Many (all) the questions have been answered during the Q&A period. Nevertheless, we ask that you provide written answers below so students can come back to read them again. Thanks!

1. Can you explain again at the beginning of 2.2, why u has a *?

   I've added some remarks on this to the notes (in green)

2. In note, why is there interference only when their momentum or energy uncertainties are larger than Ei-Ej?

   If the uncertainties in energy and momentum were small enough to resolve the kinematic differences between different neutrino mass eigenstates, interference patterns would disappear because we would know which mass eigenstate has propagated. This is analogous to the double slit experiment: if you devise an apparatus that tells you which path the photon has taken, interference will be gone.

3. Can you please explain how can we cancel the normalization constant with the delta function?

   You can view this delta-function as being the delta-function that appears in Fermi's golden rule. So it's not really part of the oscillation probability, but of the production rate. I agree that in the QM formalism this normalization is ad-hoc. In a QFT calculation that treats neutrino production, propagation, and detection as a single Feynman diagram, one can factorize the resulting rate into a production rate, an oscillation probability, and a detection cross-section. It is then more clear where each factor belongs.

4. You mentioned a paper about propagator over long distances. Can you provide the reference? And can you say a few words here about the topic?

   The paper that first derived this approximation was, to my knowledge, https://arxiv.org/abs/hep-ph/9603430 (see the appendix of that paper). That paper is also one of the many works that treat neutrino oscillations in QFT.

   A paper on this topic which I find very useful is https://arxiv.org/abs/hep-ph/0109119, a long report on many aspects of neutrino (and neutral meson) oscillations in QFT.

   Finally, https://arxiv.org/abs/1001.4815 deals with the correspondence between quantum mechanical and quantum field theoretical approaches to neutrino oscillations.

5. Does the matter effect in neutrino propagation change if the wave packet of the neutrino is much smaller than the propagation distance from neutrino production to detection? Will the neutrino still coherently interact with all the electrons in its path, or is the coherence limited by the extent of the wave packet?

   No, because the neutrino still encounters the same number of electrons (and protons and neutrons) along the way. The only difference with shorter wave packets is that you know more precisely where the neutrino is localized at any given moment. But you still cannot tell when and where it interacts, so one still has to some coherently over all background particles.

6. In the coherent forward scattering, how can there be an exchange of a W boson with no change between the initial and final states?

   This is just a very special kinematic configuration, but there is nothing wrong with it. The 4-momentum flowing through the W propagator is such that the outgoing neutrino is in exactly the same quantum state (same E and p) as the incoming neutrino, and the outgoing electron is in exactly the same quantum state as the incoming electron.

7. If the initial and final state of the particle is identical in coherent forward scattering, how do we actually know that it is taking place?

   We do not know whether the neutrino interacted via coherent forward scattering or not, and if it interacted, with which background particle it interacted. So we have to sum over all possibilities coherently. (The diagram where the neutrino did not interact corresponds to the vacuum oscillation term in the final formula of my notes.)

8. Does neutrino oscillation play any role in CP violation? apart from being there 3 generations.

   Indeed, CP violation and neutrino oscillations are intimately related. Since there are three generations, the mixing matrix generally contains a CP-violating phase δ. And because going from neutrinos to antineutrinos implies the replacement $U \to U^*$ in the derivation of the oscillation probability, the sign of that phase is different for neutrinos compared to anti-neutrinos. In other words, if δ is different from 0 or π, neutrinos will oscillate differently from antineutrinos, which amounts to CP violation.

9. A good explanation was given on the propagator in QFT reducing to QM evolution operator in large distance limit. However, intuitively, neutrinos travel very fast. Why is the QM treatment still valid? Thank you.

   The fact that neutrinos travel fast does not invalidate QM. We are using relativistic QM here, but this just means that the Hamiltonian looks different compared to the one usually found in introductory textbooks – the kinetic term now has the form $\sqrt{p^2 + m^2}$ rather than the non-relativistic form $p^2/(2m)$.

10. I don't understand the scattering when the initial and final states are the same. Isn't that like no interaction at all?

    No, there is nothing wrong with exchanging a virtual particle without changing the external states. Consider as an analogy the propagation of light through glass. The photons feel the presence of the glass (for instance their velocity in glass is different from the speed of light in vacuum). And they feel it because they constantly get absorbed and re-emitted by the little dipoles that are the atoms. But in each such absorption/re-emission process, the energy and momentum of the outgoing photon is exactly the same as that of the incoming photon.

11. Could you describe collective neutrino oscillations in brief.

    In environments where the neutrino density becomes really high (such as the cores of supernovae or neutron star–neutron star mergers), each neutrino feels the presence of the other neutrinos. So there is an extra MSW potential (let's call is V_v) that is proportional not to the electron density n_e like V_CC in the lecture, but is proportional to the neutrino density. And if the neutrino density is really, really high, that term will become much larger than both the vacuum oscillation term and the standard MSW term. And because V_v is independent of the energy of the test neutrino, it affects neutrinos of all energies in the same way. So neutrinos of all energies will oscillate in the same way – unlike in vacuum, where oscillations depend strongly on energy.

    But with V_v in the Hamiltonian, the equations that describe the evolution of the neutrino ensemble in space and time are non-linear. So oscillations in this case are highly non-trivial and very different from the simple harmonic oscillations we see with neutrino–neutrino interactions.

12. Can these flavor oscillation be a process related to atomic dark matter?

    I'm not sure I understand this question. Neutrino oscillations as described in the lecture are a pure SM process (well, SM + neutrino masses of course).

    If the question is whether neutrino oscillations can be affected by atomic DM, the answer is, typically not. The reason is that matter effects are proportional to the number density of scatterers (n_e in the lecture). So if you imagine neutrino–DM interactions here, that process would suffer from the rather low DM density. Remember, n_e ~ 1e23/cm^3, while n_DM ~ 1/cm^3 unless DM is *extremely* light.

13. why the phase in the presence of matter has this form with effective potentials?

    The phase is p*L as in vacuum. But what changes in matter is the dispersion relation, that is, the relation between p and E. If E is the total neutrino energy and E_kin the kinetic energy, then p^2 = E_kin^2 - m^2 = (E - V)^2 - m^2.