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Lectures on Unification

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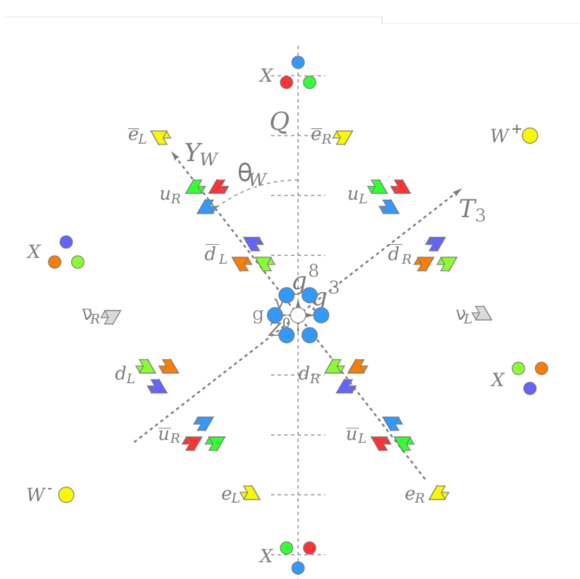
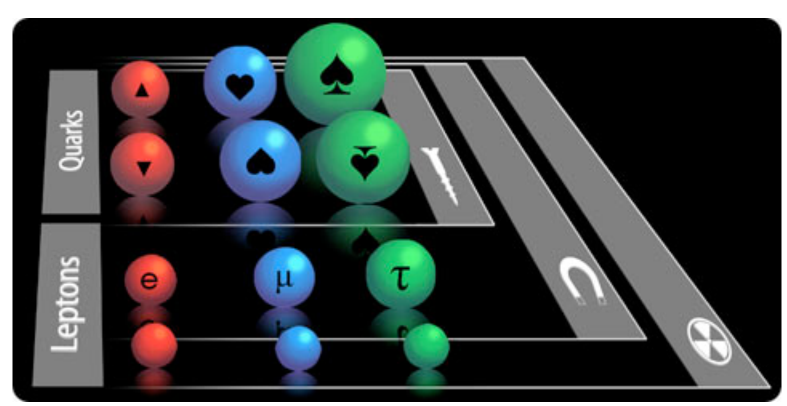
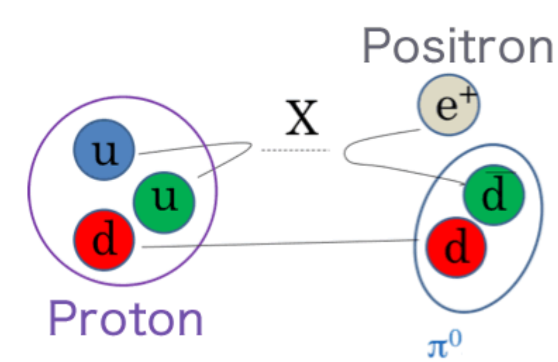
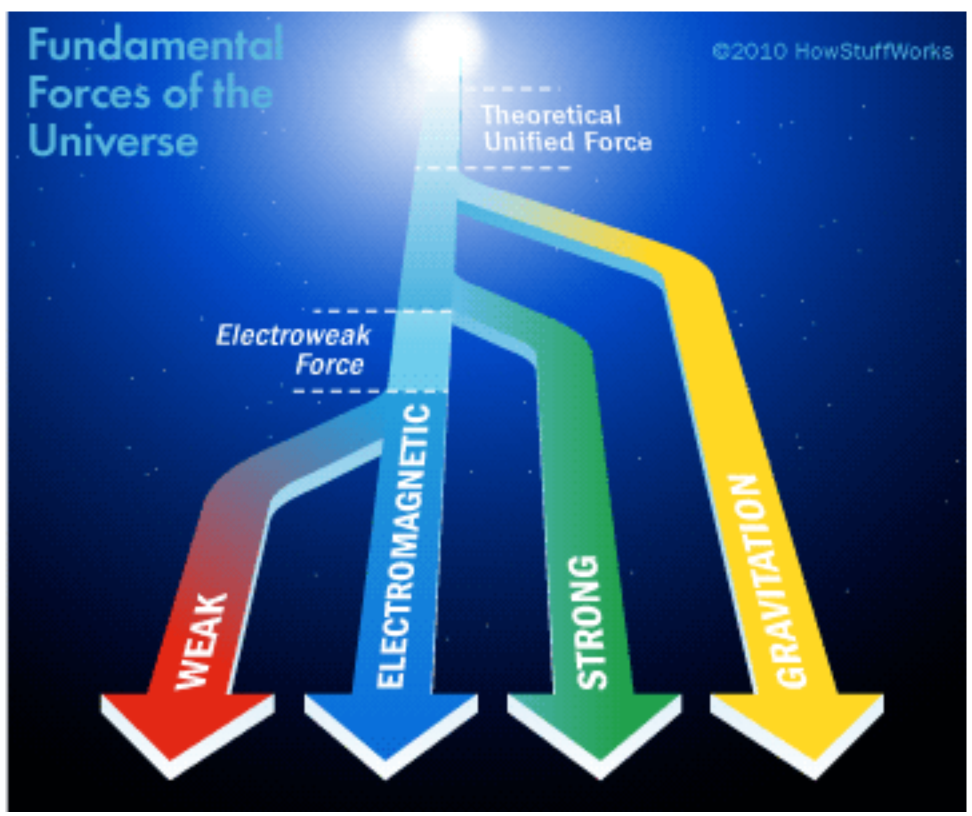
50th SLAC Summer Institute (Aug 8-19, 2022)
“Golden Opportunities: Puzzles & Surprises, Past and Present”







(image credits: Wikipedia, E. Palti, Quora)



(image credits: SLAC, HowStuffWorks)

This talk: ideas of unification, with a broad (theoretical) brush…

Grand unification:

- ✓ some history and classic results
- ✓ current status and approaches
- ✓ some general lessons and outlook

References

H. Georgi, "The Future of Grand Unification," Prog. Theor. Phys. Suppl. 170 (2007) pp.119-137

J. Pati, "Advantages of Unity with SU(4) Color," A. Salam 90th Birthday Memorial Meeting, 2016

P. Langacker, "Grand Unified Theories and Proton Decay," Phys. Rept. 72, 4 (1981) , pp.185-385

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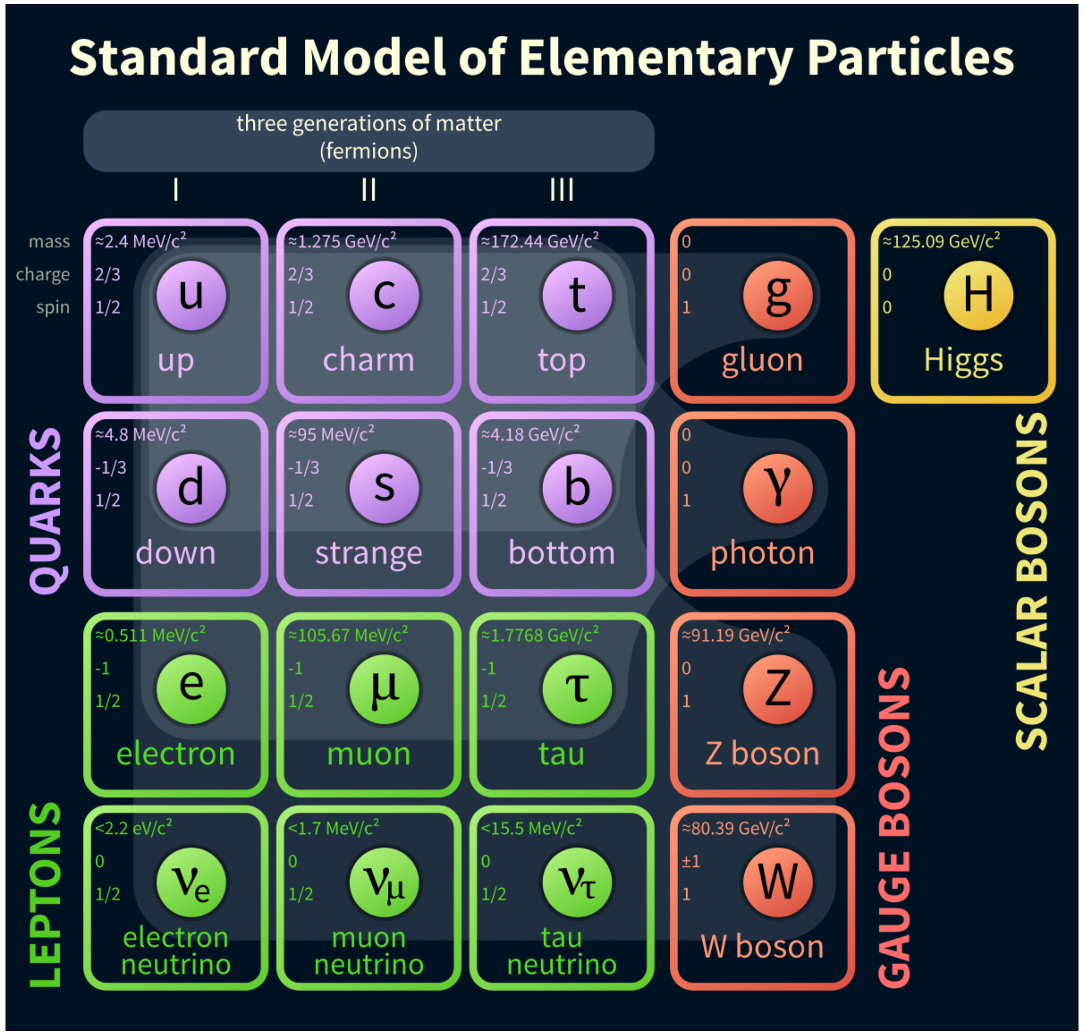
T. Ohlsson, "Proton Decay," Talk at Neutrino 2022

…

(and standard disclaimer!)

The Standard Model (SM)



Nobel Prize (NP) 1979

Glashow Salam Weinberg

Gauge Principle:

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$\xrightarrow{\langle H \rangle \sim \sqrt{2G_F}} SU(3)_c \times U(1)_{EM}$$

Matter content:

3 generations of chiral fermions

(LH— useful convention)

$$Q = (u, d)^T \sim (\mathbf{3}, \mathbf{2}, 1/6) \qquad u^c \sim (\overline{\mathbf{3}}, \mathbf{1}, -2/3)$$

$$L = (\nu, e)^T \sim (\mathbf{1}, \mathbf{2}, -1/2) \qquad d^c \sim (\overline{\mathbf{3}}, \mathbf{1}, 1/3)$$

$$e^c \sim (\mathbf{1}, \mathbf{1}, 1)$$

Higgs sector: $h = (h^+, h_0)^T \sim (\mathbf{1}, \mathbf{2}, 1/2)$

NP 2013

Englert Higgs

(Higgs mechanism)

also: Anderson (dec.); Brout (dec.); Guralnik (dec.), Hagen, Kibble

Spectacularly successful theory of the strong and electroweak interactions!

(image credits: Wikimedia commons)

Going Beyond the Standard Model…

Spectacularly successful, but many outstanding problems…

→ ✓ **Aesthetics**

- 3-2-1 gauge symmetry and quantum numbers
- fermion family replication, flavor mixing, CP violation
- fermion masses and mixings, ultralight neutrino masses
- 19 free parameters… "too complicated and arbitrary"

✓ **Naturalness**

- origin of electroweak scale and light Higgs mass
- vacuum structure…

✓ **Cosmology**

- origin of dark matter and dark energy
- origin of matter-antimatter asymmetry
- *predictive* quantized gravity…

→ **solutions require physics beyond the Standard Model!**

Grand Unification Paradigm

(image credit:Fightstar, "Grand Unification" album, 2006)

Embed SM gauge group into larger gauge symmetry structure**

$G_{\rm SM}$ $\quad SU(3)_c \times SU(2)_L \times U(1)_Y \longrightarrow G$

Basic requirements:

✓ **sufficiently large (total) group rank** ≥ 4

✓ **chiral SM fermions** →

— complex representations: $(\psi_L \neq \psi_R)$

$$\psi_L = (\mathbf{3},\mathbf{2})_{1/6} + (\overline{\mathbf{3}},\mathbf{1})_{-2/3} + (\overline{\mathbf{3}},\mathbf{1})_{1/3} + (\mathbf{1},\mathbf{2})_{-1/2} + (\mathbf{1},\mathbf{1})_1$$

$$\psi_R = (\overline{\mathbf{3}},\mathbf{2})_{-1/6} + (\mathbf{3},\mathbf{1})_{2/3} + (\mathbf{3},\mathbf{1})_{-1/3} + (\mathbf{1},\mathbf{2})_{1/2} + (\mathbf{1},\mathbf{1})_{-1}$$

— anomaly-free

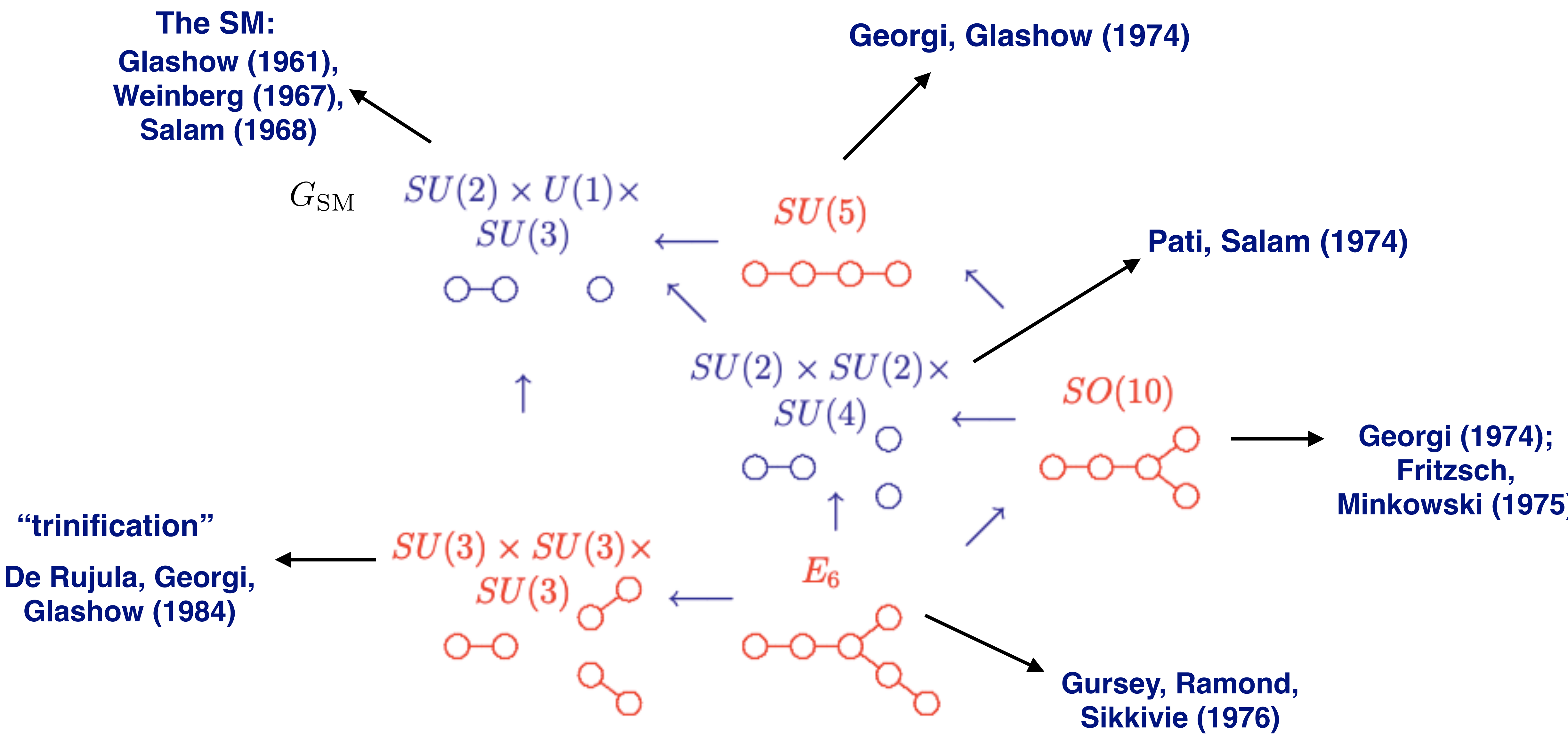
✓ **"standard" embedding of fermions in** $SU(3)_c$

Gell-Mann, Ramond, Slansky '78

Possibilities: $SU(N) \quad N > 2$ $\qquad SO(4N+2) \quad N > 1$ $\qquad E_6$

(strict interpretation — single group, not product of groups)**

Some prototypical examples of grand unified theories (GUTs):



(image credit: Georgi, Prog. Theor. Phys. Suppl. 170 (2007) 119)

These structures lead to some stunning results!

Now-classic fermion embeddings (single generation):

$SU(5)$ **Georgi, Glashow (1974)**

$$d^c \sim (\overline{\mathbf{3}}, \mathbf{1}, 1/3)$$
$$L = (\nu, e)^T \sim (\mathbf{1}, \mathbf{2}, -1/2)$$
$$\rightarrow \overline{\mathbf{5}}$$

$$Q = (u, d)^T \sim (\mathbf{3}, \mathbf{2}, 1/6)$$
$$u^c \sim (\overline{\mathbf{3}}, \mathbf{1}, -2/3)$$
$$e^c \sim (\mathbf{1}, \mathbf{1}, 1)$$
$$\rightarrow \overline{10}$$

$SO(10)$ **Georgi (1974); Fritzsch, Minkowski (1975)**

$$Q = (u, d)^T \sim (\mathbf{3}, \mathbf{2}, 1/6)$$
$$L = (\nu, e)^T \sim (\mathbf{1}, \mathbf{2}, -1/2)$$
$$u^c \sim (\overline{\mathbf{3}}, \mathbf{1}, -2/3)$$
$$d^c \sim (\overline{\mathbf{3}}, \mathbf{1}, 1/3)$$
$$e^c \sim (\mathbf{1}, \mathbf{1}, 1)$$
$$\nu^c \sim (\mathbf{1}, \mathbf{1}, 0)$$
$$\rightarrow \mathbf{16}$$

$SU(4) \times SU(2) \times SU(2)$ **Pati, Salam (1974)**

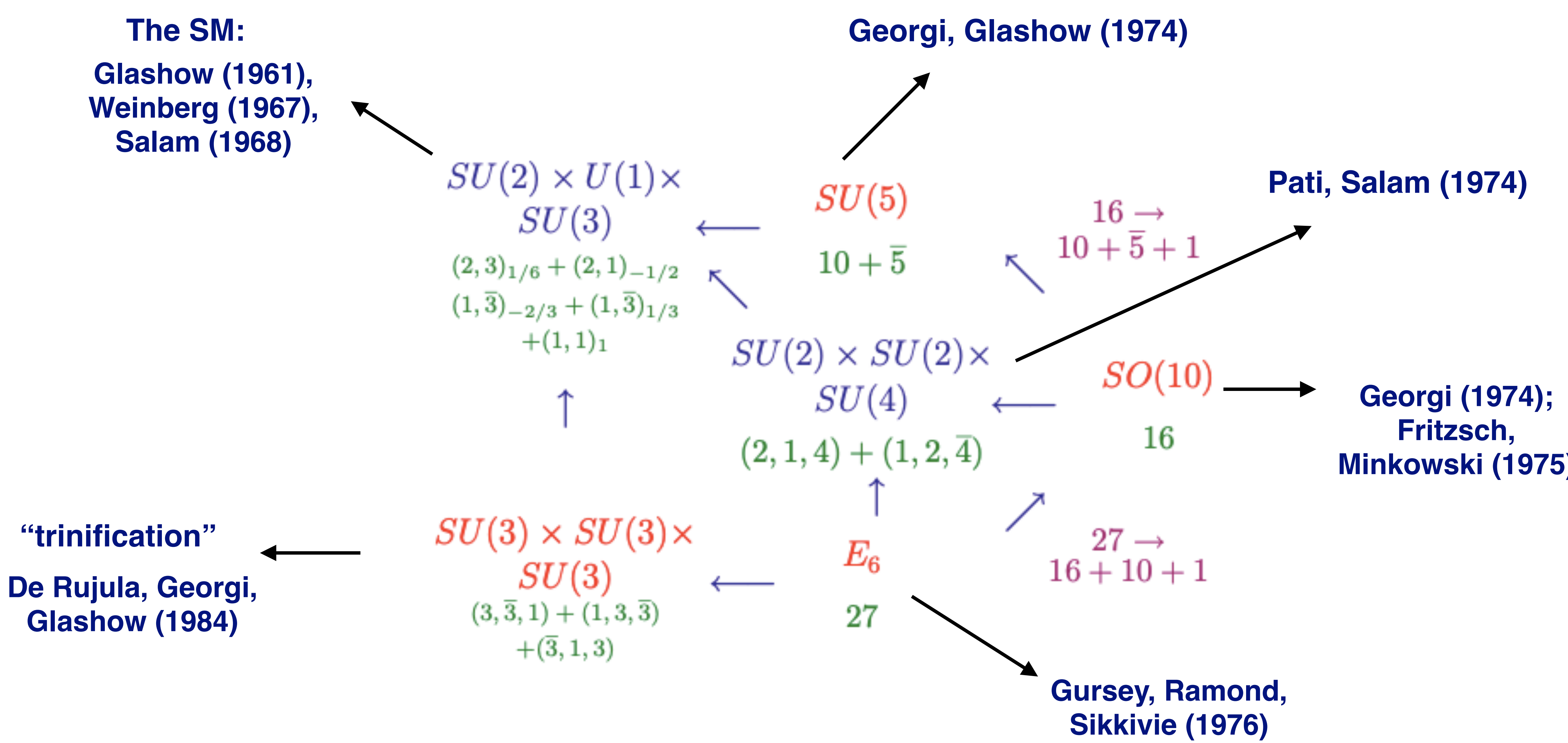
$$Q = (u, d)^T \sim (\mathbf{3}, \mathbf{2}, 1/6)$$
$$L = (\nu, e)^T \sim (\mathbf{1}, \mathbf{2}, -1/2)$$
$$\rightarrow (\mathbf{4}, \mathbf{2}, \mathbf{1})$$

$$u^c \sim (\overline{\mathbf{3}}, \mathbf{1}, -2/3)$$
$$d^c \sim (\overline{\mathbf{3}}, \mathbf{1}, 1/3)$$
$$e^c \sim (\mathbf{1}, \mathbf{1}, 1)$$
$$\nu^c \sim (\mathbf{1}, \mathbf{1}, 0)$$
$$\rightarrow (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$$

RH neutrino!

A pictorial version…

(image credit: Georgi, Prog. Theor. Phys. Suppl. 170 (2007) 119)



subgroup branchings →

patterns of spontaneous symmetry breaking (SSB)

- ✓ **Implications for SM gauge couplings…**

  - **— quantization of charge**

  - **— SM weak mixing angle:**

$$SU(2)_L \quad U(1)_Y$$
$$W^3_\mu \qquad B_\mu$$
$$(g, g')$$

→

$$Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu$$
$$A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu$$

$$\tan\theta_W = g'/g \qquad e = g\sin\theta_W$$

Unified gauge coupling → **prediction for** $\sin^2\theta_W$

- ✓ **Fermion mass relations (Yukawa unification)…**

- ✓ **New gauge bosons — what they do…** → **nucleon decay!**

Some history…

**around the time of the first SSI!

Let's go back to nearly 50 years ago**

as the grand unification paradigm developed in parallel with the development and verification of the SM itself

No attempt here to characterize this rich and interwoven history fully

Focus here on:

Georgi's retrospective of the birth of SU(5)

H. Georgi, "The Future of Grand Unification," Yukawa-Tomonaga 100th Birthday Celebration, Prog. Theor. Phys. Suppl. 170 (2007) 119

Pati's comments on the origin of Pati-Salam

J. Pati, "Advantages of Unity with SU(4) Color," Memorial Meeting for Nobel Laureate Professor Abdus Salam's 90th Birthday, Int. J. Mod. Phys A. 32 (2017) 09

A very exciting time in physics →

let's hear more of these great stories!

(are you listening, #grahamfarmelo…)

Early 1970's…

Excitement about gauge theories with spontaneously broken symmetries!

✓ **renormalizability proven by 't Hooft in 1971** NP 1999

(HG: late FNAL theorist Ben Lee influential in spreading this message)

Georgi and Glashow started model-building, as did Pati, Salam, and others

HG's account:

✓ **First, lessons/inspiration from leptons only**

— **misleading exp. situation on weak neutral currents**

(not clear in 1971-72 that they existed! seen definitively at Gargamelle in 1973)

Model-building goal: eliminate weak neutral currents →

postulate new (heavy) fermions to complete gauge multiplets

— efforts to understand/calculate m_e/m_μ

(speculation by Bjorken: ratio from radiative correction in the fine structure constant

HG: a "proto-GUT" for leptons $SU(3) \times SU(3) \longrightarrow SU(2) \times U(1)$ Weinberg '72

"hide" the new interactions →

postulate new (higher-scale) stage of symmetry breaking

"superheavy" gauge bosons — constrained by virtual effects
$(> m_W)$

prediction** for weak mixing angle: $\sin^2\theta_W = 1/4$ (**at "GUT" scale)

✓ **Now, lessons/inspiration from quarks**

— efforts to consider quarks:

concerns about incorporating fractionally charged quarks

concerns about coupling strength difference of strong vs. electroweak

Situation (both experimental and theoretical) of course evolved dramatically!

Three Nobel prize-winning developments of that era:

✓ fractionally charged quarks — Friedan, Taylor NP 1990

✓ discovery of J/ψ and "November revolution" — Richter, Ting NP 1976

✓ asymptotic freedom — Gross, Wilczek, Politzer NP 2004

HG: difficulty of fitting fractionally charged quarks+leptons into simple group

amusing anecdote about becoming a group theory expert out of "necessity"

Recognizing need for complex representation

(LH and RH fermions — different charges in SM)

anomaly constraints → restrictions on possible gauge groups

Bouchiat et al. '72
Gross, Jackiw '72

Inspired by work of Pati and Salam

Pati, Salam '73, 74

Pati-Salam: $SU(4) \times SU(2) \times SU(2)$

$$SU(4) \sim SO(6)$$

$$SU(2) \times SU(2) \sim SO(4)$$

$\longrightarrow$ $SO(10)$ **fit SM fermions +** ν^c **in complex spinor irrep** $\mathbf{16}$

quarks and antiquarks in the same representation

$\longrightarrow$ **dictated by group theory!**

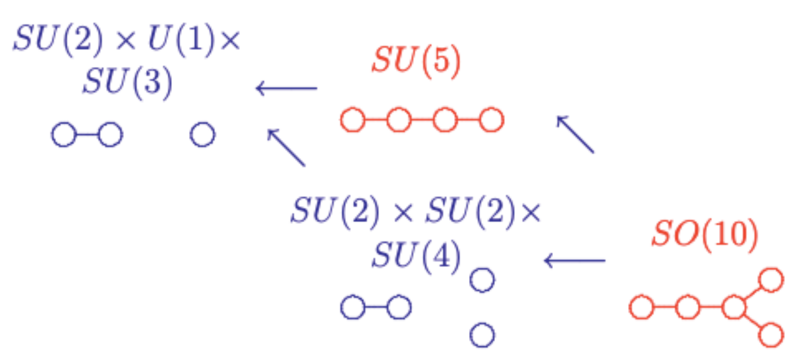
Via analogy with PS breaking,

HG: eliminate ν^c **direction in** $SO(10)$

$SO(10)$ **generators:** $45 \longrightarrow 24 = 5^2 - 1$

$\downarrow$

$SU(5)$



Georgi, Glashow '74

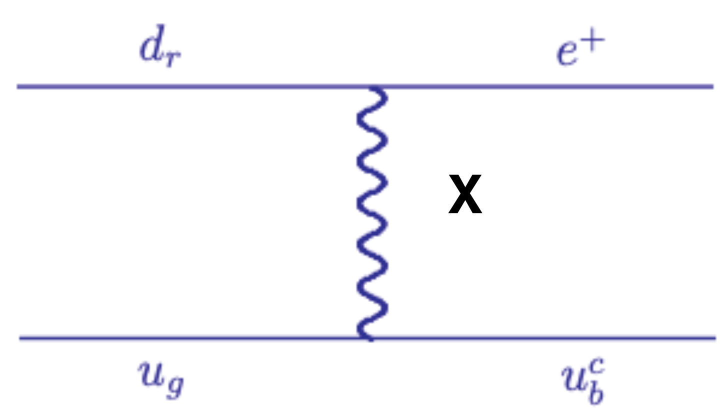
Minimal (Georgi-Glashow) $SU(5)$ followed right away: Georgi, Glashow '74

$$\overline{\mathbf{5}} = (\overline{\mathbf{3}}, \mathbf{1})_{1/3} + (\mathbf{1}, \mathbf{2})_{-1/2} \qquad \mathbf{10} = (\mathbf{3}, \mathbf{2})_{1/6} + (\overline{\mathbf{3}}, \mathbf{1})_{-2/3} + (\mathbf{1}, \mathbf{1})_1 \qquad \mathbf{16} = \mathbf{10} + \overline{\mathbf{5}} + \mathbf{1}$$

$$SU(5) \xrightarrow{\langle \mathbf{24} \rangle} SU(3)_c \times SU(2)_L \times U(1)_Y$$

HG: $SU(5)$ **contains** $SU(3)_c \times SU(2)_L \times U(1)_Y$**, because** $2 + 3 = 5$ **!**

new (leptoquark) gauge bosons mediate proton decay!



(image credits: Georgi, Prog. Theor. Phys. Suppl. 170 (2007) 119)

$$p \to e^+\pi^0 \qquad \tau_{p\to e^+\pi^0} \sim M_X^4/m_p^5$$

superheavy gauge bosons: $M_X \geq 10^{14}$ GeV

JP's account:

Back to 1972-1973

No clear understanding of the origin of the strong interactions

"superstrong" force: gauging color $SU(3)_c$

known but not widely accepted

't Hooft's proof of renormalizability of spontaneously broken gauge theories

→ **model-building excitement, building variants of** $SU(2) \times U(1)$

JP's motivation: wanted to address its major shortcomings

- ✓ **seemingly arbitrary quantum number assignments; 5 "scattered" multiplets**
- ✓ **no apparent reason for coexistence of quarks and leptons, or the 3 forces**
- ✓ **no compelling reason for charge quantization, or the relation** $Q_{\text{electron}} = -Q_{\text{proton}}$
- ✓ **bothered by the apparent putting in "by hand" the non-conservation of parity**

→ **put quarks and leptons into common multiplets of higher symmetry group**

Pati, Salam '73, 74

Pati-Salam: suggest new color group, $SU(4)_c$ $SU(4)_c \to SU(3)_c \times U(1)_{B-L}$

"lepton number as the fourth color" $\mathbf{4} = (Q, L)^T$

neutrino and electron: "up and down quarks" of lepton color

Canonical PS model: includes $SU(2)_L \times SU(2)_R$

$$SU(4) \times SU(2) \times SU(2) \longrightarrow SU(3)_c \times U(1)_{B-L} \times SU(2) \times SU(2) \longrightarrow G_{\rm SM}$$

$$(\mathbf{4},\mathbf{2},\mathbf{1}) = (\mathbf{3},\mathbf{2})_{1/6} + (\mathbf{1},\mathbf{2})_{1/2} \qquad (\bar{\mathbf{4}},\mathbf{1},\mathbf{2}) = (\bar{\mathbf{3}},\mathbf{1})_{-2/3} + (\bar{\mathbf{3}},\mathbf{1})_{-1/3} + (\mathbf{1},\mathbf{1})_1 + (\mathbf{1},\mathbf{1})_0$$

Successes:

✓ charge quantization with fractionally charged quarks

→ quark charges = (lepton charge)/(# colors)

$$Q = \frac{1}{2}\left(T_{3L} + T_{3R} + (B-L)\right) \qquad {\rm Tr}\, Q = 0$$

✓ weak interaction universality

✓ the necessity of a right-handed neutrino!

JP: then "ugly duckling," now a "beautiful swan" post-1998 (ν oscillations)

Classic results: minimal $SU(5)$

$$a = 1,\dots 5 = \begin{cases} \alpha \ (a = 1,2,3) \\ r \ (a = 4,5) \end{cases}$$

✓ **Fermion representations:** **3 copies of** $\overline{\mathbf{5}} + \mathbf{10}$

$$\chi_{5a} = (d_1^c, d_2^c, d_3^c, e^-, -\nu)_L^T \qquad \psi_{10}^{ab} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & -u^1 & -d^1 \\ -u_3^c & 0 & u_1^c & -u^2 & -d^2 \\ u_2^c & -u_1^c & 0 & -u^3 & d^3 \\ u^1 & u^2 & u^3 & 0 & -e^+ \\ d^1 & d^2 & d^3 & e^+ & 0 \end{pmatrix}$$

(or equivalently $\psi^{5a} = (d^1, d^2, d^3, e^+, -\nu^c)_R^T$ **)**

✓ **SSB and scalar representations:** $\mathbf{24}$ **(adjoint)** Φ $\qquad$ $\mathbf{5}$ **(fundamental)** H

$$H = (h^1, h^2, h^3, h^+, -h^0)^T$$

$$SU(5) \xrightarrow{\langle \mathbf{24} \rangle} SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{\langle \mathbf{5} \rangle} SU(3)_c \times U(1)_{EM}$$

✓ **Gauge bosons:** $\mathbf{24} = (\mathbf{8},\mathbf{1})_0 + (\mathbf{1},\mathbf{3})_0 + (\mathbf{1},\mathbf{1})_0 + (\mathbf{3},\mathbf{2})_{-5/6} + (\overline{\mathbf{3}},\mathbf{2})_{5/6}$

$$(G_\mu)^\alpha_\beta \qquad (W_\mu)^r_s \qquad B_\mu \qquad (A_\mu)^\alpha_r \qquad (A_\mu)^r_\alpha$$

$$(A_\mu)^4_\alpha \equiv X_\mu \qquad (A_\mu)^5_\alpha \equiv Y_\mu$$

✓ **Yukawa interactions:** $\chi^T_{5a} C \psi^{ab}_{10} H^\dagger_b$ $\qquad$ $\epsilon_{abcde} (\psi^{ab}_{10})^T C \psi^{cd}_{10} H^e$

(down quarks, charged leptons) $\qquad$ **(up quarks)**

✓ **quantization of electric charge** — Georgi, Glashow '74

Rank 4 ⟶ 4 diagonal (traceless) generators:

$SU(3)$ $T_3 = \mathrm{Diag}(1, -1, 0, 0, 0)$

$SU(3)$ $T_8 = (1/\sqrt{3})\mathrm{Diag}(1, 1, -2, 0, 0)$

$SU(2)$ $T_{11} = \mathrm{Diag}(0, 0, 0, 1, -1)$

$T_{12} = (1/\sqrt{15})\mathrm{Diag}(-2, -2, -2, 3, 3)$

Electric charge operator Q is an $SU(5)$ generator:

$$Q = \frac{1}{2}\left(T_{11} + \sqrt{\frac{5}{3}} T_{12}\right) \qquad \mathrm{Tr}\, Q = 0$$

⟶ **electric charge quantized**

e.g. $\overline{\mathbf{5}}$ ⟶ $Q_{d^c} = -Q_e/3 = -1/3$ (3 = **# of colors**) **(like PS!)**

consequence of simple group: variations possible w/o chg quantization

e.g. "flipped" $SU(5)$ $SU(5) \times U(1)$

Barr '82; Derendinger et al. '84

✓ **Prediction of** $\sin^2\theta_W$ **Georgi, Glashow '74**

$$D_\mu = \partial_\mu - (ig_5/2)(W^3_\mu T^{11} + B_\mu T^{12} + \ldots)$$

$$\downarrow$$

$$-(ig_5/2)(\sin\theta_W A_\mu + \cos\theta_W Z_\mu)T^{11} - (ig_5/2)(\cos\theta_W A_\mu - \sin\theta_W Z_\mu)T^{12}$$

$$\equiv -ieQA_\mu - igQ_Z Z_\mu$$

identify terms and use $Q = \frac{1}{2}\left(T_{11} + \sqrt{\frac{5}{3}}T_{12}\right)$

$$\tan\theta_W = g'/g = \sqrt{3/5}$$

→ $\boxed{\sin^2\theta_W = 3/8}$ **a remarkable result!**

Initially not known well experimentally, so this value seemed reasonable

but realized soon after that this holds at M_X **Georgi, Quinn, Weinberg '74**

→ **need to run to weak scale via RGE!**

gives input re size of M_X

General program of gauge coupling unification

For any theory with unified group $G \longrightarrow G_{\rm SM}$ **at scale** M_X

gauge couplings of the (properly normalized) subgroups should unify at/near M_X

→ **two independent determinations of** M_X

$\sin^2\theta_W \quad \alpha/\alpha_s$ **(measured at/below weak scale)**

Minimal (GG) $SU(5)$

$$\sin^2\theta_W = \frac{g'^2}{g'^2+g^2} = \frac{g_1^2}{g_1^2+\frac{5}{3}g_2^2} \longrightarrow \frac{3}{8} \qquad (Q^2 \geq M_X^2)$$

$$\alpha/\alpha_s = \frac{e^2}{g_s^2} = \frac{g^2\sin^2\theta_W}{g_s^2} \longrightarrow \frac{3}{8} \qquad (Q^2 \geq M_X^2)$$

Running coupling constants decrease these quantities at lower energies

$$\frac{1}{g_i^2(Q^2)} - \frac{1}{g_i^2(M_X^2)} = -\beta_i \ln\frac{Q^2}{M_X^2}$$

Georgi, Quinn, Weinberg '74

simplest 1-loop (ignore Higgs,...)

$$\sin^2\theta_W(Q^2) = \frac{3}{8}\left[1 - \frac{55\alpha}{18\pi}\ln\frac{M_X^2}{Q^2}\right] \qquad \frac{\alpha(Q^2)}{\alpha_s(Q^2)} = \frac{3}{8}\left[1 - \frac{11\alpha}{2\pi}\ln\frac{M_X^2}{Q^2}\right]$$

naively:

$M_X \sim O(10^{16}\ \mathrm{GeV})$

BUT…more to the story!

Including n_H light complex Higgs fields:

$$\sin^2\theta_W(Q^2) = \frac{3}{8}\left[1 - \frac{\alpha}{4\pi}\left(\frac{110 - n_H}{9}\right)\ln\frac{M_X^2}{Q^2}\right] \qquad \frac{\alpha(Q^2)}{\alpha_s(Q^2)} = \frac{3}{8}\left[1 - \frac{\alpha}{2\pi}\left(11 + \frac{n_H}{6}\right)\ln\frac{M_X^2}{Q^2}\right]$$

small corrections: small changes in $\ln M_X$ but large changes in M_X

Many small corrections: generally decrease M_X

(running α, careful thresholds, two-loops,...)

Buras et al. '78; Goldman, Ross '79 ...

Intense effort to compute these quantities and match with data →

consistency of M_X prediction AND proton decay rate expectations

Upshot of this important story:

nice account in talk by Haber, SUSY '97

GG:** $\sin^2\theta_W(m_W) = 0.214^{+0.004}_{-0.003}$ **4WOGU, 1983**

exp: $\sin^2\theta_W(m_W) = 0.228 \pm 0.0044$ **8WOGU, 1987**

Evidently, the strong and electroweak coupling constants do not unify in GG!**

But they appeared to do so then in supersymmetric version: nWOGU ⟶ SUSY-XX

We'll return to this soon!

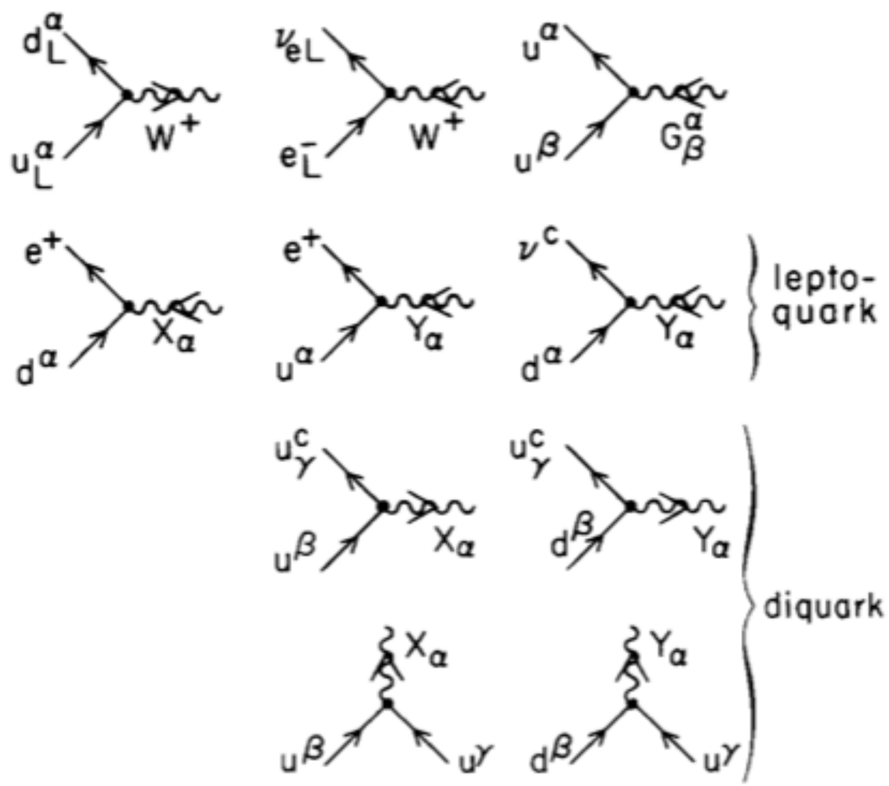
****results much more general than GG**

$(A_\mu)^4_\alpha \equiv X_\mu$

$(A_\mu)^5_\alpha \equiv Y_\mu$

✓ **Baryon and lepton number violation** → **proton decay**

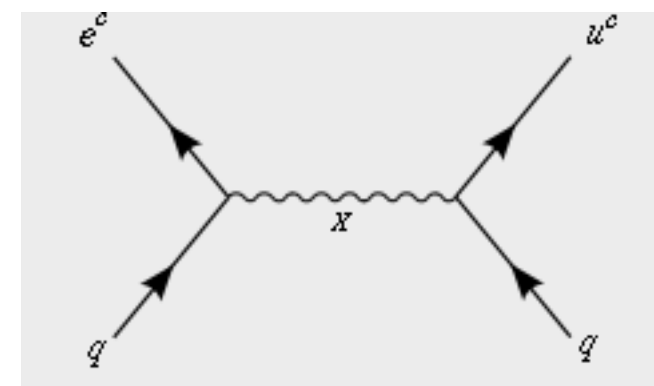
GG: mediated by gauge bosons $X_\mu \quad Q_X = 4/3 \qquad Y_\mu \quad Q_Y = 1/3$



vertices violate *B* and *L*
but preserve *B-L* w/assignment

$$X_\mu \quad Y_\mu \longrightarrow B - L \to 2/3$$

$$p \to e^+\pi^0$$



dimension 6 operators: $|\Delta B| = 1$

$$\sim (g_5^2/M_X^2)(\epsilon_{\alpha\beta\gamma}\bar{u}_L^{c\gamma}\gamma_\nu u_L^\beta)(2\bar{e}_L^+\gamma^\mu d_L^\alpha + \bar{e}_R^+\gamma^\mu d_R^\alpha)$$
$$+(g_5^2/M_Y^2)(\epsilon_{\alpha\beta\gamma}\bar{u}_L^{c\gamma}\gamma_\nu d_L^\beta)(\bar{\nu}_R^c\gamma_\mu d_R^\alpha) + \text{h.c.}$$

(one family — can generalize this)

proton lifetime: need hadronic matrix elements, anomalous dimensions, etc.

$$\tau^{\rm GG}_{p\to e^+\pi^0} \sim 10^{30-31} \text{ yrs}$$

see e.g. Bueno et al. ’07, PDG ’22

(image credits: Langacker ’81, Wikipedia)

Proton decay can also be mediated at tree level by scalars!

$\mathbf{5} \quad H = (h^1, h^2, h^3, h^+, -h^0)^T$ $\qquad\qquad \epsilon_{abcde}(\psi_{10}^{ab})^T C \psi_{10}^{cd} H^e$

suppressed by small Yukawa couplings $\qquad \tau_p \sim m_T^4 / m_p^5$

but still requires heavy color triplet scalar $\qquad m_T \gg m_h$

↓

doublet-triplet splitting problem

arises also in SSB: cross terms b/w Φ **and** H

fine-tuning (part of the famous Higgs hierarchy problem)

A preview: **in SUSY versions aspects of this will be mitigated…**

but new constraints from proton decay will also arise!

✓ **Fermion masses**

with minimal scalar content: Φ **,** H

interactions $\chi^T_{5a} C \psi^{ab}_{10} H^\dagger_b$ $\epsilon_{abcde} (\psi^{ab}_{10})^T C \psi^{cd}_{10} H^e$

(down quarks, charged leptons) **(up quarks)**

(partial) Yukawa unification:

$$m_b = m_\tau \qquad m_s = m_\mu \qquad m_d = m_e$$

holds at M_X

works well for third family but still disastrous for lighter generations:

$$m_d / m_s = m_e / m_\mu$$

(bad!)

Fixes include: non-minimal scalar sector, family symmetries,…

one famous approach: Georgi-Jarlskog

include both 5 **and** 45 **scalars**

Georgi, Jarlskog '79

$$m_b = m_\tau \qquad m_\mu = 3 m_s \qquad m_d = 3 m_e$$

holds at M_X

Chanowitz et al. '77
Buras et al. '78

tip of the hat to Mohapatra, TASI '97 lectures

Recap: G/B/U checklist

minimal (GG) $SU(5)$

G: **Beautiful and economical…**

- ✓ electric charge quantization
- ✓ elegantly accommodates the SM fermions in $\overline{\mathbf{5}} + \mathbf{10}$
- ✓ $G_{\rm SM}$ is its maximal subgroup (unique)
- ✓ correct form of SM charged and neutral weak currents
- ✓ predictive power (with desert hypothesis)

$$\alpha \ll \alpha_s \longrightarrow M_X \gg m_W \qquad \sin^2\theta_W \longleftrightarrow \tau_{p\to e^+\pi^0}$$

- ✓ *B* and *L* violation prototypical; *B-L* conservation
- ✓ Yukawa unification ($b-\tau$) for minimal Higgs sector

B: **So predictive that it's ruled out!**

- ✓ gauge coupling unification (with desert hypothesis) fails
- ✓ lighter generation masses; doublet-triplet splitting
- ✓ reducible representations; arguably still quite complicated

U: **Hierarchy problem, no connection to gravity, …**

$SO(10)$

Georgi (1974);
Fritzsch, Minkowski (1975)

simplest version of $SO(4N+2)$ $\quad N=2$

45 gauge bosons $(2N+1)(4N+1)$

each fermion family: irreducible complex spinor rep $\mathbf{16}$

SM quarks, leptons+ right-handed neutrino

minimal embedding of SM Higgs: $\mathbf{10}$ $\quad \mathbf{10} = \mathbf{5} + \overline{\mathbf{5}}$ **(2 Higgs doublets)**

Many options for symmetry breaking patterns:

$SO(10)$

(1) $SO(10) \to SU(5)$

(2) $SO(10) \to SU(5) \times U(1)_X$

$$\mathbf{16} = \mathbf{10}_{-1} + \overline{\mathbf{5}}_3 + \mathbf{1}_{-4}$$

(3) $SO(10) \to SU(4) \times SU(2) \times SU(2)$ **Pati-Salam**

$$\mathbf{16} = (\mathbf{4},\mathbf{2},\mathbf{1}) + (\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$$

$\downarrow$

$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)'$

$$\mathbf{16} = (\mathbf{3},\mathbf{2},\mathbf{1}) + (\overline{\mathbf{3}},\mathbf{1},\mathbf{2}) + (\mathbf{1},\mathbf{2},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{2})$$

hypercharge embeddings:

usual: T^{12} **of** $SU(5)$

flipped: l.c. of T^{12} **and** $U(1)_X$

can have successful gauge coupling unification

$SO(10)$ **gauge bosons:**

$SU(5)$ **decomposition:**

$$\mathbf{45} = \mathbf{24} + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{1}$$

new

$(SU(3)_c, SU(2)_L, SU(2)_R)$ **decomposition:**

$$\mathbf{45} = (\mathbf{8},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{1},\mathbf{3}) + (\mathbf{1},\mathbf{1},\mathbf{1}) + (\overline{\mathbf{3}},\mathbf{2},\mathbf{2}) + (\mathbf{3},\mathbf{2},\mathbf{2}) + (\mathbf{3},\mathbf{1},\mathbf{1}) + (\overline{\mathbf{3}},\mathbf{1},\mathbf{1})$$

$$(G_\mu)^\alpha_\beta \quad W^{\pm,3}_{\mu L} \quad W^{\pm,3}_{\mu R} \quad B_\mu \quad \begin{pmatrix} X_\mu & \bar{Y}'_\mu \\ Y_\mu & \bar{X}'_\mu \end{pmatrix} \begin{pmatrix} X'_\mu & \bar{Y}_\mu \\ Y'_\mu & \bar{X}_\mu \end{pmatrix} \quad X_{\mu s} \quad \bar{X}_{\mu s}$$

new

Higgs fields for GUT breaking:

(1) 16 **or** 126 **(2)** 45 **(3)** 54 **many possible chains!**

Fermion masses: **(minimal EW Higgs)** $\psi^T_{16L} C \psi_{16L} H^\dagger_{10}$ →

Yukawa unification! $\boxed{m_t = m_b = m_\tau}$ **holds at** M_X

(requires 2 light Higgs, can be OK for large vev ratio)

but again, bad for lighter generations

Many references!!

Many references!! See Langacker ’81, Ross ’84 for early reviews

Proton decay revisited:

Dimension 6 operator analysis, vector (gauge) boson-mediated

$$\mathcal{O}_1 = (\epsilon_{\alpha\beta\gamma}\bar{u}_L^{c\gamma}\gamma_\nu u_L^\beta)(\bar{e}_R\gamma_\mu d_R^\alpha) - (\epsilon_{\alpha\beta\gamma}\bar{u}_L^{c\gamma}\gamma_\nu d_L^\beta)(\bar{\nu}_R^c\gamma_\mu d_R^\alpha)$$

$$\mathcal{O}_2 = (\epsilon_{\alpha\beta\gamma}\bar{u}_L^{c\gamma}\gamma_\nu u_L^\beta)(\bar{e}_L^+\gamma_\mu d_L^\alpha) \qquad \mathcal{O}_3 = (-\epsilon_{\alpha\beta\gamma}\bar{u}_R^{c\gamma}\gamma_\nu d_R^\beta)(\bar{\nu}_L^c\gamma_\mu d_L^\alpha)$$

Effective Lagrangian:

$$-\mathcal{L} = (4G_1/\sqrt{2})\mathcal{O}_1 + (4G_2/\sqrt{2})\mathcal{O}_2 + \text{h.c.} \qquad \frac{G_i}{\sqrt{2}} = \frac{g_{\text{GUT}}^2}{8M_i^2}$$

$$r \equiv G_2/G_1$$

$SU(5) \longrightarrow$

$$-\mathcal{L}_{\text{SU(5)}} = (4G/\sqrt{2})(2\mathcal{O}_2 + \mathcal{O}_1) + \text{h.c.} \qquad r = 2$$

e^+ **final states preferred over** ν^c

$SO(10) \longrightarrow$

$$-\mathcal{L}_{\text{SO(10)}} = -\mathcal{L}_{\text{SU(5)}} + (4G'/\sqrt{2})(2\mathcal{O}_3 + \mathcal{O}_1 + \text{h.c.}) \qquad r = \frac{(2/M_X^2)}{(1/M_X^2 + 1/M_{X'}^2)}$$

X', Y'

$SO(10)$ **checklist:**

G: ✓ each generation in a single irreducible representation!

✓ charge quantization

✓ can get successful gauge coupling unification

✓ naturally includes right-handed neutrino

✓ full Yukawa unification with simplest Higgs sector

✓ more SSB chains — more options. can incorporate GG, PS, flipped SU(5),…

✓ proton decay: can have different patterns than GG

B: ✓ Yukawa relations problematic for light generations w/o more ingredients

✓ doublet-triplet splitting issue remains

U: ✓ same (didn't expect progress here…)

Barr '82; Derendinger et al. '84,...

Flipped $SU(5)$ $SU(5) \times U(1)_X$

hypercharge embedding: $Y = \frac{1}{5}\left(Q_X - \frac{Q_Y^{(\mathrm{GG})}}{6}\right)$

Fermion representations: **3 copies of** $\mathbf{10} + \overline{\mathbf{5}} + \mathbf{1}$

$$\psi_{10}^{ab} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & d_3^c & -d_2^c & -u^1 & -d^1 \\ -d_3^c & 0 & d_1^c & -u^2 & -d^2 \\ d_2^c & -d_1^c & 0 & -u^3 & d^3 \\ u^1 & u^2 & u^3 & 0 & -\nu^c \\ d^1 & d^2 & d^3 & \nu^c & 0 \end{pmatrix}$$

$$\chi_{5a} = (u_1^c, u_2^c, u_3^c, e^-, -\nu)_L^T$$

$$\psi = e^c$$

low dimension irreps

SSB to SM gauge group: $10, \overline{10}$ **scalars** **electroweak Higgs** $5, \overline{5}$

Proton decay: **differences from GG** $r = 0$

Larger groups **prototype:** E_6

Gürsey et al. '76,...

Fermion representations: **3 copies of** $\mathbf{27}$ ← **exotics!** **challenge: must decouple them**

many SSB branches...

$SU(3)_c \times SU(3) \times SU(3)$

$$\mathbf{27} = (\mathbf{3}, \overline{\mathbf{3}}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}, \overline{\mathbf{3}}) + (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{3})$$

$SO(10) \times U(1)$

$$\mathbf{27} = \mathbf{16} + \mathbf{10} + \mathbf{1}$$

What's up next…

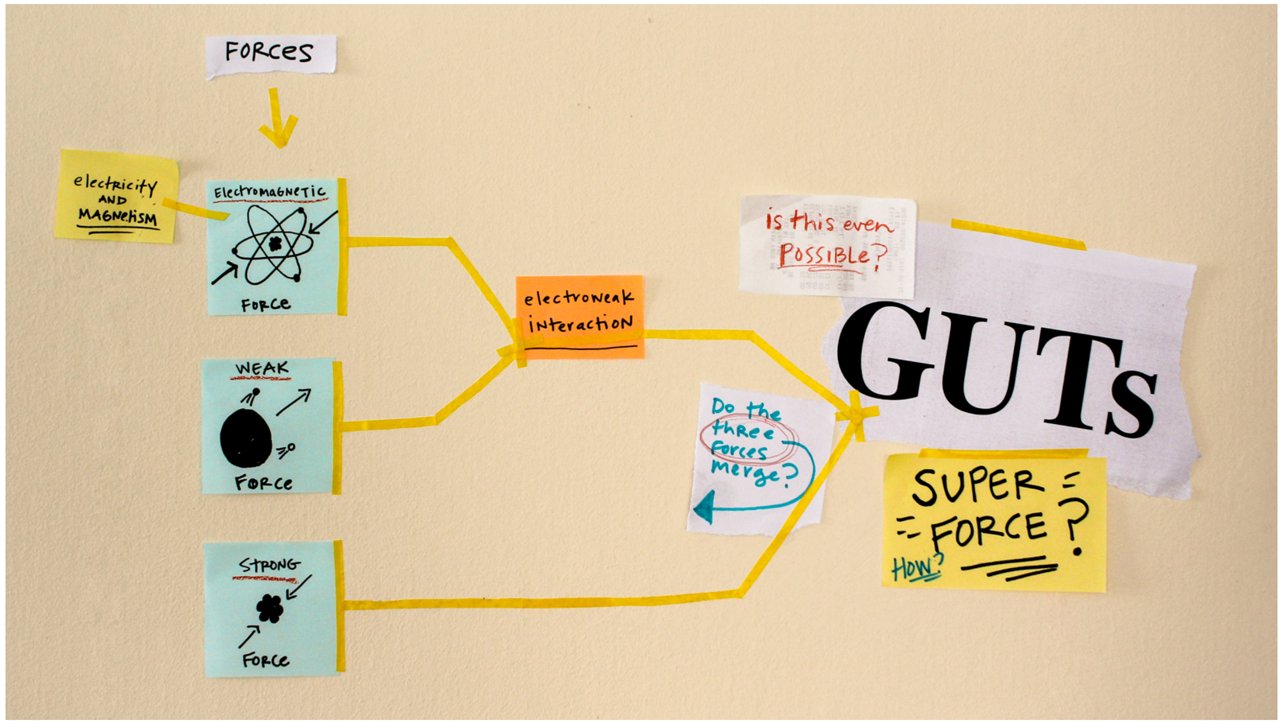
Turn to supersymmetry:

itself a "unification paradigm" of bosons and fermions

discuss successes/challenges for SUSY GUTs

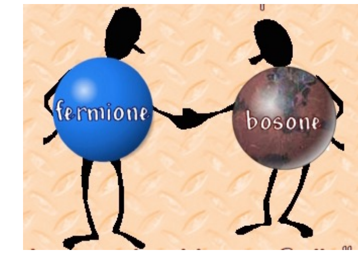
Current approaches (both with and without SUSY)

General lessons from the grand unification paradigm…

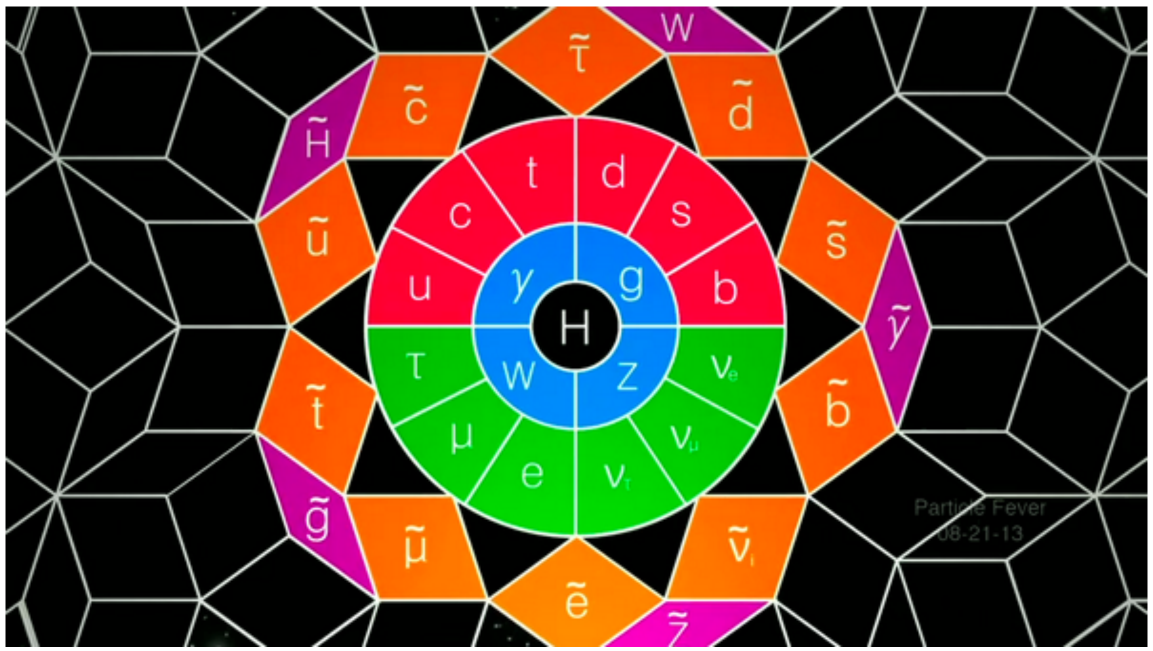


(image credit: Symmetry magazine/Sandbox Studio)

Supersymmetric Grand Unification



Supersymmetry (SUSY) → **a distinct unifying paradigm:**



bosons ↔ fermions

***N=1* SUSY**

field → **superfield**

$\Psi(x)$ $\quad\quad$ $\Psi(x,\theta,\bar{\theta})$

(space) $\quad\quad$ **(superspace)**

matter/Higgs: chiral superfields

$\Phi(x,\theta)$ $\quad$ **spin 0 + spin 1/2**

$(\Phi^\dagger(x,\bar{\theta}))$ $\quad$ **+ aux (F)**

gauge: vector superfields

$V(x,\theta,\bar{\theta})$ $\quad$ **spin 1 + spin 1/2**

+ aux (D)

chiral superfield interactions: $(W(\Phi))_F$ ← **"superpotential"** **(holomorphic)**

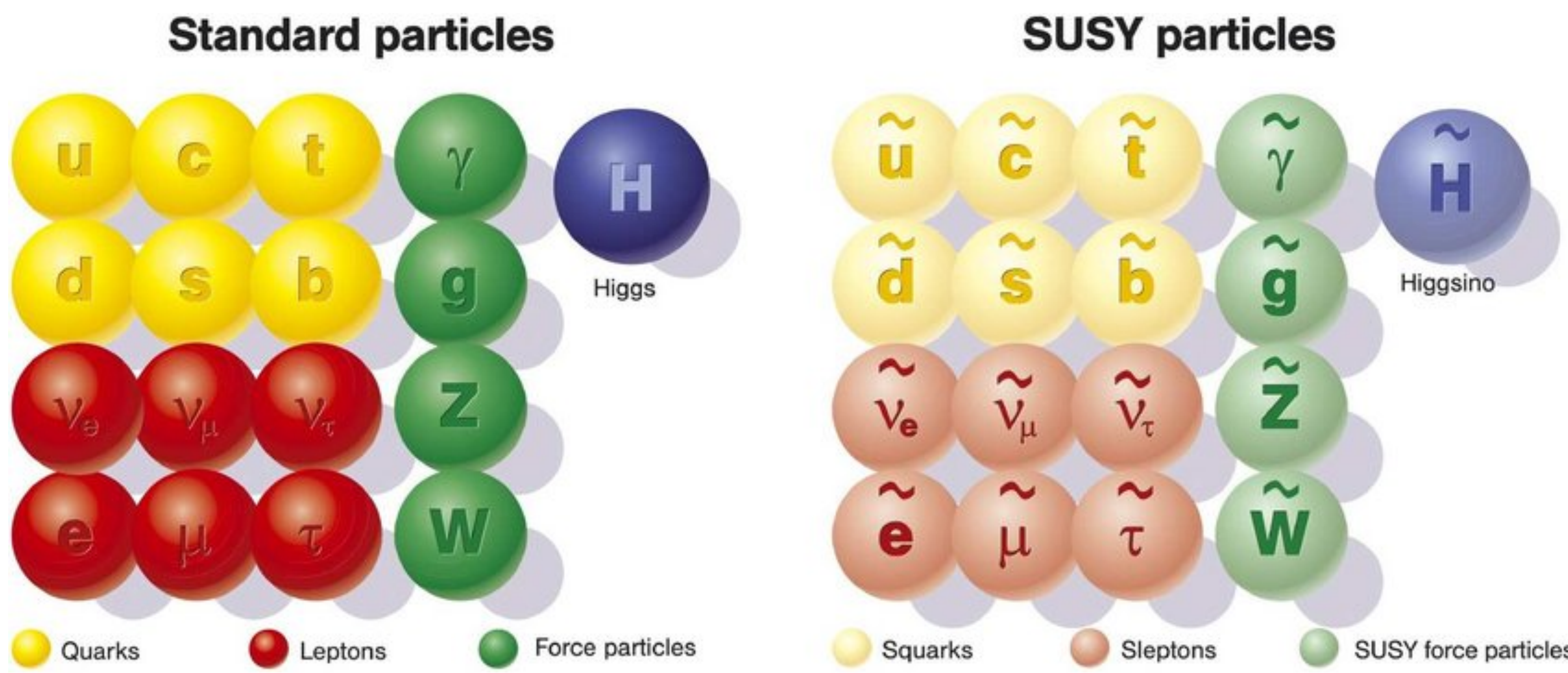
gauge-matter interactions: $(\Phi^\dagger e^{gV}\Phi)_D$

(image credits: "Particle Fever"/Mark Levinson via quantumdiaries.org,...)

Reviews: Martin ’97; Chung et al. ’03, many others…

Building (low/TeV-scale) SUSY models…

Example: minimal supersymmetric extension of SM (MSSM)



SM matter: chiral superfields $\hat{Q}, \hat{u}, \hat{d}, \hat{L}, \hat{e}$

conserved “R-parity” **(more soon!)**

Higgs sector: 2 chiral superfields $\hat{H}_u, \hat{H}_d$ **gauge anomalies, holomorphy of** W $\tan\beta = \langle H_u \rangle / \langle H_d \rangle$

Superpotential **up to cubic order in chiral superfields (renorm.)**

“mu term”: Higgsino mass parameter

$$W_{\rm MSSM} = Y_{u_{ij}} \hat{Q}_i \hat{u}_j \hat{H}_u + Y_{d_{ij}} \hat{Q}_i \hat{d}_j \hat{H}_d + Y_{e_{ij}} \hat{L}_i \hat{e}_j \hat{H}_d + \mu \hat{H}_u H_d$$

(image credit: M. Grefe/DESY)

Soft supersymmetry breaking →

parametrized by soft supersymmetry breaking Lagrangian

$$
\begin{aligned}
-\mathcal{L}_{soft} &= \frac{1}{2}\left[M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B}\right] \\
&+ \epsilon_{\alpha\beta}[-bH_d^\alpha H_u^\beta - H_u^\alpha \widetilde{Q}_i^\beta \widetilde{A}_{u_{ij}} \widetilde{U}_j^c + H_d^\alpha \widetilde{Q}_i^\beta \widetilde{A}_{d_{ij}} \widetilde{D}_j^c + H_d^\alpha \widetilde{L}_i^\beta \widetilde{A}_{e_{ij}} \widetilde{E}_j^c + \text{h.c.}] \\
&+ m_{H_d}^2 |H_d|^2 + m_{H_u}^2 |H_u|^2 + \widetilde{Q}_i^\alpha m_{Q_{ij}}^2 \widetilde{Q}_j^{\alpha *} \\
&+ \widetilde{L}_i^\alpha m_{L_{ij}}^2 \widetilde{L}_j^{\alpha *} + \widetilde{U}_i^{c*} m_{U_{ij}}^2 \widetilde{U}_j^c + \widetilde{D}_i^{c*} m_{D_{ij}}^2 \widetilde{D}_j^c + \widetilde{E}_i^{c*} m_{E_{ij}}^2 \widetilde{E}_j^c. \qquad (2.10)
\end{aligned}
$$

gaugino masses $M_{1,2,3}$

bilinear scalar coupling $b \equiv B\mu$

trilinear scalar couplings $\widetilde{A}_{\alpha_{ij}} = A_{\alpha_{ij}} Y_{\alpha_{ij}}$

scalar mass-squares $m^2_{f_{ij}}$

105 new parameters!

Need models and/or simplifying assumptions …

e.g. universality: $M_{1,2,3} = M_{1/2}$ $\quad m^2_{f_{ij}} = m_0^2 \delta_{ij}$ $\quad A_{\alpha_{ij}} = A_0 \delta_{ij}$

(minimal supergravity/mSUGRA…)

“no-scale”: $m^2_{f_{ij}} = 0 = A_{\alpha_{ij}}$

(at high scale)

but many other possibilities!

Minimal SUSY $SU(5)$

Dimopoulos, Georgi '81 Sakai '81
Dimopoulos, Raby, Wilczek '81
Ibanez, Ross '81,...

supersymmetric sector: well-defined!

✓ **chiral superfields:**

matter: $\overline{\mathbf{5}}+\mathbf{10}$ **(3 copies)**

$\hat{\Phi}_{\bar{5}}=\bar{F}$ $\hat{\Phi}_{10}=T$

Higgs: $\mathbf{24}$ $\hat{\Phi}_{24}=\Phi$

$\mathbf{5},\overline{\mathbf{5}}$ $\hat{\Phi}_{5H}=H_u$ $\hat{\Phi}_{\bar{5}H}=H_d$

✓ **vector superfields:**

gauge bosons: $\mathbf{24}$

$\hat{V}_{24}$

$$SU(5)\xrightarrow[\langle\mathbf{24}\rangle]{} SU(3)_c\times SU(2)_L\times U(1)_Y \xrightarrow[\langle\mathbf{5}\rangle,\langle\overline{\mathbf{5}}\rangle]{} SU(3)_c\times U(1)_{EM}$$

✓ **superpotential:**

$\overline{\mathbf{5}}\,\mathbf{10}\,\overline{\mathbf{5}}_H$ $\mathbf{10}\,\mathbf{10}\,\mathbf{5}_H$

Yukawa: $(Y_d)_{ij}\bar{F}_{ai}T_j^{ab}H_{db}+(Y_u)_{ij}\epsilon_{abcde}T_i^{ab}T_j^{cd}H_u^e$ **(SM fermion masses)**

Higgs/SSB: $x\,\mathrm{Tr}\,\Phi^2+y\,\mathrm{Tr}\,\Phi^3+\lambda_1(H_u^a\Phi_a^bH_{db}+MH_u^aH_{da})\ +z\,\mathrm{Tr}\,\Phi$

(constraint)

supersymmetry breaking sector:

here is where the model-dependence leaks in!

At unification scale $\sim M_X$

universality: $M_{1,2,3} = M_{1/2}$ $\quad m^2_{fij} = m^2_0 \delta_{ij}$ $\quad A_{\alpha_{ij}} = A_0 \delta_{ij}$

(minimal supergravity/mSUGRA…)

no-scale: $m^2_{f_{ij}} = 0 = A_{\alpha_{ij}}$

(no-scale supergravity)

Can instead have GUT-motivated mass-squares of SM fermion partners:

$$m^2_{d,L} = m^2_{\bar{5}} \qquad m^2_{Q,u,e} = m^2_{10}$$

and/or GUT-motivated mass-squares of Higgs fields:

$$m^2_{H_u} = m^2_{5H} \qquad m^2_{H_d} = m^2_{\bar{5}H}$$

or further parameter relations from a specific SUSY-breaking model

Note: any of these can be called "minimal" SUSY $SU(5)$

More SUSY GUTs…

We can of course use the same "recipe," more generally

- ✓ choose gauge group G
- ✓ assign SM chiral superfields to irreps of G (usually same as non-SUSY case)
- ✓ do the same for Higgs chiral superfields (must ensure anomaly cancellation)
- ✓ fix (parametrize, assume) form of soft SUSY breaking sector
- ✓ can include other ingredients (e.g. family/horizontal symmetries,…)

Many examples!

Most-studied examples:

- ✓ SUSY *SO(10)* (many varieties)
- ✓ SUSY flipped SU(5)
- ✓ SUSY Pati-Salam,…

Many references!!

Advantages of SUSY GUTs

Back to minimal SU(5), examine SSB:

$$SU(5) \xrightarrow[\langle \mathbf{24} \rangle]{} SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow[\langle \mathbf{5} \rangle, \langle \overline{\mathbf{5}} \rangle]{} SU(3)_c \times U(1)_{EM}$$

SUSY scalar potential: "F terms" $F_\Phi \equiv \dfrac{\partial W(\hat{\Phi})}{\partial \Phi}$

SUSY-preserving vacuum → $\langle F_\Phi \rangle = 0$

$$(F_\Phi)^a_b = z\delta^a_b + 2x\Phi^a_b + 3y\Phi^a_c\Phi^c_b \qquad \mathrm{Tr}\,\Phi = 0 \longrightarrow z = -(3/5)y\,\mathrm{Tr}\,\Phi^2$$

$\langle \mathbf{24} \rangle$ **preserve SM subgroup:** $(1/2)\mathrm{Diag}(2a, 2a, 2a, -3a, -3a) \longrightarrow a = (4x/3y) \sim M_X$

Get the MSSM below the GUT scale:

unpacking the $5, \overline{5}$ **Higgs** $\hat{H}_u = (\xi_u, H_u)^T \qquad \hat{H}_d = (\bar{\xi}_d, H_d)^T$

interactions with Φ → $\lambda_1(a + M)\xi_u\bar{\xi}_d + \lambda_1(-3a/2 + M)H_uH_d$

→ **doublet-triplet Splitting problem again!**

Triplets need to be heavy, doublets need to be light →

Fine-tune: $M - 3a/2 = \mu$ $\mu \ll M_X$

triplet Higgs masses $\simeq M_X$ doublet Higgs mass parameter: μ

Doublet-Triplet Splitting Problem of GUTs: quite generic

arises whenever need large mass splittings of fields within single GUT multiplet

fine-tuning not optimal (better solutions, e.g. “sliding singlet”, “missing partner”***)

but a still great advantage in SUSY compared to non-SUSY case:

non-renormalization theorem of SUSY superpotential

tuning at tree level maintained with radiative corrections!

well-known property of SUSY: mitigation of the gauge hierarchy problem!

(**return to this later!)

Another famous SUSY advantage: **gauge coupling unification in MSSM**

$G \longrightarrow \text{MSSM}$ **at** $Q \sim M_X$ **"desert"**

$b_i^{\text{SM}} = (41/10, -19/6, -7)$
$b_i^{\text{MSSM}} = (33/5, 1, -3)$

$$\frac{1}{\alpha_i(m_Z)} = \frac{1}{\alpha_G(m_X)} + \frac{b_i}{2\pi} \ln\left(\frac{M_X}{m_Z}\right) + \delta_i$$

thresholds

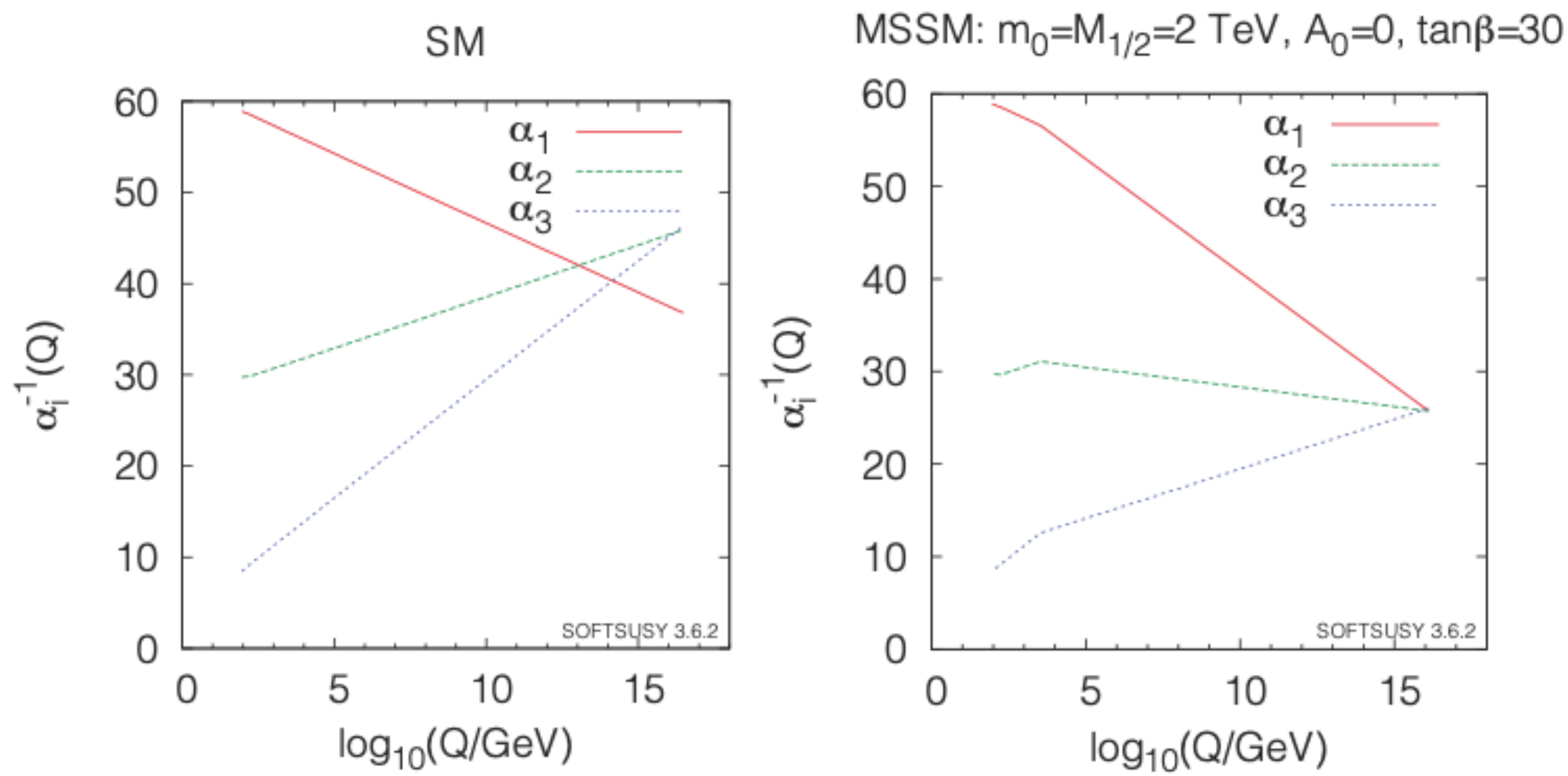


Figure from PDG 2022 GUT review (Hebecker and Hisano; plots by Allanach using SOFTSUSY)

(two-loop RGE; universality assumption for soft SUSY masses)

More details about this famous story: Amaldi et al. '91; Ellis et al. '91; Langacker, Luo '91;...

Inputs: $1/\alpha(m_Z)$ $\sin^2\theta_W(m_Z)$ **(exceedingly well-measured!)**

extrapolate to unified value for $1/\alpha_{1,2}(M_X)$ →

gives unification scale and unified GUT gauge coupling

Then run down to electroweak scale and predict $1/\alpha_3(m_Z)$

An amazing result:

LO (without thresholds) analysis in great agreement with experiment!

$$1/\alpha_{\rm GUT}(M_X) \simeq 24 \qquad M_X \simeq 2\times 10^{16}\ {\rm GeV}$$

More accurately:

2-loop + thresholds (SUSY and high-scale GUT) →

depends both on details of SUSY partner masses

and masses of superheavy states (color triplet Higgs, etc.)

Here constraints from baryon number violation are of great importance!

and the constraints are of a new type than in non-SUSY GUTs

B and *L* violation in supersymmetric grand unification

✓ **dimension 4 (!)** Weinberg '82; Sakai, Yanagida '82,…

Recall in the minimal SUSY $SU(5)$ superpotential (cubic terms)

$\overline{\mathbf{5}}\,\mathbf{10}\,\overline{\mathbf{5}}_H$ (down quark + lepton masses)

$\mathbf{10}\,\mathbf{10}\,\mathbf{5}_H$ (up quark masses)

but gauge invariance also allows $\overline{\mathbf{5}}\,\mathbf{10}\,\overline{\mathbf{5}}$ → ***B*, *L* violation**

A problem in supersymmetrizing the SM even without a GUT:

usual terms:** $Q\,u\,H_u$ $Q\,d\,H_d$ $L\,e\,H_d$

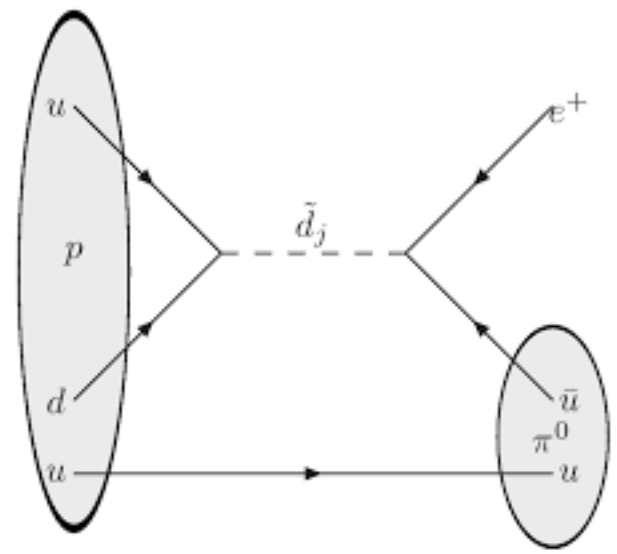
but also allowed:** $u\,d\,d$ $Q\,L\,d$ $L\,L\,e$ ***B*, *L* violation!**

(reason: L, H_d have same quantum numbers!)

(note: family indices suppressed)**

With both

$udd \qquad QLd$ → **rapid proton decay!**



(image credit: Allanach '16 via researchgate.net)

Way out:

forbid some (or all) of these terms via symmetries

(primary constraints from first generation)

Safest (and most common):

R-parity:

quark, lepton superfields: odd
Higgs superfields: even

Farrar, Fayet '78

or matter parity

Dimopoulos, Raby, Wilczek '82; Ellis et al. '82

Note: in some SUSY GUTs

effective R-parity follows from GUT irrep assignment

e.g. $SO(10)$ → **matter fields:** $\mathbf{16}$

Higgs fields: $\mathbf{10}_H, \overline{\mathbf{10}}_H$

✓ dimension 5

Weinberg ’82; Sakai, Yanagida ’82,…

minimal SUSY $SU(5)$ **non-renormalizable operators**

$\mathbf{10}\ \mathbf{10}\ \mathbf{10}\ \bar{\mathbf{5}}$ → $QQQL \qquad uude$

generated by color triplet Higgs superfields at $\sim M_X$

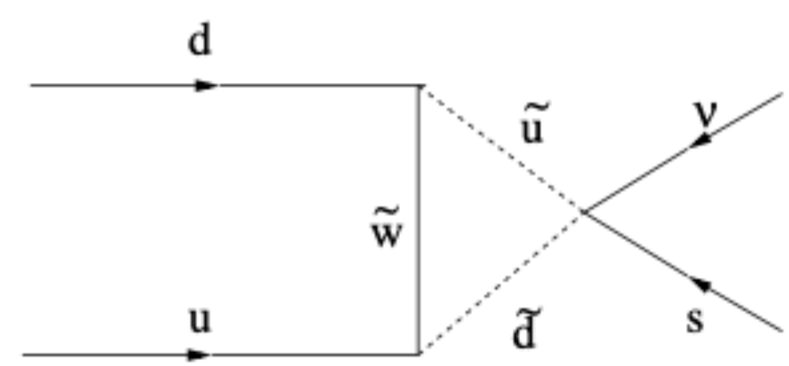
from SUSY F-terms: include scalars (squarks/sleptons)

“dressed” by exchange of MSSM gauginos or Higgsinos

suppressions: light generation Yukawas, loop factor

Generation Structure:

figure from Murayama, Pierce ’01



$Q_i Q_j Q_k L_\ell \qquad u_i u_j d_k e_\ell$

(i) **(ii)**

overall symmetric (bosonic superfields)

antisymmetric index contraction: $SU(3)_c, SU(2)_L$

→ **(i) antisymmetric in** i, j, k → **2nd or 3rd gen in proton decay operator!**

(ii) antisymmetric in i, j

$p \to K^+ \bar{\nu}$

Extremely strong constraint on SUSY GUT models:

$$\tau_{p\to K^+\bar{\nu}} \sim M_T^2 m_{\rm SUSY}^2/m_p^5$$

proton decay bounds on GUT-scale color triplet mass

affect size of GUT thresholds in gauge coupling unification

But dependent on details of the masses and mixings of the sfermions —

i.e. the SUSY breaking sector (least well understood)

e.g. minimal SUSY *SU(5)* — **Murayama, Pierce '01**

degenerate masses $m_{\rm SUSY} = 1$ TeV

irreconcilable bounds from gauge coupling unification and proton decay

→ **minimal SUSY *SU(5)* ruled out**

But can survive for tuned sfermion mass/mixing patterns — **Bajc et al. '02,…**

Non-minimal models:

suppress via accidental symmetries, heavy sfermions, missing partner mech…

many references!

Current status and modern approaches

GUT models in light of present experimental constraints

SuperK bounds (2020)

Kearns SSI 2022

e.g. $\tau/B(p \to e^+\pi^0) > 2.4 \times 10^{34}$ yr

LHC bounds, DM bounds,…

Continued work on both SUSY and non-SUSY GUTs!

T. Ohlsson's talk at Neutrino 2022

general proton lifetime upper bound (d = 6, superheavy vector-mediated):

Dorsner, Fileviez Perez '05
Nath, Fileviez Perez '07

$$\tau_p < 6.0 \times 10^{39} \frac{1}{\alpha_{\rm GUT}^2} \left(\frac{M_X}{10^{16}\ {\rm GeV}}\right)^4 \left(\frac{0.003\ {\rm GeV}^3}{\alpha_{\rm ChPT}}\right)^2 \ {\rm yr}$$

non-SUSY: generally $\tau_p < \mathcal{O}(10^{36})$ yr

non-SUSY GUTs still can be viable (non-minimal implementations)

non-SUSY GUTs:

Many recent examples: $SU(5)$ $SO(10)$

with extra ingredients/symm

e.g. couplings to axions,…

more flexibility: multi-stage SSB

unification can be readily achieved

some "minimal" cases remain

many references!

T. Ohlsson's talk at Neutrino 2022

One example: $SO(10) \longrightarrow SU(4) \times SU(2) \times SU(2)$

Babu, Bajc, Saad '16

economical (non-minimal) Higgs sector

full fit to entire fermion spectrum

can obtain $M_X \sim \text{few} \times 10^{15-16}\ \text{GeV}$

Another: $SO(10) \longrightarrow SU(5) \times U(1)_{\text{PQ}}$ (flipped)

Ohlsson et al. '20

can achieve unification, $M_X \sim 7 \times 10^{15}\ \text{GeV}$

And another:

Lee, Mohapatra '16

$SU(5) \times SU(5)$

successful unification,

intermediate-scale vectorlike states

many interesting directions for exploration…

SUSY GUTs:

LHC era: heavier superpartners, alleviate dim 5 proton decay bounds

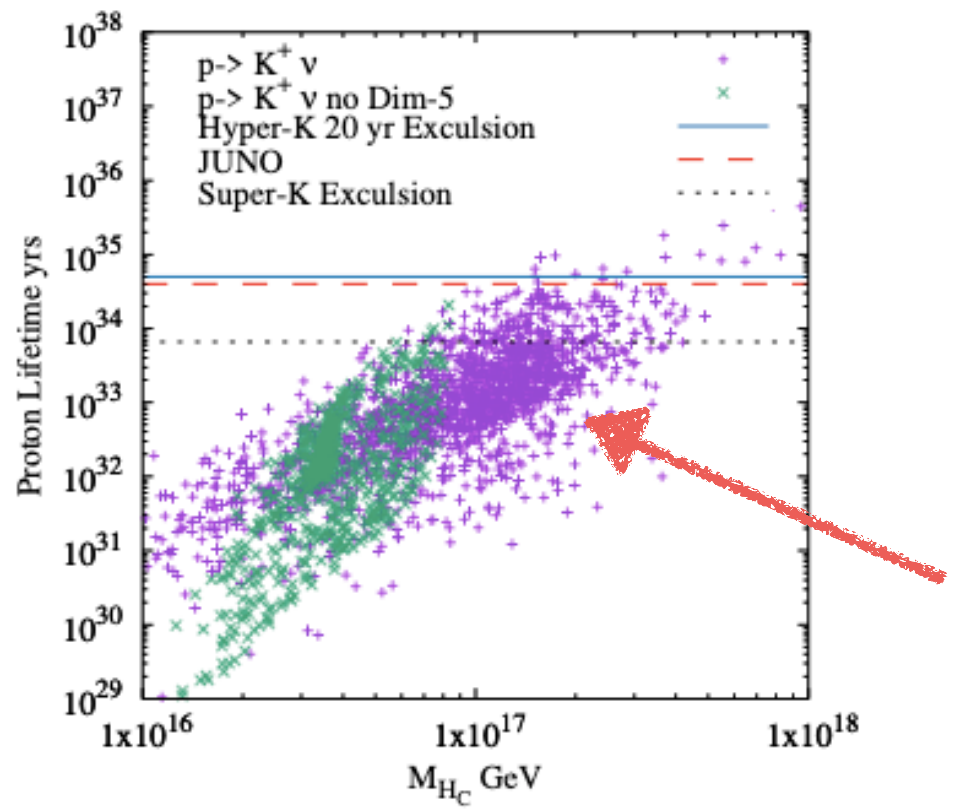
many references!

✓ **new "minimal" versions of SUSY** $SU(5)$

include Planck-suppressed NR operators, superpartners ~ tens of TeV range

correct generation of lighter fermion masses — **Babu et al. '20**

CMSSM version — **Evans, Yanagida '21**



still viable, general limits of $\tau_{p\to K^+\bar{\nu}} < \mathcal{O}(10^{35})$ yr color triplet Higgs "paired" with additional state

can have different story in flipped/"missing partner" $SU(5)$

dimension 5 ops suppressed, dim 6 dominate

Many recent analyses:

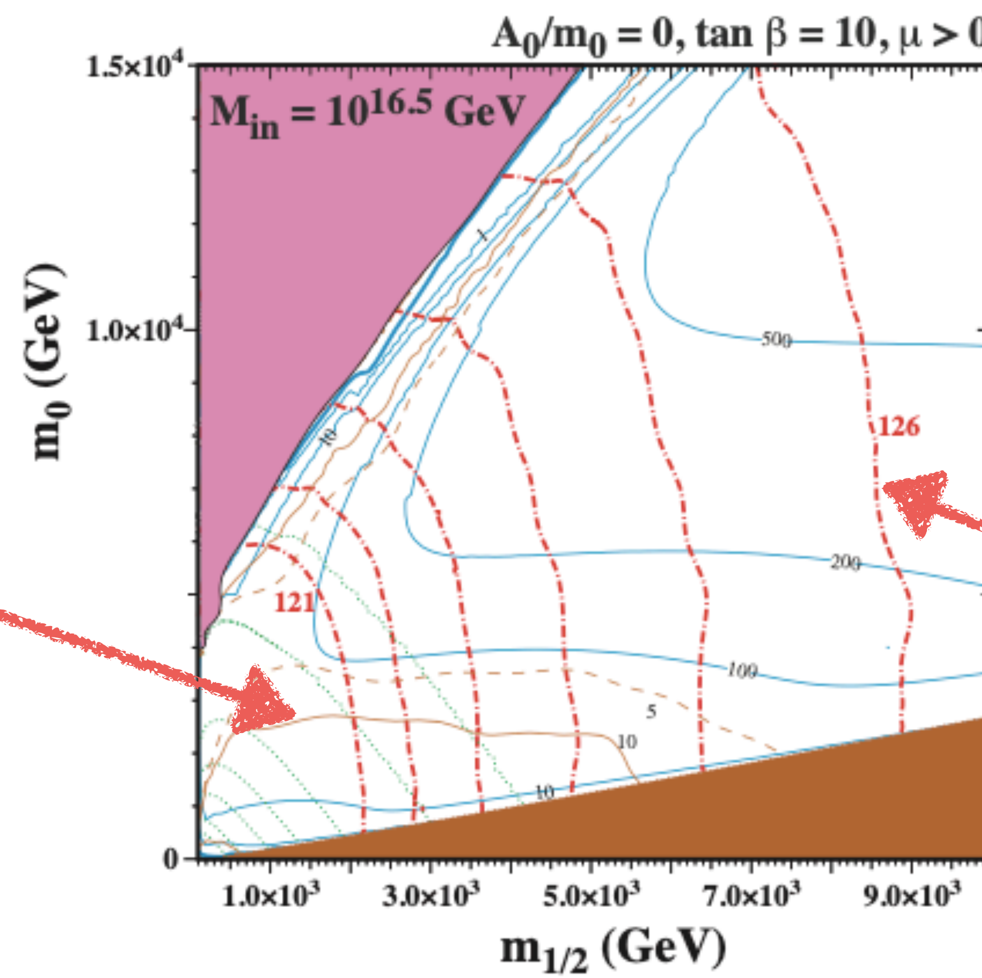
Ellis et al. '20,'21,...

$\tau_p \sim 10^{35-37}$ yr

no-scale flipped SU(5)

$\tau_{p \to e^+\pi^0} \sim 10^{36}$ yr

figure from Ellis et. al '21



Higgs mass

✓ **SUSY $SO(10)$, Pati-Salam**

many possibilities:

One example:

$SO(10)$ Yukawa textures

Mohapatra, Severson '18

Another: Pati-Salam

Yukawa unification — Poh, Raby, Wang '17

axion DM, RPV — Kawamura, Raby '20

a rich framework for continued explorations!

Making the proton stable…

Early ideas: "kinematic blocking" (nontrivial Higgs sector needed) — see e.g. Langacker '81

combined gauge + global symmetries

Explore splitting within GUT multiplets:

"missing partner" mechanism for doublet-triplet splitting problem: — Grinstein '82, Masiero et al. '82

$SU(5)$ $\mathbf{5}, \bar{\mathbf{5}}$ Higgs $\hat{H}_u = (\xi_u, H_u)^T$ $\hat{H}_d = (\bar{\xi}_d, H_d)^T$

new multiplets: "partners" of triplets but not doublets: $\mathbf{50}, \overline{\mathbf{50}}$

couple them to $5, \bar{5}$ via other new field(s) $\mathbf{75}$ $\mathbf{75}'$ (also forbid bare mass terms) (acquire vevs)

Non-minimal GUT models: → identification of SM fermions — Barr '14,…

Example with stable proton: $SU(5)$ — Fornal, Grinstein '17

only leptons in the $\bar{5} + 10$ quarks embedded in new vectorlike pairs

ex. $\bar{\mathbf{5}} = (D^c_5, L)$ $\overline{\mathbf{50}} = (d^c, \ldots)$

$\mathbf{50}, \overline{\mathbf{50}}$ (two copies)

no tree-level proton decay!

exploring similar splitting ideas in $SU(7)$ — Popov, LE '21,…

Connections with higher dimensions, string theory

(another "unification paradigm" to incorporate gravity)

✓ **orbifold GUTs:**

Kawamura '99
Altarelli, Feruglio '01
Hall, Nomura '01
...

SUSY GUT in 5 or 6 dimensions, compactified on orbifold

Orbifold → breaks symmetries
projects out unwanted states/couplings

bulk gauge group splits into "local groups" at different orbifold fixed points

provides rationale for split vs. unsplit multiplets in 4d (bulk vs. localized)

Features:

- gauge coupling unification due to bulk physics
- doublet-triplet splitting
- suppress d=5 proton decay (project out color triplet Higgs)

✓ **string GUTs:** explicit "top-down" constructions — Many references!

heterotic string: $E_8 \times E_8, SO(32)$ compactified on Calabi-Yau manifolds

→ SUSY GUTs/MSSM — e.g. Aldabazal et al. '94,… Anderson et al. '11,…

SUSY flipped $SU(5)$ — Antoniadis et al. '87,…

(breaking by lower-dimensional representations)

gauge coupling unification at string scale $\sim 10^{17}$ GeV

even without GUT group in 4d — Dienes '96 (review)

(famous GUT/string scale mismatch)

Type IIB/braneworlds:

gauge group from brane stacks — e.g. Blumenhagen et al. '07,…

→ SUSY GUTs/MSSM

F theory realizations — Beasley, Heckman, Vafa '08… Cvetic et al. '19,…

recent (non-technical) review: Cvetic et al. '22 (Snowmass)

General Lessons from Grand Unification Paradigm

✓ **Violation of baryon number, lepton number**

global symmetries in the SM (renormalizable level)

→ **violated by nonrenormalizable operators**

Weinberg ’79
Wilczek, Zee ’79
Weinberg ’80

Enumerate such operators:

operator	dimension	*B*	*L*	
$\ell\ell hh$	5	0	2	← Weinberg operator (neutrino mass)
$qqq\ell$	6	1	1	← usual proton decay ops; cutoff: $\sim 10^{15}$ GeV
$qqq\ell^c H$ $qqq\ell^c D$	7	1	-1	
$qqqqqq$	9	2	0	
$qqq\ell^c\ell^c\ell^c H$	10	1	-3	
$qqq\ell\ell\ell HH$	11	1	3	

GUT models: such violations generic →

quarks + leptons, quarks + antiquarks in same GUT multiplet

specific operators: depend on GUT embeddings

General conclusion:

***B*, *L* violation general extension of unification paradigm**
important for experiments to continue to push this frontier!

Example: $\Delta B = 2 \quad \Delta L = 0 \qquad qqqqqq$

→ **neutron — antineutron oscillations**

Kearns SSI 2022

$\Delta m = \langle \bar{n} | \mathcal{H}_{\rm eff} | n \rangle \qquad P_{n \to \bar{n}}(t) = \sin^2(|\Delta m| t)$

simplified models: no $\Delta B = 1 \quad \Delta L = 1$

(color-sextet scalars)

Arnold, Fornal, Wise '12,…

recent (non-technical) reviews: **Fileviez Perez et al. '22 (Snowmass)**

Dev et al. '22 (Snowmass)

✓ **Implications for the flavor puzzle of the SM** →

origin of quark and lepton masses and mixings

GUT paradigm:

intriguing framework in which to address this difficult issue

(Yukawa unification, intricacies of GUT Higgs sector,…)

true even before the discovery of neutrino oscillations in 1998!

Now precision-level measurements in both quark and lepton sectors:

quark (CKM) and lepton (PMNS) mixings very different!

"challenge" for GUT paradigm?

But GUTs (Pati-Salam, *SO(10)*,…) → **right-handed neutrinos**

neutrino seesaw (Type I) at/near unification scale

Theme of quark-lepton unification:

perhaps it will showcase itself in the neutrino sector!

Conclusions

Aim here was to showcase ideas from the grand unification paradigm

and to recall/appreciate/contemplate their origins as the SM itself developed

As GUTs approach their 50th birthday →

GUTs are alive and well, and the framework remains compelling

some minimal/elegant implementations ruled out:

Experiments probing *B* violation will continue to probe this important paradigm

Conclude with a quote from Langacker's '81 review:

"…Grand unified theories have many attractive features and dramatic consequences. Even if it turns out that they are wrong in detail there is a good chance that they are directing our attention in the right direction."

Backup

SM fermion mixing

PDG 2022

$$|V_{\rm CKM}| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix},$$

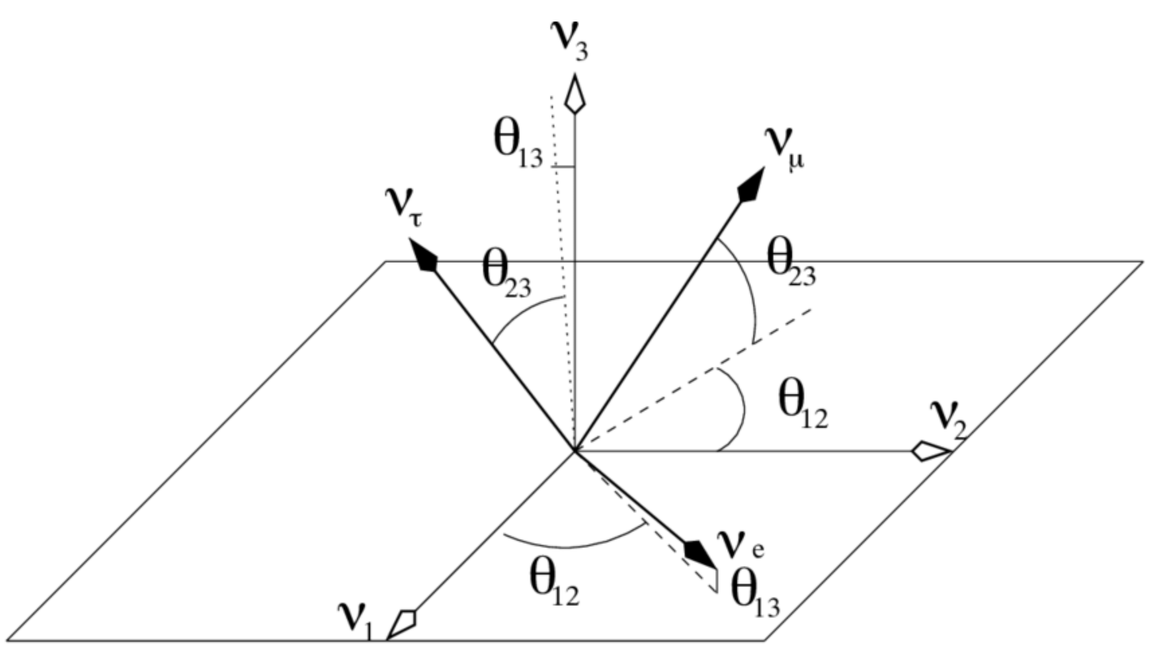
$$V_{\rm CKM} = \mathcal{R}_1(\theta^q_{23})\mathcal{R}_2(\theta^q_{13}, \delta_{\rm CKM})\mathcal{R}_3(\theta^q_{12})$$

NuFIT 5.1 (2021)

$$|U|^{\text{w/o SK-atm}}_{3\sigma} = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.143 \to 0.156 \\ 0.232 \to 0.507 & 0.459 \to 0.694 & 0.629 \to 0.779 \\ 0.260 \to 0.526 & 0.470 \to 0.702 & 0.609 \to 0.763 \end{pmatrix}$$

$$|U|^{\text{with SK-atm}}_{3\sigma} = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.144 \to 0.156 \\ 0.244 \to 0.499 & 0.505 \to 0.693 & 0.631 \to 0.768 \\ 0.272 \to 0.518 & 0.471 \to 0.669 & 0.623 \to 0.761 \end{pmatrix}$$

$$U_{\rm PMNS} = \mathcal{R}_1(\theta_{23})\mathcal{R}_2(\theta_{13}, \delta)\mathcal{R}_3(\theta_{12})\mathcal{P}$$



2 large PMNS angles: $\theta_{23},\ \theta_{12}$