Lecture 1

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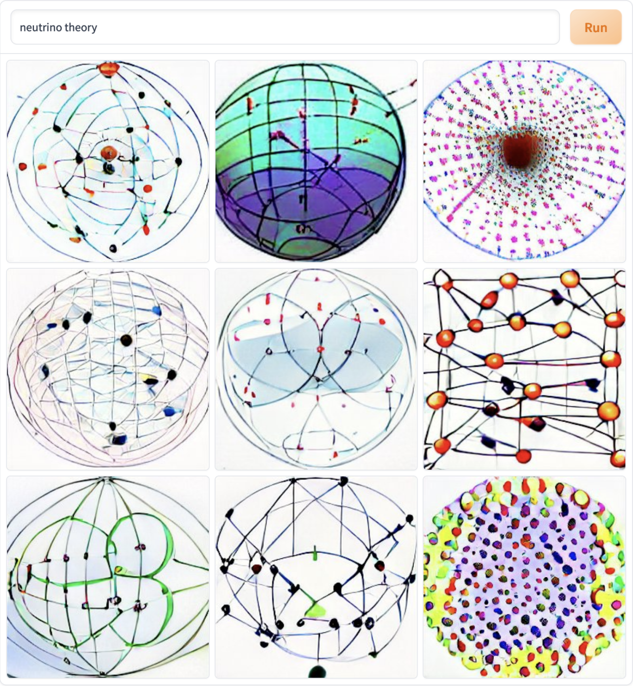
Neutrino Theory

Joachim Kopp
(CERN & JGU Mainz)



1. Neutrino Masses

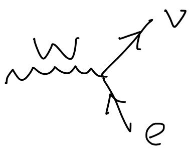
1.1 Neutrinos in the SM

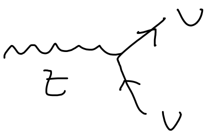
Quarks	u	c	t
	d	s	b
Leptons	e	μ	τ
	ν_e	ν_μ	ν_τ

- may elucidate origin of flavor
- produced in nuclear reactions
  e.g. $\pi \to \mu + \nu$
- may help explain baryon asymmetry
- may teach us about DM

....

$$\mathcal{L} \supset \sum_{\alpha=e,\mu,\tau} \Big[\bar{\nu}_{\alpha L}\, i\not{\partial}\, \nu_{\alpha L}$$
$$+ \frac{g}{\sqrt{2}} \left(W^{\mu +}\, \bar{\nu}_{\alpha L} \gamma_\mu e_{\alpha L} + h.c. \right)$$
$$+ \frac{g}{2\cos\theta_W} Z^\mu\, \bar{\nu}_{\alpha L} \gamma_\mu \nu_{\alpha L} \Big]$$
$$+ \text{mass terms}$$





1.2 Dirac Masses

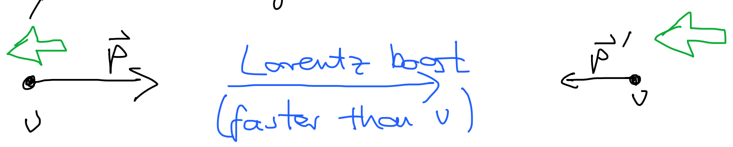
In the SM, only LH neutrinos exist

$$\nu_L = \frac{1-\gamma^5}{2}\nu = \begin{pmatrix} \chi_1 \\ \chi_2 \\ 0 \\ 0 \end{pmatrix}$$

Dirac mass term makes $\nu_R = \frac{1+\gamma^5}{2}\nu = \begin{pmatrix} 0 \\ 0 \\ \chi_3 \\ \chi_4 \end{pmatrix}$

$$\mathcal{L}_m \supset -\sum_{\alpha,\beta} \underbrace{\boxed{m_{\alpha\beta}}}_{y_{\alpha\beta}\langle v\rangle} \bar{\nu}_{\alpha L} \nu_R \qquad \mathcal{L} \supset -y\, \bar{L} \tilde{H} \nu_R$$

4 physical d.o.f.



Problem: why are $m_{\alpha\beta}$ so small

1.3 Majorana Masses

Mass terms couple LH and RH states

The antiparticle of ν_L is a RH state

Could ν_R be identical to the antiparticle of ν_L?

More formally: charge conjugation

$$\hat{C}: \psi \to \psi^c \equiv -i\gamma^2\gamma^0\bar{\psi}^T$$

Effect on chirality:

$$\gamma^5\psi^c = \gamma^5(-i\gamma^2\gamma^0)\psi^*$$
$$= +i\gamma^2\gamma^5\psi^*$$
$$= -(\gamma^5\psi)^c$$

↳ $\hat{C}$ flips chirality

$$\boxed{\text{Identify } \nu_R \equiv (\nu_L)^c}$$

In 4-component notation: $\nu_L = \begin{pmatrix}\chi_1\\ \chi_2\\ 0\\ 0\end{pmatrix} \equiv \begin{pmatrix}\chi\\ 0\end{pmatrix}$

$$(\nu_L)^c = -i\begin{pmatrix}0 & \sigma^2\\ -\sigma^2 & 0\end{pmatrix}\begin{pmatrix}\chi^*\\ 0\end{pmatrix} = \begin{pmatrix}0\\ i\sigma^2\chi^*\end{pmatrix}$$

$\Rightarrow \nu = \begin{pmatrix}\chi\\ i\sigma^2\chi^*\end{pmatrix}$ 4-component Majorana spinor but only 2 d.o.f.

A new type of mass term:

$$\mathcal{L} \supset -\frac{1}{2}\sum_{\alpha,\beta} m_{\alpha\beta}\,\overline{(\nu_{L\alpha})^c}\,\nu_{L\beta} + h.c.$$

Complex symmetric 3×3 matrix

Problem:

- why is $m_{\alpha\beta}$ so small
- How can this arise from an SU(2)-invariant UV completion

1.4 The Seesaw Mechanism

Augment SM with 3 RH neutrinos N_R, singlet under the SM gauge group

For simplicity, consider first 1 LH ν_L, 1 RH N_R:

$$\mathcal{L} \supset -m_D\,\bar{\nu}_L N_R - \frac{1}{2}m_M\,\overline{(N_R)^c}\,N_R$$

from Higgs mechanism
$\mathcal{L} \supset -y\bar{L}\tilde{H}N_R$
natural scale is $m_D \sim m_H$

no natural scale
could be very large

Write $n = \begin{pmatrix}\nu_L\\ N_R{}^c\end{pmatrix}$

$$\mathcal{L} \supset -\frac{1}{2} n^c \underbrace{M}_{\begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix}} n + h.c.$$

$$= -\frac{1}{2} \begin{pmatrix} \overline{\nu_L^c} & \overline{N_R} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R^c \end{pmatrix}$$

Diagonalize M to obtain physical states (with definite mass)

eigenvalues $\sim m_M, \underbrace{-\frac{m_D^2}{m_M}}$ (for $m_M \gg m_D$) absorbed by rephasing $\nu_L' \to i\nu_L'$

eigenvectors $N_R' = N_R + \mathcal{O}\left(\frac{m_D}{m_M}\right)$

$\nu_L' = \nu_L + \mathcal{O}\left(\frac{m_D}{m_M}\right)$

Effective low-E Lagrangian

$$\mathcal{L} \supset -\frac{1}{2} \frac{m_D^2}{m_M} \overline{(\nu_L')^c} \nu_L'$$

With $m_D \sim 100$ GeV and $m_M \sim 10^{14}$ GeV

↳ $m_\nu = \frac{m_D^2}{m_M} \sim 0.1$ eV

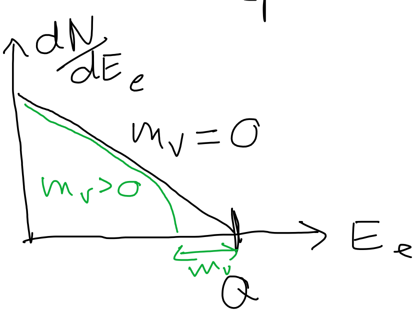
1.5 Measuring ν masses

1.5.1 Kinematics

β-decay $^3H \to {}^3He + e^- + \bar{\nu}_e$

$$E_{e,max} = \underbrace{Q}_{m_H - m_{He}} - m_\nu$$

Measure e^- spectrum, look at endpoint



1.5.2 Neutrinoless Double Beta Decay

↳ see experimental lecture

1.5.3 Cosmology

Sensitive through large-scale structure

ν carry energy from overdense to underdense regions of the Universe

the larger m_ν, the larger this effect

↳ look for suppression on small scales

current limits

$$\sum m_\nu < 0.12\,\text{eV}$$