Standard Model Higgs basics (1st class)

Stefania Gori
UC Santa Cruz

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Overview

Chapter 1: Introduction and electroweak symmetry breaking

Chapter 2: Interactions of the Higgs boson with
- gauge bosons
- fermions
- another Higgs boson

TODAY

Chapter 3:

Chapter 4:

Phenomenology of the Higgs at colliders (LHC)

TOMORROW

Goal: Introduce the basics of the Standard Model Higgs boson (theory + pheno)

Please interrupt to ask questions! We do not have to go through all slides! :)

You can also contact me per email: sgori@ucsc.edu

An (incomplete) collection of references

Books:

"The Higgs hunter's guide", J.F.Gunion, H.E.Haber, G.L.Kane, S.Dawson, *Front.Phys.* 80 (2000) 1-404.

Review articles:

"The Anatomy of electro-weak symmetry breaking. I: The Higgs boson in the standard model", A. Djouadi, Phys.Rept. 457 (2008)

"Handbook of LHC Higgs Cross Sections: 4. Deciphering the Nature of the Higgs Sector", LHC Higgs Cross Section Working Group, de Florian et. al., 1610.07922

"The Higgs program and open questions in particle physics and cosmology", B.Heinemann, Y.Nir, Phys.Usp. 62 (2019) 9, 920-930

(**"CPsuperH: a Computational Tool for Higgs Phenomenology in the Minimal Supersymmetric Standard Model with Explicit CP Violation"**, J.S.Lee, A.Pilaftsis, M.Carena, S.Y.Choi, M.Drees, J.Ellis, C.E.M.Wagner, Comput.Phys.Commun.156:283-317,2004)

Lectures:

Tasi lectures on the Higgs boson:
S. Dawson, hep-ph/9411325, 1712.07232,
H. Logan, 1406.1786,
L. Reina, hep-ph/0512377,
…

First: what is the Standard Model (SM)?

The SM is

- a remarkably successful description of nature
- a Quantum Field Theory
- based on symmetry principles
- ~minimal
- a model with an enormous predictive power

But we do not understand why it works so well. . .

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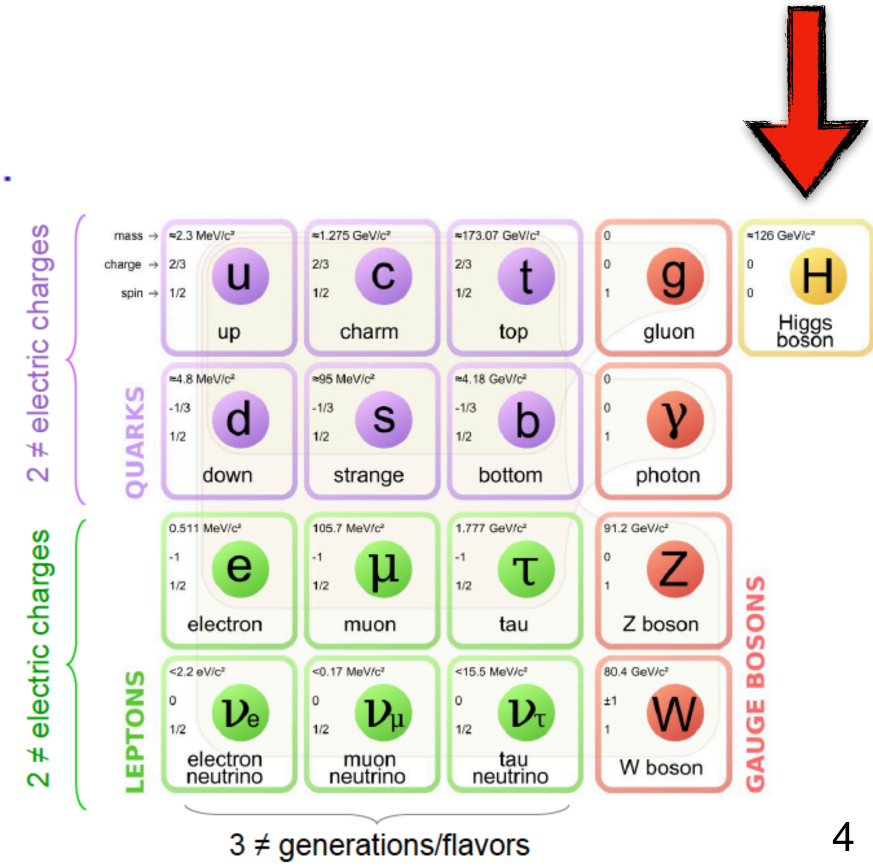
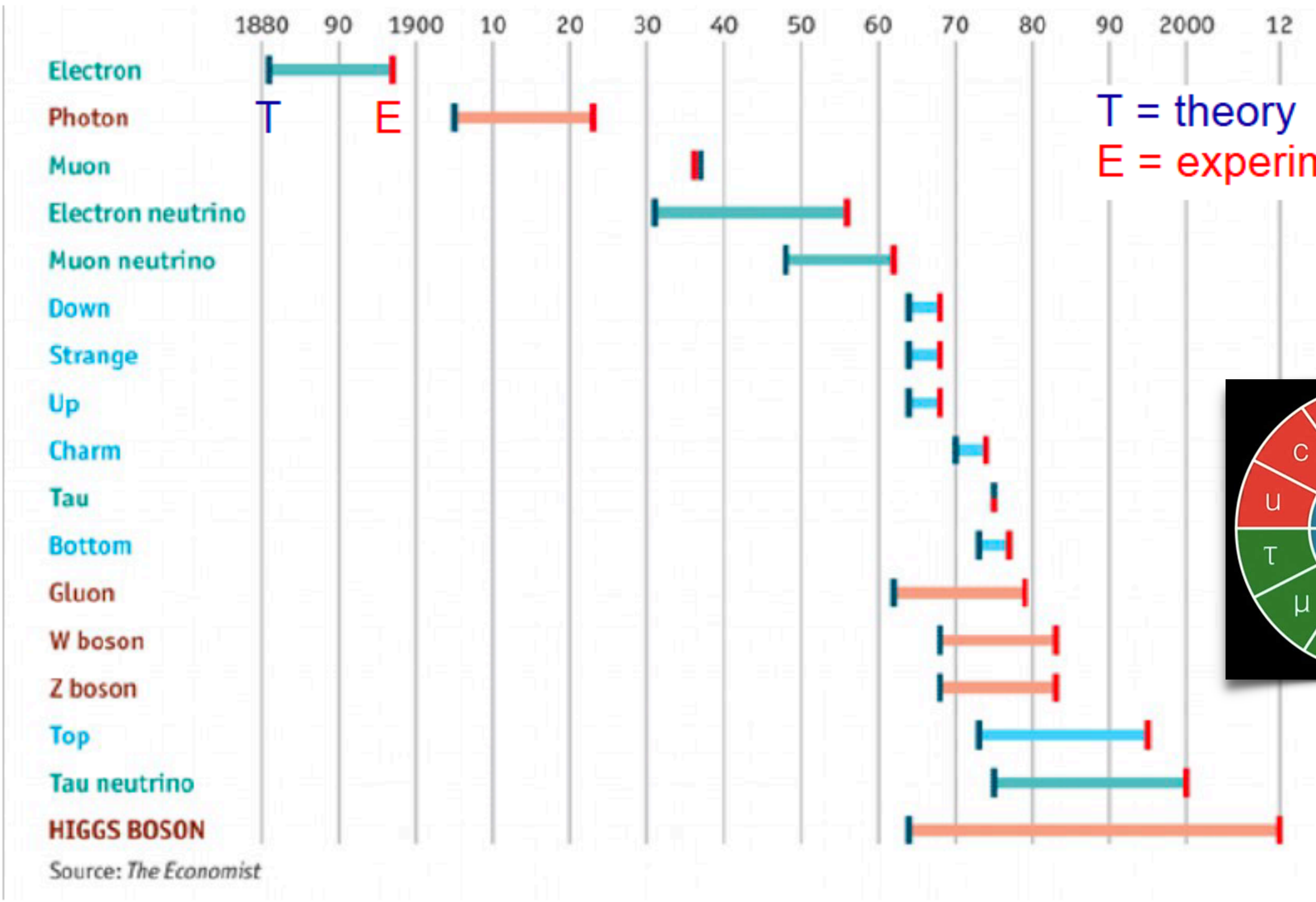
Particle content:

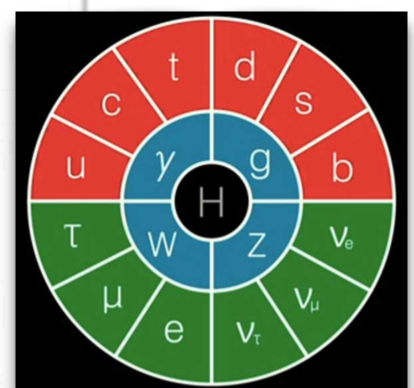


Discoveries!

year



T = theory
E = experiment



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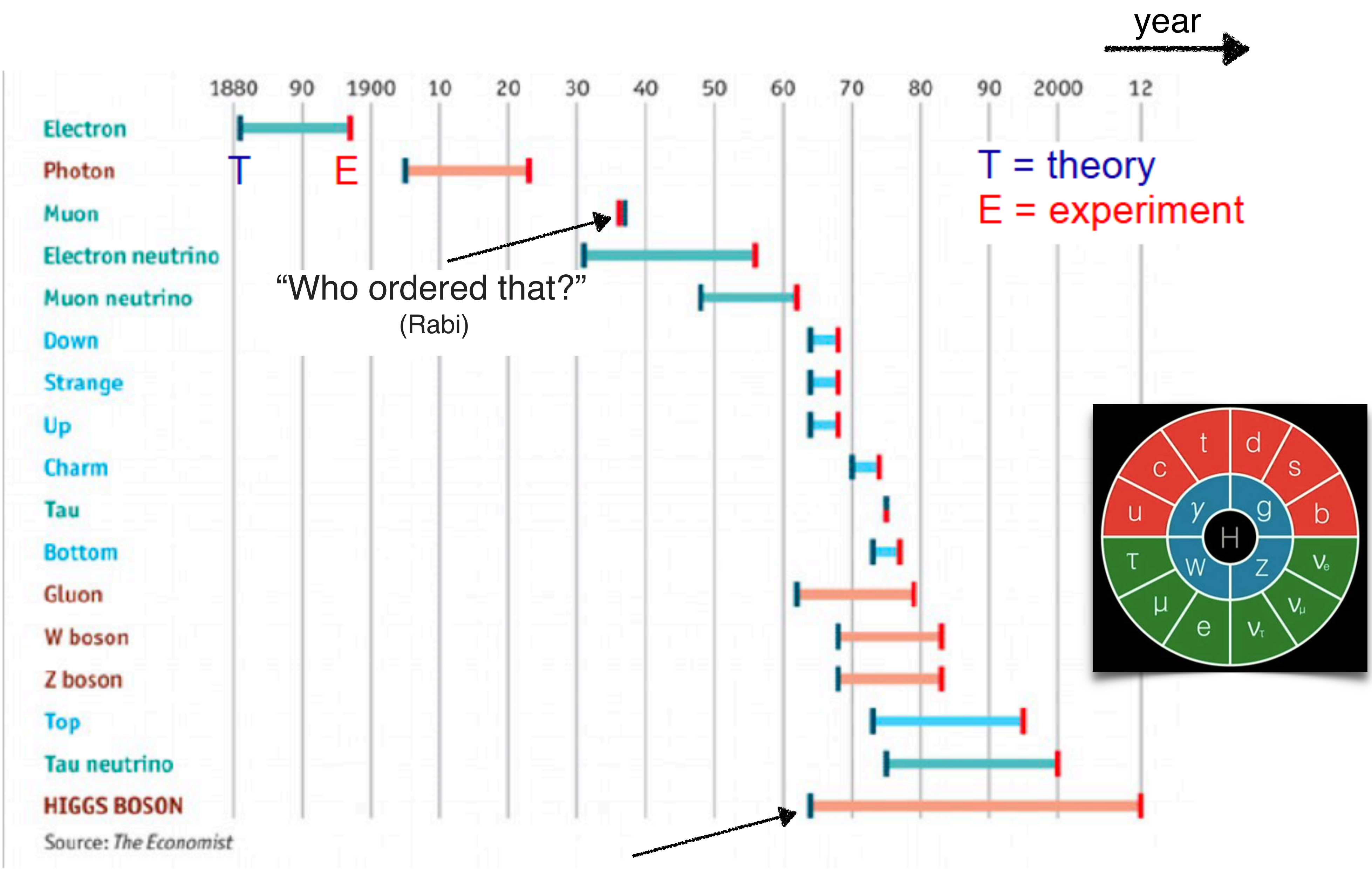
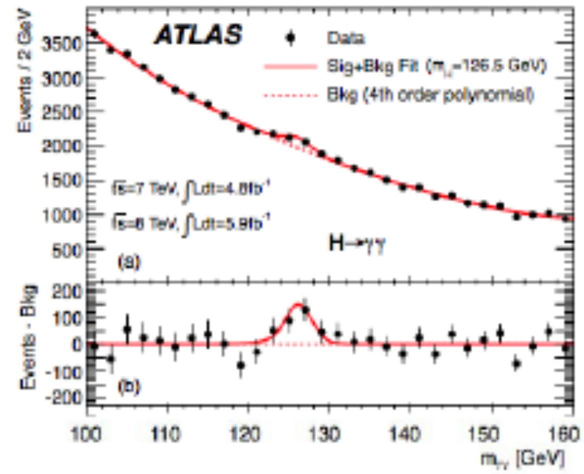
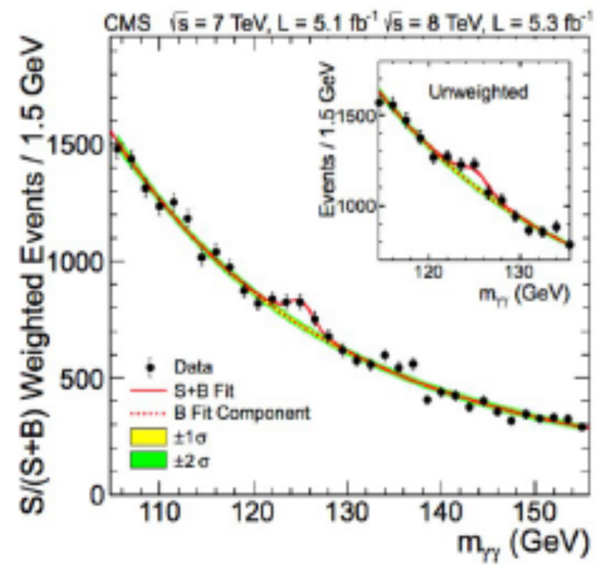


The Higgs discovery

The first elementary particle discovery of 21st century



CERN, July 4th 2012, ~11am



After ~30 years of experimental searches
(LEP, SLC, Tevatron, LHC)

Fundamental principles of the SM

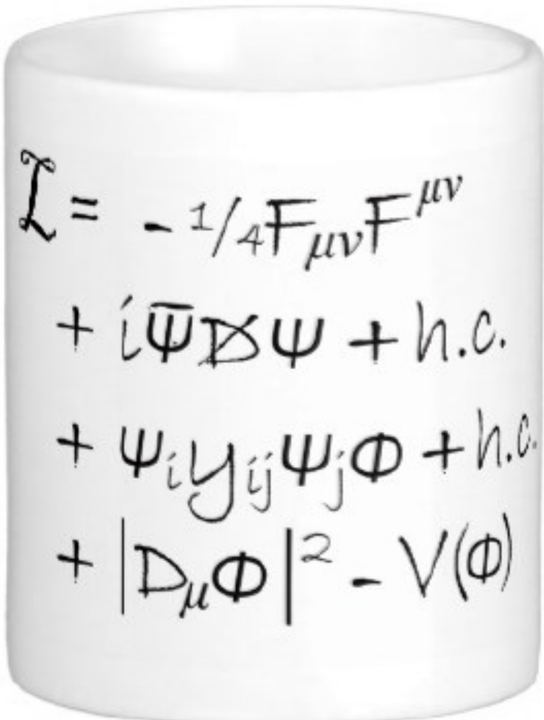
We write down a Lagrangian based on

- minimality: only observed and/or unavoidable objects
- unitarity
- renormalizability: finite predictions for the physical observables
- symmetries

Symmetries:

- Lorentz symmetry
- Gauge symmetries: $SU(3) \times SU(2) \times U(1)_Y$
- we do not impose global symmetries. They are "accidental": e.g.
  - ✔ $SU(3)^5$ flavor symmetry broken by the Higgs interactions with fermions
  - ✔ Lepton and baryon number
  - ✔ ...

Free parameters of the SM Lagrangian



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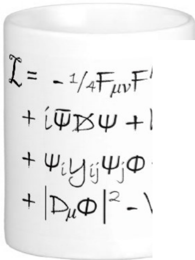
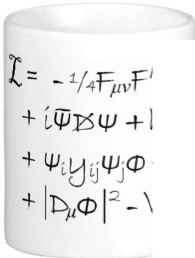


$$-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$i\bar{\Psi}\not{D}\psi + h.c.$$

- Describes the gauge interactions of quarks and leptons
- Parametrized by 3 gauge couplings g_1, g_2, g_3
- Stable with respect to quantum corrections
- Highly symmetric

Gauge sector

Free parameters of the SM Lagrangian



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$$+\left|D_\mu\Phi\right|^2 - V(\Phi)$$

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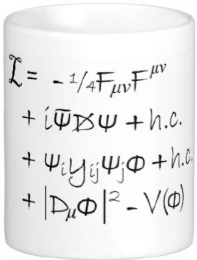
Gauge sector

- Breaks electro-weak symmetry and gives mass to the *W* and *Z* bosons
- 2 free parameters: Higgs mass Higgs vev
- Not stable with respect to quantum corrections

Higgs sector

Free parameters of the SM Lagrangian



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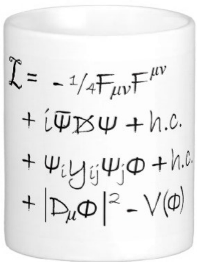
Higgs sector

$$+\Psi_i y_{ij}\Psi_j\Phi + h.c.$$

- Leads to masses and mixings of the quarks and leptons
- 10+10 free parameters in the quark+lepton sector (12 in the lepton sector in case of Majorana masses)
- Stable with respect to quantum corrections

Flavor sector

Free parameters of the SM Lagrangian



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Higgs sector (1)

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Flavor sector (3)

The Higgs couples to ~everything!

Breaking a U(1) gauge symmetry

Let's take a U(1) gauge symmetry with A the associated gauge boson.

Let's add a complex scalar, ϕ, with charge -e

$$\boxed{\phi = a(x) + ib(x)}$$

How to write the Lagrangian for the scalar?

(1) Higgs sector

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$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(\phi)$$

Massless gauge boson, A

$$\begin{aligned} D_\mu &= \partial_\mu - ieA_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \\ V(\phi) &= \mu^2|\phi|^2 + \lambda\left(|\phi|^2\right)^2 \end{aligned}$$

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Higgs sector

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The Lagrangian is is invariant under the gauge U(1) transformations:

$$\begin{aligned} A_\mu(x) &\rightarrow A_\mu(x) - \partial_\mu\eta(x), \\ \phi(x) &\rightarrow e^{-ie\eta(x)}\phi(x) \end{aligned}$$

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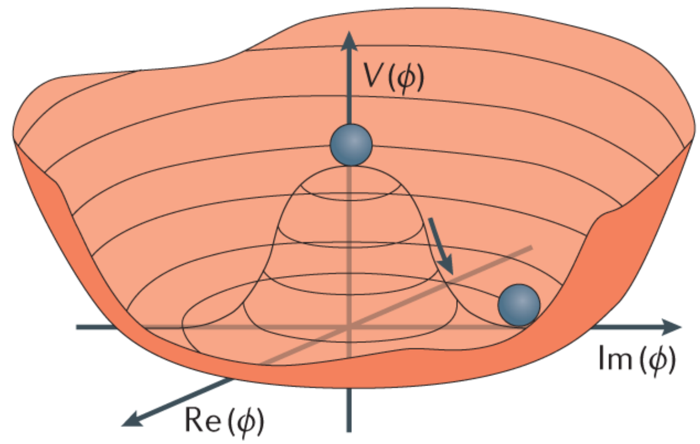
$$V(\phi) = \mu^2|\phi|^2 + \lambda\,(|\phi|^2)^2$$

If $\mu^2 < 0$:



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$$A_\mu(x) \to A_\mu(x) - \partial_\mu\eta(x),$$
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$$\langle\phi\rangle = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$$

Vacuum breaks the U(1) symmetry!

v=vacuum expectation value (VEV)

The mass of the U(1) gauge boson

We can rewrite the complex scalar field as:

$$\phi = a(x) + ib(x) = \frac{1}{\sqrt{2}} e^{i\frac{\chi}{v}}(v + h)$$

- χ and h are the 2 degrees of freedom of the complex Higgs field
- h has minimum at 0

(1)
Higgs sector

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Using these two real scalar fields, the Lagrangian becomes:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e^2v^2}{2}A^\mu A_\mu - evA_\mu\partial^\mu\chi + \frac{1}{2}(\partial_\mu h\partial^\mu h + 2\mu^2h^2)$$
$$+ \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + (h, \chi \text{ interactions})$$

A gets a mass = ev

The **Higgs**, h,
has a mass^2 = $-2\mu^2$

Massless scalar field, χ
(**Goldstone Boson**)

(1) Higgs sector

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Massless scalar field, χ (**Goldstone Boson**)

χ field disappears from the Lagrangian if we "fix the gauge":

$$A'_\mu \equiv A_\mu - \frac{1}{ev}\partial_\mu\chi \qquad \text{(unitary gauge)}$$

"χ is eaten by the gauge field, A"

(1)
Higgs sector

The mass of the U(1) gauge boson

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Using th

Summary

Spontaneous breaking of a gauge symmetry by a non-zero VEV of a scalar field results in the disappearance of a Goldstone boson and its transformation into the longitudinal component of a massive gauge boson

Consequence: Physical Higgs particle

= ev

χ field di
if we “fix

2μ²

Massless scalar field, χ (**Goldstone Boson**)

$$A'_\mu \equiv A_\mu - \frac{1}{ev} \partial_\mu \chi$$ (unitary gauge)

“χ is eaten by the gauge field, A”

(1)
Higgs sector

EWSB in the SM

Gauge symmetry & particle content:

SU(3) x SU(2) x U(1)$_Y$

- Gauge bosons:
  - SU(3): G^i_μ, i=1…8
  - SU(2): W^i_μ, i=1,2,3
  - U(1): B_μ
- Gauge couplings: g_s, g, g′
- Complex SU(2) Higgs doublet (with hypercharge 1/2): Φ

$$\Phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Electroweak symmetry breaking (EWSB)
SU(2) x U(1)$_Y$ → U(1)$_{em}$

"Extended version" of the breaking of the abelian U(1) symmetry seen before

(1)
Higgs sector

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"Extended version" of the breaking of the abelian U(1) symmetry seen before

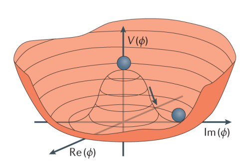
The scalar potential allowed by the gauge symmetry:

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

Note: SU(3) is not broken! (the Higgs field is not charged under SU(3))

If $\mu^2 < 0$, then spontaneous symmetry breaking

Possible minimum: $\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$



Choice of minimum breaks gauge symmetry

(1)
Higgs
sector

Higgs potential & Higgs mass, self interactions

We can rewrite the complex SU(2) doublet scalar field as:

$$\Phi = e^{i\sigma_i \frac{\omega_i}{v}} \begin{pmatrix} 0 \\ \frac{h+v}{\sqrt{2}} \end{pmatrix}$$

h, ω_i
real scalars

generators of SU(2)
(Pauli matrices)

(1)

Higgs sector

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h, ω_i real scalars

generators of SU(2)
(Pauli matrices)

The scalar potential becomes:

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

Higgs mass

$$V(h, \omega_i) = \frac{m_h^2}{2} h^2 + \frac{m_h^2}{2v} h^3 + \frac{m_h^2}{8v^2} h^4$$

Disappearance of ω_i fields (Goldstones)

Higgs self-interactions

In terms of the initial parameters:

$$v^2 = \frac{-4\mu^2}{\lambda}$$
$$m_h^2 = -2\mu^2$$

The Higgs mass in the Standard Model

A PHENOMENOLOGICAL PROFILE OF THE HIGGS BOSON

John Ellis, Mary K. Gaillard *) and D.V. Nanopoulos +)

CERN -- Geneva

Nucl. Phys. B 106, 292 (1976)

We should perhaps finish with an apology and a caution. We apologize to experimentalists for having no idea what is the mass of the Higgs boson, unlike the case with charm [3),4)] and for not being sure of its couplings to other particles, except that they are probably all very small. For these reasons we do not want to encourage big experimental searches for the Higgs boson, but we do feel that people performing experiments vulnerable to the Higgs boson should know how it may turn up.

We need to measure the Higgs mass!

LHC: m_h=125 GeV

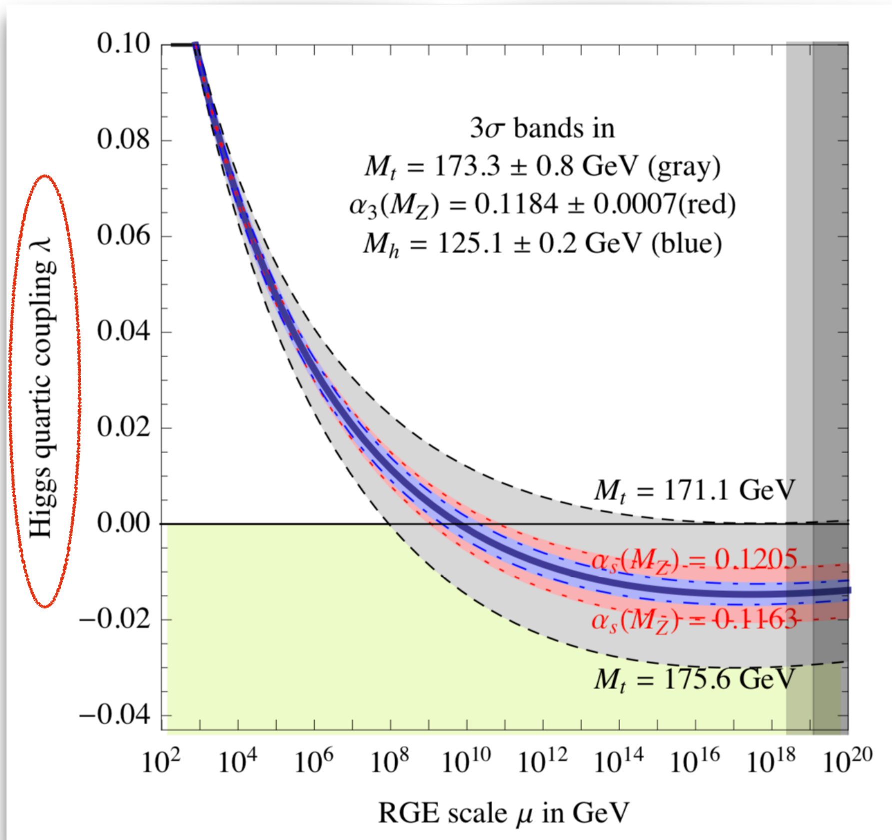
(1)
Higgs sector

Vacuum stability

Once we fix the two free parameters of the Higgs potential, we can compute the running of the quartic coupling, λ, as a function of the energy scale

$$V(\Phi) = \mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

Buttazzo et al, 1307.3536



Assuming no New Physics

(1) Higgs sector

Vacuum stability

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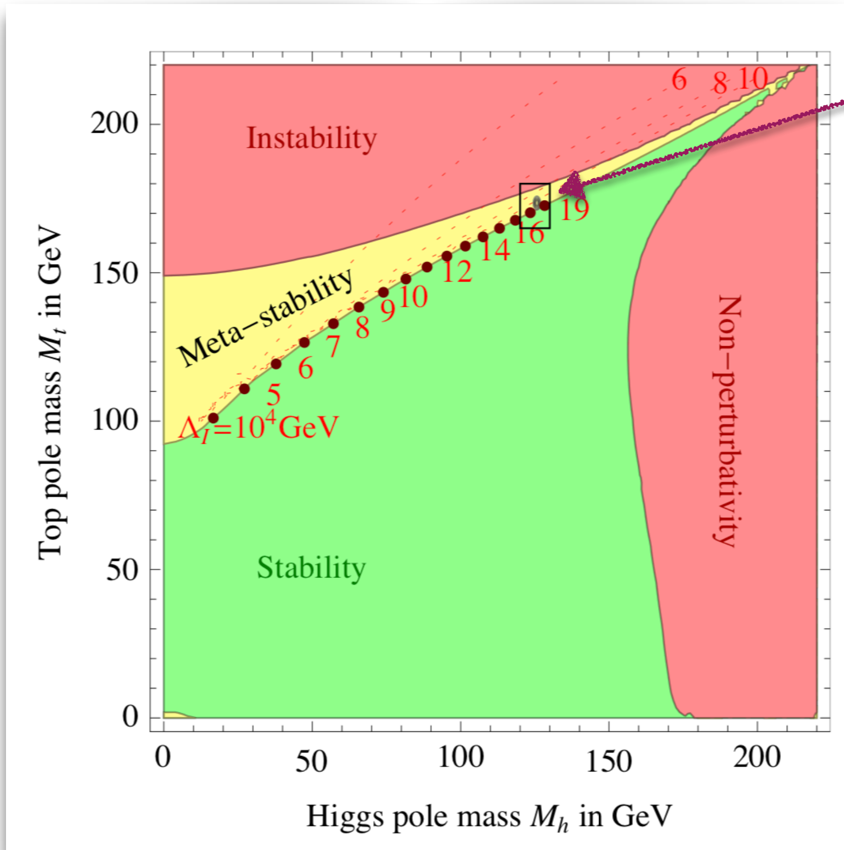
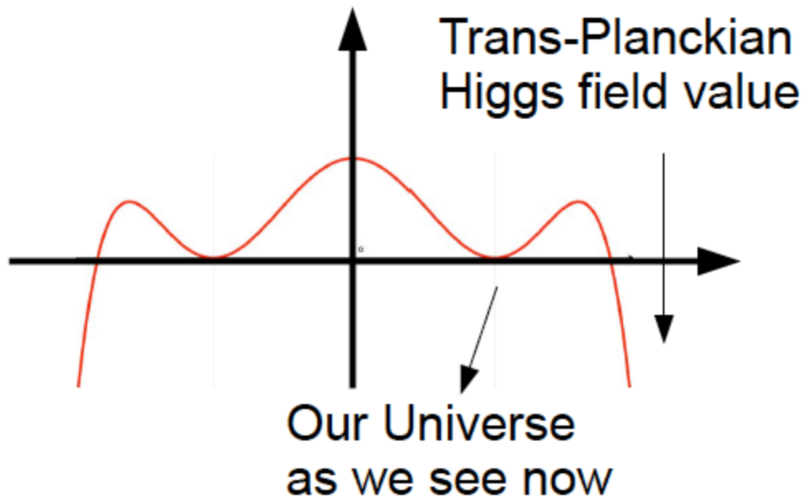


We are here. Is this a coincidence?



Stability condition:

$$M_h > (129.6 \pm 1.5)\ \mathrm{GeV}$$

Assuming no New Physics

(2) Gauge sector

The (Higgs) gauge sector of the SM

SU(2) U(1)$_Y$

$$\mathcal{L}^{\text{gauge}} = (D_\mu\Phi)^\dagger D^\mu\Phi, \quad D_\mu = \partial_\mu - i\frac{g}{2}\sigma^i W_\mu^i - i\frac{g'}{2}B_\mu$$

We can write this explicitly at the minimum of the Higgs potential:

$$(D_\mu\Phi)^\dagger D^\mu\Phi \to \frac{v^2}{8}\left(g^2(W_\mu^1)^2 + g^2(W_\mu^2)^2 + (-gW_\mu^3 + g'B_\mu)^2\right) + \cdots$$

Charged gauge boson masses

Neutral gauge boson masses

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

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We can diagonalize the system and find:

Eigenstates:

$$W_\mu^\pm \equiv \frac{W_\mu^1 \mp W_\mu^2}{\sqrt{2}}$$

$$Z_\mu \equiv \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2+(g')^2}}$$

$$A_\mu \equiv \frac{g'W_\mu^3 + gB_\mu}{\sqrt{g^2+(g')^2}}$$

Eigenvalues:

$$m_w = \frac{gv}{2}$$

$$m_z = \sqrt{g^2+(g')^2}\frac{v}{2}$$

$$m_A = 0$$

Masses vanish when v=0

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Let's define the Weinberg angle

$$\sin\theta = \frac{g'}{\sqrt{g^2 + (g')^2}}$$

$$Z_\mu = -\sin\theta B_\mu + \cos\theta W_\mu^3$$
$$A_\mu = \cos\theta B_\mu + \sin\theta W_\mu^3$$

$$m_w = m_z \cos\theta$$
$$\rho \equiv \frac{m_w}{m_z \cos\theta} = 1 \quad \text{(tree level)}$$

Custodial symmetry

Recap on the SM EWSB

The Higgs mechanism generates the mass of W, Z

- Higgs VEV breaks SU(2) x $U(1)_Y$
- $U(1)_{em}$ is left unbroken (the photon is massless)
- Single Higgs SU(2) doublet is the minimal model to achieve this breaking pattern

Recap on the SM EWSB

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Before spontaneous symmetry breaking:

Massless:
- W_i (i=1,2,3): 6 degrees of freedom
- B: 2 degree of freedom
- Complex Φ doublet: 4 degrees of freedom

6+2+4 = 2+6+3+1

After spontaneous symmetry breaking:

Massless:
- A (photon): 2 degree of freedom

Massive:
- $W^{\pm}$: 6 degrees of freedom
- Z: 3 degrees of freedom
- Real scalar (Higgs): 1 degree of freedom

(2)
Gauge sector

Higgs-gauge boson couplings

SU(2) U(1)$_Y$

$$\mathcal{L}^{\text{gauge}} = (D_\mu \Phi)^\dagger D^\mu \Phi, \quad D_\mu = \partial_\mu - i\frac{g}{2}\sigma^i W^i_\mu - i\frac{g'}{2}B_\mu$$

Now we can insert the physical Higgs boson: $\Phi = \begin{pmatrix} 0 \\ \frac{h+v}{\sqrt{2}} \end{pmatrix}$

(2) Gauge sector

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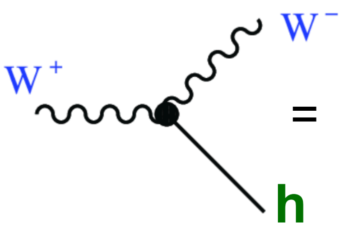
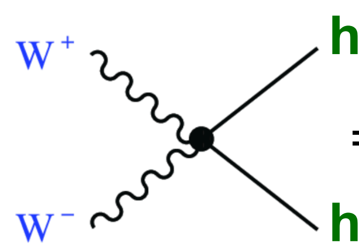
Couplings are proportional to the **mass**

(1) $\mathcal{L} \supset \frac{g^2}{4}(v+h)^2 W^+_\mu W^{\mu-} \to \frac{g^2 v}{2} h W^+_\mu W^{\mu-} + \frac{g^2}{4} hh W^+_\mu W^{\mu-}$

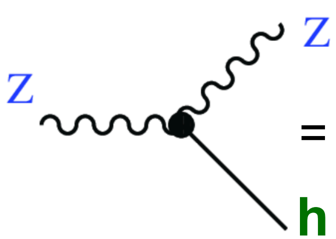


$= \quad i\frac{g^2 v}{2} = 2i\frac{m_w^2}{v}$

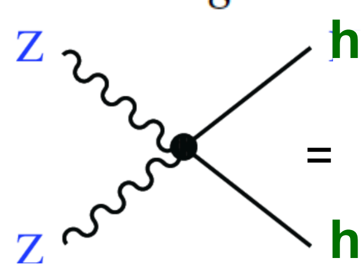


$= \quad i\frac{g^2}{4}2! = 2i\frac{m_w^2}{v^2}$

(2) $\mathcal{L} \supset \frac{g^2+g'^2}{8}(v+h)^2 Z_\mu Z^\mu \to \frac{(g^2+g'^2)v}{4} h Z_\mu Z^\mu + \frac{g^2+g'^2}{8} hh Z_\mu Z^\mu$



$= \quad i\frac{(g^2+g'^2)v}{4}2! = 2i\frac{m_z^2}{v}$



$= \quad i\frac{g^2+g'^2}{8}2!2! = 2i\frac{m_z^2}{v^2}$

(2)
Gauge sector

A predictive model

Four free parameters in gauge-Higgs sector (g, g', μ, λ)

Conventionally chosen to be

- α
- G_F (Fermi constant)
- M_Z
- M_h

All observables can be expressed
in terms of these parameters

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Examples:

1. W mass

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_w^2} = \frac{\pi\alpha}{2\left(1-\frac{m_w^2}{m_z^2}\right)m_w^2}$$

2. Z coupling to leptons

$$\frac{g}{\cos\theta}\left(-\frac{1}{2}+\sin^2\theta\right)Z_\mu\bar{\ell}_L\gamma^\mu\ell_L = \sqrt{\frac{4\pi\alpha}{1-\frac{m_w^2}{m_z^2}}}\frac{m_z}{m_w}\left(\frac{1}{2}-\left(\frac{m_w}{m_z}\right)^2\right)Z_\mu\bar{\ell}_L\gamma^\mu\ell_L$$

Many observables have been measured!

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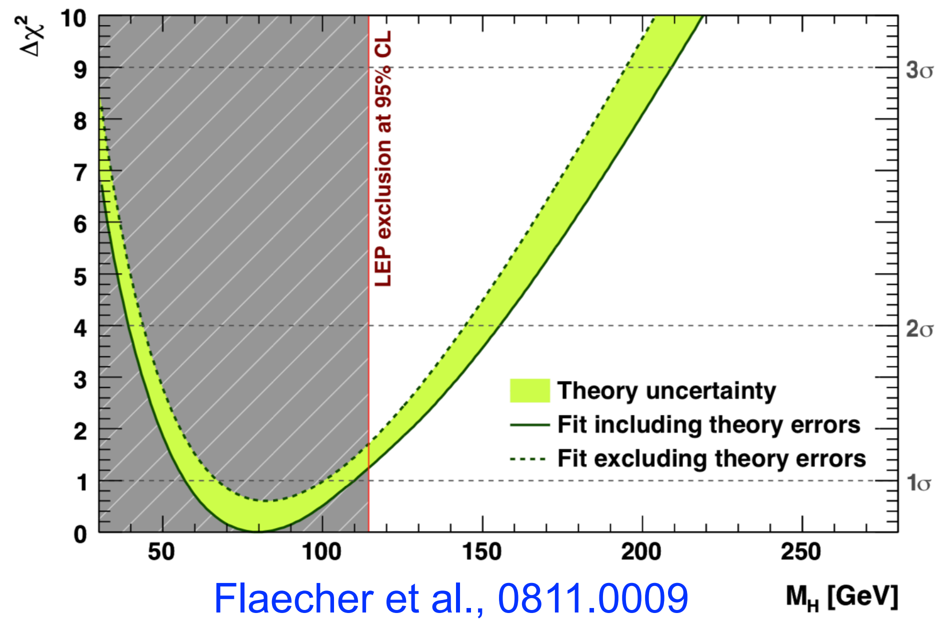
In fact, we had hints for the value of the Higgs mass **before** the Higgs discovery!



Flaecher et al., 0811.0009

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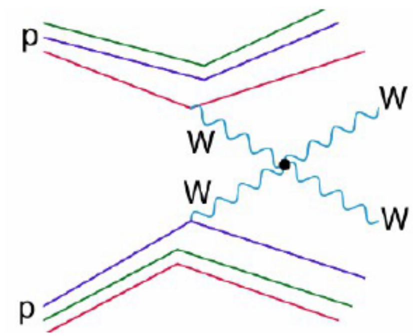
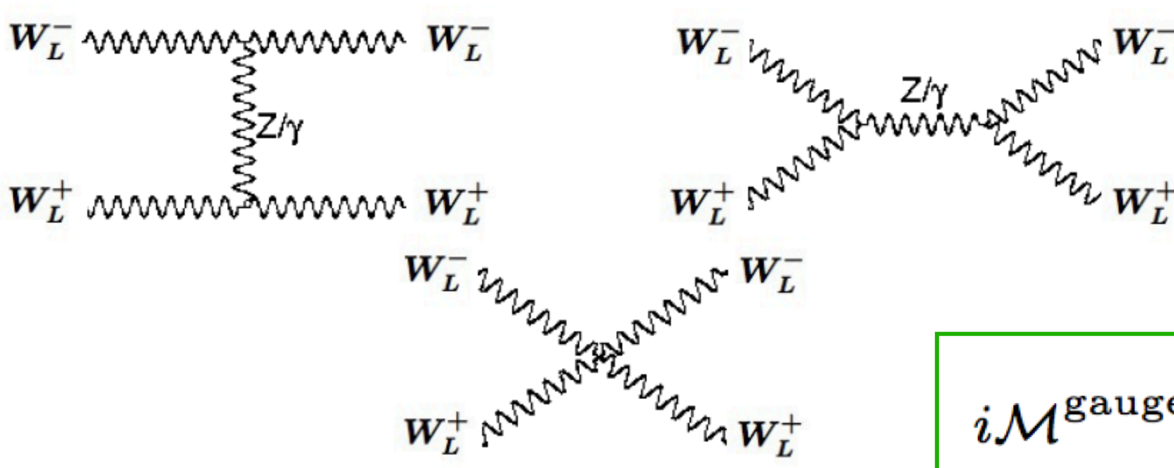
(2) Gauge sector

The need for a Higgs: unitarity
(beyond mass generation)

In our theory of weak interactions, if we do not have a Higgs, the scattering of two longitudinal W bosons:





$$\boxed{i\mathcal{M}^{\text{gauge}} = -i\frac{g^2}{4m_W^2}u + \mathcal{O}((E/m_W)^0)}$$

WW scattering will essentially grow with energy until violating unitarity at the ~TeV scale

We expect some new dynamics should show up at TeV scale

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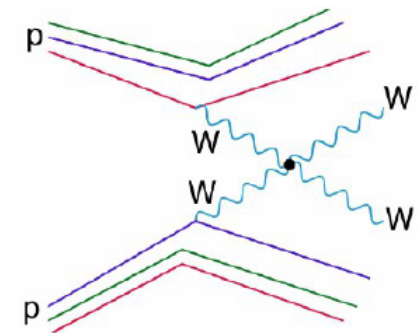

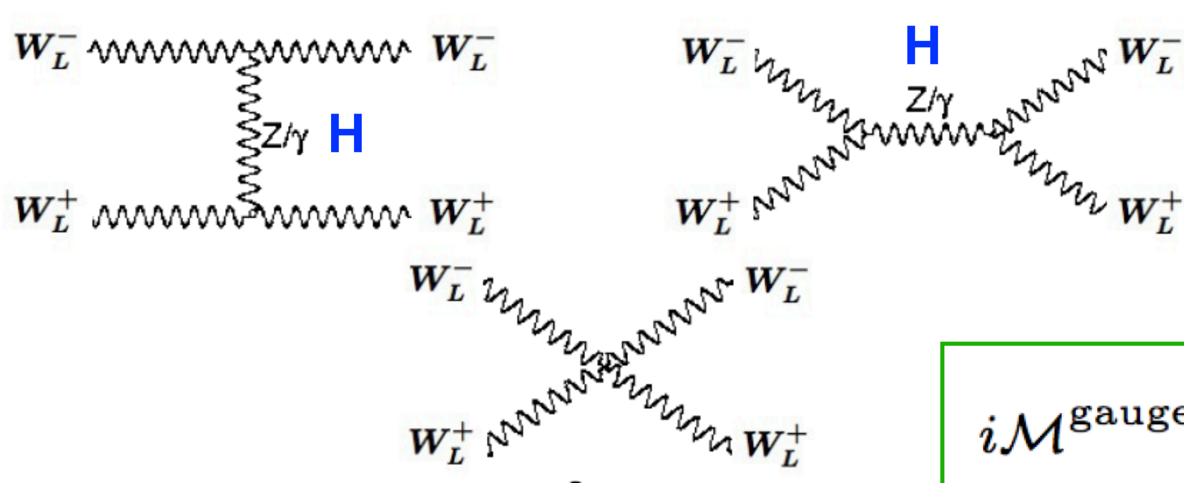

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The leading term cancels introducing the SM Higgs

(3)

Fermion sector

Generation of fermion masses

Fermion representations under SU(3) x SU(2) x U(1)$_Y$:

$$Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix} = (3,2,1/6),\ u_R^i = (3,1,2/3),\ d_R^i = (3,1,-1/3)$$

$$L_L^i = \begin{pmatrix} \nu_L^i \\ \ell_L^i \end{pmatrix} = (1,2,-1/2),\ e_R^i = (1,1,-1)$$

(i=1,2,3 = flavor index)

What Yukawa interaction can I write down that is invariant under the SM gauge symmetry?

Reminder: Φ=(1,2,1/2)

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$$\bar{Q}_L^i Y_D^{ij} d_R^j \Phi + \bar{Q}_L^i Y_D^{ij} u_R^j \tilde{\Phi} + \bar{L}_L^i Y_E^{ij} e_R^j \Phi + \text{h.c.}$$

$$\tilde{\Phi} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Phi^*$$

After EWSB
(at the minimum):

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\bar{d}_L^i \frac{Y_D^{ij} v}{\sqrt{2}} d_R^j$$

down-quark mass term, M_d

(similar for the other quarks and leptons)

Without the Higgs field **no fermion mass term would be allowed** in the Lagrangian

CKM matrix and Higgs-quark interactions

$$\bar{d}_L^i M_D^{ij} d_R^j + \bar{u}_L^i M_U^{ij} u_R^j$$

In the quark sector, four rotation matrices are needed to diagonalize the system:

eigenvalues: $M_f^{\text{diag}} = U_{fL} M_f U_{fR}^\dagger \quad (f = u, d)$

eigenstates: $f_{Li}^{\text{mass}} = U_{fL}^{ij} f_{Lj}, \quad f_{Ri}^{\text{mass}} = U_{fR}^{ij} f_{Rj}$

Mass matrices diagonalized by different transformations for u_L and d_L, which are part of the same SU(2) doublet, Q_L

$$\begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix} = (U_{uL}^\dagger)_{ij} \begin{pmatrix} u_{Lj}^{\text{mass}} \\ (U_{uL} U_{dL}^\dagger)_{jk} d_{Lk}^{\text{mass}} \end{pmatrix}$$

CKM (unitary) matrix

(3) Fermion sector

CKM matrix and Higgs-quark interactions

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CKM (unitary) matrix

The physical Higgs boson couples to fermion mass eigenstates:

$$\bar{Q}^i_L Y_D^{ij} d^j_R \Phi + \bar{Q}^i_L Y_D^{ij} u^j_R \tilde{\Phi} + \bar{L}^i_L Y_E^{ij} e^j_R \Phi + \text{h.c.} \qquad \Phi = \begin{pmatrix} 0 \\ \frac{h+v}{\sqrt{2}} \end{pmatrix}$$

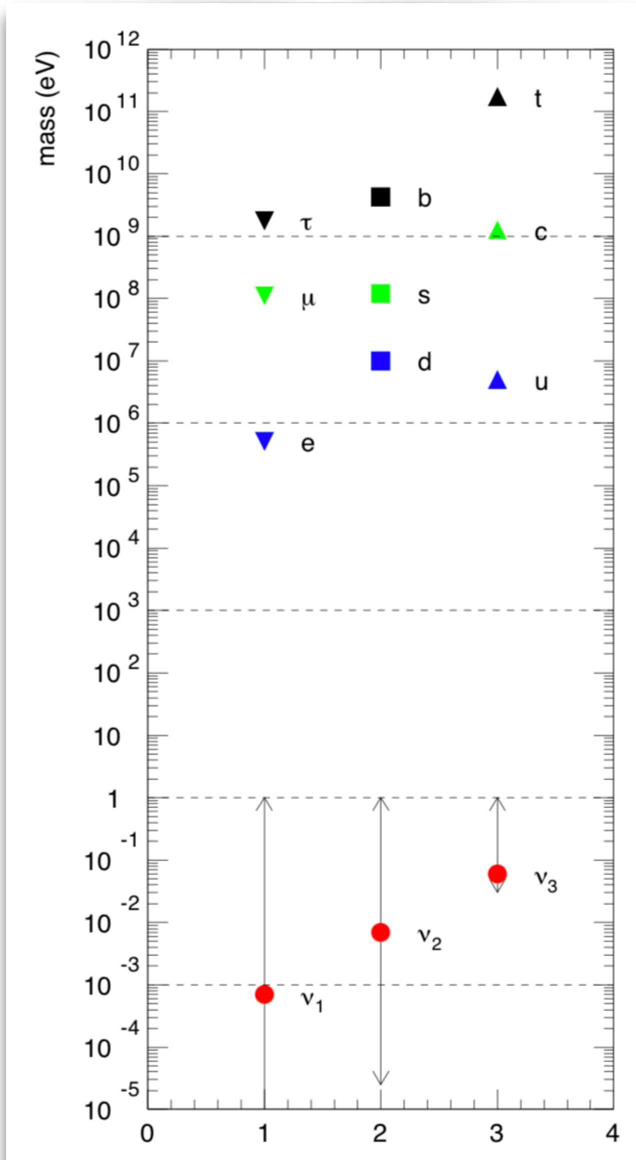
$$\bar{d}^i_L \frac{(M_D^{\rm diag})_{ii}}{v} d^i_R h + \bar{u}^i_L \frac{(M_U^{\rm diag})_{ii}}{v} u^i_R h + \bar{\ell}^i_L \frac{(M_L^{\rm diag})_{ii}}{v} \ell^i_R h$$

Couplings are proportional to the **mass**

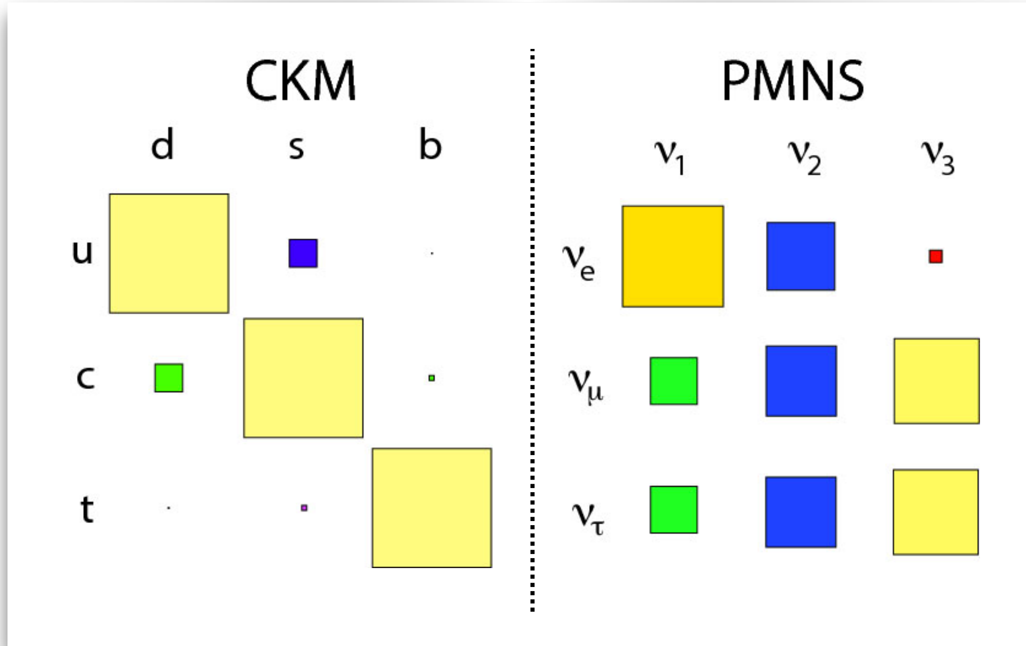
flavor diagonal interactions!

(3) Fermion sector

The SM flavor puzzle



De Gouvea, 0902.4656



Why such large hierarchies?

Does the Higgs couple so hierarchically to quarks and leptons?

(3)
Fermion sector

Flavor changing neutral currents

$$\mathcal{L}_f^{\rm gauge} \supset \bar{f}_i \gamma^\mu D_\mu f_i, \qquad D_\mu = \partial_\mu - i\frac{g}{2}\sigma^i W_\mu^i - i\frac{g'}{2}B_\mu$$

diagonal
in flavor space

mass
eigenstates

$$-\frac{g}{\sqrt{2}}\begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix}\gamma^\mu W_\mu^+ V_{\rm CKM}\begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \text{h.c.}$$

non-diagonal
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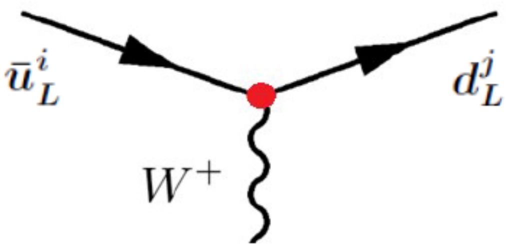

Not to get confused:

this mixing originates only from the Higgs sector:
$V_{\rm CKM} \to \delta$ if we switch-off the Yukawa interactions

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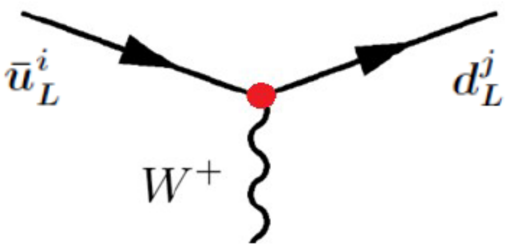

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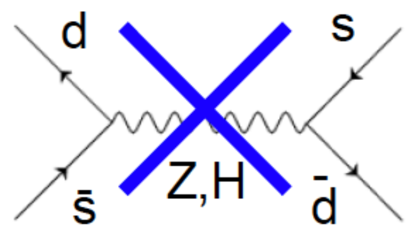
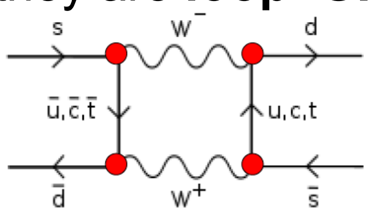
Exercise: prove that neutral γ, Z and g currents stay flavor universal, since they do not mix the chiralities



(Note for the experts:
At one loop, neutral flavor transitions are generated. However they are **loop+GIM** suppressed.)



No flavor changing neutral currents (FCNCs) at tree level in the SM

Recap on Higgs couplings

- Higgs couples to fermion mass:

$$\mathcal{L} \supset -\frac{m_f}{v}\bar{f}fh = -\frac{m_f}{v}(\bar{f}_L f_R + \bar{f}_R f_L)h$$

Largest coupling is to heaviest fermion
Is the top quark special?

We do not know if
the Higgs couples to neutrinos:

$$\bar{L}_L^i Y_N^{ij} N_R^j \tilde{\Phi}$$

do they exist?

- Higgs couples to gauge boson mass:

$$\mathcal{L} \supset 2\frac{m_w^2}{v}W^{\mu+}W_\mu^- h + \frac{m_z^2}{v}Z^\mu Z_\mu h + \cdots$$

2 Higgs - 2 gauge boson interactions

Only free parameter is Higgs mass!

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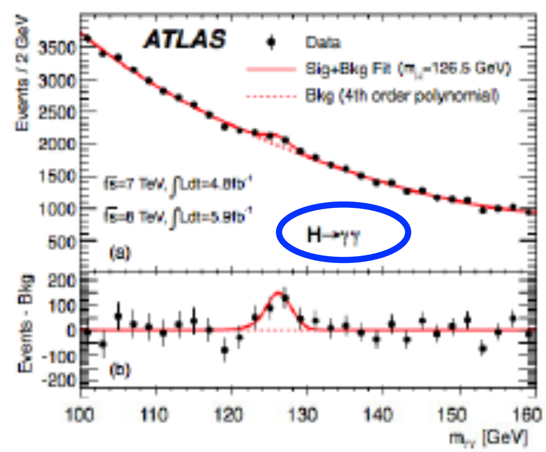

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This is only for tree-level couplings…
<u>tomorrow</u> we will discuss
loop-induced couplings &
Higgs phenomenology