Naturalness: Why is the Higgs Light?
The Naturalness Strategy

The naturalness strategy: an analogy from E&M

\[ \Delta E_C = \frac{1}{4\pi \varepsilon_0} \frac{e^2}{r_e} \]

\[ (m_e c^2)_{\text{obs}} = (m_e c^2)_{\text{bare}} + \Delta E_C \]

Experimentally \( r_e \lesssim 10^{-18} \text{ cm} \Rightarrow \Delta E_C \gtrsim 100 \text{ GeV} \)

If so, \[ 0.511 = -99999.489 + 100000.000 \text{ MeV} \]

To avoid fine-tuning, i.e. for the theory to be “natural”, need picture to change on scales below \( 2.8 \times 10^{-13} \text{ cm} \)
The Naturalness Strategy

Naively, quantum mechanics only makes the problem worse (Weisskopf (1939)):

Quantum fluctuations of the electric field within the “volume” $2r_e$ of the electron: $|\vec{E}|^2 \sim \frac{hc}{r_e^4}$ with mean frequency $\nu \sim \frac{c}{r_e}$.

Induces electron to vibrate with amplitude $x \sim \frac{e|\vec{E}|}{m\nu^2}$ and energy $E \sim \frac{e^2|\vec{E}|^2}{m\nu^2} \sim \frac{e^2h}{mc^2}r_e^2$.

Seven orders of magnitude larger than the classical problem!
The Naturalness Strategy

**Dirac (1928/29):** There is a new state in the relativistic quantum theory

**Weisskopf (1939):** Compute the self-energy including the positron

\[ \Delta t \sim \frac{\hbar}{\Delta E} \sim \frac{\hbar}{(2m_e c^2)} \]

\[ \Delta E = \Delta E_C + \ldots \]

\[ d \sim c\Delta t \sim 200 \times 10^{-13} \text{ cm} \]

\[ \Delta E = -\Delta E_C + \ldots \]

\[ \Delta E = \Delta E_C - \Delta E_C + \frac{3\alpha}{4\pi} m_e c^2 \log \frac{\hbar}{m_e c r_e} \]
The Naturalness Strategy

What about scalars?

Consider the pion...

Another divergence...

\[ m_{\pi^\pm}^2 - m_{\pi^0}^2 = \frac{3\alpha}{4\pi} \Lambda^2 \]

Given observed splitting, *predict* scale of new physics:

\[ m_{\pi^\pm}^2 - m_{\pi^0}^2 = (35.5 \text{ MeV})^2 \Rightarrow \Lambda \lesssim 850 \text{ MeV} \]

Another (more predictive) example: K\text{L}-K\text{S} mass difference.
The “Hierarchy Problem”

The Higgs is an apparently elementary scalar

Assuming the Standard Model is valid down to some length scale $r_{\text{new}} \approx c$ then we have

$$r_{\text{new}} \equiv \frac{\hbar c}{\Lambda}$$

$$\Delta m_H^2 = \frac{\Lambda^2}{16\pi^2} \left[ -6y_t^2 + \frac{9}{4}g_2^2 + \frac{3}{4}g_Y^2 + 6\lambda + \ldots \right]$$

Expecting NP at $\Lambda$ such that $\Delta m_H^2 \sim m_H^2$ is a strategy.
# The Naturalness Strategy

<table>
<thead>
<tr>
<th>Param</th>
<th>UV sensitivity</th>
<th>Natural if</th>
<th>NP</th>
<th>Scale</th>
<th>Natural?</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;mₑ&quot;</td>
<td>$e^2 \Lambda$</td>
<td>$\Lambda \lesssim 5$ MeV</td>
<td>Positron</td>
<td>511 keV</td>
<td>✓</td>
</tr>
<tr>
<td>$m_{\pi\pm}^2 - m_{\pi0}^2$</td>
<td>$\frac{3\alpha}{4\pi} \Lambda^2$</td>
<td>$\Lambda \lesssim 850$ MeV</td>
<td>Rho</td>
<td>770 MeV</td>
<td>✓</td>
</tr>
<tr>
<td>$m_{KL}-m_{KS}$</td>
<td>$\frac{s_c f_K^2 m_{K^0_L}}{24\pi^2 v^4} \Lambda^2$</td>
<td>$\Lambda \lesssim 2$ GeV</td>
<td>Charm</td>
<td>1.2 GeV</td>
<td>✓</td>
</tr>
<tr>
<td>$m_H^2$</td>
<td>$\frac{6y_t^2}{16\pi^2} \Lambda^2$</td>
<td>$\Lambda \lesssim 500$ GeV</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>
A Reminder

Often said that the quadratic divergence “is” the hierarchy problem:

\[ \Delta m_H^2 = \frac{\Lambda^2}{16\pi^2} \left[ -6y_t^2 + \frac{9}{4}g_2^2 + \frac{3}{4}g_Y^2 + 6\lambda + \ldots \right] \]

But this is misleading, as it depends on regularization scheme (try dim reg!).

Only meaningful if \( \Lambda \) is a **physical scale**. From the bottom-up, \( \Lambda \) a proxy for the scale of new physics. Given a UV completion, “quadratic divergence” replaced by finite, calculable, regularization scheme-independent contributions.

Tempting to “solve” hierarchy problem by positing no physical scale above weak scale. Still need to explain why scale of quantum gravity, hypercharge Landau pole do not contribute.
From the “naturalness strategy” to new physics

At this level, we expect

- New physics around the TeV scale…
- …coupling to the Higgs

Strong motivation for BSM Higgs physics! But maybe too broad to be useful guidance to experiment.

To make further progress, we can come up with models. **Easiest path:** work by analogy with known examples.
Supersymmetry

How did the electron self-energy work out in detail?

Appearance of positron associated with new symmetry, chiral symmetry.

Dirac fermion: \[ \Psi = \begin{pmatrix} \chi \\ \xi^\dagger \end{pmatrix} \quad \text{Lagrangian: } \quad i \overline{\Psi} \phi \Psi + m \overline{\Psi} \Psi \]

Invariant under U(1)\(\nu\) global symmetry

As \( m \to 0 \) a new U(1)\(\Lambda\) symmetry emerges:

\[ \Psi \to e^{-i\alpha} \Psi \]

\[ \Psi \to e^{-i\alpha \gamma_5} \Psi \]

Implies quantum corrections to the mass must be proportional to the mass itself, \( \delta m \propto m \)

\[ \Rightarrow \text{Fermion masses are “technically natural”, dependence on short-distance physics at most logarithmic. Explains Weisskopf’s result...} \]
Supersymmetry

Clever Student: But the Higgs is a scalar, not a fermion, so how can chiral symmetry help?

SUSY Enthusiast: Introduce a symmetry that relates bosons and fermions (supersymmetry!); relate the Higgs to a fermionic counterpart (“higgsino”) and exploit its chiral symmetry. Then SUSY solves the hierarchy problem in analogy with the electron mass.

Clever Student: But why don’t we see degenerate superpartners?

SUSY Enthusiast: Can break the symmetry by dimensionful parameters (“soft breaking”) giving extra mass to superpartners; per earlier discussion, quantum corrections proportional to symmetry-breaking terms, rather than arbitrarily short distances.

\[
\Delta m_{H}^{2} = -\frac{6y_{t}^{2}}{16\pi^{2}}\Lambda^{2} + \frac{6y_{t}^{2}}{16\pi^{2}}\Lambda^{2} - \frac{6y_{t}^{2}}{16\pi^{2}}(m_{\tilde{t}}^{2} - m_{t}^{2}) \log \frac{\Lambda^{2}}{m_{\tilde{t}}^{2}}
\]
Global Symmetry

How did the charged/neutral pion case work out?

*Pions are composite, “come apart” at the scale $\Lambda$.***

**Clever Student:** But why weren’t the pion masses themselves at the scale $\Lambda$ in the first place?

**Composite Higgs Enthusiast:** Pions are pseudo-goldstone bosons of spontaneously broken chiral symmetry.

Would be massless if symmetry were exact; soft breaking by quark masses (dimensionful) and hard breaking by QED (dimensionless).

Pion masses well below $\Lambda$ provided soft & hard breaking are “small”.

$$m_\pi^2 \propto m_q f_\pi \quad \delta m_\pi^2 = \frac{3\alpha}{4\pi} \Lambda^2$$
Global Symmetry

Clever Student: Ah, so we can make the Higgs “like a pion” — let’s call that a Composite Higgs Model. But what is the global symmetry? Do Standard Model couplings respect it?

Composite Higgs Enthusiast: Extend the SM so that SU(2)xU(1) is part of a larger global symmetry, preferably one that preserves SO(4) custodial symmetry. Many options, starting w/ SO(5)…

Gauge couplings & light fermion yukawas can break the symmetry, but lightness of Higgs suggests top yukawa should ~respect the symmetry.

Explicit and soft breaking of the global symmetry (as long as both are sufficiently “small”) can lead to a light Higgs well below the scale of compositeness.

\[ \Delta m_H^2 = -\frac{6y_t^2}{16\pi^2} \Lambda^2 + \frac{6y_t^2}{16\pi^2} \Lambda^2 - \frac{6y_t^2}{16\pi^2} (m_T^2 - m_t^2) \log \frac{\Lambda^2}{m_T^2} \]
We (collectively) spend much of our time looking for solutions to the hierarchy problem. We have yet to find evidence for these solutions, (not for lack of outstanding experimental effort). Natural question: have we exhausted the solutions?
22 Ways to Solve the Hierarchy Problem?

Necessarily incomplete, with apologies for the many omissions…
The Hierarchy Problem Cartoon

- Higgs sector cutoff
- Quantum gravity cutoff
- Uninteresting
- RG flow to IR
- (~unique vacuum)
- Standard Model

\[ m_H \text{ is not technically natural} \Rightarrow \text{hierarchy problem} \]
Adding a symmetry

...and (sometimes) breaking it softly

1. Supersymmetry (*a la* the electron)
2. Global symmetry (*a la* the pion)
3. Discrete symmetry
4. Modular invariance

[Dienes et al. ’94-’01, …]

Experimental signals: partner particles

- The familiar host of prompt signals (with or without missing energy)
- Rich variety of displaced decays (RPV, twin higgs, folded SUSY, …)
Discrete Symmetries

Consider a scalar $H$ transforming as a fundamental under a global $SU(4)$:

$$V(H) = -m^2|H|^2 + \lambda|H|^4$$

Potential leads to spontaneous symmetry breaking,

$$|\langle H \rangle|^2 = \frac{m^2}{2\lambda} \equiv f^2$$

$SU(4) \rightarrow SU(3)$ yields seven goldstone bosons.
Discrete Symmetries

Now gauge $SU(2)_A \times SU(2)_B \subset SU(4)$, w/ 

$$H = \begin{pmatrix} H_A \\ H_B \end{pmatrix}$$

Then 6 goldstones are eaten, leaving one behind.

Explicitly breaks the $SU(4)$; expect radiative corrections.

$$V(H) \supset \frac{9}{64\pi^2} \left( g_A^2 \Lambda^2 |H_A|^2 + g_B^2 \Lambda^2 |H_B|^2 \right)$$

But these become $SU(4)$ symmetric if $g_A = g_B$ from a $Z_2$

*Quadratic potential has accidental $SU(4)$ symmetry.*
Discrete Symmetries

Now gauge $SU(2)_A \times SU(2)_B \subset SU(4)$, w/

$$H = \begin{pmatrix} H_A \\ H_B \end{pmatrix}$$

Then 6 goldstones are eaten, leaving one behind.

Explicitly breaks the $SU(4)$; expect radiative corrections.

$$V(H) \supset \frac{9}{64\pi^2} g^2 \Lambda^2 \left( |H_A|^2 + |H_B|^2 \right)$$

But these become $SU(4)$ symmetric if $g_A = g_B$ from a $Z_2$

*Quadratic potential has accidental $SU(4)$ symmetry.*
Twin Higgs

[Chacko, Goh, Harnik ‘05]

Radiative corrections to mass-squared are SU(4) symmetric thanks to $Z_2$:

$$V(H) \supset \frac{\Lambda^2}{16\pi^2} \left(-6y_t^2 + \frac{9}{4}g^2 + \ldots\right) (|H_A|^2 + |H_B|^2)$$

Higgs is a PNGB of ~SU(4), but partner states neutral under SM.

$$\mathcal{L} \supset -y_t H_A Q_3^A \bar{u}_3^A - y_t H_B Q_3^B \bar{u}_3^B$$
<table>
<thead>
<tr>
<th>strong direct production</th>
<th>scalar</th>
<th>fermion</th>
</tr>
</thead>
<tbody>
<tr>
<td>QCD</td>
<td>SUSY</td>
<td>Composite Higgs/RS</td>
</tr>
<tr>
<td>DY direct production</td>
<td>EW</td>
<td>Quirky Little Higgs</td>
</tr>
<tr>
<td>Higgs portal direct production</td>
<td>singlet</td>
<td>Twin Higgs</td>
</tr>
<tr>
<td></td>
<td></td>
<td>hyperbolic Higgs</td>
</tr>
</tbody>
</table>

Mirror Glueballs
Higgs portal observables

Higgs coupling shifts ~ tuning

Table 1. The "theory space" of solutions to the hierarchy problem with top partners, organized by SM gauge charge and spin, with a representative model example in each field. The gauge charge dictates the direct top partner production mode, which makes the LHC suitable for discovery of colored top partners. For uncolored top partners, mirror glueballs are highly favored for EW-charged mirror sectors, and possible for singlet top partners. Higgs coupling shifts of same order as tuning are present in all known fermionic top partner theories. Together, these two signatures allow discovery of all known uncolored top partner theories. A hypothetical "singlet-stop" theory is indicated with a question mark, and would have to be discovered by either probing the UV completion or, for partner masses of a few 100 GeV, with Higgs portal observables (see text).

As exciting as this experimental signature is, it is not a requirement for generic Twin-Higgs type models—the SM-singlet sector could easily have relatively light quarks, making for a hadron spectrum more like that of the visible sector. On the other hand, mirror glueballs, and their associated signals, are a requirement for uncolored naturalness theories with EW-charged mirror sectors, like Folded SUSY or Quirky Little Higgs. This is due to LEP limits forbidding BSM particles with EW charge lighter than about 100 GeV

It is interesting to think about the empty square in Table 1. So far, no explicit theory with SM-singlet scalar top partners has been proposed. If such a theory existed, and there were no other SM-charged states required near the weak scale, discovery could be quite difficult. In a Folded-SUSY like spectrum with weak-scale soft masses we might again expect the existence of mirror glueballs, with their accompanying experimental signatures. If, however, the mirror sector contains light matter or mirror-QCD was broken, discovery would have to proceed through Higgs-portal observables: invisible direct top partner production $h \rightarrow \tilde{t} \tilde{t}$ $[60,61]$, Higgs cubic coupling shifts $[60,62]$ at a 100–4–

\[21\]
Modular Invariance

[Dienes et al. ’94-’01, …]

At heart, solving hierarchy problem is about controlling \( \delta m_H^2 \sim (\text{Str}\mathcal{M}^0)\Lambda^2 + (\text{Str}\mathcal{M}^2)\log \Lambda + \ldots \)

For finite # of states \( \text{Str}\mathcal{M}^{2\beta} \equiv \sum_{\text{states } i} (-1)^F(M_i)^{2\beta} \) Cancellations require degenerate boson/fermion pairs

For infinite # of states, suitably regularize: \( \text{Str}\mathcal{M}^{2\beta} \equiv \lim_{y \to 0} \sum_{\text{states } i} (-1)^F(M_i)^{2\beta} e^{-yM_i^2} \)

Then many nondegenerate spectra have vanishing supertraces

E.g. for masses \( M_n = \sqrt{n\mu} \) and degeneracies \( g_n \)

\[
\begin{align*}
  g_n &= \begin{cases} 
    (-1)^n n^{2k} & \text{for any } k \geq 1, \ k \in \mathbb{Z} \\
    (-1)^n (n^5 - n) & \text{n even: bosons} \\
    (-1)^n (n^5 + 2n^3) & \text{n odd: fermions}
  \end{cases}
\end{align*}
\]

Generally satisfied when modular invariance controls the spectrum, degeneracies \( g_n \) given by the envelope of some suitable function \( \Phi(n) \)
Lowering the cutoff

...in diverse dimensions

5. RS / Technicolor
   [Randall, Sundrum '99; Weinberg '79; Susskind '79]

6. LED / $10^{32}$ x SM
   [Arkani-Hamed, Dimopoulos, Dvali '98; Antoniadis + ibid. '98; Dvali, Redi '09]

7. LST / Clockwork
   [Antoniadis, Dimopoulos, Giveon '01; Kaplan, Rattazzi '15; Giudice, McCullough '16]

8. Classicalization
   [Dvali, Giudice, Gomez, Kehagias '10]

9. Disorder
   [Rothstein '12]

Experimental signals: resonances, ...

- Primary distinctions are in spacing & coupling of resonances
- Potential goldmine of relatively unexplored signals for LST — e.g. perturbative string excitations
A Cutoff Solution?: Disorder

How does RS solve hierarchy problem?

Curvature localizes the graviton zero mode.

→ Fields localized at different points in 5th dimension see different fundamental scales

[Rothstein '12]: Can achieve the same outcome in a flat fifth dimension by localizing graviton w/ disorder

\[
S = - \int d^5x \sqrt{G} (M_3^3 R) + \sum_{\langle ij \rangle} M_4^4 V(|X_i - X_j|) - \sum_i \int d^4x \sqrt{g} f_i
\]

In this case disorder = randomly spaced & tensioned branes

But: not obvious that it works in detail

An interesting source of exponential hierarchies for scalars [NC, Sutherland '17]
Selecting a vacuum

Vacuum is one of many; end up in observed vacuum through dynamical process or anthropic constraint.

10. Anthropics (pressure)

11. Relaxation (rolling) [Graham, Kaplan, Rajendran ‘15]

12. Naturalness (reheating) [Arkani-Hamed et al ‘16]

13. Crunching away (collapse) [Csaki et al ‘20, see also Geller, Hochberg, Kuflik ‘18, Cheung & Saraswat ‘18, …]

Experimental signals: Diverse, but typically

- Cosmology (Bubble collisions; axions; contributions to $N_{\text{eff}}$ and $\sum m_\nu$)
- Exotic lab signals (displaced decays, hidden sector confinement, intensity frontier, …)
Relaxion

What if the weak scale is selected by dynamics, not symmetries?

The idea: couple Higgs to field whose minimum sets $m_H=0$
The problem: How to make $m_H=0$ a special point of potential?

Vev gives quark masses which give axion potential.

"Relaxion"

[Graham, Kaplan, Rajendran '15]

But: immense energy stored in evolving field, need dissipation.
Relaxion

**Simplest version: an axion coupled to QCD during inflation.**

\[ V(\phi) = \Lambda^4(H) \cos(\phi/f) + F(g\phi) + (-M^2 + g\phi)|H|^2 \]

- Axion-like potential
- Scanning term
- Large bare Higgs mass

Viable for Higgs + non-compact axion + inflation w/

- Very low Hubble scale (\ll \Lambda_{QCD})
- 10 Giga-years of inflation

Why not? Various other subtleties regarding technical naturalness, trans-Planckian field excursions, CC, fine-tuning to inflationary sector; need to solve strong CP problem. *New UV considerations.*

Extensive development, e.g. [Espinosa et al. ’15; Hardy ’15; Gupta et al ’15; Batell, Giudice, McCullough ’15; Choi, Im ’15; Kaplan, Rattazzi ’15; Di Chiara et al. ’15; Ibanez et al. ’15; Hook, Marques-Tavares ’16; Nelson, Prescod-Weinstein ’17; …]
5.2.2 The space are bounded by LEP and LHC searches that we describe in detail in the next section. To the veto around the charged pion mass, 100 MeV we have assumed a 10% theoretical error. The gap in the excluded region is again due to information about the di-lepton and b-bjarrel where the mixing angle $\sin \theta$ can become large. Finally, for GeV-scale masses we see from figure 3 that the mixing angle $\sin \theta$ becomes significant for relaxions with mass $\lesssim 100$ MeV.

The bands corresponds to a choice of $\sin^2 \theta = \frac{\Lambda^4}{v f m_h^2}$ for the relaxion, arising from direct and indirect probes at various hadron and lepton colliders, as in Fig. 4. Using only this information about the total rate and differential distribution of the SM and background events, we present. Here $\Lambda^4$ is the upper bound on the 4th force for $m_\phi = 10^4$ GeV (gray, thick, solid), $10^5$ GeV (gray, dashed), $10^6$ GeV = 10 GeV (gray, thin, solid), $10^7$ GeV = 100 GeV (gray, thin, dashed), $10^8$ GeV = 10 TeV (gray, dotted), $10^9$ GeV = 1 Te Pe (gray, dot-dashed), $10^{10}$ GeV = 10 TePe (gray, long dashed), $10^{11}$ GeV = 1 PePe (gray, long dot-dashed). Astrophysical and cosmological probes include the Supernova SN1987a (pale violet, labelled as SN), LHCb search for $B^+ \to J/\psi K^*$ (turquoise), LEP search for $B^0 \to J/\psi \phi$ (green). The bands correspond to a choice of $\sin^2 \theta$ as functions of $\Lambda^4$.

The vertical light gray line corresponds to the contour for the 95% CL excluded region in light blue in figure 3, and the mass, $m_\phi$.

Our estimate of the 95% CL excluded region is the upper bound on $\sin^2 \theta = \frac{\Lambda^4}{v f m_h^2}$ for $m_\phi = 10^4$ GeV (gray, thick, solid), $10^5$ GeV (gray, dashed), $10^6$ GeV = 10 GeV (gray, thin, solid), $10^7$ GeV = 100 GeV (gray, thin, dashed), $10^8$ GeV = 10 TeV (gray, dotted), $10^9$ GeV = 1 TePe (gray, dot-dashed), $10^{10}$ GeV = 1 PePe (gray, long dashed), $10^{11}$ GeV = 1 PePe (gray, long dot-dashed). Astrophysical and cosmological probes include the Supernova SN1987a (pale violet, labelled as SN), LHCb search for $B^+ \to J/\psi K^*$ (turquoise), LEP search for $B^0 \to J/\psi \phi$ (green). The bands correspond to a choice of $\sin^2 \theta$ as functions of $\Lambda^4$. Finally, for GeV-scale masses we see from figure 3 that the mixing angle $\sin \theta$ becomes significant for relaxions with mass $\lesssim 100$ MeV.
**Naturalness**

N copies of the SM

High Higgs cutoff $\Lambda_H$, high gravity cutoff $\Lambda_G$

**Two effects:**

1. Random UV contributions $\xrightarrow{}$ flat distribution of $m_H^2$ between $\pm \Lambda_H^2$

   \[
   \rho(m_H) \xrightarrow{\Lambda_H/\sqrt{N}} m_H \xrightarrow{\Lambda_H} \Lambda_H
   \]

   At least 1 copy w/ $|m_H| \sim \Lambda_H/\sqrt{N}$

2. Large number of species renormalizes Planck scale (e.g. graviton wavefunction renorm.)

   $$M_{Pl}^2 \sim N \Lambda_G^2$$

   Gravitational strong coupling scale $\Lambda_G$ below $M_{Pl}$

**Abstract**

[Arkani-Hamed, Cohen, D’Agnolo, Hook, Kim, Pinner ‘16]
**NNaturalness**

**Scale separation**

For example:
One copy w/ weak-scale Higgs for

\[ \begin{align*}
N &= 10^{16}: \\
\Lambda_H &= 10^{10} \text{ GeV} \\
\Lambda_G &= 10^{10} \text{ GeV} \\
(\text{That’s it.})
\end{align*} \]

\[ \begin{align*}
N &= 10^4: \\
\Lambda_H &= 10^4 \text{ GeV} \\
\Lambda_G &= 10^{16} \text{ GeV} \\
(\text{SUSY/compositeness @ } \Lambda_H)
\end{align*} \]

Why does copy w/ smallest \( m_H \) dominate?

*Cosmology.*

Reheaton \( \phi \) starts universe via \( \phi |H|^2 \) couplings

Decays (provided \( m_\phi < |m_{H,i}| \))

\[ \begin{align*}
\frac{m_{H,i}^2}{m_{H,i}^0} < 0 \\
\frac{m_{H,i}^2}{m_{H,i}^0} \geq 0
\end{align*} \]

\[ \begin{align*}
\Gamma &\propto \frac{1}{m_{H,i}^2} \\
\Gamma &\propto \frac{1}{m_{H,i}^4}
\end{align*} \]

Preferentially reheats copy w/ smallest \( |m_H| \) & \( m_{H,i}^2 < 0 \)
N Higgses...in the sky

All sectors reheated by some amount ⇒ dark radiation

Primary signals in dark radiation, extensive coverage by CMB-S4

\[ \frac{\rho_i}{\rho_{us}} = \frac{\Gamma_i}{\Gamma_{us}} \]

Dominated by sectors with similar scales

[Arkani-Hamed, Cohen, D’Agnolo, Hook, Kim, Pinner ‘16]

[\phi, m_\phi = 100 \text{ GeV}]

\[ \Delta N_{\text{eff}}, \phi, N = 10^4 \]

\( (r=1 \leftrightarrow \text{flat } m_\phi^2; r<1 \leftrightarrow \text{larger splitting}) \)
Complicating the flow

SM is reached from some intermediate fixed point where, say, a generalized Veltman condition is satisfied

\[ \delta m_H^2 = \sum_i c_i \frac{g_i^2}{16\pi^2} \Lambda_i^2 = 0 \]

This is a sense in which

14. Conformal symmetry

could address the hierarchy problem

Top-down: Embed SM in orbifold of N=4 SYM
[Frampton, Vafa ’99; Csaki, Skiba, Terning ‘99]

Bottom-up: “Little conformal symmetry”
[Houtz, Colwell, Terning ’16]

A challenge: how do fixed point couplings know about UV scale?

Experimental signals: Not fully explored, but expect new particles w/ SM quantum numbers around the TeV scale. Novelty is that statistics, irreps & couplings differ from more familiar solutions.
Exploding the cutoff

Gravity doesn’t provide a UV scale & the SM takes care of itself

15. Asymptotic fragility
   [Dubovsky, Gorbenko, Mirbabayi ’13]

16. Agravity [Salvio, Strumia ’14]

Scale $M_{Pl}$ not associated with relevant operator becoming strong, not “felt” by non-grav physics.

In IR, looks like CFT perturbed by irrelevant operators; in UV, no UV fixed point; cannot define local observables.

Example in 2d, no proposal for 4d.

Gravity has no intrinsic length scale and is “renormalizable”

$$S \sim \int d^4 x \sqrt{g} \left( \frac{R^2}{f^2} + \frac{1}{3} R - \frac{R_{\mu\nu}^2}{f^2} + \ldots \right)$$

(E-H term via vev of some field)

Can be re-written in terms of 2-derivative fields w/ ghosts.

Experimental signals: Details of gravity sector might be irrelevant. Crucially, must render SM couplings asymptotically free. Not a property of the SM itself, so entails low-scale unification.
Not actually the SM

17. Lee-Wick (higher derivative scalar)
[Grinstein, O’Connell, Wise ’06]

\[ \sim \frac{1}{2} \partial_\mu \phi \partial_\mu \phi - \frac{1}{2M^2} (\partial^2 \phi)^2 + \ldots \]

\[ -\frac{1}{2} \partial_\mu \tilde{\phi} \partial^\mu \tilde{\phi} + \frac{1}{2} M^2 \tilde{\phi}^2 + \ldots \]

Lee-Wick: higher-derivative theory

Improves UV convergence of diagrams, introduce for every SM field

\[ \frac{1}{p^2 - m^2} - \frac{1}{p^2 - M^2} = \frac{m^2 - M^2}{(p^2 - m^2)(p^2 - M^2)} \]

Write as normal field + new field w/ wrong-sign quadratic action

Can be defined in a unitary, Lorentz-invariant manner with only microscopic acausality. But who ordered that?
Connecting UV & IR

Essential feature of the hierarchy problem: the UV doesn’t know about the IR…unless it does?

Two frameworks exhibiting UV/IR mixing: QG & NCQFT

QG (cartoon version): collide sufficiently energetic particles, make a black hole. More energetic particles → bigger black hole.

NCQFT (cartoon version): non-commutativity of the form $[x^\mu, x^\nu] = i \theta^{\mu\nu}$, qualitatively a position-position uncertainty principle $\Delta x^\mu \Delta x^\nu \geq \theta/2$ [Filk ’96, Minwalla, Seiberg, Van Raamsdonk ’99, NC, Koren ’19]

Two ways to put this to work for hierarchy problem:

18. Indirect UV/IR mixing

19. Direct UV/IR mixing
Indirect UV/IR

**Usual (EFT) logic** of hierarchy problem: uncorrelated UV contributions give broad distribution of possible values of $m_h$ up to cutoff; $m_h$ well below cutoff “unlikely”

**Usual (EFT) logic** of hierarchy solution: lower the cutoff or eliminate sensitivity.

**Alternately**: consistency with gravity orchestrates correlations among UV parameters to satisfy bounds, changing the distribution.
Indirect UV/IR: WGC

(Electric) weak gravity conjecture: an abelian gauge theory must contain a state of charge $q$ and mass $m$ satisfying

$$gq \geq \frac{m}{M_{Pl}}$$

[Arkani-Hamed, Motl, Nicolis, Vafa '07]

“Justification”: consider BH of charge $Q$, mass $M$ decaying to this particle

$\# \text{ particles produced} = \frac{Q}{q}$

Energy conservation: $mQ/q < M$

Then BH satisfies

$$Z = Q \frac{M_{Pl}}{M} < z = q \frac{M_{Pl}}{m}$$

Extremal BH ($Z=1$) stable unless there exists a state with $z > 1$

$\Rightarrow q > \frac{m}{M_{Pl}}$ to avoid stable black holes, remnants, in conflict w/ holography
A Family of Conjectures

Electric WGC: \[ m \leq (gq)M_{P1} \]
[Arkani-Hamed, Motl, Nicolis, Vafa ‘07]

Magnetic WGC: \[ \Lambda \lesssim gM_{P1} \]
[Arkani-Hamed, Motl, Nicolis, Vafa ‘07]

+Scalar WGC: \[ m \leq \sqrt{g^2q^2 - \mu^2M_{P1}} \]
[Palti ‘17]

dS WGC: \[ m^2 \gtrsim gqM_{P1}H \]
[Montero, Van Riet, Venken ‘19]

Axion WGC: \[ f \leq (1/S)M_{P1} \]
[Arkani-Hamed, Motl, Nicolis, Vafa ‘07]

New hierarchies from EFT + gravity.
Weak Gravity, Weak Scale?

[Cheung, Remmen '14]: If mass of WGC particle is UV sensitive, then for fixed UV-insensitive parameters, satisfying the WGC enforces fine-tuning. *(Or: would orchestrate correlations among UV contributions)*

**Application to SM:** charge SM fermions under weakly gauged (unbroken) \( U(1)_{B-L} \) (bounds currently \( q \ll 10^{-24} \)).

Cancel anomalies with RHN \( \nu_R \)

Neutrino mass from EWSB

\[
y_{\nu} H \bar{L} \nu_R \rightarrow m_\nu \sim y_{\nu} v
\]

If lightest neutrino is WGC particle,

\[
m_\nu \sim 0.1 \text{ eV, } q \gg 10^{-29}
\]

**For fixed \( y, q \), satisfying WGC places an upper bound on \( v \)**

See also: [Ibañez, Martin-Lozano, Valenzuela '17, …; March-Russell & Petrossian-Byrne '20, …]
Weak Gravity, Weak Scale?

Problem: Magnetic WGC implies $\Lambda$ well below weak scale. Simple fix...

\[ -\mathcal{L} \supset \left\{ m_L L L^c + m_N N N^c + y H^\dagger L N^x + y H L^c N \right\} + \text{h.c.} \]

Best option: $m_N < m_L$, lightest mass eigenstate $\chi_1$ is WGC particle

Then for fixed (technically natural) $g, m_L, m_N, y$,

\[ v^2 \lesssim \frac{2}{y^2} \left( m_{\chi_1}^2 + m_{\chi_1} (m_L - m_N) - m_L m_N \right) \]
Weak Gravity, Weak Scale?

Lightest particle charged under $\text{U}(1)_X$ is stable $\Rightarrow$ dark matter candidate

$\text{U}(1)_X$ gives a very weak, long-range force, too weak to influence individual collisions but relevant on scale of galaxy clusters

Galaxy cluster collisions can trigger plasma instabilities, making DM collisional on large scales [Ackerman, Buckley, Carroll, Kamionkowski '08; Heikinheimo, Raidal et al '15; Spethmann et al '16]

Timescale of plasma fluctuations set by plasma frequency,

$$\omega_p = \sqrt{\frac{g^2 \rho}{m^2}} \geq \sqrt{\frac{\rho}{M_{\text{Pl}}}} \quad \omega_p^{-1} \lesssim 10^{15} \text{ s} \times \left(\frac{0.04 \text{ GeV cm}^{-3}}{\rho}\right)^{1/2}$$

C.f. $\tau \sim 1 \text{ Gyr} \sim 10^{16} \text{ s}$ for galaxy cluster collisions

\[\text{Figure 6}\]

Estimated parameter space for which small perturbations would experience significant growth in astrophysical systems, for a vector mediator of mass $m_A$, coupling to DM particles of mass $m$ with coupling $g$. The shaded parameter space above the broken red lines, labelled with mediator mass $m_A$, is the space in which perturbations would grow for that mediator mass. The region above the solid red line is the region in which perturbations would grow for sufficiently light mediators.

The astrophysical situations we consider are cluster collisions such as the Bullet Cluster, and dense DM subhalos (such as dwarf galaxies) moving through a larger halo (see Section 3). The 'Coulomb' region shows the parameter space in which $2^2$ Coulomb collisions would have a significant impact in DM halos (see section 5.1). Note that, for DM masses $m \lesssim 10^{-25} \text{ eV}$, the DM occupation number in Galactic halos must be large, which may result in additional coherent scattering effects (Section 6).

To estimate when DM-DM plasma instabilities would arise during such collisions, we need some model for the DM distributions in the colliding clusters. [16] estimates these distributions, for the Bullet Cluster collision, using gravitational lensing information. Assuming cored DM density profiles, their best-fit parameters have core radii $\sim 100 \text{ kpc}$, and central densities $\sim \text{few} \times 10^{-1} \text{ GeV cm}^{-3}$, with inferred halo velocity dispersions of $\sim 800 \text{ km s}^{-1}$. Simulations of the gas dynamics in the collision suggest that the relative velocity of the halos is $\sim 3000 \text{ km s}^{-1}$. These parameters give a central plasma frequency of $\omega_p^{-1} \lesssim 10^{15} \text{ s} \times \left(\frac{0.04 \text{ GeV cm}^{-3}}{\rho}\right)^{1/2}$ compared to a crossing time of $\sim 100 \text{ kpc} / (3000 \text{ km s}^{-1}) \sim 3 \times 10^7 \text{ yr}$. If we take a conservative threshold of $O(100) \text{ e-folding times}$, we would expect plasma instabilities to grow significantly for $g \lesssim 10^{-16} m \text{ GeV}^{-1}$ (3.2)

Profiles with central cusps, such as NFW, would lead to higher central DM densities — to be conservative, we consider cored profiles (gravitational lensing data does not have the spatial resolution to distinguish these possibilities).
Things I can’t (yet) cleanly compartmentalize

20. Tune the CC to set the weak scale
[Arvanitaki, Dimopoulos, Gorbenko, Huang, Van Tilburg ‘16]

21. Massless moduli from explicitly broken SUSY
[Dong, Freedman, Zhao ’14, ’15]

Example: explicit marginal SUSY breaking involving U(1)_{R} gauge fields on bdy of AdS_{3}

\[ \delta S \sim \int_{\text{bdy}} A \wedge \bar{A} \sim \int d^2 z J(z) \bar{J}(\bar{z}) \]

Induces splitting in R-charged multiplets. Feed to R-neutral multiplets w/ yukawa

\[ \lambda \phi_{N} \phi_{R}^{\dagger} \phi_{R} \]

R-neutral scalars massless to all orders

Analogous to

\[ y_{i}^{2} m_{i}^{2} - y_{i}^{2} m_{i}^{2} = 0 \]

Make vacua tuning CC “dense” near weak scale

Signals
- Vector-like leptons (direct search, Higgs invisible width, precision electroweak)
- Super-light (O(10^{-10} eV)) radion

\[ \begin{align*}
N & \quad \begin{cases}
- M_{UV}^{2} & \quad M_{UV}^{2} \\
- \lambda v_{L}^{2} & \quad 0 \\
+ M_{UV}^{2} & \quad \end{cases} \\
H & \quad \begin{cases}
- v_{L}^{2} & \quad v_{L}^{2} \\
0 & \quad v_{L}^{2} \\
+ v_{L}^{2} & \quad \end{cases} \\
N_{1} & \quad \begin{cases}
0 & \quad 0 \\
0 & \quad 0 \\
0 & \quad \end{cases} \\
N_{2} & \quad \begin{cases}
0 & \quad 0 \\
0 & \quad 0 \\
0 & \quad \end{cases} \\
\text{SM} & \quad \begin{cases}
0 & \quad 0 \\
0 & \quad 0 \\
0 & \quad \end{cases} \\
\sigma_{1} & \quad \begin{cases}
0 & \quad 0 \\
0 & \quad 0 \\
0 & \quad \end{cases} \\
\sigma_{2} & \quad \begin{cases}
0 & \quad 0 \\
0 & \quad 0 \\
0 & \quad \end{cases} \\
\end{align*} \]
Self-Organized Criticality

Some systems evolve into critical states on their own (sandpiles, a la [Bak, Tang, Wiesenfeld ’84]). Wouldn’t that be nice? [Giudice ’08, Kaplan ‘97]

Vanishing Higgs mass coinciding with potential minimum for an extra-dimensional modulus field [Eroencel, Hubisz, Rigo ’18]

Localization of scalar fields exponentially close to critical points during eternal inflation [Giudice, McCullough, You ’21]
[Khoury et al. ’19-’20]
1. Supersymmetry
2. Global symmetry
3. Discrete symmetry
4. Modular invariance
5. RS/Technicolor
6. LED/10^{32}xSM
7. LST/Clockwork
8. Classicalization
9. Disorder
10. Anthropic
11. Relaxation
12. NNaturalness
13. Crunching away
14. Conformal symmetry
15. Asymptotic fragility
16. Agravity
17. Lee-Wick Theory
18. Weak gravity conjecture
19. Non-commutative QFT
20. Weak scale from CC
21. AdS magic
22. Self-organized criticality
23. …

With apologies for the many omissions…
Conclusions

• Electroweak hierarchy problem remains one of the biggest motivations for BSM physics.

• Close to comprehensively understanding conventional solutions & searching accordingly. Should obviously keep searching for these as hard as possible, but…

• …at some point data tips the balance towards truly unconventional solutions. Many of these are a way of making sense of the apparent failure of Wilsonian EFT.

• Promising places to look: conformal symmetry; UV/IR mixing; self-organized criticality. But who am I to say? Lots to explore. Lively intersection of QFT, cosmology, quantum gravity.

• Experimental possibilities vast once we understand the full space of theories, cosmology playing an increasingly central role.

• Skepticism is justified, but imagine facing the ultraviolet catastrophe in the early 20th c…

Thank you!