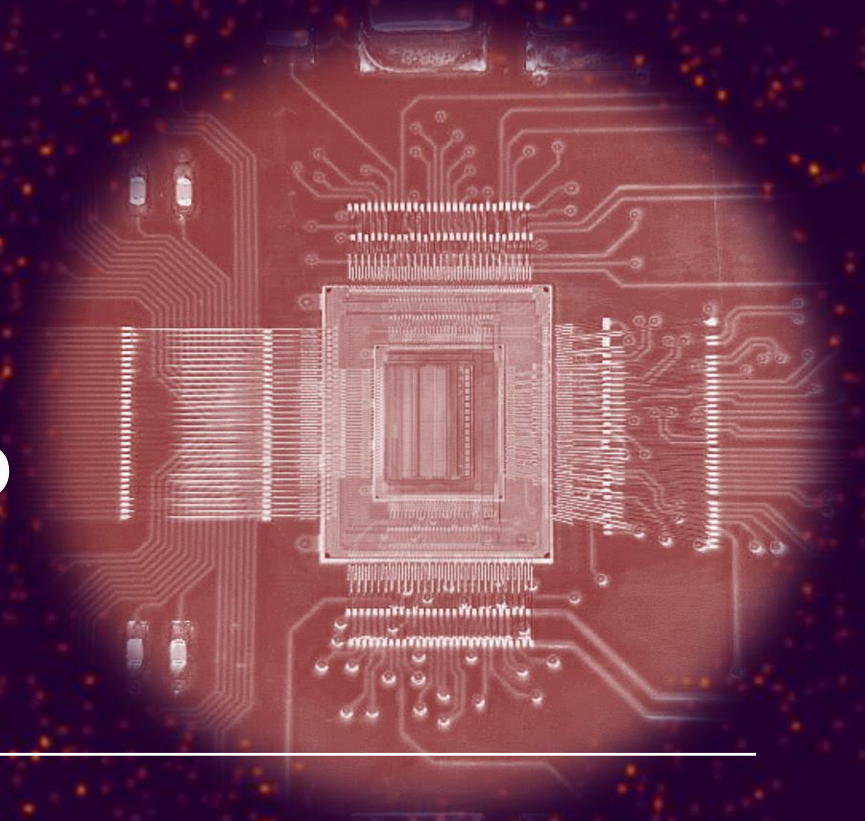
Analog Front-end for HEP

HEPIC Summer Week



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27 June 2022

Outline

From Particle to Detected Charge

- Signal Formation: Shockley-Ramo Theroem
- Current calculation in electrode
- Hybrid Vs Monolithic detectors

From Charge to Amplified Analog Signal

- Typical Front-End Channel
- Charge Sensitive Amplifier
- Shaper

Noise and Design Optimization

- Thermal, shot, Flicker
- Series Vs Parallel noise
- Noise calculation and optimization

References

Many texts, figures and equations are borrowed from theses references:

Angelo Rivetti (2015). « *CMOS Front-end Electronics for Radiation Sensors* ». CRC Press.

Marc Arques, Master course at Ecole Nationale Supérieure de Physique, Electronique et Matériaux (Grenoble INP - Phelma).

W. Snoeys et al. « *A process modification for CMOS monolithic active pixel sensors forenhanced depletion, timing performance and radiation tolerance"* Nuclear Inst. And Methods in Physics Reasearch (2017), Vol 817, p. 90 – 96.

L. Flores Sanz de Acedo et al., « *Design of large scale sensors in 180 nm CMOS process modified for radiation tolerance* », Nuclear Instruments and Methods in Physics Research Section A (2020), Volume 980.

M. Dyndal et al., *"Mini-MALTA: Radiation hard pixel designs for small-electrode monolithic CMOS sensors for the High Luminosity LHC"*, JINST (2020), Vol 15.

R. Cardella et al. "MALTA: an asynchronous readout CMOS monolithic pixel detector for the ATLAS High-Luminosity upgrade", JINST (2019), Vol 14.

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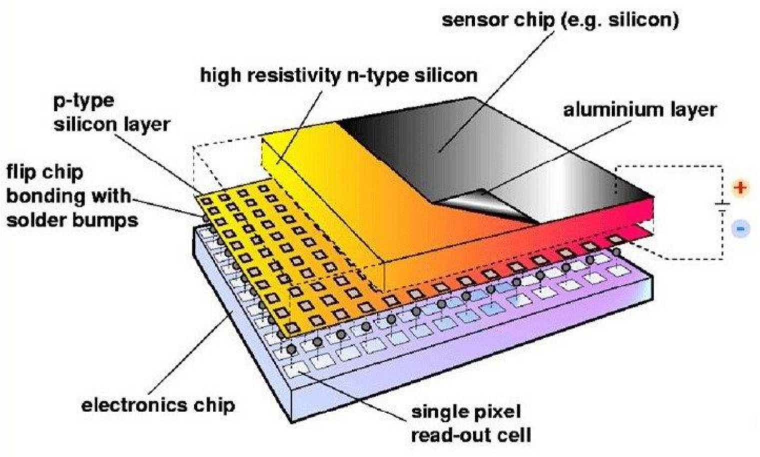
Particle Detectors

Particle passing through a detector lose energy by ionization (charged particles) or by absorption (photons)

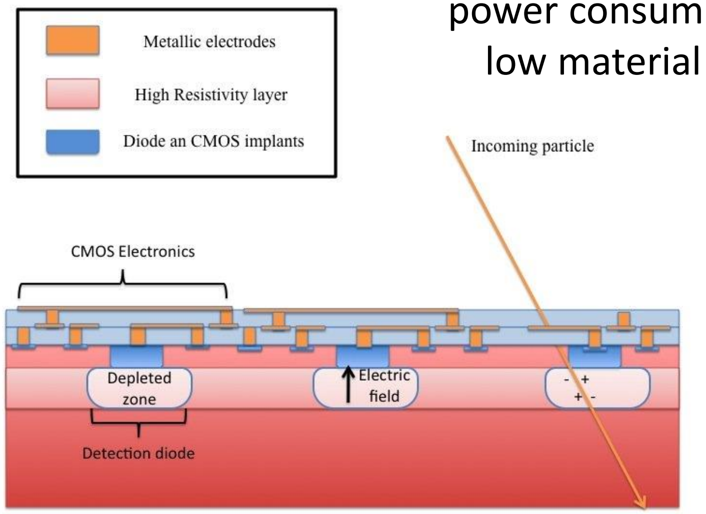
There exists many particle detectors: Gaz electrons multipliers, vacuum tube Photomultipliers (PM), Silicon Photomultiplier (SiPM), silicon strips, scintillator detectors, CCD, **hybrid pixel detectors**, **monolithic pixel detectors (MAPS)**

used in Particle Trackers

Very good spatial and temporal resolution, low power consumption and low material budget



Hybrid Detector



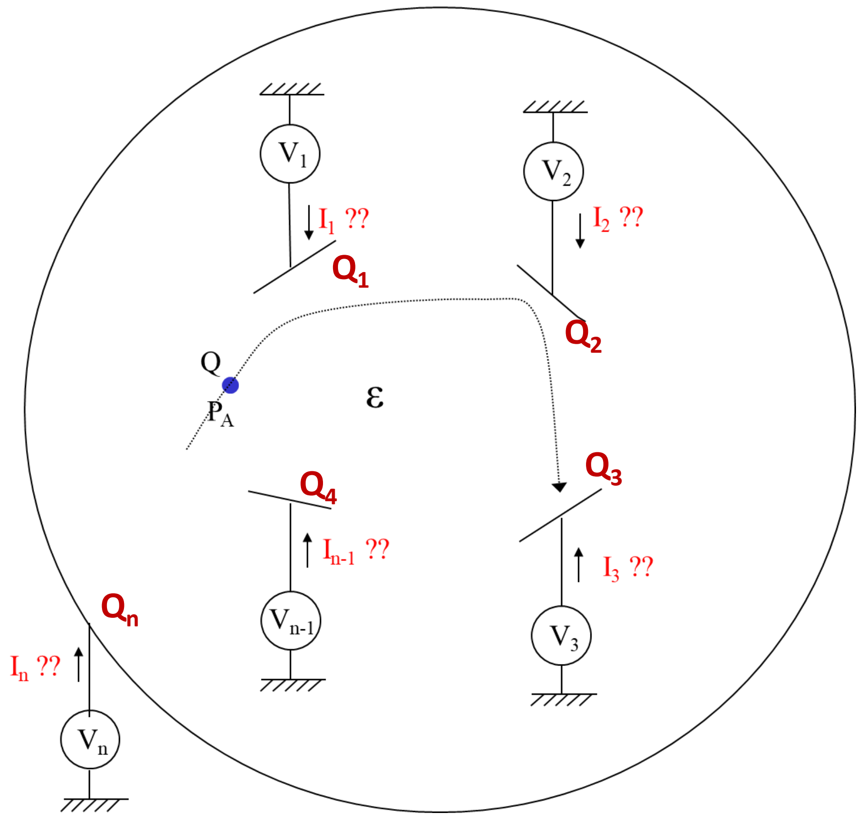
Monolithic Detector

Signal Formation: Shockley-Ramo Theorem

- This theorem is valid for all detectors based on charge detection by electrodes.

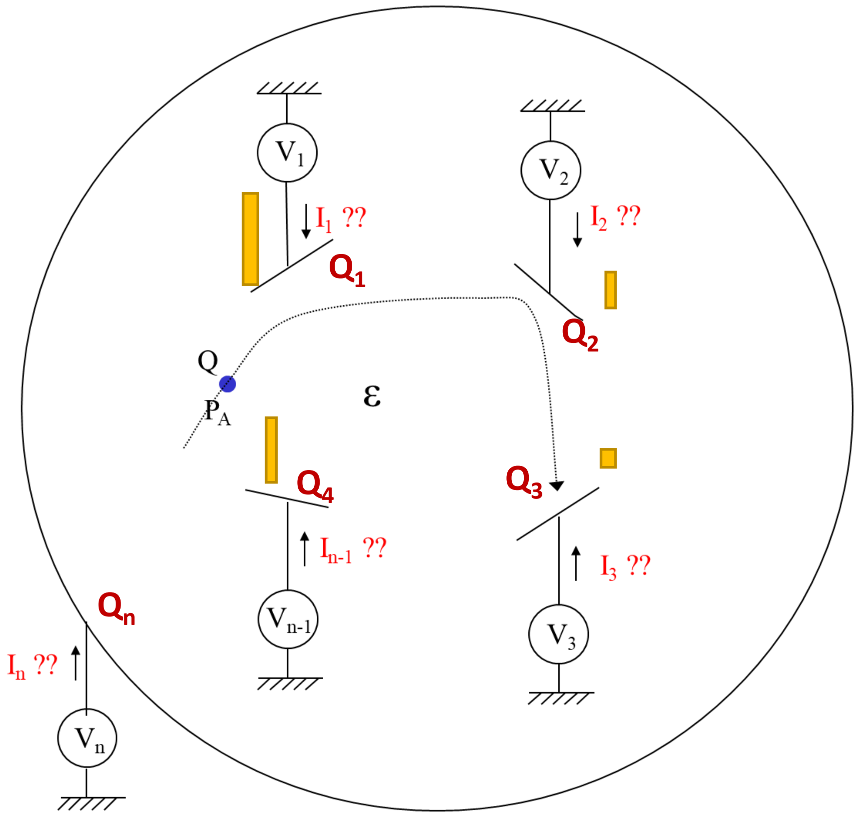
- When a charge Q is created in a system containing N electrodes, it induces a counter-charge –Q on the N electrodes; keeping the system's neutrality:
  - $Q = -(Q_1 + Q_2 + Q_3 + \cdots . + QN)$

- Knowing the potentials (V) and the system's geometry, we can calculate the electric field (E) at each point:
  - $\vec{E} = -\nabla V$
- The particle's speed (v) is thus known:
  - Dans un semiconducteur: $\vec{v} = \mu\vec{E}$



Shockley-Ramo Theorem

- Intuitive, qualitative approach:
  - A given electrode is more influenced if the charge is close to it.
  - At all times: $Q = -(Q_1 + Q_2 + Q_3 + \cdots . + QN)$

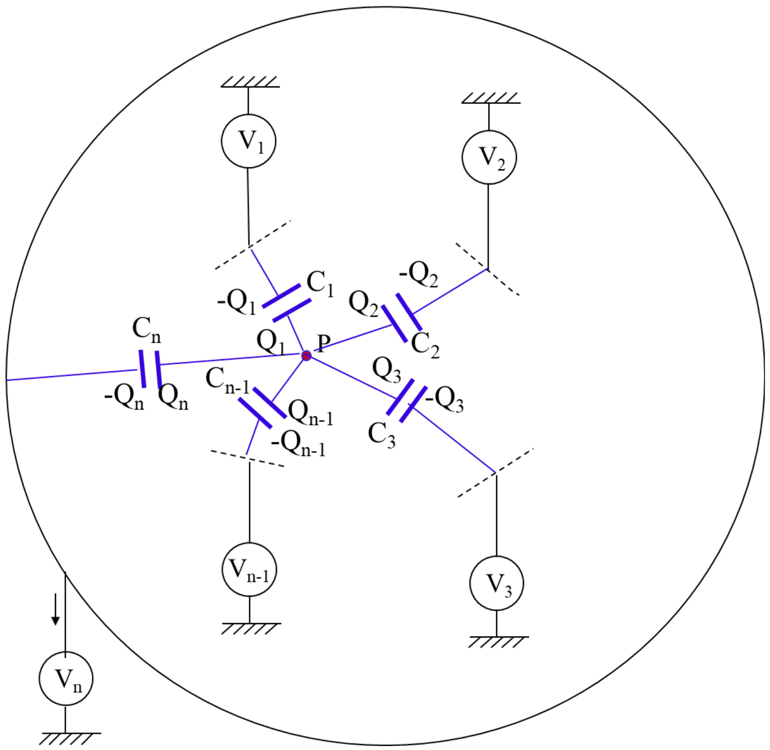


Shockley-Ramo Theorem

- We can calculate a capacitor value C between the charge Q and each electrode.
- Knowing that Q =C.V, we can write:

$$\Delta V(P) = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3} = \cdots = \frac{Q_n}{C_n} = \frac{Q_1 + Q_2 + Q_3 + \cdots + Q_n}{C_1 + C_2 + C_3 + \cdots + C_n} = \frac{Q}{C}$$

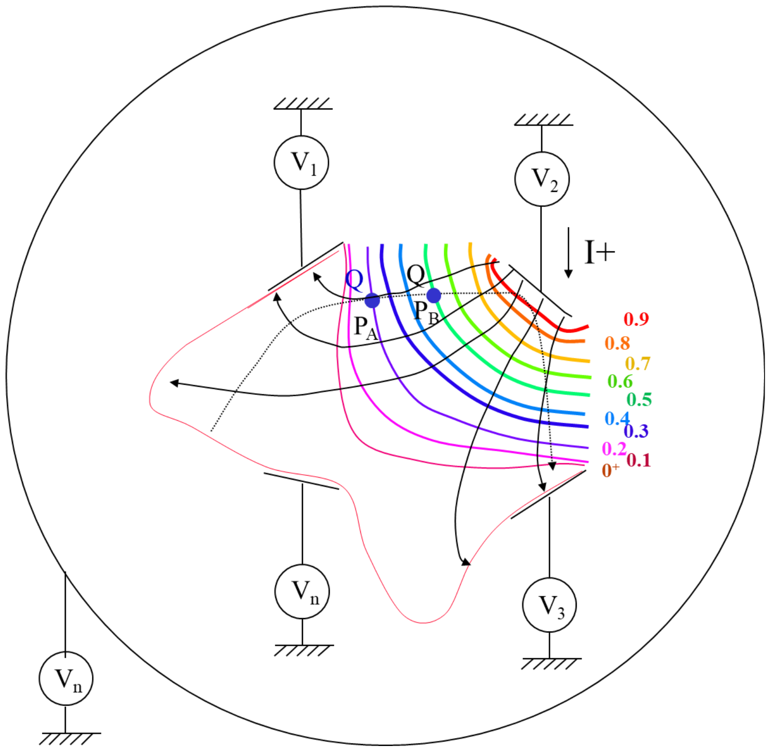
- We can calculate the fraction of charge on a given electrode; also know as a Weighting Potential:
  - $W_2 = \frac{-Q_2}{-Q} = \frac{C_2}{C_1+C_2+C_3+\cdots+C_n}$



Shockley-Ramo Theorem

$$W_2 = \frac{-Q_2}{-Q} = \frac{C_2}{C_1 + C_2 + C_3 + \cdots + C_n}$$

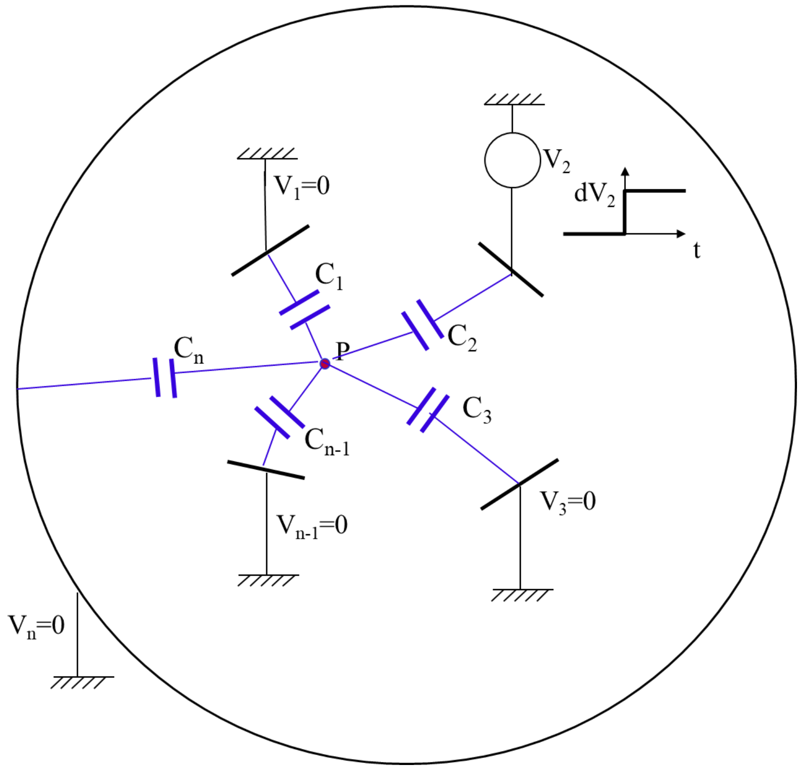
- We can calculate planes of 'iso_counter-charge' in a weighting potential, i. e. plans where the induced charge on a particular electrode is constant.
- If the moving charge crosses many equipotential planes, it induces a current on the electrode.
- Knowing the charge's speed , the gradient of W, (also known as E_W), one can calculate the change rate of induced charge , i. e the current.:



$$I = -Q\left(\overrightarrow{grad}\, W\right).\vec{v} = Q\, \overrightarrow{E_W}.\vec{v}$$

Shockley-Ramo Theorem

- To calculate W of a given electrode (electrode 2 for example), on can apply a voltage variation dV_2 (= 1V by convention)on electrode 2, and a null potential at all other electrodes.
- The variation of the potential of point P is given by a voltage divider:
  - $\frac{dV_p}{dV_2} = \frac{C_2}{C_1+C_2+C_3+\cdots+C_n} = W_2$



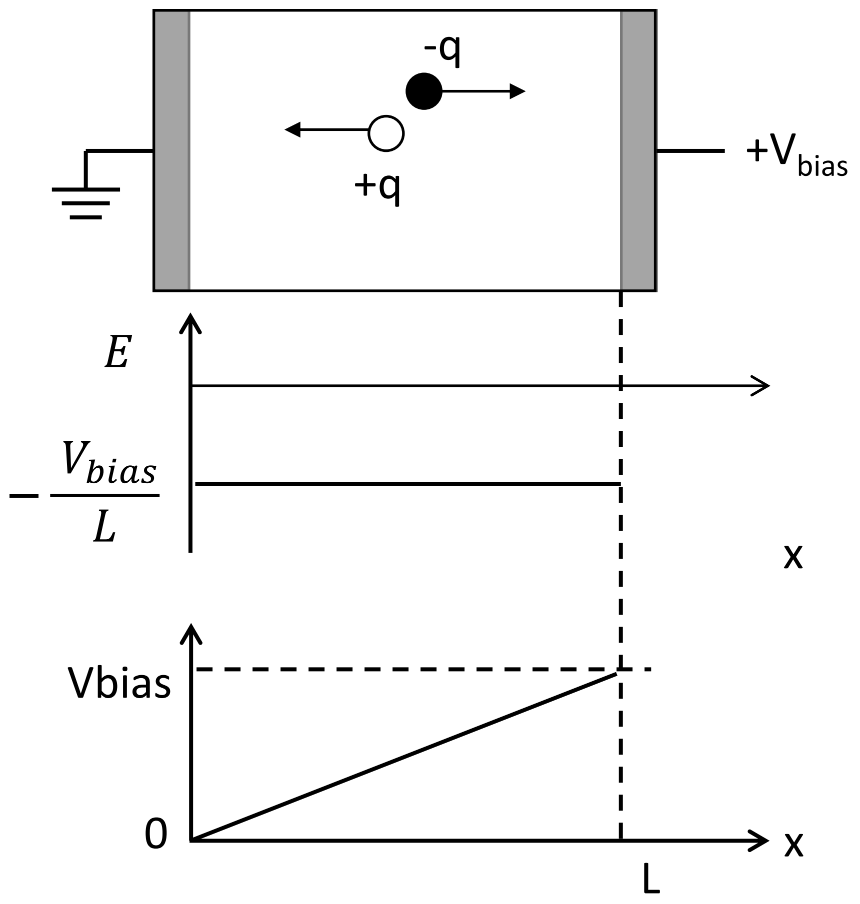
The weighting potential is function of the **geometry** of the detector, it is sufficient to calculate it only one time for a given detector and then use it to calculate the induced current for any charge moving through the detector.

$$I = -Q\left(\overrightarrow{grad}\,W\right).\vec{v} = Q\,\overrightarrow{E_W}.\vec{v}$$

Current calculation in electrode

- Let's examine the simple case of a semiconductor detector polarized by 2 metal electrodes..
  - *(we assume a very high resistivity semiconductor, with absence of charges in the detector volume)*
- the electric $field\ is\ constant\ \vec{E} = -\frac{V_{bias}}{L}\vec{u}_x$
- the potential increase linearly $V = \frac{V_{bias}}{L}x$
- The weighting filed is obtained by applying a potential of 1V on the electrode of interest:
- ➔ Weighting potential $\mathrm{W} = \frac{1}{L}x$
- ➔ Weighting field $\overrightarrow{E_w} = -\frac{1}{L}\vec{u}_x$
- Charge speed : $\vec{v} = \mu\vec{E}$
- We have all the elements to calculate the current:

$$I = -Q\left(\overrightarrow{grad}\,W\right).\vec{v} = Q\,\overrightarrow{E_W}.\vec{v}$$



Current calculation in electrode

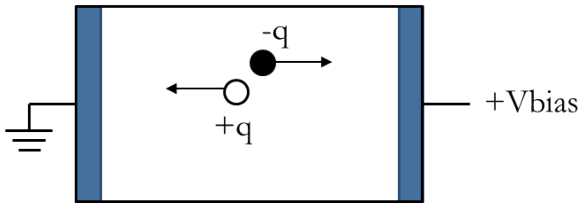
Note that électrons and holes have different mobilities

➔ $I = q(\mu_n + \mu_p)\frac{V_{bias}}{L}\frac{1}{L}$

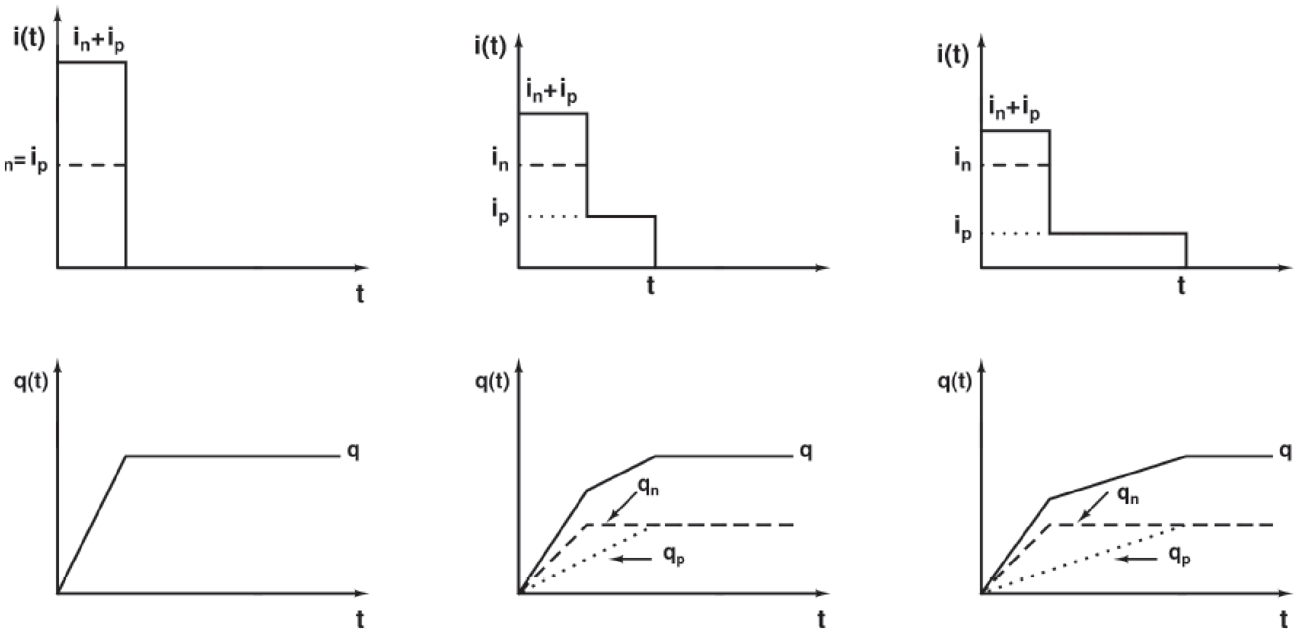
Le temps de collecte de la charge est

$t_c = \frac{L}{v}$

$$Q_{tot} = \int (I_e + Ih)dt = q$$

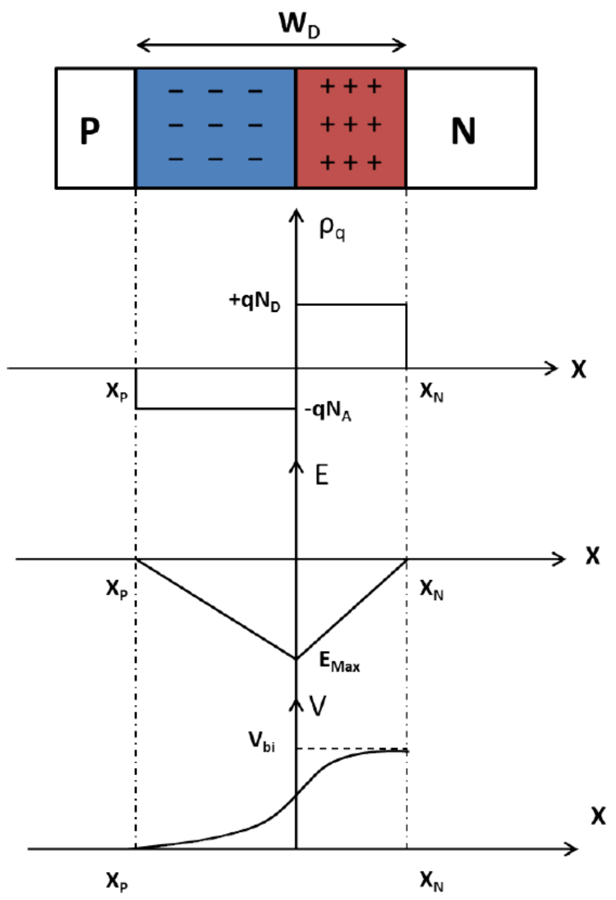


If charge is created at L/2



If $\mu_n = \mu_p$ **If $\mu_n = 2\ \mu_p$** **If $\mu_n = 3\ \mu_p$**

Reminder: PN Junction



A PN junction is formed by a P-crystal doped with acceptor atoms (N_A), in conctact with N-crystal doped with donnor atoms(N_D).

$$N_A\ .\ X_P = N_D\ .\ X_N$$

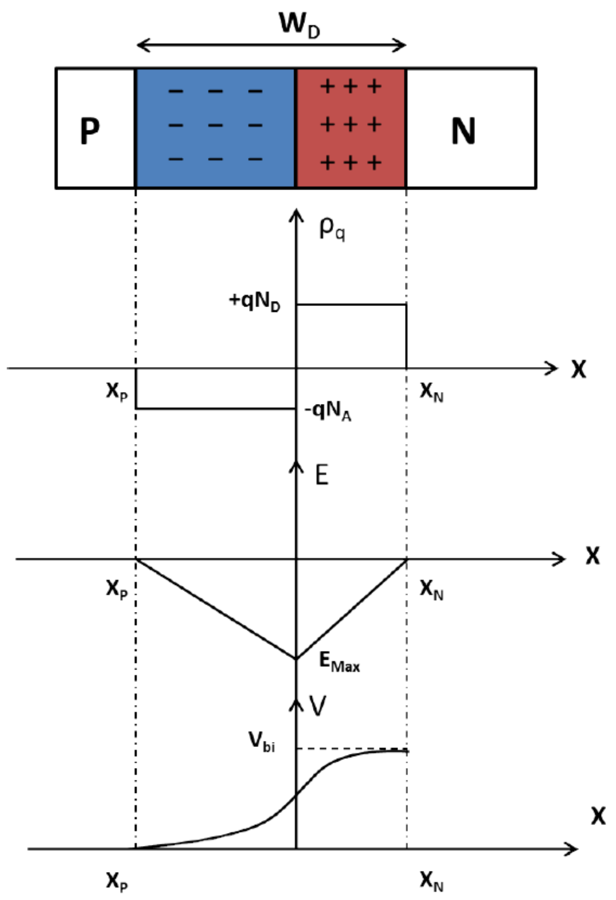
The depletion zone is more extended on the lower doped side

$$|E_{Max}| = \frac{qN_AN_D}{\epsilon_0\epsilon_r(N_D+N_A)}W_D$$

$$V_{bi} = \frac{KT}{q}\ln\frac{N_AN_D}{n_i^2}$$

Note: if a charge is created outside of the depletion zone, it can still be collected by diffusion, provided it reaches the depletion zone before recombination. However, this a slow non-preferred method of detection

Reminder: PN Junction



If we suppose that one side is more doped that the other ($N_D >> N_A$), which is generlly the case, then the depeletion width can be approximated to:

$$W_D = \sqrt{\frac{2\epsilon_0\epsilon_r}{qN_A}(\frac{KT}{q}\ln\frac{N_AN_D}{n_i^2} + V_A)}$$

W_D increase if V_A increases, et if N_A decreases, that's why detectors are built on high resistivity substrate, and preferrably with junction that can support high reverse bias before breakdown.

Finally, the collection time is inversely proportional to the applied bias ($V_A >> V_{bi}$)

$$T_e = \frac{W_D^2}{\mu_n\ (V_{bi} + V_A)}$$

In conclusion, a good detector has:

- High resistivity
- Supports high reverse voltage

Hybrid Detectors

Currently used in the ATLAS ITK in LHC

- Hybrid detectors will also be used in the next upgrade

92 million pixels of 500 µm x 400 µm

Advantages of hybrid detectors:

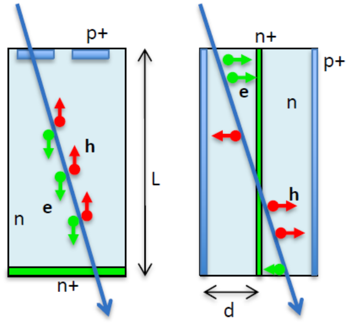
Detector optimization independently from the readout ASIC

Very good radiation tolerance (especially 3D detectors)

Limits:

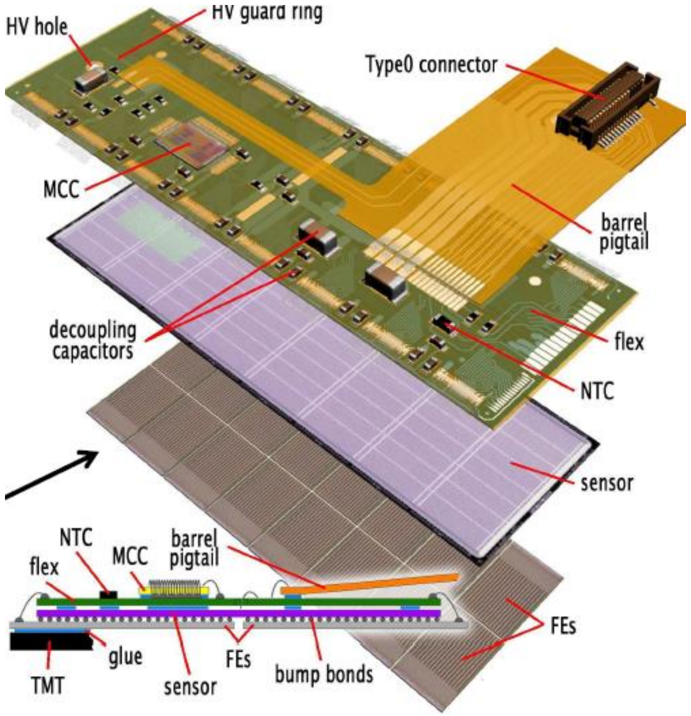
Long process, complex and expensive

Relatively high material budget



Planar sensor　3D sensor

ASIC



Monolithic Detectors

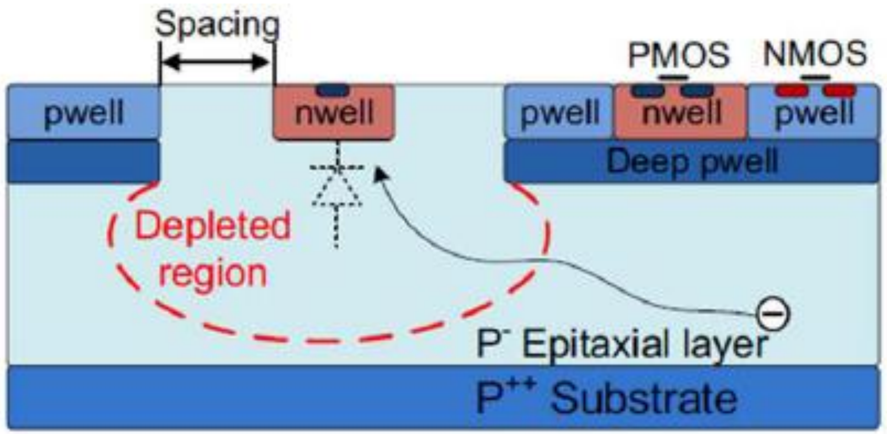
Detection occurs in the readout ASIC epitaxial layer. ➔ same ASIC for sensing and signal processing.

Advantages:

- Relatively less expensive
- Fast production using available commercial technologies
- Low sensor capacitance→ better performance, or same performance for less power
- Low material budget

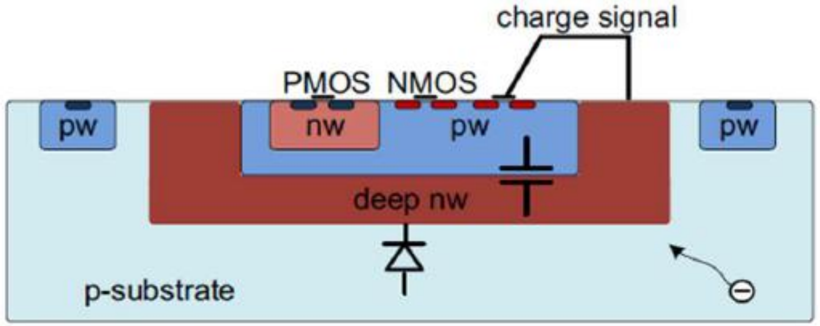
Limits:

- Reverse bias limited by technology
- Less radiation tolerant than hybrid detectors



DMAPS: Large Electrode Vs Small Electrode

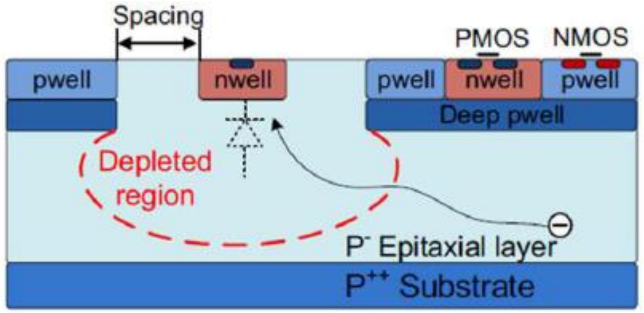
Large electrode design (ex: LF technology)



- Fast charge collection by drift
- Good radiation tolerance

- Relatively big sensor capacitance (around 150 fF for a pixel of 50 um x 150 um)
- Possible coupling between readout and sensor node

Small electrode design (ex: TJ technology)

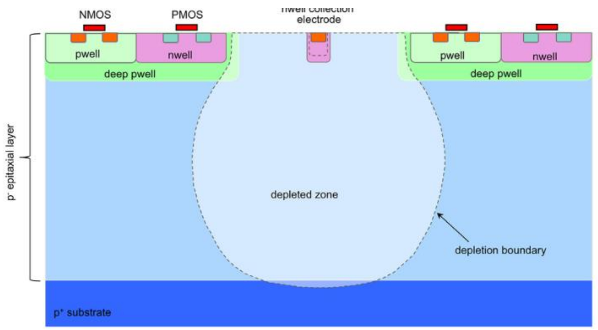


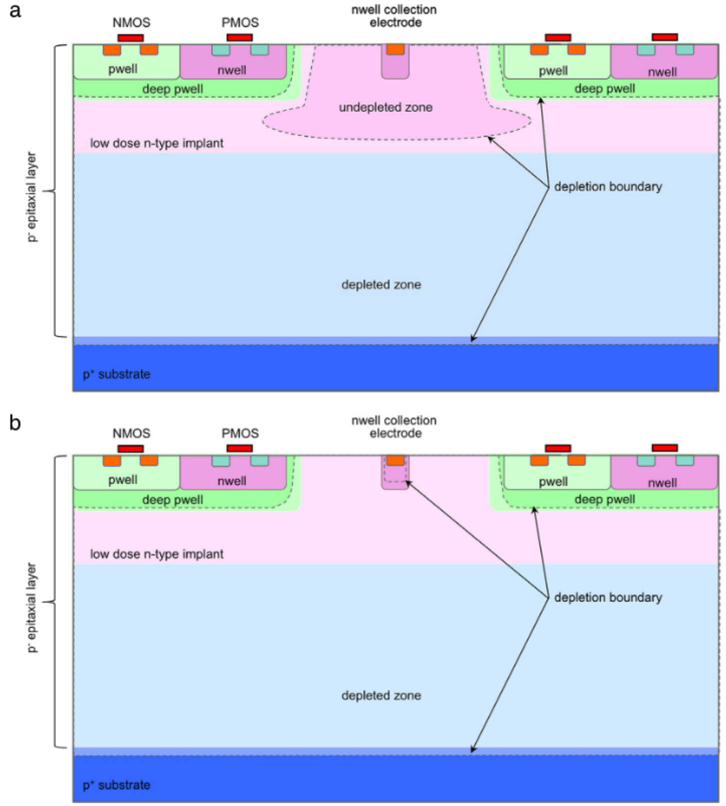
- Very small sensor capacitance $(\sim 3\ fF)$
- Convenient for small pixels

- Limited depletion zone, slow charge collection, partly by diffusion
- Not very good radiation tolerance

Small Electrode 1st Optimization

Within CERN collaboration developing MAPS on TowerJazz technology

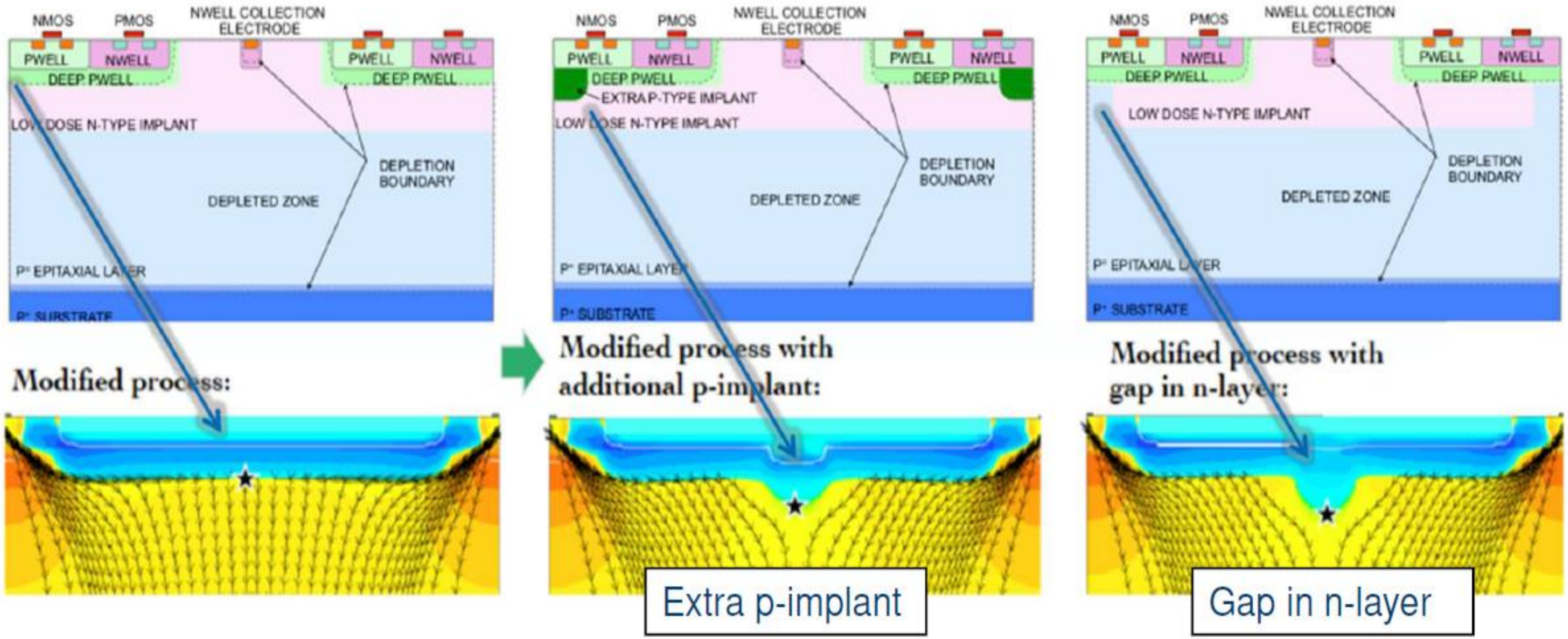




- Adding a low-doped N layer that is completely depleted under bias, gives a big collection area by drift while maintaining small sensor capacitance.

Small Electrode 2nd Optimization

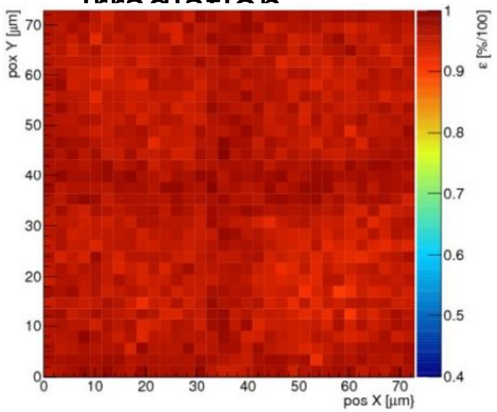
- Additional implants create lateral electric field for fast charge collection at pixel corners



Irradiation Results for TJ small electrode pixels

Before irradiation

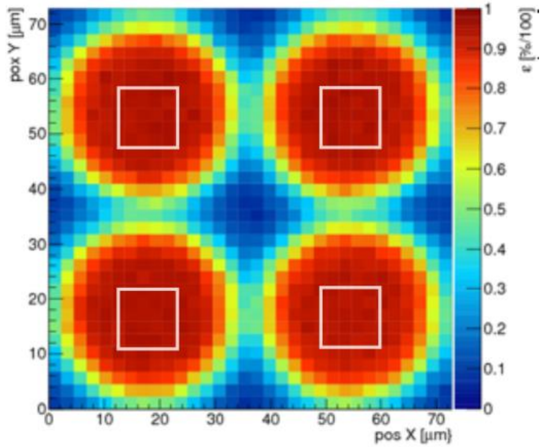
2×2 pixels

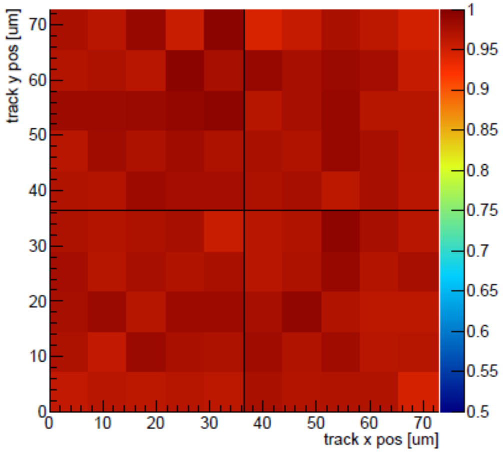


After 1st optimization

After 2nd optimization

After 10^{15} n_{eq}/cm2





Towards the 'Golden' Detector?

Bent wafer scale ($\sim 27\ cm \times 10\ cm$) stitched sensor, based on MAPS

Almost no 'dead' areas, or overhead materials

Still in R&D for the ALICE detector upgrade

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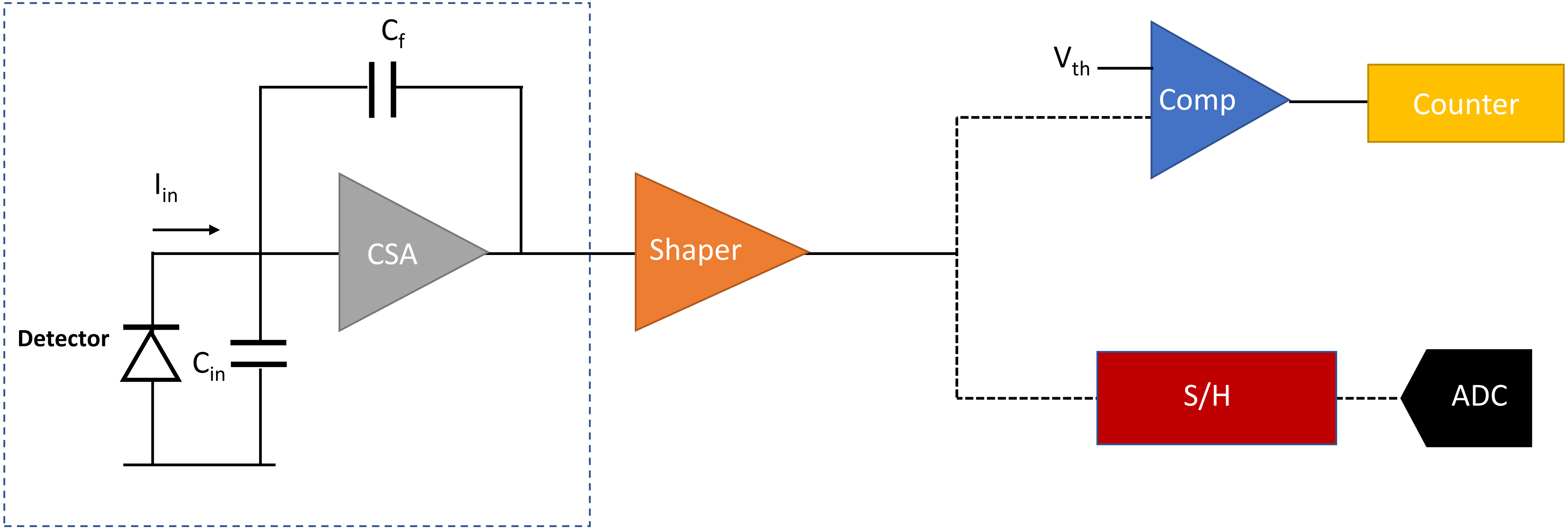
Typical Front-End Channel: 1- CSA



Chare Sensitive Amplifier
Integrates the incoming charge on a feedback capacitor

Shaper
Shapes (Filters) the signal for an optimal Signal to Noise Ratio

Comparator to detect an event above a certain threshold
And/Or
a Sample and Hold circuit for an ADC further in the channel

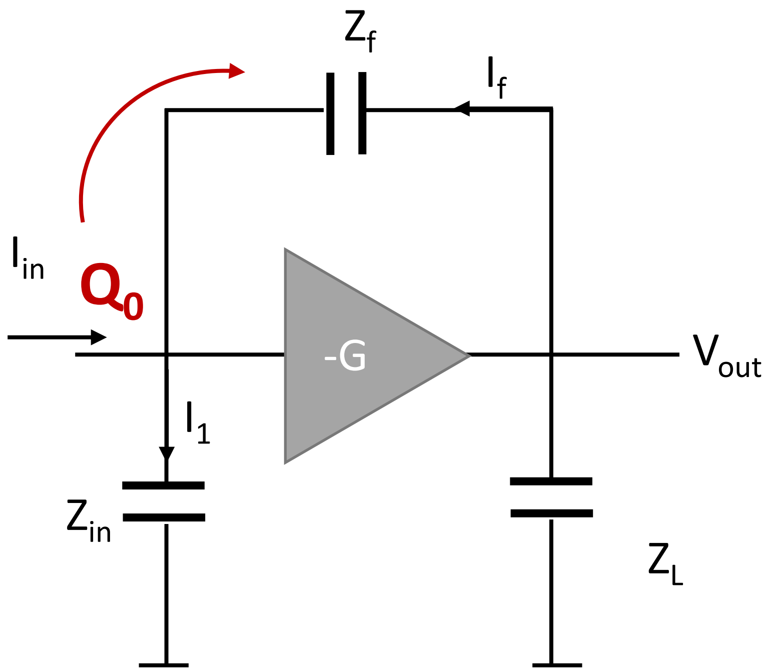
Charge Amplification

The signal that we want to detect is a charge Q_0, which is created by an ionizing particle or a photon. Normally charge creation is much faster than the amplifier bandwidth so we can model it by a current spike $I(t) = Q_0\delta(t)$

The input impedance is generally the detector capacitance C_{in}, whose value can range up to several hundreds of fF.

The voltage $V_{in} = \frac{Q_0}{C_{in}}$ is generally very small, and we have a big incertitude about the intrinsic C_{in}.

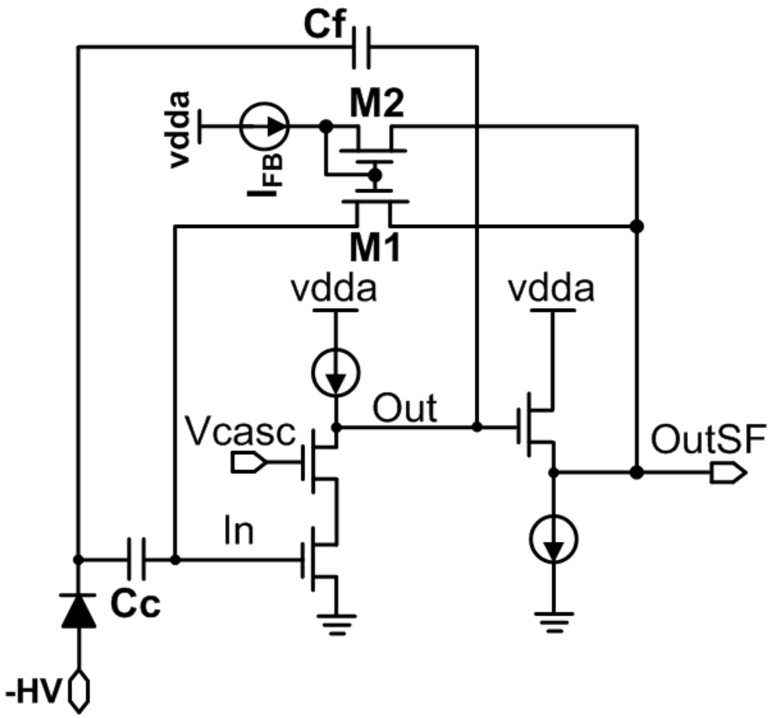
A widely used method consists in integrating the incoming charge on a low feedback capacitor C_f of know value, producing a measurable V_{out} proportional to Q_0. This architecture is called a Charge Sensitive Amplifier (CSA),



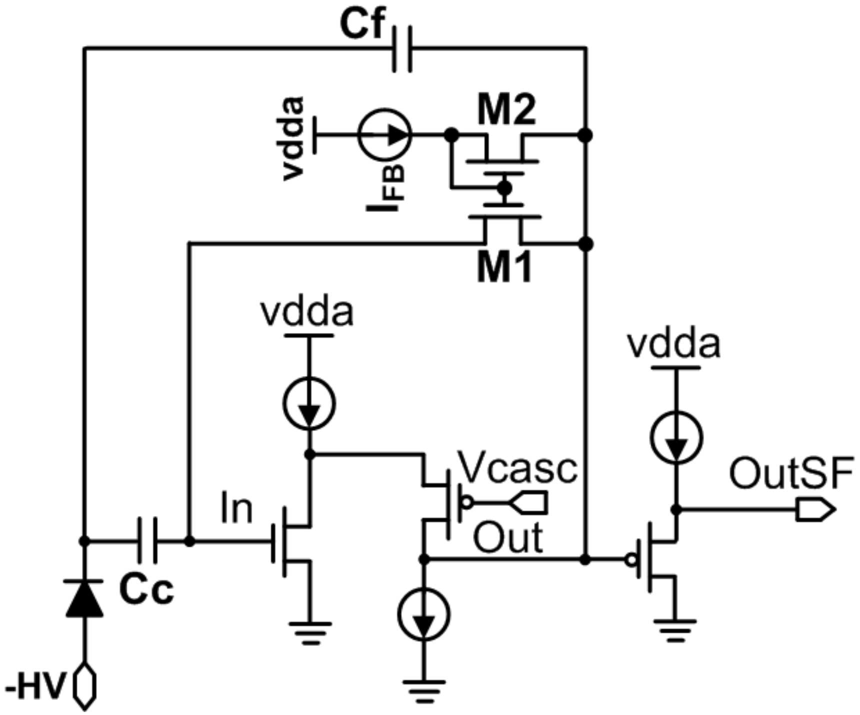
CSA Examples

LF Monopix chip

Cascode Architecutre



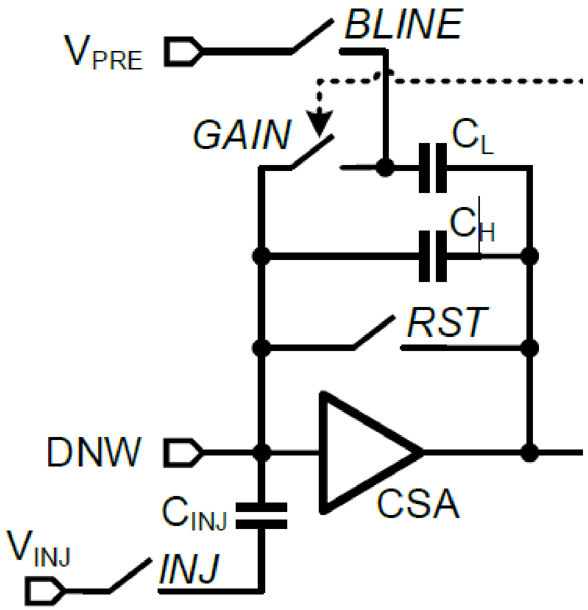
Folded-Cascode Architecutre



AC coupling, current feedback, Source Follower stage

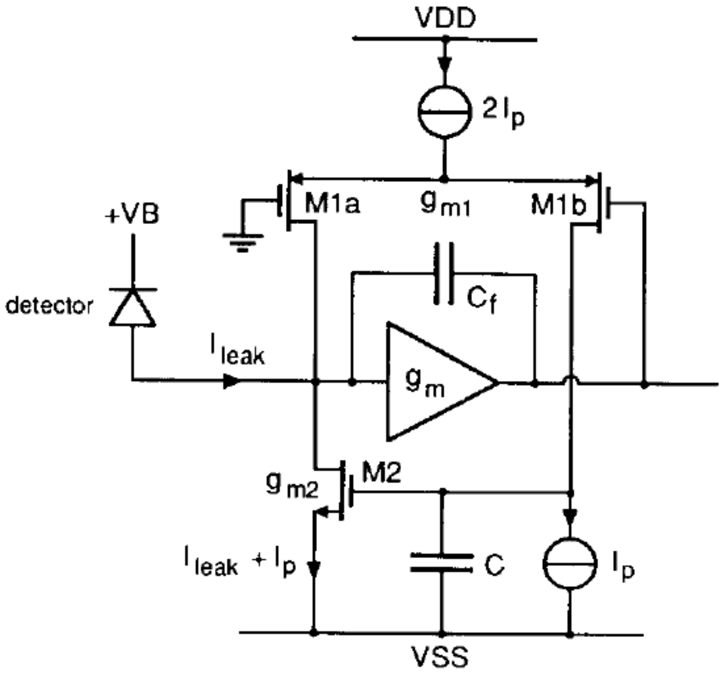
CSA Examples

ePixM for LCLSII



Synchronous reset, 2 gain modes

Krummenacher Architecture



Leakage current compensation

There are many other architectures: differential, inverter based, ….
with different reset mechanisms and leakage current compensation

How to model a simple CSA?

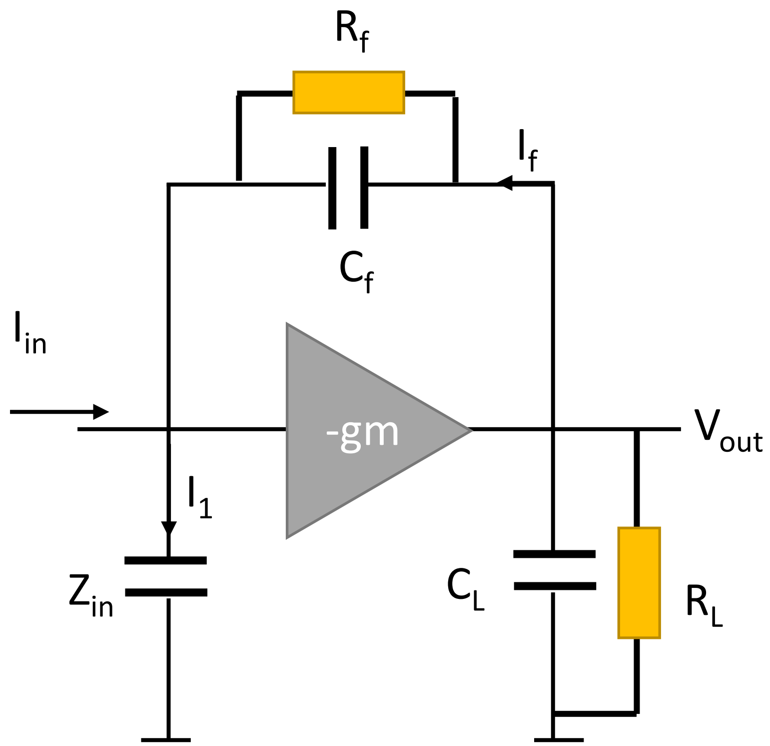
How to calculate the transfer function?

How to derive temporal behavior

How to choose design parameters gm, C_f , R_f , C_L?

How to minimize noise?

How to have a design intuition?



Laplace Transform - Reminder

A Laplace transform of function f(t) in a time domain, where t is the real number greater than or equal to zero, is given as F(s)

$$F(s) = \int_0^{\infty} f(t)e^{-st}dt$$

Where s is a complex number in the frequency domain $s = \sigma + j\omega$

Main advantage: transforms complex differential equation to simple algebraic equations.

$$\frac{df(t)}{dt} \overset{\mathcal{L}}{\rightarrow} \mathrm{sF(s)}$$

Difference with Fourier Transform: Fourier transform is useful to study stable permanent systems (typically to find the harmonics of a periodic signal)

Laplace is more general, and can study any system (stable or unstable). If we reduce $s = j\omega$ we end up with Fourier transform. The real part σ introduces real exponentials in the solution.

Laplace Transform Table

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	$\mathbf{e}^{at}$	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p, p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$\mathbf{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$\mathbf{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$\mathbf{e}^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$\mathbf{e}^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n\mathbf{e}^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{\mathbf{e}^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$
27.	$u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28.	$u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$\mathbf{e}^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			

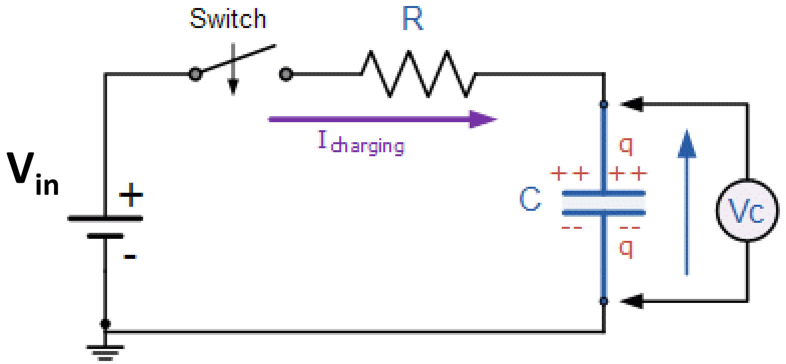
1

Laplace Transform – Example RC Circuit

$$I(t) = C\frac{dV_c(t)}{dt} \xrightarrow{\mathcal{L}} I(s) = CsV_c(s) = \frac{V_c(s)}{Z_c} \Rightarrow Z_c = \frac{1}{sC}$$

$V_c = V_{in} - IR$

$$\frac{V_c}{V_{in}} = \frac{1}{1+sRC}$$

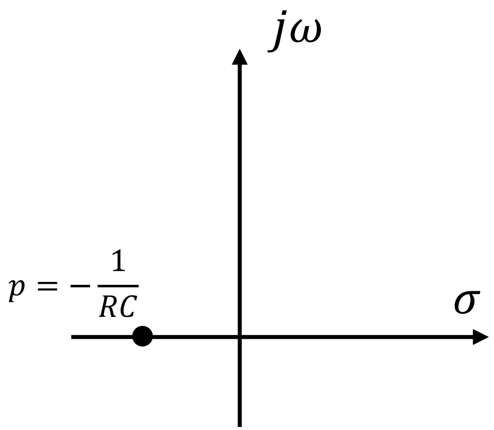


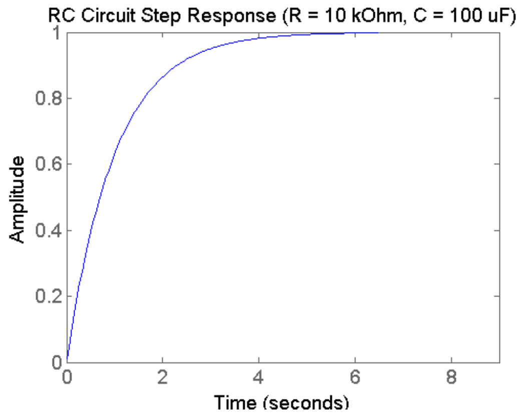
Assume that $V_{in}(t) = U(t)$ that goes from 0 to 1 at t=0

➔ $V_{in}(s) = \frac{1}{s}$

➔ $V_c(s) = \frac{1}{s(1+sRC)} = \frac{1}{s} - \frac{1}{\frac{1}{RC}+s}$

➔ $V_c(t) = 1 - e^{-\frac{t}{RC}}$

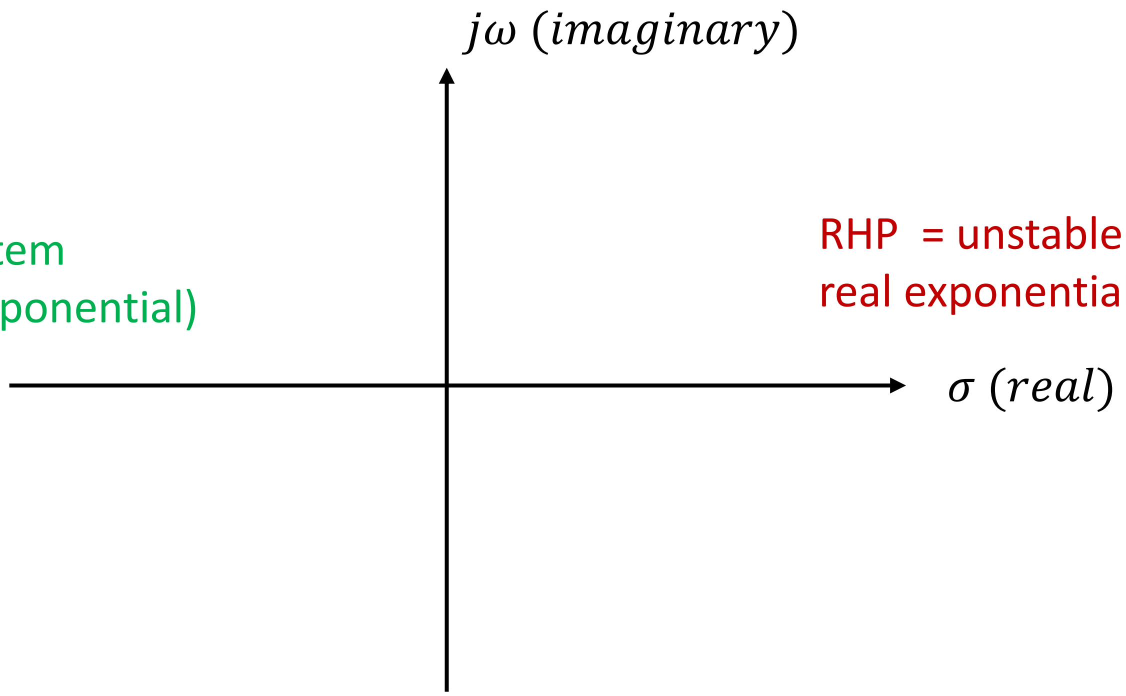




Laplace Poles Diagram

Pure imaginary poles (complex conjugate) = oscillator

LHP = stable system
(decaying real exponential)

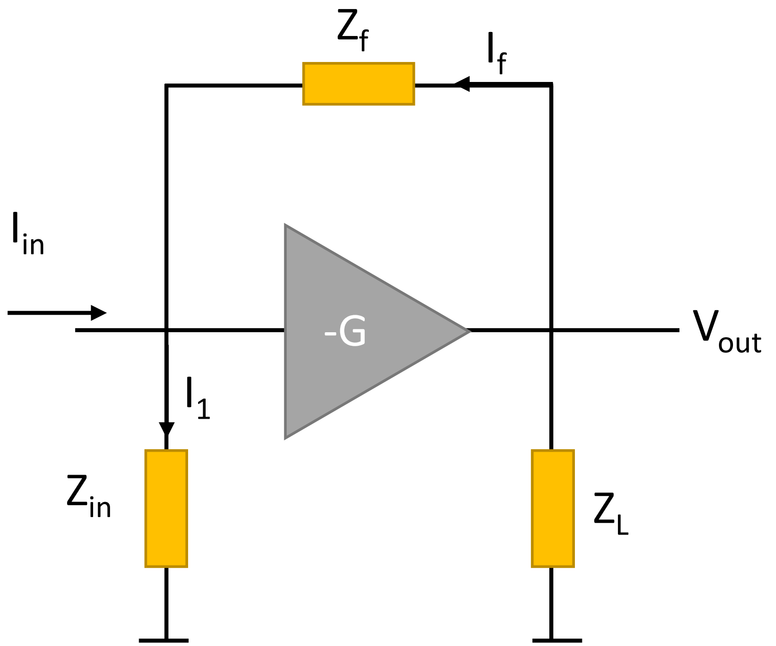


RHP = unstable system (rising
real exponential)

Charge Sensitive Amplifier (CSA)

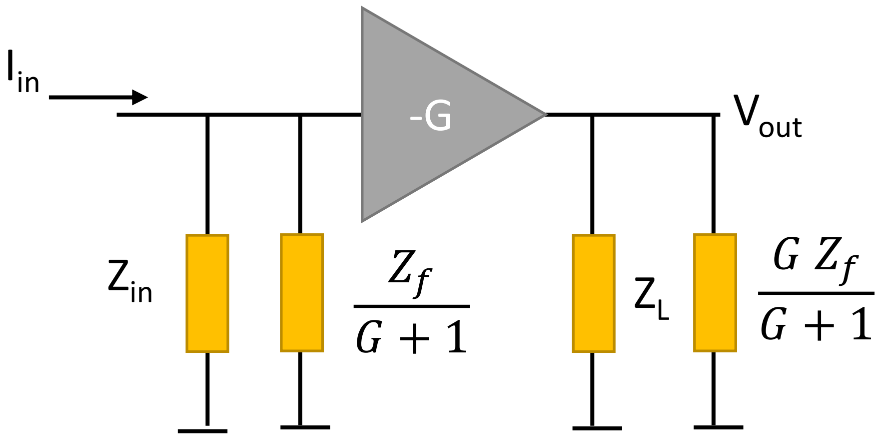
$$V_{out} = -G\ V_{in}$$
$$I_{in} = I_1 - I_f$$

➔ $\frac{V_{out}}{I_{in}} = \frac{-G\ Z_{in} Z_f}{Z_f + Z_{in}(G+1)}$



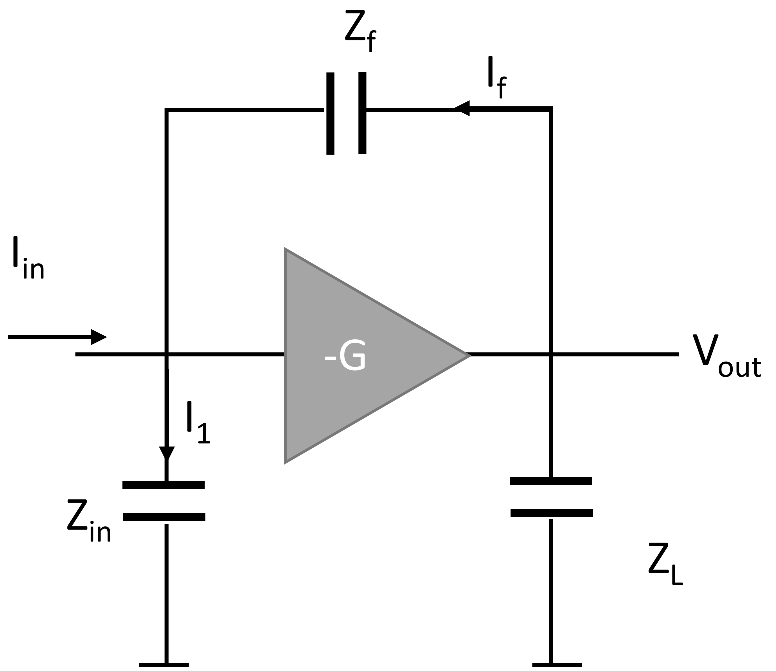
We could reach the same result using the Miller Theorem

(This approach assumes an ideal amplifier)



CSA: Case1 capacitive feedback

$$\frac{V_{out}}{I_{in}} = \frac{-G\, Z_{in} Z_f}{Z_f + Z_{in}(G+1)} = \frac{-G \frac{1}{sC_{in}} \frac{1}{sC_f}}{\frac{1}{sC_f} + \frac{1}{sC_{in}}(G+1)}$$



If G → ∞ *then*

$$\frac{V_{out}}{I_{in}} = -\frac{1}{sC_f}$$

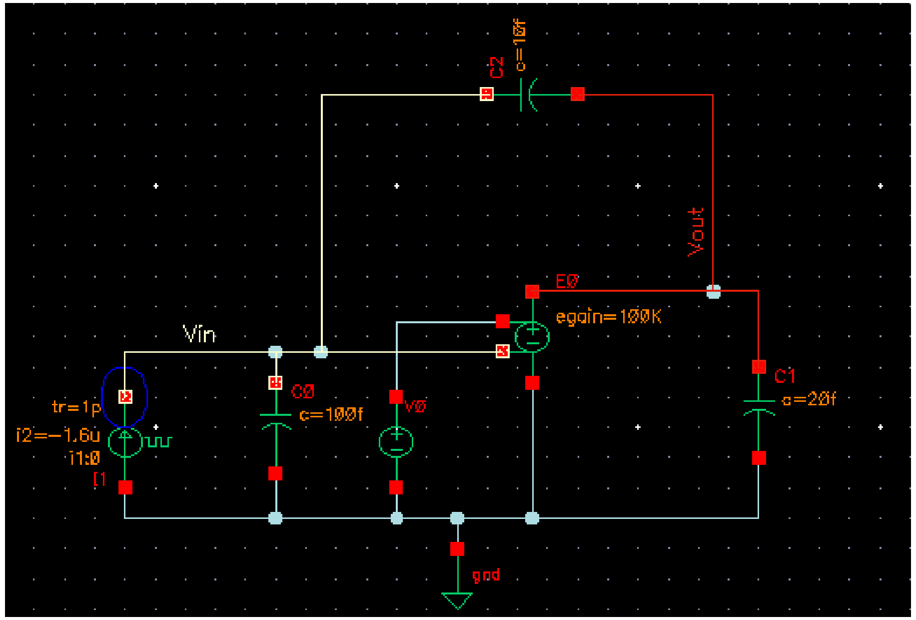
Let's consider an instantaneous injection of a charge Q_0

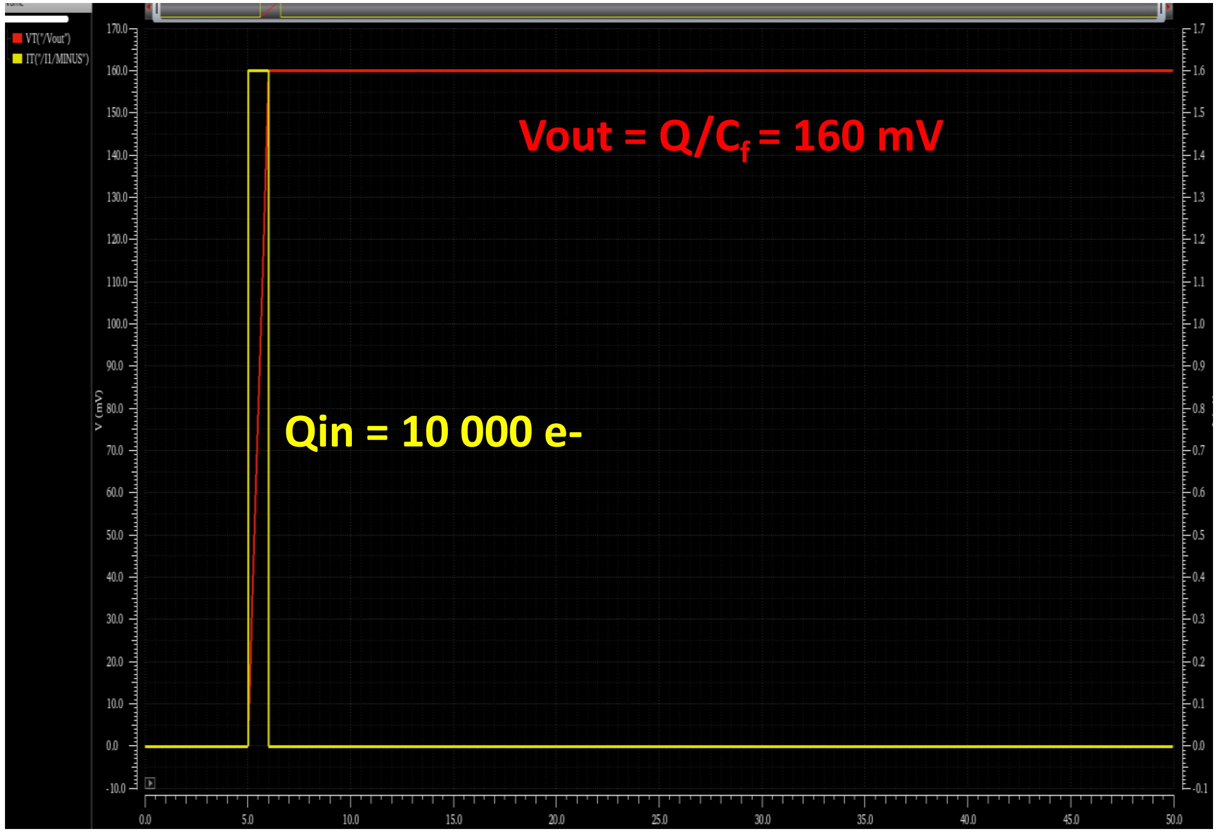
$$I(t) = Q_0 \delta(t) \rightarrow I(s) = Q_0$$

$$Vout\ (s) = -\frac{Q_0}{sC_f} \rightarrow Vout(t) = -\frac{Q_0}{C_f}\mathrm{U(t)}$$

Instantaneous rise time, and amplitude maintained forever

CSA: Case1 capacitive feedback





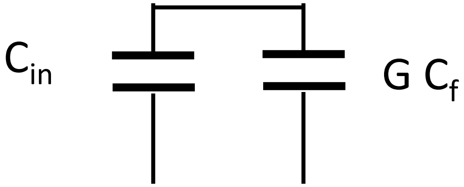
- C_{in} = 100fF C_f= 10fF C_L = 20fF
- G = 100k Q = 10 000 e- over 1 ns

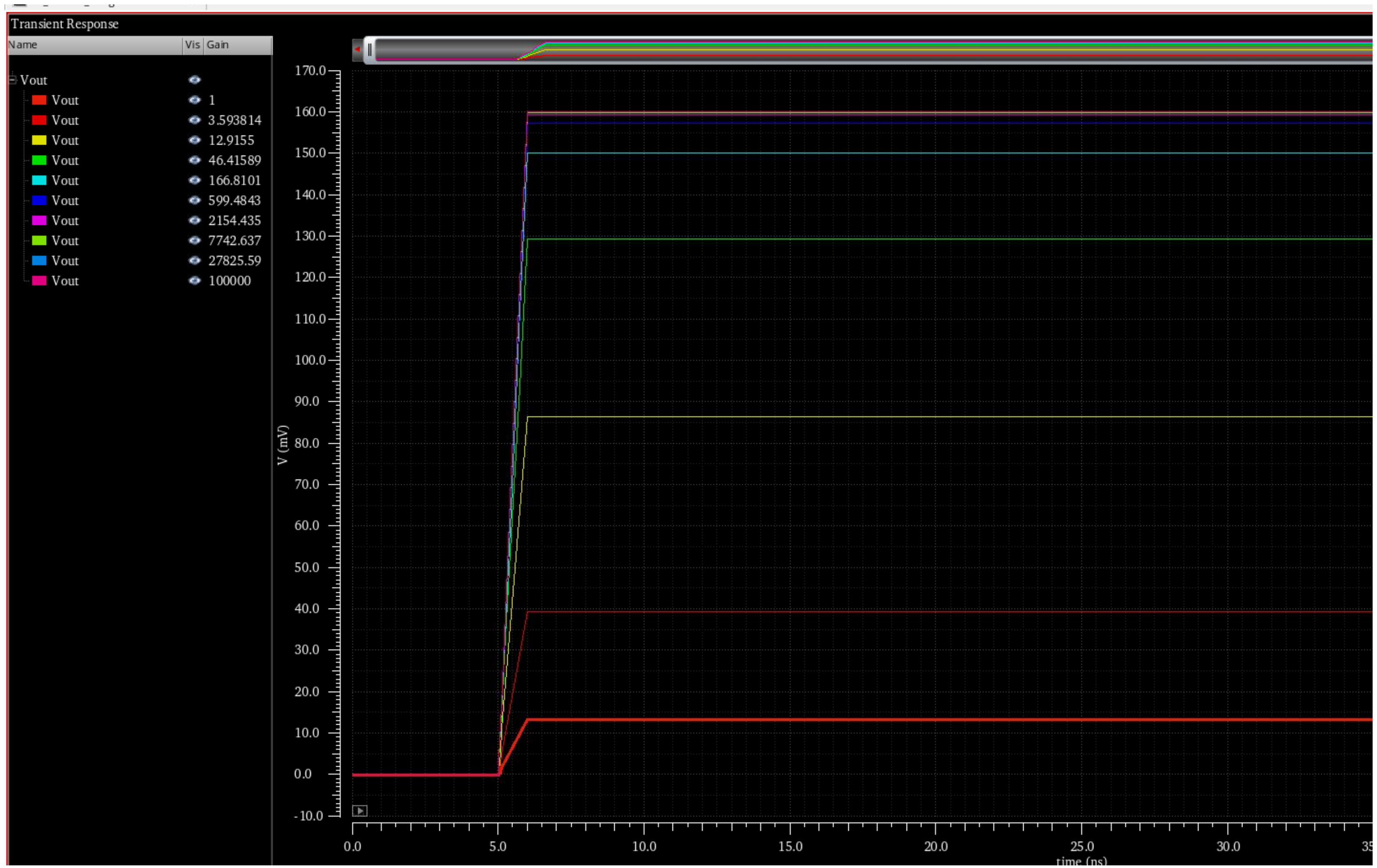
What's a high gain?

Sweep G from 1 to 10^5

For very low gains, V_{out} is dependent on G and C_{in}
In order for the output to be only dependant on Cf, the gain must be high so that:

$$G \gg \frac{C_{in}}{C_f}$$





CSA: Case2 RC feedback

If G → ∞ *then*

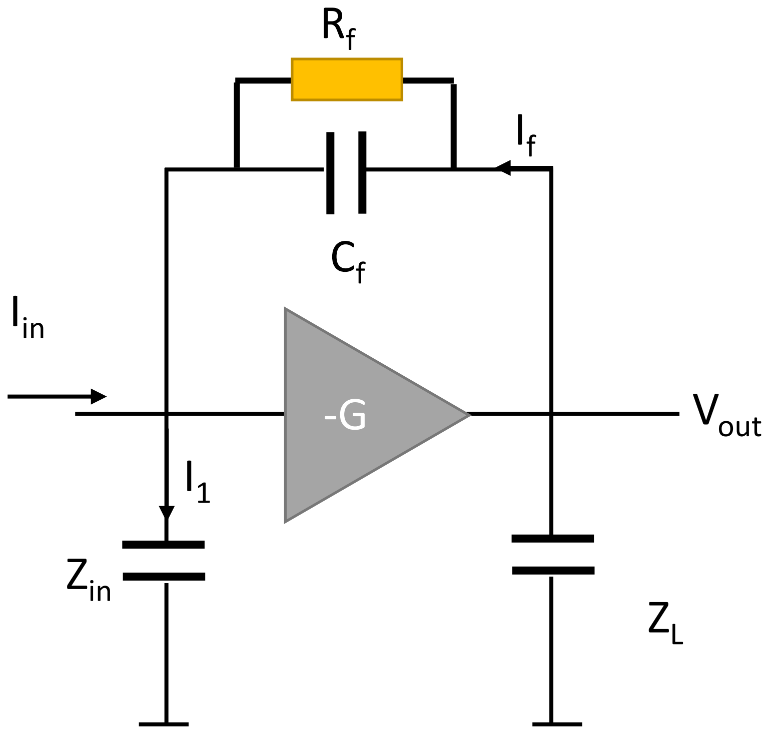
$$\frac{V_{out}}{I_{in}} = -Z_f = -\frac{R_f}{1 + sR_f C_f}$$

$$V_{out}(s) = -\frac{R_f\ Q_0}{1 + sR_f C_f}$$

T domain — S domain

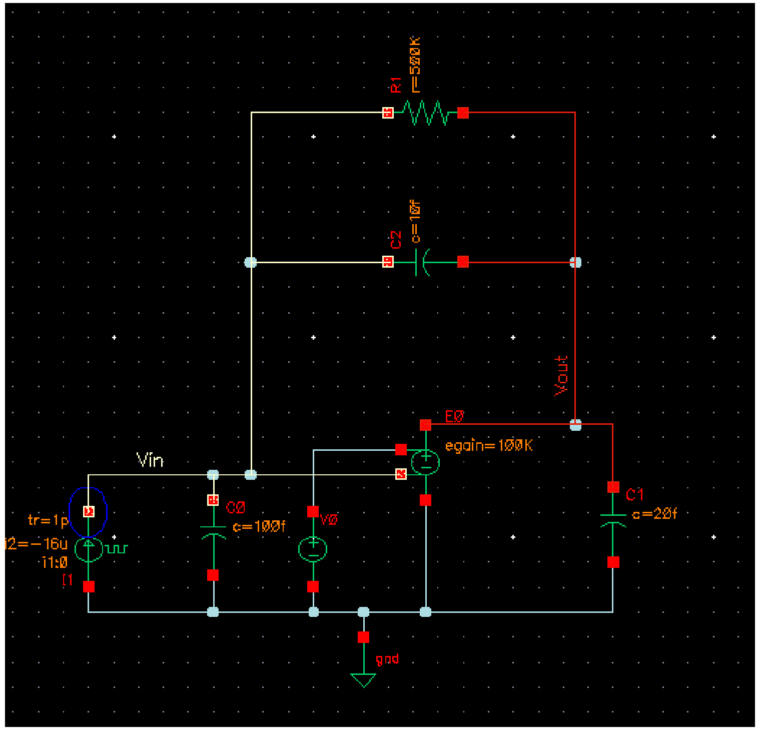
$$e^{at} \longleftrightarrow \frac{1}{s-a}$$

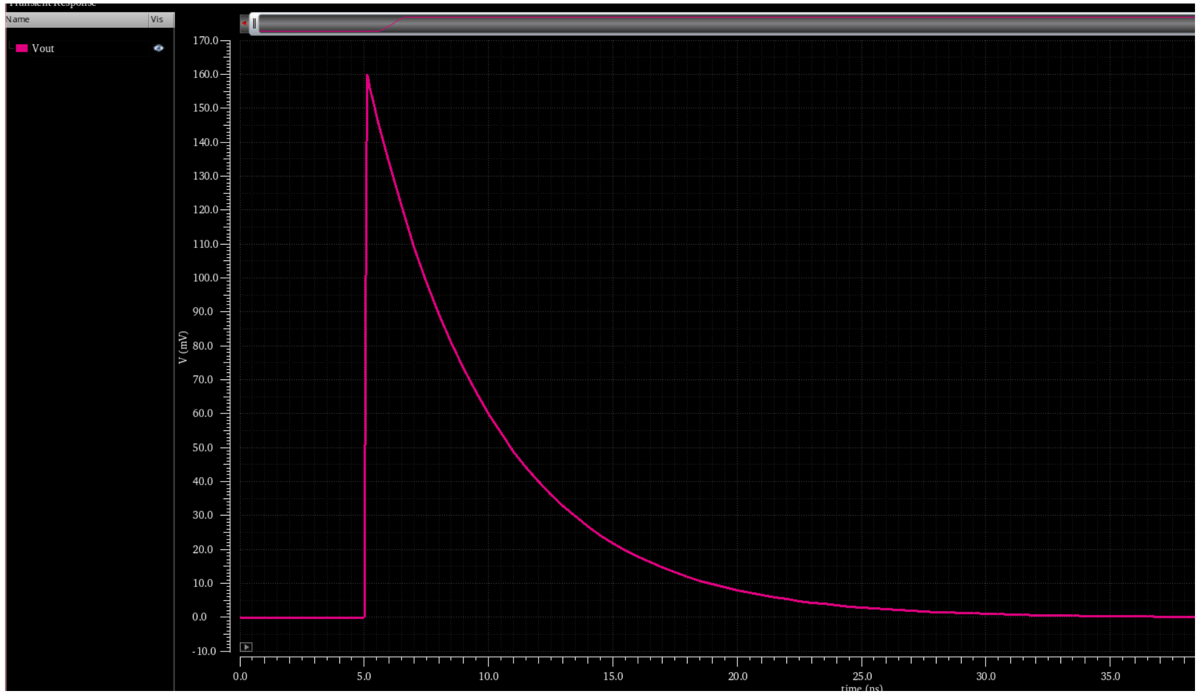
$$V_{out}(t) = -\frac{Q_0}{C_f} e^{-\frac{t}{R_f C_f}}$$



Instantaneous rise time (ideal amp), and discharge with a time constant $R_f C_f$

CSA: Case2 RC feedback



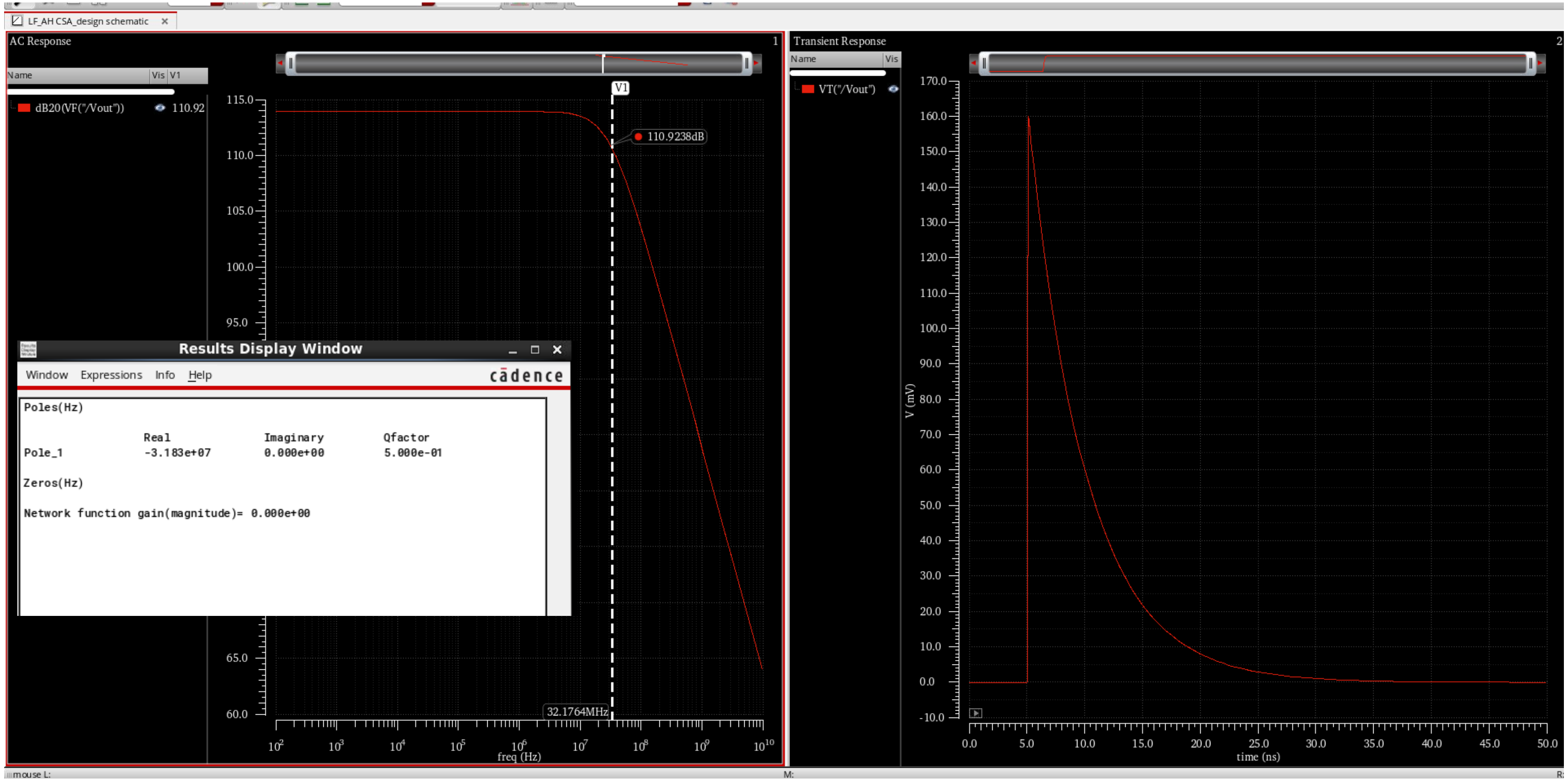


Rf = 500k

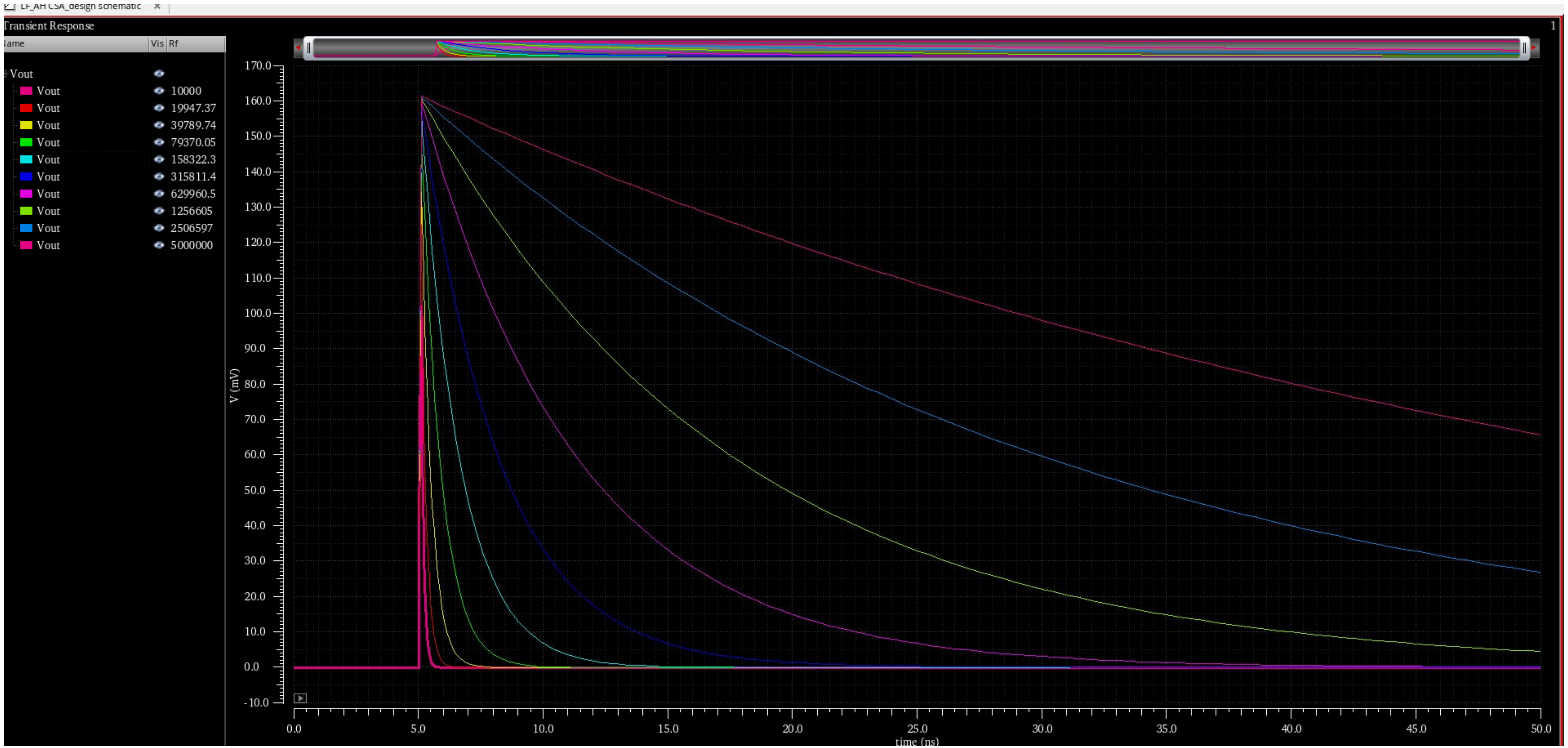
RxC = 5 ns

Pole = 1/(2*pi*RC) = 31.8 MHz

CSA: Case2 RC feedback



The effect of R_f : sweep 10 kΩ - 5 MΩ



CSA: Case3 capacitive with Real Amp

Open loop Transfer Function:

$$\frac{V_{out}}{V_{in}} = -\frac{gmR_L}{1 + s\,R_L C_L} = -G = -\frac{G_0}{1 + \frac{s}{P_0}}$$

$$Close\ loop\ Transfer\ Function:$$

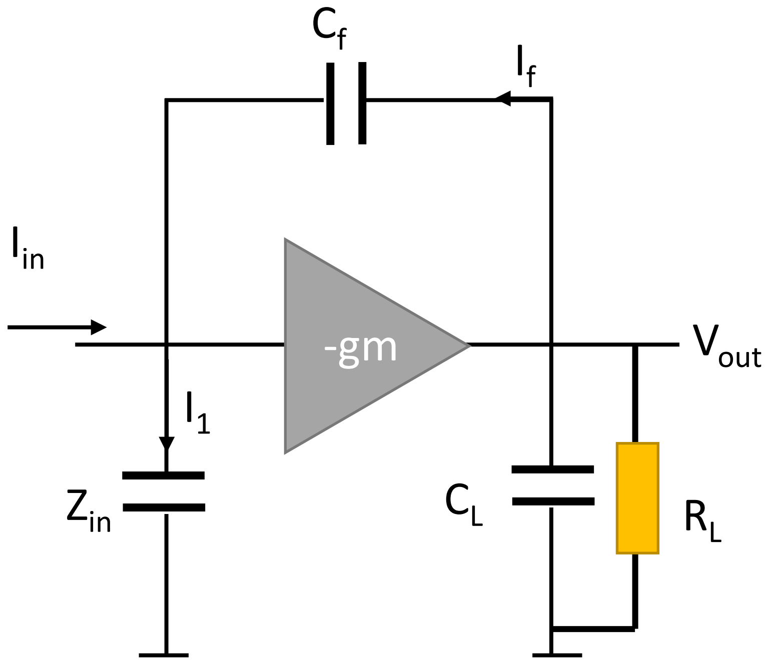
$$\frac{V_{out}}{I_{in}} = \frac{-G\,Z_{in}Z_f}{Z_f + Z_{in}(G+1)}$$

$$Assuming\ GC_f \gg C_{in} we\ end\ up\ with:$$

$$\frac{V_{out}}{I_{in}} = -\frac{\frac{1}{sC_f}}{1 + s\frac{C_L(C_{in}+C_f)}{gmC_f}}$$

→ Final gain

→ Rising time constant



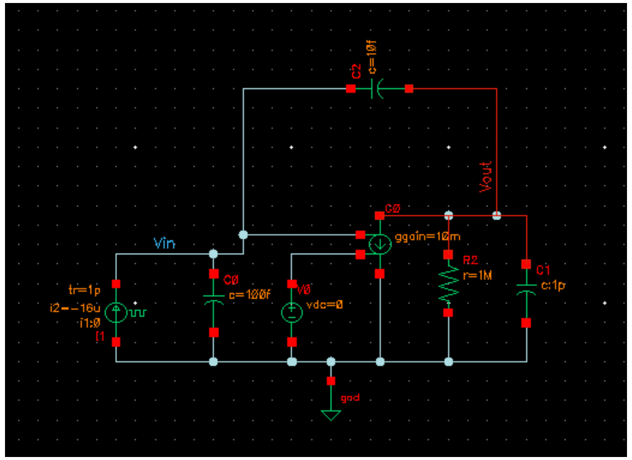
NB: C_{in} represents the sum of all input capacitances $C_{det} + C_{gs} + C_{gd} + \ldots$

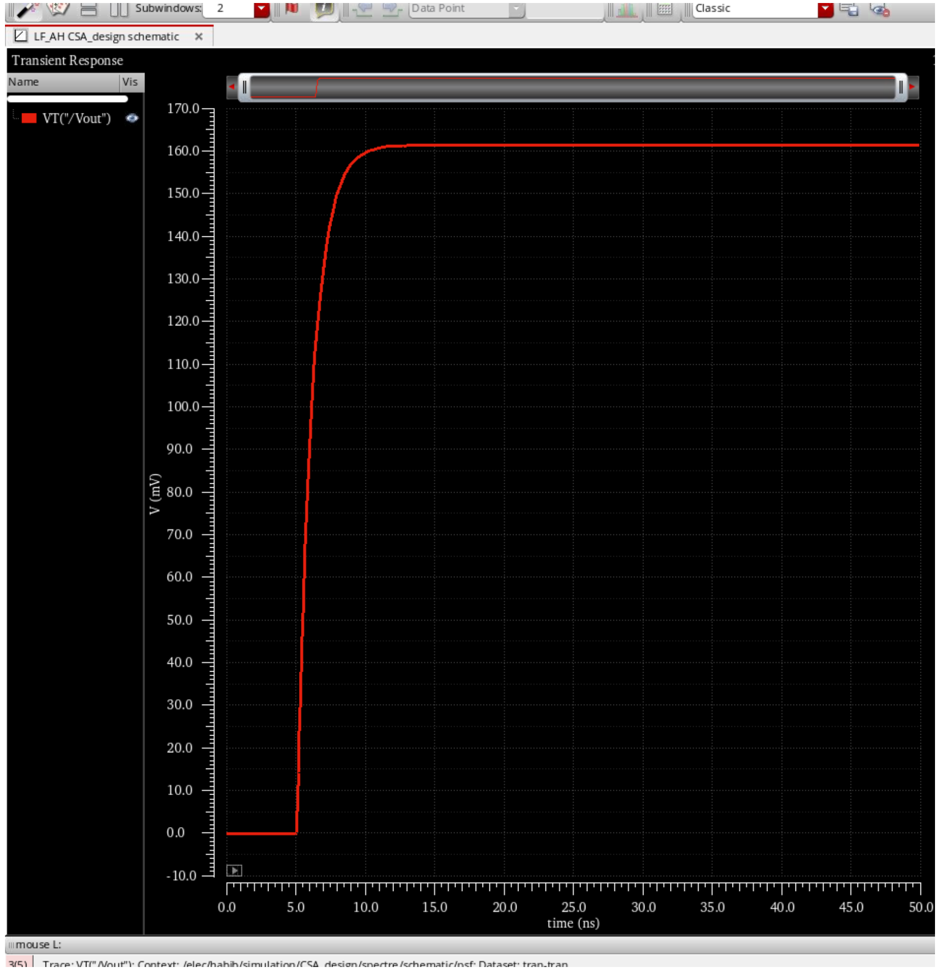
CSA: Case3 capacitive with Real Amp

$$\frac{V_{out}}{I_{in}} = -\frac{\frac{1}{sC_f}}{1 + s\frac{C_L(C_{in}+C_f)}{gmC_f}}$$

In time domain:

$$V_{out}(t) = -\frac{Q_0}{C_f}\left(1 - e^{-\frac{t\,gm\,C_f}{C_L(C_{in}+C_f)}}\right)$$





CSA: Case4 R_fC_f with Real Amp

$$We\ can\ prove\ that:$$

$$\frac{V_{out}}{I_{in}} = \frac{R_f}{1 + sR_fC_f + s^2\frac{R_fC_{in}C_L}{gm}}$$

If the system has a dominant pole ($R_fC_f \gg \frac{C_{in}C_L}{gm\,C_f}$) then the TF can be written as:

$$\frac{V_{out}}{I_{in}} = \frac{R_f}{(1 + sR_fC_f)(1 + s\frac{C_{in}C_L}{gm\,C_f})}$$

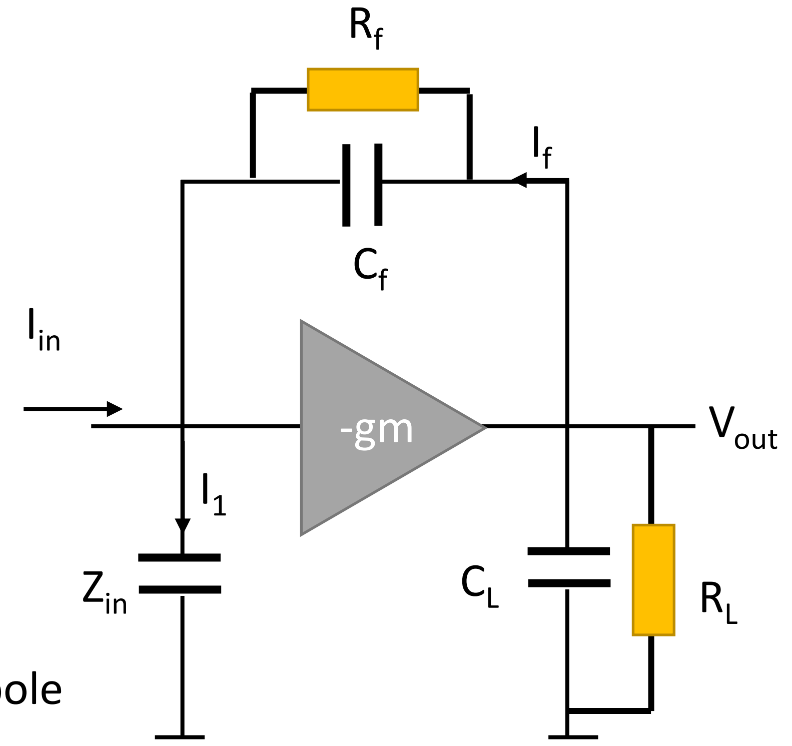
Feedback pole

Output pole

$$\frac{V_{out}}{I_{in}} = \frac{R_f}{(1 + s\tau_f)(1 + s\tau_r)}$$

Falling time constant

Rising time constant



CSA: Case4 $R_f C_f$ with Real Amp

$$\frac{V_{out}}{I_{in}} - \frac{R_f}{(1 + sR_f C_f)(1 + s\frac{C_{in} C_L}{gm\ C_f})}$$

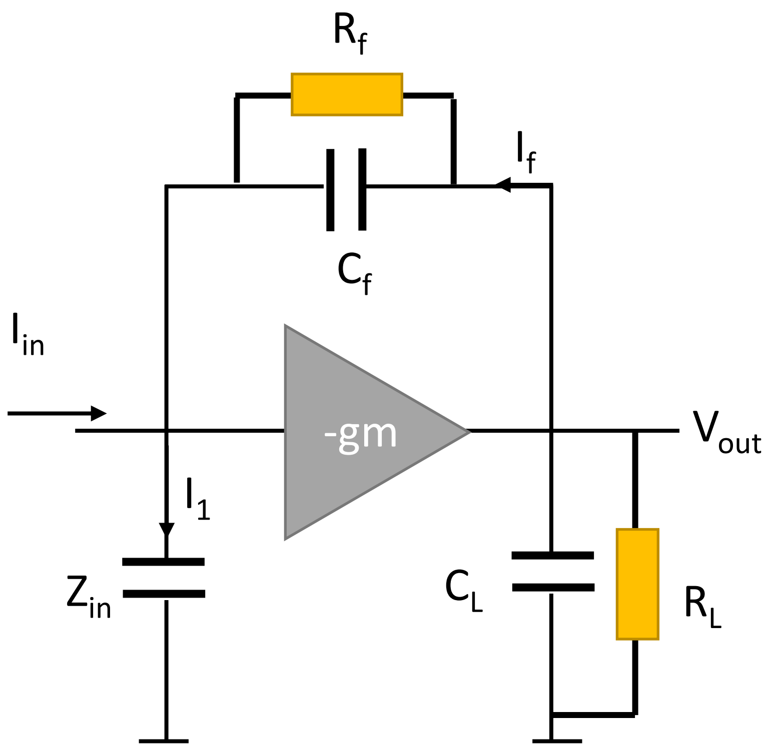
In time domain:

$$Vout(t) = -\frac{Q_0\ R_f C_f}{R_f C_f^2 - \frac{C_{in} C_L}{gm}}(e^{-\frac{t}{R_f C_f}} - e^{-\frac{t\ gm\ C_f}{C_{in} C_L}})$$

It is already assumed that $R_f C_f \gg \frac{C_{in} C_L}{gm}$ then the TF approximates to :

$$Vout(t) = -\frac{Q_0}{C_f}(e^{-\frac{t}{R_f C_f}} - e^{-\frac{t\ gm\ C_f}{C_{in} C_L}})$$

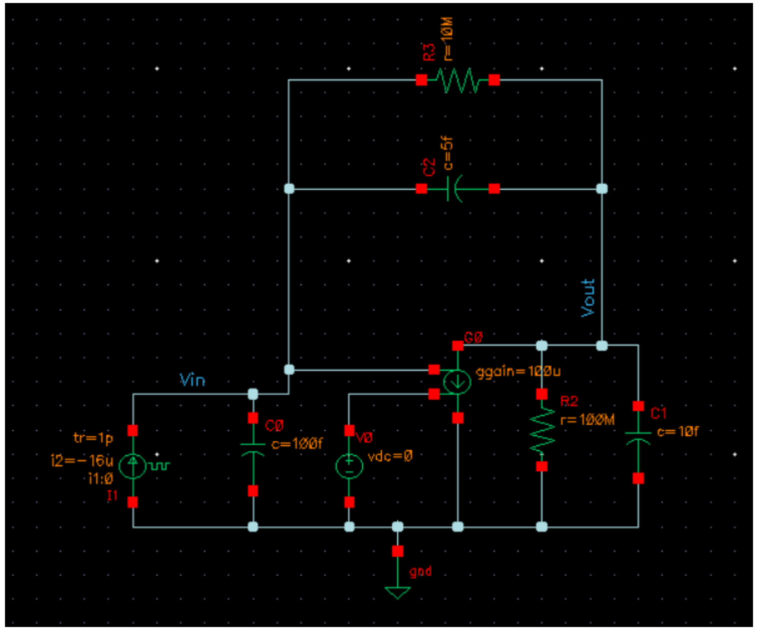
$$Vout(t) = -\frac{Q_0}{C_f}(e^{-\frac{t}{\tau_f}} - e^{-\frac{t}{\tau_r}})$$

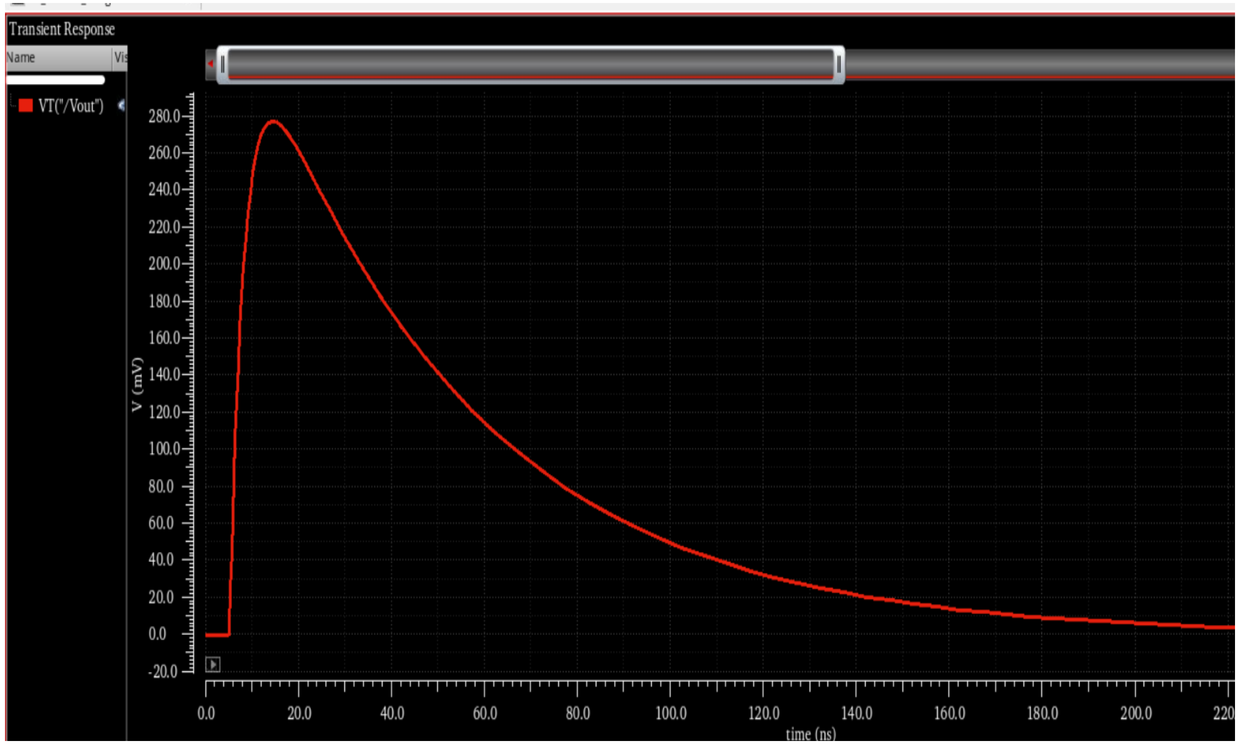


CSA: Case4 R_fC_f with Real Amp

$$Vout(t) = -\frac{Q_0}{C_f}\left(e^{-\frac{t}{R_f C_f}} - e^{-\frac{t\, gm\, C_f}{C_{in} C_L}}\right)$$

$$Vout(t) = -\frac{Q_0}{C_f}\left(e^{-\frac{t}{\tau_f}} - e^{-\frac{t}{\tau_r}}\right)$$





NB: The signal reaches max amplitude only if $R_fC_f \gg \frac{C_{in}C_L}{gm}$

Stability of a closed loop system

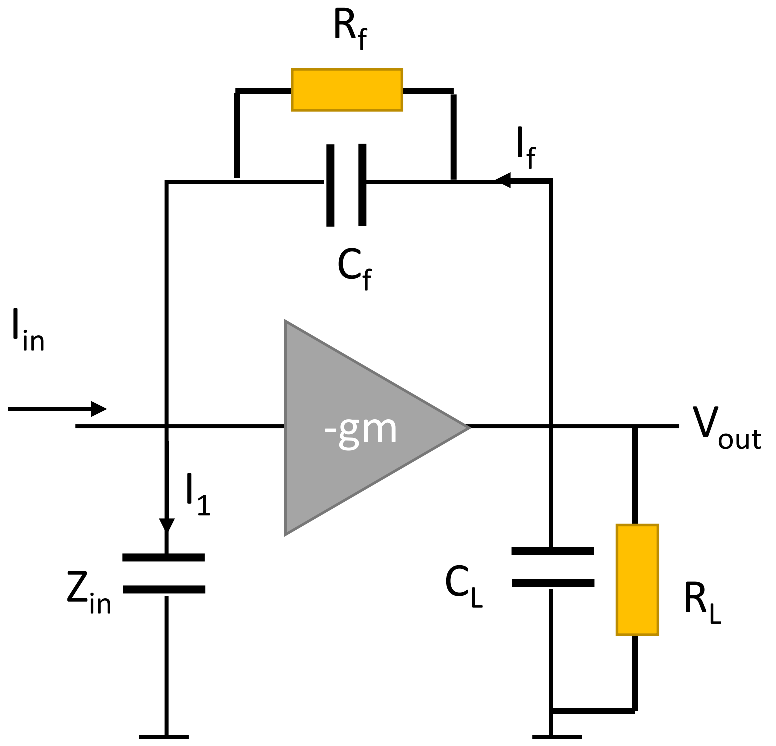
The transfer function can be written under this form:

$$H = \frac{G_0}{1 + s\frac{2\zeta}{\omega_0} + \frac{s^2}{\omega_0^2}}$$

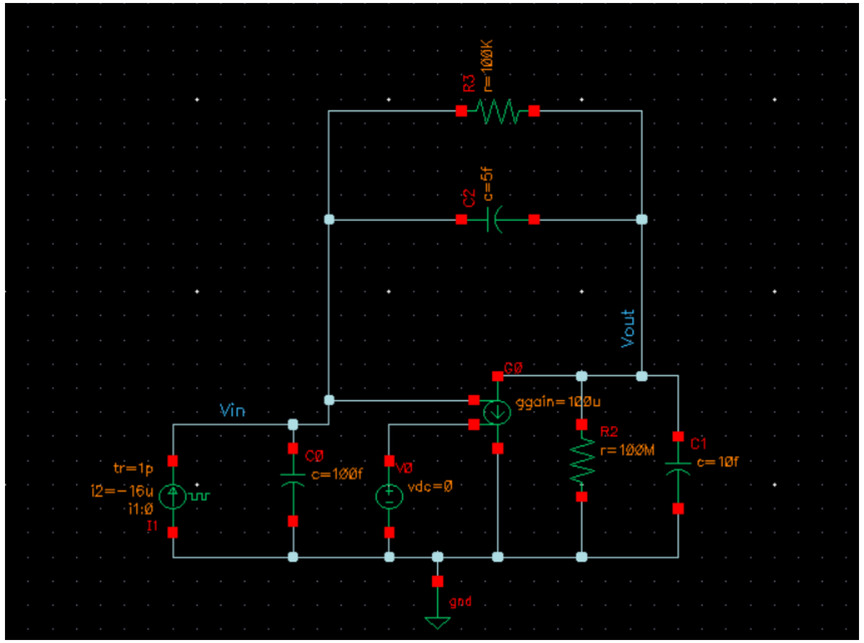
ω_0 is called the natural frequency, and ζ the damping factor. The denominator admits the following roots:

$$P_{1,2} = -\zeta\omega_0 \pm \omega_0\sqrt{\zeta^2 - 1}$$

- If ζ>1 then the roots are real and the system is over-damped (stable)
- If $0 < \zeta < 1$ then the root are complex conjugate and the system exhibits damped oscillations
- If $\zeta = 0$ then the system is pure oscillator (unstable)



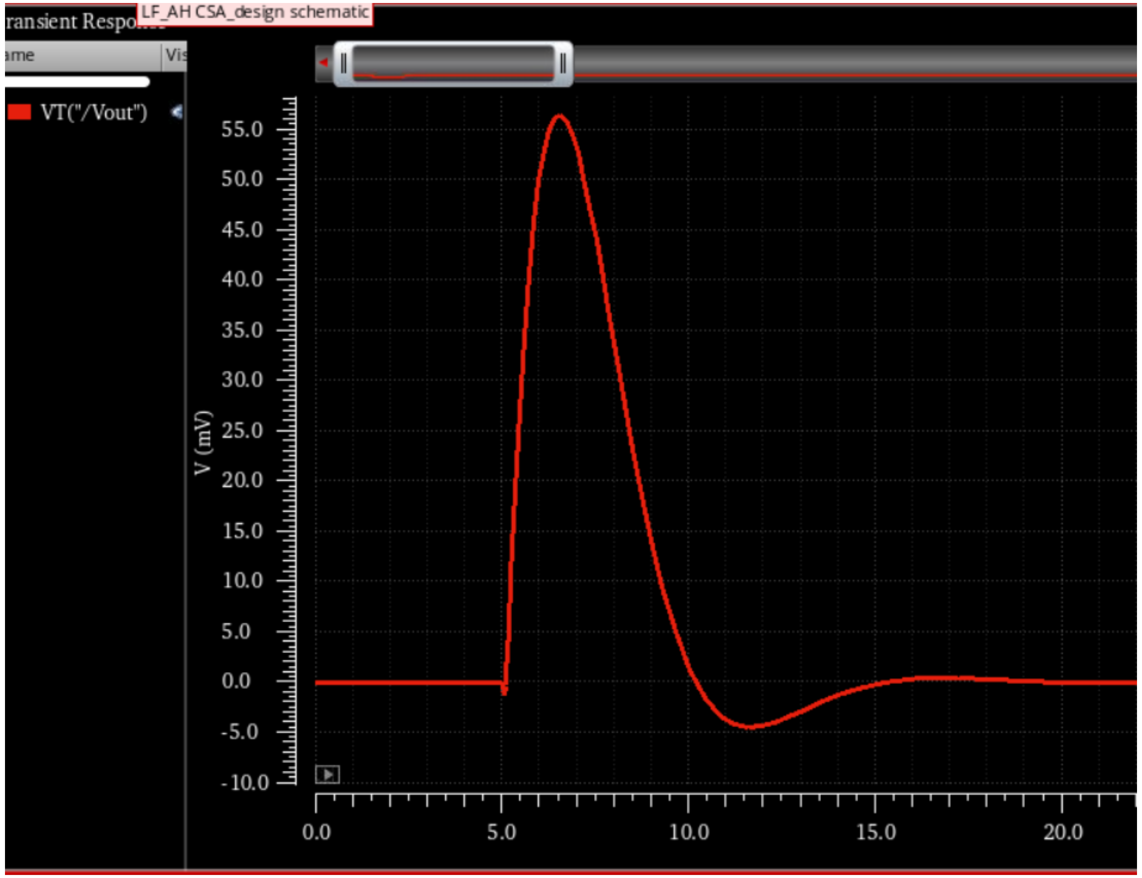
Case of complex conjugate poles



P1 = -82 + j 97 MHz

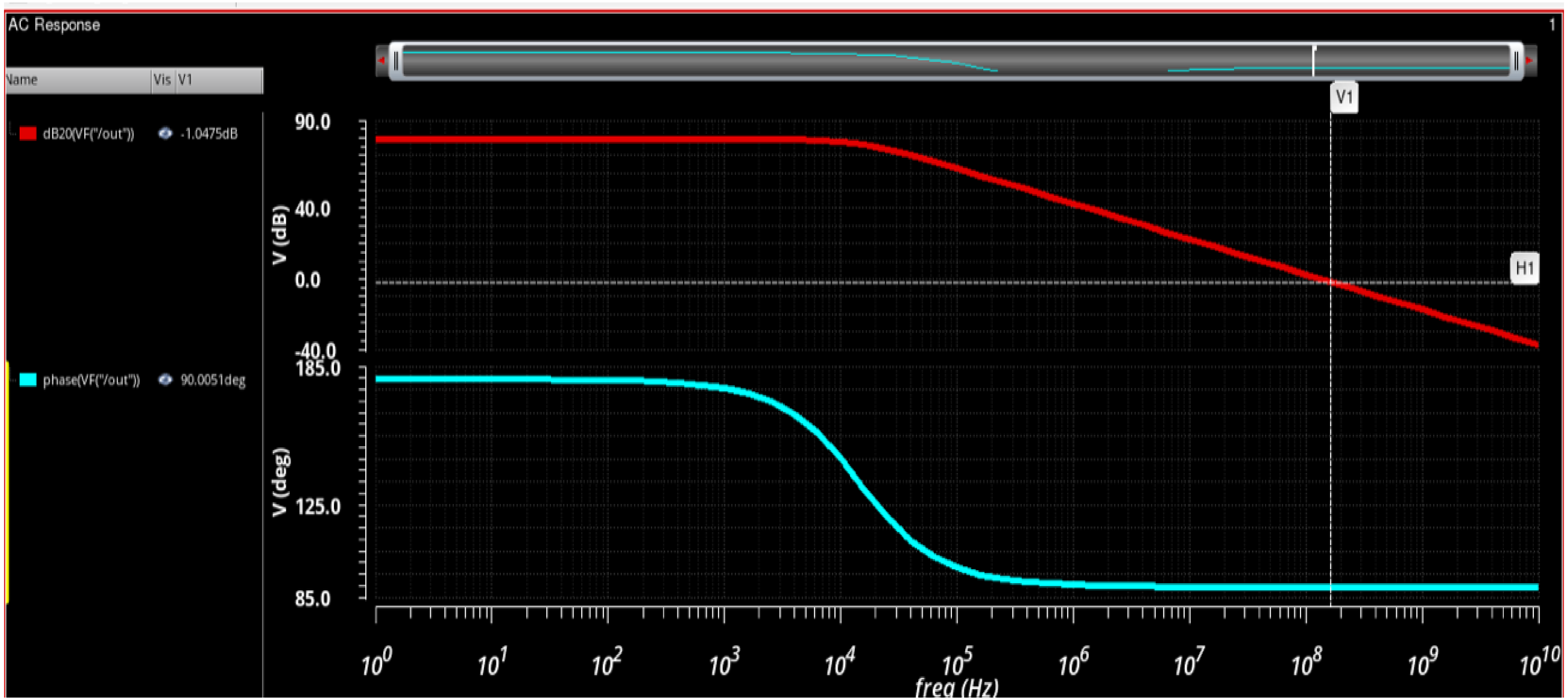
P2 = -82 – j97 MHz

Damped oscillation

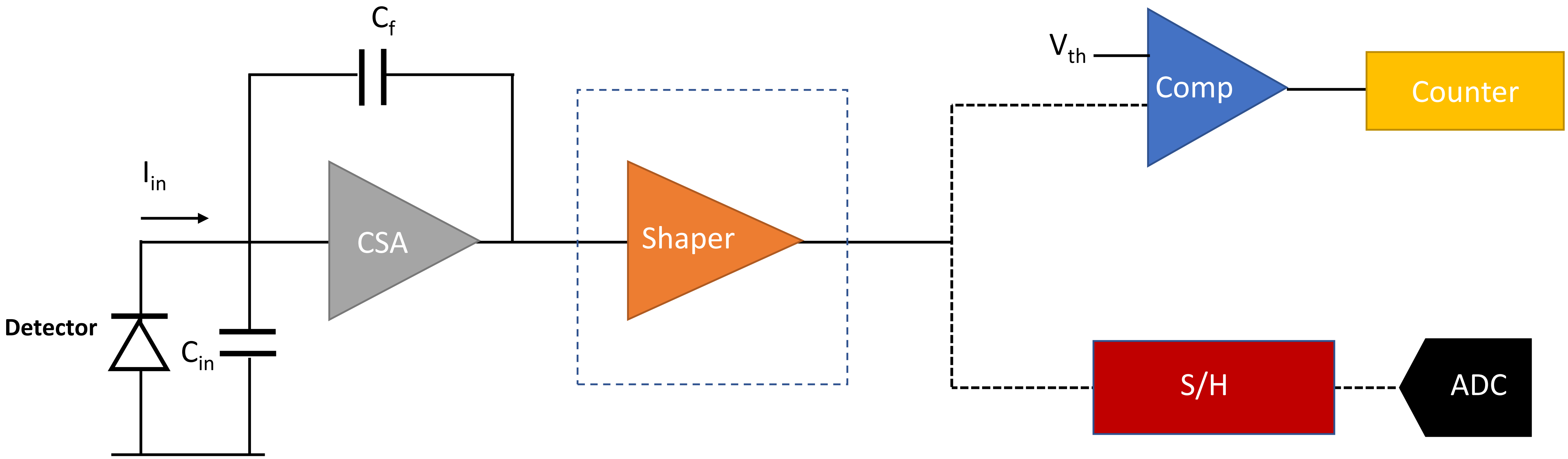


Stability Study by Simulation

Simulate open loop response, and check for phase margin. Good practice PM > 45°



Typical Front-End Channel: 2- Shaper

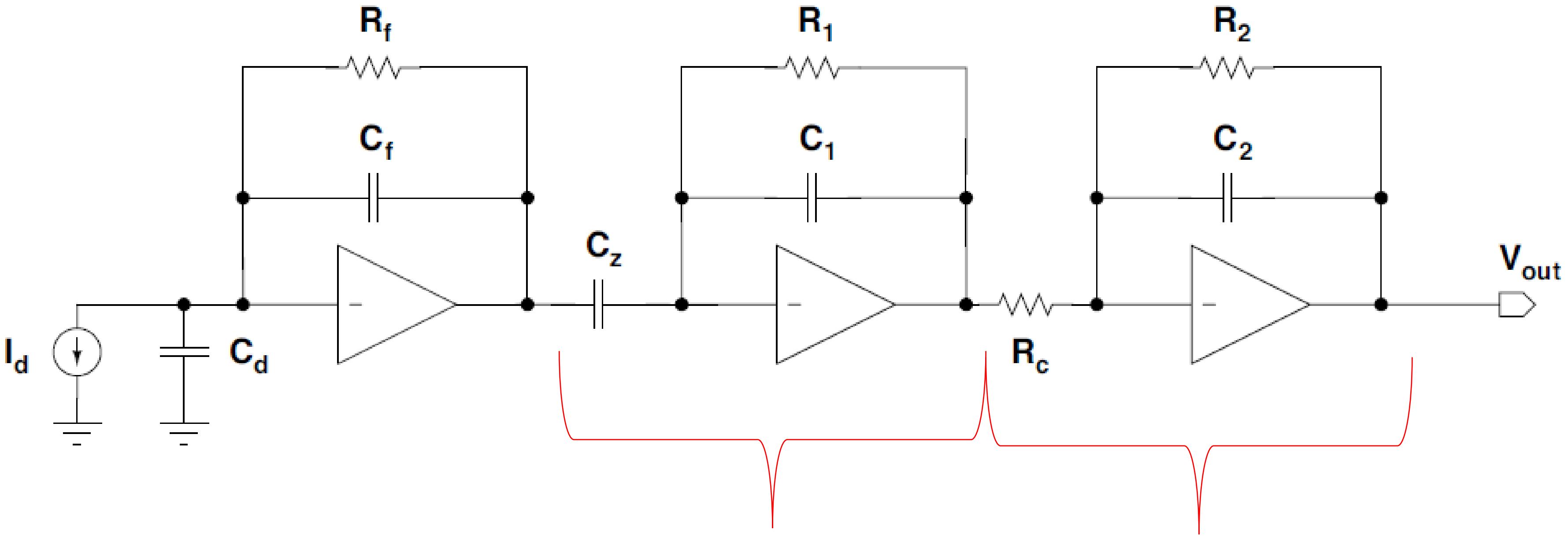


Chare Sensitive Amplifier
Integrates the incoming charge on a feedback capacitor

Shaper
Shapes (Filters) the signal for an optimal Signal to Noise Ratio

Comparator to detect an event above a certain threshold
And/Or
a Sample and Hold circuit for an ADC further in the channel

Shaper: CR-RC filter

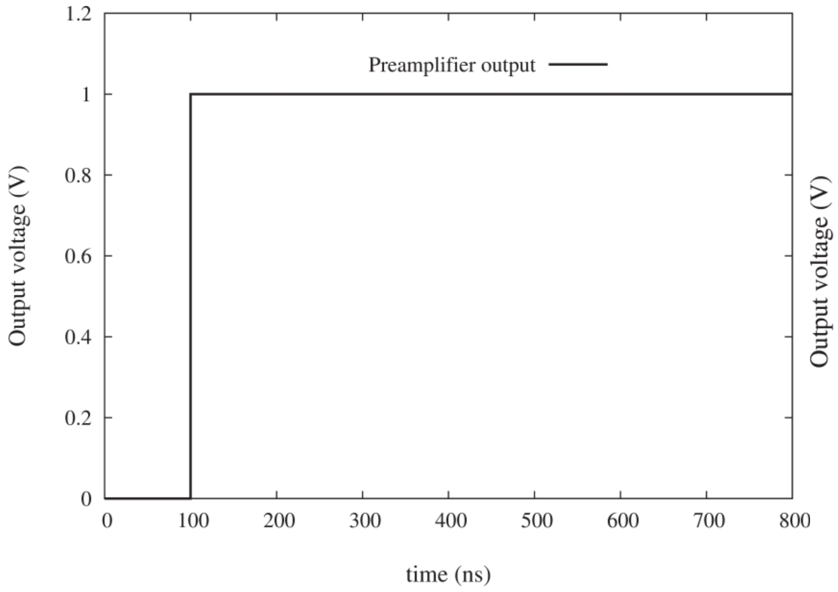


Differentiator
= High pass filter
$\tau_z = R_1 C_1$

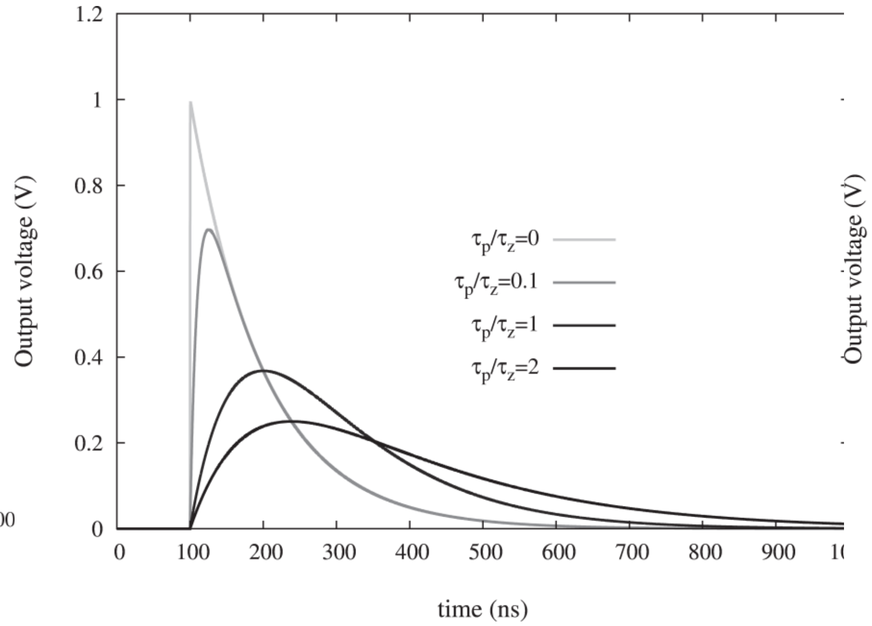
Integrator
= Low pass filter
$\tau_p = R_2 C_2$

Shaper = Band pass filter

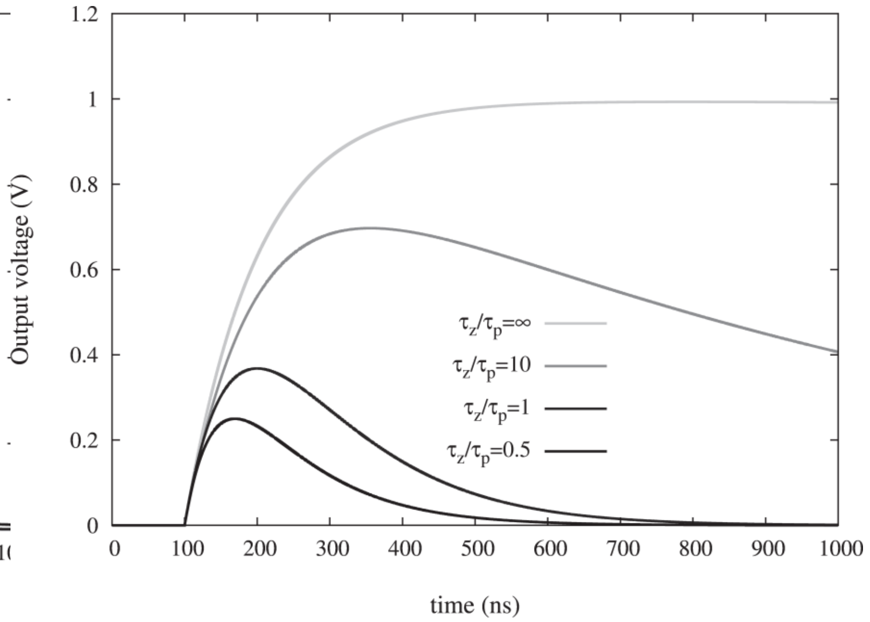
Qualitative Approach



CSA output
(assuming $R_f = \infty$)



Effect of τ_z while τ_p = constante



Effect of τ_p while τ_z = constante

Normally the choice $\tau_z = \tau_p$ is the best compromise for signal amplitude within a fixed duration, and for filtering of high and low noise frequencies

CR-RC Shaper Modeling

It can be demonstrated that the CR-RC transfer function is

$$V_{out}(s) = Q_{in}\frac{C_z}{C_f}R_1\frac{R_2}{R_c}\frac{1}{(1+s\tau)^2}$$

$Where\ \tau = \tau_z = \tau_p$

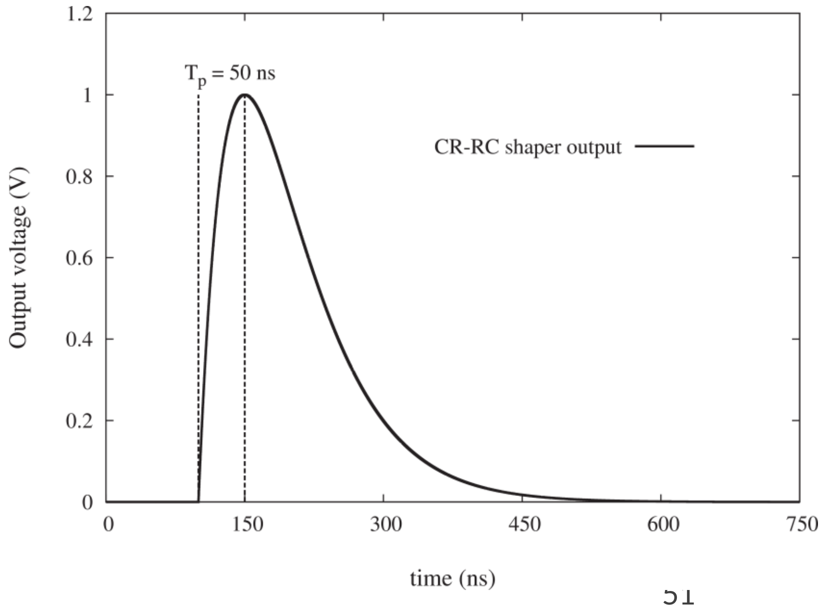
Taking inverse Laplace Transform yields:

$$V_{out}(t) = Q_{in}\frac{C_z}{C_f}R_1\frac{R_2}{R_c}\frac{1}{\tau}\left(\frac{t}{\tau}\right)e^{-\frac{t}{\tau}}$$

The peaking time is $T_{peak} = \tau$

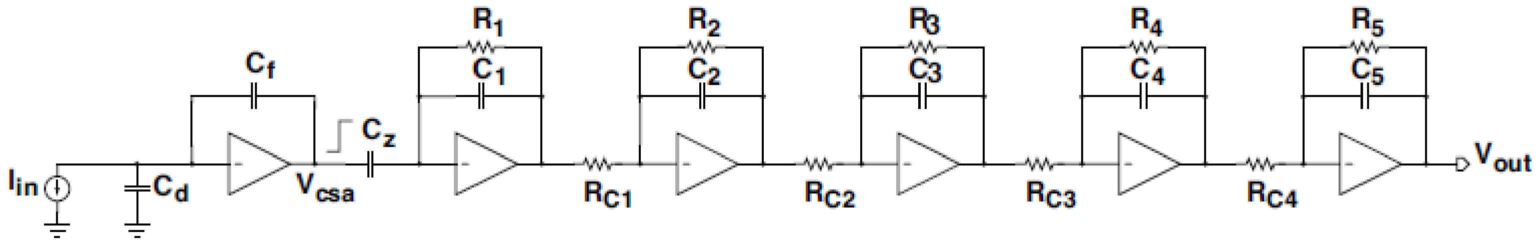
And the maximum amplitude is:

$$V_{out,peak} = \frac{Q_{in}}{C_f}\frac{C_z}{C_1}\frac{R_2}{R_c}\frac{1}{e}$$



NB: The ratio of resistors and capacitors can introduce gain in addition to filtering

CR-RC^N shaper

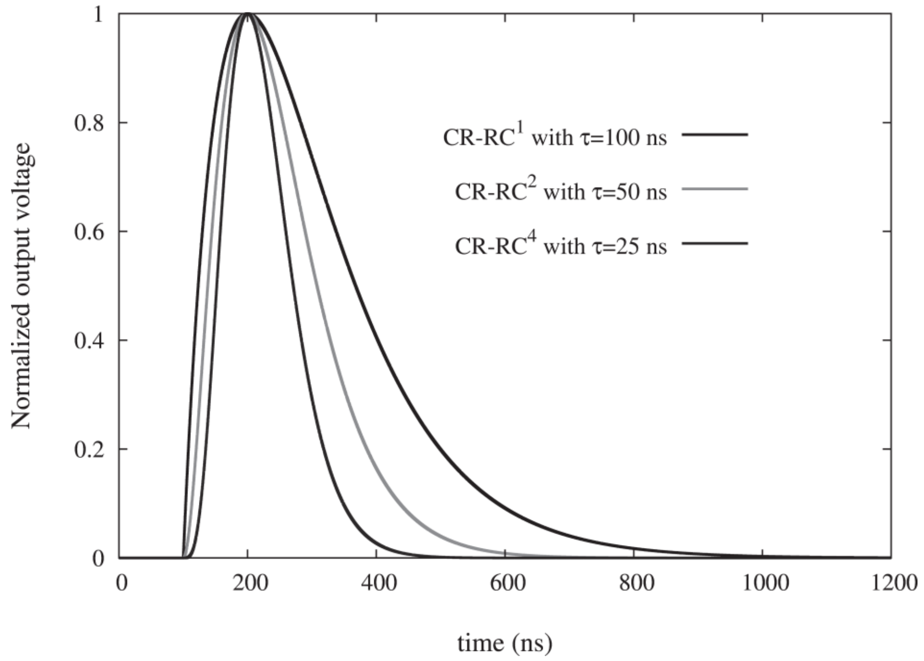


Introducing 'N' number of integrator stages yields a higher the order of the shaper

$$V_{out}(s) = \frac{Q_{in}}{C_f} \frac{C_z R_1}{(1+s\tau)^{n+1}}$$

$$V_{out}(t) = \frac{Q_{in}}{C_f} \frac{C_z}{C_1} \frac{1}{n!} \left(\frac{t}{\tau}\right)^n e^{-\frac{t}{\tau}}$$

Higher order filters give faster return to baseline, useful for high-rate applications, at the expense of power and circuit surface



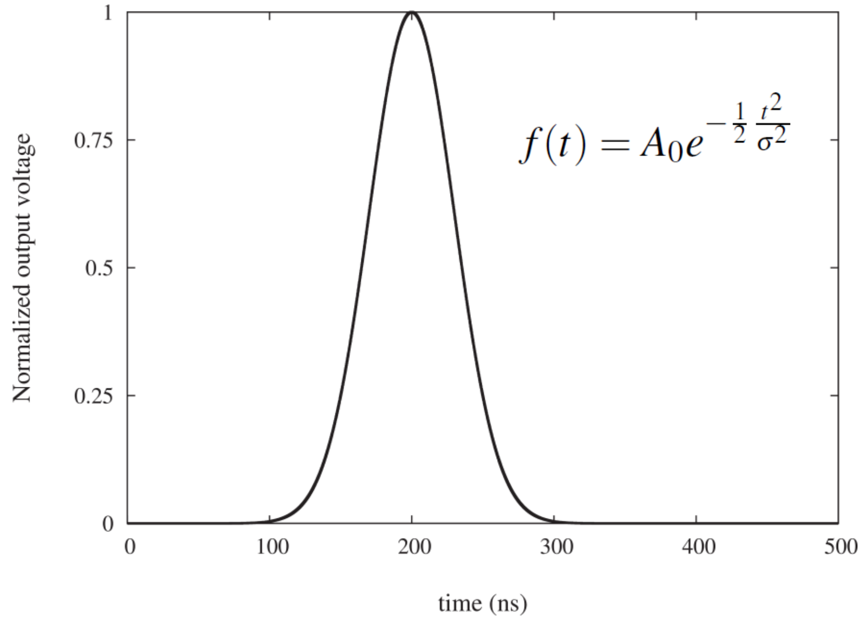
Complex Conjugate Shapers

It can be demonstrated that a perfect filter's response approaches a Gaussian shape.

This response can better be approximated using complex conjugate poles in the transfer function.

The math can be complex, so there are tables that give the poles positions as a function of the shaper order. (Poles are normalized to σ ➔ $p = \sigma s$)

	n=2	n=3	n=4	n=5	n=6	n=7
R_0		1.2633		1.4767		1.6610
R_1	1.0985	1.1491	1.3554	1.4167	1.5601	1.6230
I_1	0.4551	0.7864	0.3278	0.5979	0.2687	0.5008
R_2			1.1810	1.2037	1.4614	1.4950
I_2			1.0604	1.2995	0.8330	1.0455
R_3					1.2207	1.2344
I_3					1.5145	1.7113



Theoretical Gaussian pulse shape

Outline

From Particle to Detected Charge

- Signal Formation: Shockley-Ramo Theroem
- Current calculation in electrode
- Hybrid Vs Monolithic detectors

From Charge to Amplified Analog Signal

- Typical Front-End Chain
- Charge Sensitive Amplifier
- Shaper

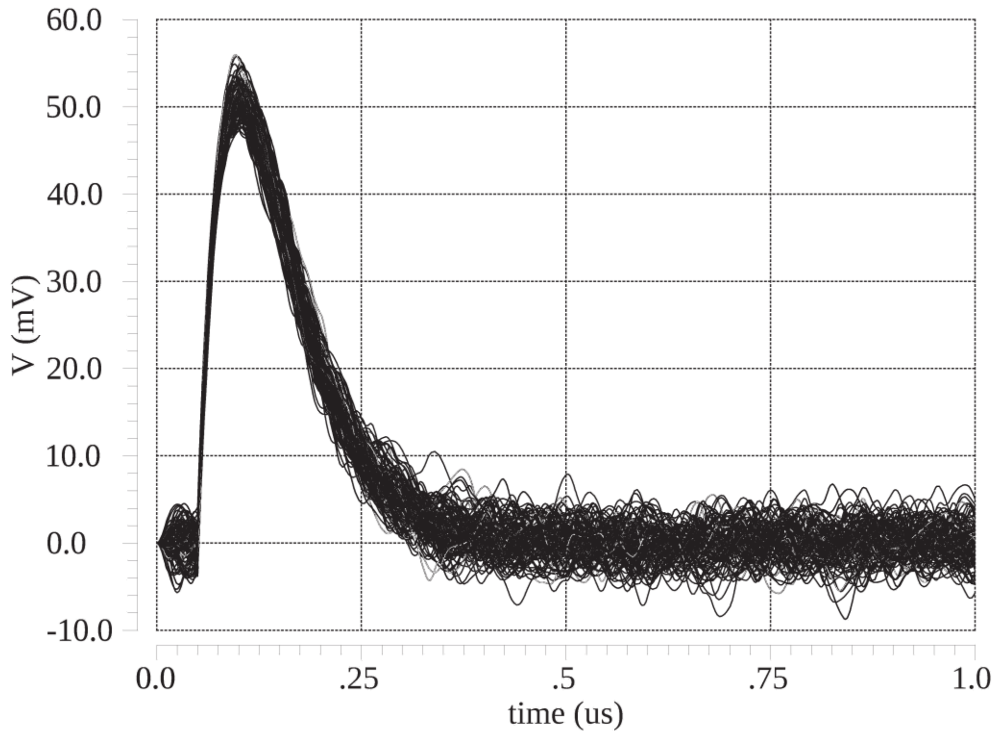
Noise and Design Optimization

- Thermal, shot, Flicker
- Series Vs Parallel noise
- Noise calculation and optimization

Noise

Practically, the signal fluctuates around a mean value for baseline and amplitude due to noise. This limits the capability of the CSA for detecting small charges, which must be distinguishable from the noise.
In order to optimize our circuit, we need to understand the origin of noise sources.

Noise ca be represented by a voltage source whose power spectral density is noted $S_v(f)$ measured in $\left[\frac{V^2}{Hz}\right]$. For brevity, the same quantity can be noted in several references as $v_n^2(f)$. The average output power obtained after integration over the BW is then noted $< v^2 >$.
Noise can also be represented as current sources.



Noise calculation

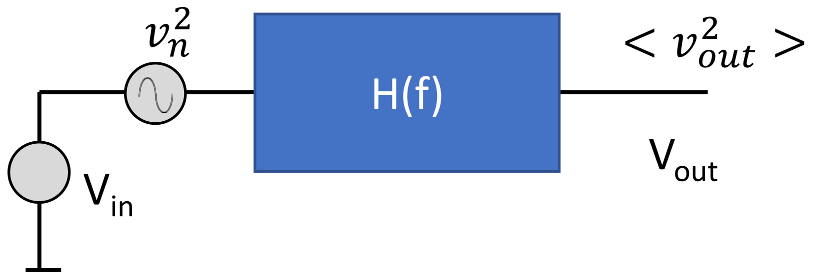
Considering a noise spectral density of

v_n^2 at circuit input, whose unit is $\left[\frac{V^2}{Hz}\right]$

$H(f)$ is the system's transfer function

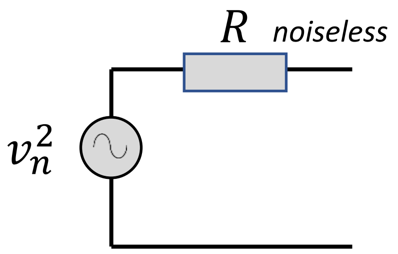
The average output noise is:

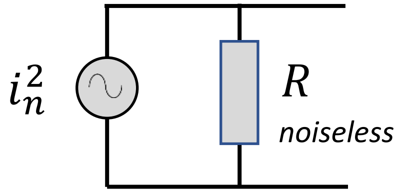
$$< v_{out}^2 > = \int_0^\infty v_n^2 . |H(f)|^2 df \qquad [V^2]$$



Noise Sources: 1- Thermal Noise

One of the primary noise sources in electronic circuits is due to the thermal agitation of charge carriers in conductors.



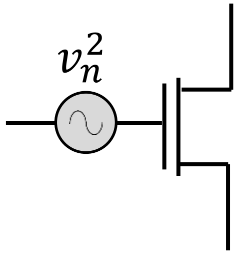


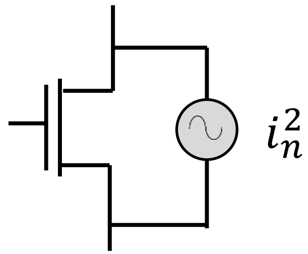
<u>Resistor</u>: In a resistor of value R, the thermal noise :

- $v_n^2 = 4k_BTR$ or $i_n^2 = \frac{4k_BT}{R}$

<u>MOS transistor</u>: In a transistor thermal noise is given by

- $v_n^2 = \frac{4\gamma k_BT}{gm}$ or $i_n^2 = 4\gamma k_BTgm$
- $\gamma = \frac{2}{3}$ for strong inversion and $\frac{1}{2}$ for weak inversion





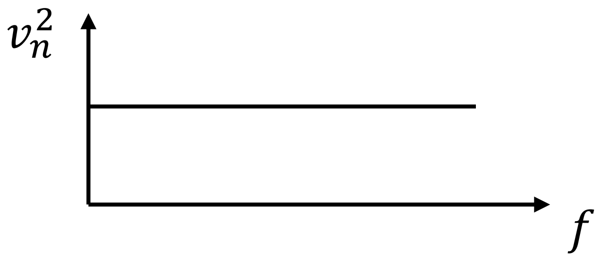
Noise Sources: Shot noise

stems from the fact that the electrical current is quantized in fundamental packets, the electrons, each having a charge q and it is present, for instance, when a photo-current is generated by an incident beam of light or when a potential barrier needs to be crossed, like in diodes.

If the leakage current through a diode is I_{leak} then the shot noise is.

$$i_n^2 = 2qI_{leak}$$

Both thermal noise and shot noise are considered **White Noise**, that is their power spectral density is constant for all frequencies



Flicker noise

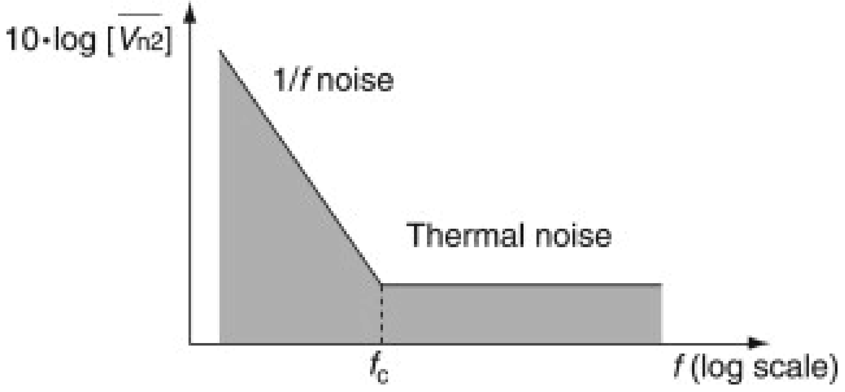
Flicker noise or $1/f$ noise is due to traps located at Si-SiO_2 interface, that can temporarily capture and then release the charge carriers.

There are different models for Flicker noise:

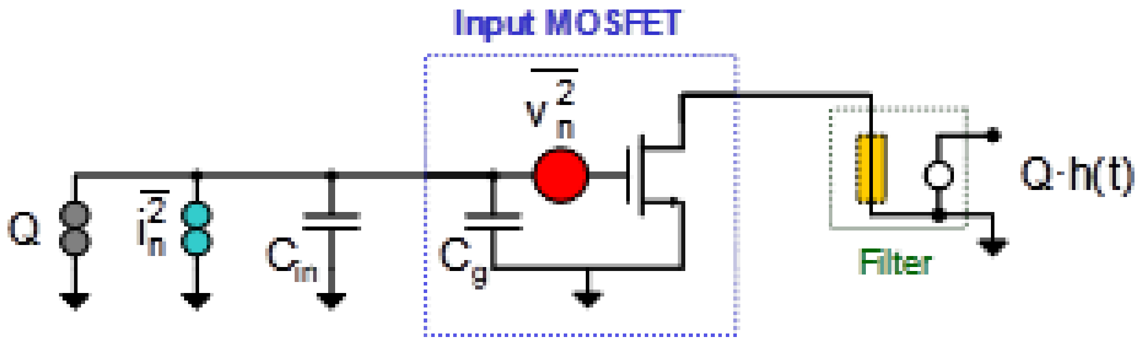
$$v_{nf}^2 = \frac{K_{fb}}{C_{ox}^2 WL}\frac{1}{f}$$

Or

$$v_{nf}^2 = \frac{K_f}{C_{ox} WL}\frac{1}{f^\alpha} \quad with\ 1.3 > \alpha > 0.8$$



Series Vs Parallel noise



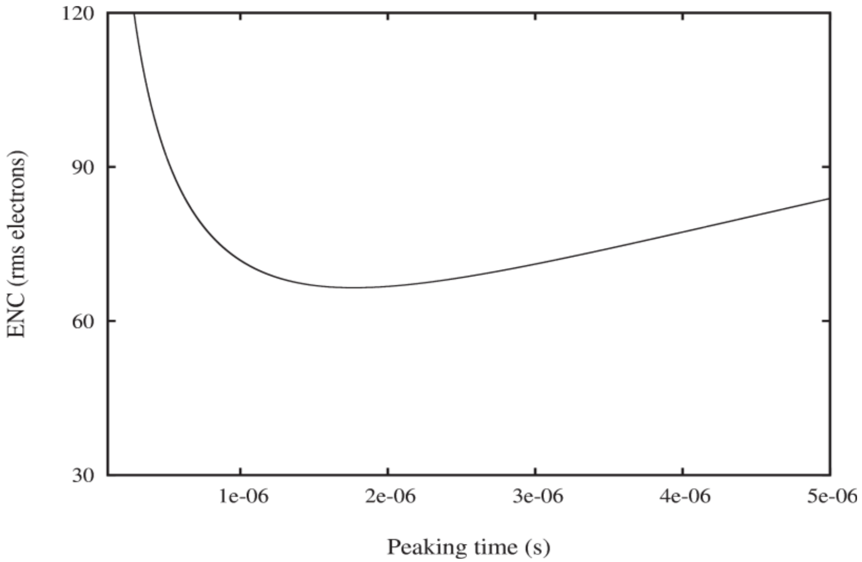
We see that, when referred to the input, the noise can be represented either by voltage sources connected in series with the input or by current sources connected in parallel as they are different in nature.

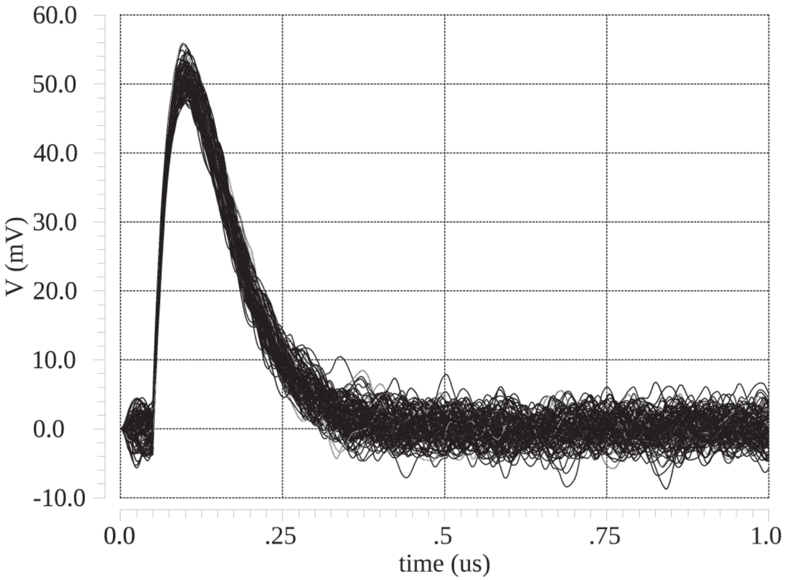
- Voltage sources e.g. input MOS thermal noise, 1/f noise. They are directly applied to input of amplifier and are generally reduced as you reduce the amplifier bandwidth (i.e. increase peaking time).
- Current sources e.g. : leakage current, feedback circuitry noise current. These currents are integrated on C_{in} (like input charge) and increase as the integration window increases (increase with slow peaking time).
- Noise optimization consists in reducing the overall noise of the system

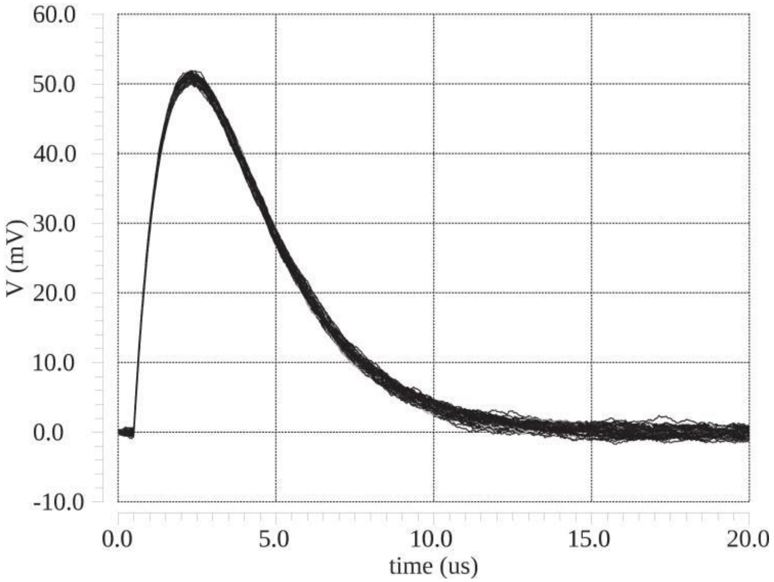
Normally the resolution of the front-end is limited by the noise from the input MOSFET

Series Vs Parallel noise: Example

By calculations or simulations, we can optimize the peaking time in order to have a minimum ENC for the system



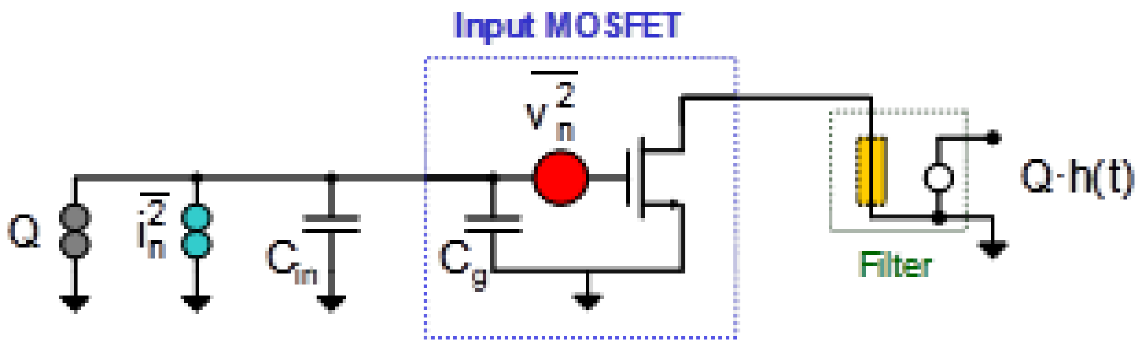




Peaking time = 50 ns
ENC =275 e-rms

Peaking time = 1.8 us
ENC = 67 e-rms

Series Vs Parallel noise: calculation



Since the transfer function $H(j\omega)$ is calculated for an input current, we can directly use it to calculate the output noise due to current noise sources.

The voltage noise sources can be treated as current using the Norton equivalence: $(I_{eq})^2 = \left(\frac{V_{in}}{Z_{in\_eq}}\right)^2$

By assuming a total series and parallel unilateral noise spectral densities of S_{vn} and S_{in} respectfully, then the total output noise N is:

$$N^2 = \frac{1}{2\pi}\int_0^{\infty}\left[S_{in}|H(j\omega)|^2 + \left(C_{in} + C_g + C_f\right)^2 s^2 S_{vn}|H(j\omega)|^2\right] d\omega$$

Equivalent Noise Charge

The Equivalent Noise Charge is the input charge that generates a S/N ratio at the output equal to 1

Assuming that the front-end implements a time invariant filter with overall impulse response h(t). If the input charge is $Q_0\ \partial(t)$. Then the output response is V_{out} (t) = $Q_0\ \partial(t) * \mathrm{h(t)} = Q_0 h(t)$ which admits a maximum at $t=t_0$

$$Signal = S = Vout(t_0) = \ Q\, h(t)\Big|_{max} = Vout\_max$$

We have calculated that the output noise is:

$$N^2 = \frac{1}{2\pi}\int_0^{\infty}\left[S_{in}\left|H\left((j\omega)\right)\right|^2 + \left(C_{in} + C_g + C_f\right)^2 s^2 S_{vn}\left|H\left((j\omega)\right)\right|^2\right] d(\omega)$$

Thus the SNR is:

$$\left(\frac{S}{N}\right)^2 = \frac{Q^2 h(t)|^2_{max}}{\frac{1}{2\pi}\int_0^{\infty}\left[S_{in}|H(j\omega)|^2+\left(C_{in}+C_g+C_f\right)^2\omega^2 S_{vn}|H(j\omega)|^2\right]d\omega} = 1$$

The charge that would make SNR = 1 is:

$$Q^2 = ENC^2 = \frac{\frac{1}{2\pi}\int_0^{\infty}\left[S_{in}|H(j\omega)|^2+\left(C_{in}+C_g+C_f\right)^2\omega^2 S_{vn}|H(j\omega)|^2\right]d\omega}{h(t)|^2_{max}}$$

(in Coulombs)

Equivalent Noise Charge – Shaperless case

It can be demonstrated that there is a theoretical optimal filter function that minimize the ENC. Filters that approximate the optimal filter are called shapers. Let's assume first the case of a shaperless CSA. It can be demonstrated solving the ENC formula that:

$$ENC^2 = \left(C_{in} + C_g + C_f\right)^2 \left[\frac{S_w}{4\tau_r} + A_f \ln\left(\frac{\tau_f}{\tau_r}\right)\right] + \frac{S_p}{4}\,\tau_f$$

Using a classical model (only approximation in deep technologies)

$$C_g = C_{ox}WL,\ S_w = \frac{4kT\gamma}{g_m},\ A_f = \frac{K_f}{C_{ox}WL},\ \ S_p = \frac{4kT}{Rf} + 2qIleak$$

And knowing that

$$\tau_r = \frac{\left(C_{in} + C_g + C_f\right)C_L}{gmC_f} \qquad \tau_f = R_f\,C_f$$

Equivalent Noise Charge – Shaperless case

Let's first assume first the case of a shaperless CSA.

We've already seen that the transfer function of a CSA with RC feedback can be modeled as:

$$H(s) = -\frac{R_f}{(1+s\tau_f)(1+s\tau_r)}$$

By integrating the different noise sources' spectral densities over all frequencies, we can demonstrate that:

$$\boldsymbol{ENC^2 = \left(C_{in} + C_g + C_f\right)^2 \left[\frac{S_w}{4\tau_r} + A_f \ln\left(\frac{\tau_f}{\tau_r}\right)\right] + \frac{S_p}{4}\,\tau_f}$$

Using a classical model (only approximation in deep technologies)

$$C_g = C_{ox}WL,\ S_w = \frac{4kT\gamma}{g_m},\ A_f = \frac{K_f}{C_{ox}WL},\ \ S_p = \frac{4kT}{Rf} + 2qIleak$$

And knowing that

$$\tau_r = \frac{\left(C_{in} + C_g + C_f\right)C_L}{gmC_f} \qquad \tau_f = R_f\, C_f$$

Equivalent Noise Charge – Shaperless case

The ENC is thus expressed as:

$$ENC^2 = \left(C_{in} + C_g + C_f\right)^2 \left[\frac{\gamma k T C_f}{\left(C_{in} + C_g + C_f\right) C_L} + \frac{K_f}{C_{ox} W L} \ln\left(\frac{R_f\, {C_f}^2 gm}{\left(C_{in} + C_g + C_f\right) C_L}\right)\right] + \frac{\left(\frac{4kT}{R_f} + 2qIle_{ak}\right)}{4} R_f\, C_f$$

In this case, one could note that:

- MOS thermal noise is not reduced by increasing gm
- Parallel noise is not reduced by increasing Rf
- Flicker is reduced by increasing the transistor area WL
- Overall noise is reduced by reducing C_{in} , C_f

Equivalent Noise Charge – with Shapers

When shaping filters are used, we can generalize this equation:

$$Q^2 = ENC^2 = \frac{\frac{1}{2\pi}\int_0^\infty \left[S_{in}|H(j\omega)|^2 + \left(C_{in}+C_g+C_f\right)^2 \omega^2 S_{vn}|H(j\omega)|^2\right] d\omega}{h(t)|_{max}^2}$$

Let's consider a generic fliter function, characterized by shaping time constant τ, it is convenient to represent it as:

$$h\left(\frac{t}{\tau}\right) \xrightarrow{Laplace} \tau.H(\tau s)$$

In this case the peaking time, the max value,… are normalized and generic for any τ

Now Introducing the typical spectral densities previously discussed, the ENC is calculated as:

$$ENC^2 = \frac{\left(C_{in}+C_g+C_f\right)^2\left[S_w \int_0^\infty |\tau H(j\omega\tau)|^2 \omega^2 d\omega + A_f 2\pi \int_0^\infty |\tau H(j\omega\tau)|^2 \omega d\omega\right]}{2\pi \cdot h(t/\tau)|_{max}^2} + \frac{S_{in}\int_0^\infty |\tau H(j\omega\tau)|^2 d\omega}{2\pi \cdot h(t/\tau)|_{max}^2}$$

Equivalent Noise Charge – with Shapers

$$ENC^2 = \frac{\left(C_{in} + C_g + C_f\right)^2 \left[S_w \int_0^{\infty} |\tau H(j\omega\tau)|^2 \omega^2 d\omega + A_f 2\pi \int_0^{\infty} |\tau H(j\omega\tau)|^2 \omega d\omega\right]}{2\pi \cdot h(t/\tau)|_{max}^2} + \frac{S_{in} \int_0^{\infty} |\tau H(j\omega\tau)|^2 d\omega}{2\pi \cdot h(t/\tau)|_{max}^2}$$

Thus:

$$ENC^2 = \left(C_{in} + C_g + C_f\right)^2 \left[\frac{S_w}{\tau_p} a_w + A_f 2\pi a_f\right] + S_p \tau_p a_p$$

$$a_w = \frac{\int_0^{\infty} |H(x)|^2 x^2 dx}{2\pi \cdot h(t/\tau)|_{max}^2}, \quad a_f = \frac{\int_0^{\infty} |H(x)|^2 x dx}{2\pi \cdot h(t/\tau)|_{max}^2}, \quad a_p = \frac{\int_0^{\infty} |H(x)|^2 dx}{2\pi \cdot h(t/\tau)|_{max}^2}$$

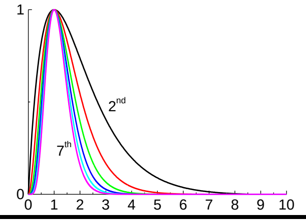
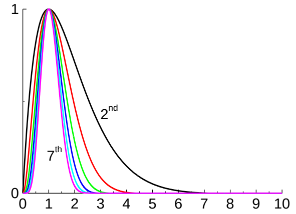
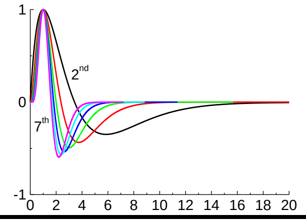
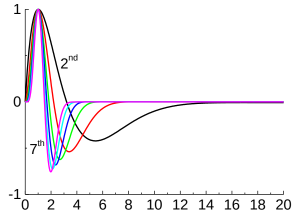
Where a_w, a_f and a_p are constants that depend on the characteristic of the filter

Equivalent Noise Charge – with Shapers

$$ENC^2 = \left(C_{in} + C_g + C_f\right)^2 \left[\frac{S_w}{\tau_p} a_w + A_f 2\pi a_f\right] + S_p \tau a_p$$

$$a_w = \frac{\int_0^\infty |H(x)|^2 x^2 dx}{2\pi \cdot h(t/\tau)|^2_{max}}, \quad a_f = \frac{\int_0^\infty |H(x)|^2 x dx}{2\pi \cdot h(t/\tau)|^2_{max}}, \quad a_p = \frac{\int_0^\infty |H(x)|^2 dx}{2\pi \cdot h(t/\tau)|^2_{max}}$$

In the table, the normalized coefficients for some commonly adopted time invariant filters are reported, along with the ratio between the pulse width τ_w, calculated from 1% to 1% of the curve, and the peaking time t_p, calculated from 1% to the peak)

Filter	Shape	a_w	a_f	a_p	τ_w/τ_p
Triang.		1	0.44	0.33	2
RU-2		0.92	0.59	0.92	7.66
RU-3		0.82	0.54	0.66	5.04
RU-4		0.85	0.53	0.57	4.17
RU-5		0.89	0.52	0.52	3.73
RU-6		0.92	0.52	0.48	3.46
RU-7		0.94	0.51	0.46	3.27
CU-2		0.93	0.59	0.88	6.31
CU-3		0.85	0.54	0.61	3.92
CU-4		0.91	0.53	0.51	3.16
CU-5		0.96	0.52	0.46	2.84
CU-6		1.01	0.52	0.42	2.66
CU-7		1.04	0.51	0.40	2.55
RB-2		1.03	0.75	1.01	16.6
RB-3		1.11	0.77	0.76	9.87
RB-4		1.30	0.81	0.66	7.68
RB-5		1.47	0.84	0.62	6.60
RB-6		1.61	0.87	0.59	5.94
RB-7		1.74	0.89	0.57	5.53
CB-2		1.08	0.79	1.02	12.9
CB-3		1.27	0.86	0.76	7.29
CB-4		1.58	0.93	0.67	5.60
CB-5		1.86	0.98	0.63	4.81
CB-6		2.11	1.02	0.59	4.37
CB-7		2.31	1.06	0.58	4.11

R=Real Pole, C=Complex Pole, U=Unipolar, B=Bipolar

Equivalent Noise Charge – with Shapers

Using a classical model (only approximation in deep technologies)

$$C_g = C_{ox}WL,\ S_w = \frac{4\gamma kT}{g_m},\ A_f = \frac{K_f}{C_{ox}WL},\ \ S_p = \frac{4kT}{Rf} + 2qIleak$$

$$\boldsymbol{ENC^2 = \left(C_{in} + C_{ox}WL + C_f\right)^2 \left[\frac{1}{\tau}\frac{4\gamma kT}{g_m}a_w + 2\pi\frac{K_f}{C_{ox}WL}a_f\right] + \left(\frac{4kT}{Rf} + 2qIlea_k\right)\tau a_p}$$

In this case, one could note that:

- Series and parallel noise are weighted inversely WRT $\boldsymbol{\tau}$
- MOS thermal noise is reduced by increasing gm
- Parallel noise is reduced by increasing R_f
- Flicker is reduced by increasing the transistor area WL
- Overall noise is reduced by reducing C_{in} , C_f

While optimizing for noise, one must keep an eye on other specifications, like gain, speed, count rate, maximum area, power, etc

Conclusion

- In a hybrid or monolithic detector, detected particles create charges in the semiconductor by ionization or absorption.
- These charges move under the effect of the electric field in the depleted volume and induce a current in the electrode according to Shockley-Ramo Theorem.
- The signal is amplified by a Charge Sensitive Amplifier, where the input charge is integrated on a feedback capacitor.
- Shapers are used to optimize the signal shape to maximize the Signal to Noise Ratio.
- There exists several types of noise sources (thermal, shot, Flicker) which could be represented in a series or parallel configuration at the input.
- Theoretical modeling of CSA helps understand the key factors to optimize speed, noise, surface, power consumption,…
- Optimization with and without shaper yields different conclusions regarding noise reduction factors.
- Simulation is needed to tune and optimize the circuit with respect to a particular design technology.

Thank You For Your Attention!