Higgs Precision Physics – Decays

A. Freitas
University of Pittsburgh

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What does decay have to do with precision?
What does decay have to do with precision?

1. Intro: Higgs precision studies
2. SM prediction for Higgs decays
3. Higgs production at $e^+e^-$ colliders
4. SM input parameters
5. Theory calculations
6. Beyond the SM
Higgs physics is entering the precision era:

Determination of Higgs couplings from Higgs production and decay:

<table>
<thead>
<tr>
<th>Higgs Decay</th>
<th>LHC* (2020)</th>
<th>HL-LHC*</th>
<th>CEPC</th>
<th>FCC-ee</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h\gamma\gamma$</td>
<td>9%</td>
<td>1.8%</td>
<td>1.6%</td>
<td>1.5%</td>
</tr>
<tr>
<td>$h\tau\tau$</td>
<td>15%</td>
<td>1.9%</td>
<td>1.2%</td>
<td>0.5%</td>
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<tr>
<td>$h\mu\mu$</td>
<td>–</td>
<td>4.3%</td>
<td>5%</td>
<td>6%</td>
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<tr>
<td>$hWW$</td>
<td>9%</td>
<td>1.7%</td>
<td>1.1%</td>
<td>0.2%</td>
</tr>
<tr>
<td>$hZZ$</td>
<td>8%</td>
<td>1.5%</td>
<td>0.25%</td>
<td>0.15%</td>
</tr>
<tr>
<td>$hgg$</td>
<td>10%</td>
<td>2.5%</td>
<td>1.2%</td>
<td>0.8%</td>
</tr>
</tbody>
</table>

* assuming no exotic decays
New physics reach

Precision $\equiv$ large mass scales

- Example: mixing with a heavy particle, $\theta \sim \frac{m_{SM}}{M}$

- Model-independent formulation through effective field theory (EFT)

→ lecture by V. Sanz

de Blas, Durieux, Grojean, Gu, Paul ’19
Access to Higgs interactions through production and decay

Most couplings only accessible in decays ($b, c, \tau, \mu, \gamma, Z$)

Decay ratios can reduce systematics, e.g.

$$\frac{\sigma[pp \rightarrow H \rightarrow \gamma\gamma]}{\sigma[pp \rightarrow H \rightarrow ZZ]}$$

$\mathcal{O}(\%)$ BSM effects $\rightarrow$ need SM predictions with higher-order corrections
SM predictions for Higgs decays

**Reviews:** 1404.0319, 1906.05379

**hbb:** [HL-LHC: 7%, FCC-ee: 0.8%]

- $\mathcal{O}(\alpha_s^4)$ QCD corrections
  
  $21\% + 4\% + 0.2\% - 0.15\%$

  Baikov, Chetyrkin, Kühn '05

- $\mathcal{O}(\alpha)$ QED+EW, $< 0.5\%$

  Dabelstein, Hollik '92; Kniehl '92

- leading $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha\alpha_s)$ for large $m_t$

  $< 0.5\%$ (use for error estimate)

  Kwiatkowski, Steinhauser '94

  Butenschoen, Fugel, Kniehl '07

**Current theory error:** $\Delta_{\text{th}} < 0.4\%$

**With full 2-loop:** $\Delta_{\text{th}} \sim 0.2\%$

**hττ:** [HL-LHC: 3.8%, FCC-ee: 1.1%] → Similar, but no QCD
SM predictions for Higgs decays

\( h_{WW^*/hZZ^*} \): [HL-LHC: 3.4%, FCC-ee: 0.4%]

- \( m_H < 2m_W \) → one or both W/Z must be off-shell
- Need full process \( h \rightarrow 4f \)
- Differential distributions contain more info
  → S. Gori
  
  - complete \( \mathcal{O}(\alpha) + \mathcal{O}(\alpha_s) \) for \( h \rightarrow 4f \)
    → MC implementation for distributions
    Bredenstein, Denner, Dittmaier, Weber '06
  - leading \( \mathcal{O}(\alpha^2), \mathcal{O}(\alpha \alpha_s) \) and \( \mathcal{O}(\alpha \alpha_s^2) \) for large \( m_t \)
    → Small (0.2%) effect
    Kniehl, Spira '95; Kniehl, Steinhauser '95; Djouadi, Gambino, Kniehl '97; Kniehl, Veretin '12

Theory error: \( \Delta_{th,EW} < 0.3\%, \Delta_{th,QCD} < 0.5\% \)

With NNLO final-state QCD corrections: \( \Delta_{th,QCD} < 0.1\% \)
SM predictions for Higgs decays

hWW*/hZZ*:

- Non-trivial effects in distributions
  - Bredenstein, Denner, Dittmaier, Weber ’06
  - Larger theory uncertainty?

- Non-perturbative color reconnection effects
  - No impact on decay rate, but on distributions
  - Difficult to model, currently done with MC programs such as PYTHIA/HERWIG
hWW*/hZZ*: 

- Intermediate W/Z can become on-shell
  \[ \rightarrow \text{regularize divergencies in } \frac{1}{p^2 - m_V^2} \quad (V = W, Z) \]

- One option: *complex mass scheme*
  \[ m_V^2 \rightarrow \mu_V^2 \equiv m_V^2 - i m_V \Gamma'_V \] everywhere,
  incl. relations like
  \[ g = \frac{e}{s_W} = \frac{e}{\sqrt{1-m_W^2/m_Z^2}} \]
  \[ \text{Denner, Dittmaier, Roth, Wieders '05} \]
  \[ \rightarrow \text{preserves gauge invariance, consistent to all orders} \]
  \[ \frac{d\Gamma}{d(p^2)} \propto \frac{1}{(p^2 - m_V^2)^2 + m_V^2 \Gamma'_V} \]

- In experimental analyses:
  \[ \frac{d\Gamma}{d(p^2)} \propto \frac{1}{(p^2 - M_V^2)^2 + p^4 M_V^2 / \Gamma'_V} \]
  \[ m_Z = M_Z / \sqrt{1 + \Gamma_Z^2 / M_Z^2} \approx M_Z - 34 \text{ MeV} \]
  \[ \Gamma'_Z = \Gamma_Z / \sqrt{1 + \Gamma_Z^2 / M_Z^2} \approx \Gamma_Z - 0.9 \text{ MeV} \]
SM predictions for Higgs decays

hgg: [HL-LHC: 5%, FCC-ee: 1.6%]

- Loop-induced process in the SM

\[
\Gamma[h \rightarrow gg] = \frac{y_t^2 \alpha_s^2 m_t^2}{4\pi^3 m_H} \left[ 1 + \left(1 - \frac{4m_t^2}{m_H^2}\right) \arcsin^2 \frac{m_H}{2m_t} \right]^2
\]

\[
\approx \frac{\alpha_s^2 m^3_H}{72\pi^3 v^2} \quad \left( m_H \ll 2m_t, \ y_t = \frac{\sqrt{2}}{v} m_t \right)
\]

For heavy BSM quark:

\[
\delta \Gamma[h \rightarrow gg] \approx \frac{y_h^2 \alpha_s^2 m_H^3}{144\pi^3 M_Q^2}
\]

- \(N^n\)LO corrections require \((n+1)\)-loop diagrams
  - \(O(\alpha_s)\) exact result
  - Use large \(m_t\) expansion, used for \(O(\alpha_s^2)\)
    \(\rightarrow\) fast convergence \((m_t^{-4} \text{ term is } \sim 1\% \text{ of } m_t^0 \text{ term})\)
  - \(O(\alpha_s^3)\) with low-energy theorem (equiv. to 1st term in large \(m_t\) exp.)

Djouadi, Graudenz, Spira, Zerwas ’95

Schreck, Steinhauser ’07

Baikov, Chetyrkin ’06
hgg:

- **Low-energy theorem:** for $m_t \to \infty$ top loop becomes effective $hgg$ interaction:

  \[ L_{\text{eff}} = \frac{C_{hg}}{v} h G_{\mu\nu} G_{\mu\nu} \]

  \[ \Gamma[h \to gg] = \frac{2C_{hg}^2 m_H^3}{\pi v^2} \]  

  \[ \text{LO SM res.:} \quad C_{hg} = -\frac{\alpha_s}{12\pi} \]  

  \[ (1) \]

  \[ (2) \]

  Gauge-invariant form:

  \[ L_{\text{eff}} = \frac{C_{hg}}{v^2} (\phi^\dagger \phi) G_{\mu\nu} G_{\mu\nu}, \quad \phi = (G^+, \frac{v+h+G^0}{\sqrt{2}})^\top \]  

  \[ (3) \]

- **Higher orders:**
  - corrections to $C_{hg}$ (zero external momentum)
  - corrections to (2) with effective $hgg$ cpl.
SM predictions for Higgs decays

**hgg:**

- **EW corrections:**
  - Can use expansion or low-energy theorem for top-quark contributions
    
    Degrassi, Maltoni ’04
  
  - Direct 2-loop calculation for (dominant) light-quark contributions
    
    Aglietti, Bonciani, Degrassi, Vicini ’04; Degrassi, Maltoni ’04

QCD corrections: $65\% + 20\% + 2\%$

EW corrections: $\sim 5\%$

Theory error (dominated by QCD): $\Delta_{th} \approx 3\%$

With $O(\alpha_s^4)$ in large $m_t$-limit (4-loop massless QCD diags.): $\Delta_{th} \approx 1\%$
SM predictions for Higgs decays

\[ h\gamma\gamma: \quad [\text{HL-LHC: 3.6\%, FCC-ee: 3.0\%}] \]

- Top+W loop at LO
  - \( \mathcal{O}(\alpha_s^2) \) QCD corrections (with large \( m_t \) expansion)
    - Zheng, Wu ’90; Djouadi, Spira, v.d.Bij, Zerwas ’91
    - Dawson, Kauffman ’93; Maierhöfer, Marquard ’12
  - NLO EW
    - Aglietti, Bonciani, Degrassi, Vicini ’04; Degrassi, Maltoni ’04
    - Actis, Passarino, Sturm, Uccirati ’08

Theory error: \( \Delta_{\text{th}} < 1\% \)
Real vs. pseudo observables

- $\Gamma \to XX$ is not a real observable
- Effect of detector acceptance and isolation criteria, in particular for extra gluon (jet) and photon radiation
- Effect of selection cuts to reduce backgrounds
- Monte-Carlo tools needed to simulate these effects

**Current LHC studies:**
- rad. corrections in decay, only parton shower for $\gamma, g$ emission; differential rates corrected (rewighted) with fixed-order results

**More accurate procedure:**
- Implement fixed-order rad. corr. in MC program, **matching** to avoid double counting with parton shower

Catani, Krauss, Kuhn, Webber ’01
Nason ’04; Frixione, Nason, Oleari ’07
Higgs production at $e^+e^-$ colliders

- **hZ production**: dominant at $\sqrt{s} \sim 240$ GeV
- **WW fusion**: sub-dominant but useful for constraining $h$ width  Han, Liu, Sayre ’13
SM predictions for Higgs production

**hZ production:** [CEPC: 0.5%, FCC-ee: 0.3%]

- $\mathcal{O}(\alpha)$ corr. to $hZ$ production and $h, Z$ decay
  
  - $\Gamma_H/m_H \approx 4 \times 10^{-5}$, $\Gamma_Z/m_Z \approx 0.025$  
  
  ⇒ include off-shell $Z$ effects

  Technology for $\mathcal{O}(\alpha)$ corr. to $hf\bar{f}$ production available

  - $\mathcal{O}(\alpha\alpha_s)$ corrections

  Gong et al. ’16;  Chen, Feng, Jia, Sang ’18

  Theory error: $\Delta_{th} \sim O(1\%)$

  With full 2-loop corrections for $ee \rightarrow HZ$:

  $\Delta_{th} \lesssim O(0.3\%)$
SM predictions for Higgs production

WW fusion:

- $\mathcal{O}(\alpha)$ corrections

Theory error: $\Delta_{\text{th}} \sim O(1\%)$?

Full NNLO calc. for $2 \rightarrow 3$ process is very challenging, but may not be needed

Belanger et al. '02; Denner, Dittmaier, Roth, Weber '03
SM predictions for Higgs decays need measured input parameters

Reviews: 1906.05379, 2012.11642

- $M_Z, M_W$: current precision $< 0.1\%$ $\rightarrow$ negligible impact

- $m_t$: Most precise measurement at LHC: $\delta m_t \sim 0.3 \text{ GeV}$

Additional theory error from scheme translation Hoang, Plätzer, Samitz ’18
$\rightarrow$ Total uncertainty $\delta m_t \sim 0.5 \text{ GeV}$
$\rightarrow$ Negligible impact because $m_t$ only appears in loops
SM predictions for Higgs decays need measured input parameters.

**Reviews:** 1906.05379, 2012.11642

- $M_H$: high precision important for $h \rightarrow WW^*, ZZ^*$

  amplitude $\propto \frac{1}{p_1^2 - m_V^2}$, \quad $p_1 \sim m_H - m_V$ (both $V, V^*$ at rest)

  \[ \Gamma_{VV^*} = \text{[energy]} \]

  \[ \Rightarrow \Gamma_{VV^*} \propto \frac{[\text{energy}]^5}{|p_1^2 - m_V^2|^2} \sim \frac{m_H^5}{[(m_H - m_V)^2 - m_V^2]^4} \]

  Uncertainty $\delta m_H$ \quad $\Rightarrow$ \quad $\frac{\delta \Gamma_{VV^*}}{\Gamma_{VV^*}} = \frac{m_H - 6m_V}{m_H - 2m_V} \frac{\delta m_H}{m_H} \approx 10 \frac{\delta m_H}{m_H}$

  $\delta m_H = 0.2 \text{ GeV}$ \quad $\Rightarrow$ \quad $\frac{\delta \Gamma_{VV^*}}{\Gamma_{VV^*}} = 1.6\%$

  CEPC/FCC-ee/ILC can achieve $\delta m_H \lesssim 20 \text{ MeV}$ \quad $\Rightarrow$ \quad $\frac{\delta \Gamma_{VV^*}}{\Gamma_{VV^*}} \lesssim 0.2\%$
SM input parameters

• $\alpha_s$: important for $h \rightarrow gg$ (also $h \rightarrow q\bar{q}$)

$$\delta \alpha_s = 0.001 \Rightarrow \frac{\delta \Gamma_{gg}}{\Gamma_{gg}} \approx 3\%$$

Methods for $\alpha_s$ determination:

• Most precise determination using Lattice QCD:
  $\alpha_s = 0.1184 \pm 0.0006$  HPQCD '10
  $\alpha_s = 0.1185 \pm 0.0008$  ALPHA '17
  $\alpha_s = 0.1179 \pm 0.0015$  Takaura et al. '18
  $\alpha_s = 0.1172 \pm 0.0011$  Zafeiropoulos et al. '19

→ Difficulty in evaluating systematics

• $e^+e^-$ event shapes and DIS: $\alpha_s \sim 0.114$
  Alekhin, Blümlein, Moch '12; Abbatte et al. ’11; Gehrmann et al. ’13

→ Subject to sizeable non-pertubative power corrections
→ Systematic uncertainties in power corrections?
SM input parameters

\( \alpha_s \):

- Hadronic \( \tau \) decays: \( \alpha_s = 0.119 \pm 0.002 \) \hspace{1cm} PDG '18
  \( \rightarrow \) Non-perturbative uncertainties in OPE and from duality violation
  \hspace{1cm} Pich '14; Boito et al. '15,18

- Electroweak precision \( (R_\ell = \Gamma_{Z}^{\text{had}}/\Gamma_{\ell}^{Z}) \):
  \( \alpha_s = 0.120 \pm 0.003 \) \hspace{1cm} PDG '18
  \( \rightarrow \) No (negligible) non-perturbative QCD effects
  FCC-ee: \( \delta R_\ell \sim 0.001 \)
  \hspace{1cm} \Rightarrow \hspace{1cm} \delta \alpha_s < 0.0001

Theory input: N\(^3\)LO EW corr. + leading N\(^4\)LO
  to keep \( \delta_{\text{th}} R_\ell \lesssim \delta_{\text{exp}} R_\ell \)

Caviat: \( R_\ell \) could be affected by new physics
SM input parameters

\begin{itemize}
\item \(\alpha_s\):
  \begin{itemize}
  \item \( R = \frac{\sigma[ee\rightarrow\text{had.}]}{\sigma[ee\rightarrow\mu\mu]} \) at lower \( \sqrt{s} \)
  \end{itemize}
  
  e.g. CLEO \((\sqrt{s} \sim 9 \text{ GeV})\): \(\alpha_s = 0.110 \pm 0.015\)

  Kühn, Steinhauser, Teubner '07

  \(\rightarrow\) dominated by \(s\)-channel photon, less room for new physics

  \(\rightarrow\) QCD still perturbative

  naive scaling to 50 \(\text{ab}^{-1}\) (BELLE-II): \(\delta\alpha_s \sim 0.0001\)

\item \(m_b, m_c\): From quarkonia spectra using Lattice QCD
  \(\delta m_b^{\overline{\text{MS}}} \sim 30 \text{ MeV}, \, \delta m_c^{\overline{\text{MS}}} \sim 25 \text{ MeV}\)

  LHC HXSWG '16

  \(\Rightarrow\) \(\frac{\delta\Gamma_{bb}}{\Gamma_{bb}} \approx 1.4\%, \, \frac{\delta\Gamma_{cc}}{\Gamma_{cc}} \approx 4.0\%\)

  \(\rightarrow\) estimated improvements \(\delta m_b^{\overline{\text{MS}}} \sim 13 \text{ MeV}, \, \delta m_b^{\overline{\text{MS}}} \sim 7 \text{ MeV}\)

  Lepage, Mackenzie, Peskin '14
\end{itemize}
Theory calculations

Theory uncertainties

- Theory error estimate is not well defined, ideally $\Delta_{\text{th}} \ll \Delta_{\text{exp}}$

- Common methods:
  - Count prefactors ($\alpha$, $N_c$, $N_f$, ...)
  - Extrapolation of perturbative series
    e.g. assume $\frac{\Gamma_{\text{NNNLO}}}{\Gamma_{\text{NNLO}}} \sim \frac{\Gamma_{\text{NNLO}}}{\Gamma_{\text{NLO}}}$
  - Renormalization scale dependence
    (for $\overline{\text{MS}}$ renormalization, widely used for QCD)
  - Renormalization scheme dependence
    e.g. compare $\overline{\text{MS}}$ and OS renormalization
Experimental precision requires inclusion of multi-loop corrections in theory.

Integrals over loop momenta:

$$\int d^4q_1 d^4q_2 \, f(q_1, q_2, p_1, k_1, \ldots, m_1, m_2, \ldots)$$

Computer algebra tools:

- Generation of diagrams, $O(1000) - O(10000)$
- Lorentz and Dirac algebra
- Integral simplification (e.g. symmetries)

$\begin{cases} \text{not a limiting factor} \end{cases}$

Evaluation of loop integrals:

- Analytical
- Approximate (expansions)
- Numerical
Analytic calculations

- Mostly used for diagrams with few mass scales
- Reduce to **master integrals** with integration-by-parts and other identities
  
  Chetyrkin, Tkachov ’81; Gehrmann, Remiddi ’00; Laporta ’00; ...

Public programs:  
  
  Reduze  von Manteuffel, Studerus ’12
  FIRE  Smirnov ’13,14
  LiteRed  Lee ’13
  KIRA  Maierhoefer, Usovitsch, Uwer ’17

  → Large need for computing time and memory

- Evaluate master integrals with differential equations or Mellin-Barnes rep.
  
  Kotikov ’91; Remiddi ’97; Smirnov ’00,01; Henn ’13; ...

  → Result in terms of Goncharov polylogs / multiple polylogs

  → Some problems need iterated elliptic integrals / elliptic multiple polylogs
  
  Broedel, Duhr, Dulat, Trancredi ’17,18
  Ablinger er al. ’17

  → Even more classes of functions needed in future?
Asymptotic expansions

- Exploit large mass/momentum ratios, e.g. \( \frac{M_Z^2}{m_t^2} \approx \frac{1}{4} \)
- Evaluate coeff. integrals analytically
- Fast numerical evaluation

→ Public programs:
  exp Harlander, Seidensticker, Steinhauser '97
  asy Pak, Smirnov '10

→ Possible limitations:
  - no appropriate mass/momentum ratios
  - bad convergence
  - impractical if too many mass/mom. scales
Challenge 1: presence of UV/IR divergencies

- Remove through subtraction terms
  \[
  \int d^4 q_1 d^4 q_2 (f - f_{\text{sub}}) + \int d^4 q_1 d^4 q_2 f_{\text{sub}}
  \]
  finite\hspace{1cm} \text{solve analytically}

- Remove through variable transformations:
  a) Sector decomposition
     Public programs: (py)\text{SecDec} \hspace{1cm} \text{Carter, Heinrich '10; Borowka et al. '12,15,17}
     \text{FIESTA} \hspace{1cm} \text{Smirnov, Tentyukov '08; Smirnov '13,15}
  
  b) Mellin-Barnes representations
     Public programs: \text{MB/MBresolve} \hspace{1cm} \text{Czakon '06; Smirnov, Smirnov '09}
     \text{AMBRE/MBnumerics} \hspace{1cm} \text{Gluza, Kajda, Riemann '07}
     \text{Dubovyk, Gluza, Riemann '15}
     \text{Usovitsch, Dubovyk, Riemann '18}
Challenge 2: stability and convergence

- Integration in momentum space: $4L$ dimensions ($L =$ # of loops)
- Integration in Feynman parameters: $P - 1$ dimensions ($P =$ # of propagators)

→ Multi-dim. integrals need large computing resources and converge slowly

- Variable transformations to avoid singularities and peaks

![Graphs showing variable transformations](image-url)
Loop calculations: Summary

Analytical techniques and expansions:
Complexity increases with ...
... more loops;
... more external particles;
... more different masses

Numerical techniques:
Complexity increases with ...
... more loops;
... more external particles;
... fewer masses
Deviations from SM predictions

"Kappa framework"

- Multiply Higgs Feynman rules with factor $\kappa_X \neq 1$:

$$
\begin{align*}
H - \rightarrow - h \rightarrow f & = - \frac{igm_f}{2m_w} \kappa_f \\
H - \rightarrow - W^+ & = igm_w g_{\mu \nu} \kappa_W
\end{align*}
$$

- breaks gauge-invariance
- in general not possible to compute EW corrections

Effective field theory framework

$$
\mathcal{L} = \mathcal{L}_{SM} + \sum_{d=5}^{\infty} \frac{1}{\Lambda^{d-4}} \sum_i c_i \mathcal{O}^{(d)}_i
$$

- Operators required to satisfy SM gauge invariance
- Valid description for energies $E \ll \Lambda$ ($\Lambda \sim$ mass of heavy particles)
- Leading contribution to Higgs physics: $d = 6$
- **SMEFT**: Higgs doublet as in SM
- **HEFT**: Higgs and Goldstone bosons treated independently

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Deviations from SM predictions

"Kappa framework"

- Multiply Higgs Feynman rules with factor $\kappa_X \neq 1$:

\[
  H \rightarrow f = -\frac{ig_{w} \kappa_f}{2m_{w}} \, K_f \\
  H \rightarrow W = ig_{w} g_{\mu} \kappa_{w}
\]

- breaks gauge-invariance
  → in general not possible to compute EW corrections

Effective field theory framework

\[
  \mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_{d=5}^{\infty} \frac{1}{\Lambda^{d-4}} \sum_{i} c_{i} \mathcal{O}_{i}^{(d)}
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- Operators required to satisfy SM gauge invariance
- Valid description for energies $E \ll \Lambda$ ($\Lambda \sim$ mass of heavy particles)
- Leading contribution to Higgs physics: $d = 6$
- **SMEFT**: Higgs doublet as in SM
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No conclusive BSM evidence in Higgs physics
→ BSM effects are small
→ ignore rad. corr. to BSM contributions (?)

But some higher-order effects can be significant:

- Large SM QCD corrections to LO operator insertions:
  \[ \delta_{QCD} \sim 20\% \]
  \[ \delta_{QCD} \sim 50\ldots100\% \]
  → Practical calculations similar to SM

- New operators appearing in loops:
  e.g. 

\[ \begin{array}{c}
\text{H} \rightarrow W^+ b \\
\end{array} \]
Example 1: $hgg$ operator in $h \rightarrow b\bar{b}$

\[
\frac{\delta \Gamma_{bb}}{\Gamma_{bb}} = C_F \frac{\alpha_s}{\pi} \frac{v^2}{\Lambda^2} c_{\phi G} \ln^2 \frac{m_b^2}{m_H^2} \approx 2.4 \frac{v^2}{\Lambda^2} c_{\phi G}
\]

[Note: $c_{\phi G}$ expected to be loop-induced]

Example 2: $ttbb$ operator in $h \rightarrow b\bar{b}$

\[
\frac{\delta \Gamma_{bb}}{\Gamma_{bb}} \approx 1.75 \frac{v^2}{\Lambda^2} c_{qtqb}^{(1)}
\]

[Note: $c_{qtqb}^{(1)}$ also makes a contribution to $m_b$, and fine-tuning arguments suggest it shouldn’t be large]
Example 3: dipole-type $\gamma$-top operator in $h \rightarrow \gamma\gamma$

$$\frac{\delta \Gamma_{bb}}{\Gamma_{bb}} \approx \mathcal{O}(10-20) \times \frac{v^2}{\Lambda^2} c_{tB}^{(1)}$$

[Note: $c_{tB}$ expected to be loop-induced]
Backup slides
SMEFT operators in Warsaw basis

Grzadkowski, Iskrzynski, Misiak, Rosiek ’10

<table>
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<tr>
<th>$X^3$</th>
<th>$\varphi^6$ and $\varphi^4 D^2$</th>
<th>$\psi^2 \varphi^3$</th>
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<td>$Q_G$</td>
<td>$Q_\varphi$</td>
<td>$Q_{\varphi \varphi}$</td>
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<tr>
<td>$Q_{\tilde{G}}$</td>
<td>$(\varphi^\dagger \varphi)^3$</td>
<td>$(\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi)$</td>
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<td>$Q_W$</td>
<td>$Q_{\varphi \varphi}$</td>
<td>$Q_{u \varphi}$</td>
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<tr>
<td>$Q_{\tilde{W}}$</td>
<td>$(\varphi^\dagger \varphi)(\bar{q}_p u_r \tilde{\varphi})$</td>
<td>$Q_{d \varphi}$</td>
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<tr>
<td>$Q_{\bar{W} B}$</td>
<td>$Q_{dB}$</td>
<td>$Q_{\varphi u d}$</td>
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</tbody>
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* $\varphi^\dagger \varphi$ is the scalar product of two vectors $\varphi$. * $\bar{l}_p e_r \varphi$ and $\bar{q}_p u_r \tilde{\varphi}$ are the products of quark and lepton fields with their respective Pauli matrices. * $\bar{q}_p d_r \varphi$ and $\bar{q}_p u_r d_r \varphi$ are higher-order interactions involving multiple quark fields.
SMEFT operators in Warsaw basis

Grzadkowski, Iskrzynski, Misiak, Rosiek ’10

<table>
<thead>
<tr>
<th>( (\bar{L}L)(\bar{L}L) )</th>
<th>( (\bar{R}R)(\bar{R}R) )</th>
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<tbody>
<tr>
<td>( Q_{lt} )</td>
<td>( (\bar{l}<em>p \gamma</em>\mu l_r)(\bar{l}_s \gamma^\mu l_t) )</td>
<td>( Q_{le} )</td>
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<tr>
<td>( Q_{qq}^{(1)} )</td>
<td>( (\bar{q}<em>p \gamma</em>\mu q_r)(\bar{q}_s \gamma^\mu q_t) )</td>
<td>( Q_{lu} )</td>
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<tr>
<td>( Q_{qq}^{(3)} )</td>
<td>( (\bar{q}<em>p \gamma</em>\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t) )</td>
<td>( Q_{ld} )</td>
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<td>( Q_{lq}^{(1)} )</td>
<td>( (\bar{l}<em>p \gamma</em>\mu l_r)(\bar{q}_s \gamma^\mu q_t) )</td>
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<td>( Q_{qu}^{(1)} )</td>
</tr>
<tr>
<td>( Q_{ud}^{(1)} )</td>
<td>( \bar{u}<em>p \gamma</em>\mu u_r)(\bar{d}_s \gamma^\mu d_t) )</td>
<td>( Q_{qu}^{(8)} )</td>
</tr>
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<td>( Q_{jadq}^{(8)} )</td>
<td>( \bar{u}<em>p \gamma</em>\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t) )</td>
<td>( Q_{qd}^{(1)} )</td>
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<th>( \mathcal{B})-violating</th>
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<td>( \bar{q}<em>p^j u_r \varepsilon</em>{jk}(\bar{q}_s^k d_t) )</td>
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