

# Fermion Generations Puzzle: Lecture 1

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**AKA the still answering Rabi's question  
lectures almost a century later in response to  
the discovery of the muon: "Who ordered  
that?"**



# I love muons



I think we should make a collider out of them  
I still have no idea why they exist...

To order ***THE*** fashion statement of Snowmass or perhaps all of HEP do it by **Friday**:

<https://www.customink.com/g/srs0-00cm-kvb4/>

**WHO ORDERED THAT?**



**ANYONE? ANYONE?**



# Rough goals of these lectures

- Lecture 1 (Me): Introduce flavor/ways of thinking about it from theory perspective
- Lecture 2 (Mat): Experimental POV/much more detailed overview of flavor
- Lecture 3 (Me): Models of flavor/models of flavorful physics



**FLAVOR IS ONE OF THE  
GREATEST PUZZLES IN THE UNIVERSE**



**YOU'RE GOING TO GIVE  
US SOME INSIGHT RIGHT?**



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**YOU'RE GOING TO GIVE  
US SOME INSIGHT RIGHT?**

# What is flavor?

- Not only do we have the muon which is the heavier copy of the electron, we also have the tau (3 copies of the same particle only differing in mass, different “flavors”)
- That would be weird enough as is, but ok...
- Where it really gets weird is that we have apparently 3 copies of everything in the universe for no apparent\* reason
  - up, charm, top quarks
  - down, strange, bottom quarks
  - electron, muon, tau
  - electron neutrino, muon neutrino, tau neutrino

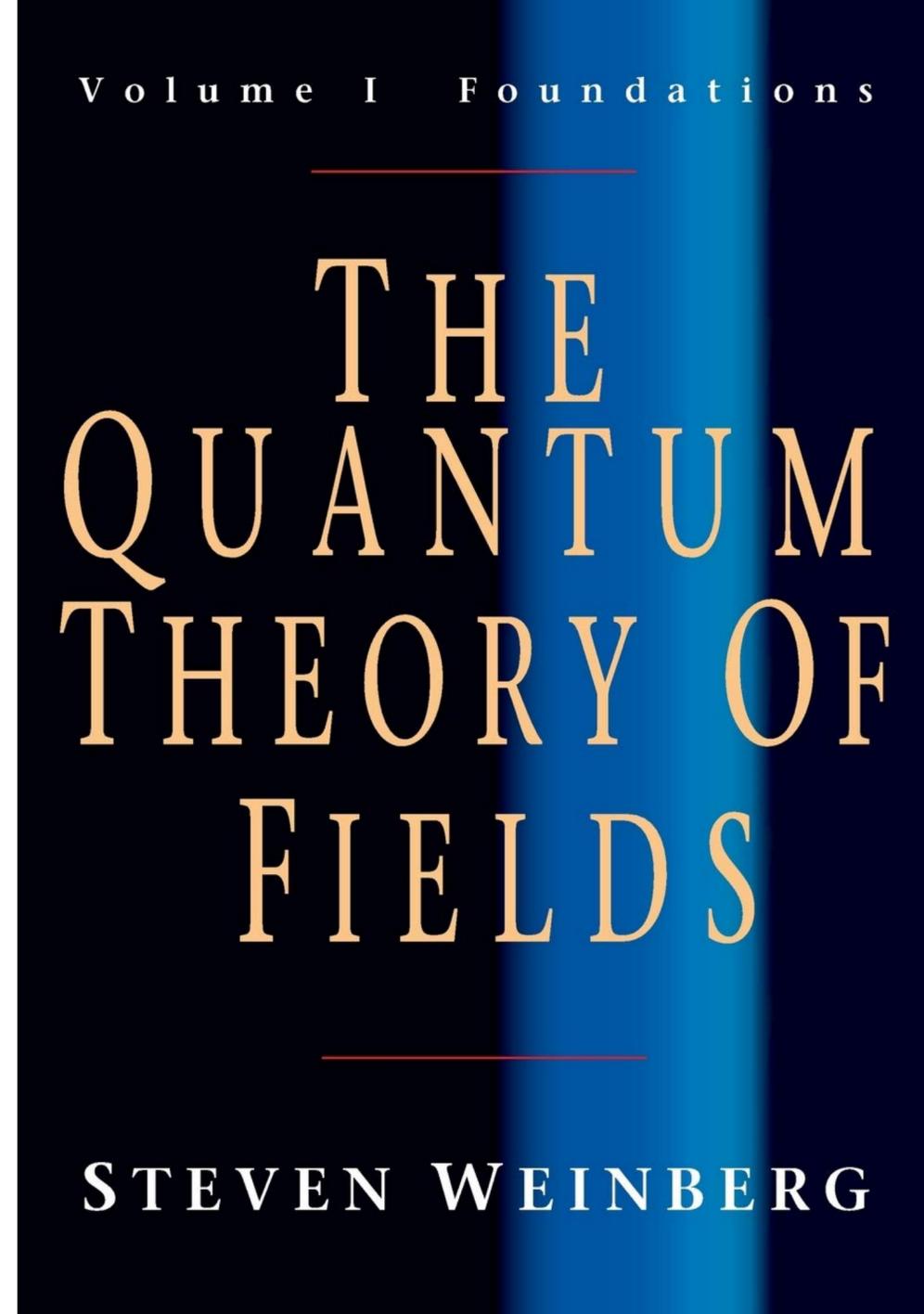
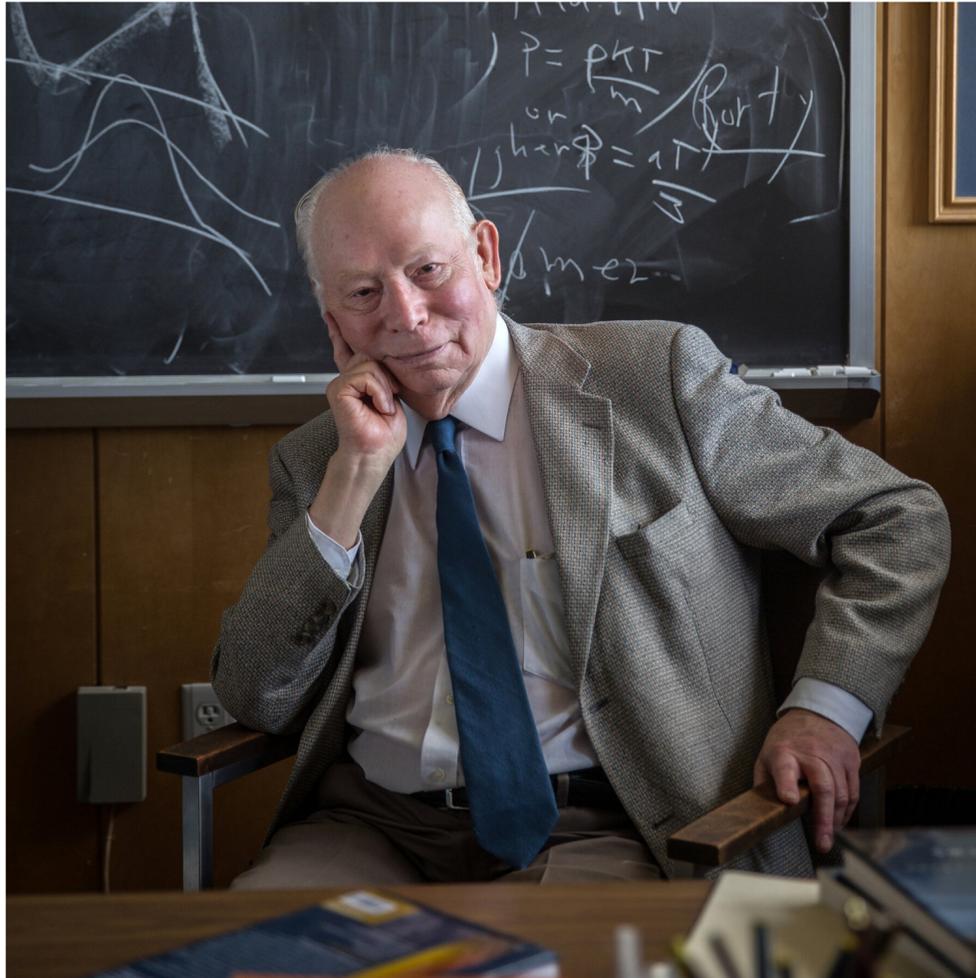
**Since everything is copied we call these generations or families**

**Flavor:  
Most bizarre thing ever  
or mundane?**

**Depends on your perspective**

**What's the framework for particle physics?**

# Quantum Field Theory



**QM+SR=only game in town**

**All particles are representations of spacetime  
symmetry + internal symmetries**

**Alternatively put, you give me the representations  
I can give you the most general Lagrangian**

**A single Dirac fermion  $\psi$**

$$\mathcal{L} = i\bar{\psi}\not{\partial}\psi$$

**N Dirac fermions  $\psi_i$  w/slight abuse of notation**

$$\mathcal{L} = i\bar{\psi}\not{\partial}\psi$$

**More fermions = more “flavors”**

**From classical POV only change is the symmetry structure**

# Symmetries of N Dirac fermions

$$\mathcal{L} = i\bar{\psi}\not{\partial}\psi$$

Clearly there exists a  $SU(N) \times U(1)$  symmetry rotating amongst flavors: “flavor symmetry” + fermion number

$$\psi \rightarrow U\psi$$

At this level there are also chiral flavor symmetries

$$\mathcal{L} = i\bar{\psi}_L\not{\partial}\psi_L + i\bar{\psi}_R\not{\partial}\psi_R$$

$$\psi_L \rightarrow U_L\psi_L$$

$$\psi_R \rightarrow U_R\psi_R$$

**Global Chiral Flavor Symmetries**  
 $SU(N)_L \times SU(N)_R \times U(1) \times U(1)$

“Flavors” and Flavor Symmetry are *ubiquitous* from QFT perspective... so what’s the big deal about flavor physics?



In QFT kinetic terms always have the most symmetry possible - every “interaction” only potentially reduces it

$$\mathcal{L}_{mass} = -M\bar{\psi}\psi$$

Generic matrix

$$\mathcal{L}_{mass} = -M\bar{\psi}\psi \rightarrow -M\bar{\psi}_L\psi_R$$

Inherently breaks global chiral flavor symmetries, but to what?

Could be  $SU(N) \times U(1)$  could be  $U(1)^N$  or could be something in between, it *depends* on the actual masses!

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$$\mathcal{L}_{mass} = - M \bar{\psi} \psi$$

Generic matrix

Can use chiral flavor rotation to diagonalize mass matrix

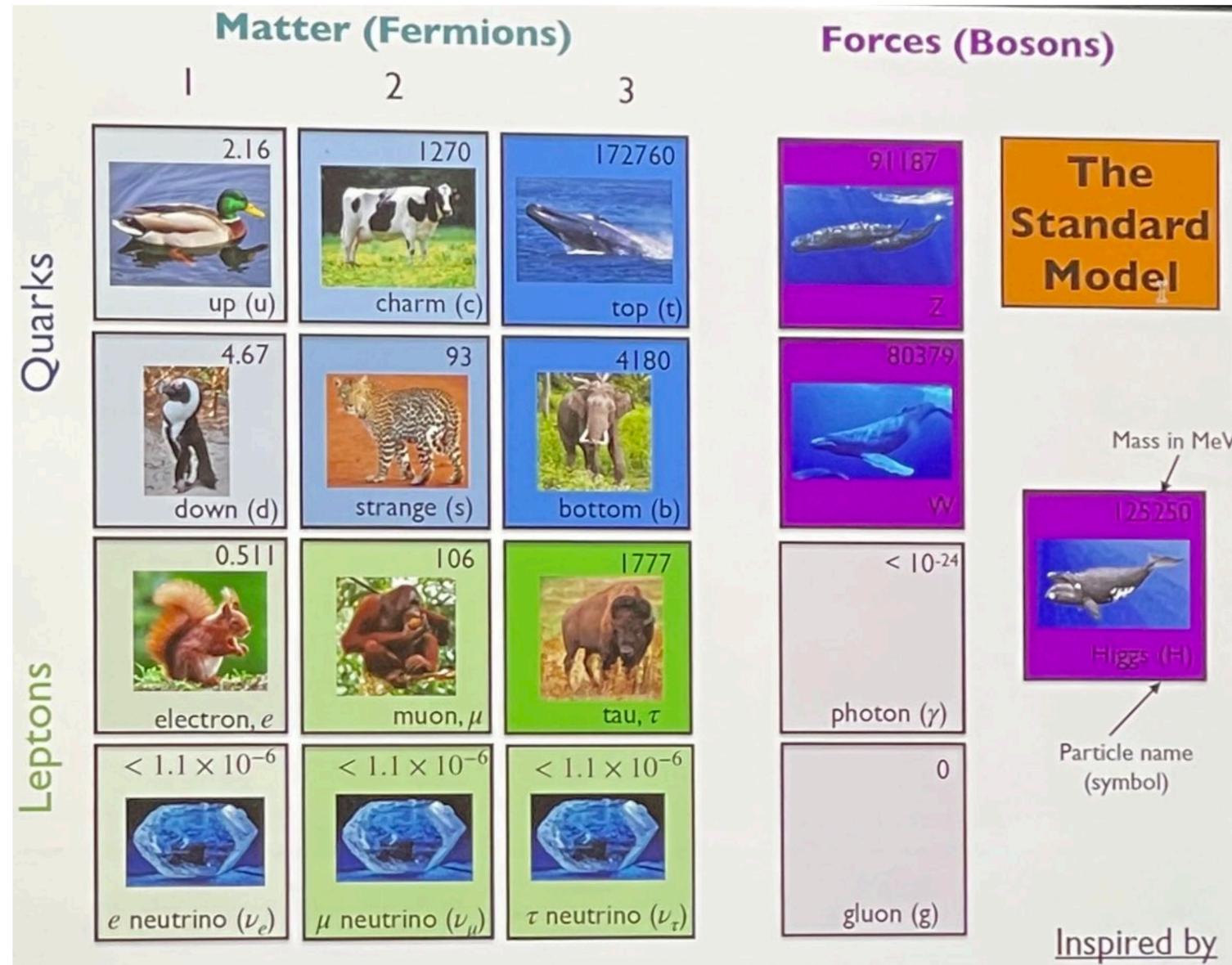
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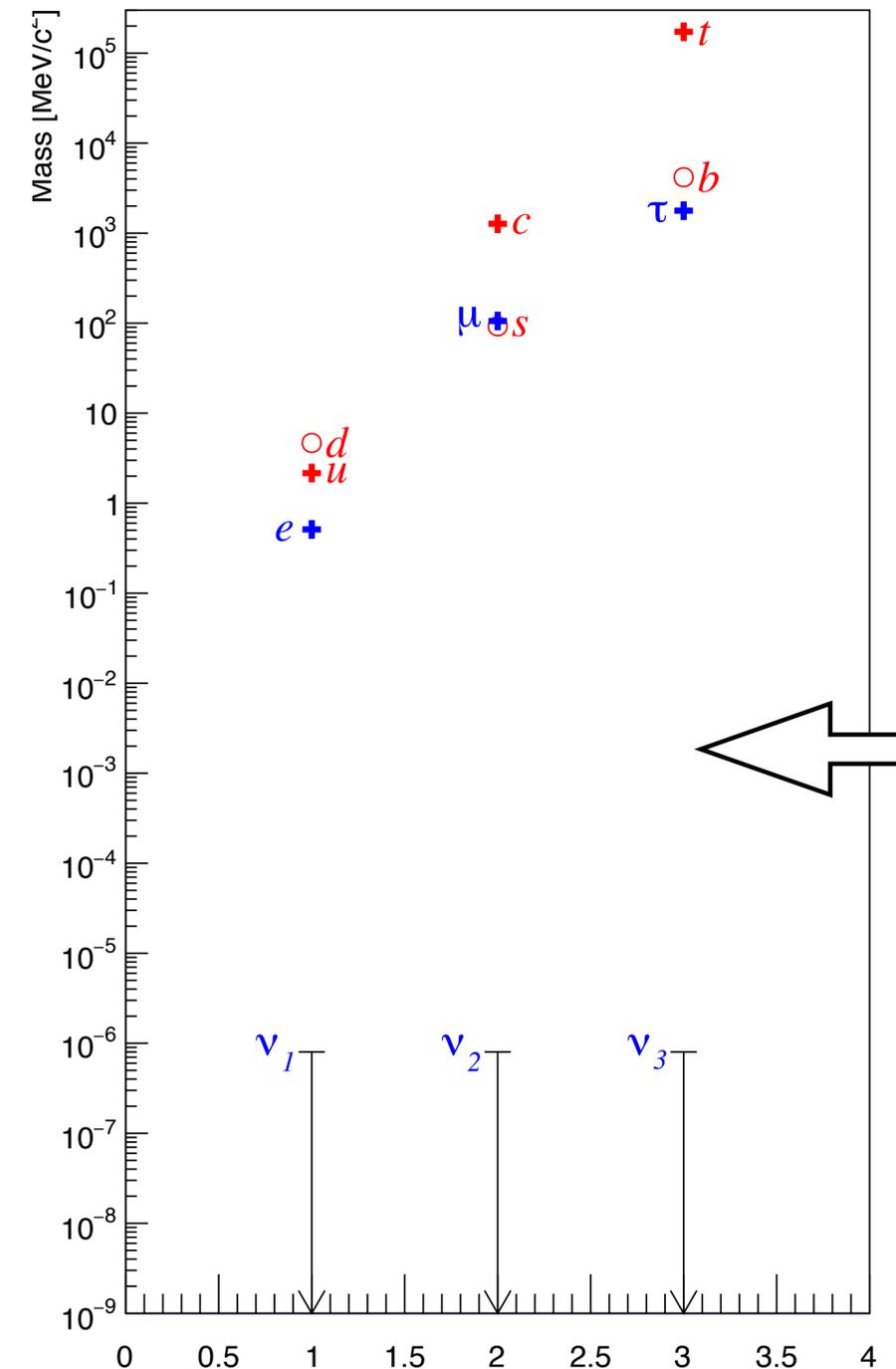
$$U_L^\dagger M U_R$$

Could be  $SU(N) \times U(1)$  could be  $U(1)^N$  or could be something in between, it *depends* on the actual masses!

# Standard Model appears to have the least residual flavor symmetry possible!

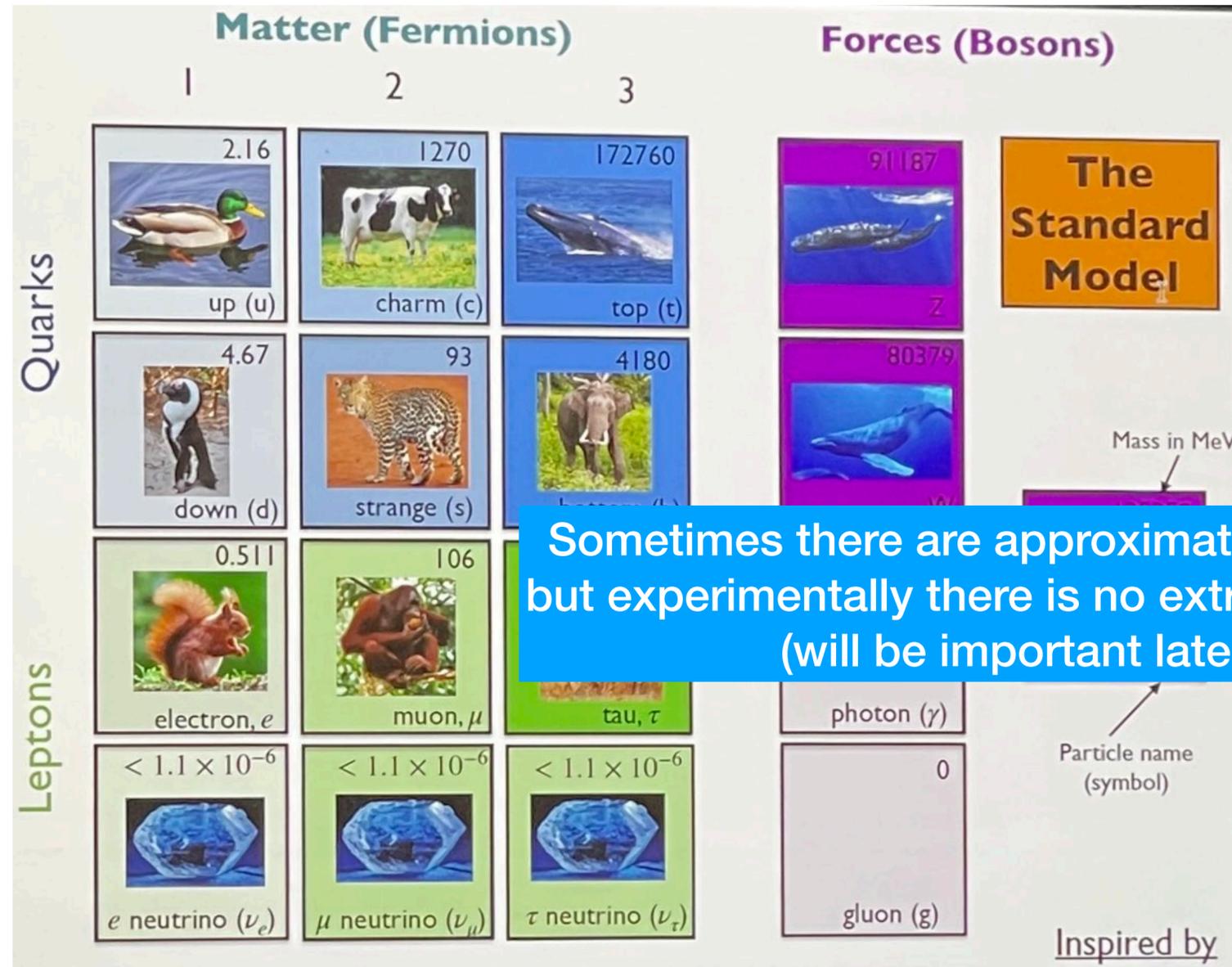


From Heather Gray

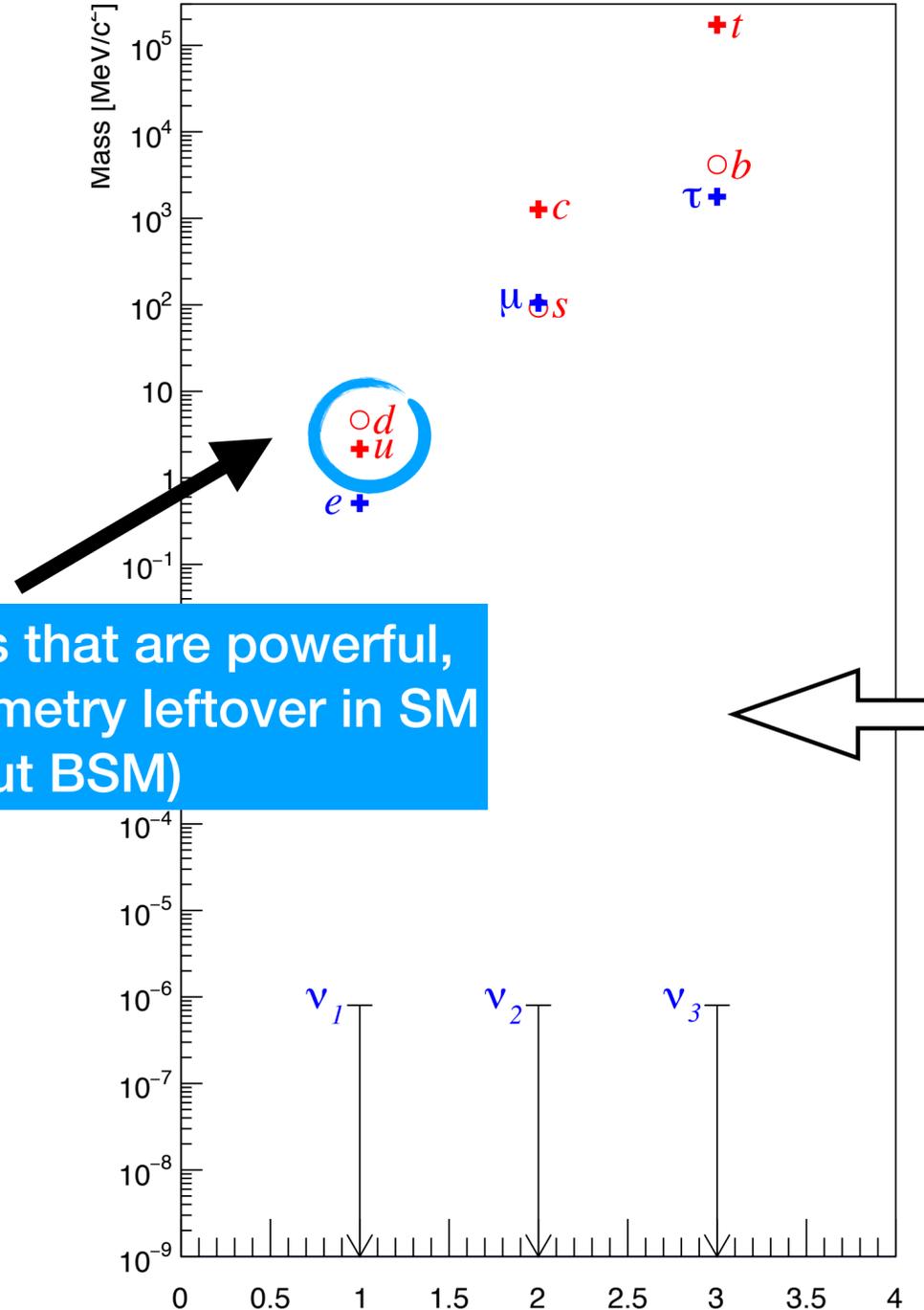


From Mat Charles lecture

# Standard Model appears to have the least residual flavor symmetry possible!



Sometimes there are approximate symmetries that are powerful, but experimentally there is no extra flavor symmetry leftover in SM (will be important later talking about BSM)



From Heather Gray

From Mat Charles lecture

# Where are we now?

- Flavor symmetries are everywhere in QFT, no big deal to add more particles...
- The \*SM\* apparently has no extra flavor symmetry even though it could have
- What about the CKM matrix/flavor mixing and gauge interactions, isn't that what we normally talk about with Flavor?
- Why 3 instead of N?

# Where are we now?

- Flavor symmetries are everywhere in QFT, no big deal to add more particles...
- The \*SM\* apparently has no extra flavor symmetry even though it could have
- What about the CKM matrix/flavor mixing and gauge interactions, isn't that what we normally talk about with Flavor physics? **Let's talk about this next, but I want to stress mass/mixing are not independent**
- Why 3 instead of N? **Good question...**

**How does the picture change in gauge theories?**

“Vector-like” gauge theories everything here is the same

$$\mathcal{L} = i\bar{\psi}\partial\psi \rightarrow i\bar{\psi}D\psi$$

Clearly there exists a  $SU(N) \times U(1)$  symmetry rotating amongst flavors: “flavor symmetry” + fermion number

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**Global Chiral Flavor Symmetries**  
 $SU(N)_L \times SU(N)_R \times U(1) \times U(1)$

# “Vector-like” gauge theories everything stays the same

$$\mathcal{L}_{mass} = -M\bar{\psi}\psi$$

Generic matrix

Can use chiral flavor rotation to diagonalize mass matrix

$$\mathcal{L}_{mass} = -M\bar{\psi}\psi \rightarrow -M\bar{\psi}_L\psi_R$$

Inherently breaks global chiral flavor symmetries, but to what?

$$U_L^\dagger M U_R$$

Could be  $SU(N) \times U(1)$  could be  $U(1)^N$  or could be something in between, it *depends* on the actual masses!

**Isn't the SM a chiral gauge theory?**

Chiral gauge theories, not everything fills a Dirac fermion so starting point slightly different (LH and RH can transform differently)

$$\mathcal{L} = i\bar{\psi}_L \not{D}\psi_L + i\bar{\psi}_R \not{D}\psi_R$$

$$\psi_L \rightarrow U_L \psi_L$$

**Global Chiral Flavor Symmetries are basically the same**

$$\psi_R \rightarrow U_R \psi_R$$

$$SU(N_L) \times SU(N_R) \times U(1) \times U(1)$$

**Flavor subtlety shows up in mass!**

# Flavor/Mass in chiral gauge theories

$\mathcal{L}_{mass} = -M\bar{\psi}_L\psi_R$   
doesn't work in a  
chiral gauge theory!

Chiral flavor  
symmetries  
broken,  
no big deal

$$\psi_L \rightarrow U_L\psi_L$$

$$\psi_R \rightarrow U_R\psi_R$$

We now also have  
**GAUGE** symmetries  
that we don't want to break!

$$\psi_L \rightarrow U_L^g\psi_L$$

$$\psi_R \rightarrow U_R^g\psi_R$$

## Enter the Higgs!

$$\mathcal{L}_{Yukawa} = -y\bar{\psi}_L H\psi_R$$

A Higgs in the appropriate representation under the gauge symmetry  
can give Fermion masses in chiral gauge theories after SSB

**In the SM the Higgs does double duty!**

**No reason the same Higgs had to be the predominant source of gauge boson mass and fermion mass...**

*For example:*

**SM gauge boson masses could have come mostly from a Higgs triplet of SU(2)**

**SM fermion masses could have come from “standard” Higgs doublet of SU(2)**

**The fact that the *minimal* structure works and is so intertwined is kind of amazing and dictated by experiment and symmetry**

# So now to SM flavor explicitly

Table 29.1 Charges of Standard Model fields.  
 indicates that the field transforms in the fundamental representation, and  $-$  indicates that a field is uncharged.

Field	$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$e_R$	$\nu_R$	$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$u_R$	$d_R$	$H$
SU(3)	$-$	$-$	$-$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	$-$
SU(2)	<input type="checkbox"/>	$-$	$-$	<input type="checkbox"/>	$-$	$-$	<input type="checkbox"/>
U(1) <sub>Y</sub>	$-\frac{1}{2}$	$-1$	$0$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

Open whatever QFT book you like, e.g. Schwartz here (or Peskin since this is SSI)

SM classically has a huge global flavor symmetry of the kinetic terms of the QUDLE fermion fields  $U(3)^5$  (assuming 3 “generations”) but also is a chiral gauge theory

$$\mathcal{L}_{Yukawa} = -Y_L^{ij} \bar{L}_i H e_{Rj} - Y_D^{ij} \bar{Q}_i H d_{Rj} - Y_U^{ij} \bar{Q}_i \tilde{H} u_{Rj} + h.c.$$

**All of SM flavor and fermion masses are because of Yukawa couplings**



**This is *even* true for neutrino physics!**

# Wait a second, what about CKM?

$$\mathcal{L}_{Yukawa} = - Y_L^{ij} \bar{L}_i H e_{Rj} - Y_D^{ij} \bar{Q}_i H d_{Rj} - Y_U^{ij} \bar{Q}_i \tilde{H} u_{Rj} + h.c.$$

Lots of parameters here, 3 3x3 complex matrices, 54 parameters, way more than you might have heard?

Can play the same game using chiral flavor rotation/field redefinition to diagonalize “mass matrix”/Yukawas - subtlety is gauge sector and can't make everything diagonal everywhere

$$U_L^\dagger M U_R$$

**Flavor Basis**



**Mass Basis**

Yukawas non-diagonal in flavor space  
Gauge interactions family diagonal!

Yukawas diagonal  
CC interactions non-diagonal  $V_{CKM}$

# Wait a second, what about CKM?

$$\mathcal{L}_{Yukawa} = - Y_L^{ij} \bar{L}_i H e_{Rj} - Y_D^{ij} \bar{Q}_i H d_{Rj} - Y_U^{ij} \bar{Q}_i \tilde{H} u_{Rj} + h.c.$$

We can redefine our fields with unitary matrices that would diagonalize the Yukawas

What remains is essentially dictated by

$$\begin{aligned} u_L &\rightarrow U_u u_L \\ d_L &\rightarrow U_d d_L \end{aligned}$$

that defines the Unitary CKM matrix  $V_{CKM} \equiv U_u^\dagger U_d$

When “masses” are diagonal but different there is still our  $U(1)^{N=6 \text{ quarks}}$  symmetry so we can get rid of a few more parameters

$V_{CKM}$  is dictated by 3 angles  
and a phase in SM

CC interactions given by ~

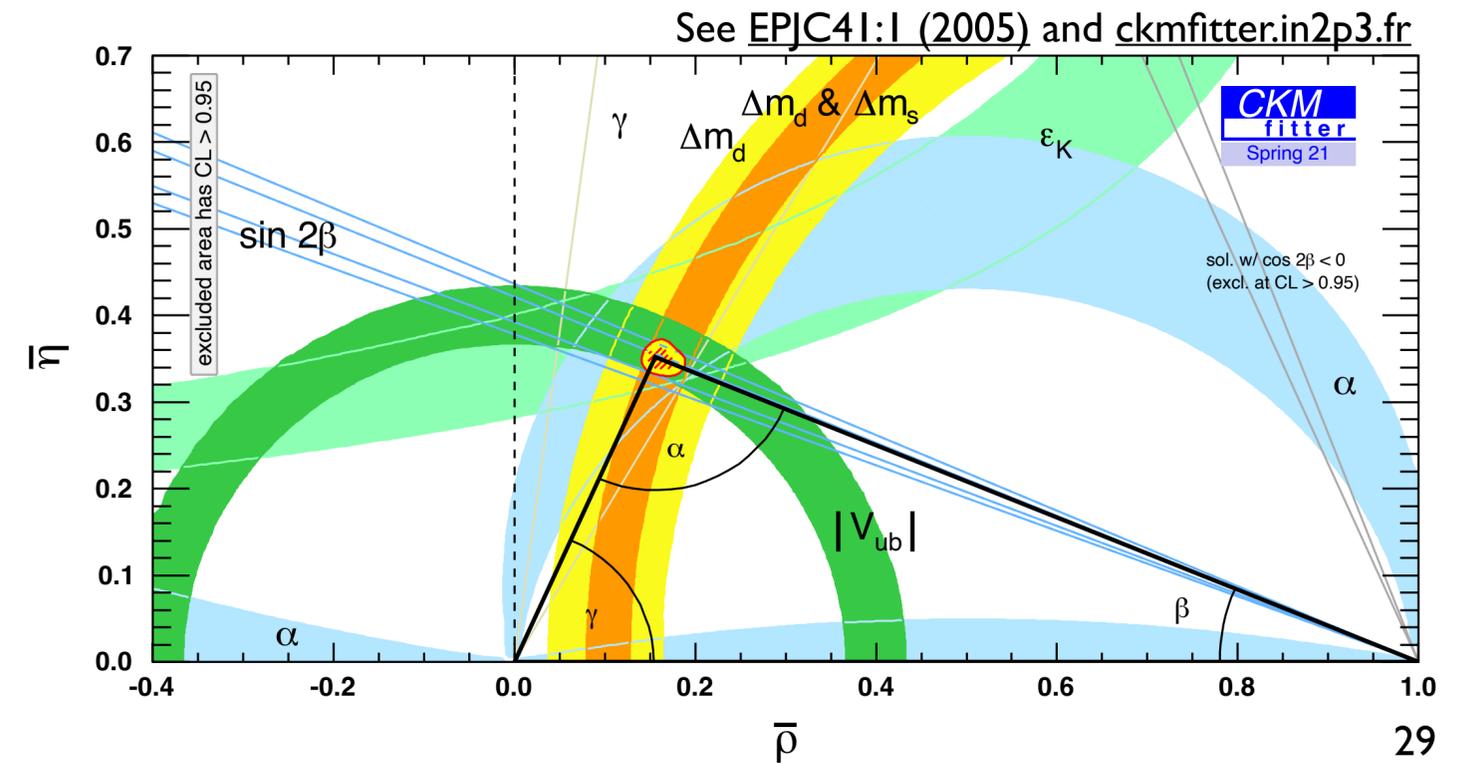
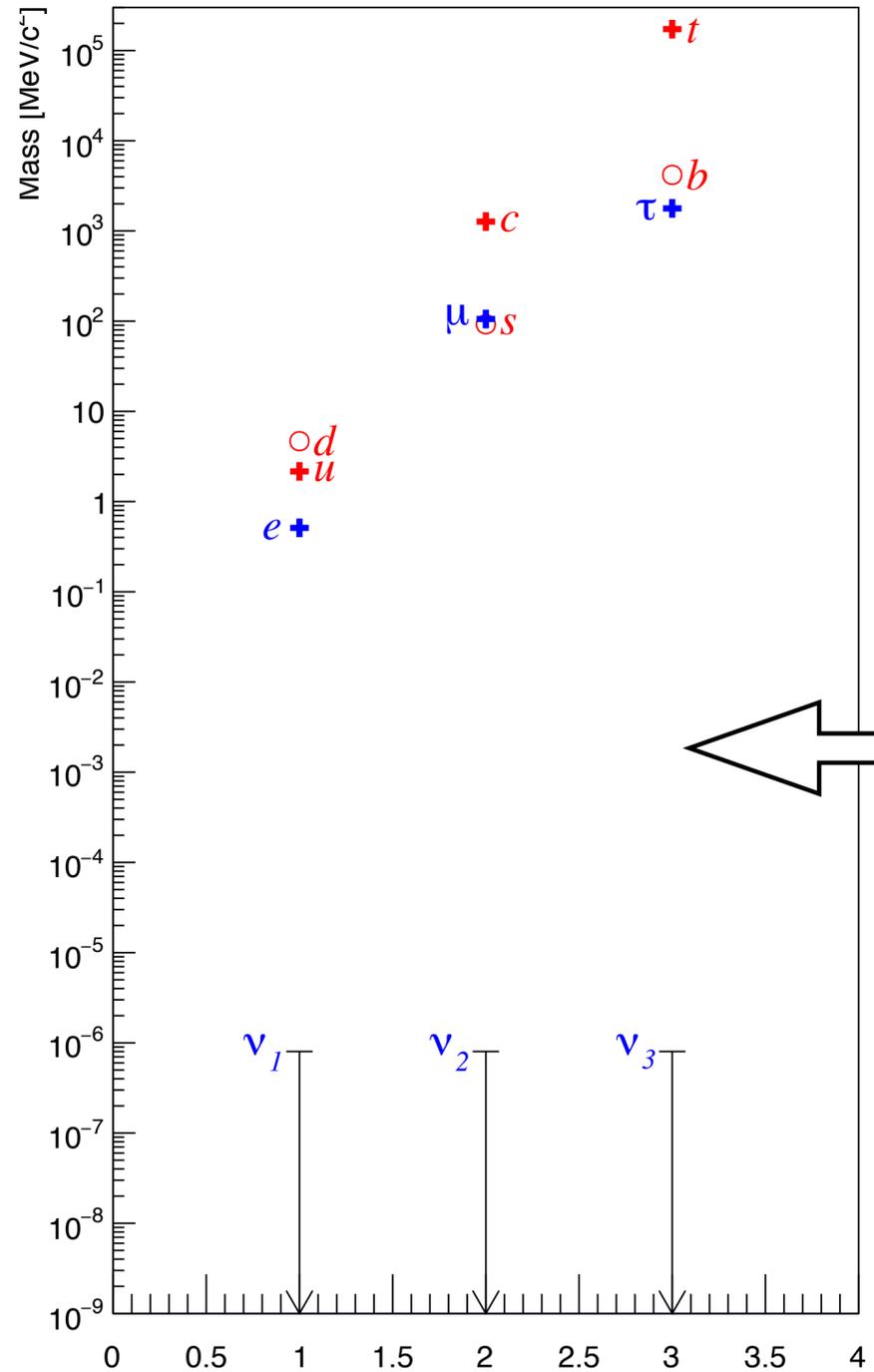
$$\mathcal{L}_{gauge} \supset \bar{u}_L^i W_\mu^+ V_{CKM}^{ij} d_L^j$$

You'll hear lots more about CKM in Mat's lecture tomorrow!

# Few quick CKM/SM flavor comments

- When you go to mass basis, all of “flavor” now exists in gauge interactions - you can effectively forget the Higgs exists and do everything in low energy EFT - “standard” flavor physics - Meson mixings, rare decays etc.
- SM being a chiral gauge theory added these complications, but also why we talk about number of families/generations not  $X$  up quarks and  $Y$  down quarks - Anomaly cancelation (no time sorry!) and naturalness
- CKM is also first hint at why  $N_{generations} \geq 3$  you can always make it real otherwise so CPV starts at 3 generations (Nobel Prize)

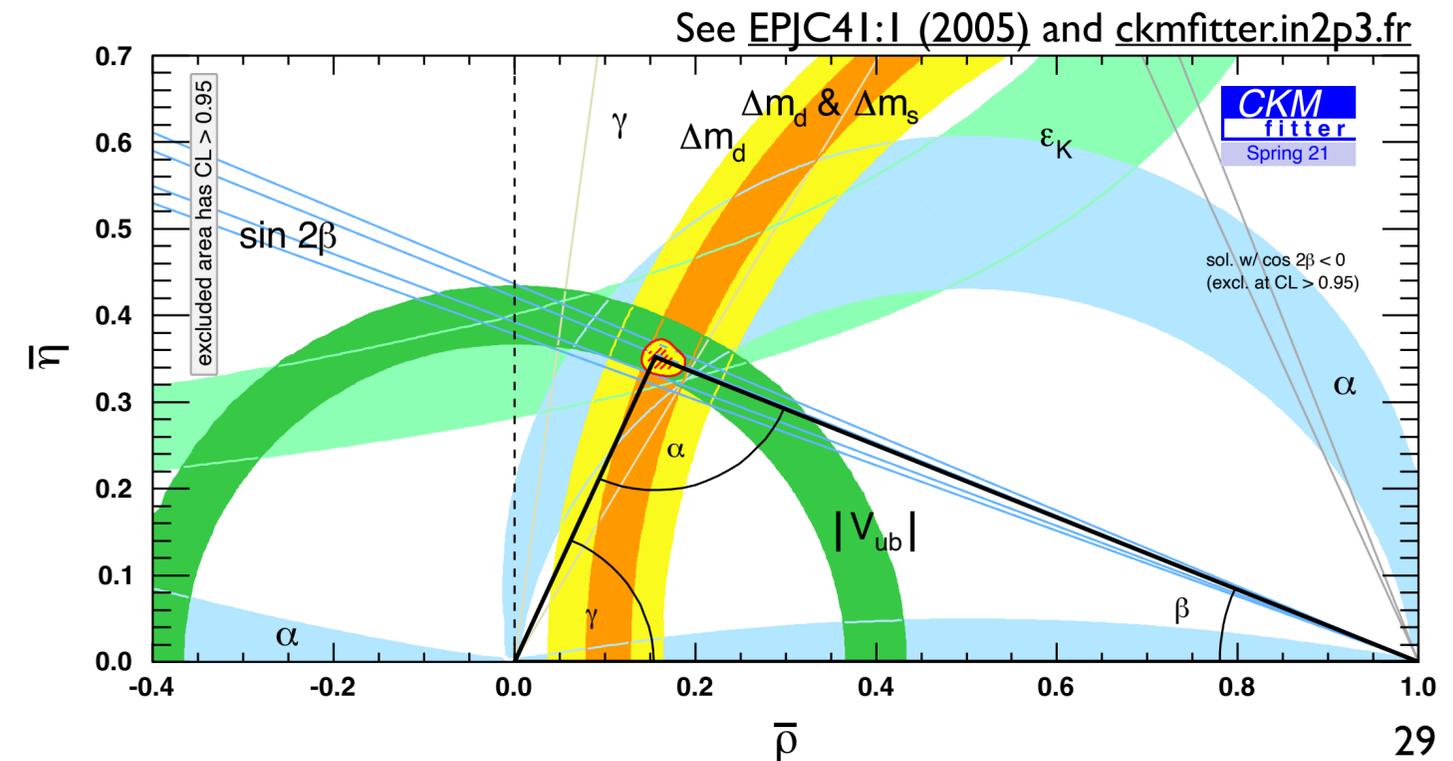
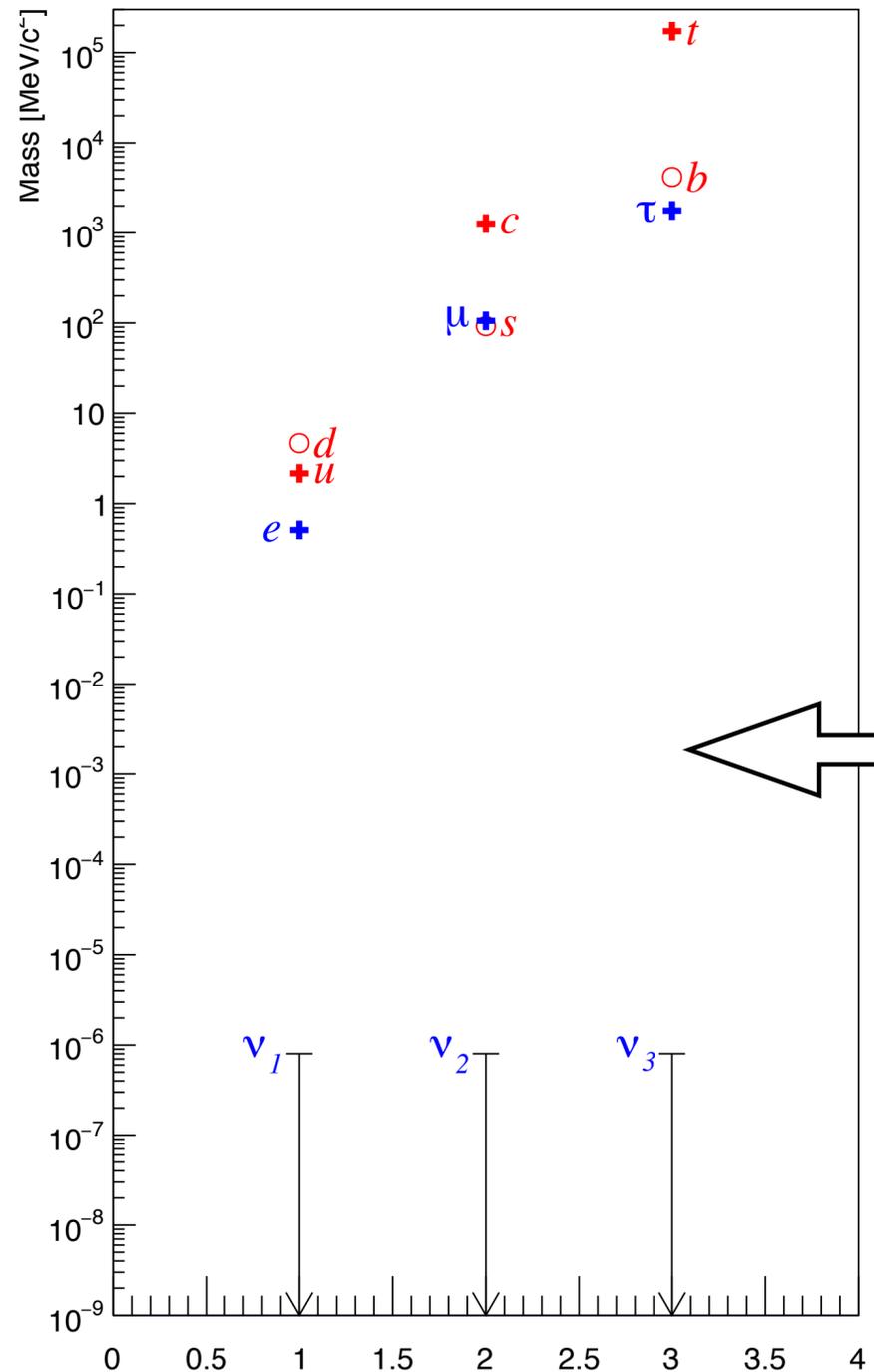
So despite that flavor looks completely crazy as measured and dominates number of parameters in Standard Model...



From Mat Charles lecture



So despite that flavor looks completely crazy as measured and dominates number of parameters in Standard Model...

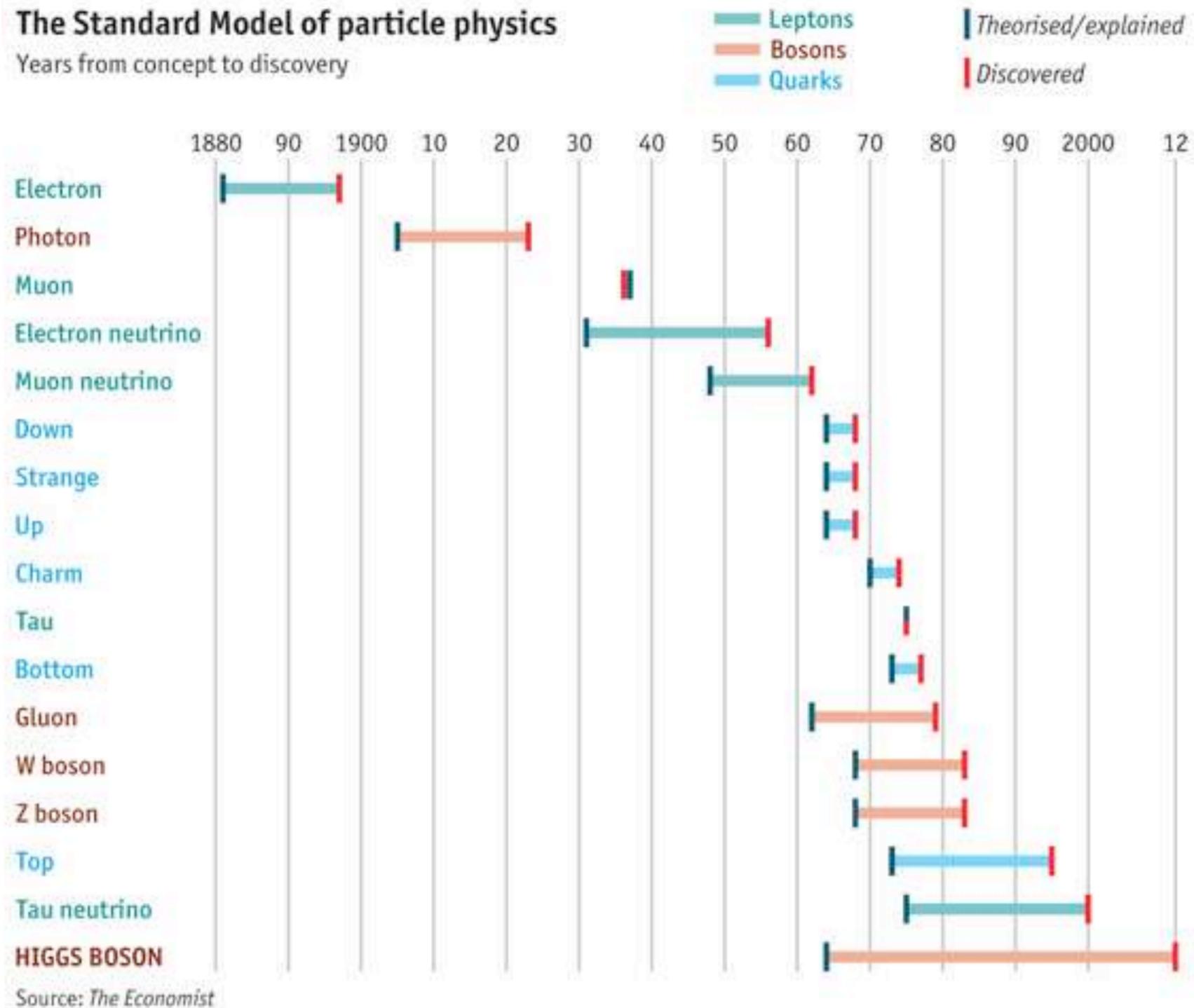


From Mat Charles lecture

There's still a lot of theoretical insight and a delicate structure inherent in SM

# The Standard Model of particle physics

Years from concept to discovery



From Jesse Thaler Snowmass talk

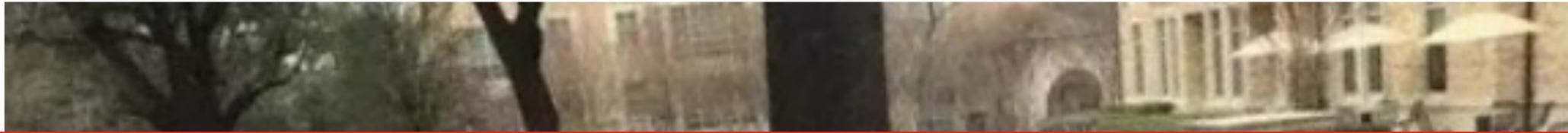
Rigorous speculation has a pretty good track record in high-energy physics

# Is all this a basis dependent statement?



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Is all this a basis dependent statement?



**NO, not all...**

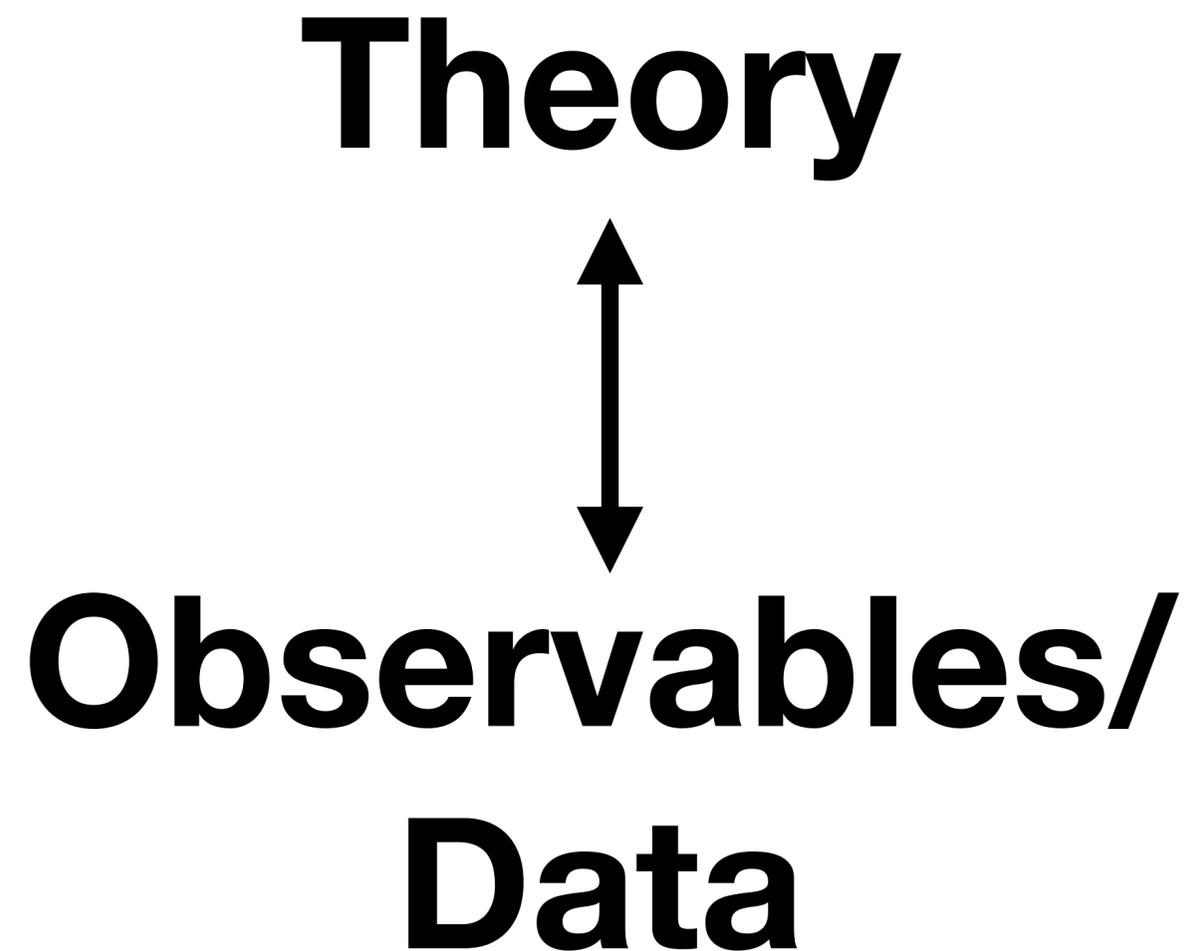
**In SM if the Yukawa interactions *didn't* exist I could make gauge interactions flavor blind!**



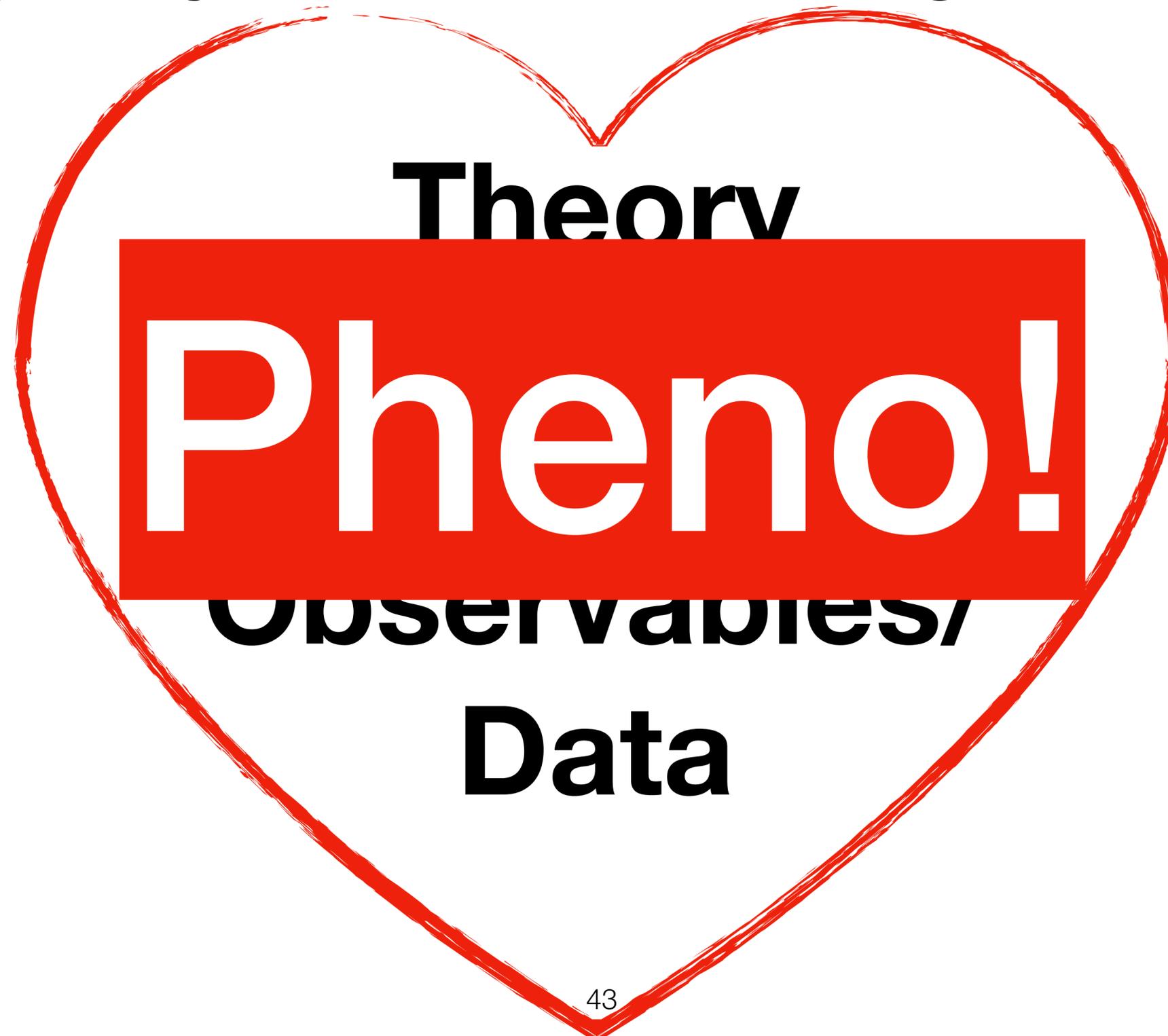
# So now that we've established that Flavor Physics is Higgs physics where do we go from here?

- Why are there 3 generations? **Technically we've only made a case for  $N \geq 3$  so far**
- The masses and mixings we observe are absolutely bizarre and arbitrary as far as we know - it would be really nice to have some explanation explanation for them! **As we've seen SM theory has something to say so hopefully there is something more**
- What's the role of BSM physics and flavor? **Two separate questions, what causes flavor and or if BSM physics for another reason (e.g. DM) how does it interact with flavor**

So now that we've established that Flavor Physics is Higgs physics where do we go from here?



So now that we've established that Flavor Physics is Higgs physics where do we go from here?



# I don't know for sure there are only 3 generations but I am highly confident there are, why?

- “Normal flavor physics experiments” told me? No, but they suggested at least 3
- I have beautiful models of flavor that single out 3? No
- String theory told me there are? No
- Electroweak precision physics told me? Suggestive (see Mat's talk), but no
- Higgs physics told me!



There's a lower bound on generations, but not an upper necessarily, 4, 5, 6 generations?

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} & \cdots \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} & \cdots \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} & \cdots \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \ddots \end{pmatrix}$$

Stealing one last figure from Mat Charles lecture...

From low energy CKM point of view I can always add more rows/columns  
e.g. (0,0,0,1) may be unattractive but who knows from this point of view?

Let's take a trip down memory lane to before I was born, and definitely before most of you all were (apologies to any organizers)

LEP SUMMER STUDY

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SCENARIOS FOR PHYSICS AT LEP\*

Sheldon L. Glashow

Lyman Laboratory of Physics  
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Cambridge, Massachusetts 02138

# Let's take a trip down memory lane to before I was born, and definitely before most of you all were (apologies to any organizers)

Fermions come in families, each of which involves fifteen Weyl fields. Each family consists of a doublet of colored quarks and a doublet of leptons. One lepton in each family has zero charge, and (in the minimal scenario) zero mass. The first family consists of  $(u, d, \nu_e, e)$ . These are the "relevant" particles, just the ones that are needed to make up the universe, and to make it work as well. As for the other fundamental fermions, as far as we can see, they are simply put in for spite: we can find no essential role for them, and apparently could get along quite well without them. Not only are we descended of slime found upon an obscure bit of the universe, but we are built up out of those elementary building-blocks which by accident of birth are the lightest--the runts.

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All else (excepting gravitational phenomena) is calculable in terms of these parameters. Of the seventeen parameters, only the t-quark mass and the Higgs boson mass remain quite unmeasured. Beyond these, there is simply no room for further surprises. *There will be nothing new in high-energy physics until such a time as we can study collisions at energies of order  $10^{17}$  GeV, surely an unlikely occurrence.* The arrogance of the physicist knows no bound, but let him consider the complacent words of George Gamow in 1947: "...we have now much sounder reasons for believing that our elementary particles are actually the basic units and cannot be subdivided further.

**Something to remember  
for anyone who tries to  
tell you particle physics  
is over...**

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II. The Scenario of Many Fermions

The only difference between this scenario and the last is that here we admit the possible existence of fermions beyond those in the three known families. The conventional  $SU(3) \times SU(2) \times U(1)$  gauge group is still assumed. The simplest thing to do is to conjecture that all fermions come in isomorphic 15-dimensional families, but to let their number  $N$  be larger than three. It is relevant to mention here some of the arguments that put limits on  $N$ :

Let's take a trip down memory lane to before I was born, and definitely before most of you all were (apologies to any organizers)

not upset universality. Let  $n(m)$  denote the number of neutrino states with mass less than  $m$ . We obtain:

$$n(1 \text{ MeV}) \leq 3$$

$$n(10 \text{ MeV}) \leq 13$$

$$n(100 \text{ MeV}) \leq 10^6.$$

Evidently, astrophysics tells us very little about  $N$  if we are willing to tolerate neutrinos with mass.

# Let's take a trip down memory lane to before I was born, and definitely before most of you all were (apologies to any organizers)

This brings to mind an awful scenario put forward by John Ellis. Suppose that there are not very many *charged* quarks or leptons lying below  $1/2 M_Z$  in mass, but that there are a considerable number of light neutrinos. Consequently, the  $Z^0$  resonance will be quite broad, and it will decay primarily into unobservable neutrino channels. LEP would not see the resonance at all. Ellis shows that this scenario is unlikely, and that the many light neutrinos should be accompanied by many relatively light charged leptons. The absence of a distinct  $Z^0$  resonance is then compensated by the existence of many observable charged-lepton thresholds leading to a large value of  $R$ . Were these charged leptons much heavier than their neutrinos, large radiative corrections would change the relative strength of charged current and neutral-current effects. This would upset the empirical success of the gauge theory. Of course, this could be repaired by an appropriate change in the Higgs sector. The possibility of "neutrino wipeout" with no easily observable compensation remains a remote possibility.

**What Mat will talk about**

# Let's take a trip down memory lane to before I was born, and definitely before most of you all were (apologies to any organizers)

A simple and attractive model of CP violation puts the blame on the structure of the quark mass matrix. As first realized by Kobayashi and Maskawa, this requires  $N \geq 3$ . Unfortunately, no upper bound on  $N$  is obtained. On the other hand, some people argue that the asymptotic freedom of SU(3) must be preserved at arbitrarily high energy. It is not established that this is truly necessary, but if it is, it would seem to require that  $N \leq 8$ . If there exists a grand unification, this condition may be weakened depending upon how large is the unifying group. For SU(5), the constraint becomes  $N \leq 13$ . Since I don't know how seriously to regard this constraint, I choose to ignore it.

Let's take a trip down memory lane to before I was born, and definitely before most of you all were (apologies to any organizers)

No argument yet presented *proves* that  $N = 3$ . Indeed, it is not even clear that  $N$  may not be infinite. Of course,

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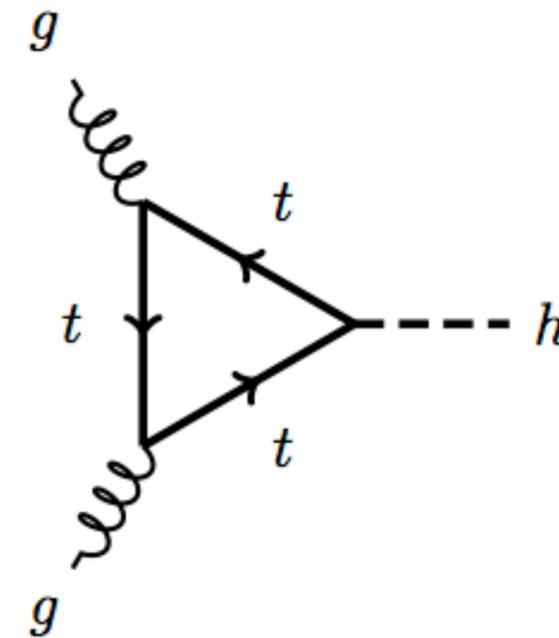
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From low energy CKM point of view I can always add more rows/columns  
e.g. (0,0,0,1) may be unattractive but who knows from this point of view?

If you remember that CKM comes from Yukawas  
though...

I can't just escape by making things heavier and  
diagonal!

$$y_{t'} \sim \frac{M_{t'}}{v}$$



Higgs properties get altered and a 4th generation  
was ruled out early on!



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# Mantra from today: Flavor Physics is Higgs Physics (Of course you can also do great Flavor physics without the Higgs)

- We will also separate out the key concepts behind what is tied to the *origin* of flavor, versus what does it just mean that there can be other types of physics could be *flavorful* as well
- Next time we will talk more about actual models of Flavor - Frogatt-Nielsen, Radiative models, Geometrical, and their “generic” predictions
- Overview an example of *flavorful* physics and why there still could be drastic changes in flavor even at low energy for current and future experiments!