Standard Model
Higgs basics
(1st class)

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Overview

Chapter 1: Introduction and electroweak symmetry breaking

Chapter 2: Interactions of the Higgs boson with
- gauge bosons
- fermions
- another Higgs boson

Chapter 3: Phenomenology of the Higgs at colliders (LHC)

Chapter 4: 

Goal: Introduce the basics of the Standard Model Higgs boson (theory + pheno)
Please interrupt to ask questions! We do not have to go through all slides! :)
Books:


Review articles:


"Handbook of LHC Higgs Cross Sections: 4. Deciphering the Nature of the Higgs Sector”, LHC Higgs Cross Section Working Group, de Florian et. al., 1610.07922

“The Higgs program and open questions in particle physics and cosmology”, B.Heinemann, Y.Nir, Phys.Usp. 62 (2019) 9, 920-930


Lectures:

First: what is the Standard Model (SM)?

The SM is:
- a remarkably successful description of nature
- a Quantum Field Theory
- based on symmetry principles
- ~minimal
- a model with an enormous predictive power

But we do not understand why it works so well...
First: what is the Standard Model (SM)?

The SM is

- a remarkably successful description of nature
- a Quantum Field Theory
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- ~minimal
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But we do not understand why it works so well...
Discoveries!

T = theory
E = experiment

Source: The Economist
Discoveries!

“Who ordered that?”
(Rabi)

T = theory
E = experiment

Source: The Economist
The Higgs discovery

The first elementary particle discovery of 21st century

CERN, July 4th 2012, ~11am

After ~30 years of experimental searches
(LEP, SLC, Tevatron, LHC)
Fundamental principles of the SM

We write down a Lagrangian based on

- minimality: only observed and/or unavoidable objects
- unitarity
- renormalizability: finite predictions for the physical observables
- symmetries

Symmetries:

- Lorentz symmetry
- Gauge symmetries: SU(3) × SU(2) × U(1)\_Y

we do not impose global symmetries. They are "accidental": e.g.

- SU(3)\(^5\) flavor symmetry broken by the Higgs interactions with fermions
- Lepton and baryon number
- ...
Free parameters of the SM Lagrangian

\[ L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \gamma^\mu \psi + h.c. + \psi_i Y_{ij} \psi_j \phi + h.c. + |D_\mu \phi|^2 - V(\phi) \]
Free parameters of the SM Lagrangian

\[ L = \frac{-1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \gamma^\mu \gamma^\nu \psi F_{\mu\nu} + \frac{1}{2} \phi^2 + i \bar{\psi} \gamma^\mu \gamma^\nu \psi F_{\mu\nu} + i \bar{\psi} \gamma^\mu \gamma^\nu \psi F_{\mu\nu} + E \phi |D_\mu \phi|^2 - V(\phi) \]

- Describes the gauge interactions of quarks and leptons
- Parametrized by 3 gauge couplings \( g_1, g_2, g_3 \)
- Stable with respect to quantum corrections
- Highly symmetric

**Gauge sector**
Free parameters of the SM Lagrangian

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- Breaks electro-weak symmetry and gives mass to the \( W \) and \( Z \) bosons
- 2 free parameters:
  - Higgs mass
  - Higgs vev
- Not stable with respect to quantum corrections

Gauge sector \hspace{2cm} Higgs sector
Free parameters of the SM Lagrangian

\[ L = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + i \bar{\psi} D_{\mu} \psi + h.c. + \frac{|D_{\mu} \phi|^2 - V(\phi)}{\phi} + \bar{\psi}_i y_{ij} \psi_j \phi + h.c. \]

- Describes the gauge interactions of quarks and leptons
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- 2 free parameters: Higgs mass, Higgs vev
- Not stable with respect to quantum corrections

- Leads to masses and mixings of the quarks and leptons
- 10+10 free parameters in the quark+lepton sector (12 in the lepton sector in case of Majorana masses)
- Stable with respect to quantum corrections

Gauge sector  Higgs sector  Flavor sector
Free parameters of the SM Lagrangian

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- 10+10 free parameters in the quark+lepton sector (12 in the lepton sector in case of Majorana masses)
- Stable with respect to quantum corrections

(2) Gauge sector  Higgs sector (1)  Flavor sector (3)

The Higgs couples to \( \sim \)everything!
Breaking a U(1) gauge symmetry

Let’s take a U(1) gauge symmetry with A the associated gauge boson.
Let’s add a complex scalar, \( \phi \), with charge -e

How to write the Lagrangian for the scalar?

\[ \phi = a(x) + ib(x) \]
Breaking a U(1) gauge symmetry

Let’s take a U(1) gauge symmetry with $A$ the associated gauge boson.
Let’s add a complex scalar, $\phi$, with charge $-e$

How to write the Lagrangian for the scalar?

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - V(\phi)
\]

\[
D_\mu = \partial_\mu - ieA_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu
\]

\[
V(\phi) = \mu^2 |\phi|^2 + \lambda (|\phi|^2)^2
\]
Breaking a U(1) gauge symmetry

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$$V(\phi) = \mu^2 |\phi|^2 + \lambda (|\phi|^2)^2$$

The Lagrangian is invariant under the gauge U(1) transformations:

$$A_\mu(x) \rightarrow A_\mu(x) - \partial_\mu \eta(x),$$

$$\phi(x) \rightarrow e^{-ie\eta(x)} \phi(x)$$
Breaking a U(1) gauge symmetry

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Let’s add a complex scalar, \( \phi \), with charge -e

How to write the Lagrangian for the scalar?

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_{\mu} \phi|^2 - V(\phi)
\]

\[
D_{\mu} = \partial_{\mu} - ieA_{\mu}, \quad F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}
\]

\[
V(\phi) = \mu^2 |\phi|^2 + \lambda (|\phi|^2)^2
\]

The Lagrangian is is invariant under the gauge U(1) transformations:

\[
A_{\mu}(x) \rightarrow A_{\mu}(x) - \partial_{\mu} \eta(x),
\phi(x) \rightarrow e^{-ie\eta(x)} \phi(x)
\]

Vacuum breaks the U(1) symmetry!

\[
\langle \phi \rangle = \sqrt{-\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}
\]

v=vacuum expectation value (VEV)
The mass of the U(1) gauge boson

We can rewrite the complex scalar field as:

$$\phi = a(x) + ib(x) = \frac{1}{\sqrt{2}} e^{i\frac{\chi}{v}} (v + h)$$

• $\chi$ and $h$ are the 2 degrees of freedom of the complex Higgs field
• $h$ has minimum at 0
The mass of the U(1) gauge boson

We can rewrite the complex scalar field as:

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• \(\chi\) and \(h\) are the 2 degrees of freedom of the complex Higgs field
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Using these two real scalar fields, the Lagrangian becomes:

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{e^2 v^2}{2} A^\mu A_\mu - evA_\mu \partial^\mu \chi + \frac{1}{2} \left( \partial_\mu h \partial^\mu h + 2\mu^2 h^2 \right) + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + (h, \chi \text{ interactions}) \]

A gets a mass = \(ev\)

The Higgs, \(h\), has a mass\(^2 = -2\mu^2\)

Massless scalar field, \(\chi\) (Goldstone Boson)
The mass of the U(1) gauge boson

We can rewrite the complex scalar field as:

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$$+ \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + (h, \chi \text{ interactions})$$

$\chi$ field disappears from the Lagrangian if we “fix the gauge”:

$$A'_\mu \equiv A_\mu - \frac{1}{ev} \partial_\mu \chi$$

(unitary gauge)

“A is eaten by the gauge field, A”

The Higgs, $h$, has a mass$^2 = -2\mu^2$

Massless scalar field, $\chi$ (Goldstone Boson)
The mass of the U(1) gauge boson

We can rewrite the complex scalar field as:

\[ \phi = a(x) + ib(x) = \frac{1}{\sqrt{2}} e^{i\frac{\chi}{\nu}} (v + h) \]

- \( \chi \) and \( h \) are the 2 degrees of freedom of the complex Higgs field
- \( h \) has minimum at 0

Using the massless scalar field, \( \chi \), the Lagrangian becomes:

\[ A \text{ gets a mass} = ev \]

The Higgs, \( h \), has a mass:

\[ 2 = -2\mu^2 \]

Massless scalar field, \( \chi \) (Goldstone Boson)

\( \chi \) field disappears from the Lagrangian if we “fix the gauge”:

\[ \chi \text{ is eaten by the gauge field, } A' \]

(\( A' = A_\mu - \frac{1}{ev} \partial_\mu \chi \) (unitary gauge))

Summary

Spontaneous breaking of a gauge symmetry by a non-zero VEV of a scalar field results in the disappearance of a Goldstone boson and its transformation into the longitudinal component of a massive gauge boson

Consequence: Physical Higgs particle
EWSB in the SM

Gauge symmetry & particle content:
\( \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_Y \)

- Gauge bosons:
  - SU(3): \( G^i_{\mu}, i=1\ldots8 \)
  - SU(2): \( W^i_{\mu}, i=1,2,3 \)
  - U(1): \( B_{\mu} \)
- Gauge couplings: \( g_s, g, g' \)
- Complex SU(2) Higgs doublet (with hypercharge 1/2): \( \Phi \)

\[
\Phi = \begin{pmatrix}
\phi_1 + i\phi_2 \\
\phi_3 + i\phi_4
\end{pmatrix} = \begin{pmatrix}
\phi^+ \\
\phi^0
\end{pmatrix}
\]

Electroweak symmetry breaking (EWSB)
\( \text{SU}(2) \times \text{U}(1)_Y \rightarrow \text{U}(1)_{\text{em}} \)

“Extended version” of the breaking of the abelian U(1) symmetry seen before
EWSB in the SM

Gauge symmetry & particle content: $SU(3) \times SU(2) \times U(1)_Y$

- **Gauge bosons:**
  - $SU(3)$: $G^i_\mu, i = 1\ldots8$
  - $SU(2)$: $W^i_\mu, i = 1,2,3$
  - $U(1)$: $B_\mu$
- **Gauge couplings:** $g_s, g, g'$
- **Complex $SU(2)$ Higgs doublet** (with hypercharge $1/2$): $\Phi$

$$\Phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

The scalar potential allowed by the gauge symmetry:

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

If $\mu^2 < 0$, then spontaneous symmetry breaking

**Possible minimum:**

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix}$$

**Choice of minimum breaks gauge symmetry**

Note: $SU(3)$ is not broken! (the Higgs field is not charged under $SU(3)$)

Electroweak symmetry breaking (EWSB)

$SU(2) \times U(1)_Y \rightarrow U(1)_{em}$

“Extended version” of the breaking of the abelian $U(1)$ symmetry seen before
We can rewrite the complex SU(2) doublet scalar field as:

$$
\Phi = e^{i \sigma_i \frac{\omega_i}{v}} \begin{pmatrix} 0 \\ \frac{h + v}{\sqrt{2}} \end{pmatrix}
$$

$h, \omega_i$ real scalars

generators of SU(2) (Pauli matrices)
Higgs potential & Higgs mass, self interactions

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generators of SU(2) (Pauli matrices)

Real scalars: \( h, \omega_i \)

The scalar potential becomes:

\[ V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \]

\[ V(h, \omega_i) = \frac{m_h^2}{2} h^2 + \frac{m_h^2}{2v} h^3 + \frac{m_h^2}{8v^2} h^4 \]

Higgs mass

Disappearance of \( \omega_i \) fields (Goldstones)

Higgs self-interactions

In terms of the initial parameters:

\[ v^2 = -\frac{4\mu^2}{\lambda} \]

\[ m_h^2 = -2\mu^2 \]
The Higgs mass in the Standard Model

We need to measure the Higgs mass!

LHC: $m_h=125$ GeV
Vacuum stability

Once we fix the two free parameters of the Higgs potential, we can compute the running of the quartic coupling, $\lambda$, as a function of the energy scale.

\[ V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \]

Buttazzo et al, 1307.3536
Vacuum stability

Once we fix the two free parameters of the Higgs potential, we can compute the running of the quartic coupling, $\lambda$, as a function of the energy scale

\[ V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \]

We are here.
Is this a coincidence?

Trans-Planckian
Higgs field value

Our Universe
as we see now

Stability condition:
$M_h > (129.6 \pm 1.5) \text{ GeV}$

Assuming no New Physics
The (Higgs) gauge sector of the SM

\[ \mathcal{L}^{\text{gauge}} = (D_\mu \Phi)^\dagger D^\mu \Phi, \quad D_\mu = \partial_\mu - i\frac{g}{2} \sigma^i W^i_\mu - i\frac{g'}{2} B_\mu \]

We can write this explicitly at the minimum of the Higgs potential:

\[ (D_\mu \Phi)^\dagger D^\mu \Phi \rightarrow \frac{v^2}{8} \left( g^2 (W_\mu^1)^2 + g^2 (W_\mu^2)^2 + (-g W_\mu^3 + g' B_\mu)^2 \right) + \cdots \]

\[ \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \]

### Charged gauge boson masses

### Neutral gauge boson masses
The (Higgs) gauge sector of the SM

We can write this explicitly at the minimum of the Higgs potential:

\[
(D_\mu \Phi)\dagger D^\mu \Phi \rightarrow \frac{v^2}{8} \left( g^2 (W^1_\mu)^2 + g^2 (W^2_\mu)^2 + (-gW^3_\mu + g'B_\mu)^2 \right) + \cdots
\]

Eigenstates:

\[
W^\pm_\mu = \frac{W^1_\mu \mp W^2_\mu}{\sqrt{2}}
\]

\[
Z_\mu = \frac{gW^3_\mu - g'B_\mu}{\sqrt{g^2 + (g')^2}}
\]

\[
A_\mu = \frac{g'W^3_\mu + gB_\mu}{\sqrt{g^2 + (g')^2}}
\]

Eigenvalues:

\[
m_w = \frac{gv}{2}
\]

\[
m_z = \sqrt{g^2 + (g')^2} \frac{v}{2}
\]

\[
m_A = 0
\]

Masses vanish when v=0
The (Higgs) gauge sector of the SM

\[ \mathcal{L}_{\text{gauge}} = (D_\mu \Phi)^\dagger D^\mu \Phi, \quad D_\mu = \partial_\mu - ig^2 / 2 \sigma^i W^i_\mu - ig'/2 B_\mu \]

We can write this explicitly at the minimum of the Higgs potential:

\[ (D_\mu \Phi)^\dagger D^\mu \Phi \to \frac{v^2}{8} \left( g^2 (W^1_\mu)^2 + g^2 (W^2_\mu)^2 + (-gW^3_\mu + g' B_\mu)^2 \right) + \cdots \]

Neutral gauge boson masses

<table>
<thead>
<tr>
<th>Charged gauge boson masses</th>
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\[ \langle \Phi \rangle = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v \end{array} \right) \]

We can diagonalize the system and find:

**Eigenvalues:**

\[ m_{w} = \frac{g v}{2} \]

\[ m_{z} = \sqrt{g^2 + (g')^2} \frac{v}{2} \]

\[ m_{A} = 0 \]

**Masses vanish when \( v = 0 \)**

Let's define the Weinberg angle

\[ \sin \theta = \frac{g'}{\sqrt{g^2 + (g')^2}} \]

\[ Z_\mu = -\sin \theta B_\mu + \cos \theta W^3_\mu \]

\[ A_\mu = \cos \theta B_\mu + \sin \theta W^3_\mu \]

\[ m_{w} = m_{z} \cos \theta \]

\[ \rho \equiv \frac{m_{w}}{m_{z} \cos \theta} = 1 \quad \text{(tree level)} \]

Custodial symmetry
Recap on the SM EWSB

The Higgs mechanism generates the mass of $W, Z$
- Higgs VEV breaks $SU(2) \times U(1)_Y$
- $U(1)_{em}$ is left unbroken (the photon is massless)
- Single Higgs $SU(2)$ doublet is the minimal model to achieve this breaking pattern
Recap on the SM EWSB

The Higgs mechanism generates the mass of W, Z
- Higgs VEV breaks SU(2) x U(1)\(_Y\)
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- Single Higgs SU(2) doublet is the **minimal model** to achieve this breaking pattern

**Before** spontaneous symmetry breaking:

**Massless:**
- \(W_i\) (i=1,2,3): 6 degrees of freedom
- B: 2 degree of freedom
- Complex \(\Phi\) doublet: 4 degrees of freedom

**After** spontaneous symmetry breaking:

**Massless:**
- A (photon): 2 degree of freedom

**Massive:**
- \(W^\pm\): 6 degrees of freedom
- Z: 3 degrees of freedom
- Real scalar (Higgs): 1 degree of freedom

\[6+2+4 = 2+6+3+1\]
Higgs-gauge boson couplings

\[ \mathcal{L}^{\text{gauge}} = (D_\mu \Phi)^\dagger D^\mu \Phi, \quad D_\mu = \partial_\mu - i\frac{g}{2} \sigma^i W^i_\mu - i\frac{g'}{2} B_\mu \]

Now we can insert the physical Higgs boson:

\[ \Phi = \begin{pmatrix} 0 \\ \frac{h + v}{\sqrt{2}} \end{pmatrix} \]
Gauge sector

Higgs-gauge boson couplings

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Now we can insert the physical Higgs boson:

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Couplings are proportional to the mass

(1) \[ \mathcal{L} \supset \frac{g^2}{4} (v + h)^2 W^\mu W^\nu \rightarrow \frac{g^2 v}{2} h W^\mu W^\nu - \frac{g^2}{4} hh W^\mu W^\nu \]

(2) \[ \mathcal{L} \supset \frac{g^2 + g'^2}{8} (v + h)^2 Z^\mu Z^\nu \rightarrow \frac{(g^2 + g'^2) v}{4} h Z^\mu Z^\nu + \frac{g^2 + g'^2}{8} hh Z^\mu Z^\nu \]
A predictive model

Four free parameters in gauge-Higgs sector \((g, g', \mu, \lambda)\)

Conventionally chosen to be

- \(\alpha\)
- \(G_F\) (Fermi constant)
- \(M_Z\)
- \(M_h\)

All observables can be expressed in terms of these parameters
A predictive model

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**Examples:**

1. W mass

\[
\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_w^2} = \frac{\pi\alpha}{2 \left(1 - \frac{m_w^2}{m_Z^2}\right)m_w^2}
\]

2. Z coupling to leptons

\[
\frac{g}{\cos \theta \left(-\frac{1}{2} + \sin^2 \theta\right)} Z_{\mu} \bar{\ell}_L \gamma^\mu \ell_L = \sqrt{\frac{4\pi\alpha}{1 - \frac{m_w^2}{m_Z^2}} \frac{m_Z}{m_w} \left(\frac{1}{2} - \left(\frac{m_w}{m_Z}\right)^2\right)} Z_{\mu} \bar{\ell}_L \gamma^\mu \ell_L
\]

Many observables have been measured!
A predictive model

Four free parameters in gauge-Higgs sector \((g, g', \mu, \lambda)\)

Conventionally chosen to be

- \(\alpha\)
- \(G_F\) (Fermi constant)
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All observables can be expressed in terms of these parameters

Examples:

1. **W mass**

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\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_w^2} = \frac{\pi \alpha}{2 \left(1 - \frac{m_w^2}{m_Z^2}\right)m_w^2}
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2. **Z coupling to leptons**

\[
\frac{g}{\cos \theta \left(\frac{1}{2} + \sin^2 \theta\right)} Z_{\mu} \bar{\ell}_L \gamma^\mu \ell_L = \sqrt{\frac{4\pi \alpha}{1 - \frac{m_w^2}{m_Z^2}} m_z \left(\frac{1}{2} - \left(\frac{m_w}{m_Z}\right)^2\right)} Z_{\mu} \bar{\ell}_L \gamma^\mu \ell_L
\]

In fact, we had hints for the value of the Higgs mass **before** the Higgs discovery!

Many observables have been measured!
The need for a Higgs: unitarity (beyond mass generation)

In our theory of weak interactions, if we do not have a Higgs, the scattering of two longitudinal W bosons:

\[ \mathcal{M}^{\text{gauge}} = -i \frac{g^2}{4m_W^2} u + \mathcal{O}\left(\frac{E}{m_W}\right)^0 \]

WW scattering will essentially grow with energy until violating unitarity at the \( \sim \text{TeV} \) scale.

We expect some new dynamics should show up at TeV scale.
The need for a Higgs: unitarity (beyond mass generation)

In our theory of weak interactions, if we do not have a Higgs, the scattering of two longitudinal W bosons:

WW scattering will essentially grow with energy until violating unitarity at the ~TeV scale.

We expect some new dynamics should show up at TeV scale.

The leading term cancels introducing the SM Higgs.

\[ i \mathcal{M}^{\text{gauge}} = -i \frac{g^2}{4m_W^2} u + \mathcal{O}\left(\frac{E}{m_W}\right)^0 \]
What Yukawa interaction can I write down that is invariant under the SM gauge symmetry?

Reminder: $\Phi = (1, 2, 1/2)$
Generation of fermion masses

Fermion representations under SU(3) x SU(2) x U(1)$_Y$:

\[
Q^i_L = \begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix} = (3, 2, 1/6), \quad u^i_R = (3, 1, 2/3), \quad d^i_R = (3, 1, -1/3)
\]

\[
L^i_L = \begin{pmatrix} \nu^i_L \\ \ell^i_L \end{pmatrix} = (1, 2, -1/2), \quad e^i_R = (1, 1, -1)
\]

What Yukawa interaction can I write down that is invariant under the SM gauge symmetry?

Reminder: $\Phi = (1, 2, 1/2)$

\[
\bar{Q}^i L Y_D^{ij} d^j_R \Phi + \bar{Q}^i L Y_D^{ij} u^j_R \tilde{\Phi} + \bar{L}^i L Y_E^{ij} e^j_R \Phi + \text{h.c.}
\]

After EWSB (at the minimum):

\[
\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \quad \text{down-quark mass term, } M_d
\]

Without the Higgs field no fermion mass term would be allowed in the Lagrangian
In the quark sector, **four rotation matrices** are needed to diagonalize the system:

- **eigenvalues:** \( M_f^{\text{diag}} = U_{fL} M_f U_{fR}^\dagger \) (\( f = u,d \))
- **eigenstates:** \( f_{Li}^{\text{mass}} = U_{fL}^{ij} f_{Lj}, \quad f_{Ri}^{\text{mass}} = U_{fR}^{ij} f_{Rj} \)

Mass matrices diagonalized by different transformations for \( u_L \) and \( d_L \), which are part of the same SU(2) doublet, \( Q_L \)

\[
\begin{pmatrix}
  u_{Li} \\
  d_{Li}
\end{pmatrix}
= (U_{uL}^\dagger)^{ij}
\begin{pmatrix}
  u_{Lj}^{\text{mass}} \\
  (U_{uL} U_{dL}^\dagger)_{jk} d_{Lk}^{\text{mass}}
\end{pmatrix}
\]

**CKM (unitary) matrix**
In the quark sector, four rotation matrices are needed to diagonalize the system:

eigenvalues: $M_{f}^{\text{diag}} = U_{fL} M_{f} U_{fR}^\dagger \quad (f = u, d)$

eigenstates: $f_{Li}^{\text{mass}} = U_{fL}^{ij} f_{Lj}, \quad f_{Ri}^{\text{mass}} = U_{fR}^{ij} f_{Rj}$

Mass matrices diagonalized by different transformations for $u_L$ and $d_L$, which are part of the same SU(2) doublet, $Q_L$

$$
\begin{pmatrix}
  u_{Li} \\
  d_{Li}
\end{pmatrix} = (U_{uL}^\dagger)_{ij} \begin{pmatrix}
  u_{Lj}^{\text{mass}} \\
  (U_{uL} U_{dL}^\dagger)_{jk} d_{Lk}^{\text{mass}}
\end{pmatrix}
$$

The physical Higgs boson couples to fermion mass eigenstates:

$$
\bar{Q}^i_L Y_D^{ij} d_R^j \tilde{\Phi} + \bar{Q}^i_L Y_D^{ij} u_R^j \tilde{\Phi} + \bar{L}^i_L Y_E^{ij} e_R^j \tilde{\Phi} + \text{h.c.}
$$

$$
\Phi = \begin{pmatrix}
  0 \\
  \frac{h + \nu}{\sqrt{2}}
\end{pmatrix}
$$

Couplings are proportional to the mass

$\Phi$ has flavor diagonal interactions!
The SM flavor puzzle

Why such large hierarchies?

Does the Higgs couple so hierarchically to quarks and leptons?
Flavor changing neutral currents

$\mathcal{L}_{\text{gauge}} \supset f_i \gamma^\mu D_\mu f_i$, \quad $D_\mu = \partial_\mu - ig^i W^i_\mu - ig'^i B_\mu$

Not to get confused:
this mixing originates only from the Higgs sector:
$V_{\text{CKM}} \rightarrow \delta$ if we switch-off the Yukawa interactions

Fermion sector
Not to get confused:
this mixing originates only from the Higgs sector:
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**Exercise:** prove that neutral \( \gamma, Z \) and \( g \) currents stay flavor universal, since they do not mix the chiralities

(Note for the experts:
At one loop, neutral flavor transitions are generated. However they are **loop+GIM** suppressed.)

No flavor changing neutral currents (FCNCs) at tree level in the SM
Recap on Higgs couplings

• Higgs couples to fermion mass:

\[ \mathcal{L} \supset - \frac{m_f}{v} \bar{f} f h = - \frac{m_f}{v} (\bar{f}_L f_R + \bar{f}_R f_L) h \]

Largest coupling is to heaviest fermion
Is the top quark special?

• Higgs couples to gauge boson mass:

\[ \mathcal{L} \supset 2 \frac{m_w^2}{v} W^{\mu} W_{\mu}^- h + \frac{m_z^2}{v} Z^{\mu} Z_{\mu} h + \cdots \]

Only free parameter is Higgs mass!

We do not know if
the Higgs couples to neutrinos:

\[ \bar{L}_L^i Y_N^{ij} N_R^j \Phi \]

do they exist?

2 Higgs - 2 gauge boson interactions
Recap on Higgs couplings

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This is only for tree-level couplings…
tomorrow we will discuss
loop-induced couplings &
Higgs phenomenology

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