x} - f_{\bar{\nu}_e} & \text{if } E < 0 \end{cases}$$



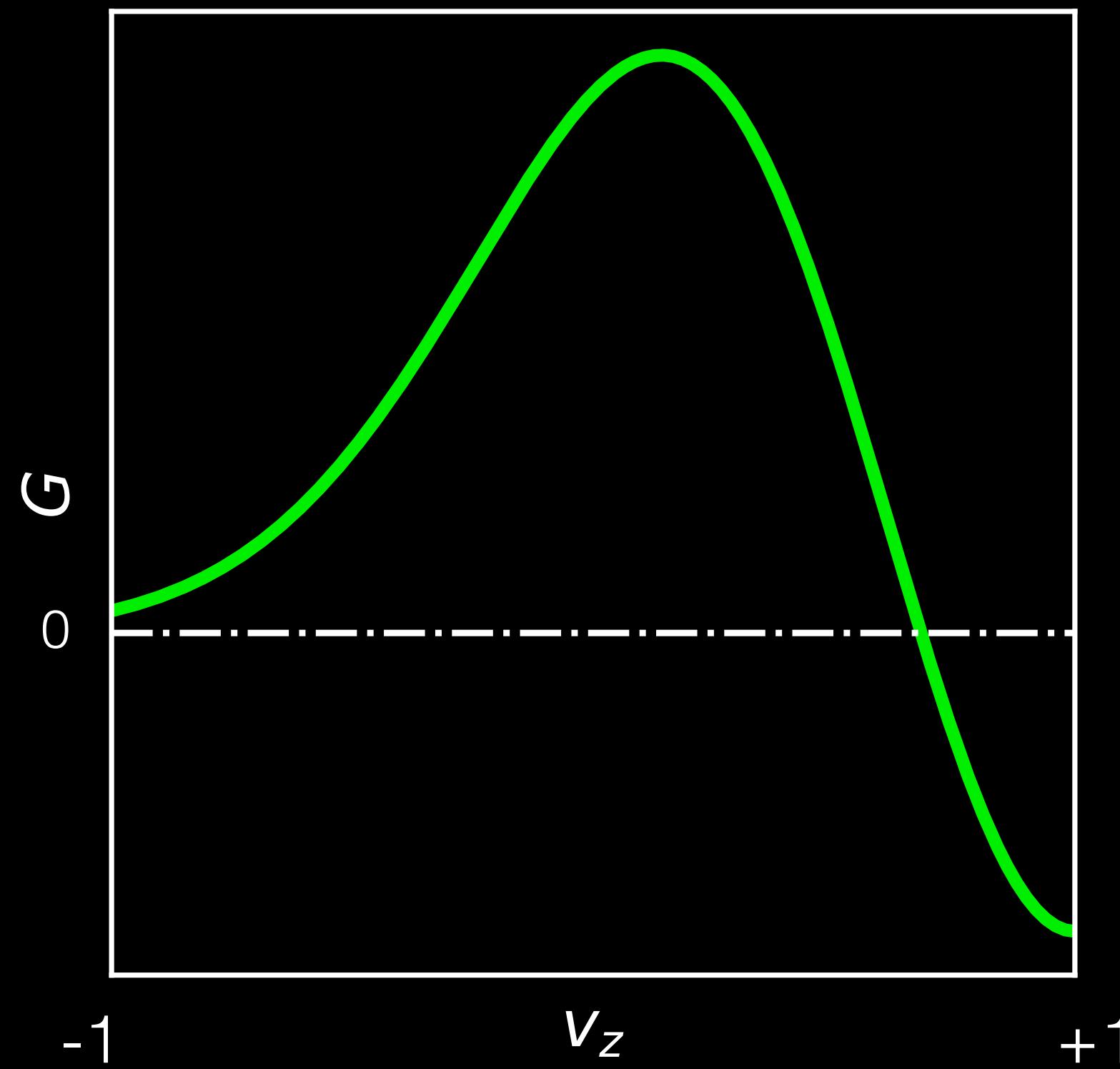
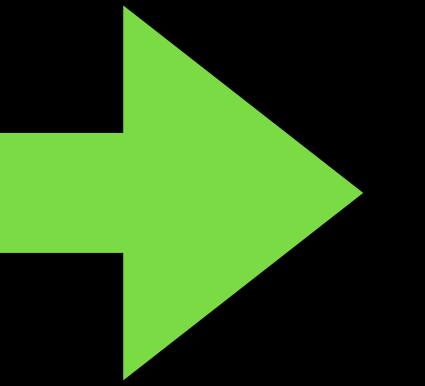
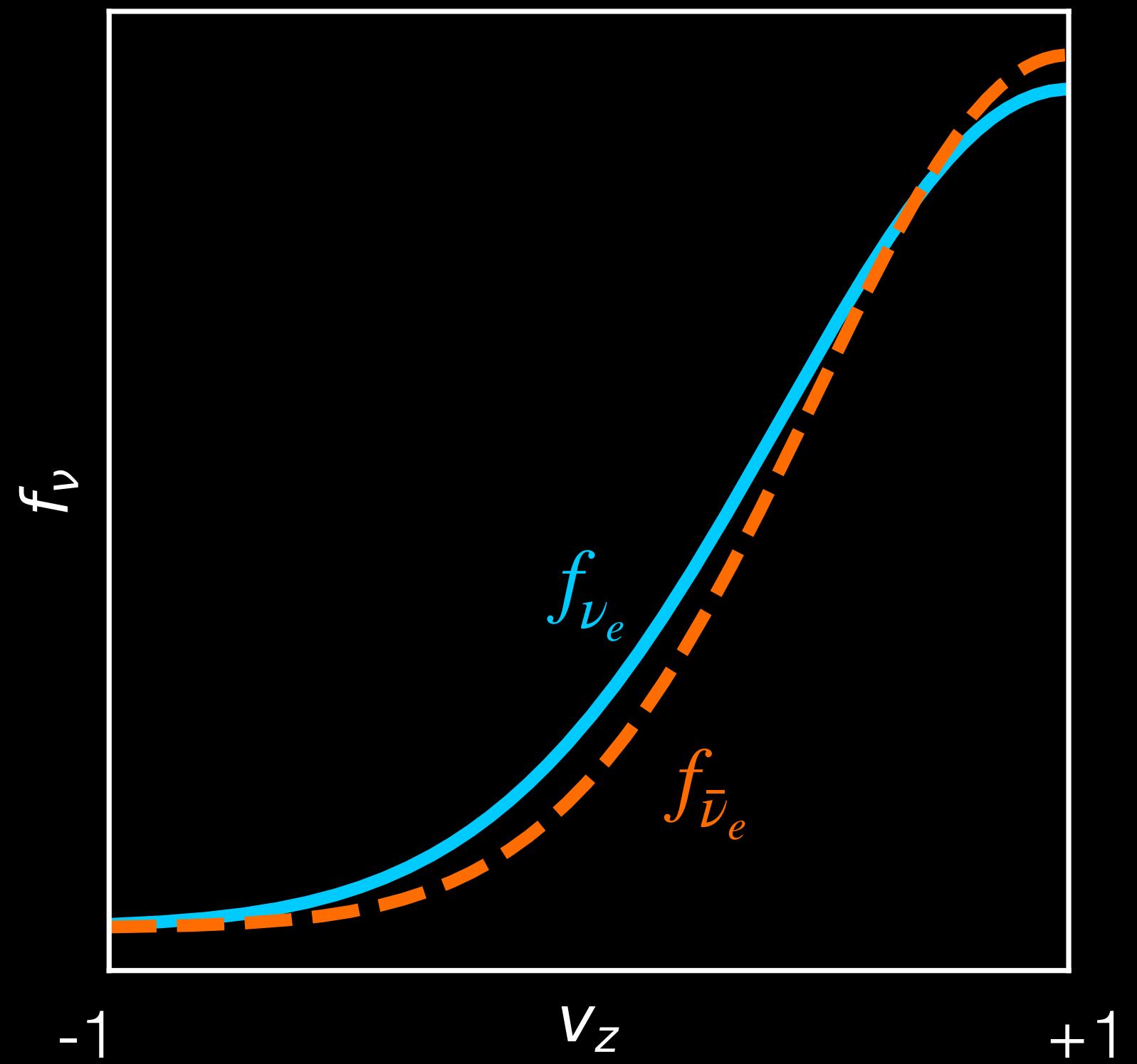
Flavor Instability and Crossing

$$G(E, \hat{v}) = \begin{cases} f_{\nu_e} - f_{\nu_x} & \text{if } E > 0 \\ f_{\bar{\nu}_x} - f_{\bar{\nu}_e} & \text{if } E < 0 \end{cases}$$

- **Slow** flavor instability requires crossing in $G(E)$: identical neutrino angular distribution.
- **Fast** flavor instability requires crossing in $G(\hat{v})$: independent of neutrino energy.

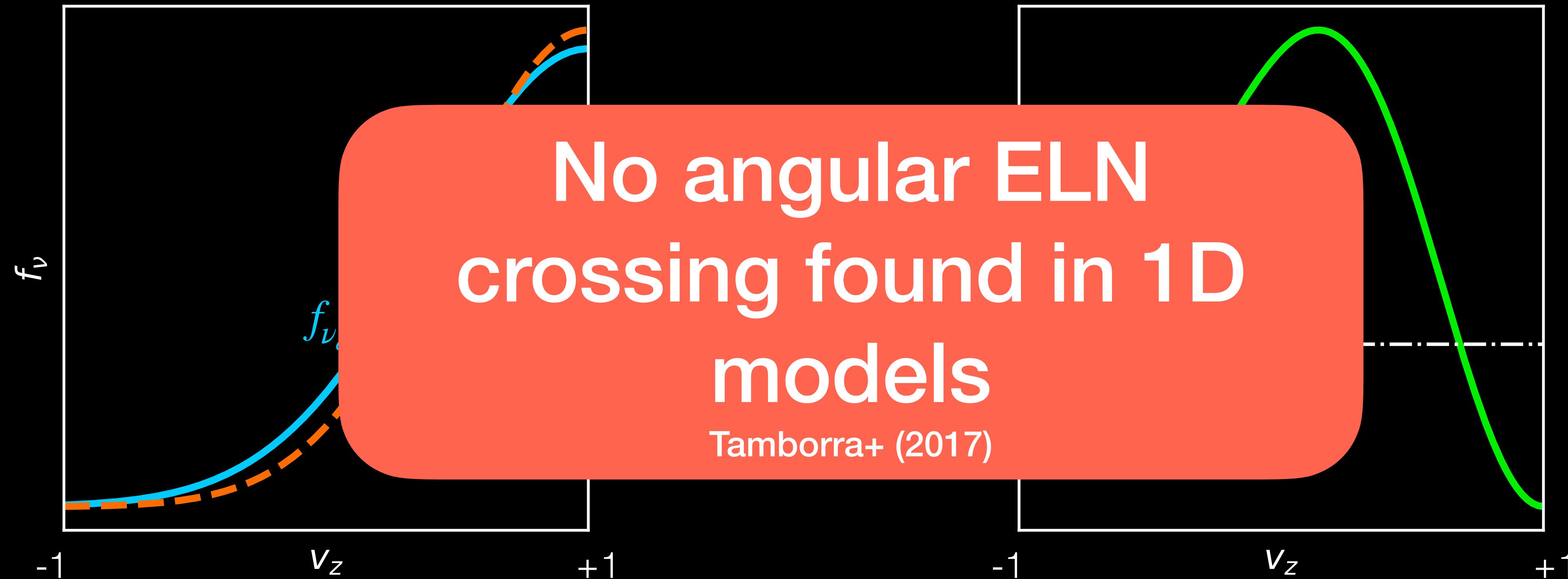
Angular ELN Crossing

$$G \sim (f_{\nu_e} - f_{\bar{\nu}_e}) - (f_{\nu_x} - f_{\bar{\nu}_x})$$



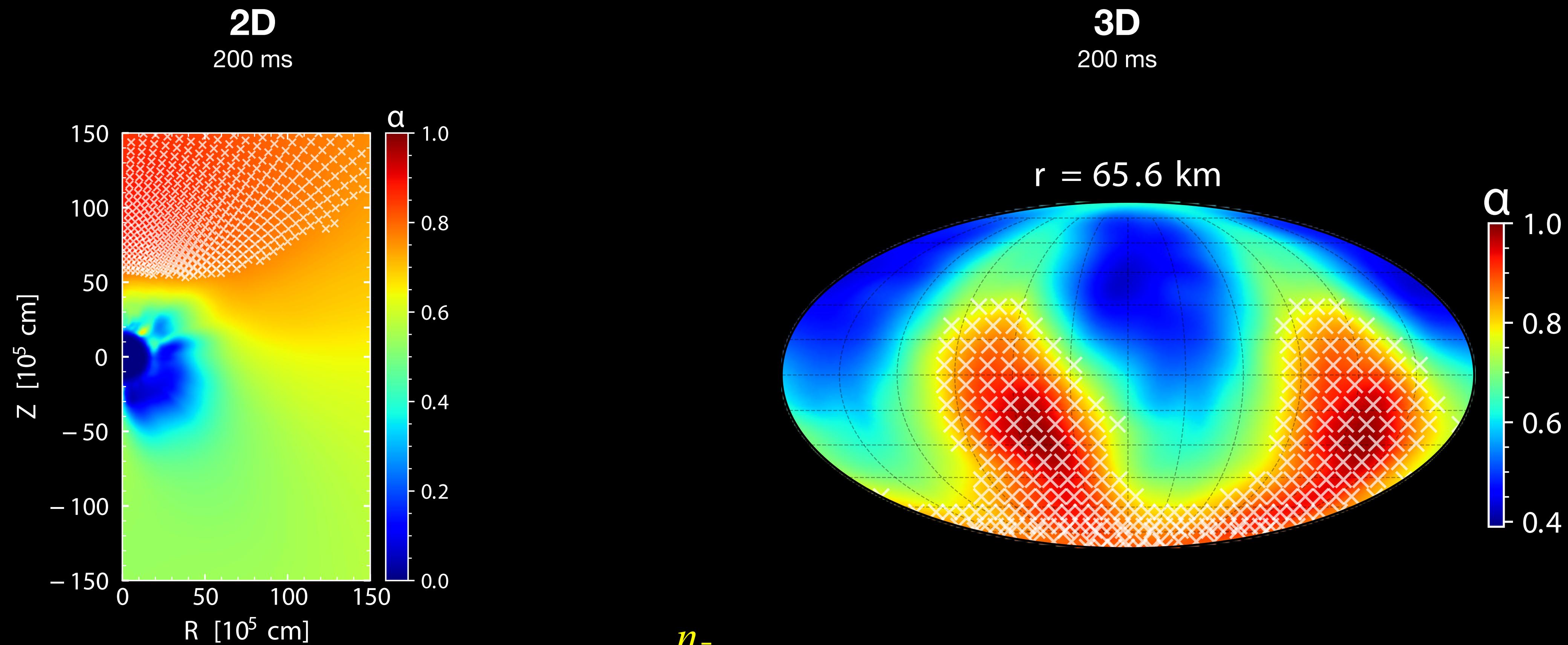
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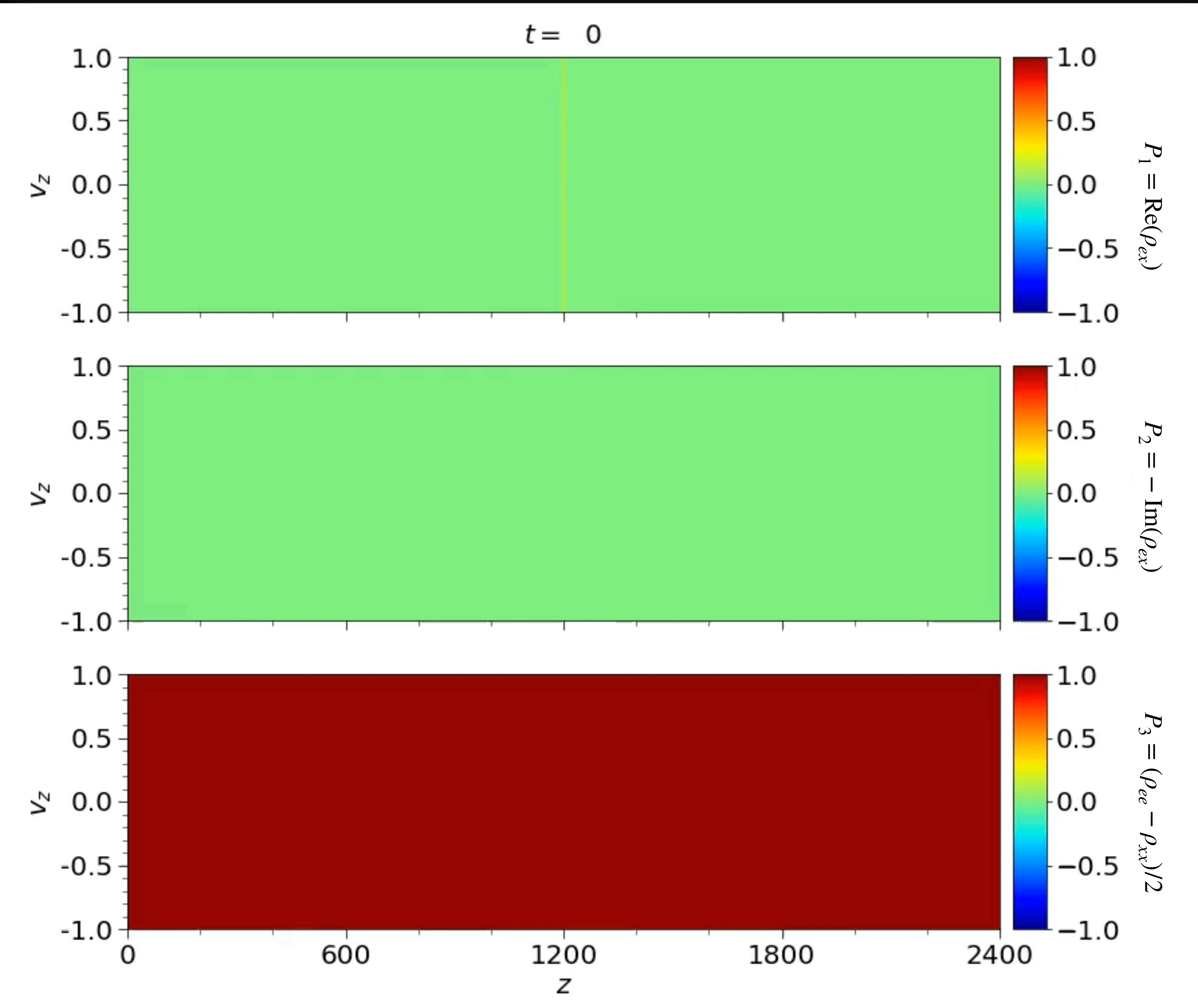


Angular ELN Crossing

2D and 3D models

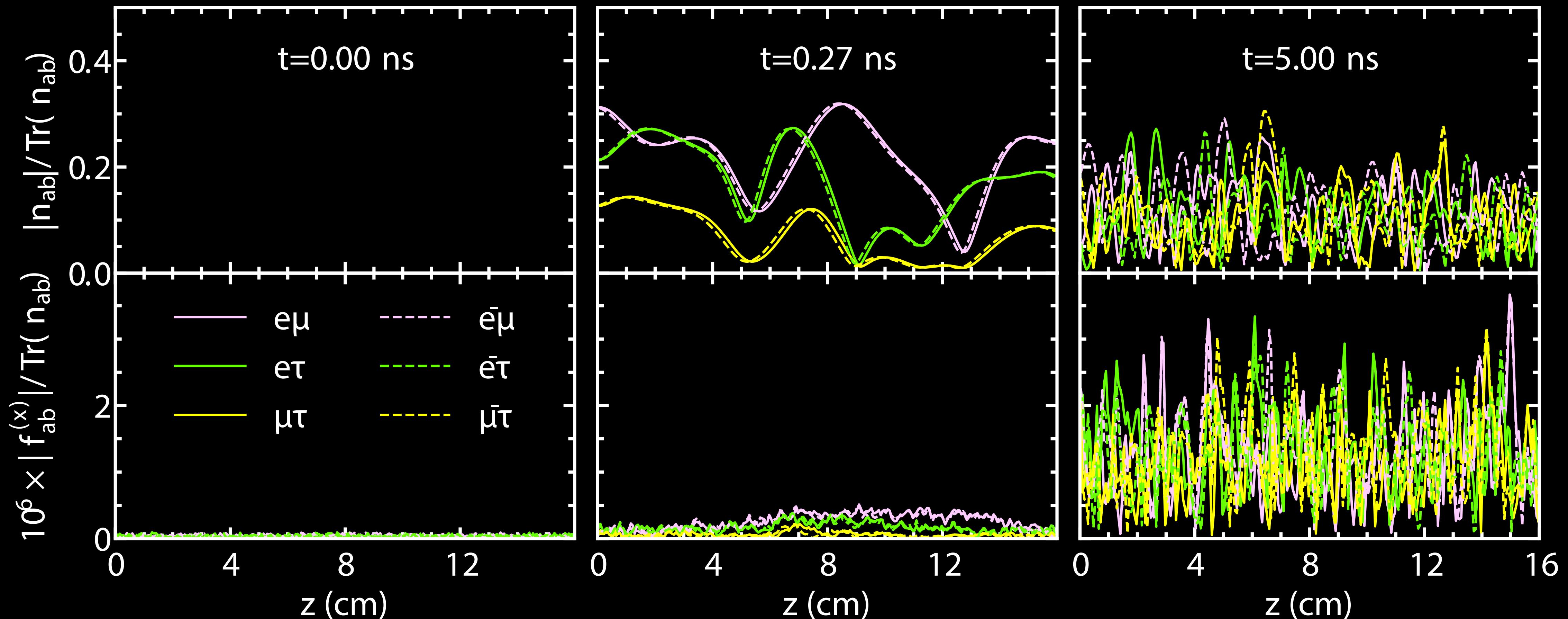


$$\alpha = \frac{n_{\bar{\nu}_e}}{n_{\nu_e}}$$



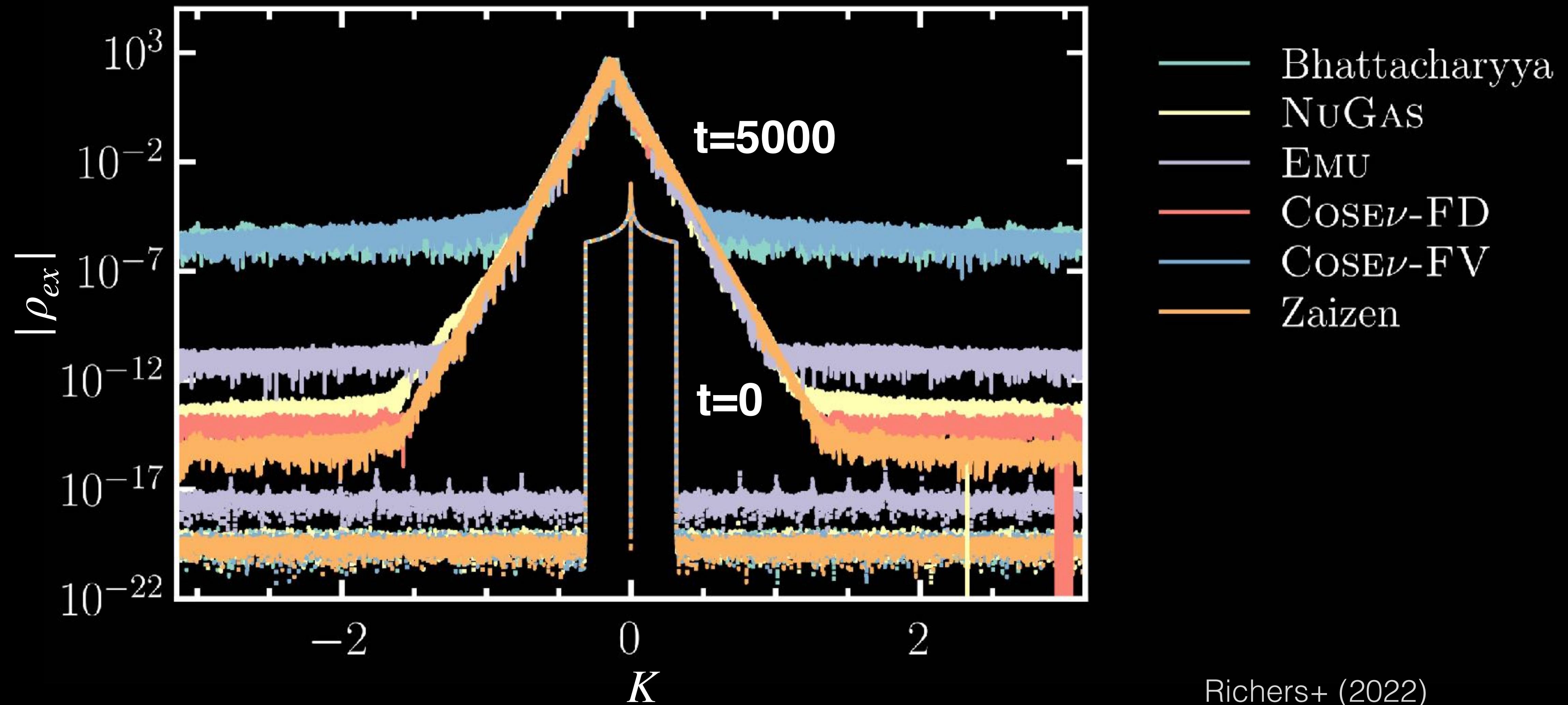
Long-Term Behavior

Kinematic decoherence



Long-Term Behavior

Persistent spatial coherence

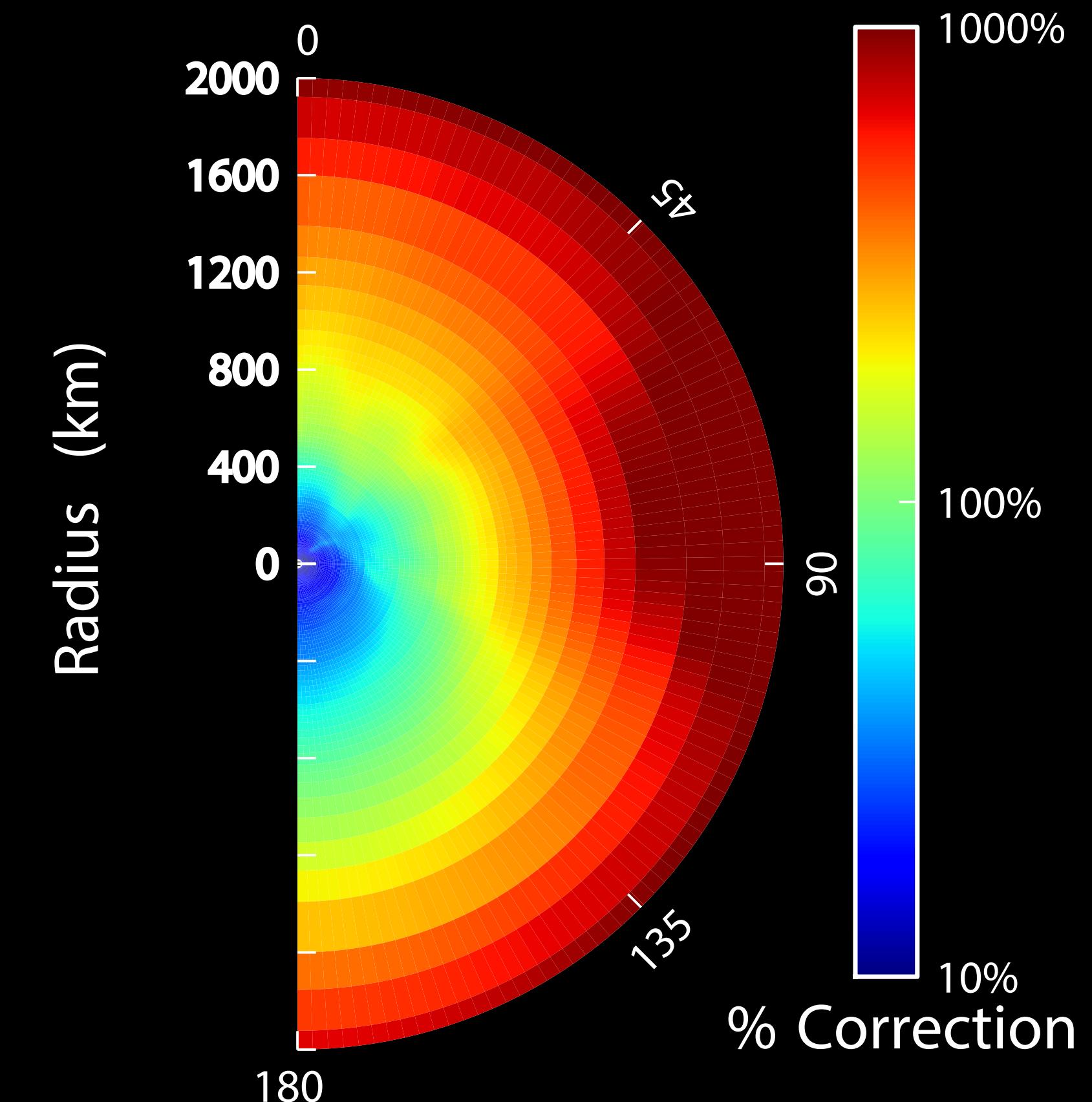
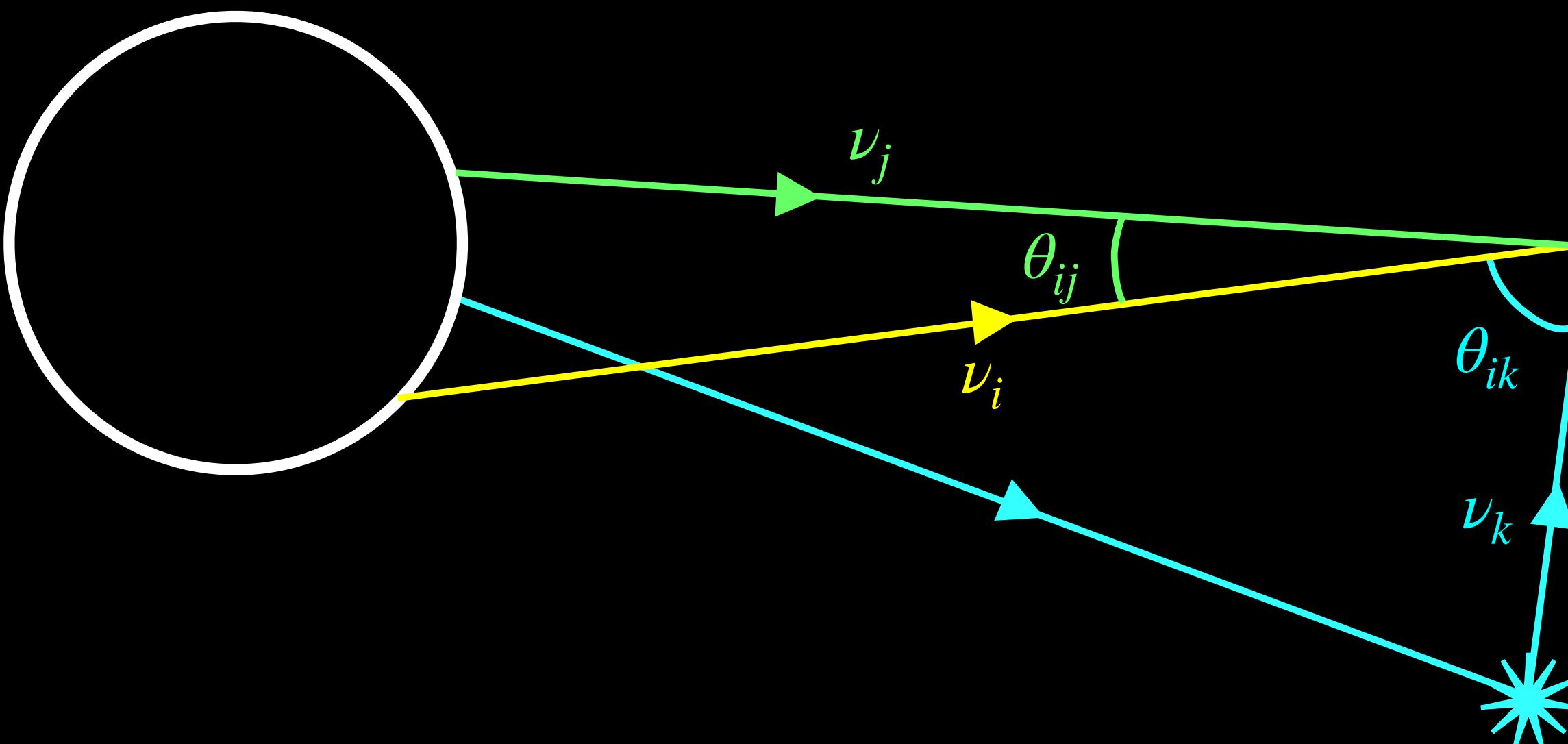


Effects of Collisions on Collective Oscillations

Neutrino Halo

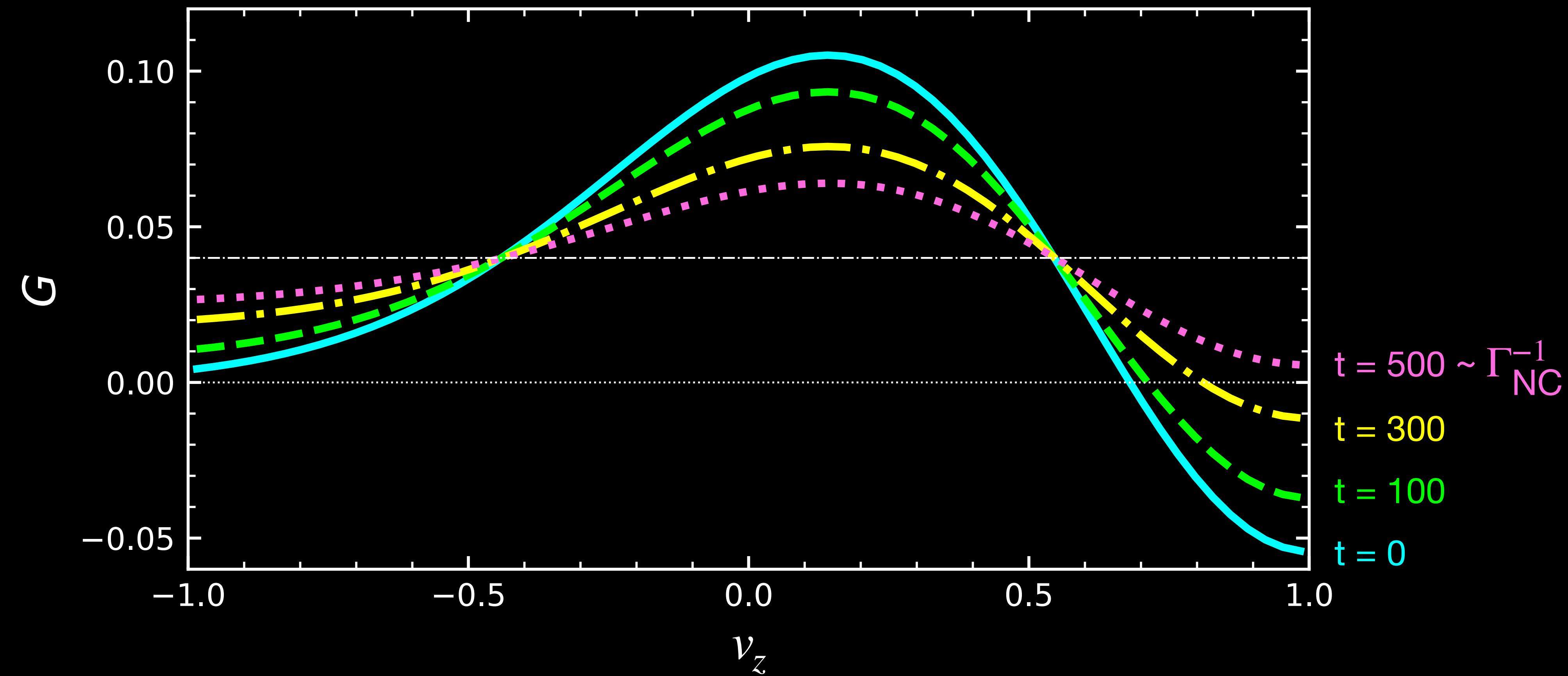
νN neutral-current scattering

$$H_{\nu\nu} = \sqrt{2} G_F \int d^3 p' (1 - \hat{v} \cdot \hat{v}') (\rho_{p'} - \bar{\rho}_{p'})$$



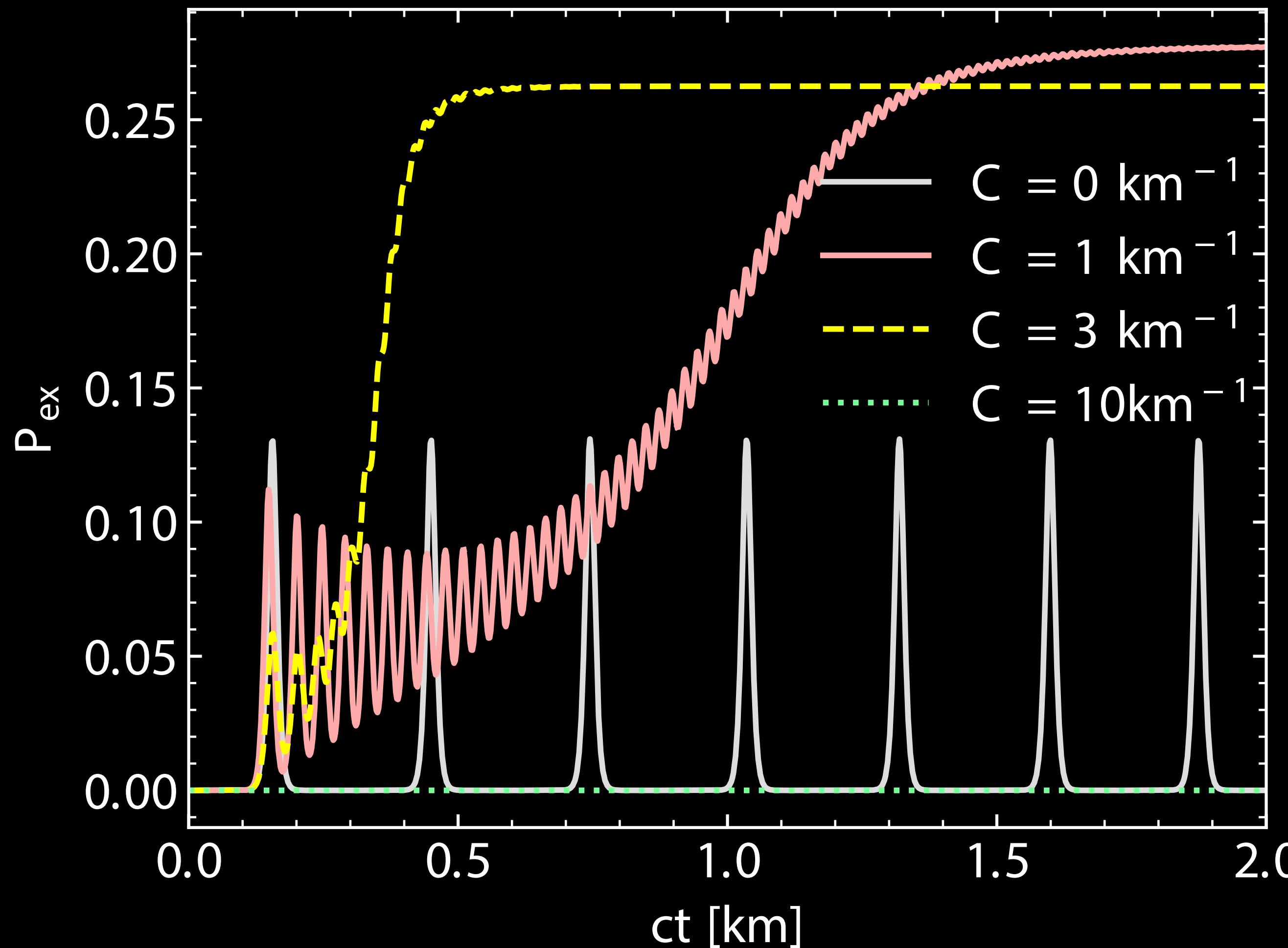
Collision and Fast Oscillations

νN neutral-current scattering



Collision and Fast Oscillations

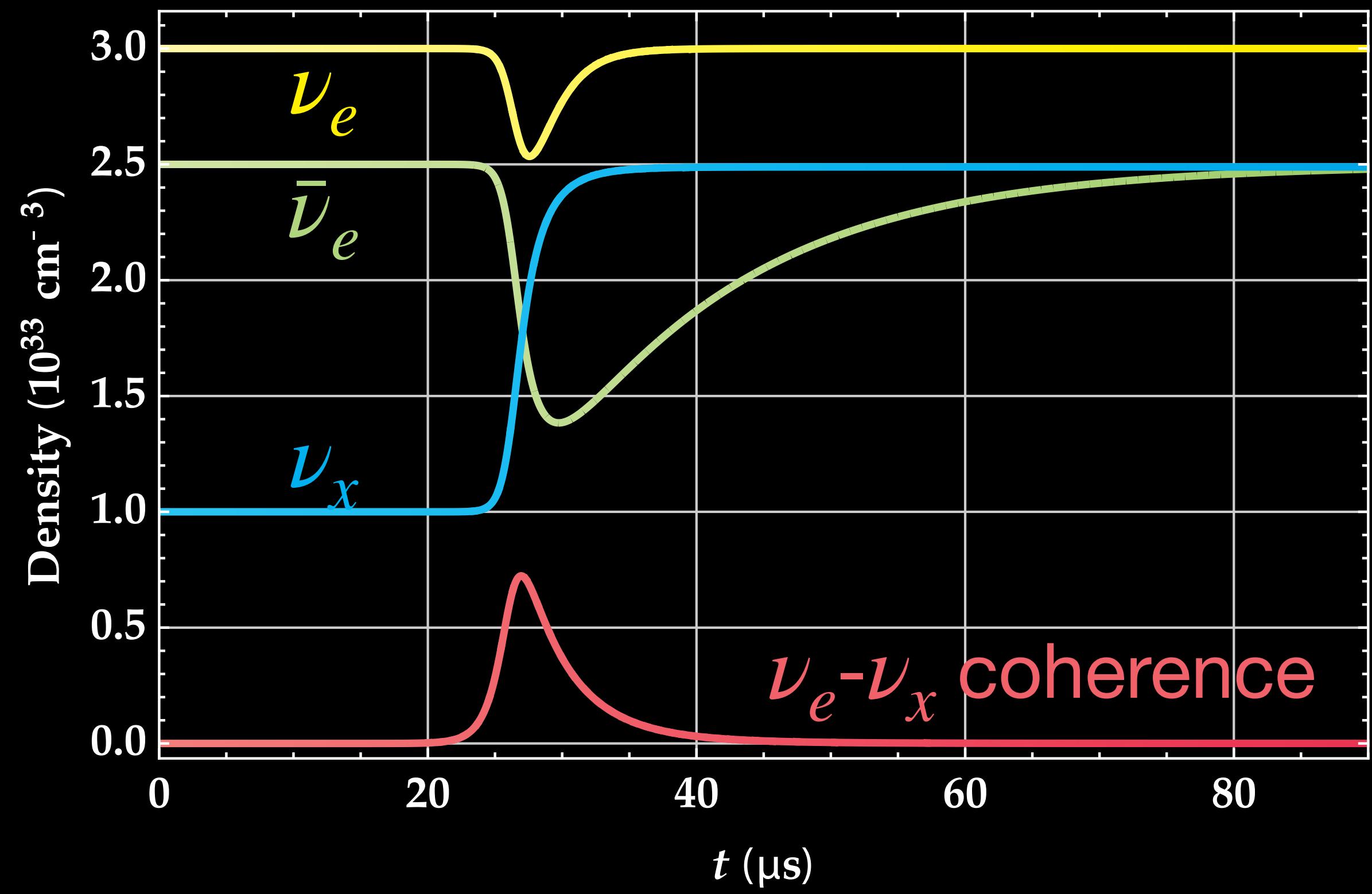
νN neutral-current scattering



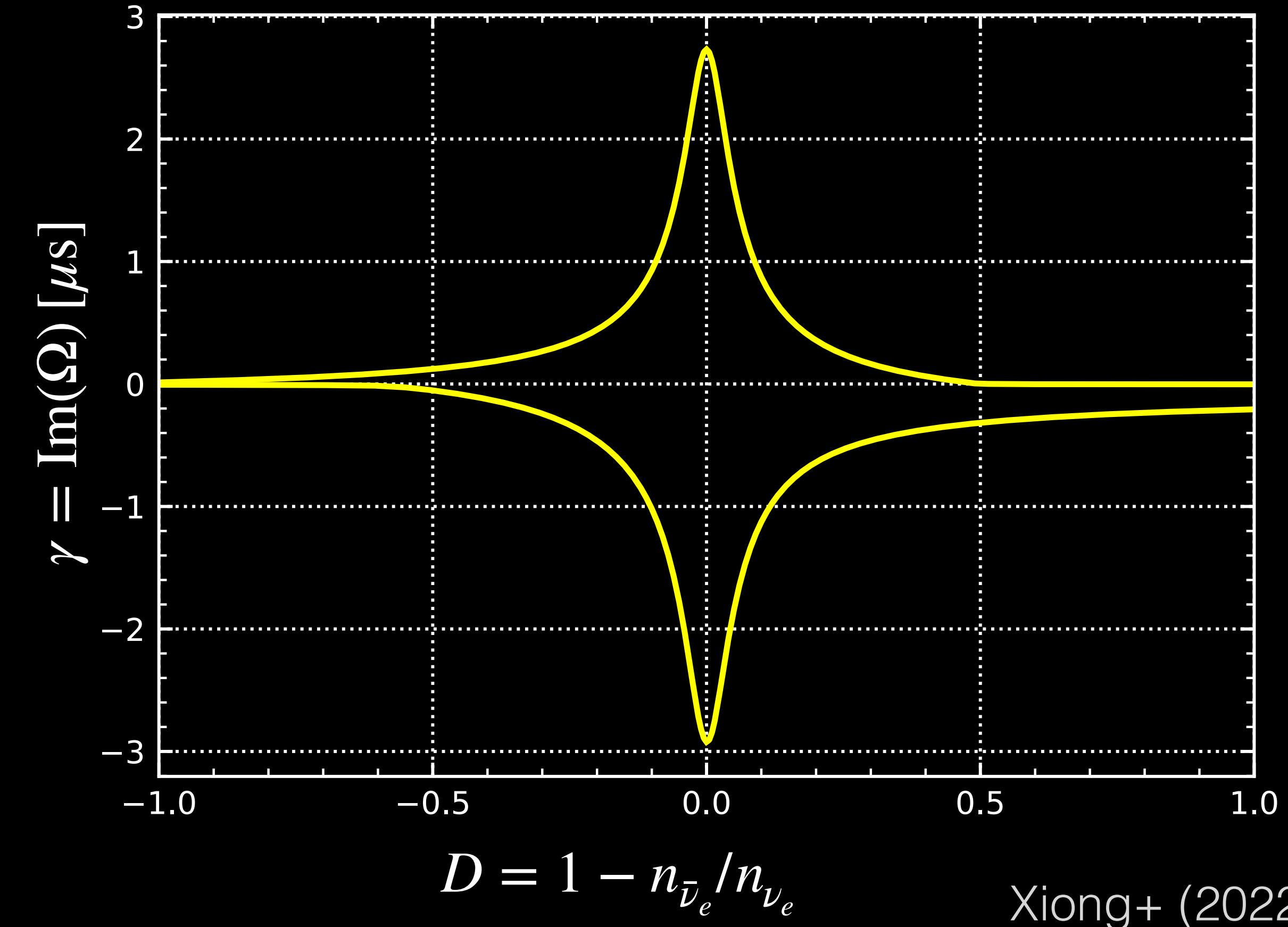
Shalgar & Tamborra (2021)
Sasaki & Takiwaki (2021)
Hansen+ (2022)

Collisional Flavor Instability

Effects of charged-current interactions



Johns (2021)



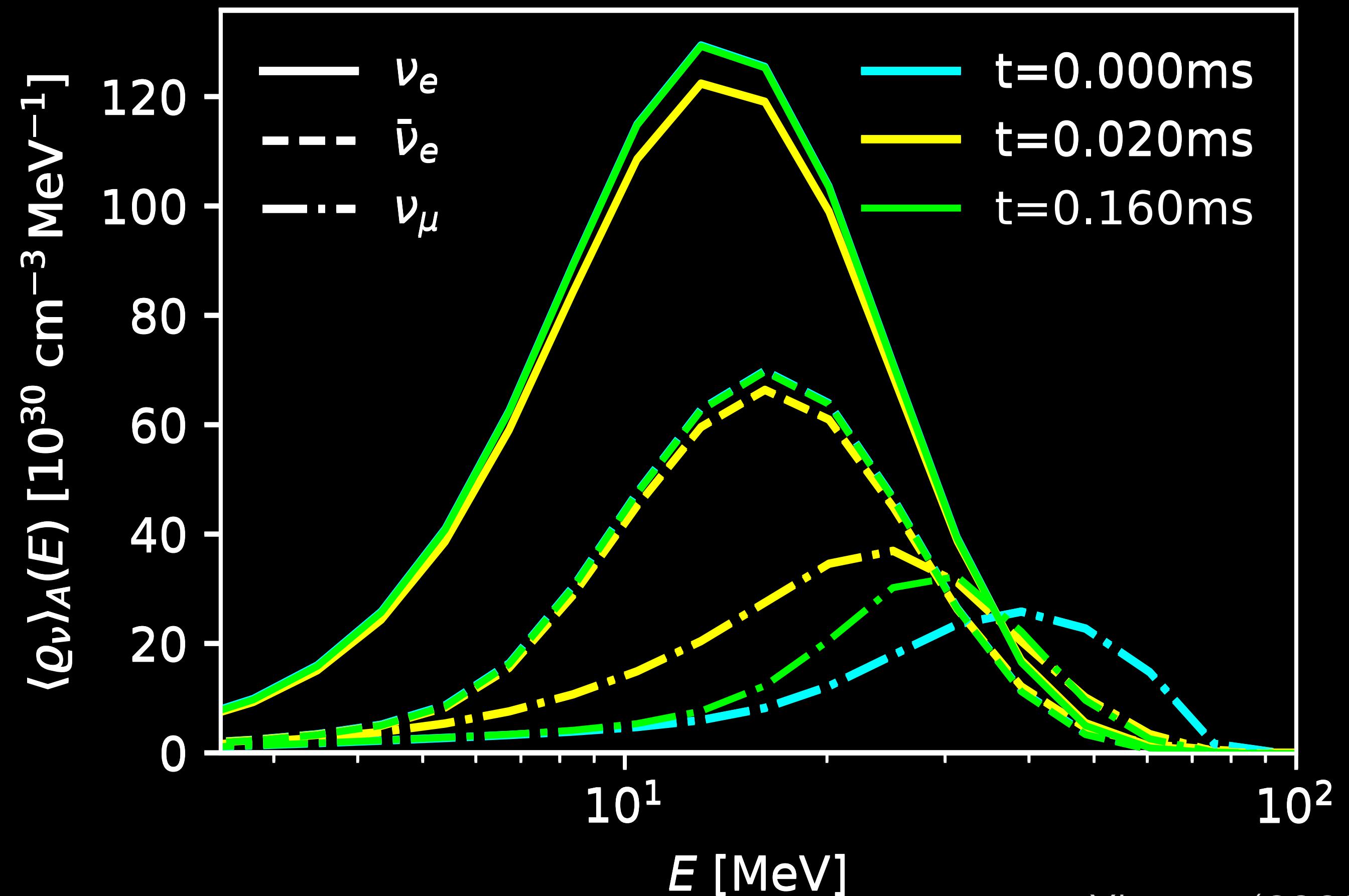
$$D = 1 - n_{\bar{\nu}_e}/n_{\nu_e}$$

Xiong+ (2022)

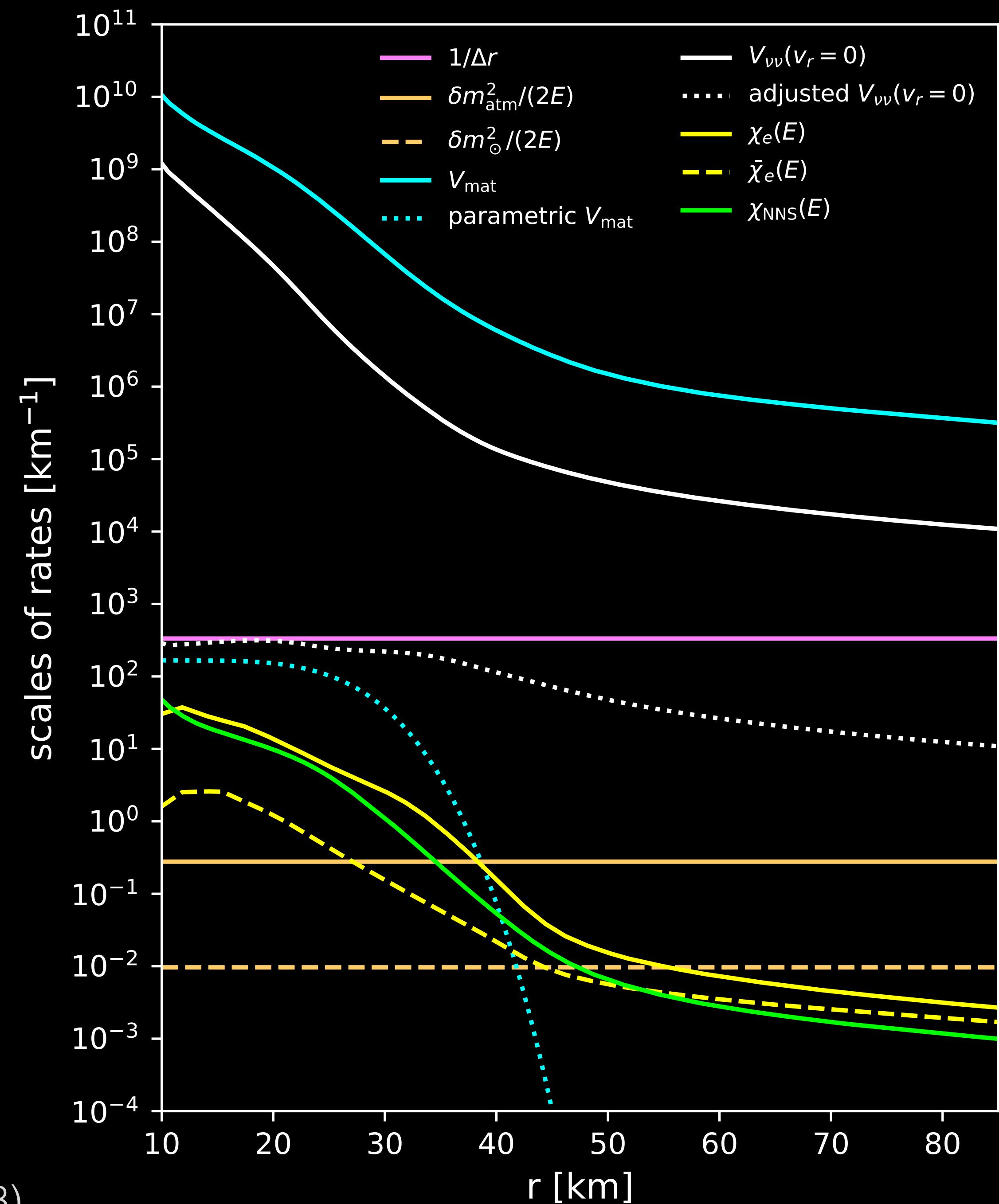
“Integrating” Oscillations in SN Simulations

Simulations w/ Scaling

CFI in 1D models

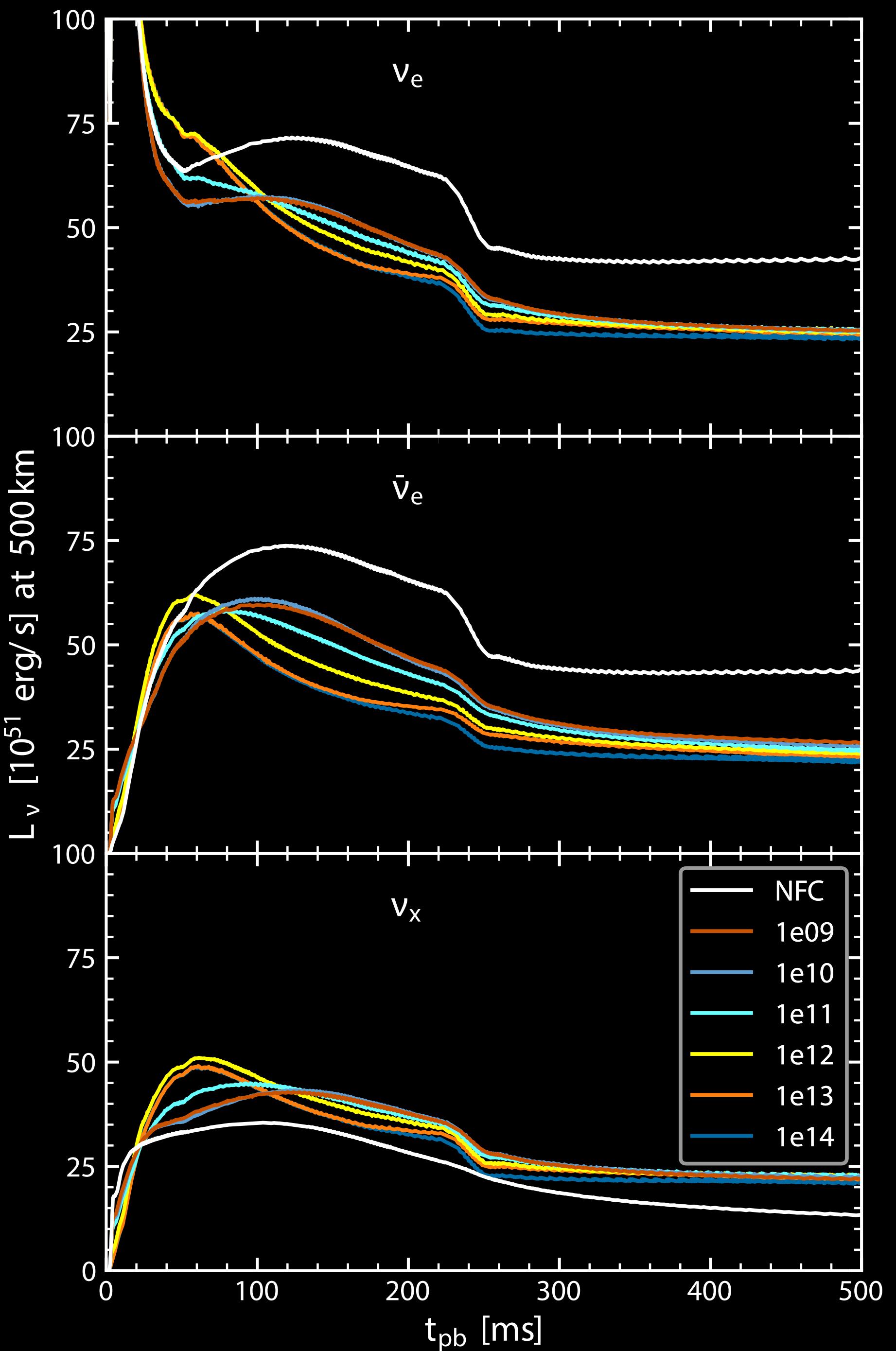
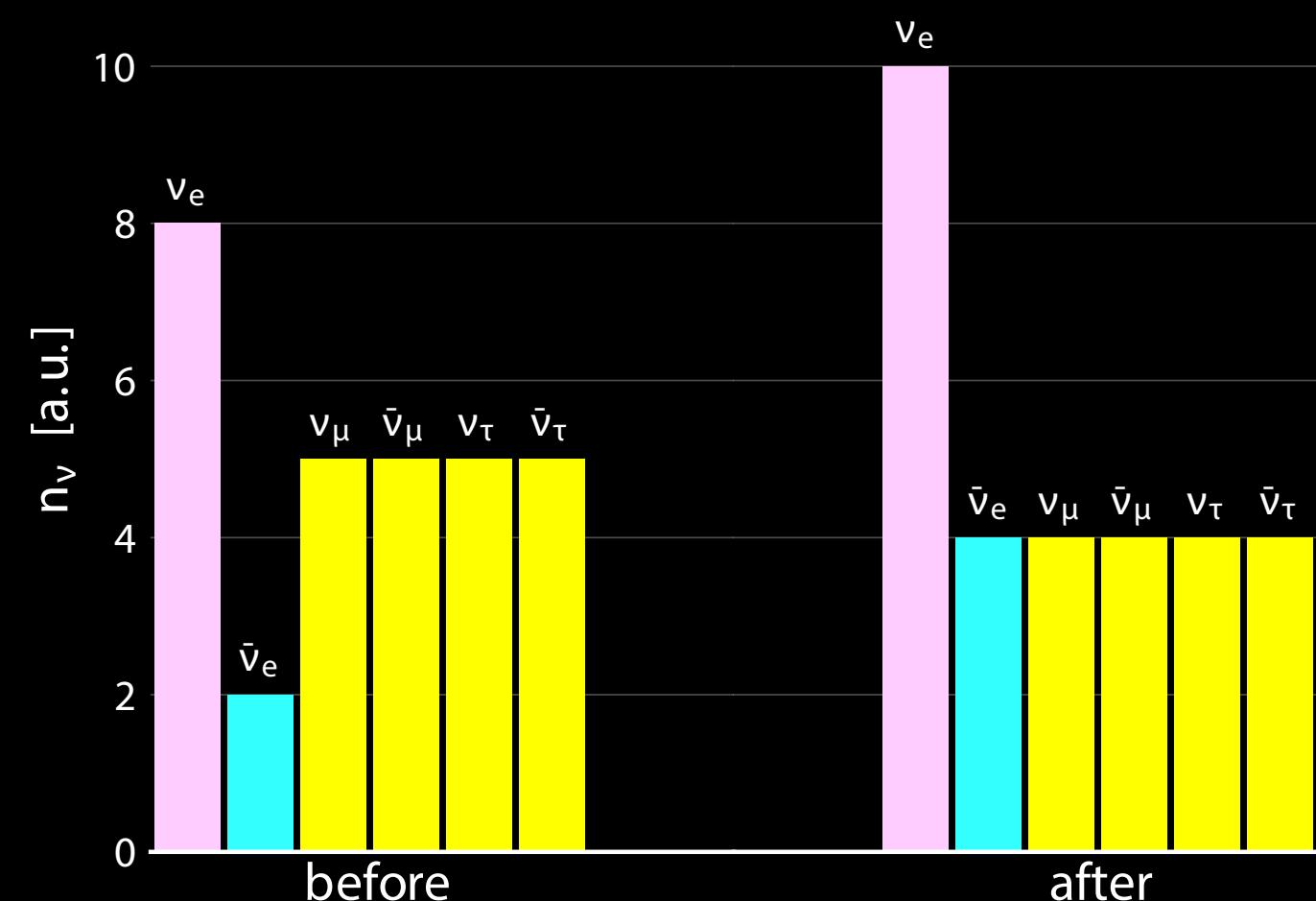
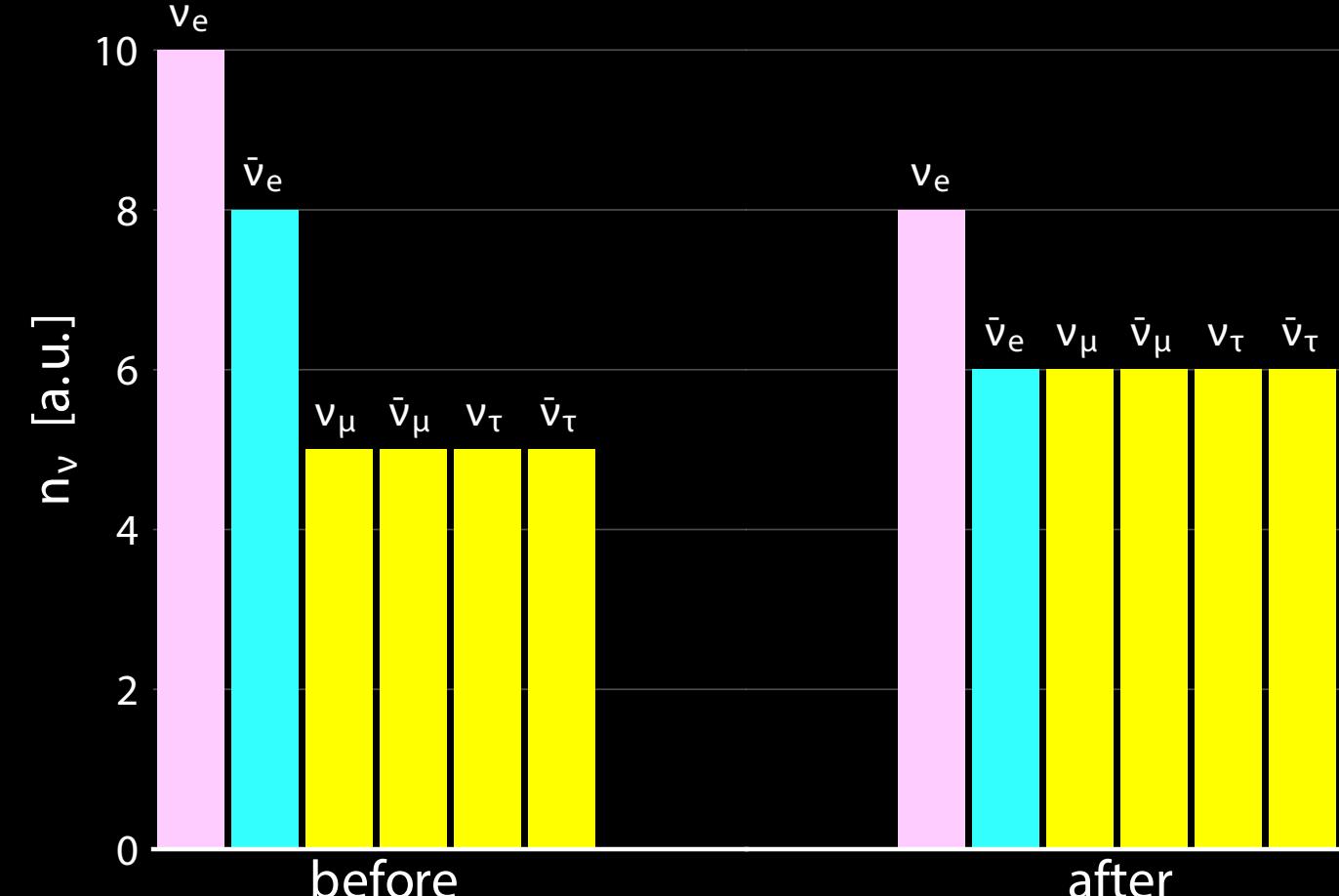


Xiong+ (2023)



Parametric Study

FFC in 1D simulations



Summary

- Neutrinos can experience collective flavor transformation in core-collapse supernovae and neutron-star mergers.
- Crossing(s) in the neutrino electron lepton number distribution can induce (fast) neutrino oscillation waves that transport/redistribute the lepton number.
- Neutrino “collisions” can also induce flavor instability and flavor conversions.
- More work needs to be done to implement neutrino oscillations in simulations.

Thank You!