On-Shell Symmetry Breaking and the Higgs Mechanism

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Introduction

We show that 'on-shell' techniques are sufficient to reproduce the physics of spontaneous symmetry breaking and the Higgs mechanism, without reference to Lagrangians, quantum fields, and a scalar field acquiring a vacuum expectation value.

In particular, we use helicity and spin spinors, along with group factors (required by consistent factorization) to specify three-particle amplitudes in the IR and UV. We then discover familiar constraints by demanding that high energy (HE) limit of the IR amplitudes match onto UV amplitudes. This work generalizes and extends that presented in Refs. [1, 2]

Scattering Amplitudes as Little Group Tensors

A particle is labeled by its momentum, p, and representation under global symmetry groups, σ, and transforms under some representation of the **little group**.

- For **massless** particles, the little group is $SO(2) = U(1)$, and its representations are specified by its helicity, h. 'Helicity spinors' transform under the little group as,

$$|\lambda\rangle_\alpha \to w^{-1}|\lambda_\alpha\rangle \quad \text{and} \quad |\tilde{\lambda}]_{\dot{\alpha}} \to w|\tilde{\lambda}_{\dot{\alpha}}]\,. \tag{1}$$

- For **massive** particles, the little group is $SO(3) = SU(2)$, and its representations are labeled by its spin, S. 'Spin spinors' transform under the little group as,

$$|\lambda\rangle^I_\alpha \to (W^{-1})^I{}_J|\lambda\rangle^J_\alpha \quad \text{and} \quad |\tilde{\lambda}]^I_\alpha \to W^I{}_J|\tilde{\lambda}]^J_\alpha\,. \tag{2}$$

Scattering amplitudes, $\mathcal{M}$, constructed from helicity and spin spinors, are Lorentz invariant and under the little group,

$$\mathcal{M}(p_a, \rho_a) \to \prod_a (D_{\rho_a\rho'_a}(W))\mathcal{M}((\Lambda p)_a, \rho'_a) \tag{3}$$

where, for massless particles $D_{\rho_a\rho'_a}(W) = \delta_{\sigma_a,\sigma'_a}\delta_{h_a,h'_a}w^{-2h_a}$ and $\rho_a = (h_a, \sigma_a)$, and for massive particles $D_{\rho_a\rho'_a}(W) = \delta_{\sigma_a,\sigma'_a}W^{I_1}_{I'_1}\dots W^{I_{2S}}_{I'_{2S}}$ and $\rho_a = (\{I_1,\dots,I_{2S}\},\sigma_a)$.

The IR

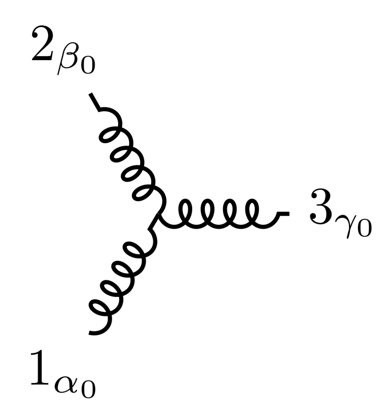
There are d_H massless adjoint gluons from the symmetry group $H \subset G$, with an associated Lie algebra spanned by,

$$\mathfrak{h} = \{X^1, X^2, \dots, X^{\alpha_1}, X^{\beta_1}, X^{\gamma_1}, \dots, X^{d_H}\}\,, \tag{4}$$

such that

$$[X^{\alpha_1}, X^{\beta_1}] = h^{\alpha_1\beta_1}{}_{\gamma_1}X^{\gamma_1}\,. \tag{5}$$

There are $(d_G - d_H)$ massive vectors arising from the 'broken' generators $\{X^{(d_H+1)_0}, \dots, X^{\alpha_0}, X^{\beta_0}, X^{\gamma_0}, \dots, X^{(d_G-d_H)_0}\}$. A general three massive vector amplitude,



$$\frac{h^{\alpha_0\beta_0\gamma_0}}{m_{\alpha_0}m_{\beta_0}m_{\gamma_0}}\langle\mathbf{12}\rangle[\mathbf{12}]\langle\mathbf{3}|p_1 - p_2|\mathbf{3}] + \text{ cyc. }]\,. \tag{6}$$

The UV

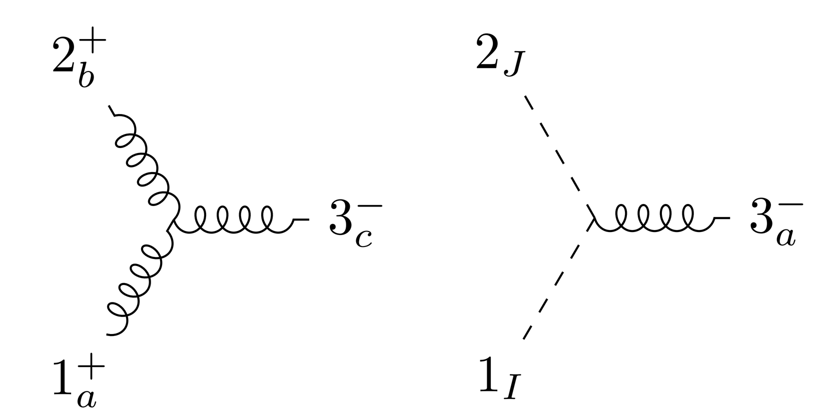
There are d_G massless adjoint gluons from the symmetry group $G = G_1 \times G_2 \times \dots \times G_n$. Each subgroup G_i has an associated coupling g_i, which we use to rescale the generators $g_i\tilde{T}^i = T^i$. The Lie algebra is then spanned by,

$$\mathfrak{g} = \{T^1, T^2, \dots, T^a, T^b, T^c, \dots, T^{d_G}\}\,, \tag{7}$$

which follow the commutation relation

$$[T^c, T^b] = f^{cb}{}_d T^d\,. \tag{8}$$

We have N_ϕ massless scalars, labeled by $\{I, J\}$. Three particle amplitudes in the UV are,

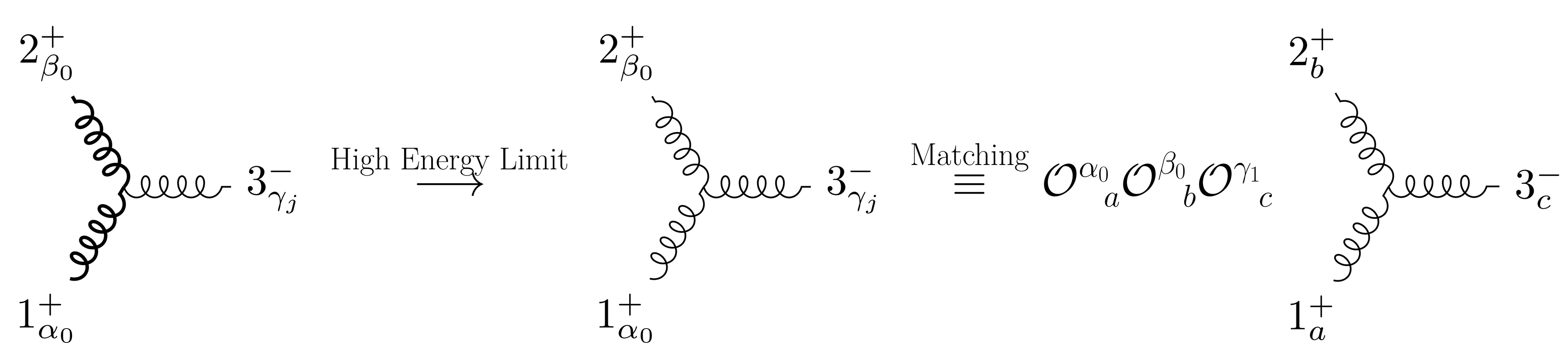


$$f^{abc}\frac{\langle 12\rangle^3}{\langle 31\rangle\langle 23\rangle} \qquad (T^a)_{IJ}\frac{\langle 31\rangle\langle 23\rangle}{\langle 12\rangle} \tag{9}$$

Matching the High Energy Limit of the IR onto UV

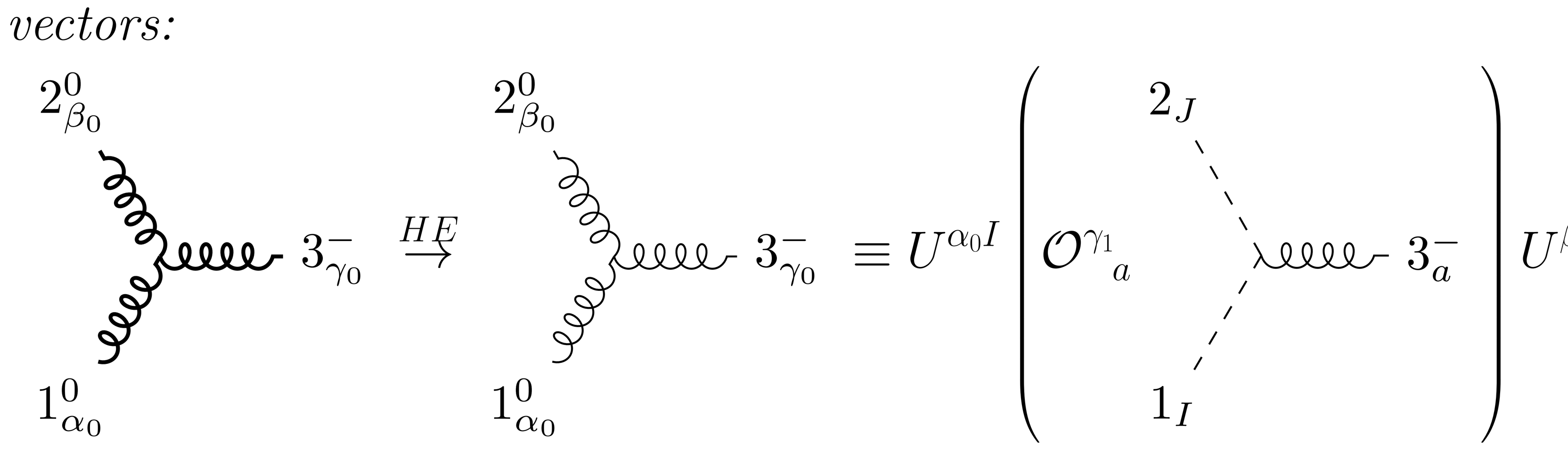
$\mathcal{O} \in SO(d_G)$ matches a linear combination of adjoint labels in the UV to the IR. $U \in SO(N_\phi)$ matches a linear combination of scalars in the UV to the longitudinal component of a massive vector in the IR.

Two massive vectors and one massless gluon:



$$h^{\alpha_1\beta_1\gamma_1}\frac{\langle 12\rangle^3}{\langle 31\rangle\langle 23\rangle} \equiv \mathcal{O}^{\alpha_0}{}_a\mathcal{O}^{\beta_0}{}_b\mathcal{O}^{\gamma_1}{}_c f^{abc}\frac{\langle 12\rangle^3}{\langle 31\rangle\langle 23\rangle} \tag{10}$$

Three massive vectors:



$$h^{\alpha_0\beta_0\gamma_0}\frac{\left((m^{\gamma_0})^2 - (m^{\alpha_0})^2 - (m^{\beta_0})^2\right)}{m_{\alpha_0}m_{\beta_0}}\frac{[23][31]}{[12]} \equiv U^{\alpha_0 I}\left(\mathcal{O}^{\gamma_0}{}_a T^a\right)_{IJ}U^{\beta_0 J}\frac{[23][31]}{[12]} \tag{11}$$

Results

From Eq. (10), we learn that, the coupling constants and generators in the IR are related to those of the UV via,

$$h^{\alpha\beta\gamma} = \mathcal{O}^\alpha{}_a\mathcal{O}^\beta{}_b\mathcal{O}^\gamma{}_c f^{abc}$$
$$X^\alpha = \mathcal{O}^\alpha{}_a T^a\,. \tag{12}$$

From Eq. (11), we have that

$$h^{\alpha_0\beta_0\gamma_0}\frac{\left((m^{\gamma_0})^2 - (m^{\alpha_0})^2 - (m^{\beta_0})^2\right)}{m_{\alpha_0}m_{\beta_0}} \equiv U^{\alpha_0 I}\left(\mathcal{O}^{\gamma_0}{}_a T^a\right)_{IJ}U^{\beta_0 J}\,, \tag{13}$$

which is solved by the ansatz

$$m^{\alpha_0}U^{\alpha_0 I} = (X^{\alpha_0})^{IJ}V_J\,, \tag{14}$$

for some $d_R = N_\phi$ dimensional vector V_J. This is the on-shell incarnation of the massive vector 'eating' the combination $(T^a)^{IJ}\phi_J V_I$. Furthermore, for massless vectors γ_1,

$$m^{\gamma_1} = 0 \Rightarrow (X^{\gamma_1})^{IJ}V_J = 0 \tag{15}$$

which tell us massless particles in the IR correspond to generators of unbroken symmetries. The mass matrix, for massive vectors and massless gluons in the IR are collectively given by,

$$(m^\alpha)^2\delta^{\alpha\beta} = VX^\alpha X^\beta V \tag{16}$$

Outlook

A similar analysis is executed for massive fermions in the IR and massless fermions in the UV, which requires the introduction of Yukawa couplings and mass mixing matrices.

References

[1] Nima Arkani-Hamed, Tzu-Chen Huang, and Yu-tin Huang. Scattering Amplitudes For All Masses and Spins. 9 2017.

[2] Brad Bachu and Akshay Yelleshpur. On-Shell Electroweak Sector and the Higgs Mechanism. *JHEP*, 08:039, 2020.

Acknowledgements

I thank Akshay Yelleshpur and Nima Arkani-Hamed for guidance at all stages of the project. I am supported in part by the Government of the Republic of Trinidad and Tobago.