A Century of Particle Physics

• Success # 1: discovery of 6 quarks and 6 leptons

• 12 fundamental fermions: matter particles (and their antimatter counterparts) derived by combining quantum mechanics and special relativity

But the intriguing pattern of mass values is assigned to their Higgs interaction

### Quarks

\begin{align*}
\text{u} &< 1 \text{ GeV} \\
\text{c} &\sim 1.5 \text{ GeV} \\
\text{t} &\sim 175 \text{ GeV} \\
\text{d} &< 1 \text{ GeV} \\
\text{s} &< 1 \text{ GeV} \\
\text{b} &\sim 4.5 \text{ GeV}
\end{align*}

### Leptons

\begin{align*}
\nu_e &< 1 \text{ eV} \\
\nu_\mu &< 0.17 \text{ MeV} \\
\nu_\tau &< 24 \text{ MeV} \\
e & 0.5 \text{ MeV} \\
\mu & 106 \text{ MeV} \\
\tau & 1.8 \text{ GeV}
\end{align*}
How to Predict Fundamental Forces

“fictitious” forces observed in accelerating frame of reference
Manifestation of Coriolis Force

Hurricanes appear to rotate in Earth’s frame of reference
A Century of Particle Physics

- Success # 2: principle of gauge invariance for predicting the nature of fundamental forces
  - matter particles (quarks and leptons) transform in curved internal spaces
  - The equations of motion predict terms that describe particle interactions with force fields

\[
L = i \bar{\psi} \gamma^\mu D_\mu \psi - \frac{1}{2} F_{\mu\nu} F^{\mu\nu}
\]
Weak Nuclear Decay

The force causing this interaction is described by particles making transitions on a "mathematical sphere"
How does the W boson Acquire Mass?

- Fill all of space with “Higgs” field

- Particles propagating through “empty space” actually propagating through Higgs field

- Interaction of particles with Higgs field slows down the particle \( \Leftrightarrow \) imparting the property of mass to it
Light versus Heavy Particles – like moving through water

Streamlined
- Moves fast through water
- analogous to light particle

Not streamlined
- Moves slowly through water
- analogous to heavy particle
Quantum Ground State Breaks Gauge Symmetry

- Gauge Symmetry predicts all particles should be massless
- Solution: scalar Higgs field develops a ground state that violates the symmetry and generates particle masses via Higgs interactions
- Phase transition → vacuum state possesses non-trivial quantum numbers
  - Dynamical origin of this phase transition is not known
  - Implies vacuum is a condensed, superconductor-like state
Fundamental vs Parametric Physics

- Fundamental principles lead to
  - Chiral fermions from irreducible representations of Lorentz group
    - fermions as spin $\frac{1}{2}$ representations of Lorentz group
    - Fermi-Dirac statistics $\rightarrow$ Pauli Exclusion Principle
    - why matter occupies volume
  - Massless force mediators (gauge bosons) from gauge invariance
  - Massive gauge bosons and fermions from spontaneous breaking of gauge symmetry

- In comparison, the breaking of gauge symmetry by the Higgs is parametrically induced
  - No dynamic or underlying principle behind it in the Standard Model
Why is Higgs Puzzling

**Gauge sector**

\[ L = i \bar{\psi} \gamma^\mu D_\mu \psi - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} \]

<table>
<thead>
<tr>
<th>particle</th>
<th>spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>quark: u, d,...</td>
<td>1/2</td>
</tr>
<tr>
<td>lepton: e...</td>
<td>1/2</td>
</tr>
<tr>
<td>photon</td>
<td>1</td>
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<tr>
<td>W, Z</td>
<td>1</td>
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<tr>
<td>gluon</td>
<td>1</td>
</tr>
<tr>
<td>Higgs</td>
<td>0</td>
</tr>
</tbody>
</table>

*h*: a new kind of elementary particle

**Higgs sector**

\[ L = (h_{ij} \bar{\psi}_i \psi_j H + h.c.) - \lambda |H|^4 + \mu^2 |H|^2 - \Lambda_{CC}^4 \]
Why is Higgs Puzzling

Ad-hoc potential, similar to and motivated by Landau-Ginzburg theory of superconductivity

Standard Model Higgs potential can be extrapolated to the high-energy of quantum gravity without additional parameters
  but no a-priori reason for a parameterization to respect this condition

\[ V(h) = \frac{1}{2} \mu^2 h^2 + \frac{\lambda}{4} h^4 \text{ or } V(h) = \frac{1}{2} \mu^2 h^2 + \frac{\lambda}{4} h^4 + \frac{1}{\Lambda^2} h^6 \]
Why is the Higgs Boson so Light?

\[ m_H^2 - m_{\text{bare}}^2 = \left( \frac{\Lambda}{\bar{H} H} \right) + \left( \frac{\lambda}{\bar{H} H} \right) + \left( \frac{\lambda}{\bar{H} H} \right) \]

\[ \lambda \int d^4k \frac{1}{(k^2 - m_H^2)^{-1}} \sim \Lambda^2 \lambda \]

For the first time, we have additive corrections to parameters which are quadratically divergent.

The Higgs boson ought to be a very heavy particle, naturally.

However, observed \( m_H << \Lambda \)
Fine-tuning Problem of Higgs Boson Mass

- The large quantum corrections must be regulated by some very high-energy physics such as energy associated with quantum gravity, $M_{\text{planck}} \sim 10^{19} \text{ GeV}$
  - Loop calculation gives Higgs boson mass correction $\sim M_{\text{planck}}^2$

- physical Higgs boson mass $\sim 125 \text{ GeV}$

- Therefore need extreme “fine-tuning” of theoretical parameters at high energy
  - Conceptual weakness of Higgs theory as a quantum theory
Higgs boson puzzles

- First fundamental (?) scalar field to be discovered
- Spontaneous symmetry breaking by development of a ground state
  - But ground state is induced parametrically by ad-hoc Higgs potential, no dynamics
- Parameters of Higgs potential are not stable under quantum corrections
  - First time that the quantum correction to a particle mass is additive and quadratically divergent
  - Gauge boson masses are protected by gauge invariance
  - Fermion masses are protected by chiral symmetry of massless fermions
- Single scalar Higgs field is a strange beast, compared to fermions and gauge bosons
- Additional symmetries and/or dynamics strongly motivated by Higgs discovery
Detecting New Physics through Precision Measurements

- Willis Lamb (Nobel Prize 1955) measured the difference between energies of $^2S_{1/2}$ and $^2P_{1/2}$ states of hydrogen atom
  - 4 micro electron volts difference compared to few electron volts binding energy
  - States should be degenerate in energy according to tree-level calculation

- Harbinger of vacuum fluctuations to be calculated by Feynman diagrams containing quantum loops
  - Modern quantum field theory of electrodynamics followed (Nobel Prize 1965 for Schwinger, Feynman, Tomonaga)
Parameters of Electro-Weak Interactions

- Gauge symmetries related to the electromagnetic and weak forces in the standard model, extension of QED
  - $U(1)_{\text{hypercharge}}$ gauge group with gauge coupling $g$
  - $SU(2)_{\text{weak}}$ gauge group with gauge coupling $g'$

- And gauge symmetry-breaking via vacuum expectation value of Higgs field $\nu \neq 0$

- Another interesting phenomenon in nature: the $U(1)$ generator and the neutral generator of $SU(2)$ get mixed (linear combination) to yield the observed gauge bosons
  - Photon for electromagnetism
  - $Z$ boson as one of the three gauge bosons of weak interaction

- Linear combination is given by Weinberg mixing angle $\theta_W$
Parameters of Electro-Weak Interactions

At tree level, all of the observables can be expressed in terms of three parameters of the SM Lagrangian: $v$, $g$, $g'$ or, equivalently, $v$, $e$, $s \equiv \sin \theta_W$ (also $c \equiv \cos \theta_W$)

$$\alpha = \frac{e^2}{4\pi}, \quad G_F = \frac{1}{2\sqrt{2}v^2}, \quad m_Z = \frac{ev}{\sqrt{2}sc}, \quad m_W = \frac{ev}{\sqrt{2}s}, \quad s_{\text{eff}}^2 = s^2,$$

Radiative corrections to the relations between physical observables and Lagrangian params:

$$m_Z^2 = \frac{e^2v^2}{2s^2c^2} + \Pi_{ZZ}(m_Z^2)$$

$$m_W^2 = \frac{e^2v^2}{2s^2} + \Pi_{WW}(m_W^2)$$

$$G_F = \frac{1}{2\sqrt{2}v^2} \left[ 1 - \frac{\Pi_{WW}(0)}{m_W^2} + \delta_{\text{VB}} \right]$$

$\mu \rightarrow \nu_\mu W$ + $\nu_\mu \rightarrow \mu e$ + ...
Radiative Corrections to Electromagnetic Coupling

\[ \alpha = \frac{e^2}{4\pi} \left[ 1 + \lim_{q^2 \to 0} \frac{\Pi_{\gamma\gamma}(q^2)}{q^2} \right] \]

\[ e^- \rightarrow \gamma \rightarrow e^- + \left( e^- \rightarrow \gamma \rightarrow e^- \right) \]

\[ \Pi_{\gamma\gamma} \]

this one is tricky: the hadronic contribution to \( \Pi_{\gamma\gamma}(0) \) cannot be computed perturbatively

We can however trade it for another experimental observable:

\[ R_{\text{had}}(q^2) = \frac{\sigma_{\text{had}}(q^2)}{\sigma_{\ell^+\ell^-}(q^2)} \]

\[ \alpha(m_Z) = \frac{e^2}{4\pi} \left[ 1 + \frac{\Pi_{\gamma\gamma}(m_Z)}{m_Z^2} \right] = \frac{\alpha}{1 - \Delta \alpha(m_Z)} \]

\[ \Delta \alpha(m_Z) = \Delta \alpha_{\ell}(m_Z) + \Delta \alpha_{\text{top}}(m_Z) + \Delta \alpha_{\text{had}}^{(5)}(m_Z) \]

\[ \Delta \alpha_{\text{had}}^{(5)}(m_Z) = -\frac{m_Z^2}{3\pi} \int_{4m_{\pi}^2}^{\infty} \frac{R_{\text{had}}(q^2) dq^2}{q^2 (q^2 - m_Z^2)} = 0.02758 \pm 0.00035 \]

(This hadronic contribution is one of the biggest sources of uncertainty in EW studies)
Radiative Corrections to W Boson Mass

All these corrections can be combined into relations among physical observables, e.g.:

\[ m_W^2 = m_Z^2 \left[ \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{2\sqrt{2} \pi \alpha}{G_F m_Z^2} (1 + \Delta r)} \right] \]

\( \Delta r \) can be parametrized in terms of two universal corrections and a remainder:

\[ \Delta r = \Delta \alpha(m_Z) - \frac{c^2}{s^2} \Delta \rho + \Delta r_{\text{rem}} \]

The leading corrections depend quadratically on \( m_t \) but only logarithmically on \( m_H \):

\[ \Delta \rho = \frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2} \approx \frac{3 \alpha}{16 \pi c^2} \left( \frac{m_t^2}{s^2 m_Z^2} + \log \frac{m_H^2}{m_W^2} + \ldots \right) \]

\[ \frac{\delta m_W^2}{m_W^2} \approx \frac{c^2}{c^2 - s^2} \Delta \rho , \quad \delta \sin^2 \theta_{\text{eff}} \approx -\frac{c^2 s^2}{c^2 - s^2} \Delta \rho \]
Contributions from Supersymmetric Particles

- Quantum correction to $W$ boson mass depends on mass splitting ($\Delta m^2$) between supersymmetric quarks

- SUSY loops can contribute tens of MeV to $M_W$
  - Even with significant exclusions from Large Hadron Collider
  - Supersymmetric particle could constitute dark matter
Motivation

- Generic parameterization of new physics contributing to W and Z boson self-energies through radiative corrections in propagators
  
  \[ \Pi_{VV}(q^2) \]
  
  \[ \Pi_{WW}(q^2) \]
  
  \[ \Pi_{ZZ}(q^2) \]
  
- \( S, T, U \) parameters (Peskin & Takeuchi, Marciano & Rosner, Kennedy & Langacker, Kennedy & Lynn)
**A\textsubscript{FB} and A\textsubscript{LR} Observables**

- Asymmetries definable in electron-positron scattering sensitive to Weinberg mixing angle $\vartheta_W$

- Fermions, Higgs (and possible new physics) also contribute radiative corrections to $\vartheta_W$ via quantum loops

- $A_{FB}$ is the angular (forward – backward) asymmetry of the final state

- $A_{LR}$ is the asymmetry in the total scattering probability for different polarizations of the initial state (measured very precisely at SLAC's SLC by SLD)
• Generic parameterization of new physics contributing to W and Z boson self-energies: $S$, $T$, $U$ parameters

$U=0$ assumed

Additionally, $M_W$ is the only measurement which constrains $U$

$M_W$ and Asymmetries are the most powerful observables

(From PDG 2021)
Motivation for Precision Measurements

- The electroweak gauge sector of the standard model is constrained by precisely known parameters

  - $\alpha_{EM}(M_Z) = 1 / 127.918(18)$
  - $G_F = 1.16637 (1) \times 10^{-5}$ GeV$^{-2}$
  - $M_Z = 91.1876 (21)$ GeV
  - $m_{\text{top}} = 172.89 (59)$ GeV
  - $M_H = 125.25 (17)$ GeV

- At tree-level, these parameters are related to $M_W$

  - $M_W^2 = \pi\alpha_{EM} / \sqrt{2} G_F \sin^2 \vartheta_W$

  - Where $\vartheta_W$ is the Weinberg mixing angle, defined by

    $\cos \vartheta_W = M_W / M_Z$
Motivation for Precision Measurements

- Radiative corrections due to heavy quark and Higgs loops and (potentially) undiscovered particles

Motivate the introduction of the $\rho$ parameter: $M_W^2 = \rho [M_W(\text{tree})]^2$

with the predictions $\Delta \rho = (\rho - 1) \sim M_{\text{top}}^2$ and $\Delta \rho \sim \ln M_H$
Motivation for Precision Measurements

- The mass of the W boson is tightly constrained by the symmetries of the standard model, in conjunction with $M_{\text{top}}$ and $M_{\text{Higgs}}$
  - The Higgs boson was the last missing component of the model
  - Following the observation of the Higgs boson, a measurement of the W-boson mass provides a stringent test of the model

- The W boson mass is presently constrained by SM global fits to a relative precision of 0.01%
  - provides a strong motivation to test the SM by measuring the mass to the same level of precision
  - SM expectation $M_w = 80,357 \pm 4_{\text{inputs}} \pm 4_{\text{theory}}$ MeV
  - Inputs include Z- and Higgs boson and top-quark masses, EM coupling and muon lifetime measurements
Beyond-SM Modifications to Expected $M_W$

- Hypotheses to provide a deeper explanation of the Higgs field, its potential and the Higgs boson, include
  - Supersymmetry
  - Compositeness
  - New strong interactions
  - Extended Higgs sector

- Hypothetical sources of particulate dark matter

- Extended gauge sector
W mass measurement – decay kinematics

- Main complication: invariant mass cannot be reconstructed from 2-body leptonic decay mode
  - Because neutrino is not detectable directly
- Exploit the “Jacobian edge” in lepton transverse momentum spectrum

\[
\frac{d\sigma}{d \cos \hat{\theta}} = \sigma_0(\hat{s}) \left[ \frac{1}{2} (1 + \cos \hat{\theta})^2 + \frac{1}{2} (1 - \cos \hat{\theta})^2 \right] \\
= \sigma_0(\hat{s})(1 + \cos^2 \hat{\theta})
\]

W boson rest frame
W mass measurement – decay kinematics

- Main complication: invariant mass cannot be reconstructed from 2-body leptonic decay mode
  - Because neutrino is not detectable directly

- Exploit the “Jacobian edge” in lepton transverse momentum spectrum

\[
\frac{d\sigma}{dp_T} = \frac{d\sigma}{d((m_W/2)\sin \hat{\theta})} \\
= \frac{2}{m_W} \frac{d\sigma}{d \sin \hat{\theta}} \\
= \frac{2}{m_W} \frac{d\sigma}{d \cos \hat{\theta}} \left| \frac{d \cos \hat{\theta}}{d \sin \hat{\theta}} \right| \\
= \frac{2}{m_W} \sigma_0(\hat{s})(1 + \cos^2 \theta) |\tan \hat{\theta}| \\
= \sigma_0(\hat{s}) \frac{4p_T}{m_W^2} \left( 2 - \frac{4p_T^2}{m_W^2} \right) \left( \frac{1}{\sqrt{1 - 4p_T^2/m_W^2}} \right)
\]
W mass measurement – decay kinematics

- Main complication: invariant mass cannot be reconstructed from 2-body leptonic decay mode
  - Because neutrino is not detectable directly
- Exploit the “Jacobian edge” in lepton transverse momentum spectrum

We can transfer $\frac{d\sigma}{dp_T}$ to $\frac{d\sigma}{dm_T}$ by using $m_T = 2p_T$:

$$
\frac{d\sigma}{dm_T} = \frac{1}{2} \frac{d\sigma}{dp_T}
$$

$$
= \sigma_0(\hat{s}) \frac{m_T}{m_W} \left(2 - \frac{m_T^2}{m_W^2}\right) \left(\frac{1}{\sqrt{1 - m_T^2/m_W^2}}\right)
$$
W mass measurement – decay kinematics

- Lepton transverse momentum not invariant under transverse boost
- But measurement resolution on leptons is good

Black curve: truth level, no $p_T(W)$

Blue points: detector-level with lepton resolution and selection, But no $p_T(W)$

Shaded histogram: with $p_T(W)$
W mass measurement – decay kinematics

- Define “transverse mass” → approximately invariant under transverse boost
- But measurement resolution of “neutrino” is not as good due to recoil

Black curve: truth level, no $p_T(W)$

Blue points: detector-level with lepton resolution and selection, But no $p_T(W)$

Shaded histogram: with $p_T(W)$

\[
m_T = \sqrt{(E_T^l + E_T^\nu)^2 - (\vec{p}_T^l + \vec{p}_T^\nu)^2} = \sqrt{2p_T^l p_T^\nu (1 - \cos \Delta \phi)}
\]
Signal Simulation and Template Fitting

- All signals simulated using a Custom Monte Carlo
  - Generate finely-spaced templates as a function of the fit variable
  - Perform binned maximum-likelihood fits to the data
- Custom fast Monte Carlo makes smooth, high statistics templates
  - And provides analysis control over key components of the simulation

\[ L = \prod_{i=1}^{N} \frac{e^{-m_i m_i n_i}}{n_i!} \]

\[ M_W = 80 \text{ GeV} \]

\[ M_W = 81 \text{ GeV} \]

- We will extract the W mass from six kinematic distributions: Transverse mass, charged lepton \( p_T \) and missing \( E_T \) using both electron and muon channels
W Boson Production at the Tevatron

Quark-antiquark annihilation dominates (80%)

Lepton $p_T$ carries most of $W$ mass information, can be measured precisely (achieved 0.004%)

Initial state QCD radiation is $O(10 \text{ GeV})$, measure as soft 'hadronic recoil' in calorimeter (calibrated to $\sim 0.2\%$) dilutes $W$ mass information, fortunately $p_T(W) \ll M_W$
W Boson Production at the Tevatron

Quark-antiquark annihilation dominates (80%)

Lepton $p_T$ carries most of $W$ mass information, can be measured precisely (achieved 0.004%)

Initial state QCD radiation is $O(10 \text{ GeV})$, measure as soft 'hadronic recoil' in calorimeter (calibrated to $\sim 0.2\%$) dilutes $W$ mass information, fortunately $p_T(W) << M_W$
Quadrant of Collider Detector at Fermilab (CDF)

EM calorimeter provides precise electron energy measurement

COT provides precise lepton track momentum measurement

Calorimeters measure hadronic recoil particles

Select W and Z bosons with central ( | \eta | < 1 ) leptons
Collider Detector at Fermilab (CDF)

- Muon detector
- Central hadronic calorimeter
- Central EM calorimeter
- Central outer tracker (COT)
CDF Particle Tracking Chamber

Reconstruction of particle trajectories, calibration to \(\sim 1 \ \mu \text{m} \) accuracy:


C. Hays et al, NIM A538, 249 (2005)
W boson Production Event

electron calorimeter

hadronic calorimeter

inferred neutrino

Central outer tracker (COT)
Event Selection

- **Goal:** Select events with high $p_T$ leptons and small hadronic recoil activity
  - to maximize $W$ mass information content and minimize backgrounds

- **Inclusive lepton triggers:** loose lepton track and muon stub / calorimeter cluster requirements, with lepton $p_T > 18$ GeV
  - Kinematic efficiency of trigger $\sim 100\%$ for offline selection

- **Offline selection requirements:**
  - Electron cluster $E_T > 30$ GeV, track $p_T > 18$ GeV
  - Muon track $p_T > 30$ GeV
  - Loose identification requirements to minimize selection bias

- **$W$ boson event selection:** one selected lepton, $|u| < 15$ GeV & $p_T(\nu) > 30$ GeV
  - $Z$ boson event selection: two selected leptons
W & Z Data Samples

- **Integrated Luminosity** (collected between February 2002 – September 2011):
  - Electron and muon channels: $L = 8.8 \text{ fb}^{-1}$
  - Identical running conditions for both channels, guarantees cross-calibration

- **Event selection gives fairly clean samples**
  - Mis-identification backgrounds $\sim 0.5\%$

<table>
<thead>
<tr>
<th>Sample</th>
<th>Candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W \rightarrow \text{ electron}$</td>
<td>1 811 700</td>
</tr>
<tr>
<td>$Z \rightarrow \text{ electrons}$</td>
<td>66 180</td>
</tr>
<tr>
<td>$W \rightarrow \text{ muon}$</td>
<td>2 424 486</td>
</tr>
<tr>
<td>$Z \rightarrow \text{ muons}$</td>
<td>238 534</td>
</tr>
</tbody>
</table>
Analysis Strategy
Maximize the number of internal constraints and cross-checks

Driven by three goals:

1) Robustness: constrain the same parameters in as many different ways as possible

2) Precision: combine independent measurements after showing consistency

3) minimize bias: blinded measurements of $M_z$ and $M_w$
Outline of Analysis

Energy scale measurements drive the $W$ mass measurement

- **Tracker Calibration**
  - alignment of the COT (2,520 cells; 30,240 sense wires) using cosmic rays
  - COT momentum scale and tracker non-linearity constrained using $J/\psi \rightarrow \mu \mu$ and $Y \rightarrow \mu \mu$ mass fits
  - Confirmed using $Z \rightarrow \mu \mu$ mass fit

- **EM Calorimeter Calibration**
  - COT momentum scale transferred to EM calorimeter using a fit to the peak of the $E/p$ spectrum, around $E/p \sim 1$
  - Calorimeter energy scale confirmed using $Z \rightarrow ee$ mass fit

- **Tracker and EM Calorimeter resolutions**

- **Hadronic recoil modeling**
  - Characterized using $p_T$-balance in $Z \rightarrow ll$ events
Drift Chamber (COT) Alignment

COT endplate geometry
Internal Alignment of COT

- Use a clean sample of $\sim 480k$ cosmic rays for cell-by-cell internal alignment

- Fit COT hits on both sides simultaneously to a single helix (AVK, H. Gerberich and C. Hays, NIMA 506, 110 (2003))
  - Time of incidence is a floated parameter in this 'di-cosmic fit'
Residuals of COT cells after alignment

(AVK & CH, NIM A 762 (2014) pp 85-99)

before alignment

after alignment

Final relative alignment of cells ~1 µm (initial alignment ~50 µm)
Consistency check of COT alignment procedure


Fit separate helices to cosmic ray tracks

Compare track parameters of the two tracks: a measure of track parameter bias
Consistency check of COT alignment procedure

(AVK & CH, NIM A 762 (2014) pp 85-99)

track parameter bias versus azimuth

solid = before alignment

open = after alignment
Cross-check of COT alignment

- Cosmic ray alignment removes most deformation degrees of freedom, but “weakly constrained modes” remain.
- Final cross-check and correction to beam-constrained track curvature based on difference of $<E/p>$ for positrons vs electrons.
- Smooth ad-hoc curvature corrections as a function of polar and azimuthal angle: statistical errors $\Rightarrow \Delta M_W = 1$ MeV.

\[
\frac{q}{p_T} \text{ (measured)} = c_0 + c_1 \frac{q}{p_T} + c_2 \left( \frac{q}{p_T} \right)^2 + \ldots
\]

$c_1$ measures momentum scale
$c_2$ includes energy loss

$c_0 = 0$
Signal Simulation and Fitting
Generator-level Signal Simulation

- Generator-level input for W & Z simulation provided by RESBOS (C. Balazs & C.-P. Yuan, PRD56, 5558 (1997) and references therein), which
  - Fully differential production and decay distributions
  - Benchmarked to RESBOS2 (J. Isaacson, Y. Fu & C.-P. Yuan, arXiv:2205.02788)

- Multiple radiative photons generated according to PHOTOS (P. Golonka and Z. Was, Eur. J. Phys. C 45, 97 (2006) and references therein)
  - Calibrated to HORACE (C.M. Carloni Calame, G. Montagna, O. Nicrosini and A. Vicini, JHEP 0710:109,2007)
Constraining Boson $p_T$ Spectrum

- Fit the non-perturbative parameter $g_2$ and QCD coupling $\alpha_S$ in RESBOS to $p_T(ll)$ spectra:

$$\Delta M_W = 1.8 \text{ MeV}$$

Position of peak in boson $p_T$ spectrum depends on $g_2$

Tail to peak ratio depends on $\alpha_S$

Fig. S2
Outline of Analysis

*Energy scale measurements drive the W mass measurement*

- **Tracker Calibration**
  - alignment of the COT (~2400 cells, ~30k sense wires) using cosmic rays
  - COT momentum scale and tracker non-linearity constrained using $J/\psi \rightarrow \mu \mu$ and $\Upsilon \rightarrow \mu \mu$ mass fits
  - Confirmed using $Z \rightarrow \mu \mu$ mass fit

- **EM Calorimeter Calibration**
  - COT momentum scale transferred to EM calorimeter using a fit to the peak of the E/p spectrum, around $E/p \sim 1$
  - Calorimeter energy scale confirmed using $Z \rightarrow ee$ mass fit

- **Tracker and EM Calorimeter resolutions**
- **Hadronic recoil modeling**
  - Characterized using $p_T$-balance in $Z \rightarrow ll$ events
Custom Monte Carlo Detector Simulation

- A complete detector simulation of all quantities measured in the data
- First-principles simulation of tracking
  - Tracks and photons propagated through a high-resolution 3-D lookup table of material properties for silicon detector and COT
  - At each material interaction, calculate
    - Ionization energy loss according to detailed formulae and Landau distribution
    - Generate bremsstrahlung photons down to 0.4 MeV, using detailed cross section and spectrum calculations
    - Simulate photon conversion and Compton scattering
    - Propagate bremsstrahlung photons and conversion electrons
    - Simulate multiple Coulomb scattering, including non-Gaussian tail
  - Deposit and smear hits on COT wires, perform full helix fit including optional beam-constraint
Custom Monte Carlo Detector Simulation

- A complete detector simulation of all quantities measured in the data
- First-principles simulation of tracking
  - Tracks and photons propagated through a high-resolution 3-D lookup table of material properties for silicon detector and COT
Tracking Momentum Scale
Tracking Momentum Scale

Set using $J/\psi \rightarrow \mu\mu$ and $\Upsilon \rightarrow \mu\mu$ resonance and $Z \rightarrow \mu\mu$ masses

- Extracted by fitting $J/\psi$ mass in bins of $1/p_T(\mu)$, and extrapolating momentum scale to zero curvature
- $J/\psi \rightarrow \mu\mu$ mass independent of $p_T(\mu)$ after 2.6% tuning of energy loss

![Graph showing $J/\psi \rightarrow \mu\mu$ mass fit (bin 8)](image)

Fig. 2

![Graph showing $J/\psi \rightarrow \mu\mu$ mass fit](image)

Fig. S9
Tracking Momentum Scale

$\gamma \rightarrow \mu\mu$ resonance provides

- Cross-check of non-beam-constrained (NBC) and beam-constrained (BC) fits
- Consistent measurements after incorporating silicon detector passive energy loss in extrapolator code of track reconstruction

![Graph showing data and simulation comparison with fit parameters $\Delta p/p = (-1380 \pm 10_{\text{stat}})$ ppm and $\chi^2$/dof = 82/70.](image-url)
### Tracking Momentum Scale Systematics

**Systematic uncertainties on momentum scale (parts per million)**

<table>
<thead>
<tr>
<th>Source</th>
<th>$J/\psi$ (ppm)</th>
<th>$\Upsilon$ (ppm)</th>
<th>Correlation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED</td>
<td>1</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>Magnetic field non-uniformity</td>
<td>13</td>
<td>13</td>
<td>100</td>
</tr>
<tr>
<td>Ionizing material correction</td>
<td>11</td>
<td>8</td>
<td>100</td>
</tr>
<tr>
<td>Resolution model</td>
<td>10</td>
<td>1</td>
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</tr>
<tr>
<td>Background model</td>
<td>7</td>
<td>6</td>
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<tr>
<td>COT alignment correction</td>
<td>4</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>Trigger efficiency</td>
<td>18</td>
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<tr>
<td>Fit range</td>
<td>2</td>
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<tr>
<td>$\Delta p/p$ step size</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>World-average mass value</td>
<td>4</td>
<td>27</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total systematic</strong></td>
<td>29</td>
<td>34</td>
<td>16 ppm</td>
</tr>
<tr>
<td>Statistical NBC (BC)</td>
<td>2</td>
<td>13(10)</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>29</td>
<td>36</td>
<td>16 ppm</td>
</tr>
</tbody>
</table>

$\Delta M_{W,Z} = 2$ MeV

Uncertainty dominated by magnetic field non-uniformity, passive material energy loss, low $p_T$ modeling and $\Upsilon$ mass world average
Z→μμ Mass Cross-check & Combination

- Using the J/ψ and Υ momentum scale, performed “blinded” measurement of Z boson mass
  - Z mass consistent with PDG value (91188 MeV) (0.7σ statistical)
  - \( M_Z = 91192.0 \pm 6.4 \text{ stat} \pm 2.3 \text{ momentum} \pm 3.1 \text{ QED} \pm 1 \text{ alignment} \) MeV

![Graph showing distribution of M(μμ) and m_μμ values with Data and Simulation curves.](image)

- \( \chi^2/\text{dof} = 33 / 30 \)
- \( P_{\chi^2} = 29\% \)
- \( P_{KS} = 88\% \)
Tracker Linearity Cross-check & Combination

- Final calibration using the J/ψ, ϒ and Z bosons for calibration

- Combined momentum scale correction:

\[ \Delta p/p = ( -1389 \pm 25_{\text{syst}} ) \text{ parts per million} \]

\[ \Delta M_W = 2 \text{ MeV} \]
EM Calorimeter Response
EM Calorimeter Scale

- E/p peak from $W \rightarrow \nu\ell$ decays provides measurements of EM calorimeter scale and its ($E_T$-dependent) non-linearity

$$\Delta S_E = (43_{\text{stat}} \pm 30_{\text{non-linearity}} \pm 34_{X0} \pm 45_{\text{Tracker}}) \text{ parts per million}$$

Setting $S_E$ to 1 using E/p calibration from combined $W \rightarrow \nu\ell$ and $Z \rightarrow \ell\ell$ samples

$$\Delta M_W = 6 \text{ MeV}$$

![Graph showing data and simulation comparison]

- Low tail used for tuning calorimeter thickness
- High tail used for tuning model of radiative material

Data

Simulation

$\chi^2$/dof = 39 / 33
$P_{\chi^2} = 21\%$
$P_{KS} = 69\%$
Measurement of EM Calorimeter Non-linearity

- Perform E/p fit-based calibration in bins of electron $E_T$
- GEANT-motivated parameterization of non-linear response:
  \[ S_E = 1 + \beta \log(E_T / 39 \text{ GeV}) \]
- Tune on W and Z data: $\beta = (7.2 \pm 0.4_{\text{stat}}) \times 10^{-3}$
  \[ \Rightarrow \Delta M_W = 2 \text{ MeV} \]
**Z→ee Mass Cross-check and Combination**

- Performed “blind” measurement of Z mass using E/p-based calibration
  - Consistent with PDG value (91188 MeV) within 0.5σ (statistical)
    - $M_Z = 91194.3 \pm 13.8 \text{ (stat)} \pm 6.5 \text{ (calorimeter)} \pm 2.3 \text{ (momentum)} \pm 3.1 \text{ (QED)} \pm 0.8 \text{ (alignment)}$ MeV

- Combine E/p-based calibration with $Z\rightarrow ee$ mass for maximum precision

\[ \Delta M_W = 5.8 \text{ MeV} \]

\[ \Delta S_E = -14 \pm 72 \text{ ppm} \]
Hadronic Recoil Model
Exploit similarity in production and decay of $W$ and $Z$ bosons

Detector response model for hadronic recoil tuned using $p_T$-balance in $Z \rightarrow ll$ events

Transverse momentum of Hadronic recoil ($u$) calculated as 2-vector-sum over calorimeter towers
Lepton Tower Removal

- We remove the calorimeter towers containing lepton energy from the hadronic recoil calculation
  - Lost underlying event energy is measured in $\phi$-rotated windows in W boson data
    \[ \Delta M_W = 1 \text{ MeV} \]

Figs. S17 & S18
Lepton Tower Removal

Fig. S20
Constraining the Hadronic Recoil Model

Exploit similarity in production and decay of $W$ and $Z$ bosons

Detector response model for hadronic recoil tuned using $p_T$-balance in $Z \rightarrow ll$ events

Transverse momentum of Hadronic recoil ($u$) calculated as 2-vector-sum over calorimeter towers
Hadronic Recoil Simulation

Recoil momentum 2-vector $u$ has

- a soft 'spectator interaction' component, randomly oriented
  - Modeled using minimum-bias data with tunable magnitude
- A hard 'jetty' component, directed opposite the boson $p_T$
  - $p_T$-dependent response and resolution parameterizations
  - Hadronic response $R = u_{\text{reconstructed}} / u_{\text{true}}$ parameterized as a logarithmically increasing function of boson $p_T$ motivated by $Z$ boson data
Tuning Recoil Response Model with $Z$ events

Project the vector sum of $p_T(ll)$ and $u$ on a set of orthogonal axes defined by boson $p_T$

Mean and rms of projections as a function of $p_T(ll)$ provide information on hadronic model parameters

FIG. S3: (left) Sketches of typical transverse vectors associated to quantities reconstructed in a $W$-boson event, with the recoil hadron momentum ($\vec{u}_T$) separated into axes parallel ($u_\parallel$) and perpendicular ($u_\perp$) to the charged lepton. (right) Illustration of the $\eta$ and $\xi$ axes in $Z$ boson events.
Tuning Recoil Response Model with $Z$ events

Project the vector sum of $p_T(ll)$ and $u$ on a set of orthogonal axes defined by boson $p_T$

Mean and rms of projections as a function of $p_T(ll)$ provide information on hadronic model parameters

$\chi^2 / \text{dof} = 14 / 14$

$\Delta M_W = 2 \text{ MeV}$
Tuning Recoil Resolution Model with $Z$ events

At low $p_T(Z)$, $p_T$-balance constrains hadronic resolution due to underlying event

At high $p_T(Z)$, $p_T$-balance constrains jet resolution
Tuning Recoil Resolution Model with $Z$ events

NEW: model of boson + dijet events

As a function of $p_T(Z)$, dijet event fraction varies between 0.4 % & 1.2 %
Tuning Recoil Resolution Model with $Z$ events

Model of $p_T$-dependent collimation of jet(s) recoiling against boson
Tuning Recoil Resolution Model with Z events

NEW: Fine-tuning model for resolution along $p_T(Z)$ axis

**Simulation**
- $\mu = -19$ MeV
- $\sigma = 4604$ MeV
- $\lambda = -0.05$
- $\kappa = 0.97$

**Data**
- $\mu = -14 \pm 13$ MeV
- $\sigma = 4598 \pm 9$ MeV
- $\lambda = -0.02 \pm 0.01$
- $\kappa = 1.03 \pm 0.01$

$\chi^2 / \text{dof} = 54 / 35$
$P_{KS} = 55\%$

**Simulation**
- $\mu = 99$ MeV
- $\sigma = 4706$ MeV
- $\lambda = -0.05$
- $\kappa = 0.93$

**Data**
- $\mu = 146 \pm 24$ MeV
- $\sigma = 4719 \pm 17$ MeV
- $\lambda = -0.07 \pm 0.01$
- $\kappa = 0.91 \pm 0.03$

$\chi^2 / \text{dof} = 41 / 35$
$P_{KS} = 1.9\%$

**Simulation**
- $\mu = -20$ MeV
- $\sigma = 5478$ MeV
- $\lambda = -0.01$
- $\kappa = 0.58$

**Data**
- $\mu = 1 \pm 17$ MeV
- $\sigma = 5456 \pm 12$ MeV
- $\lambda = 0.01 \pm 0.01$
- $\kappa = 0.56 \pm 0.02$

$\chi^2 / \text{dof} = 36 / 35$
$P_{KS} = 74\%$

**Simulation**
- $\mu = 117$ MeV
- $\sigma = 5544$ MeV
- $\lambda = -0.02$
- $\kappa = 0.56$

**Data**
- $\mu = 119 \pm 33$ MeV
- $\sigma = 5544 \pm 23$ MeV
- $\lambda = -0.02 \pm 0.01$
- $\kappa = 0.53 \pm 0.03$

$\chi^2 / \text{dof} = 42 / 35$
$P_{KS} = 49\%$
Tuning Recoil Resolution Model with Z events
NEW: Fine-tuning model for resolution perpendicular to \( p_T(Z) \) axis

**Simulation**
- \( \mu = -6 \text{ MeV} \)
- \( \sigma = 4558 \text{ MeV} \)
- \( \lambda = 0 \)
- \( \kappa = 1.05 \)

**Data**
- \( \mu = 2 \pm 12 \text{ MeV} \)
- \( \sigma = 4548 \pm 9 \text{ MeV} \)
- \( \lambda = -0.01 \pm 0.01 \)
- \( \kappa = 1.08 \pm 0.01 \)

\( \chi^2 / \text{dof} = 30 / 35 \)

**Simulation**
- \( \mu = -3 \text{ MeV} \)
- \( \sigma = 4651 \text{ MeV} \)
- \( \lambda = 0 \)
- \( \kappa = 1.01 \)

**Data**
- \( \mu = 2 \pm 24 \text{ MeV} \)
- \( \sigma = 4642 \pm 17 \text{ MeV} \)
- \( \lambda = 0 \pm 0.01 \)
- \( \kappa = 1.02 \pm 0.03 \)

\( \chi^2 / \text{dof} = 27 / 35 \)

**Simulation**
- \( \mu = -5 \text{ MeV} \)
- \( \sigma = 4914 \text{ MeV} \)
- \( \lambda = 0 \)
- \( \kappa = 0.87 \)

**Data**
- \( \mu = 24 \pm 15 \text{ MeV} \)
- \( \sigma = 4934 \pm 11 \text{ MeV} \)
- \( \lambda = -0.02 \pm 0.01 \)
- \( \kappa = 0.88 \pm 0.02 \)

\( \chi^2 / \text{dof} = 29 / 35 \)

**Simulation**
- \( \mu = -2 \text{ MeV} \)
- \( \sigma = 4986 \text{ MeV} \)
- \( \lambda = 0 \)
- \( \kappa = 0.83 \)

**Data**
- \( \mu = -1 \pm 29 \text{ MeV} \)
- \( \sigma = 4974 \pm 21 \text{ MeV} \)
- \( \lambda = 0.01 \pm 0.01 \)
- \( \kappa = 0.84 \pm 0.03 \)

\( \chi^2 / \text{dof} = 35 / 35 \)

**Low \( p_T^Z \)**
- \( P_{KS} = 84\% \)
- \( P_{KS} = 99\% \)
- \( P_{KS} = 9.3\% \)
- \( P_{KS} = 95\% \)
Testing Hadronic Recoil Model with $W$ boson events

Recoil projection (GeV) on lepton direction

Recoil projection (GeV) perpendicular to lepton
Additional Constraint on $p_T(W)$ Model with $W$ boson events

- **NEW**: In addition to the $p_T(Z)$ data constrain on the boson $p_T$ spectrum, the ratio of the $p_T(W) / p_T(Z)$ spectra is also constrained from the $p_T(W)$ data.

- **DyqT**: triple-differential cross section calculation at NNLO-QCD used to model scale variation of ratio.

- $p_T(W)$ data is used as constraint on ratio model.

- Correlation with hadronic recoil model is taken into account.

**Graphs:**
- $p_T(W)$, muon channel
- $p_T(W)$, electron channel

**Legend:**
- **Data**
- **Simulation**
Parton Distribution Functions and Backgrounds
Parton Distribution Functions

• Affect W kinematic lineshapes through acceptance cuts

• In the rest frame, $p_T = m \sin \theta^* / 2$

• Longitudinal cuts on lepton in the lab frame sculpt the distribution of $\theta^*$, hence biases the distribution of lepton $p_T$
  
  – Relationship between lab frame and rest frame depends on the boost of the W boson along the beam axis

• Parton distribution functions control the longitudinal boost

• Uncertainty due to parton distribution functions evaluated by fitting pseudo-experiments (simulated samples with the same statistics and selection as data) with varied parton distribution functions
Parton Distribution Functions

- Affect $W$ boson kinematic line-shapes through acceptance cuts
- We use NNPDF3.1 as the default NNLO PDFs
- Use ensemble of 25 'uncertainty' PDFs => 3.9 MeV
  - Represent variations of eigenvectors in the PDF parameter space
  - Compute $\delta M_W$ contribution from each error PDF
- Central values from NNLO PDF sets CT18, MMHT2014 and NNPDF3.1 agree within 2.1 MeV of their midpoint
- As an additional check, central values from NLO PDF sets ABMP16, CJ15, MMHT2014 and NNPDF3.1 agree within 3 MeV of their midpoint
- Missing higher-order QCD effects estimated to be 0.4 MeV
  - Varying the factorization and renormalization scales
  - Comparing two event generators with different resummation and non-perturbative schemes.
Backgrounds in the $W$ boson sample

- Z → ll events with only one reconstructed leptons:
  - efficiency and calorimeter response mapped using control samples of $Z \rightarrow ll$ data, and modeled in the custom simulation
  - background estimates validated using a full GEANT-based CDF detector simulation
  - the only large background is $Z \rightarrow \mu\mu$ with geometrical acceptance loss of forward muons

- W → τν → ℓν̅ν background estimated using custom simulation

- QCD jet background estimated using control samples of data, anti-selected on lepton quality requirements

- Pion and kaon decays-in-flight to mis-reconstructed muons
  - Estimated using control samples of data, anti-selected on muon track-quality requirements

- Cosmic ray muons estimated using a dedicated track-finding algorithm
# Backgrounds in the $W$ boson sample

## Muon channel

<table>
<thead>
<tr>
<th>Source</th>
<th>Fraction (%)</th>
<th>$m_T$ fit</th>
<th>$p_T^\mu$ fit</th>
<th>$p_T^\nu$ fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z/\gamma^* \rightarrow \mu\mu$</td>
<td>7.37 ± 0.10</td>
<td>1.6 (0.7)</td>
<td>3.6 (0.3)</td>
<td>0.1 (1.5)</td>
</tr>
<tr>
<td>$W \rightarrow \tau\nu$</td>
<td>0.880 ± 0.004</td>
<td>0.1 (0.0)</td>
<td>0.1 (0.0)</td>
<td>0.1 (0.0)</td>
</tr>
<tr>
<td>Hadronic jets</td>
<td>0.01 ± 0.04</td>
<td>0.1 (0.8)</td>
<td>-0.6 (0.8)</td>
<td>2.4 (0.5)</td>
</tr>
<tr>
<td>Decays in flight</td>
<td>0.20 ± 0.14</td>
<td>1.3 (3.1)</td>
<td>1.3 (5.0)</td>
<td>-5.2 (3.2)</td>
</tr>
<tr>
<td>Cosmic rays</td>
<td>0.01 ± 0.01</td>
<td>0.3 (0.0)</td>
<td>0.5 (0.0)</td>
<td>0.3 (0.3)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>8.47 ± 0.18</strong></td>
<td><strong>2.1 (3.3)</strong></td>
<td><strong>3.9 (5.1)</strong></td>
<td><strong>5.7 (3.6)</strong></td>
</tr>
</tbody>
</table>

## Electron channel

<table>
<thead>
<tr>
<th>Source</th>
<th>Fraction (%)</th>
<th>$m_T$ fit</th>
<th>$p_T^e$ fit</th>
<th>$p_T^\nu$ fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z/\gamma^* \rightarrow ee$</td>
<td>0.134 ± 0.003</td>
<td>0.2 (0.3)</td>
<td>0.3 (0.0)</td>
<td>0.0 (0.6)</td>
</tr>
<tr>
<td>$W \rightarrow \tau\nu$</td>
<td>0.94 ± 0.01</td>
<td>0.6 (0.0)</td>
<td>0.6 (0.0)</td>
<td>0.6 (0.0)</td>
</tr>
<tr>
<td>Hadronic jets</td>
<td>0.34 ± 0.08</td>
<td>2.2 (1.2)</td>
<td>0.9 (6.5)</td>
<td>6.2 (−1.1)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1.41 ± 0.08</strong></td>
<td><strong>2.3 (1.2)</strong></td>
<td><strong>1.1 (6.5)</strong></td>
<td><strong>6.2 (1.3)</strong></td>
</tr>
</tbody>
</table>

Backgrounds are small (except $Z \rightarrow \mu\mu$ with a forward muon)
W Mass Fits
Blind Analysis Technique

- All $W$ and $Z$ mass fit results were blinded with a random $[-50,50]$ MeV offset hidden in the likelihood fitter.
- Blinding offset removed after the analysis was declared frozen.
- Technique allows to study all aspects of data while keeping $Z$ boson mass and $W$ boson mass result unknown within $\pm 50$ MeV.
$W$ Transverse Mass Fits

![Graphs showing $W$ transverse mass fits for muons and electrons with chi-squared and p-values.](image)

- **muons**
  - $\chi^2$/dof = 50 / 48
  - $P_{\chi^2} = 37\%$
  - $P_{KS} = 98\%$

- **electrons**
  - $\chi^2$/dof = 39 / 48
  - $P_{\chi^2} = 79\%$
  - $P_{KS} = 76\%$

Fig. 4

![Chi-squared distribution for $W\rightarrow\mu\nu$ and $W\rightarrow e\nu$.](image)

- **$W\rightarrow\mu\nu$**
  - Chi-squared distribution

- **$W\rightarrow e\nu$**
  - Chi-squared distribution

Fig. 36
$W$ Charged Lepton $p_T$ Fits

\[ \chi^2/\text{dof} = 82 / 62 \]
\[ P_{\chi^2} = 4 \% \]
\[ P_{KS} = 89 \% \]

\[ \chi^2/\text{dof} = 83 / 62 \]
\[ P_{\chi^2} = 3 \% \]
\[ P_{KS} = 53 \% \]

Fig. 4

\[ W \rightarrow \mu \nu \]

\[ W \rightarrow e \nu \]

Fig. 37

muons

electrons
$W$ Neutrino $p_T$ Fits

\begin{align*}
\chi^2/\text{dof} & = 63 / 62 \\
P_{\chi^2} & = 43 \% \\
P_{KS} & = 70 \% \\
\text{muons}
\end{align*}

\begin{align*}
\chi^2/\text{dof} & = 69 / 62 \\
P_{\chi^2} & = 23 \% \\
P_{KS} & = 96 \% \\
\text{electrons}
\end{align*}

Fig. 4

\begin{align*}
\chi^2 & = 5 \\
W \rightarrow \mu\nu
\end{align*}

Fig. 38
Summary of $W$ Mass Fits

<table>
<thead>
<tr>
<th>Distribution</th>
<th>$W$-boson mass (MeV)</th>
<th>$\chi^2$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_T(e, \nu)$</td>
<td>$80 \ 429.1 \pm 10.3_{\text{stat}} \pm 8.5_{\text{syst}}$</td>
<td>39/48</td>
</tr>
<tr>
<td>$p^\ell_T(e)$</td>
<td>$80 \ 411.4 \pm 10.7_{\text{stat}} \pm 11.8_{\text{syst}}$</td>
<td>83/62</td>
</tr>
<tr>
<td>$p^\nu_T(e)$</td>
<td>$80 \ 426.3 \pm 14.5_{\text{stat}} \pm 11.7_{\text{syst}}$</td>
<td>69/62</td>
</tr>
<tr>
<td>$m_T(\mu, \nu)$</td>
<td>$80 \ 446.1 \pm 9.2_{\text{stat}} \pm 7.3_{\text{syst}}$</td>
<td>50/48</td>
</tr>
<tr>
<td>$p^\ell_T(\mu)$</td>
<td>$80 \ 428.2 \pm 9.6_{\text{stat}} \pm 10.3_{\text{syst}}$</td>
<td>82/62</td>
</tr>
<tr>
<td>$p^\nu_T(\mu)$</td>
<td>$80 \ 428.9 \pm 13.1_{\text{stat}} \pm 10.9_{\text{syst}}$</td>
<td>63/62</td>
</tr>
<tr>
<td>combination</td>
<td>$80 \ 433.5 \pm 6.4_{\text{stat}} \pm 6.9_{\text{syst}}$</td>
<td>7.4/5</td>
</tr>
</tbody>
</table>

Consistency between two channels and three kinematic fits
## Combinations of Fit Results

<table>
<thead>
<tr>
<th>Combination</th>
<th>$m_T$ fit</th>
<th>$p_T^\ell$ fit</th>
<th>$p_T^{\nu}$ fit</th>
<th>Value (MeV)</th>
<th>$\chi^2$/dof</th>
<th>Probability (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_T$</td>
<td>√</td>
<td>√</td>
<td></td>
<td>80 439.0 ± 9.8</td>
<td>1.2 / 1</td>
<td>28</td>
</tr>
<tr>
<td>$p_T^\ell$</td>
<td></td>
<td></td>
<td></td>
<td>80 421.2 ± 11.9</td>
<td>0.9 / 1</td>
<td>36</td>
</tr>
<tr>
<td>$p_T^{\nu}$</td>
<td></td>
<td></td>
<td></td>
<td>80 427.7 ± 13.8</td>
<td>0.0 / 1</td>
<td>91</td>
</tr>
<tr>
<td>$m_T$ &amp; $p_T^\ell$</td>
<td>√</td>
<td></td>
<td></td>
<td>80 435.4 ± 9.5</td>
<td>4.8 / 3</td>
<td>19</td>
</tr>
<tr>
<td>$m_T$ &amp; $p_T^{\nu}$</td>
<td>√</td>
<td></td>
<td></td>
<td>80 437.9 ± 9.7</td>
<td>2.2 / 3</td>
<td>53</td>
</tr>
<tr>
<td>$p_T^\ell$ &amp; $p_T^{\nu}$</td>
<td>√</td>
<td></td>
<td></td>
<td>80 424.1 ± 10.1</td>
<td>1.1 / 3</td>
<td>78</td>
</tr>
<tr>
<td>Electrons</td>
<td></td>
<td></td>
<td></td>
<td>80 424.6 ± 13.2</td>
<td>3.3 / 2</td>
<td>19</td>
</tr>
<tr>
<td>Muons</td>
<td></td>
<td></td>
<td></td>
<td>80 437.9 ± 11.0</td>
<td>3.6 / 2</td>
<td>17</td>
</tr>
<tr>
<td>All</td>
<td>√</td>
<td></td>
<td></td>
<td>80 433.5 ± 9.4</td>
<td>7.4 / 5</td>
<td>20</td>
</tr>
</tbody>
</table>

- **Combined electrons (3 fits):** $M_W = 80424.6 \pm 13.2$ MeV, $P(\chi^2) = 19\%$

- **Combined muons (3 fits):** $M_W = 80437.9 \pm 11.0$ MeV, $P(\chi^2) = 17\%$

- **All combined (6 fits):** $M_W = 80433.5 \pm 9.4$ MeV, $P(\chi^2) = 20\%$

**citation:** *Science* **376**, 170 (April 7, 2022); [DOI: 10.1126/science.abk1781]
Previous CDF Result (2.2 fb\(^{-1}\))
Combined Fit Systematic Uncertainties

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton Energy Scale</td>
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</tr>
<tr>
<td>Lepton Energy Resolution</td>
<td>2</td>
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<tr>
<td>Recoil Energy Scale</td>
<td>4</td>
</tr>
<tr>
<td>Recoil Energy Resolution</td>
<td>4</td>
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<tr>
<td>(u_{\parallel}) efficiency</td>
<td>0</td>
</tr>
<tr>
<td>Lepton Removal</td>
<td>2</td>
</tr>
<tr>
<td>Backgrounds</td>
<td>3</td>
</tr>
<tr>
<td>(p_T(W)) model</td>
<td>5</td>
</tr>
<tr>
<td>Parton Distributions</td>
<td>10</td>
</tr>
<tr>
<td>QED radiation</td>
<td>4</td>
</tr>
<tr>
<td>(W) boson statistics</td>
<td>12</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>19</strong></td>
</tr>
</tbody>
</table>
## New CDF Result (8.8 fb$^{-1}$)

### Combined Fit Systematic Uncertainties

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton energy scale</td>
<td>3.0</td>
</tr>
<tr>
<td>Lepton energy resolution</td>
<td>1.2</td>
</tr>
<tr>
<td>Recoil energy scale</td>
<td>1.2</td>
</tr>
<tr>
<td>Recoil energy resolution</td>
<td>1.8</td>
</tr>
<tr>
<td>Lepton efficiency</td>
<td>0.4</td>
</tr>
<tr>
<td>Lepton removal</td>
<td>1.2</td>
</tr>
<tr>
<td>Backgrounds</td>
<td>3.3</td>
</tr>
<tr>
<td>$p_T^Z$ model</td>
<td>1.8</td>
</tr>
<tr>
<td>$p_T^W / p_T^Z$ model</td>
<td>1.3</td>
</tr>
<tr>
<td>Parton distributions</td>
<td>3.9</td>
</tr>
<tr>
<td>QED radiation</td>
<td>2.7</td>
</tr>
<tr>
<td>$W$ boson statistics</td>
<td>6.4</td>
</tr>
<tr>
<td>Total</td>
<td>9.4</td>
</tr>
</tbody>
</table>
CDF $M_W$ vs $m_{top}$

Understanding Tevatron-LHC correlations and combination with ATLAS in progress
W Boson Mass Measurements from Different Experiments

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Mass (MeV/c²) ± Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>D0 I</td>
<td>80478 ± 83</td>
</tr>
<tr>
<td>CDF I</td>
<td>80432 ± 79</td>
</tr>
<tr>
<td>DELPHI</td>
<td>80336 ± 67</td>
</tr>
<tr>
<td>L3</td>
<td>80270 ± 55</td>
</tr>
<tr>
<td>OPAL</td>
<td>80415 ± 52</td>
</tr>
<tr>
<td>ALEPH</td>
<td>80440 ± 51</td>
</tr>
<tr>
<td>D0 II</td>
<td>80376 ± 23</td>
</tr>
<tr>
<td>ATLAS</td>
<td>80370 ± 19</td>
</tr>
<tr>
<td>CDF II</td>
<td>80433 ± 9</td>
</tr>
</tbody>
</table>

SM expectation: $M_W = 80,357 \pm 4_{\text{inputs}} \pm 4_{\text{theory}}$ (PDG 2020)

LHCb measurement: $M_W = 80,354 \pm 23_{\text{stat}} \pm 10_{\text{exp}} \pm 17_{\text{theory}} \pm 9_{\text{PDF}}$ [JHEP 2022, 36 (2022)]
1998 Status of $M_W$ vs $M_{top}$

experimental errors 68% CL:
- LEP2/Tevatron (1998)

- $M_W = 114$ GeV
- $M_W = 400$ GeV

- light SUSY
- heavy SUSY

- SM
- MSSM

both models

Heinemeyer, Hollik, Stockinger, Weber, Weiglein
2022 Status of $M_W$ vs $M_{top}$

Experimental errors 68% CL: LEP2/Tevatron (1998)

Standard Model after 2012 Higgs discovery
Epilogue

CDF W mass

2HDM: 14
2204.03693/03767/04834/04688/06485/05085/05269/05303
2204.05975/09001/05728/08406/08390/10338

SMEFT & EW data global fit: 13
2204.04805/05260/05284/05267/05992/05965/05965/08546
2204.08440/10130/04191/05283/04204

Triplet Higgs: 8
2204.05031/05760/07144/07511/07844/08266/10274/10315
SUSY: 6
2204.04286/04356/04202/05285/06541/07138

U(1)\(_x\) gauge symmetry: 6
2204.07100/08067/09487/09024/09585/10156

Vector-like fermion: 6
2204.07022/07411/08568/09477/09671/05024

Others: 9 (Non-unitarity, leptoquark, singlet scalar, ...)
2204.04559/04672/04770/04514/05302/06327/03996/05942/09031

Also related to
dark matter, neutrino masses/seesaw, flavor violation, muon g-2, flavor anomalies, gravitational waves, ...

*Preprints as of April 25th are counted.
The Future of the $M_W$ Measurement

• The experiments at the LHC have collected and are collecting a lot of data.
  • While $W$ bosons are produced slightly differently at the LHC ($pp$ collider) than the Tevatron ($p\bar{p}$ collider), the LHC experiments have the opportunity to make this measurement.

• If built, a new electron-positron collider can also measure the $W$ boson mass very precisely.

• The LHC as well as smaller, specialized experiments are sensitive to the kinds of new particles and interactions that can influence the $W$ boson mass.
  • If there is new physics which could explain the tension of our result with the SM expectation, this new physics could show up directly in these experiments.
Summary

- The $W$ boson mass is a very interesting parameter to measure with increasing precision

- New CDF result is twice as precise as previous measurements:

  \[ M_W = 80433.5 \pm 6.4_{\text{stat}} \pm 6.9_{\text{syst}} \text{ MeV} \]
  \[ = 80433.5 \pm 9.4 \text{ MeV} \]

- Difference from SM expectation of $M_W = 80357 \pm 6$ MeV
  - significance of $7.0\sigma$
  - suggests the possibility of improvements to the SM calculation or of extensions to the SM

Thank you for your attention!