# The Higgs Boson & Cosmology


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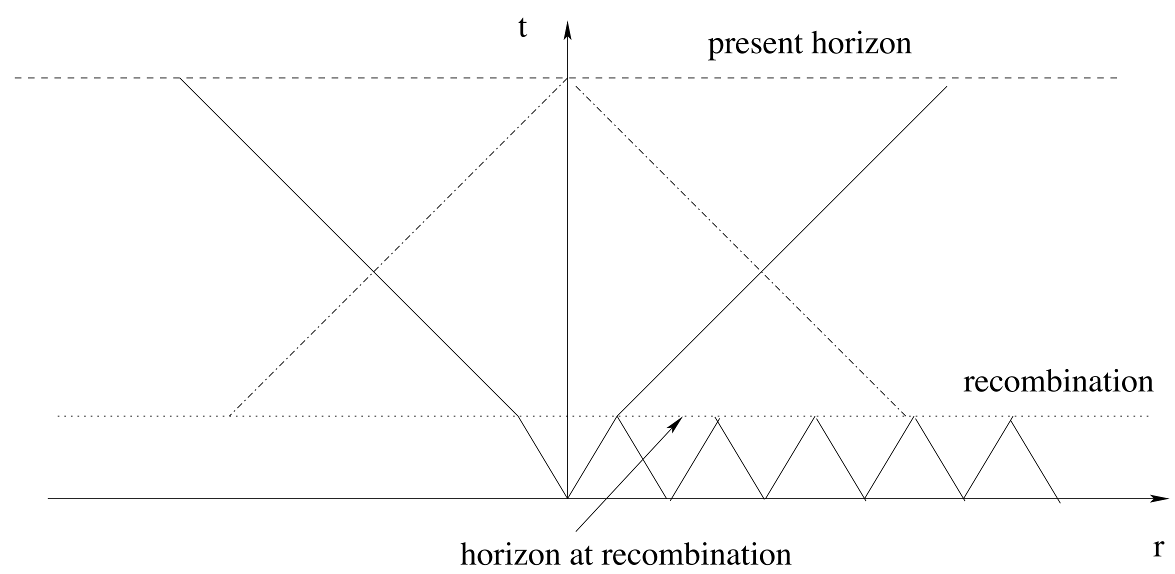
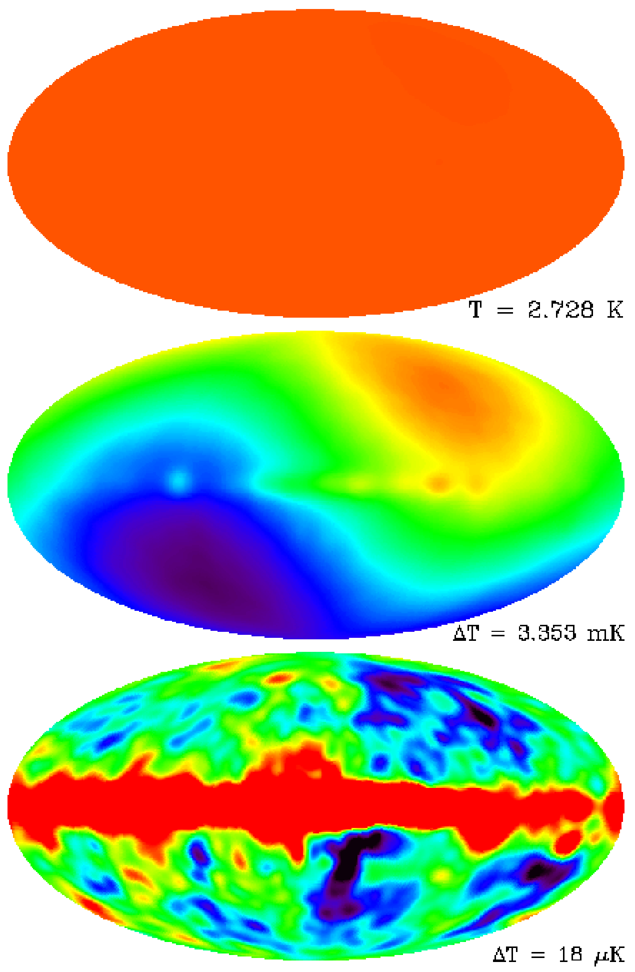
# Outline

- Coming to the Higgs boson from cosmology
- Coming to inflation from particle physics
- Higgs inflation
- Higgs inflation in different theories of gravity: Palatini and Einstein-Cartan
- Conclusions

# Coming to the Higgs boson from cosmology

## Universe at large scales: Why it is  so uniform and isotropic?




Horison :

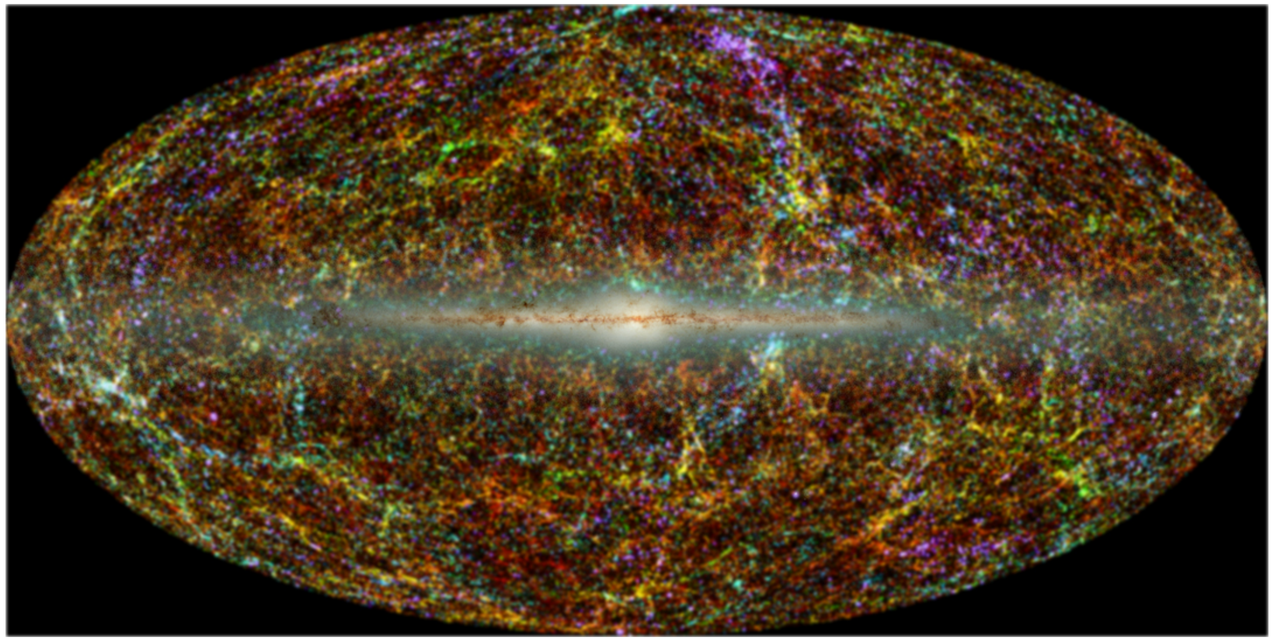
$r \propto t^{1/2}$ (radiation dominated)
$r \propto t^{2/3}$ (matter dominated Universe)

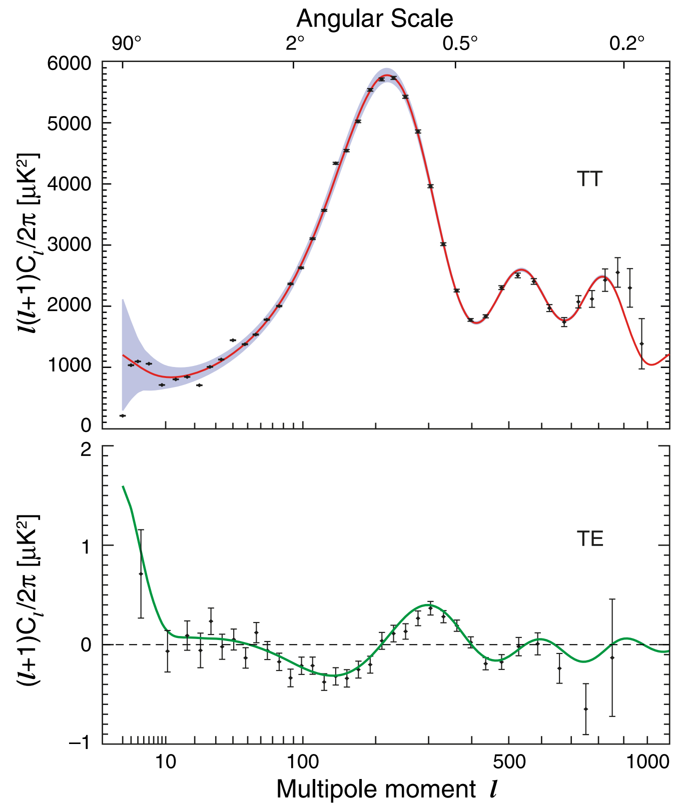
Expected temperature fluctuations at $\theta \sim 1^o : \delta T/T \sim 1$.

Observed fluctuations: $\delta T/T \sim 10^{-5}$

# Coming to the Higgs boson from cosmology

Universe at smaller scales: What is the origin of cosmological perturbations and why their spectrum is almost scale-invariant





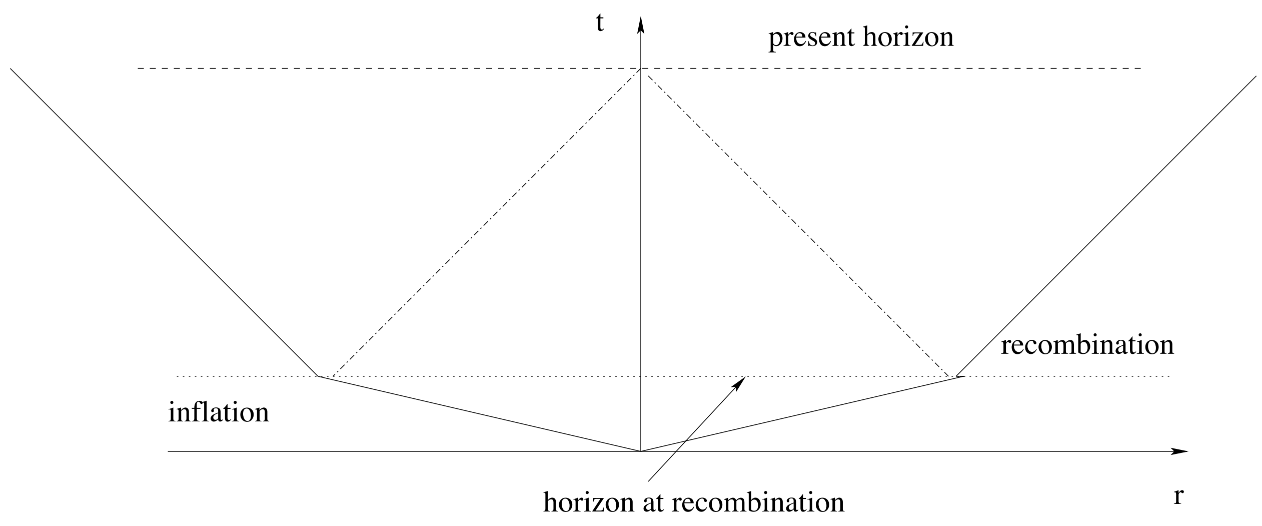
# Coming to the Higgs boson from cosmology

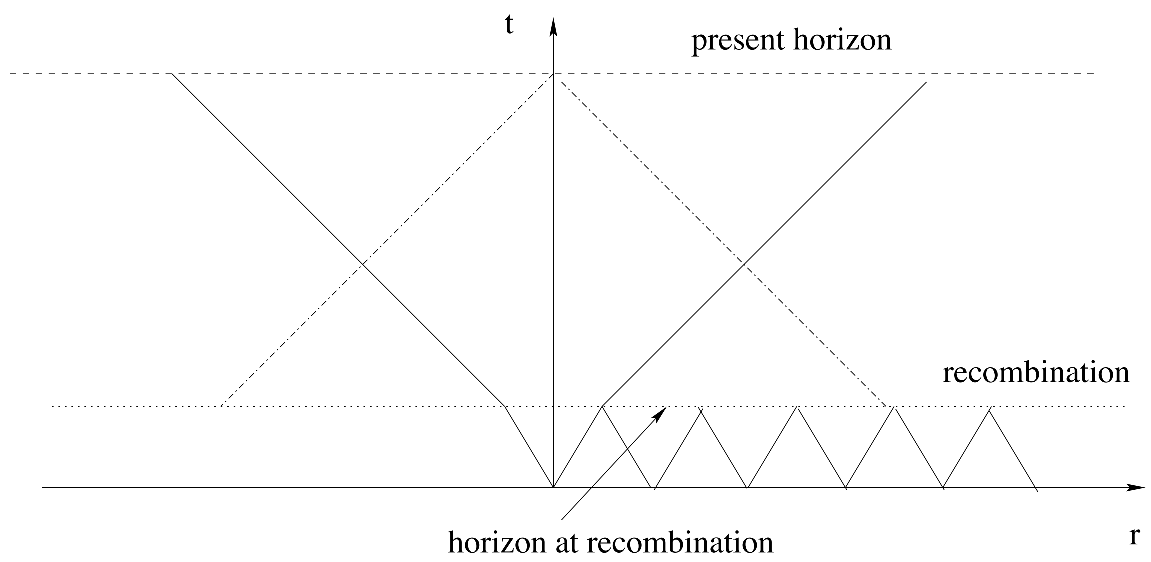
Flatness problem: Why $\Omega_M + \Omega_\Lambda + \Omega_{\rm rad}$ is so close to 1 now and was immensely close to 1 in the past?

This requires enormous fine-tuning of initial conditions (at the Planck scale?) if the Universe was dominated by matter or radiation all the time!

# Coming to the Higgs boson from cosmology

Solution: Inflation = accelerated Universe expansion in the past




# Coming to the Higgs boson from cosmology

## Mechanism: scalar field dynamics + gravity

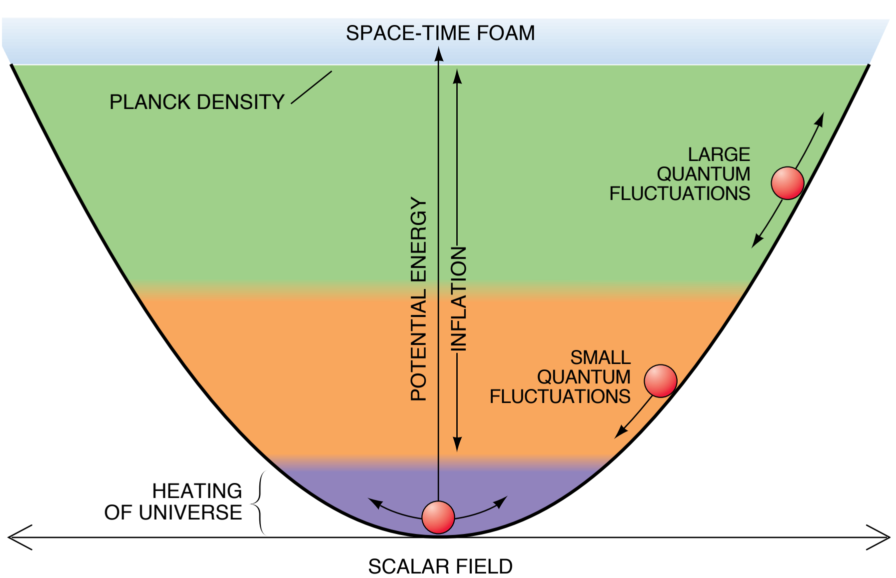
Why scalar?

- Uniform scalar condensate has an equation of state of cosmological constant and leads to exponential universe expansion

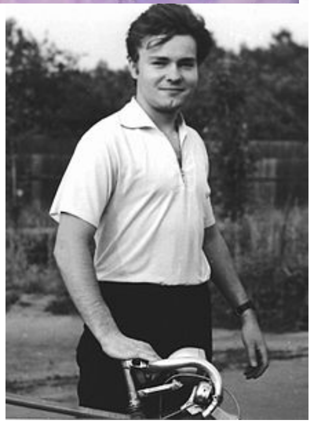
- Vectors or tensors - possible breaking of Lorentz symmetry

- Fermions - fermonic bilinears appearing in stress-energy tensors are scalars, vectors or tensors

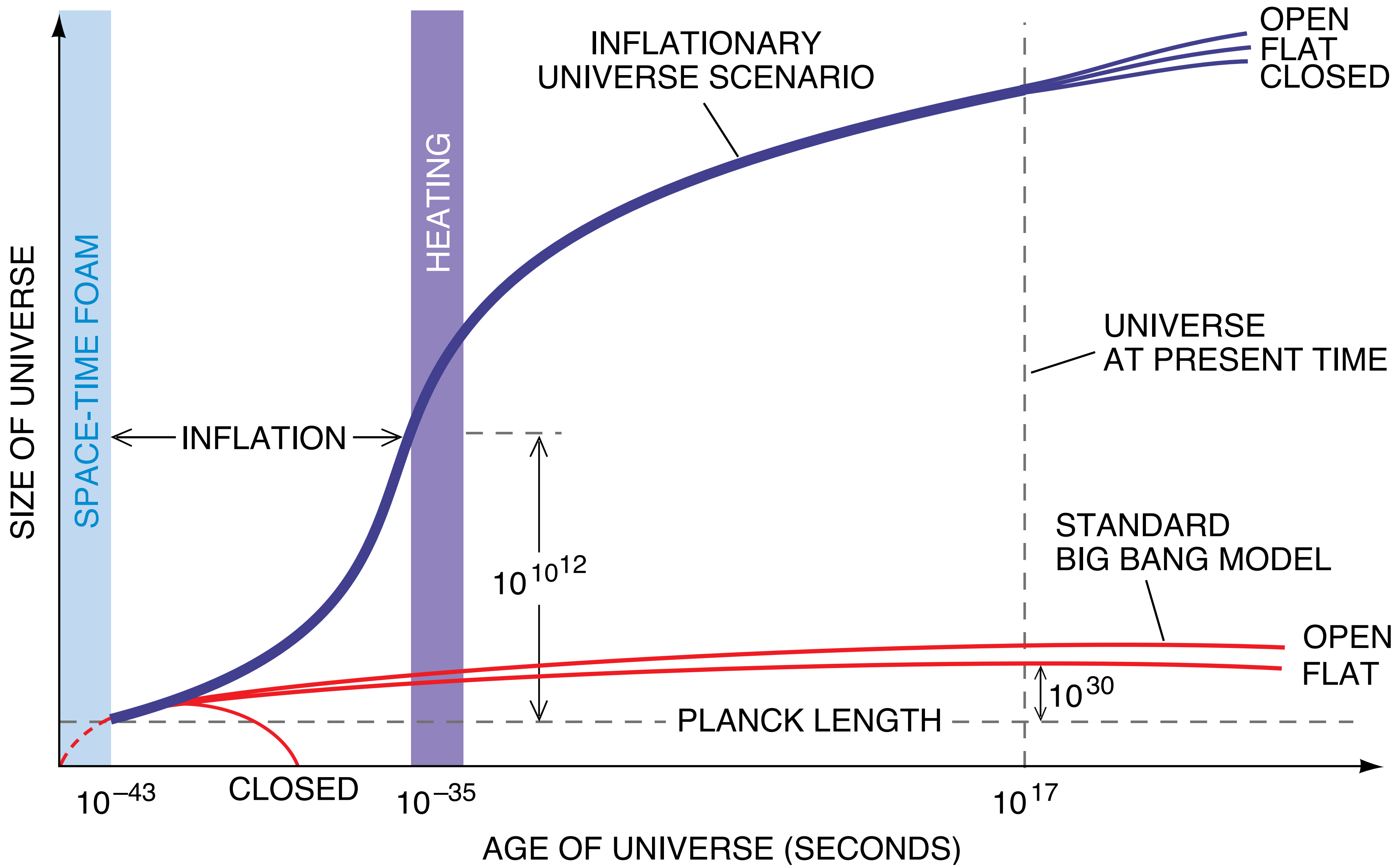
"Standard" chaotic inflation (Linde), $V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4$




Chaotic initial conditions: exponential expansion of the Universe at the beginning. Quantum fluctuations of the scalar field: generation of the density perturbations



# Coming to the Higgs boson from cosmology



# Coming to inflation from particle physics

Challenge for particle physics: what kind of scalar field drives inflation?

Inflation - is it a signal of existence of new particle?

Main requirements from cosmology:

- Nearly flat potential for large scalar fields (chaotic inflation)
- Graceful exit from inflation: sufficiently strong interactions with Standard Model particles, to heat up the Universe and make the Big Bang (radiation dominated expansion)

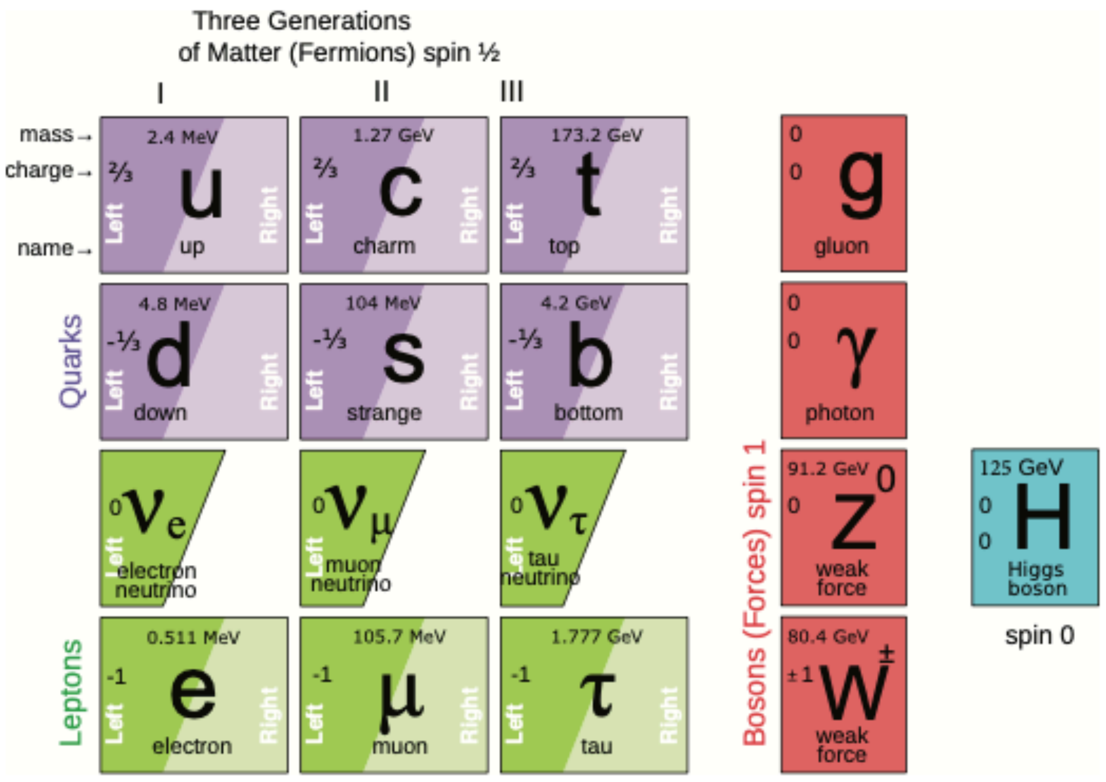
# Breakthrough in High Energy Physics: discovery of fundamental scalar field

The Standard Model (SM) of particle physics was invented in 1967 and completed with the discovery of the Higgs boson at the LHC 45 years later, in 2012.

- SM describes strong, weak and electromagnetic interactions of all known elementary particles

- it is consistent with almost all experiments in particle physics

- it is a self-consistent theory that allows to describe physics at very small and very large energies, possibly running all the way up to the Planck scale $10^{19}$ GeV (15 orders of magnitude larger than the LHC energy!).



# Higgs boson and inflation

All the properties of a new particle with mass 125 GeV are consistent with that of the Standard Model elementary Higgs boson.

Can it inflate the Universe?

Yes, if the Higgs is coupled to gravity in a “non-minimal” way.

Minimal way: just replace ordinary derivatives to covariant derivative.

# Phenomenology of Higgs Inflation

Starting point: the Standard Model and gravity Lagrangian

$$S_G = \int d^4x\sqrt{-g}\left(-\frac{M_P^2}{2}R - \frac{\xi h^2}{2}R\right)$$

should contain a non-minimal coupling of the Higgs field h to gravity, with arbitrary parameter $\xi$, to be fixed (as any other parameter of the SM) by observations.

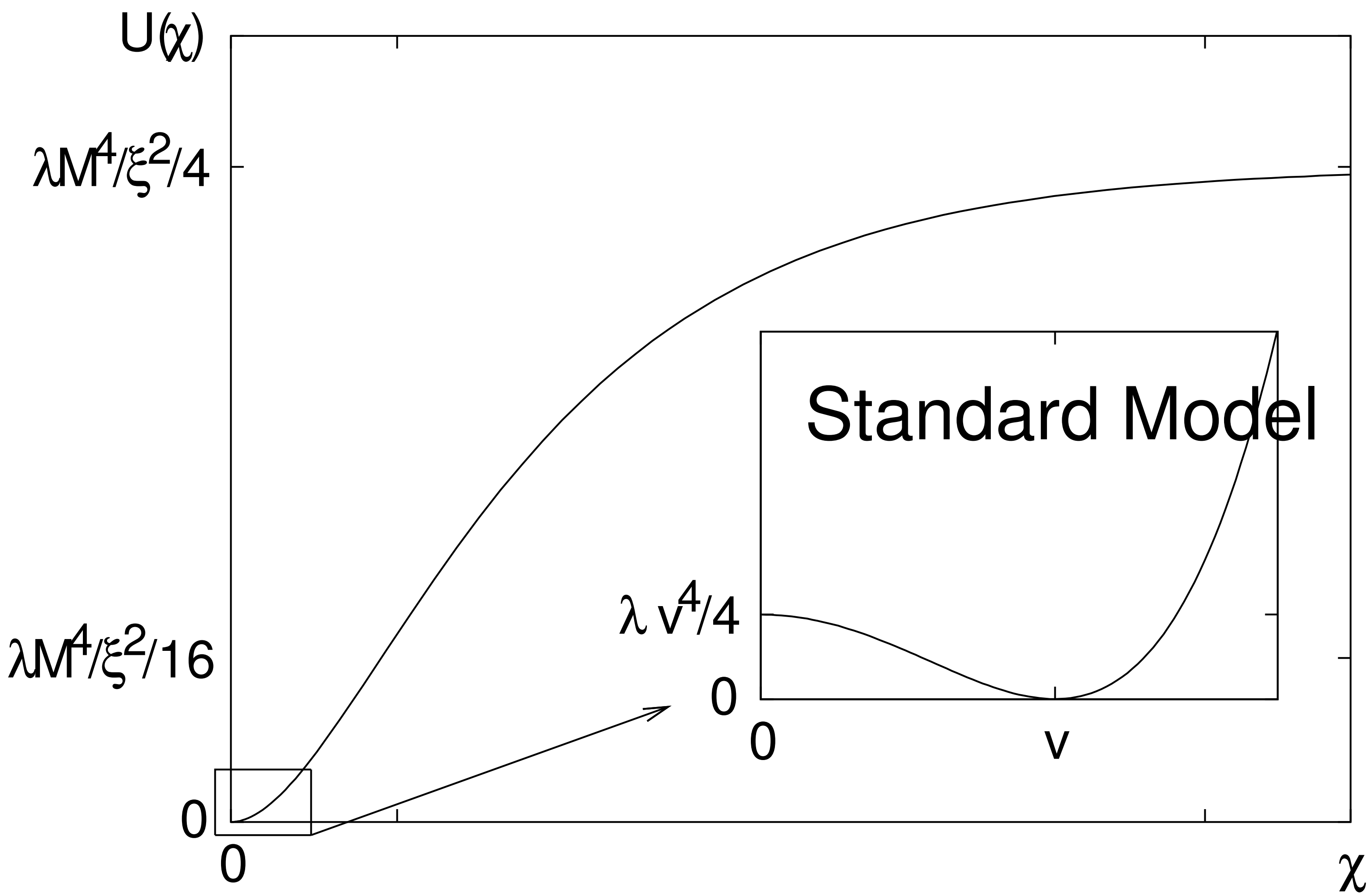
- Interesting physics: gravity strength as well as particle masses are determined by the Higgs field, if

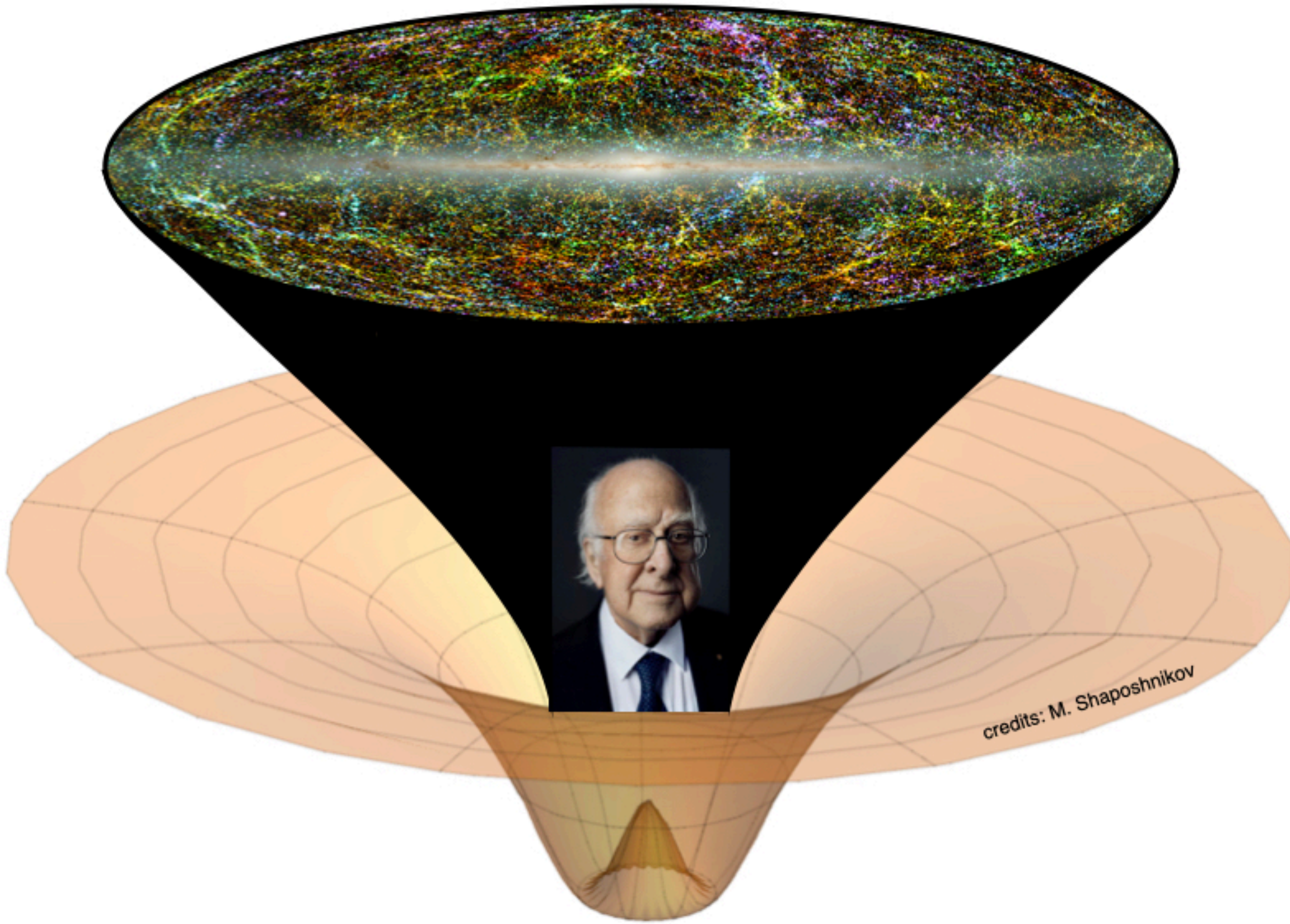
$$\xi h^2 \gg M_P^2$$

- the theory is scale invariant in this limit, leading to flat potential for the Higgs field, exactly what is needed for inflation!

Importance of non-minimal coupling for inflation: Spokoiny; Salopek, Bond, Bardeen;…
Higgs inflation: Bezrukov, MS; derivative coupling of Higgs to gravity: Germani, Kehagias

- Higgs potential in the Einstein frame, with $\chi$ being canonically normalised scalar field related to the Higgs field :






True Mexican hat potential

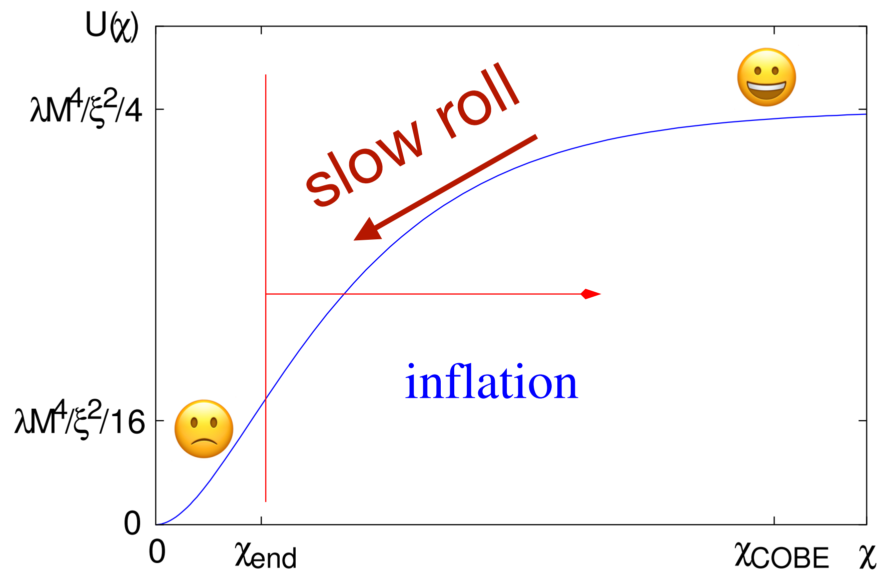
# Stages of Higgs Inflation

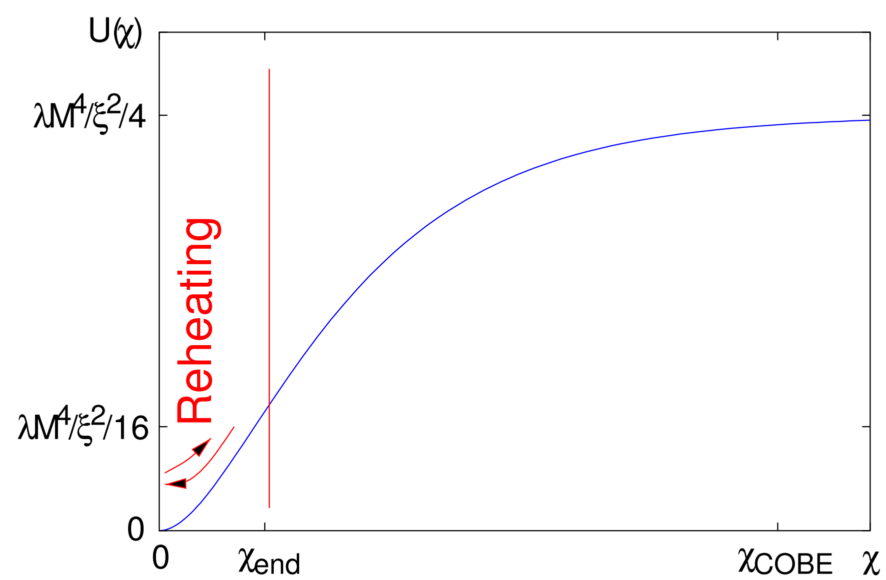
- Chaotic initial conditions: large fields on the plateau inflate, the small fields do not

- Slow roll making the universe flat, homogeneous and isotropic, and producing fluctuations leading to structure formation: clusters of galaxies, etc

- Heating of the Universe : energy stored in the Higgs field goes into the particles of the Standard Model - Higgs makes the Big Bang

- Radiation dominated stage of the Universe expansion starts, leading to baryogenesis, dark matter production, nucleosynthesis…

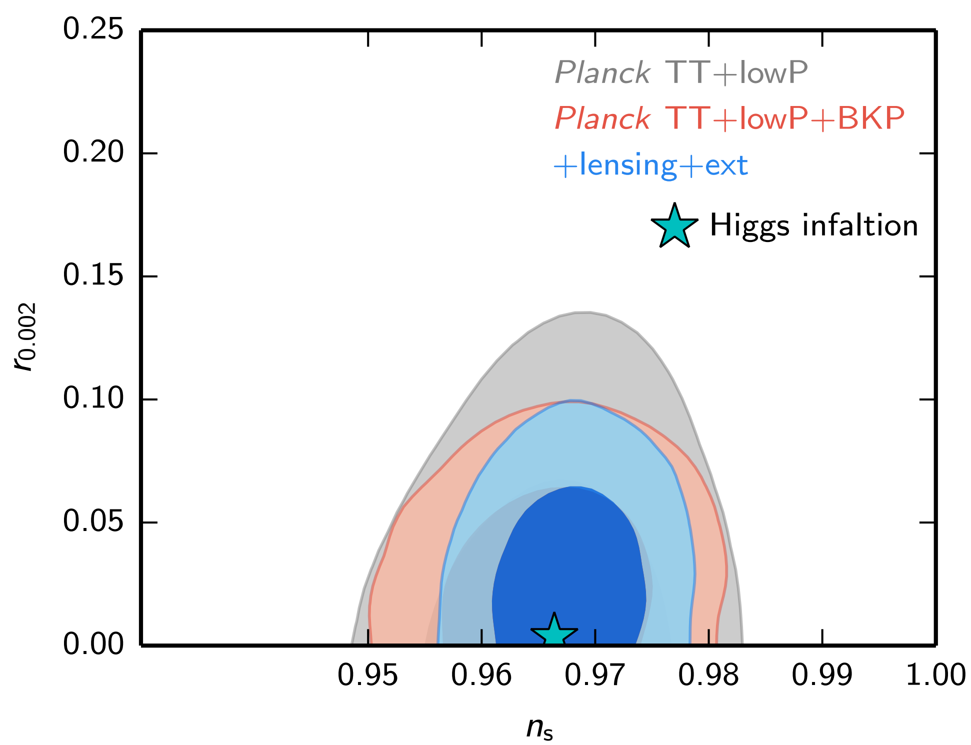


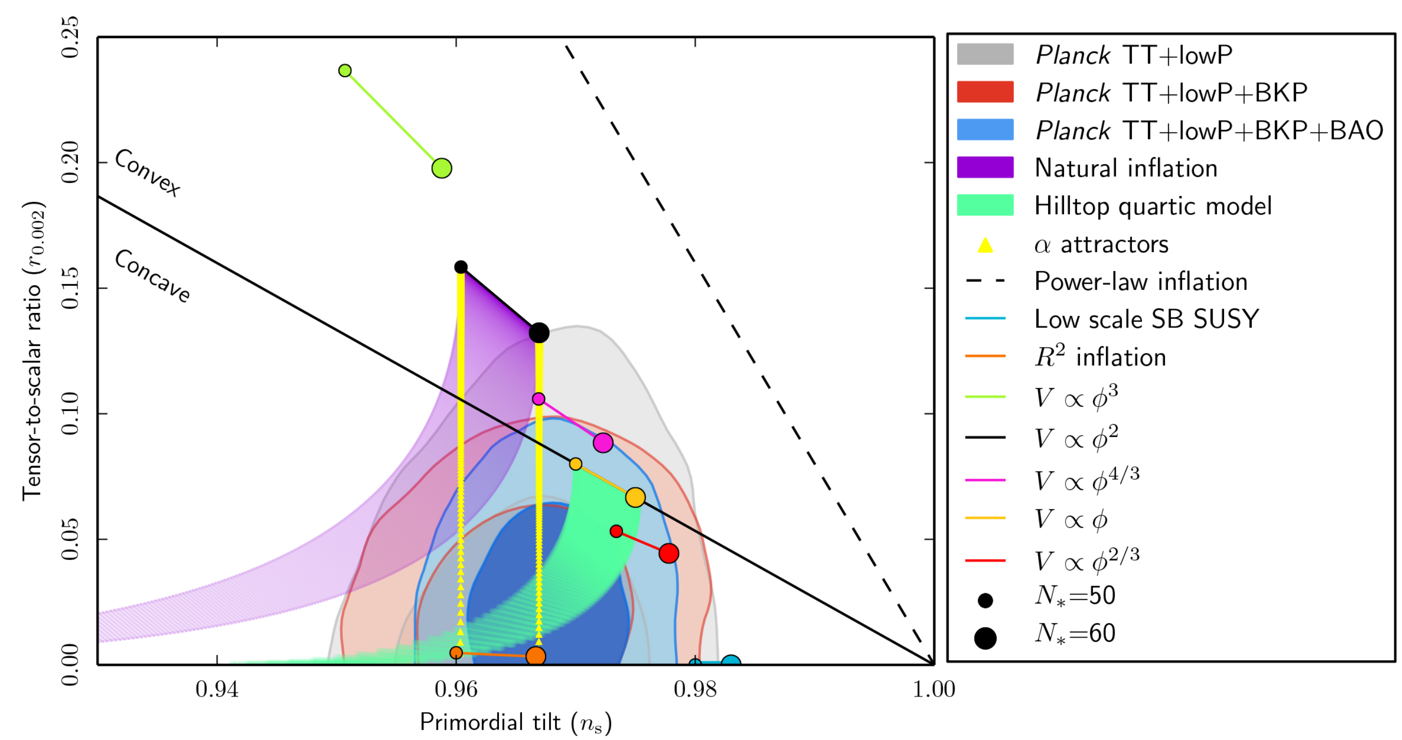



# Predictions

- COBE normalisation: $\xi = 49000\sqrt{\lambda}$ , numerically large $\xi$
- Gaussian perturbations
- Prediction for inflationary observables:

$n_s = 0.97$, $r = 0.0033$




A unique prediction of the Higgs inflation?

No, it actually depends on formulation of gravity.

# Short reminder

Flat Minkowski space-time in arbitrary coordinates $x^\mu = f^\mu(\xi^i)$ ($\xi^i$ - Cartesian coordinates). Metric $g_{\mu\nu}(x)$ and connection $\Gamma^\alpha_{\mu\nu}$ (describing the parallel transport of a vector and covariant derivatives), $dV^\mu = -\Gamma^\mu_{\nu\alpha} V^\nu dx^\alpha$ can be found from coordinate transformation $x^\mu = f^\mu(\xi^i)$ and have the following properties:

1. invariant (length) interval, $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
2. connection is symmetric, $\Gamma^\alpha_{\mu\nu} = \Gamma^\alpha_{\nu\mu}$
3. $g_{\mu\nu;\alpha} = 0$ : metricity - length of a vector is constant at the parallel transport
4. Covariant derivatives commute, $\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu = 0$

# Geometric approach to gravity, Riemann geometry

- distances: symmetric metric tensor
  $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. Same as 1.

- parallel transport of the vector, covariant derivative:
  $dV^\mu = -\Gamma^\mu_{\nu\alpha}V^\nu dx^\alpha$; symmetric connection $\Gamma^\alpha_{\mu\nu} = \Gamma^\alpha_{\nu\mu}$.
  Same as 2

- metricity, local Minkowski structure:
  $g_{\mu\nu;\alpha} = 0 \longrightarrow \Gamma^\alpha_{\mu\nu}$ is a function of the metric $g_{\mu\nu}$. Same as 3.

- Commutator of covariant derivatives: Riemann tensor
  $V_{\alpha;\mu;\nu} - V_{\alpha;\nu;\mu} = R^\beta_{\alpha\mu\nu}V_\beta$. Different from 4!

# Geometric approach to gravity, Cartan geometry, 1922-1925

- distances: symmetric metric tensor
  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$. Same as 1.

- parallel transport of the vector, covariant derivative:
  $dV^\mu = -\Gamma^\mu_{\nu\alpha} V^\nu dx^\alpha$; arbitrary connection $\Gamma^\alpha_{\mu\nu} \neq \Gamma^\alpha_{\nu\mu}$. New object: torsion tensor $T^\alpha_{\mu\nu} = \Gamma^\alpha_{\mu\nu} - \Gamma^\alpha_{\nu\mu}$. Different from 2!

- metricity, local Minkowski structure (same as 3):
  $g_{\mu\nu;\alpha} = 0 \longrightarrow \Gamma^\alpha_{\mu\nu}$ is a function of the metric $g_{\mu\nu}$ and torsion $T^\alpha_{\mu\nu}$

- Commutator of covariant derivatives: Riemann tensor
  $V_{\alpha;\mu;\nu} - V_{\alpha;\nu;\mu} = R^\beta_{\alpha\mu\nu} V_\beta$. Different from 4!

# Dynamics, Einstein-Hilbert metric action (1915)

- Lowest order action (without cosmological constant) is

$$\frac{M_P^2}{2}\int d^4x\sqrt{|g|}\,R$$

- The dynamical variable is $g_{\mu\nu}$ , variation with respect to $g_{\mu\nu}$ gives vacuum Einstein equations. (Metric is mostly positive.)

# Dynamics, Palatini action (1919)

Palatini gravity

Basic structures: metric $g_{\mu\nu}$ (distances) and symmetric connection $\Gamma^{\rho}_{\nu\sigma} = \Gamma^{\rho}_{\sigma\nu}$ . Riemann curvature tensor is expressed via connection as:

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

Same as in metric gravity

Lowest order action (without cosmological constant) is

$$\frac{M_P^2}{2}\int d^4x\sqrt{|g|}R$$

The dynamical variables are $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$ , variation with respect to $\Gamma^{\rho}_{\nu\sigma}$ gives metricity $g_{\mu\nu;\alpha} = 0$, i.e. the relation between $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$, the variation with respect to $g_{\mu\nu}$ gives vacuum Einstein equations.

Palatini pure gravity is equivalent to metric gravity

# Metric, Palatini and Einstein-Cartan gravities

Einstein-Cartan gravity

Basic structures: metric $g_{\mu\nu}$ (distances) and connection $\Gamma^{\rho}_{\nu\sigma} \neq \Gamma^{\rho}_{\sigma\nu}$ . Riemann curvature tensor is expressed via connection as:

$$R^{\rho}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

Same as in metric gravity

New object - torsion tensor: $T^{\rho}_{\nu\sigma} = \Gamma^{\rho}_{\nu\sigma} - \Gamma^{\rho}_{\sigma\nu}$

Lowest order action (without cosmological constant) is

Holst term

Same as in metric gravity

$$\frac{M_P^2}{2}\int d^4x\sqrt{|g|}R + \frac{M_P^2}{2\gamma}\int \mathrm{d}^4x\sqrt{|g|}\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$$

Barbero-Immirzi parameter

The dynamical variables are $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$ , variation with respect to $\Gamma^{\rho}_{\nu\sigma}$ gives the relation between $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$, the variation with respect to $g_{\mu\nu}$ gives vacuum Einstein equations. On the solution $g_{\mu\nu;\alpha} = 0$ and $T^{\rho}_{\nu\sigma} = 0$.

Einstein-Cartan pure gravity is equivalent to metric gravity

# EC gravity as a gauge theory

Existence of electromagnetic field - U(1) global invariance of fermion Lagrangian promoted to be local

Gluons, $W^+$, $W^-$ Z and $\gamma$ of the Standard Model - SU(3)xSU(2)xU(1) global invariance of SM fermion Lagrangian promoted to be local

Existence of gravitational field - Poincare invariance of SM fermion Lagrangian promoted to be local?

Gauging of the Poincare group: Utiyama (1956), Kibble (1961) , Sciama (1962,1964).

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**Invariant Theoretical Interpretation of Interaction**

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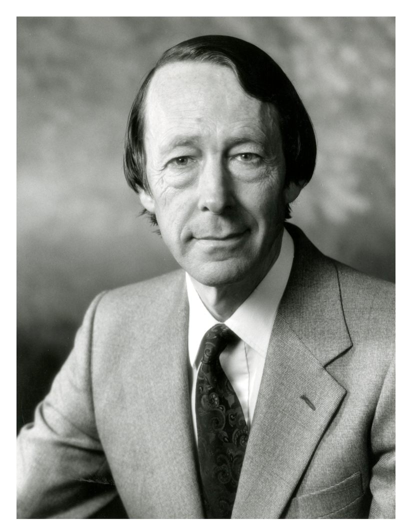
(Received July 7, 1955)



# EC gravity as a gauge theory

Basic gauge fields

- $e^I$ - tetrad one-form (frame field, translations), I=0,1,2,3
- $\omega^{IJ}$ - spin connection one form (gauge field of the local Lorentz group). Euclidean: $SO(4) \sim SU(2)_L \times SU(2)_R$
- $F^{IJ} = d\omega^{IJ} + \omega^I_K \omega^{KJ}$ : curvature 2-form

Pure gauge action:

$$\frac{M_P^2}{4}\int \epsilon_{IJKL} e^I e^J F^{KL} + \frac{M_P^2}{2\gamma}\int e^I e^J F_{IJ}$$

Again, equivalent to metric gravity

# EC gravity with matter fields

Once matter fields are added, the equivalence between different formulation of gravity is lost:

Couplings to

- scalar fields: $\phi^2 \epsilon_{IJKL} e^I e^J F^{KL}$ (or $\phi^2 R$)
- and to fermion fields via covariant derivative $D\Psi = d\Psi + \frac{1}{8}\omega_{IJ}[\gamma^I, \gamma^J]\Psi$

lead to modified relation between the spin connection and tetrad (or Christoffel symbols and the metric). Torsion is non-zero in general,
$\delta\Gamma^\rho_{\mu\nu} \propto g^{\rho\alpha} g_{\mu\nu} \partial_\alpha(\phi^2/M_P^2) + \ldots$

Physics is different!

# Bosonic action in EC gravity with Higgs field

Inclusion of the scalar field (Higgs field of the Standard Model, unitary gauge)

Scalar action

$$S_h = \int \mathrm{d}^4x \sqrt{-g} \left( -\frac{1}{2} \left(\partial_\mu h\right)^2 - U(h) \right), \quad U(h) = \frac{\lambda}{4} \left(h^2 - v^2\right)^2$$

Gravity part

Same as in metric gravity

Holst term

Nieh-Yan invariant

$$S_{\text{grav}} = \frac{1}{2} \int \mathrm{d}^4x \sqrt{-g} \left(M_P^2 + \xi h^2\right) R$$
$$+ \frac{1}{2\bar{\gamma}} \int \mathrm{d}^4x \sqrt{-g} \left(M_P^2 + \xi_\gamma h^2\right) \epsilon^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}$$
$$+ \frac{1}{2} \int \mathrm{d}^4x \xi_\eta h^2 \partial_\mu \left(\sqrt{-g} \epsilon^{\mu\nu\rho\sigma} T_{\nu\rho\sigma}\right)$$

Three non-minimal couplings: $\xi, \xi_\gamma, \xi_\eta$

For $1/\bar{\gamma} = \xi_\gamma = \xi_\eta = 0$ we get the Palatini action with non-minimal coupling.

# Bosonic action in EC gravity with Higgs field

- Torsion is not dynamical
- Same number of degrees of freedom as in the metric gravity + scalar field : 2 (graviton) + 1 (scalar)
- Equivalent metric theory : use the Weyl transformation of the metric field

$$g_{\mu\nu} \to \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

Modified kinetic term: essential for inflation

Metric action:

$$S_{\text{metric}} = \frac{M_P^2}{2} \int d^4x \sqrt{|g|} \left\{ R - \left[ \frac{1}{2\Omega^2} (\partial_\mu h)^2 + \frac{U}{\Omega^4} \right] - \frac{3M_P^2}{4(\gamma^2+1)} \left( \frac{\partial_\mu \bar{\eta}}{\Omega^2} + \partial_\mu \gamma \right)^2 \right\}$$

$$\gamma = \frac{1}{\bar{\gamma}\Omega^2} \left( 1 + \frac{\xi_\gamma h^2}{M_P^2} \right), \quad \bar{\eta} = \frac{\xi_\eta h^2}{M_P^2}$$

Flat potential: essential for inflation

# Inflation

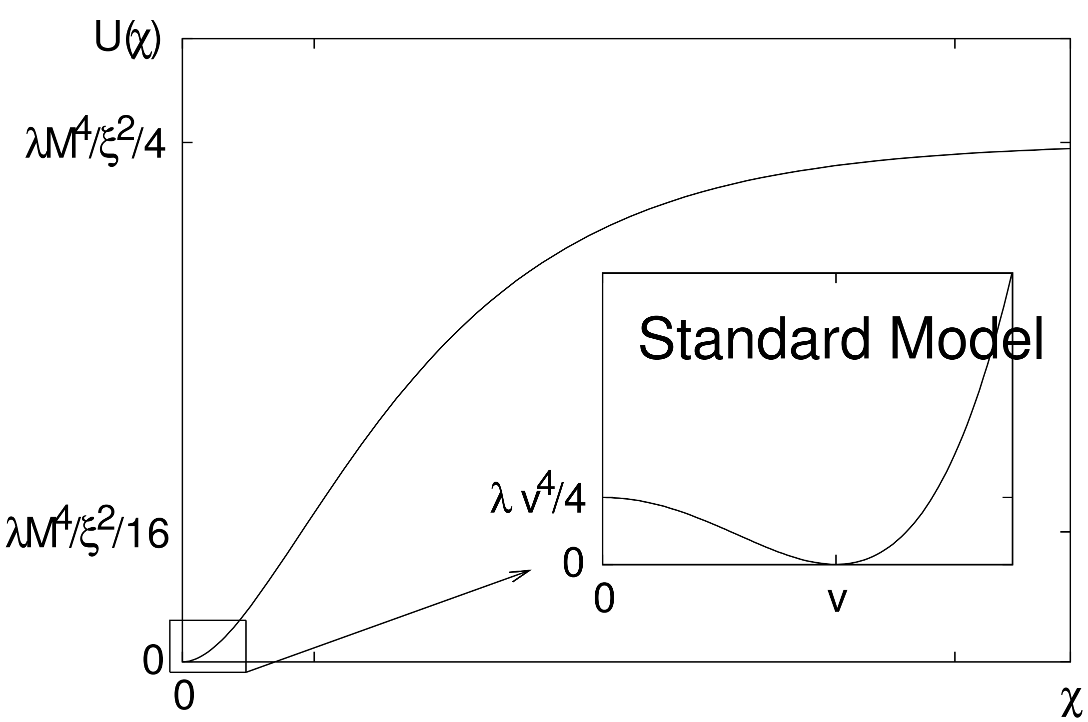
Modified kinetic term:
essential for inflation

Metric action:

$$S_{\text{metric}} = \frac{M_P^2}{2}\int d^4x\sqrt{|g|}\left\{R - \left[\frac{1}{2\Omega^2}(\partial_\mu h)^2 + \frac{U}{\Omega^4}\right] - \frac{3M_P^2}{4(\gamma^2+1)}\left(\frac{\partial_\mu\bar{\eta}}{\Omega^2} + \partial_\mu\gamma\right)^2\right\}$$

$$\gamma = \frac{1}{\bar{\gamma}\Omega^2}\left(1 + \frac{\xi_\gamma h^2}{M_P^2}\right), \quad \bar{\eta} = \frac{\xi_\eta h^2}{M_P^2}, \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

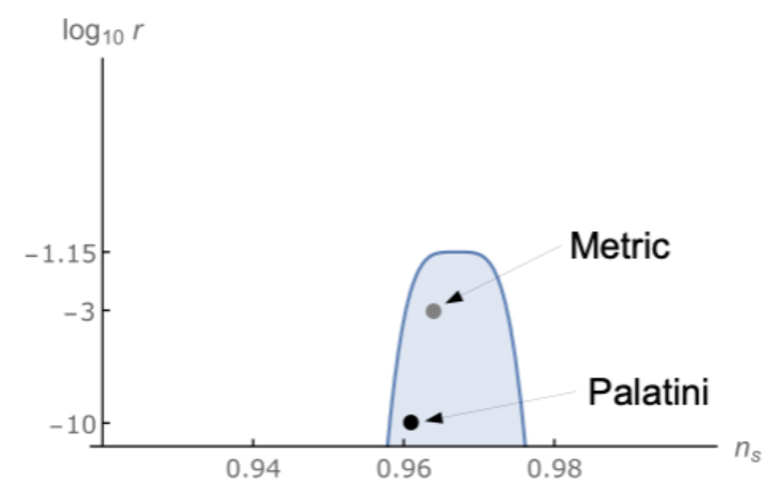
Flat potential:
essential for inflation




Metric Higgs inflation (Bezrukov, MS):
in limit of the vanishing Holst term,
$\bar{\gamma} \to \infty, \quad \xi_\gamma = 0,$ take $\quad \xi_\eta = \xi.$

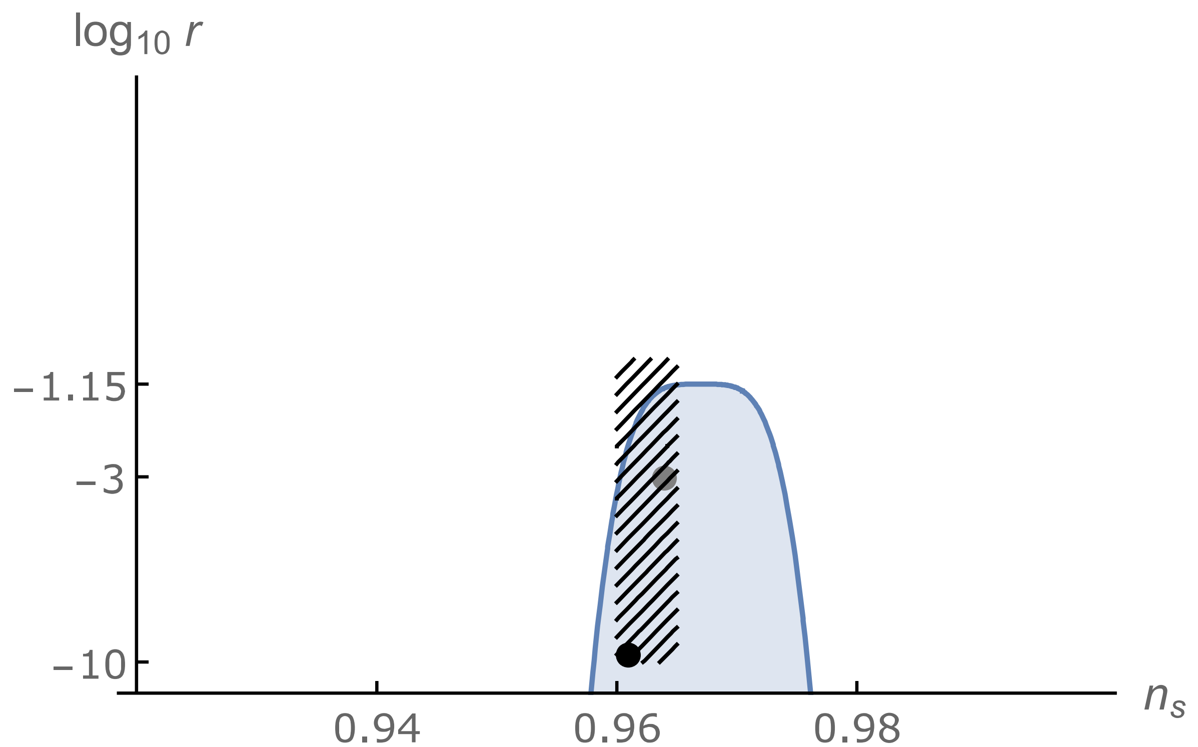
Palatini Higgs inflation (Bauer, Demir):
in limit of the vanishing Holst term,
$\bar{\gamma} \to \infty, \quad \xi_\gamma = 0,$ take $\quad \xi_\eta = 0$

# Generic Einstein-Cartan Higgs inflation




## Observations:

- Inflation is a generic phenomenon.
- Large parts of the parameter space reproduce the predictions of either metric or Palatini Higgs inflation.
- The spectral index $n_s$ is mostly independent of the choice of couplings and lies very close to $n_s = 1 - 2/N$ .
- The tensor-to-scalar ratio r can vary between 1 and $10^{-10}$. Detection of r in near future?



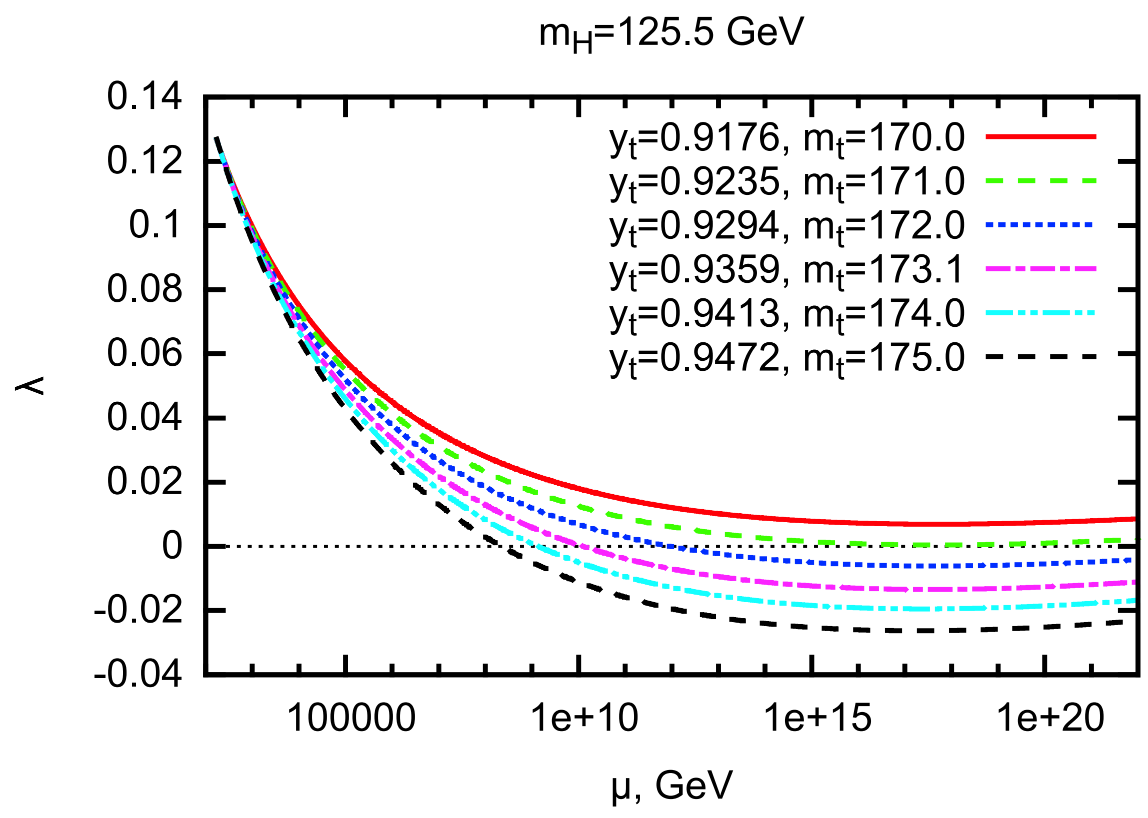
# Challenges in Higgs inflation

1. Classical theory of the Higgs inflation should be promoted to the quantum theory of Higgs inflation. Any theory of inflation involves gravity which is non-renormalisable. An approach to every type of inflation (and HI in particular) should be formulated if the framework of some effective theory and be self-consistent. Can be worked out : Bezrukov, Magnin, M.S., Sibiryakov.

2. The SM parameters are measured at small energies ~ 100 GeV, whereas inflation takes place at high energies: radiative corrections and RG running must be accounted for. What happens if the SM vacuum is metastable?

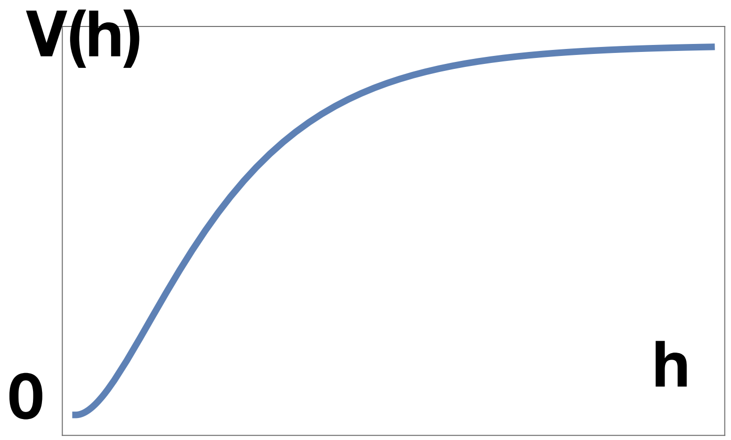
# Challenge 2: possible vacuum metastability

Behaviour of the scalar self-coupling λ: depending on the top quark Yukawa coupling, λ may cross zero at energies as small as $10^{11}$ GeV (for larger $m_t$) or never cross it (for smaller $m_t$). For all admissible SM parameters, $|\lambda| \sim 0.01$ in inflationary region, much smaller than at low energies

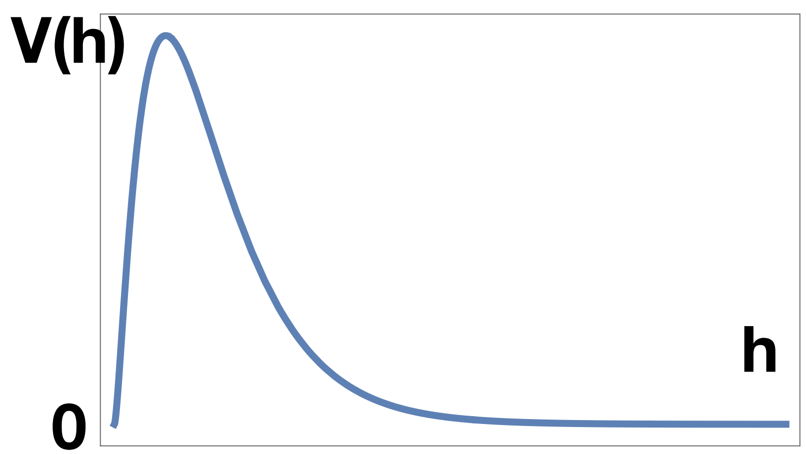



This behaviour may change the form of the Higgs potential at large h

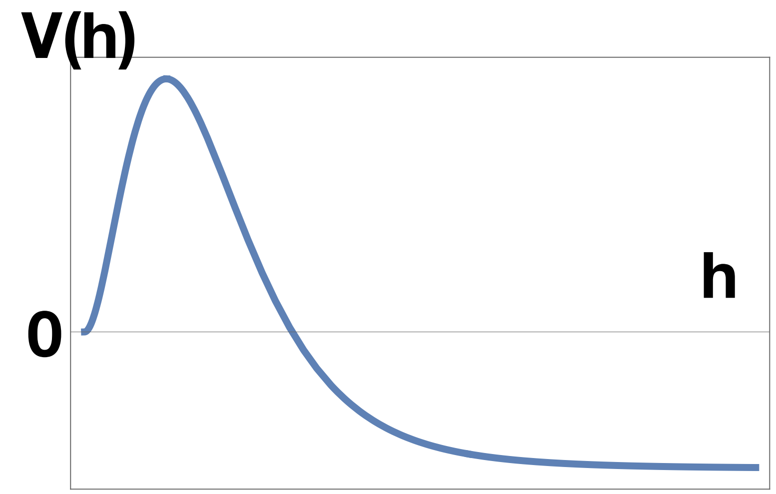
# Naive RG: $V(h)=\lambda(h)h^4$




**Stability**



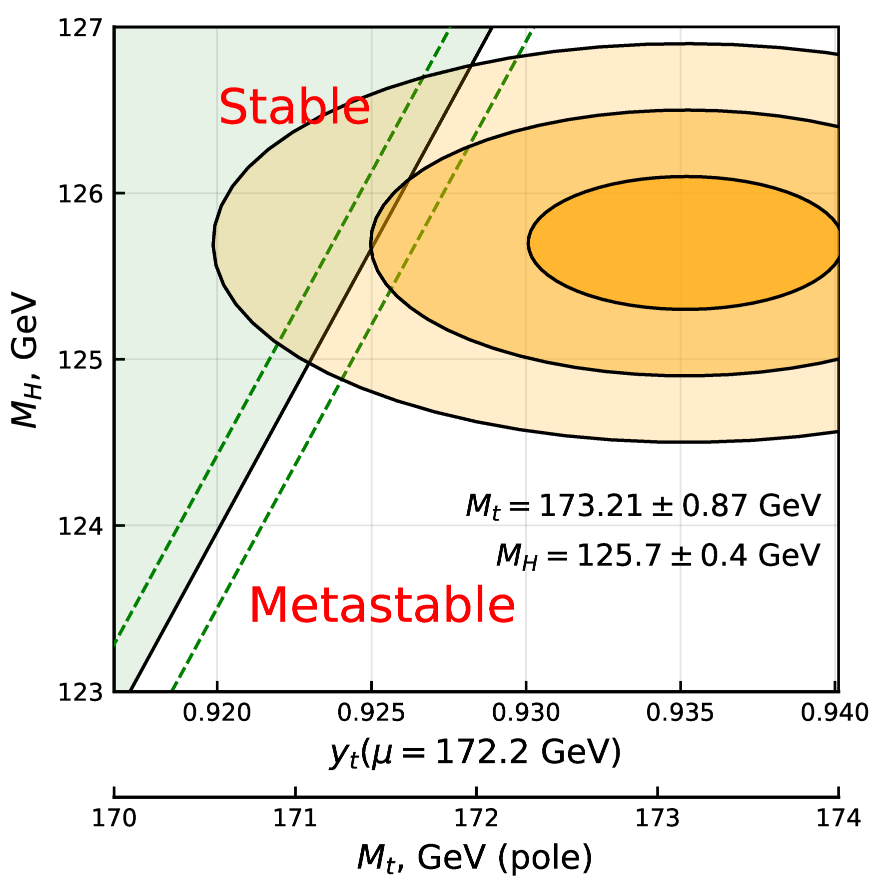

**Criticality**




**Metastability**

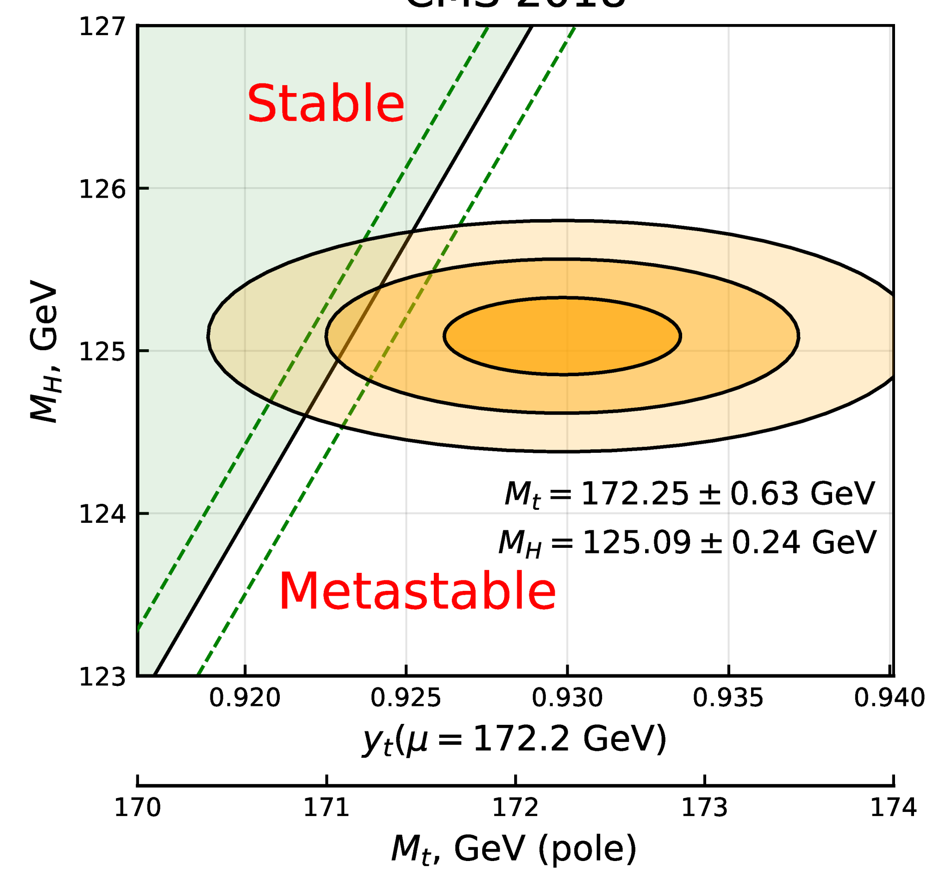
- Marginal evidence (less than 2σ) for the SM vacuum metastability given uncertainties in relation between Monte-Carlo top mass and the top quark Yukawa coupling

**2015**



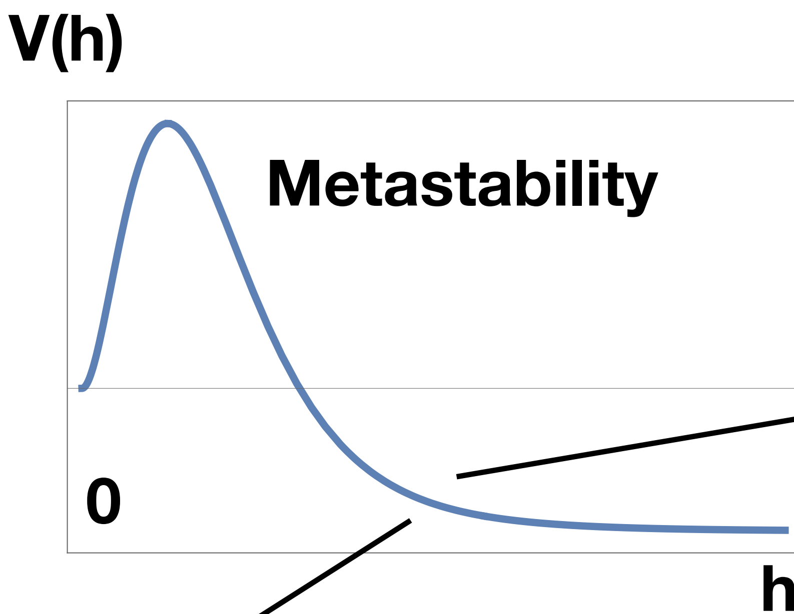

**Time evolution of SM vacuum metastability**

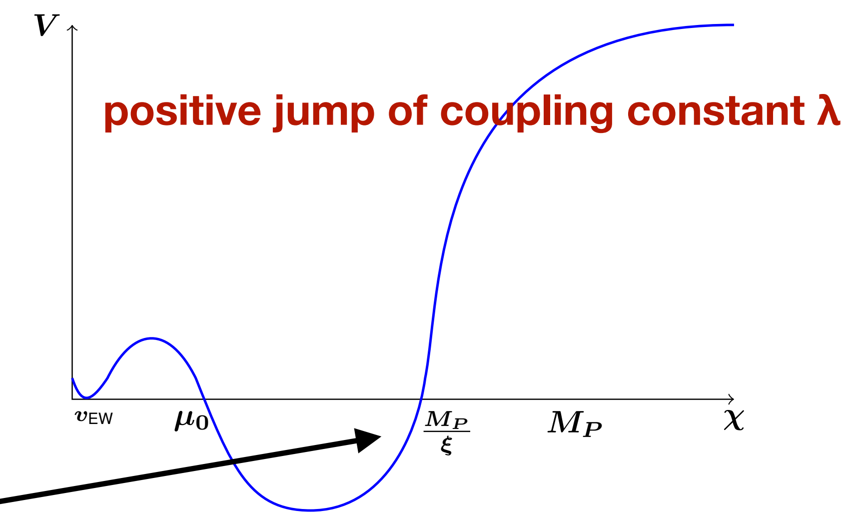
CMS 2018



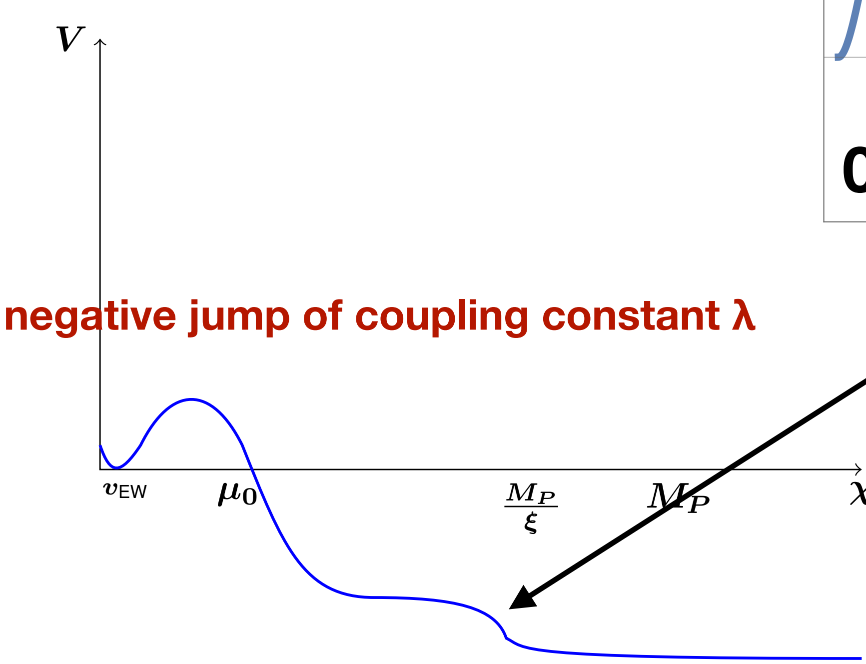
# Higgs inflation with metastable vacuum

Account for high energy corrections to naive RG running: jumps of the coupling constants at the scale $M_P/\xi$ (metric) or $M_P/\sqrt{\xi}$ (Palatini)



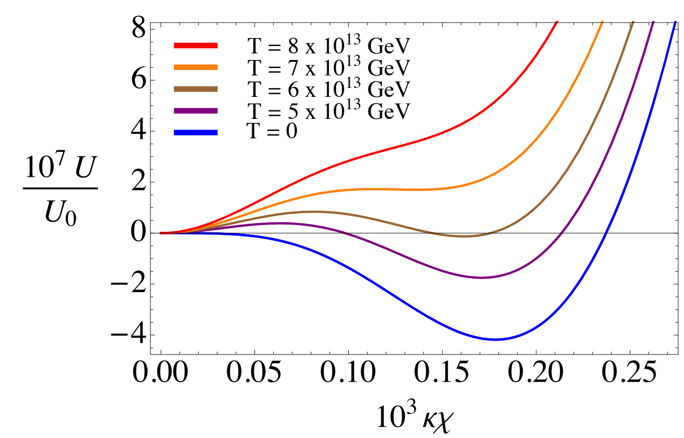




Higgs inflation works if the potential has this form, as reheating brings the Higgs field to the origin.




Higgs inflation does not work if the potential has this form

**Symmetry restoration**



# Conclusions

The SM Higgs field can play an important role in cosmology:

- It can make the Universe flat, homogeneous and isotropic.
- Quantum fluctuations of the Higgs field can lead to structure formation.
- Coherent oscillations of the Higgs field can make the Hot Big Bang and produce all the matter in the Universe.

Topics I had no time to discuss:

- Real and virtual Higgs boson can play a crucial role in baryogenesis leading to charge asymmetric Universe (lecture by Patrick Meade).
- Dark Matter production may come about as an effect of mixing between neutrinos and heavy neutral leptons, induced by the Higgs field.