Many (all) the questions have been answered during the Q&A period. Nevertheless, we ask that you provide written answers below so students can come back to read them again. Thanks!

1. (Page 30) The argument of the product mass Matrix is of orden 1?

Once the mass matrices are diagonalized the term vanishes, \( \arg(\det(MuM_d)) = 0 \). However, this term is not invariant under phase redefinitions of quark fields. If one performs a chiral rotation, let’s say of up quark fields, \( u_L' = u_L \exp(i \phi) \), \( u_R' = u_R \exp(-i \phi) \), then the mass term becomes \( \mu u_L \overline{u_R} \rightarrow \mu \exp(-2i \phi) u_L \overline{u_R} \), that is the mass parameter becomes complex, \( \mu' = \mu \exp(-2i \phi) \). This chiral transformation will also shift the theta term, \( \theta' = \theta + 2 \phi \). Any observable should be invariant under field redefinitions, which means that theta by itself is not, while \( \theta \overline{\theta} \), which is invariant, is an observable quantity.

2. (Page 39) If there is an extended Higgs sector with an additional lighter Higgs like "a", could that help the baryogensis?

Extended scalar sectors help in two ways, both by possibly making the electroweak phase transition strongly first order (unlike the SM, where it is a crossover), and by providing more sources of CP violation. For instance including a singlet scalar, one can have a two step transition, where at higher temperatures the Higgs vev is larger than at \( T=0 \). Extended Higgs sectors also provide additional sources of CP violation.

3. UV completion for the axion?

There are many concrete models for axions. The only requirement is that the global symmetry is spontaneously broken, and that it is anomalous under QCD. The two early examples are by including extra vector-like fermions that are charged under such global symmetry, or the models with extended Higgs sectors, such that SM fermions are charged under the global symmetry. This is the yellow band in figure on p. 34. However the width there is only very approximately and very easily one can fill most of the at least lower half right triangle of the plane with different models (see for instance the discussion in 1610.07593).

The common challenge for UV completions is to ensure that the global symmetry is not broken by gravitational effects (which is what one expects on general grounds). These breakings are suppressed by \( (f/M_{pl})^n \), and can be large, if the scale of the spontaneous symmetry breaking, \( f \), is large, unless the breaking only
appears at high values of n. This is the so-called axion quality problem, and is often ignored, however, there are also symmetry based solutions to it.

4. *Slide contains a term $F.F_{\text{dual}}$, which you said integrated to 0. Now it's back when we stick it in as an effective term due to some circuitous reasoning about gluons and pions. Can you explain why this doesn't integrate to 0 to all orders still.*

The $F.F_{\text{dual}}$ is in this case multiplied by the field $a(x)$, and thus is not a pure derivative, unlike just having $F.F_{\text{dual}}$ in the Lagrangian. It thus cannot be integrated by parts and pushed into the boundary term, that then vanishes.

As for the couplings of gluons generating couplings to photons at low energies: this is a consequence of $aG\tilde{G}$ leading to axion coupling to pions at low energies, including charged pions. Closing the pion loop and emitting photons will lead to $aF\tilde{F}$ effective operator at low energies. The coupling to photons is thus necessarily there for an axion that is a solution to the strong CP problem, unless one cancels this contribution by UV physics. There is no symmetry that ensures that, and can be accomplished just by tuning the parameters. That is why most of the axion searches were devised with couplings to photons in mind (since they are basically inevitable). In the last decade, on the other hand, searches using all the other couplings of axion to the SM fields, have also been developed.