Lecture 2

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2. Neutrino Mixing and Oscillations

2.1 Three Flavors of Neutrinos

$$\mathcal{L} \supset \sum_{\alpha=e,\mu,\tau} \Big[\overline{\nu_{\alpha L}}\, i\not\partial\, \nu_{\alpha L}$$
$$+ \frac{g}{\sqrt{2}} \left(W^\mu \overline{\nu_{\alpha L}} \gamma_\mu e_{\alpha L} + h.c. \right)$$
$$+ \frac{g}{2\cos\theta_W} Z^\mu \overline{\nu_{\alpha L}} \gamma_\mu \nu_{\alpha L} \Big]$$
$$- \left(\sum_{\alpha,\beta} \frac{1}{2} m_{\alpha\beta} \overline{(\nu_{\alpha L})^c} \nu_{\beta L} + h.c. \right)$$

- $m_{\alpha\beta}$ in general off-diagonal
- Diagonalize $m_{\alpha\beta}$: $\underbrace{\nu_{\alpha L}}_{\text{flavor eigenstate}} = \underbrace{U_{\alpha j}}_{\text{unitary mixing matrix}} \underbrace{\nu_{jL}}_{\text{mass eigenstate}}$

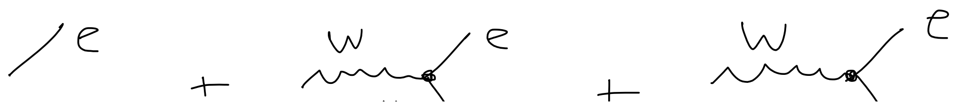
with $U^T m U = \mathrm{diag}(m_1, m_2, m_3)$

(remember that m is complex symmetric)

- In the mass basis

$$\mathcal{L} \supset \sum_j \Big[\overline{\nu_{jL}}\, i\not\partial\, \nu_{jL}$$
$$+ \left(\sum_{\alpha,j} \frac{g}{\sqrt{2}} W^\mu \overline{\nu_{jL}} U^*_{\alpha j} \gamma_\mu e_{\alpha L} + h.c. \right)$$
$$+ \frac{g}{2\cos\theta_W} \sum_j Z^\mu \overline{\nu_{jL}} \gamma_\mu \nu_{jL}$$
$$- \left(\frac{1}{2} \sum_j m_j \overline{(\nu_{jL})^c} \nu_{jL} + h.c. \right)$$

CC ν interaction involves superposition of ass eigenstates



$\frac{g}{\sqrt{2}} U_{e1}^* \quad \nu_1 \qquad \frac{g}{\sqrt{2}} U_{e2}^* \setminus \nu_2 \qquad \frac{g}{\sqrt{2}} U_{e3}^* \setminus \nu_3$

2.2 Neutrino Oscillations

Produced in CC in action is a superposition

$$|\nu_\alpha\rangle = U_{\alpha j}^* |\nu_j\rangle$$

$\nu_\alpha = U_{\alpha j} \nu_j$

$|\nu_\alpha\rangle = \nu_\alpha^+ |0\rangle = U_{\alpha j}^* \nu_j^+ |0\rangle = U$

CC detections maps neu ino state onto another flavor eigenstate

$$\langle \nu_\beta | = U_{\beta k} \langle \nu_k |$$

$$\hookrightarrow \mathcal{A} = \langle \nu_\beta | \nu_\alpha(t) \rangle$$

$$= \sum_{j,k} \langle \nu_k | U_{\beta k} U_{\alpha j}^* | \nu_j \rangle \underbrace{e^{-iE_j T + i p_j L}}_{\text{time and space evolution in QM}}$$

$$= \sum_j U_{\alpha j}^* U_{\beta j} \, e^{-iE_j T + i p_j L}$$

$$\boxed{\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= |\mathcal{A}|^2 \\ &= \sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta j}^* \\ &\quad \cdot e^{-i(E_j - E_k)T + i(p_j - p_k)L} \end{aligned}}$$

Note: States with different E and p can interfere only if E- and p-uncertainties are larger than $|E_j - E_k|$, $|p_j - p_k|$

Typically, we do not know T precisely (or we don't care) because uncertainty in ν production time is larger than $|E_j - E_k|^{-1}$

$$\hookrightarrow \bar{P}(\nu_\alpha \to \nu_\beta) = \underbrace{\frac{1}{N}}_{\text{normalization}} \int_{-\infty}^{\infty} dT \, |\mathcal{A}|^2$$

$$= \cancel{\frac{1}{N}} \sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta j}^* \cdot \cancel{2\pi} \, \delta(E_j - E_k)$$

$$\cdot \exp\left[i\left(\sqrt{E^2-m_j^2} - \sqrt{E^2-m_k^2}\right)L\right]$$

$$= \sum_{jk} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \cdot \exp\left[-i \frac{\Delta m^2_{jk} L}{2E}\right]$$

where $\Delta m^2_{jk} = m_j^2 - m_k^2$.

Consider 2-flavor approximation

$\hookrightarrow$ $U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$$\Rightarrow \overline{P} \overset{2\,fl.}{\simeq} \underbrace{\sin^2 2\theta}_{\text{osc. amplitude}} \; \underbrace{\sin^2 \frac{\Delta m^2 L}{4E}}_{\text{oscillation term with osc. length}}$$

$$\frac{\Delta m^2 L_{osc.}}{4E} = \pi$$

$$\hookrightarrow L_{osc} = \frac{4\pi E}{\Delta m^2}$$

$\Delta m^2_{21} \sim 8 \cdot 10^{-5}\,\mathrm{eV}^2 \rightarrow L_{osc} \sim 60\,\mathrm{km}$ @ $\mathcal{O}(\mathrm{MeV})$

$\Delta m^2_{31} \sim 2 \cdot 10^{-3}\,\mathrm{eV}^2 \rightarrow L_{osc} \sim 1\,\mathrm{km}$ @ $\mathcal{O}(\mathrm{MeV})$

mixing angle controls amplitude

Δm^2 controls osc. length

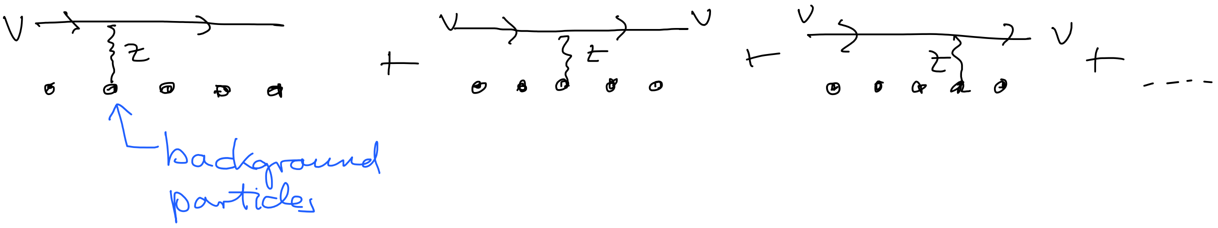
Note: in QFT, ν osc. can be described as a Feynman diagram

ν production — propagation over macroscopic distance — ν detection

Treat external particles as wave packets.

2.3 Neutrino Oscillations in Matter

Coherent forward scattering

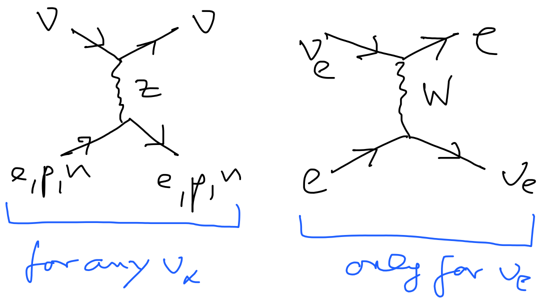


Exchange of W or Z boson with identical initial and final quantum states means that all background particles contribute coherently

[Analogy: photon travelling through matter

$$|\mathcal{M}|^2 \sim N^2 G_F^2$$

density of scatterers



quantum states of initial and final states are identical in coh. forward scattering

$$H_{eff} = \frac{G_F}{\sqrt{2}} \left[\bar{e}\gamma^\mu(1-\gamma^5)\nu_e\right]\left[\bar{\nu}_e\gamma_\mu(1-\gamma^5)e\right]$$

$$\overset{\text{Fierz}}{=} \frac{G_F}{\sqrt{2}} \left[\bar{e}\gamma^\mu(1-\gamma^5)e\right]\left[\bar{\nu}_e\gamma_\mu(1-\gamma^5)\nu_e\right]$$

Treat background e as classical

$$\langle H_{eff}\rangle = \frac{G_F}{\sqrt{2}} \underbrace{\langle \bar{e}\gamma^\mu(1-\gamma^5)e\rangle}_{= \begin{cases} n_e & \text{for } \mu=0 \\ 0 & \text{for } \mu\neq 0\end{cases}} \left[\bar{\nu}_e\gamma_\mu(1-\gamma^5)\nu_e\right]$$

$$= \underbrace{\sqrt{2}\, G_F n_e}_{V_{CC} \text{ (MSW potential)}} \overline{\nu_{eL}}\gamma^0 \nu_{eL}$$

In the derivation of $P(\nu_\alpha \to \nu_\beta)$, we had factor e^{ipL}

$$\phi = p \cdot L = \sqrt{(\hat{H} - \hat{V})^2 - \hat{M}^2}$$

2x2 matrix

MSW potential $\begin{pmatrix} V_{cc} & 0 \\ 0 & 0 \end{pmatrix}$

2x2 mass matrix

Diagonalize $$\hat{H} - \frac{\hat{M}^2}{2\hat{H}} - \hat{V} = E \cdot \mathbb{1} - U \begin{pmatrix} \frac{m_1^2}{2E} & \\ & \frac{m_2^2}{2E} \end{pmatrix} U^\dagger - \begin{pmatrix} V_{cc} & 0 \\ 0 & 0 \end{pmatrix}$$

↳ Expressions for effective mixing angle and Δm^2 in matter.

in 3f: $$U = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}$$