Lepton Flavor Violating Mixings at 2-Loop from the S-Matrix

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Some Numbers

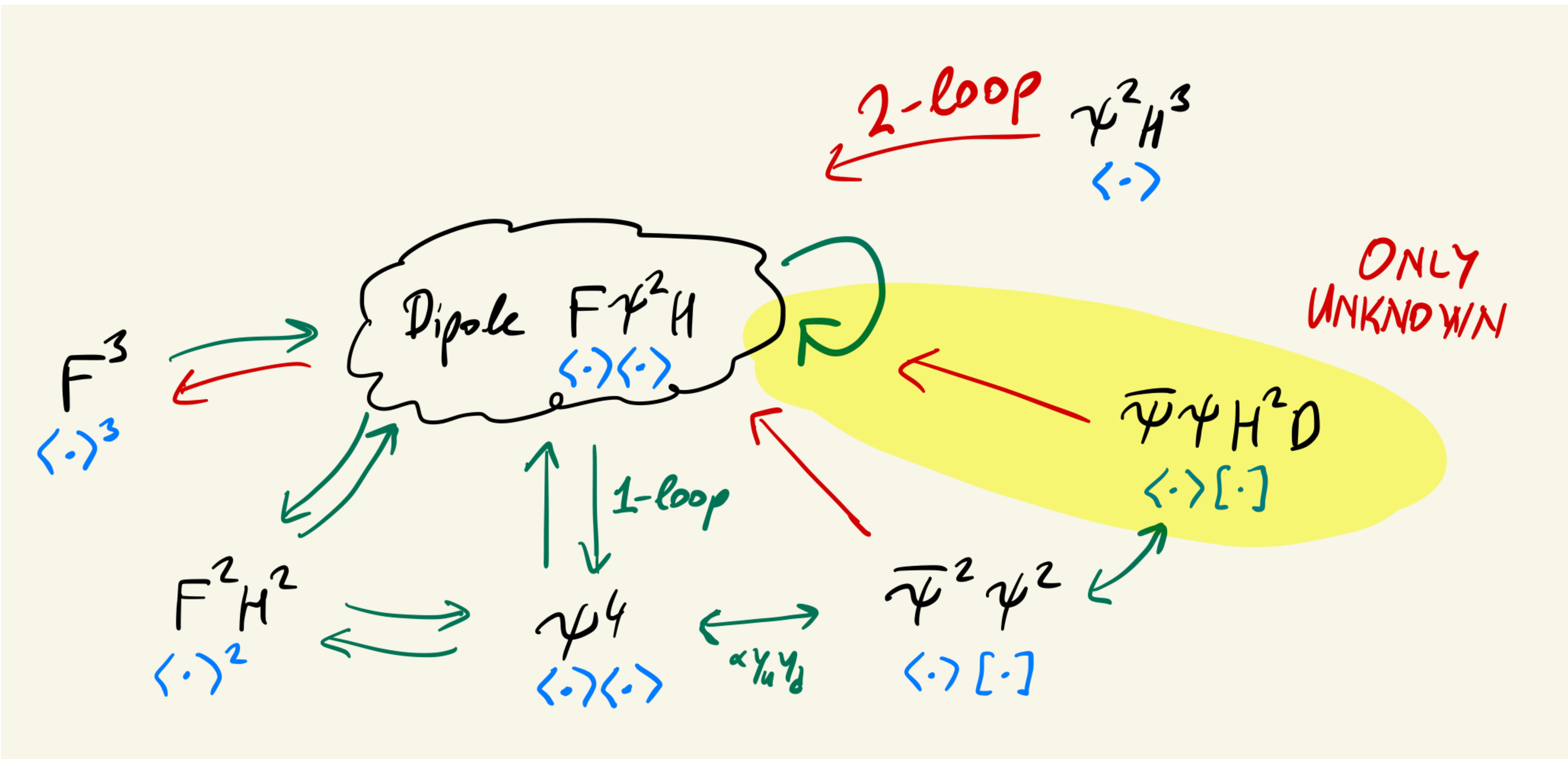
- No LFV at tree-level in Standard Model. $\mathrm{Br}(\mu \to e\gamma) \approx 10^{-54}$ with neutrino oscillations.
- Z and $H \to \mu e$ @ LHC has probed $\Lambda_{\text{New Physics}} \gtrsim 5-10$ TeV.

	$\mu \to e\gamma$ (MEG)	$\mu \to eee$ (Mu3e)
Current bound	$\mathrm{Br} < 4.2 \times 10^{-13}$	$\mathrm{Br} < 2.4 \times 10^{-12}$
Future bound	$\mathrm{Br} < 6 \times 10^{-14}$	$\mathrm{Br} < 10^{-16} \Longleftarrow \times$ 4 orders!
Current scale	$\Lambda \gtrsim 800$ TeV	$\Lambda \gtrsim 100$ TeV
Future scale	$\Lambda \gtrsim 1300$ TeV	$\Lambda \gtrsim 1000$ TeV

- Scales above are for Wilson coefficients $c_i^{(6)} \sim 1$. With 2-loop suppression $c_i \to c_i/(16\pi^2)^2$ and $\Lambda \to \Lambda/(16\pi^2) \gtrsim 6$ TeV.

LFV Mixing Picture

Renormalization relations between different groups of dimension-6 SMEFT operators:



- $\langle\cdot\rangle$ and $[\cdot]$ are spinor-helicity brackets labeling helicity structure of minimal form factors.
- All arrows above are calculated in the literature, except for the highlighted mixing going from current[2] operators into dipole operators, which starts at 2-loop.
- **An interesting case:** When our 2-loop contribution becomes as important as 1-loop ones. See the Results section for more.

Spinor-Helicity Formalism

- $P_\mu \longrightarrow \mathbf{P} = P_\mu \sigma^\mu$ with $\sigma^\mu = (1, \sigma^i)$
- Lorentz invariant: $P_\mu P^\mu = \det(P_\mu \sigma^\mu)$
- For a massless particle $\det(\mathbf{P}) = 0$. $\mathbf{P}$ is a rank-1 complex matrix.

$$P_\mu \sigma^\mu = \lambda_\alpha \tilde{\lambda}_{\dot\alpha} \equiv |P\rangle_\alpha [P|_{\dot\alpha}$$

where $\lambda, \tilde\lambda$ are left- and right-handed Weyl spinors.

- Momentum preserving rotations (*little group transformation*) $\lambda \to t\lambda$ and $\tilde\lambda \to t^{-1}\tilde\lambda$ for a $t \in \mathbb{C}$.

Some Amplitude Examples

These are all tree-level and with fixed helicities for external particles.
They are evaluated using on-shell BCFW recursion relations over smaller amplitudes.
It is useful to think these also as building blocks to contruct bigger amplitudes.

$$\mathcal{A}(1^-_\psi 2^+_\psi 3^-_\gamma) = \tilde{e}_\psi \frac{[23]^2}{[12]} \qquad \mathcal{A}(1^-_\psi 2^+_\psi 3^-_\chi 4^+_\chi) = -\tilde{e}_\psi \tilde{e}_\chi \frac{[24]^2}{[12][34]}$$

$$\mathcal{A}(1^-_\psi 2^+_\psi 3^-_\chi 4^+_\chi 5^-_\gamma) = \tilde{e}_\psi \tilde{e}_\chi^2 \frac{[24]^2}{[35][45][12]} + \tilde{e}_\psi^2 \tilde{e}_\chi \frac{[24]^2}{[15][25][34]}$$

References

1. EFT anomalous dimensions from S-Matrix, Elias-Miro, Ingoldby, Riembau, arxiv:2005.06983
2. Renormalization Group Coefficients and the S-Matrix, Caron-Huot, Wilhelm, arxiv:1607.06448
3. Renormalization of Higher-Dimensional Operators from On-shell Amplitudes, Baratella, Fernandez, Pomarol, arxiv:2005.07129
4. Structure of two-loop SMEFT anomalous dimensions via on-shell methods, Bern, Parra-Martinez, Eric Sawyer, arxiv:2005.12917
5. Renormalization Group Evolution from On-shell SMEFT, Jiang, Ma, Shu, arxiv:2005.10261

Useful Formulas to Cut & Sew

We only use the following physical quantities to calculate anomalous dimension matrix γ_{ij}.

$$F_i[1\,2\,\ldots;\mu] = {}_{\text{out}}\langle p_1, p_2, \ldots | \mathcal{O}_i | 0 \rangle_{\text{in}} \qquad \text{form factor}$$

$$\mathcal{A}(1\,2\,|\,3\,4) = {}_{\text{out}}\langle p_1, p_2, \ldots | p_3, p_4, \ldots \rangle_{\text{in}} = \langle p_1, p_2, \ldots | \mathcal{M} | p_3, p_4, \ldots \rangle \qquad \text{scattering amplitudes}$$

$$\mu \partial_\mu F_i = \left(\gamma_{ij}\right)\left(F_j\right) \qquad \text{RG equations}$$

There is a connection between on-shell S matrix and renormalization group (RG) [2].

CHW Formula 1 (exact)

$$e^{-i\pi D} F^* = F = S F^* \quad \text{with} \quad D = -\mu\partial_\mu \tag{1}$$

Extracting the interacting part of S matrix as $\mathcal{M}$ and expanding in couplings:

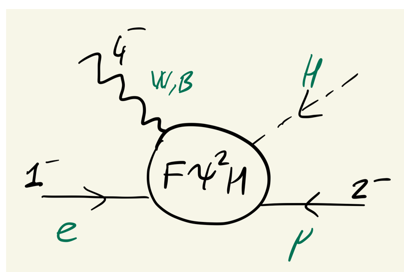
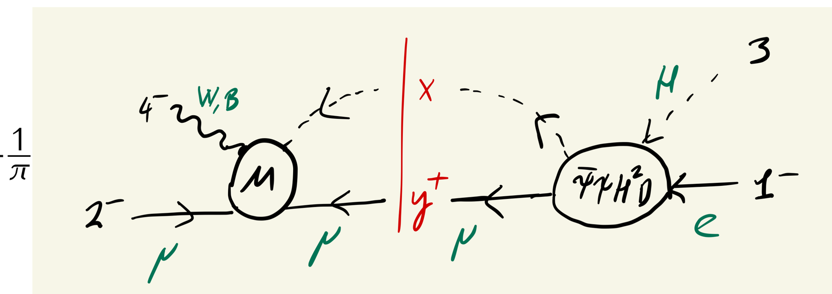
CHW Formula 2 (perturbative)

$$\underbrace{\langle \vec{n} | \mathcal{O}_j | 0 \rangle^{(0)}}_{\text{minimal FF}} \cdot \underbrace{\gamma^{(1)}_{ji}}_{\text{leading anomalous dim.}} = -\frac{1}{\pi} \sum_{\vec{m}} \underbrace{\langle \vec{n} | \mathcal{M} | \vec{m} \rangle^{(0)}}_{\text{tree-level amplitude}} \cdot \underbrace{\langle \vec{m} | \mathcal{O}_i | 0 \rangle^{(0)}}_{\text{minimal FF}} \tag{2}$$

A very efficient way to calculate anomalous dimensions in an EFT setting, by utilizing on-shell information of subdiagrams with smaller number of loops. See the ref.s: [1,3,4,5].

Let's Do the Action

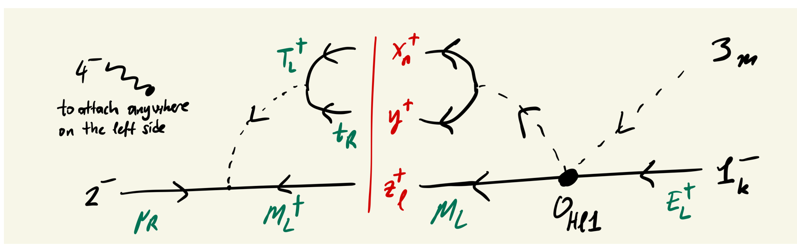
Dipoles start at 1-loop first.

$$\gamma \times$$  $$= -\frac{1}{\pi}$$ 

$$= \int d\mathrm{LIPS}_{x,y}\, \mathcal{A}(2^- 4^- | x\, y^+) \cdot F_i(x y^+ 1^- 3) = 0$$

Then there are the following groups 2-loop diagrams:

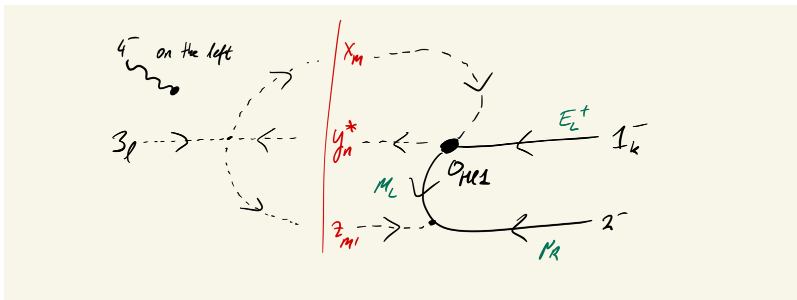
- Add a top-loop in the Higgs line.

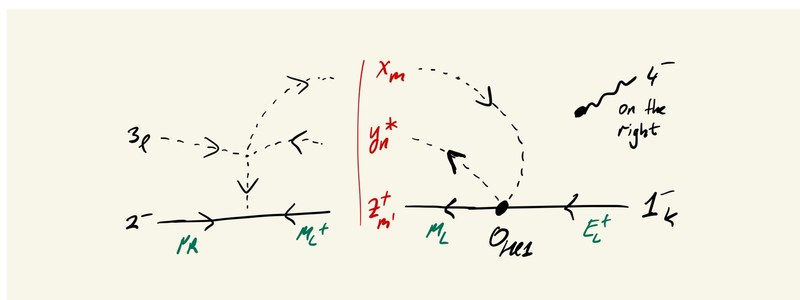


$$= \int \mathcal{A}(2^- 4^- | x_n^+ y^+ z_l^+)\, F_{Hl1}(x_n^+ y^+ z_l^+ 1_k^- 3_m)$$

$$= F_{\text{dipole}} \frac{N_{\text{Colors}} Y_\mu Y_t^2}{(16\pi^2)^2} \left(2e_H + e_{\mu R}\right)$$

- Add a vertex of quartic coupling $-\lambda(H^\dagger H)^2$.





Results

Anomalous dimensions we have got using this method:

$$\mu\partial_\mu \begin{pmatrix} F_{eB} \\ F_{eW} \end{pmatrix} = \frac{N_C Y_t^2}{(16\pi^2)^2} \begin{bmatrix} -Y_\mu(e_{\mu_R} + 2e_h) & -\frac{3}{4} Y_\mu(e_{\mu_R} + 2e_h) & -Y_e(e_{e_R} - e_h) \\ +Y_\mu(2g) & -\frac{1}{4} Y_\mu(2g) & +Y_e(g) \end{bmatrix} \begin{pmatrix} F_{Hl1} \\ F_{Hl3} \\ F_{He} \end{pmatrix}$$

$$+ \frac{\lambda}{(16\pi^2)^2} \begin{bmatrix} 6\, Y_\mu e_h & \frac{6}{4} Y_\mu e_h & 6\, Y_e e_h \\ 2\, Y_\mu g & \frac{6}{4} Y_\mu g & 2\, Y_e g \end{bmatrix} \begin{pmatrix} F_{Hl1} \\ F_{Hl3} \\ F_{He} \end{pmatrix}$$

where e_i's are hypercharges and Y_i's are Yukawa couplings.

Interestingly, it seems quite easy to come up with models (e.g. adding an extra heavy fermion singlet) where **all** 1-loop arrows in LFV Mixing Picture are zero and these 2-loop anomalous dimensions provide the leading effect!

with

$$\mathcal{O}_{Hl1} = (H^\dagger i \overleftrightarrow{\partial_\mu} H)(\bar{E}_L \gamma^\mu M_L)$$
$$\mathcal{O}_{Hl3} = (H^\dagger i \overleftrightarrow{\partial_\mu} \tau^a H)(\bar{E}_L \gamma^\mu \tau^a M_L)$$
$$\mathcal{O}_{He} = (H^\dagger i \overleftrightarrow{\partial_\mu} H)(\bar{e}_R \gamma^\mu \mu_R)$$
$$\mathcal{O}_{eB} = (\bar{E}_L \sigma^{\mu\nu} \mu_R) H B_{\mu\nu}$$
$$\mathcal{O}_{eW} = (\bar{E}_L \sigma^{\mu\nu} \mu_R) \tau^a H W^a_{\mu\nu}$$