Higgs & gravity connection?

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Current theory of (almost) all observed phenomena

Combination of the Standard Model and gravity which describes (almost: except dark matter, neutrino masses, and baryon asymmetry of the Universe) all observed phenomena. Action:

$$S = S_{\rm SM} + S_{\rm gravity}$$

In $S_{\rm SM}$:

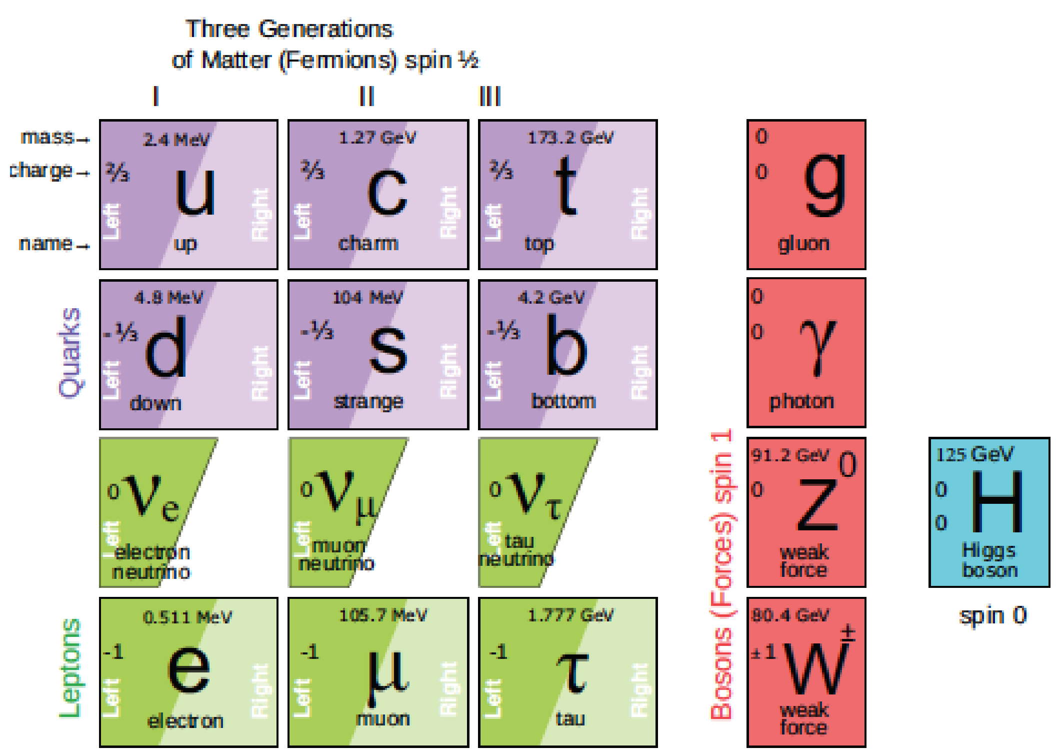
- replace Minkowski metric $\eta_{\mu\nu}$ by arbitrary metric $g_{\mu\nu}$
- replace integration measure $d^4x \to \sqrt{-g}\, d^4x$
- replace ordinary derivatives by covariant derivatives: $O_{,} \to O_{;}$

Simplest gravitational action:

$$S_{\rm gravity} = \int \sqrt{-g}\, d^4x \left(\frac{1}{2} M_P^2 R - \epsilon_{\rm vac} \right)$$

Little fix to describe all observed phenomena

νMSM- Neutrino Minimal Standard Model: 3 extra neutrinos $N_{1,2,3}$ can solve simultaneously three outstanding problems of the Standard Model. They can give masses to ordinary neutrinos, one of them can be a dark matter particle. They can also explain why the Universe contains more matter than antimatter.



Standard Model

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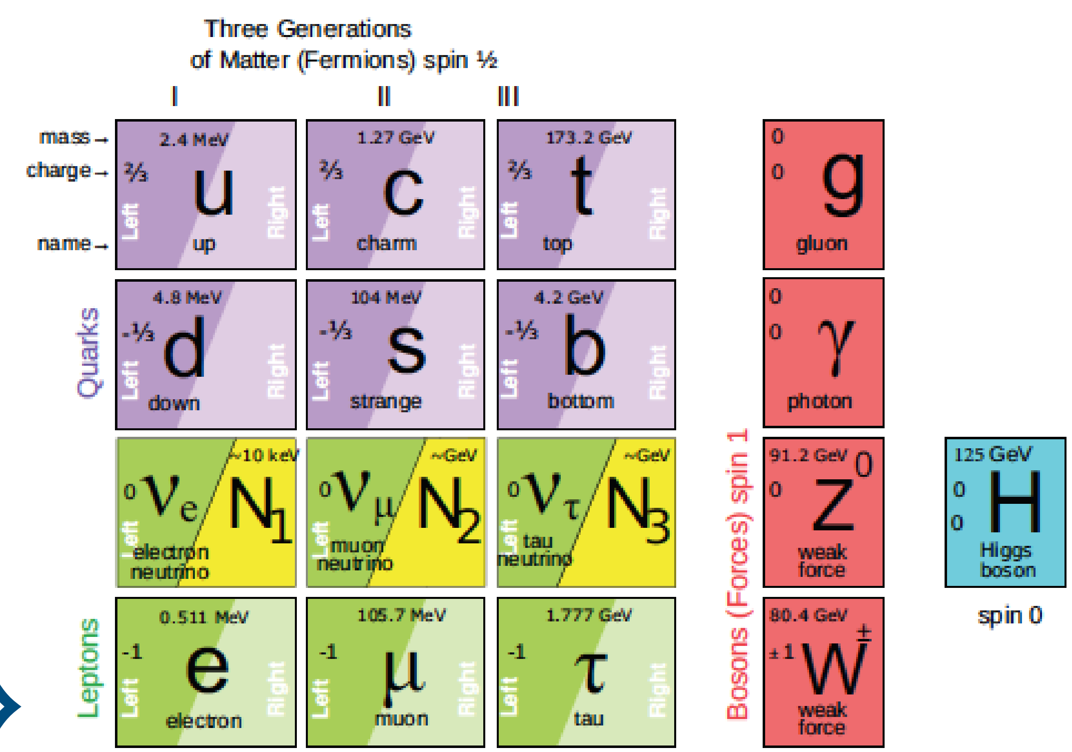


νMSM

Higgs boson can inflate and heat up the Universe, Lecture 2

Observed scales Nature

- Scale of quantum gravity, related to Newtons constant, $G_N = 6.7 \times 10^{-39}$ GeV^{-2} , $M_P = 2.435 \times 10^{18}$ GeV
- Fermi scale, associated with electroweak interactions, $G_F = 1.17 \times 10^{-5}$ GeV^{-2}, $M_W = 80.38$ GeV
- Cosmological constant, or vacuum energy, or Dark Energy, $\epsilon_{vac} = \left(2.24 \times 10^{-3}\mathrm{eV}\right)^4$

Hierarchy: $\left(\epsilon_{vac}\right)^{\frac{1}{4}} \ll M_W \ll M_P$

Are these scales independent?

Outline

- Newton constant of gravity from the Standard Model?
- Asymptotic safety of gravity and of the Standard model: Higgs self-coupling.
- Conformal symmetry, gravity, and the electroweak symmetry breaking.
- Conclusions

SM Model at high energies

Make the following exercise:

- Take the SM and forget about gravity.
- Consider the renormalisation group evolution of all the couplings of the SM, full running is available in 3-loop approximation.

Relevant RG equations

One-loop approximation, $t = log(\mu/M_W)$:

$$16\pi^2 \frac{dg}{dt} = -\frac{19}{6} g^3$$, SU(2) gauge coupling

$$16\pi^2 \frac{dg'}{dt} = \frac{41}{6} g'^3$$, U(1) gauge coupling

$$16\pi^2 \frac{dg_3}{dt} = -7 g_3^2$$, SU(3) gauge coupling

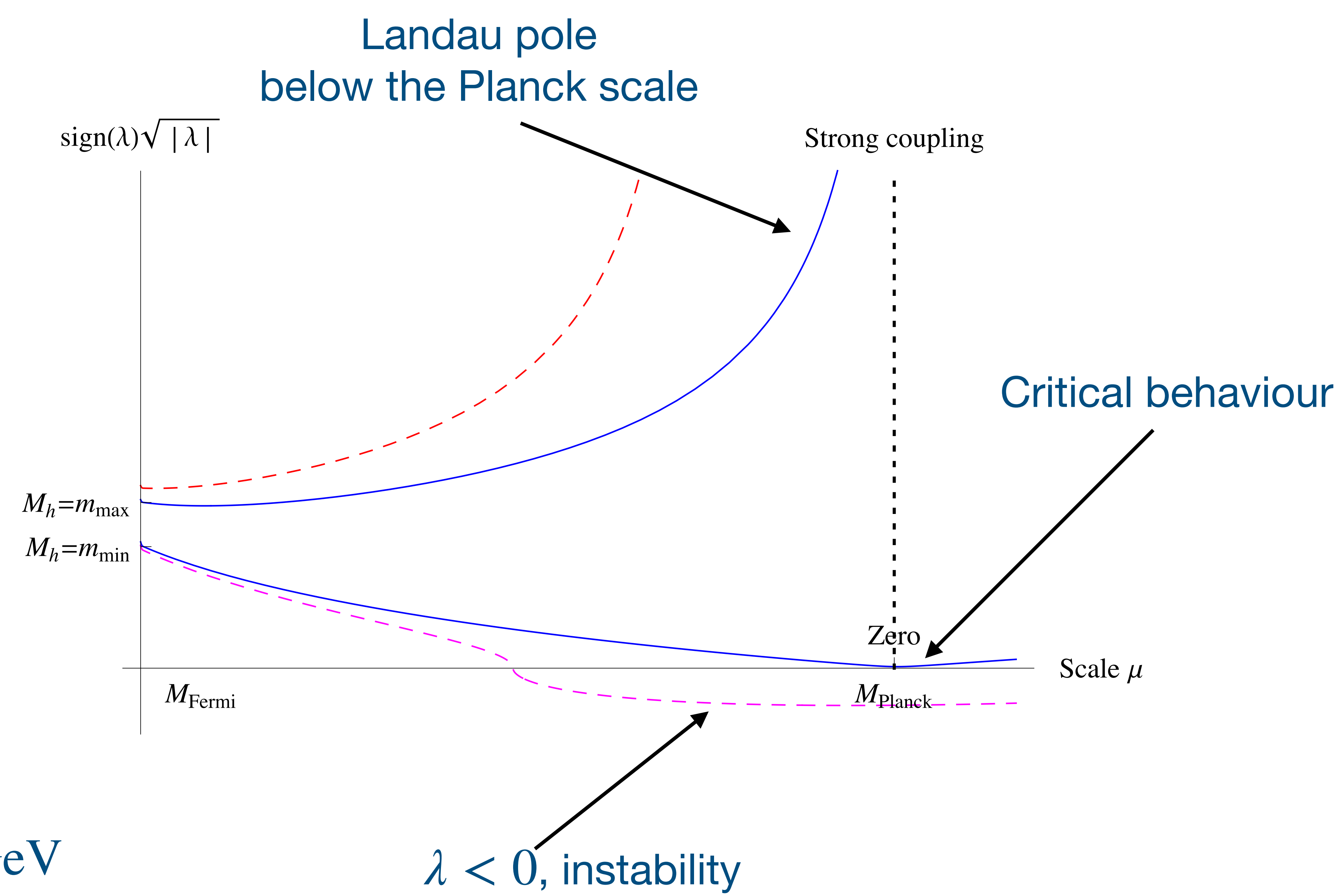
"asymptotic freedom" contributions

$$16\pi^2 \frac{dy_t}{dt} = \frac{9}{2} y_t^3 - 8 g_3^2 y_t^2 - \frac{9}{4} g^2 y_t - \frac{17}{12} g'^2 y_t,$$ top quark Yukawa coupling

$$16\pi^2 \frac{d\lambda}{dt} = 24\lambda^2 + 12\lambda y_t^2 - 9\lambda\left(g^2 + \frac{1}{3} g'^2\right) - 6 y_t^4 + \frac{9}{8} g^4 + \frac{3}{8} g'^4 + \frac{3}{4} g^2 g'^2$$

scalar self-coupling

Behaviour of the Higgs self-coupling



$m_t \simeq 173$ GeV

Self-consistency of the SM

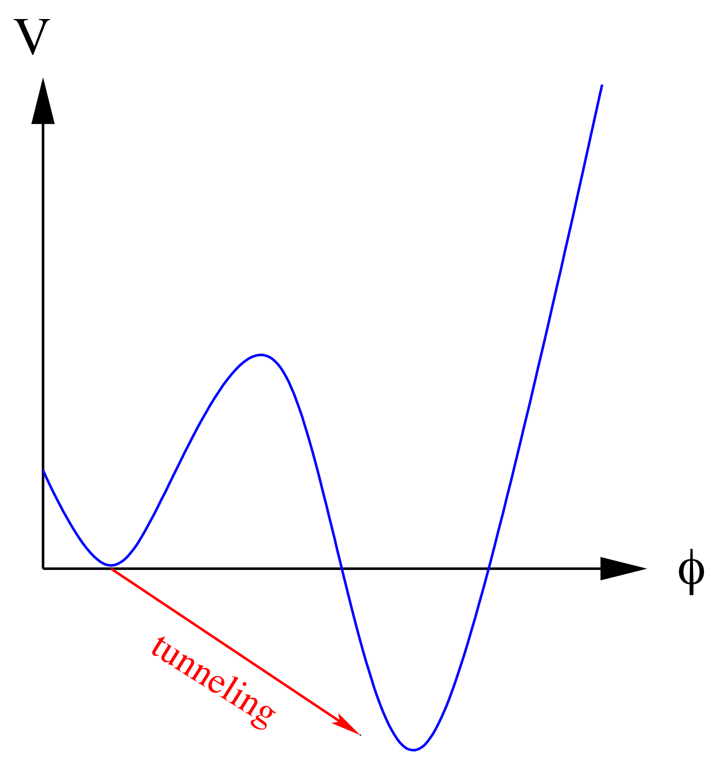
Within the SM the mass of the Higgs boson is an arbitrary parameter which can have any value (if all other parameters are fixed) from

- $m_{meta} \simeq 111$ GeV (metastability bound)

to

- $m_{triviality} \simeq 1$ TeV (triviality bound)

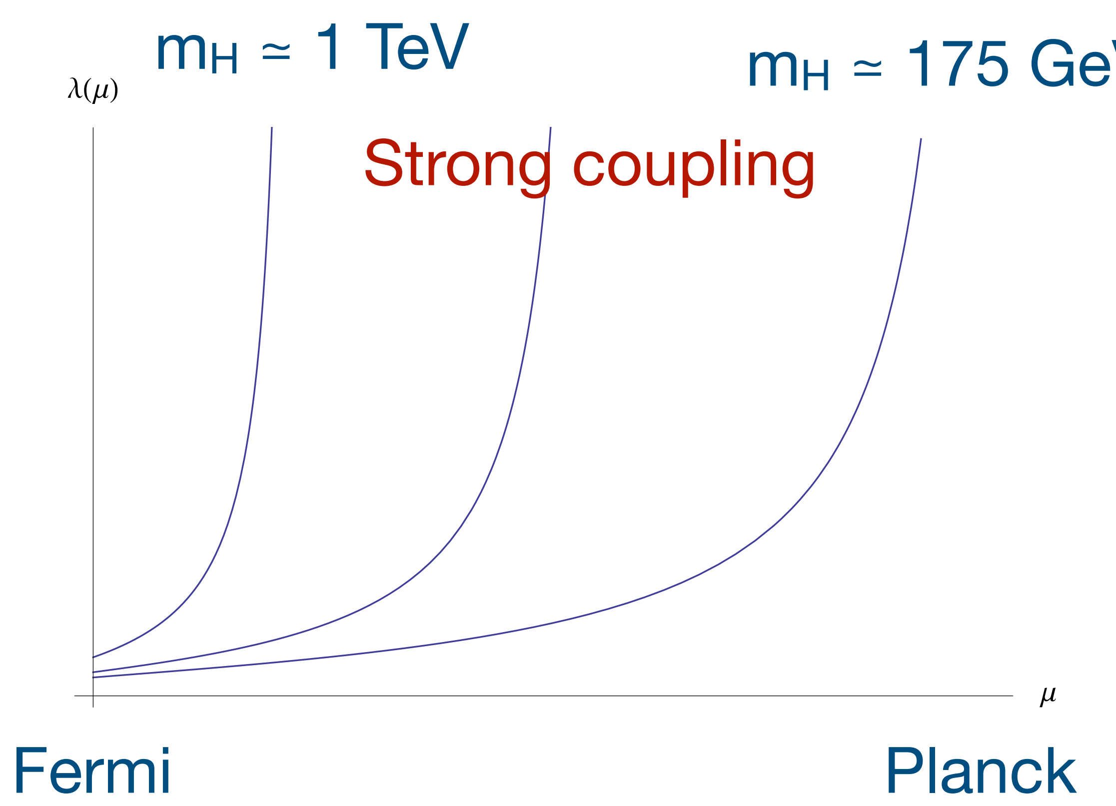
Metastability bound



The life-time of our vacuum is smaller than the age of the Universe if $m_H < m_{meta}$, with $m_{meta} \simeq 111$ GeV

Triviality bound

The Higgs boson self-coupling has a Landau pole at some energy determined by the Higgs mass. For $m_H \simeq m_{triviality} \simeq 1$ TeV the position of this pole is close to the electroweak scale. For $m_H < 175$ GeV the position of the pole is below the Planck scale.



In reality $m_{meta} < m_H < m_{triviality}$, meaning that the SM is a consistent effective theory all the way up to the Planck scale.

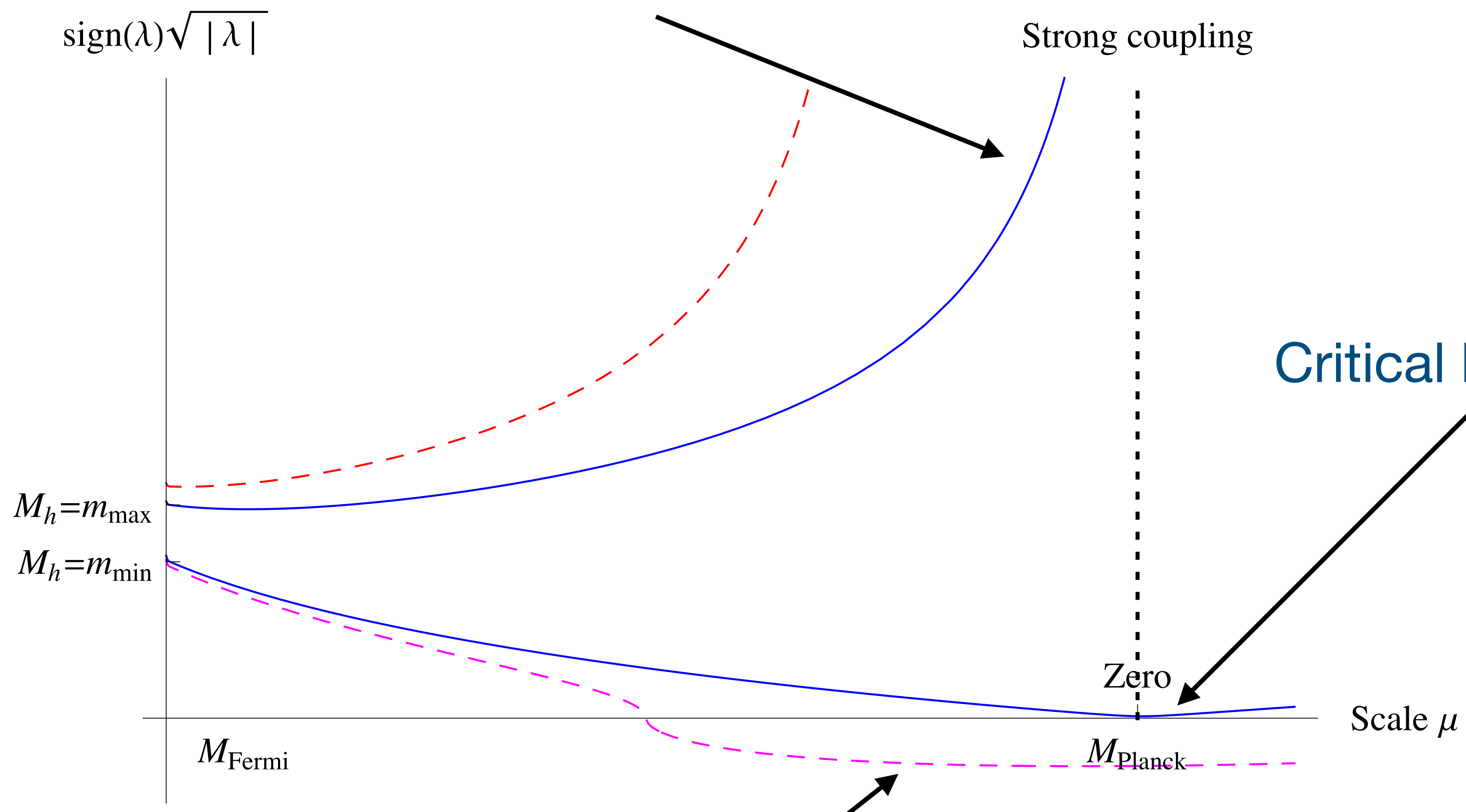
First hint that Planck scale might be related to Fermi scale

Fix all parameters of the SM, except the top quark Yukawa coupling (as the one known with lowest accuracy). Run RG equations to the high energy scale. Look at the "critical point" for the Higgs self-coupling λ: $\lambda(\mu_c) = 0,\ \beta(\lambda) = 0$. Determine two numbers from these two equations: "critical energy" μ_c and top quark Yukawa coupling.

Result (most precise computation by Bednyakov et al, '2015): pole mass of the top quark $m_t \simeq 171.4$ GeV (within error bars coinciding with the Monte-Carlo top quark mass measured at LHC ! E.g. August 23, 2021 CMS article: $m_t = 172.13^{+0.76}_{-0.77}\ \mathrm{GeV}$) and $\mu_c \simeq 7 \times 10^{17}$ GeV, just factor $\simeq 3$ smaller than the Planck scale!

Behaviour of the Higgs self-coupling

Landau pole
below the Planck scale

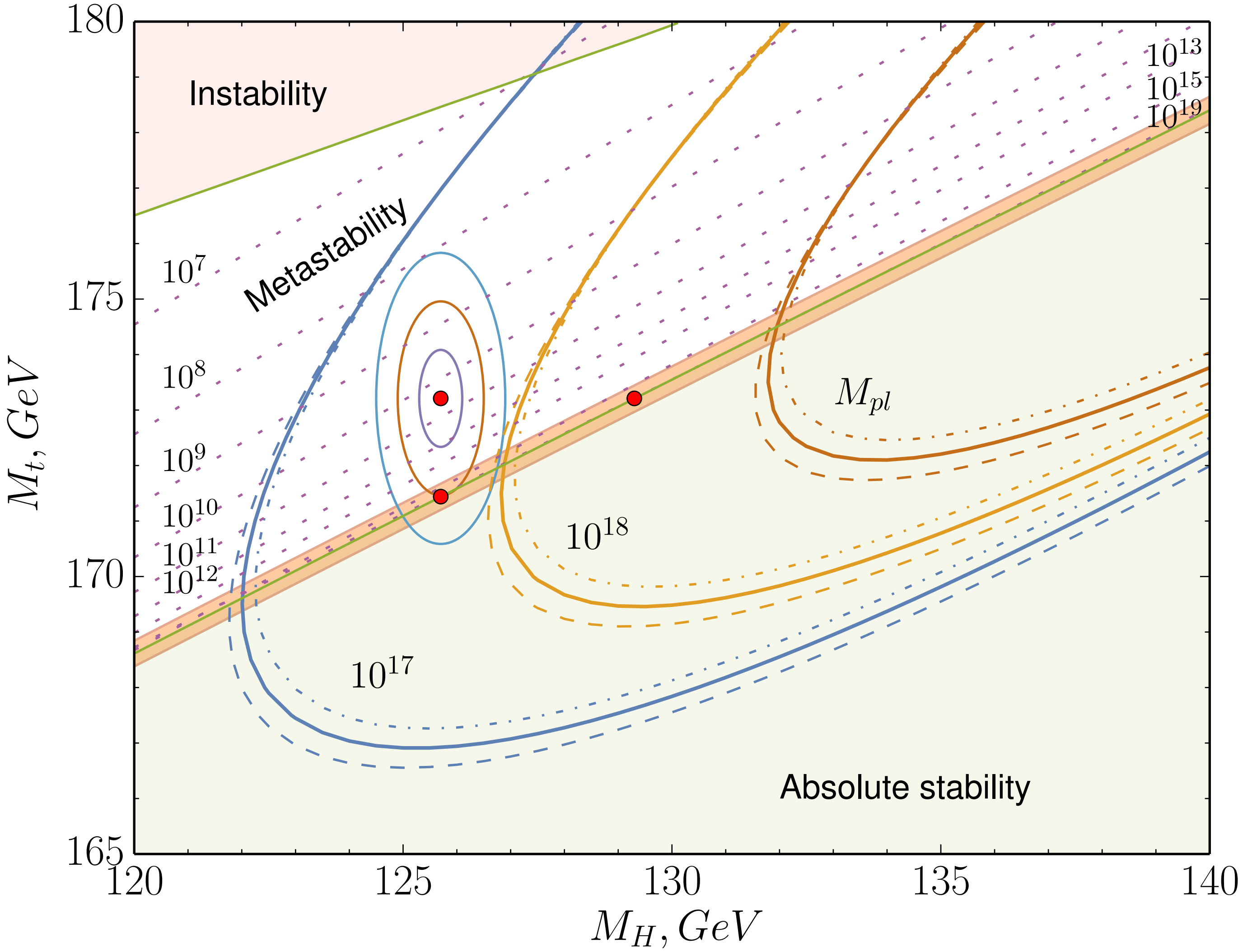


Critical behaviour

$m_t \simeq 173$ GeV

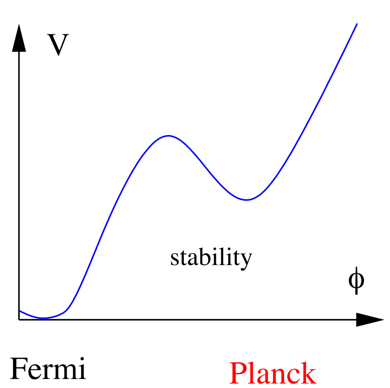
$\lambda < 0$, instability

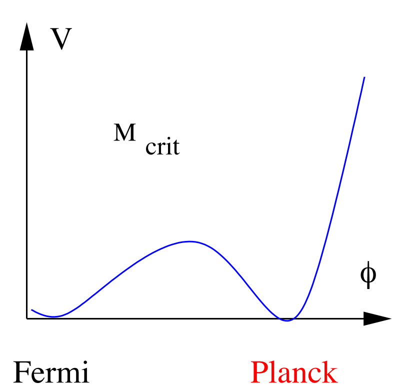
“Phase” structure of the SM

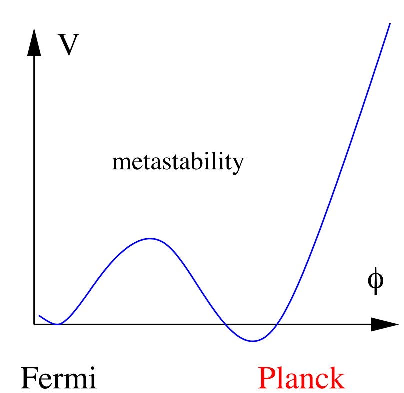


From Bednyakov et al, ‘2015,

see also Buttazzo et al ‘2013







Why $\mu_c \simeq M_P$?

The energy at which $\lambda(\mu_c) = 0,\ \beta(\lambda) = 0$ is almost the same as the Planck scale. Is this a pure coincidence or something deep telling us about the Higgs-gravity connection?

Interesting possibility: asymptotic safety of gravity and of the Standard Model.

Asymptotic safety versus renormalisability

Generic quantum field theory

- Take some field theory and write the most general Lagrangian.
- Compute all amplitudes in all orders of perturbation theory.
- Require that the theory is unitary, Lorentz - invariant, causal, etc - infinite number of conditions for infinite number of processes.
- Solve these consistency equations. Hopefully, the theory will be characterised by a finite number of essential parameters - coupling constants, making the predictions possible.

RG approach

- Introduce dimensionless coupling constants g_i constants for all terms in the action:

$g_i = \mu^D G_i, \quad G_i$ are dimensionfull in general

D is the mass dimension of coupling constant.

RG equations:

$$\mu \frac{\partial g_i}{\partial \mu} = \beta_i(g)$$

Different possibilities

- Renormalisable asymptotically free theories – Gaussian UV fixed point: essential couplings $g_i \to 0$ at $\mu \to \infty$. The number of these couplings is finite - only operators with dimension ≤ 4 are allowed.

- Asymptotically safe theories (Weinberg '1976) – non-Gaussian UV fixed point $g^* \neq 0 : \beta_i(g^*) = 0$. If the dimensionality of the critical surface in the space of coupling constants (which points are attracted to g_*) is finite, the theory is predictable.

Known solutions

Asymptotically free theories:

- QCD
- Certain GUTs
- Renormalizable theories in 2d and 3d

Asymptotically safe, but non-renormalizable theories:

- Scalar field theory in 3d at Wilson-Fischer fixed point (critical surface is 2-dimensional)
- Non-linear σ model in 3d
- Complete theory of pions and nucleons in 4d

The Standard Model is neither asymptotically free nor asymptotically safe

Gravity

Weinberg conjecture of 1979: Gravity may be asymptotically safe, and Planck scale is increasing with energy: $M_P^2(\mu) = M_P^2 + \xi_0 \mu^2$

Some evidence: ϵ - expansion, truncated solutions of exact functional RG equations, higher derivative gravity, large N (matter fields) expansion, hints from perturbation theory

Answer is not known.

What if indeed gravity is asymptotically safe?

Possible consequence:

Electroweak theory + Gravity is a final theory

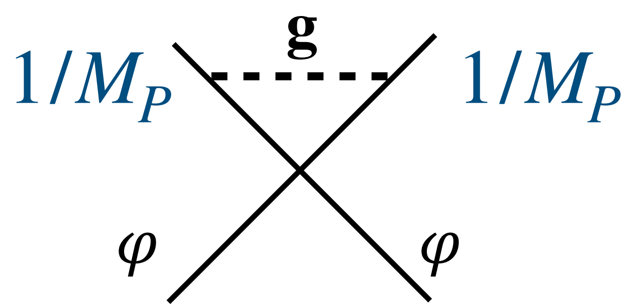
To be true: all the couplings of the SM must be asymptotically safe or asymptotically free

Gravity contribution to RG running of Higgs self-coupling

This is the problem for Higgs self-coupling - Landau pole behaviour in the SM at very large energies.

Qualitative analysis:

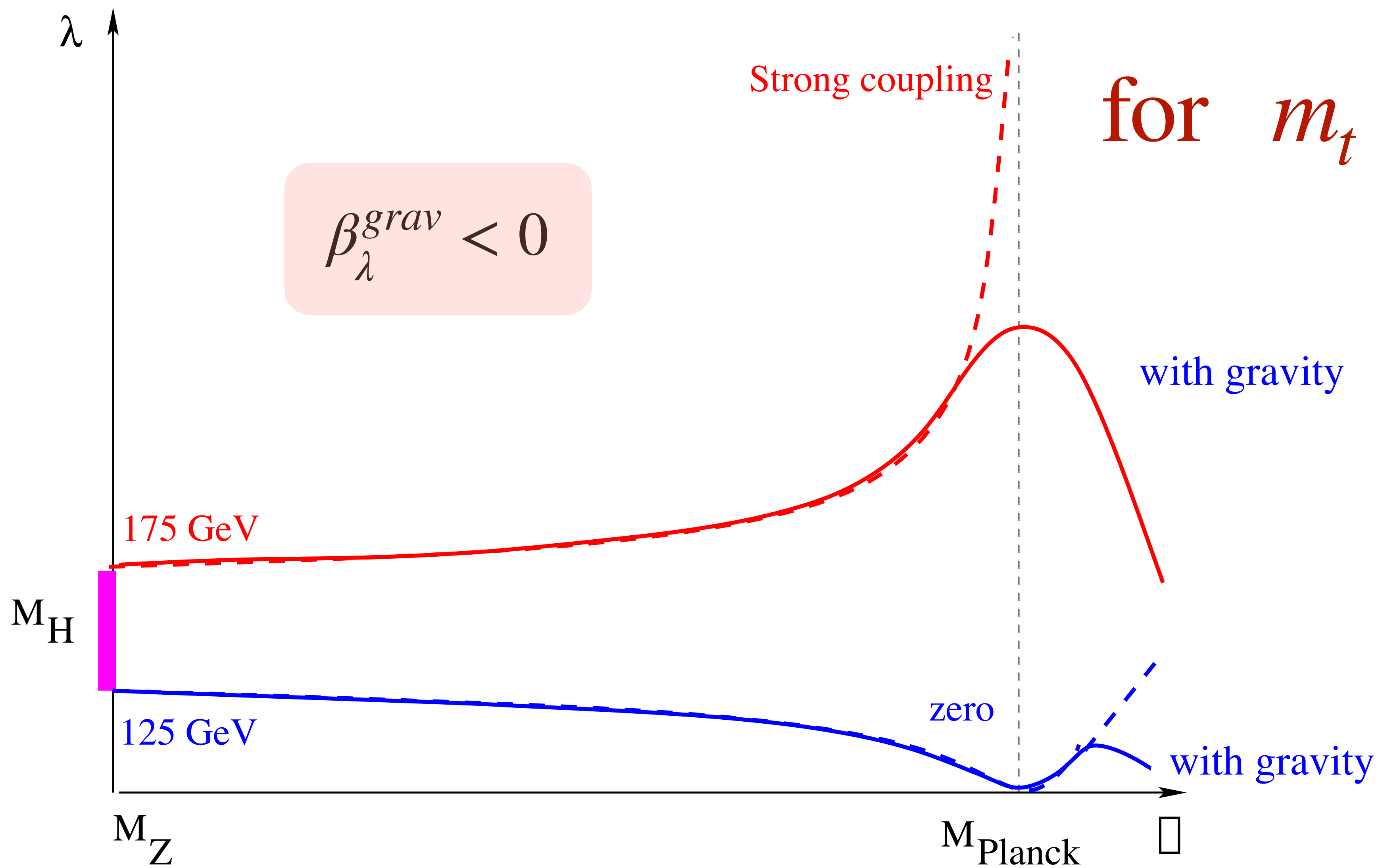
$$\mu \frac{d\lambda}{d\mu} = \beta_\lambda^{\mathrm{SM}} + \beta_\lambda^{grav}$$



On dimensional grounds:

$$\beta_\lambda^{grav} = \frac{a_\lambda}{8\pi} \frac{\mu^2}{M_P^2(\mu)} \lambda, \qquad M_P^2(\mu) = M_P^2 + \xi_0 \mu^2$$

Behaviour of the Higgs self-coupling if $a_\lambda < 0$



for $m_t \simeq 171.4$ GeV

Higgs mass is only allowed to be in the interval

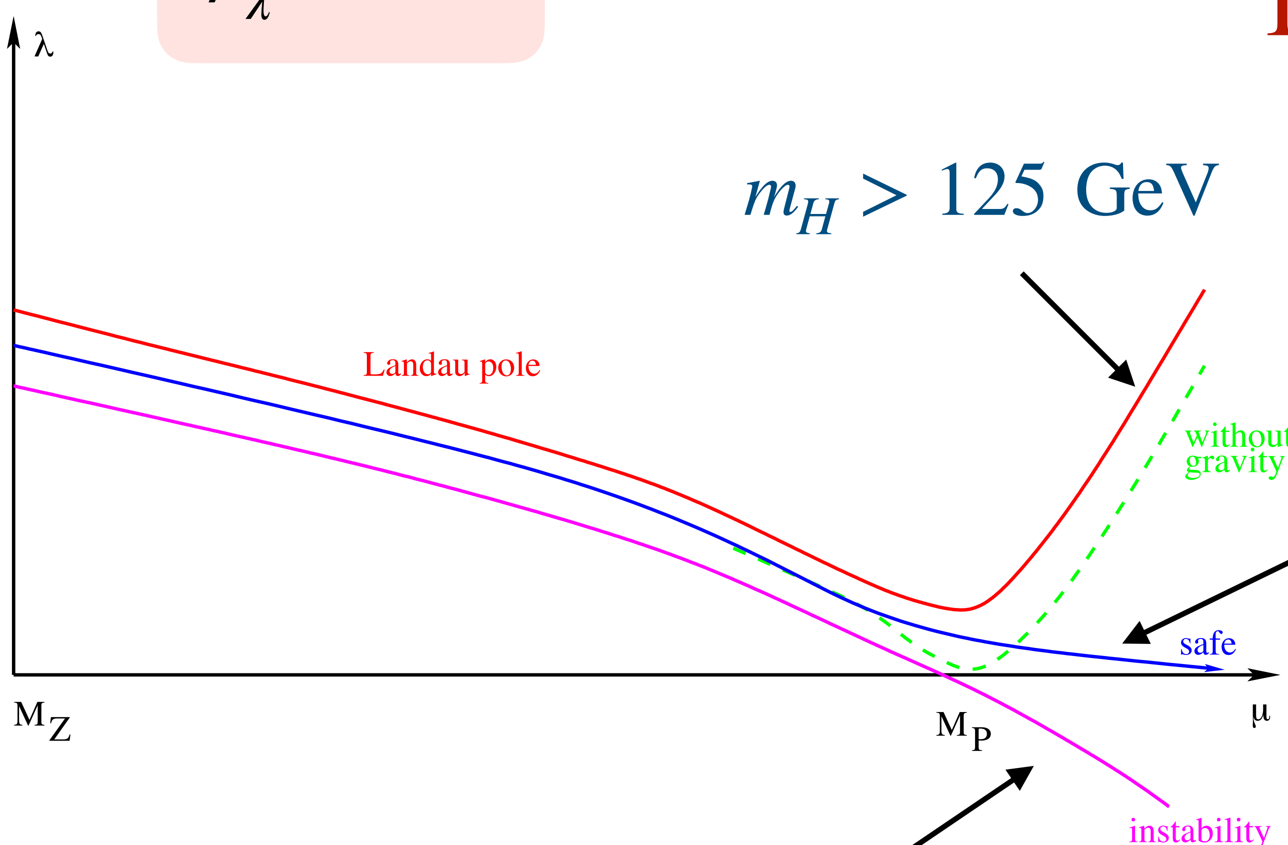
$$125 \text{ GeV} < m_{\text{H}} < 175 \text{ GeV}$$

Behaviour of the Higgs self-coupling if $a_\lambda > 0$

$\beta_\lambda^{grav} > 0$

for $m_t \simeq 171.4$ GeV

$m_H > 125$ GeV



$m_H = 125$ GeV

MS, Wetterich,
Higgs mass prediction in 2009

$m_H < 125$ GeV

Possible understanding of the amazing fact that

$$\lambda(\mu_c) = 0,\ \beta(\lambda) = 0$$

simultaneously at the Planck scale.

Conformal symmetry, gravity, and the electroweak symmetry breaking

Let's forget about gravity once more and ask the question: "Do we get enhanced symmetry if mass of the Higgs boson is put to zero?"

Why this question? t'Hooft naturalness criterion, 1980: the parameter of the theory is small "naturally" if its zero value leads to increased symmetry.

SM with M_H=0: the classical Lagrangian of the SM has a wider symmetry: it is scale invariant. Dilatations "D"- global scale transformations (σ = const) ,

$$\Psi(x) \to \sigma^n \Psi(\sigma x),$$

n = 1 for scalars and vectors, and n = 3/2 for fermions. Space-time Poincaré symmetry with 10 generators, $P = T \rtimes O(1,3)$, is enhanced to a direct product P ⇒ P x D (11 generators)

Standard Model and conformal symmetry

In fact, the resulting symmetry is even larger for M_H=0: SM gets conformally invariant: P ⇒ SO(4,2) ! 10 generators ⇒ 15

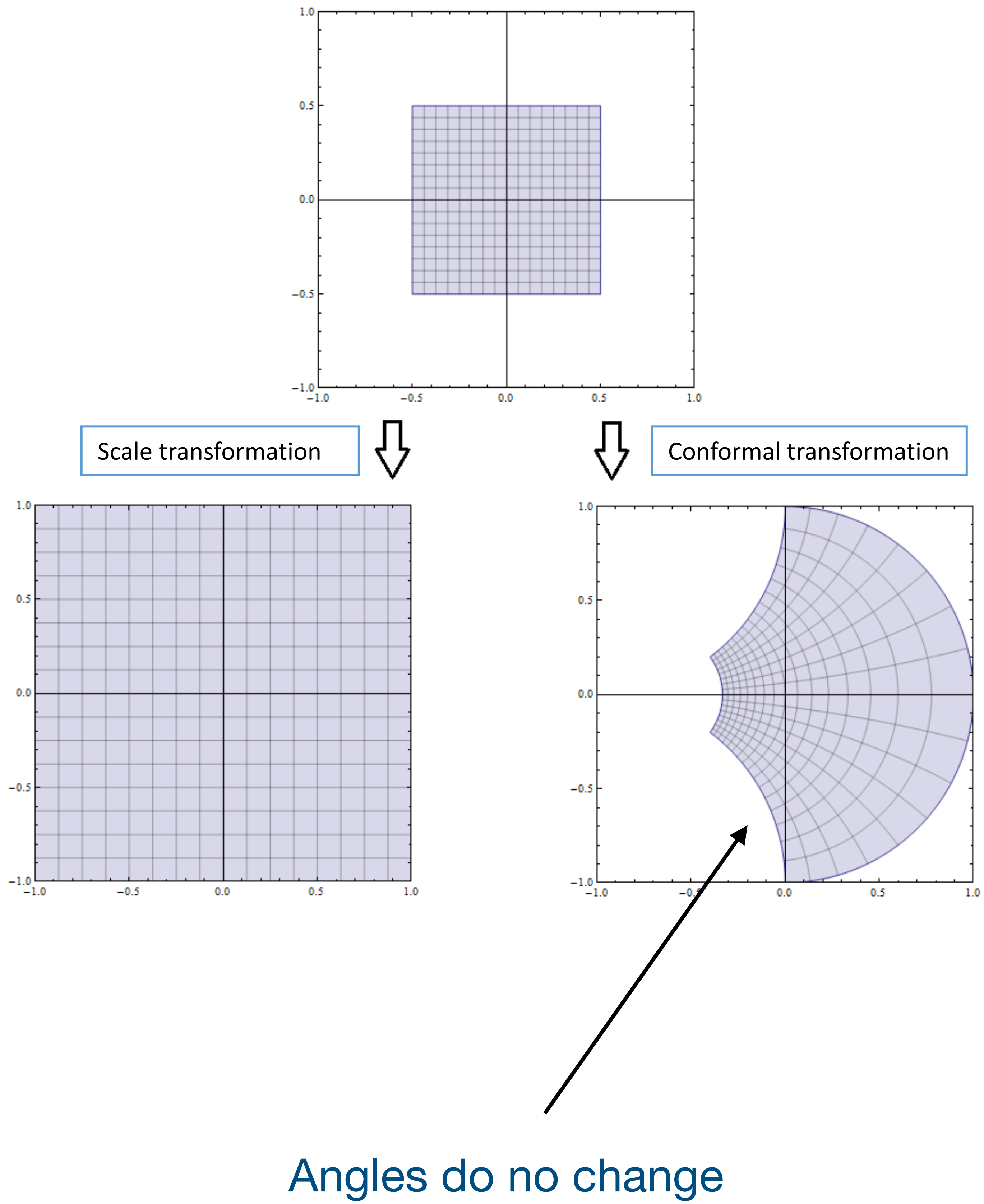
generators = 4 translations + 6 Lorentz transformations + 4 special conformal transformations + 1 scale transformation.

Conformal group: coordinate transformations $x' = F(x)$, which leave the metric $g_{\mu\nu}$ invariant up to a conformal factor $\Omega\,(x')$

$$g_{\mu\nu}(x) = \Omega\,(x')\,g'_{\lambda\sigma}(x')\,\frac{\partial F^\lambda}{\partial x^\mu}\frac{\partial F^\sigma}{\partial x^\nu}$$

Conformal symmetry of Maxwell equations: Bateman and Cunningham, 1908.

scale and conformal transformations



Conformal symmetry and quantum physics

Sidney Coleman, extract from his 1971 Erice Lectures on “Dilatations”, chapter “The death of scale invariance”:

“For scale invariance,...., the situation is hopeless; any cutoff procedure necessarily involves a large mass, and a large mass necessarily breaks scale invariance in a large way.”

No go theorem?

Conformal anomaly

The statement “ any cutoff procedure necessarily involves a large mass" is not true. Counter-example: dimensional regularsation of t’Hooft and Veltman (invented in 1972) does not involve any large mass, just a normalisation point μ which is not send to infinity. Still, the scale invariance is anomalous in realistic renormalisable theories: the renormalisation-group running of the parameters leads to a non-vanishing trace of the energy-momentum tensor, which enters the divergence of the scale current J^μ:

$$\partial_\mu J^\mu \propto \beta(g) G^a_{\alpha\beta} G^{\alpha\beta\, a} + \ \ldots$$

Conformal anomaly

The physical quantities depend on the renormalisation scale only logarithmically. Any quadratically divergent contributions to the Higgs boson mass are purely technical and are introduced by artificial explicit breaking of the conformal invariance by regulators (cutoff, Pauli-Villars, etc).

It is possible to make quantum conformal symmetry exact but spontaneously broken (non-linear realisation) in realistic setup. Consequence: existence of the exactly massless dilation (but no fifth source), non-renormalisable interactions relevant at the Planck scale.

Radiative generation of the electroweak scale

- In classically scale invariant/conformal theories the Higgs mass can be predicted :

Radiative Corrections as the Origin of Spontaneous Symmetry Breaking*

Sidney Coleman

and

Erick Weinberg

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

(Received 8 November 1972)

We investigate the possibility that radiative corrections may produce spontaneous symmetry breakdown in theories for which the semiclassical (tree) approximation does not indicate such breakdown. The simplest model in which this phenomenon occurs is the electrodynamics of massless scalar mesons. We find (for small coupling constants) that this theory more closely resembles the theory with an imaginary mass (the Abelian Higgs model) than one with a positive mass; spontaneous symmetry breaking occurs, and the theory becomes a theory of a massive vector meson and a massive scalar meson. The scalar-to-vector mass ratio is computable as a power series in e, the electromagnetic coupling constant. We find, to lowest order, $m^2(S)/m^2(V) = (3/2\pi)(e^2/4\pi)$. We extend our analysis to non-Abelian gauge theories, and find qualitatively similar results. Our methods are also applicable to theories in which the tree approximation indicates the occurrence of spontaneous symmetry breakdown, but does not give complete information about its character. (This typically occurs when the scalar-meson part of the Lagrangian admits a greater symmetry group than the total Lagrangian.) We indicate how to use our methods in these cases.

Radiative generation of the electroweak scale

The scale invariance is anomalous due to “dimensional transmutation”: the renormalisation-group running of the parameters leads to a non-vanishing trace of the energy-momentum tensor, which enters the divergence of the scale current. The physical quantities depend on the renormalisation scale only logarithmically. Take a scale-independent renormalisation, e.g. DimReg. No counter-term is needed to renormalise the scalar mass: m_H can be predicted! RG equation has a fixed point at $m_H = 0$:

$$\mu \frac{\partial}{\partial \mu} m_H^2 \propto m_H^2$$

- Procedure: compute the CW effective potential and discover that the U(1) theory is in the Higgs phase. Read off the ratio between the Higgs boson mass and the vector boson mass,

$$\frac{m_H^2}{m_W^2} = \frac{3e^2}{8\pi^2}$$

Radiative generation of the electroweak scale

Does not work for the SM (but may work in its extensions):

If the top quark mass $m_t \lesssim m_t^{crit}$, then the minimum of the effective potential is generated at $\langle H \rangle \simeq 100\ \mathrm{MeV}$ due to chiral symmetry breaking in QCD.

If the top quark mass $m_t \gtrsim m_t^{crit}$, then an extra minimum of the effective potential is generated at $\langle H \rangle \gtrsim M_P$ due to top quark loops.

$m_t^{crit} = 170 - 174$ GeV accounting for uncertainties in the relation between the Monte Carlo and pole masses of the top quark.

Radiative generation of the electroweak scale

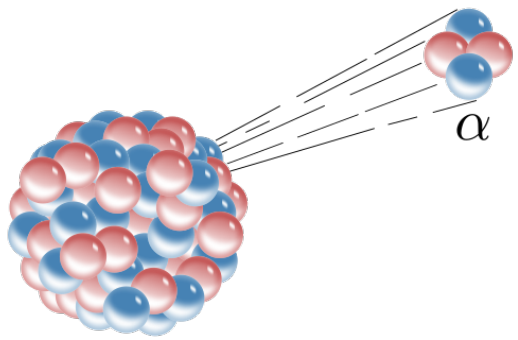
The idea fails. But we do have the breaking of scale invariance! Gravity comes with a dimensionful parameter $M_P \gg m_H$, and this must be taken into account!

- Perturbatively, with mass-independent regularisation (such as DimReg) : no gravity contribution to the Higgs mass: all corrections are suppressed by the Planck mass. The RG equation $\mu \frac{\partial}{\partial \mu} m_H^2 \propto m_H^2$ remains in force!

Gravity + conformally invariant SM is an ideal playground for looking for non-perturbative generation of the weak scale!

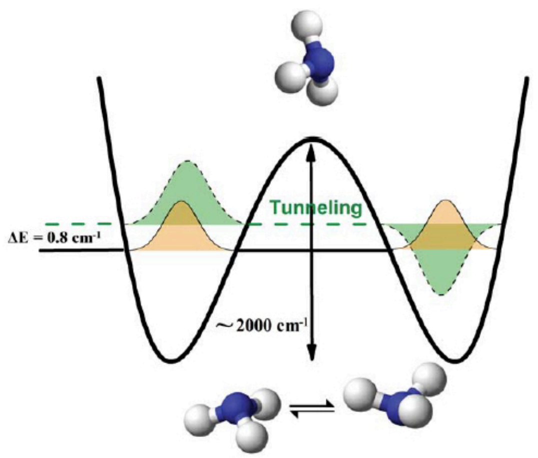
Very small numbers in quantum physics: non-perturbative effects

Well known examples:



- 1928, Gamow's theory of α-decay, uranium-238 → thorium-234 + α,

$$\Gamma = E_{\text{bounding}} e^{-S} <<< E_{\text{bounding}}$$



- 1951, Townes, Ammonia Maser,

$$\omega = E_{\text{bounding}} e^{-S} <<< E_{\text{bounding}}$$

- Mass gap in BCS superconductors

$$T_c \sim E_D e^{-1/NV}$$

Proposal : there is only one fundamental scale in Nature - M_P and the electroweak scale is generated from it non-perturbatively. The huge difference between m_H and M_P is due to the non-perturbative phenomena in gravity and the Higgs mass is related to the Planck scale as $m_H^2 = M_P^2 e^{-S}$ with $S \sim 80$.

Non-perturbative Fermi scale generation

Simplest theory that works

Scalar field with non-minimal coupling to Palatini gravity. Basic structures: metric (distances) and symmetric connection $\Gamma^{\rho}_{\nu\sigma} = \Gamma^{\rho}_{\sigma\nu}$. The action (metric -+++):

$$\frac{\mathscr{L}_{\varphi,g}}{\sqrt{g}} = \frac{1}{2}(M_P^2 + \xi\varphi^2)R - \frac{1}{2}(\partial\varphi)^2 - V(\varphi)$$

The dynamical variables are $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$, variation with respect to $\Gamma^{\rho}_{\nu\sigma}$ gives metricity, $g_{\mu\nu;\alpha} = 0$, i.e. the relation between $\Gamma^{\rho}_{\nu\sigma}$ and $g_{\mu\nu}$, the variation with respect to gives Einstein equations. Large ξ - semiclassical parameter which allows for non-perturbative estimate in Palatini gravity 😀. Metric gravity does not provide such a parameter 😢. (For details see MS, Shkerin, Zell, '2020)

"Matter" is scale invariant with $V(\varphi) = \frac{\lambda}{4}\varphi^4$. The cutoff of the theory (onset of perturbation theory breaking) $\Lambda \sim M_P/\sqrt{\xi}$

Non-perturbative Fermi scale generation

Steps of the analysis (for details see MS, Shkerin, Zell, ‘2020)

- We want to compute the Higgs vev:

$$\langle \varphi \rangle \sim \int \mathscr{D}\varphi \mathscr{D} g_{\mu\nu} \varphi e^{-S_E}$$

S_E is the euclidean action of the model. For small φ « MP - gravity is irrelevant – no contribution to the vev of the Higgs from scalar loops. Challenge: account for contributions with φ » MP . Theory for large φ:

$$\mathscr{L} = -\frac{1}{2}\xi\varphi^2 R + \frac{1}{2}(\partial\varphi)^2 + \frac{\lambda}{4}\varphi^4$$

Important properties of this action: (i) scale-invariance (ii) Planck scale is dynamical, $M_P^2 = \xi\phi^2$

Fermi scale generation

- Search for saddle points of the effective action $S_{\text{eff}} = S_E + \log(\phi)$ in the Einstein frame, where the gravity action is simply the Ricci scalar curvature R.

$$\langle \varphi(x)\,|_{x=0} \rangle \sim \int \mathscr{D}\varphi \mathscr{D} g_{\mu\nu} \varphi(x)\,|_{x=0}\, e^{-S_E}$$

S_{eff}

- Proof the existence of semiclassical parameter $S_{eff} \propto \sqrt{\xi}$, analogue of $1/\hbar$ in WKB approximation.

- Regulate singularities of the action at x=0 by higher dimensional operators.

- Convince yourself that there are classical solutions with large effective action leading to $\langle \varphi \rangle \sim M_P e^{-S}$ with $S \sim 40$

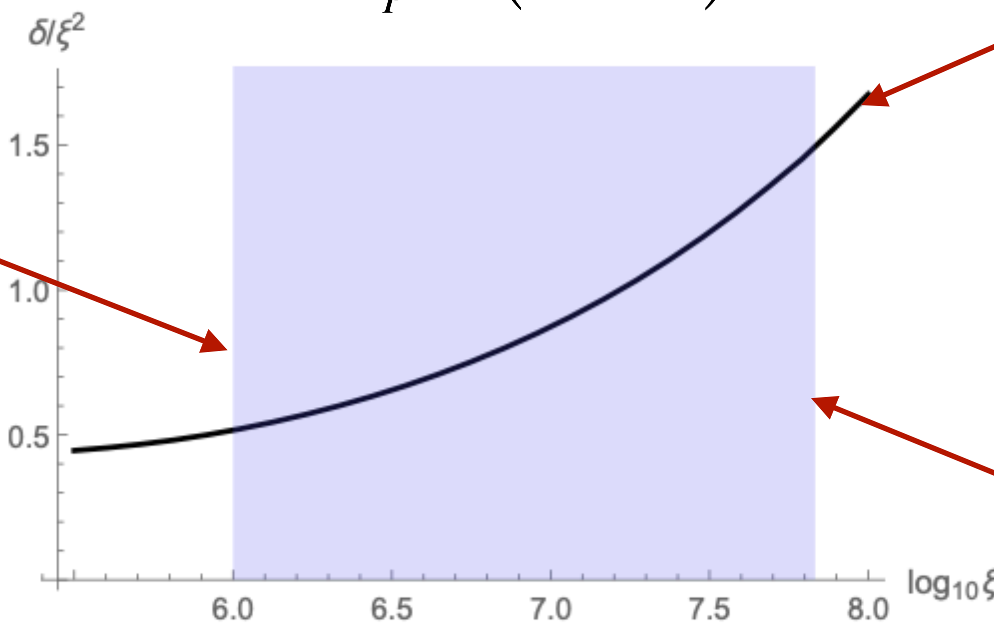
Fermi scale generation

Natural choice: $\delta \sim \xi^2$: the same cutoff $\Lambda \sim M_P/\sqrt{\xi}$

$$\delta \mathscr{L}_\delta = -\frac{\delta}{M_P^8 \Omega^8}\left(1 + \frac{\delta}{\Omega^2}\right)\left(\partial_\mu h\right)^6$$

Constraint from inflation

Requirement of the correct hierarchy. Parametrically $S \propto \sqrt{\xi}$



Constraint from t-quark mass: positive λ at the scale of inflation

Figure 1. Values of the non-minimal coupling ξ and the coupling δ of the higher-order operator (11), for which $B = \ln(M_P/(\sqrt{\xi} M_F))$. Admissible values of ξ are within the blue area, the left bound coming from inflation and the right bound coming from top quark measurements.

See also:
Rasanen and Rasanen, 1709.07853;
Rasanen, 1811.09514;
Karananas, Michel and Rubio, 2006.11290

The hierarchy between the Planck and the Fermi scales may be a natural phenomenon when the SM is classically conformal, ξ is large and the gravity is of the Palatini type! In the metric theory the source term $\sqrt{\xi}\delta(r)$ is replaced by $\delta(r)/(6 + 1/\xi)$ and the action is too small.

Conclusions

- The conjecture that there is just one scale in Nature - M_P , and that the Fermi scale is generated dynamically from it, may actually work. (No clue about the cosmological constant).

- The reason why the Fermi scale is much smaller than the Planck scale may be rooted in conformal symmetry and non-perturbative gravity effects.

- The specific value of the Higgs scalar self-coupling leading to “criticality” $\lambda(\mu_c) = 0,\ \beta(\lambda) = 0$ may be related to asymptotic safety of gravity and the Standard Model.