

Linear Compton Scattering

Experimental frame : headon collision between electron (E_e) and laser beams (ω_0)

$$e + \gamma \rightarrow e' + \gamma'$$
$$(E_e, \mathbf{p}) + (\omega_0, \mathbf{k}_0) \rightarrow (E'_e, \mathbf{p}') + (\omega'_e, \mathbf{k}'_e)$$

Electron rest frame : collision between electron ($p_e=0$) and laser photon (ω)

$$e + \gamma \rightarrow e' + \gamma'$$
$$(m_e, 0) + (\omega, \mathbf{k}) \rightarrow (m_e, \mathbf{p}') + (\omega', \mathbf{k}')$$

$$\omega = \gamma(1 + \beta)\omega_0 \approx 2\gamma\omega_0, \text{ for } \gamma = \frac{E_e}{m_e}, \beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$x = \frac{s - m_e^2}{m_e^2} = \frac{2\omega}{m_e} \approx \frac{4\omega_0 E_e}{m_e^2}$$

$$\cos \theta = m_e \left(\frac{1}{\omega} - \frac{1}{\omega'} \right) + 1, \quad \text{where } \frac{\omega}{1+x} \geq \omega' \leq \omega$$

$$\omega'_e = \gamma\omega'(1 - \beta \cos \theta)$$

$$d\omega' \approx \omega d\left(\frac{\omega'_e}{E_e}\right) = \omega dy, \text{ for } y = \frac{\omega'_e}{E_e}$$

Cross section with the incident photon and electron helicities, ξ_2 and ζ , respectively

$$d\sigma(\zeta, \xi_2) = 2\pi r_e^2 \frac{m_e}{\omega'^2} \left(\frac{\omega'}{\omega} \right)^2 \left\{ \frac{\omega}{\omega'} + \frac{\omega'}{\omega} - \sin^2 \theta + \xi_2 \zeta \left(\frac{\omega}{\omega'} - \frac{\omega'}{\omega} \right) \cos \theta \right\}$$

For $\bar{z} = \frac{z}{\ln(1+x)} = \frac{z}{L_x}$, $z = \ln \left(\frac{\omega}{\omega'} \right)$, $L_x = \ln(1+x)$ and $d\bar{z} = \frac{1}{L_x \omega'} d\omega'$

$$d\sigma(\zeta, \xi_2) = 4\pi r_e^2 \frac{L_x}{x} F(\bar{z}) d\bar{z} = 2\pi r_e^2 \frac{m_e}{\omega \omega'} F(\bar{z}) d\omega' = 2\pi r_e^2 \frac{m_e}{\omega'} F(\bar{z}) dy \quad \text{with } d\omega' = \omega dy$$

, where $F(\bar{z}) = \frac{1}{2} [1 + e^{-2z} - e^{-z} \sin^2 \theta + \xi_2 \zeta \cos \theta (1 - e^{-2z})]$

Non-Linear Compton Scattering

$e + n\gamma \rightarrow e' + \gamma'$ in a strong laser field with ξ or a_0

$$(m_e^*, 0) + n(\omega^*, \mathbf{k}) \rightarrow (m_e^*, \mathbf{p}'_n) + (\omega'_n, \mathbf{k}'_n), \quad \text{for } m_e^* = m_e \sqrt{1 + \xi^2}, \quad \omega^* = \omega / \sqrt{1 + \xi^2}$$

$$(E_e, \mathbf{p}) + n(\omega_0, \mathbf{k}_0) \rightarrow (E'_c, \mathbf{p}'_{n,c}) + (\omega'_{n,c}, \mathbf{k}'_{n,c})$$

$$\frac{d\sigma}{dy}(\zeta, \xi_2) = \frac{2\pi r_e^2}{x\xi^2} \sum_{n=1}^{\infty} (F_{1n} + \zeta\xi_2 F_{2n})$$

$$\begin{aligned} \frac{d\sigma}{d\bar{z}_n}(\zeta, \xi_2) &= \frac{2\pi r_e^2}{x\xi^2} \frac{L_{u_n} \omega'_n \sqrt{1 + \xi^2}}{n\omega} (F_{1n} + \zeta\xi_2 F_{2n}) \\ &\equiv \pi r_e^2 \frac{4L_{u_n}}{u_n} F(\bar{z}_n) \end{aligned}$$

Simple transformations : Linear $\rightarrow$ Non-Linear Compton scattering

$$x$$

$$u_n = nx/(1 + \xi^2)$$

$$\omega$$

$$n\omega/\sqrt{1 + \xi^2}$$

$$\omega' = \frac{m_e \omega}{\omega(1 - \cos \theta) + m_e}, \quad d\omega' = \omega dy$$

$$\omega'_n = \frac{nm_e \omega}{\omega(1 - \cos \theta_n)/\sqrt{1 + \xi^2} + m_e \sqrt{1 + \xi^2}}, \quad d\omega'_n = \frac{n\omega}{\sqrt{1 + \xi^2}} dy$$

$$\omega'_{min} = \frac{\omega}{1 + x}$$

$$\omega'_{n,min} = \frac{n\omega/\sqrt{1 + \xi^2}}{1 + u_n}$$

$$y_{max} = \frac{x}{1 + x}$$

$$y_n = \frac{u_n}{1 + u_n}, \quad u_n = \frac{y_n}{1 - y_n}, \quad \text{and} \quad y = \frac{\omega'_{n,c}}{E_e}$$

$$\bar{z} = \frac{\ln(\omega/\omega')}{\ln(1 + x)}, \quad d\bar{z} = \frac{\omega}{L_x \omega'} dy$$

$$\bar{z}_n = \frac{\ln(n\omega/(\omega'_n \sqrt{1 + \xi^2}))}{\ln(1 + u_n)}, \quad d\bar{z}_n = \frac{n\omega/\sqrt{1 + \xi^2}}{L_{u_n} \omega'_n} dy$$

$$\gamma = \frac{E_e}{m_e}$$

$$\frac{\gamma}{\sqrt{1 + \xi^2}}$$

Variables used for simulation in CAIN

$$u \equiv \frac{y}{1-y} \quad \therefore y = \frac{u}{1+u}$$

$$v_n \equiv \frac{u}{u_n} \quad \text{for } 0 \leq v_n \leq 1 \quad \text{and} \quad dv_n = \frac{du}{u_n}$$

$$dy = \frac{u_n}{(1+u)^2} dv_n$$

$$\therefore \frac{d\sigma}{dv_n}(\zeta, \xi_2) = \frac{2\pi r_e^2}{x\xi^2} (F_{1n} + \zeta\xi_2 F_{2n}) \frac{u_n}{(1+u)^2}$$

Transformation $\rightarrow$ variables in CAIN

$$x \rightarrow \lambda$$

$$u \rightarrow v$$

$$u_n \rightarrow v_n$$

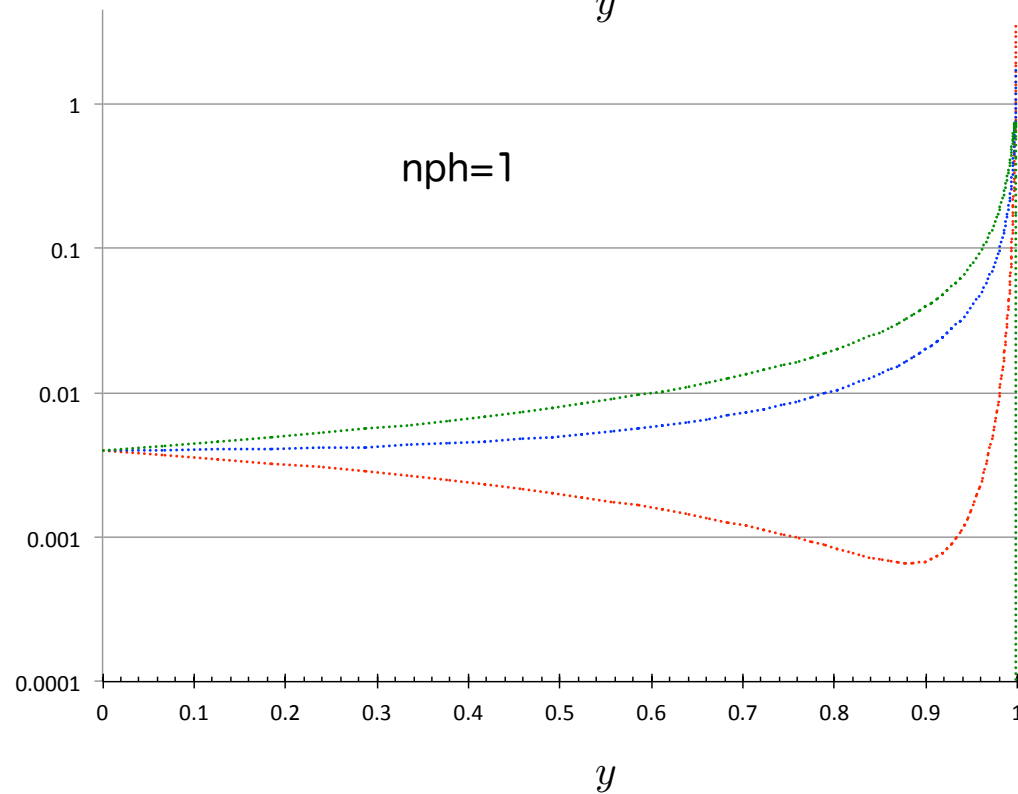
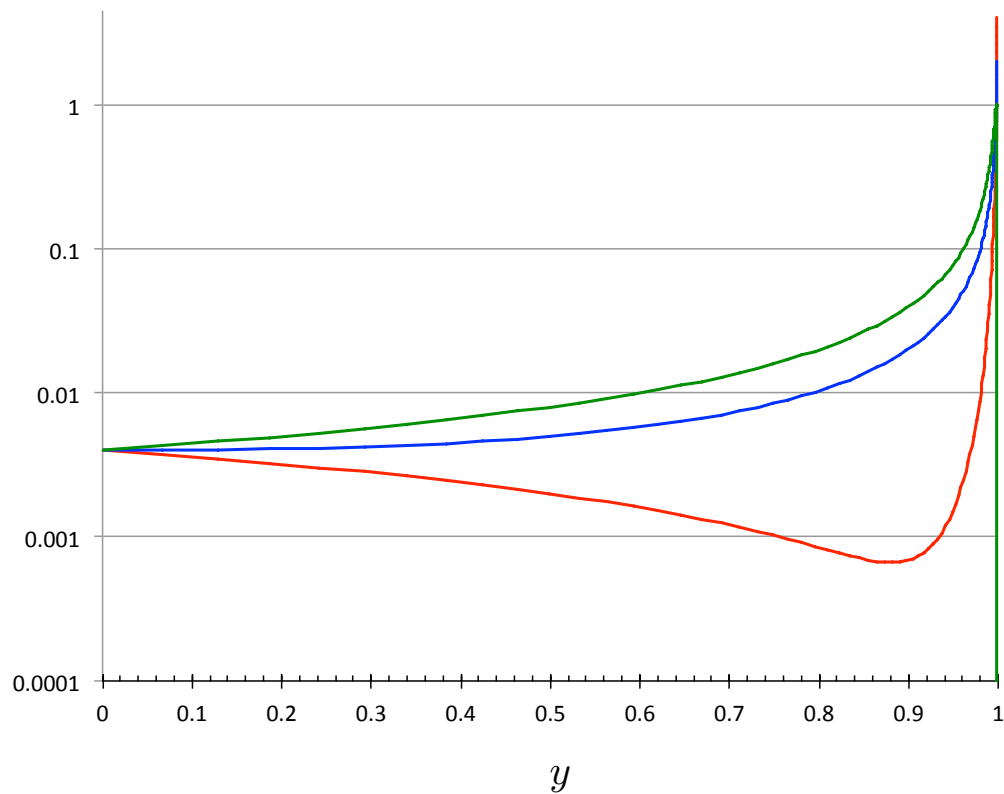
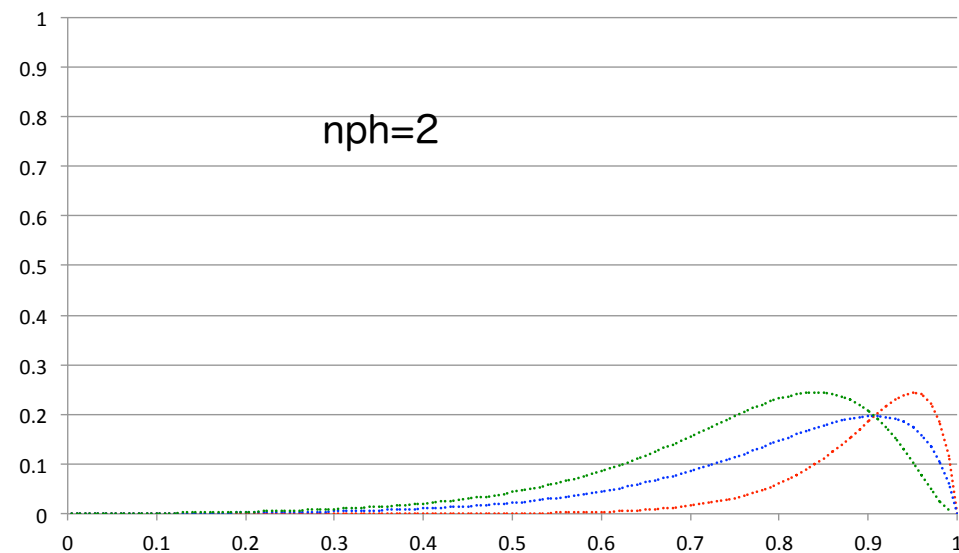
$$v_n = u/u_n \rightarrow y = v/v_n$$

Compare with following cross section formulas,

$$\frac{d\sigma}{dy}(\zeta, \xi_2) = \frac{2\pi r_e^2}{x\xi^2} \sum_{n=1}^{\infty} (F_{1n} + \zeta\xi_2 F_{2n})$$

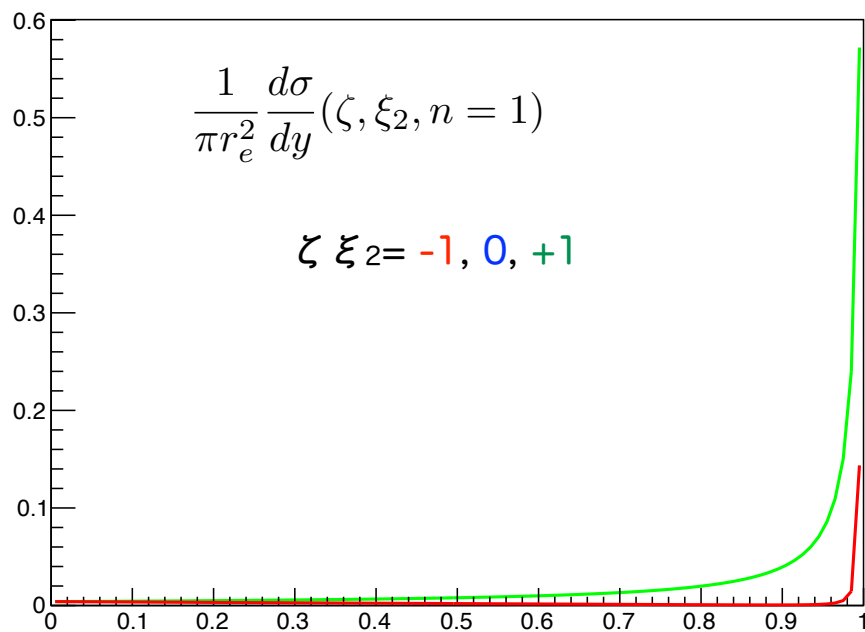
$$\frac{d\sigma}{d\bar{z}_n}(\zeta, \xi_2) = \frac{2\pi r_e^2}{x\xi^2} \frac{L_{u_n} \omega'_n \sqrt{1+\xi^2}}{n\omega} (F_{1n} + \zeta\xi_2 F_{2n}) \equiv \pi r_e^2 \frac{4L_{u_n}}{u_n} F(\bar{z}_n)$$

$$\frac{1}{\pi r_e^2} \frac{d\sigma}{dy}$$

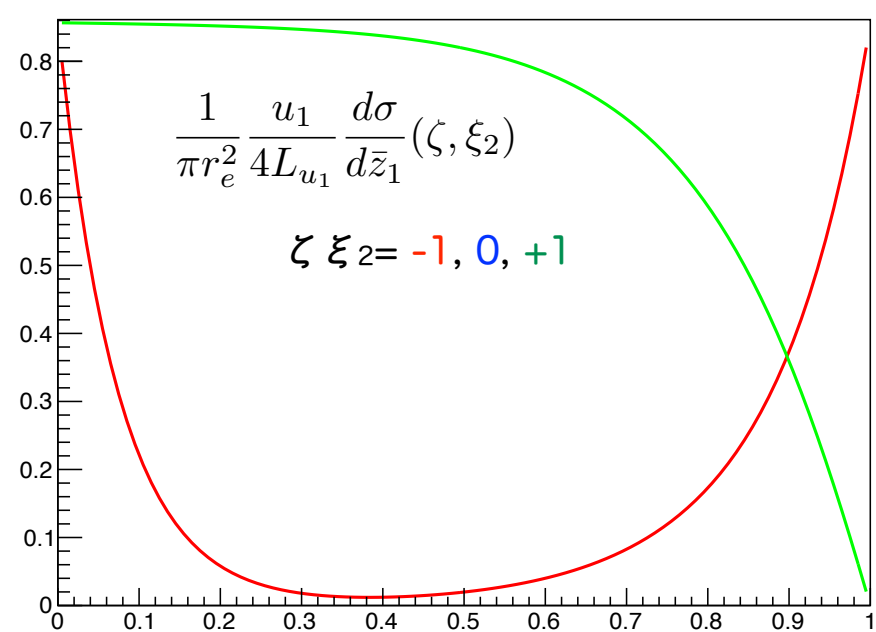


Solid lines : Linear Compton scattering, i.e. $nph=0$, $\zeta \xi_2 = -1, 0, +1$
Dotted lines : Non-linear Compton scattering, i.e. $nph=1,2$, $\zeta \xi_2 = -1, 0, +1$

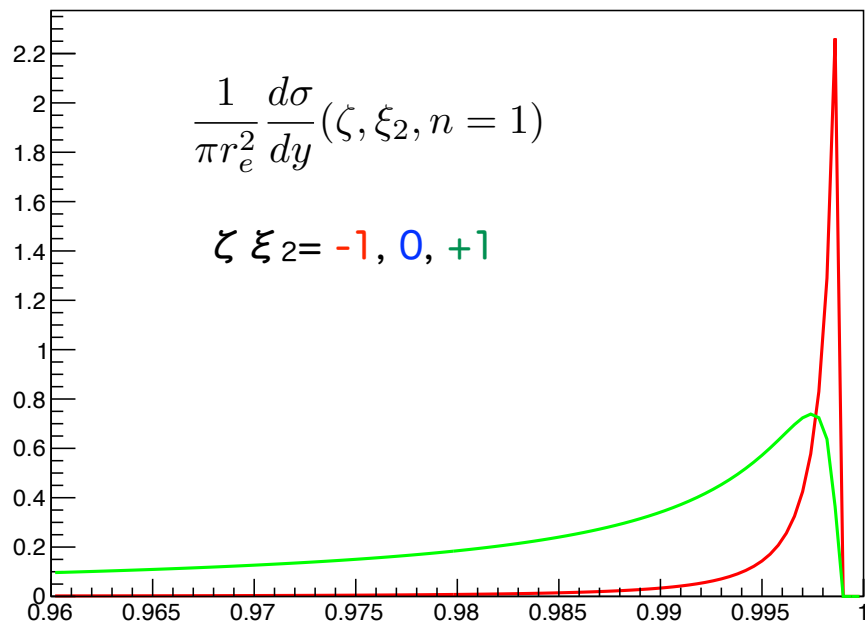
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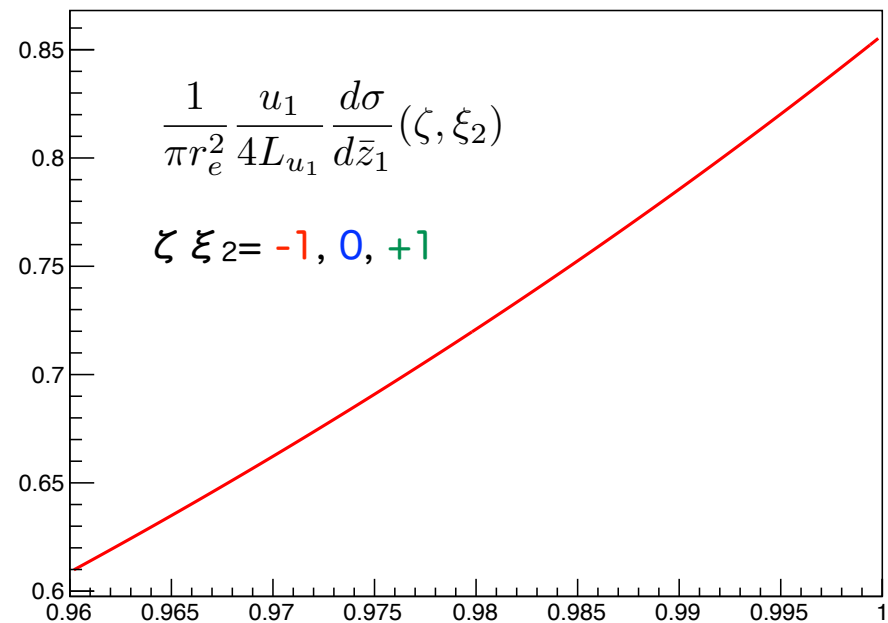
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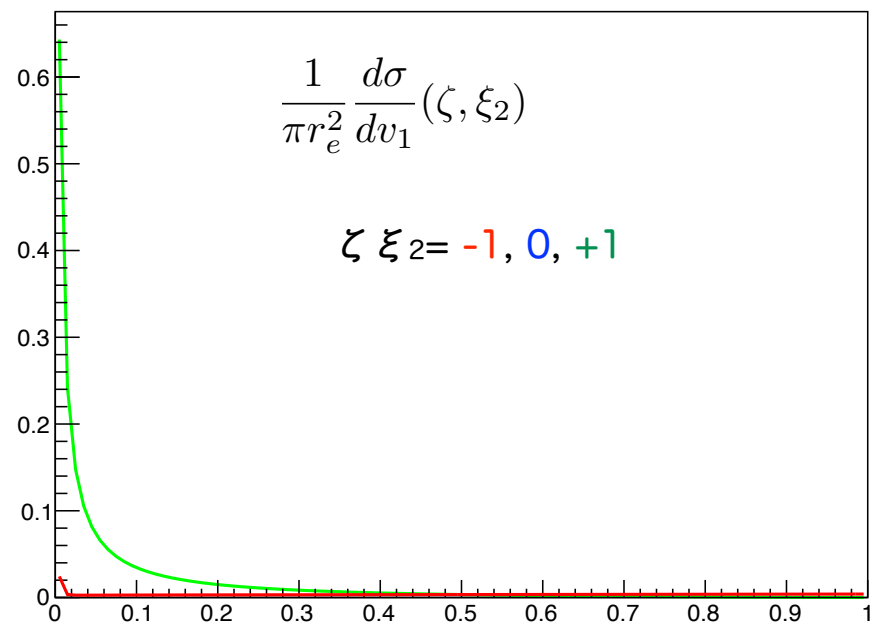
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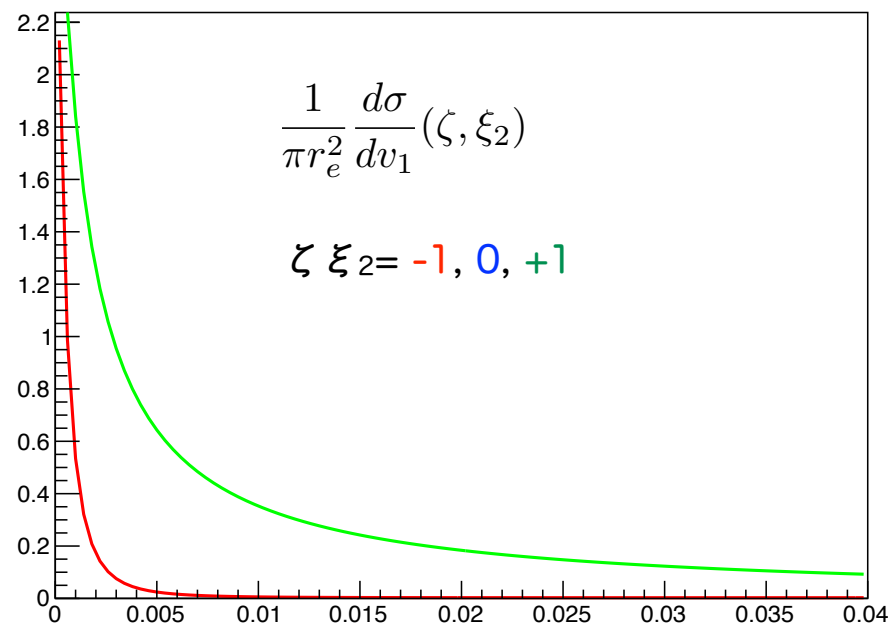
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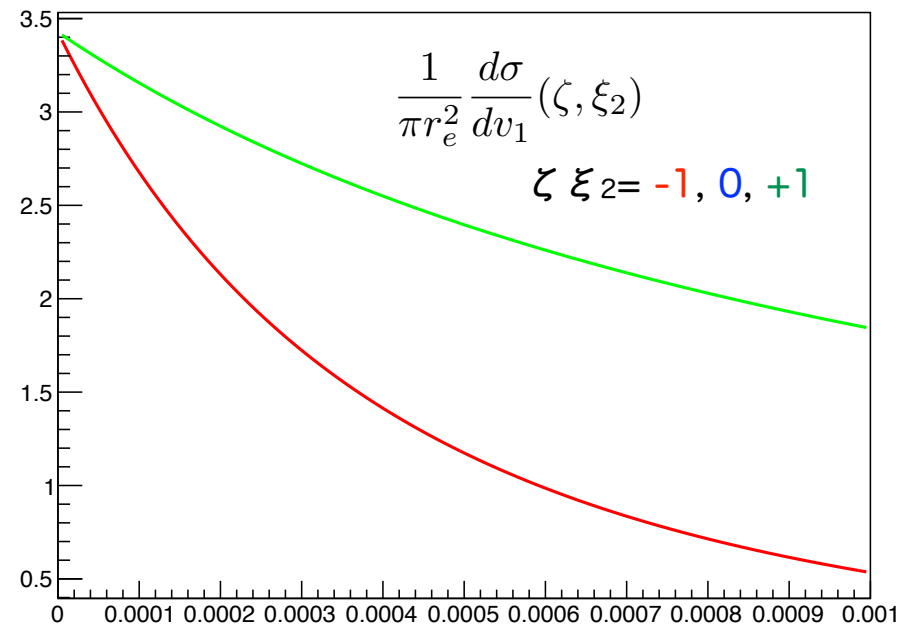
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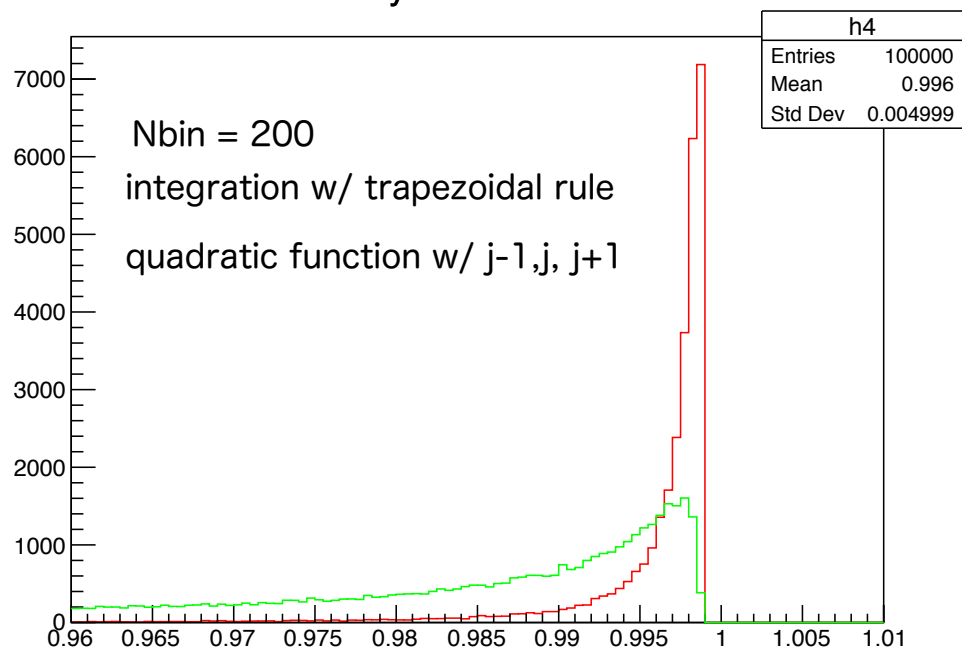
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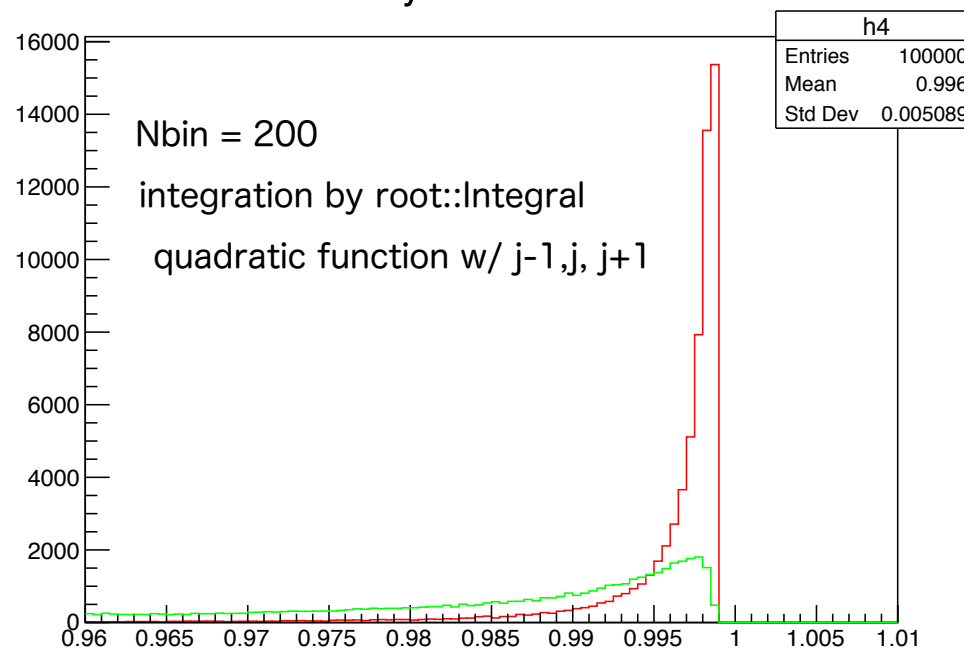
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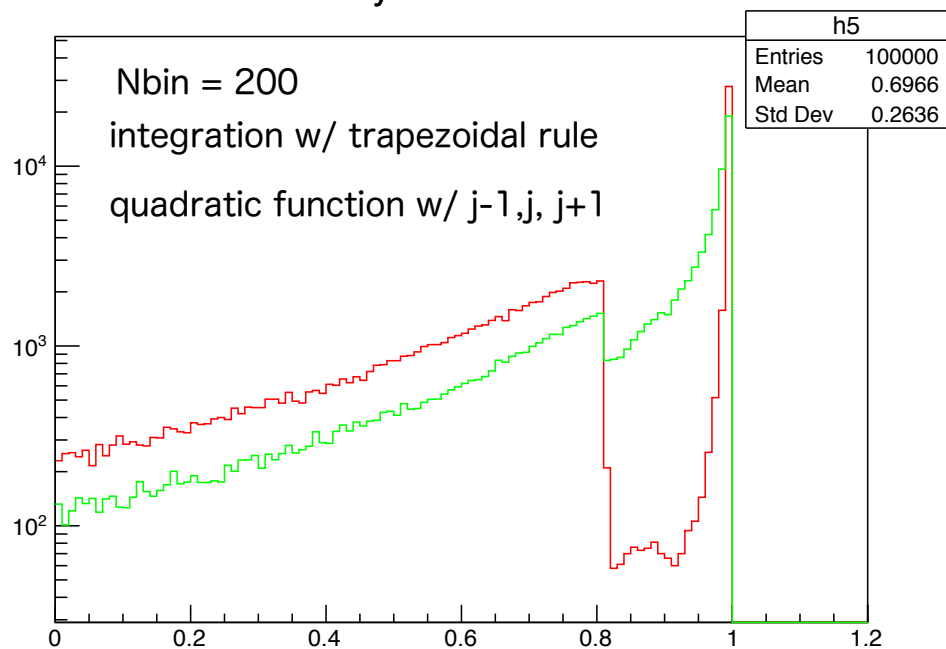
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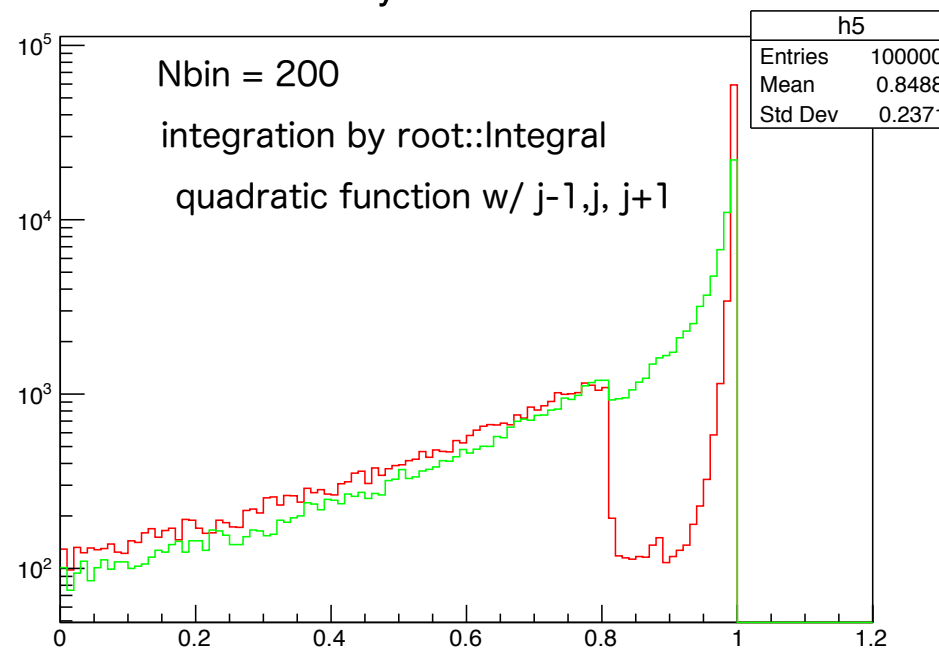
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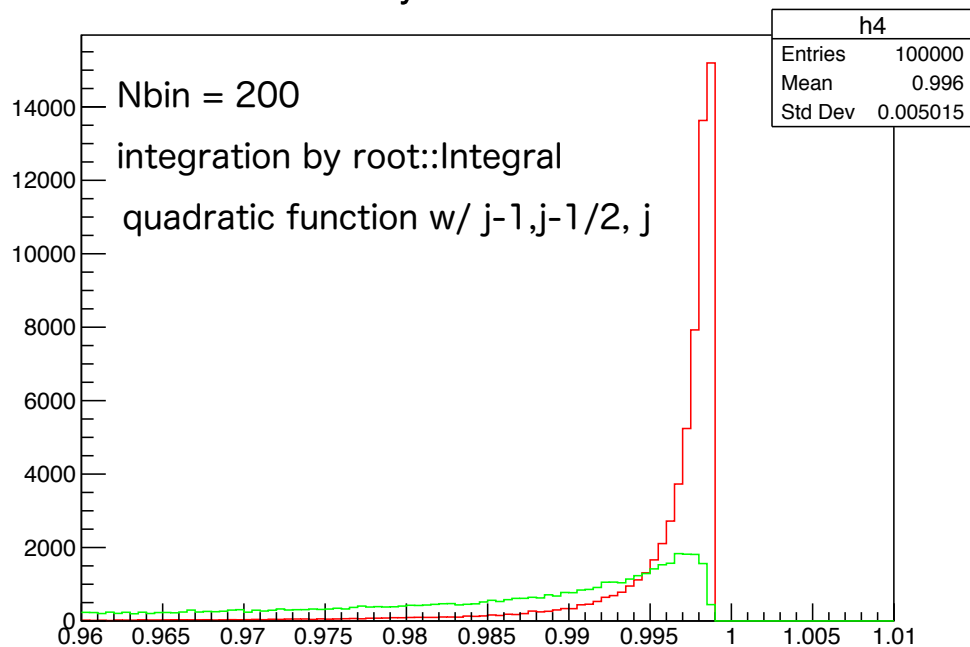
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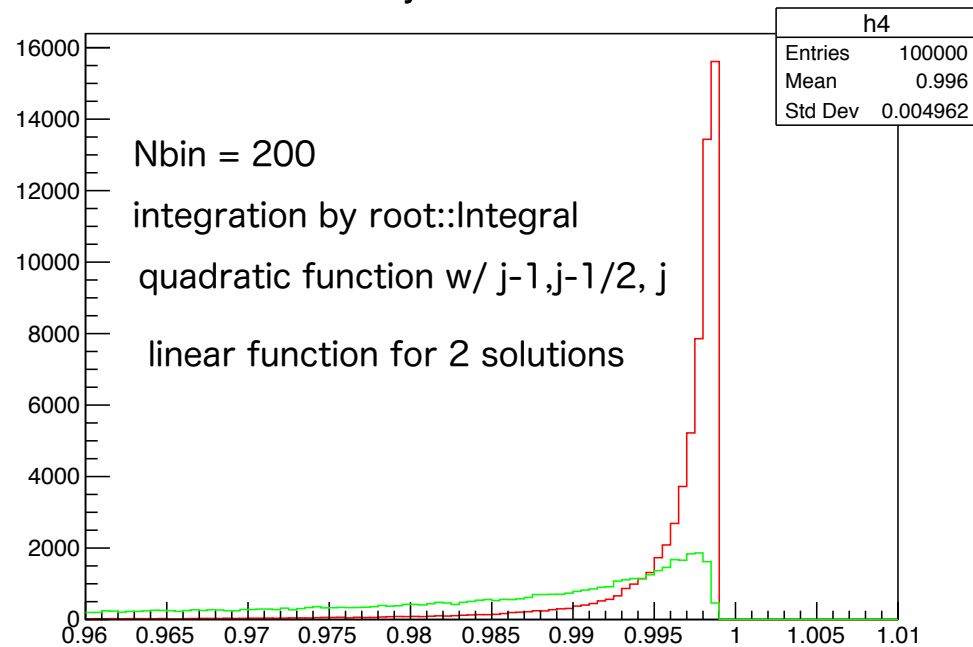
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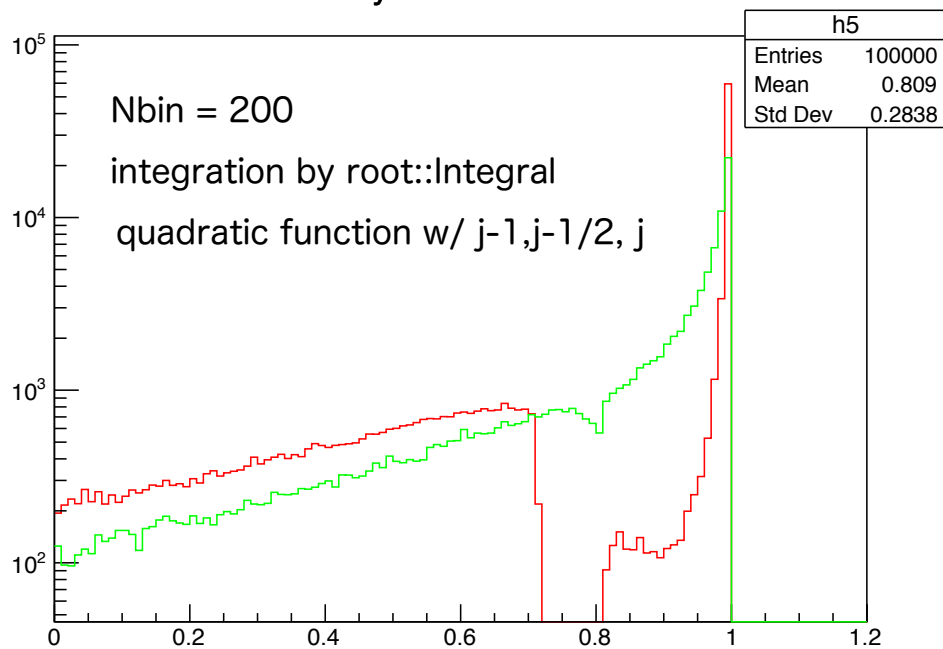
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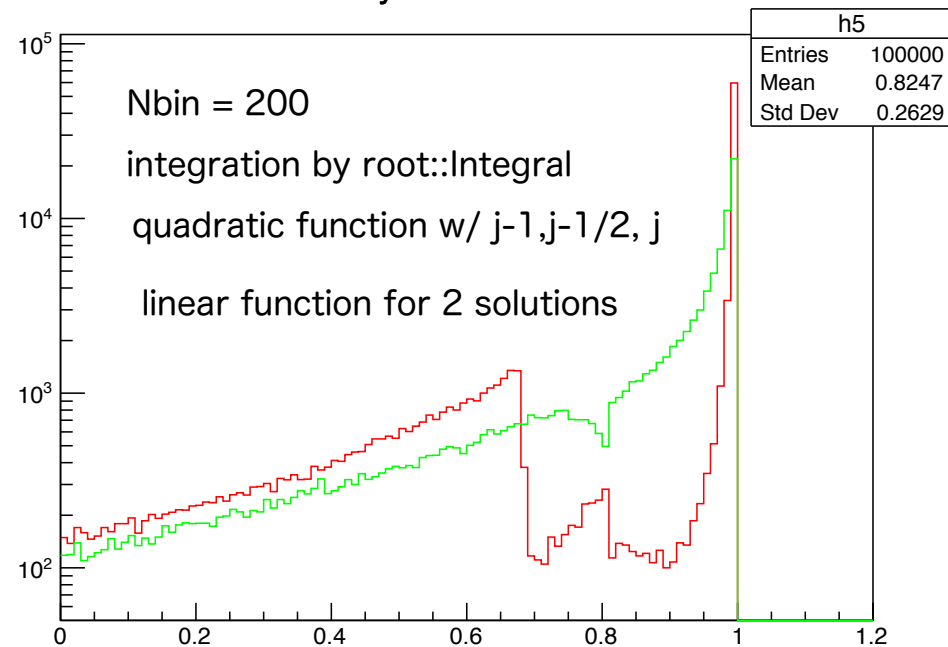
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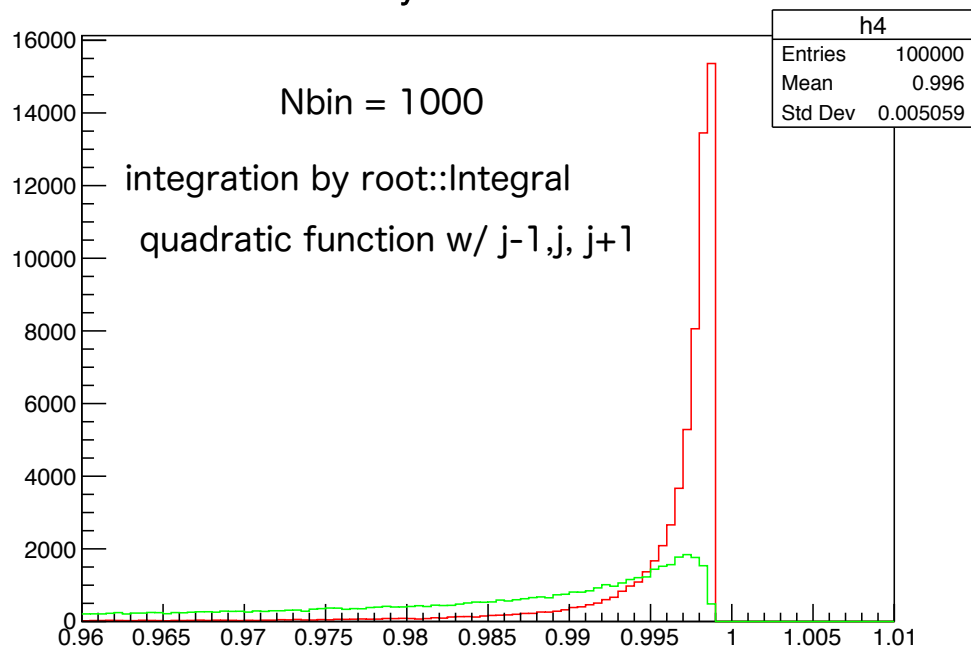
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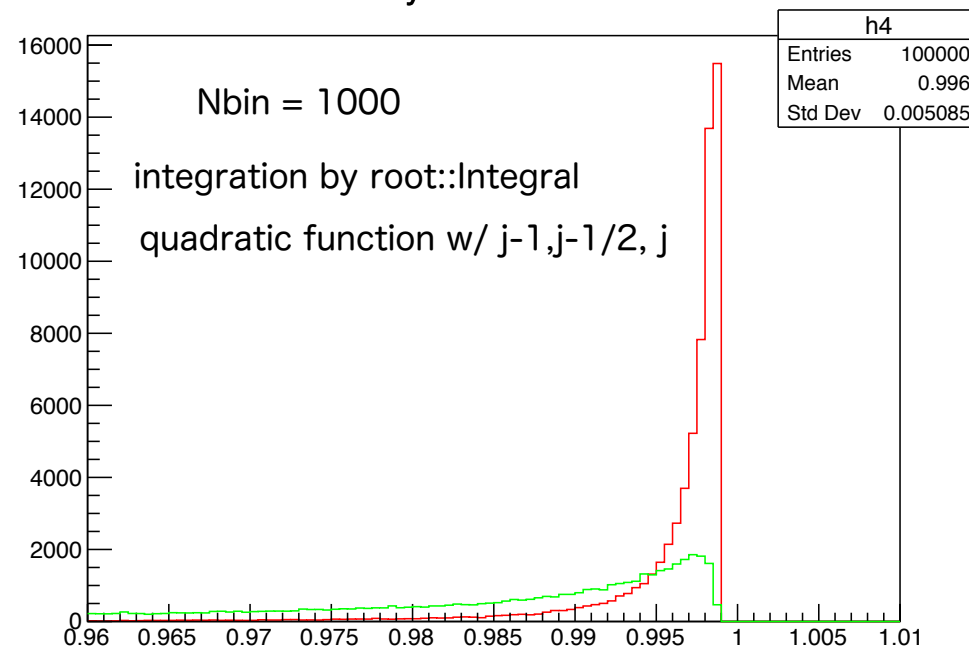
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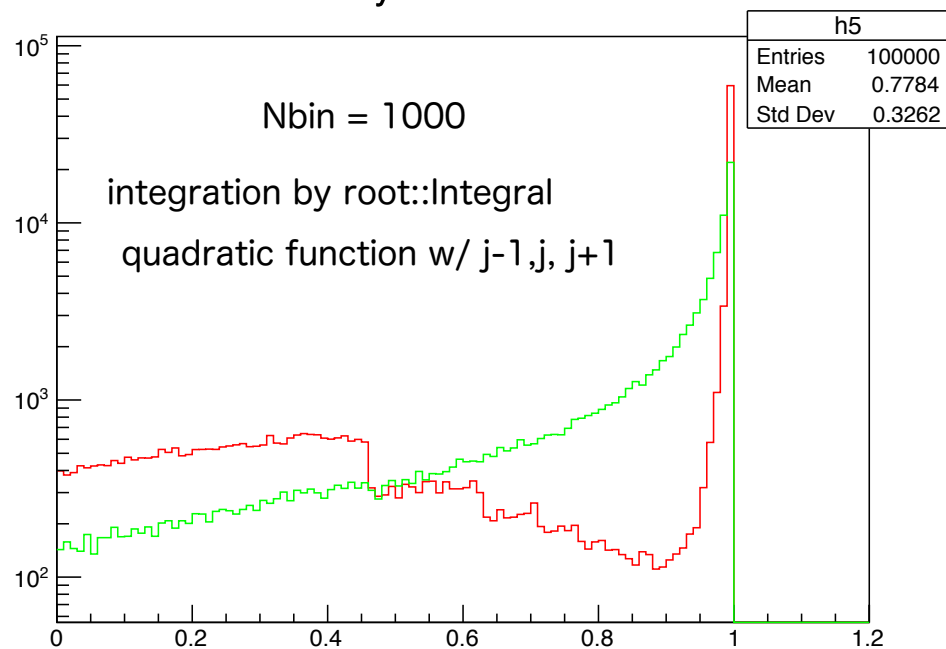
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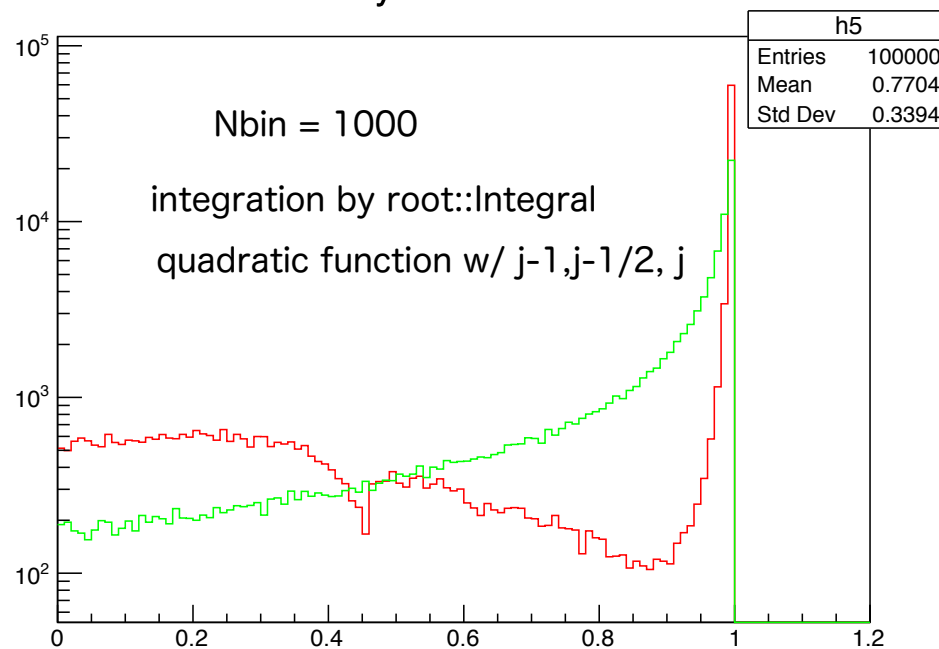
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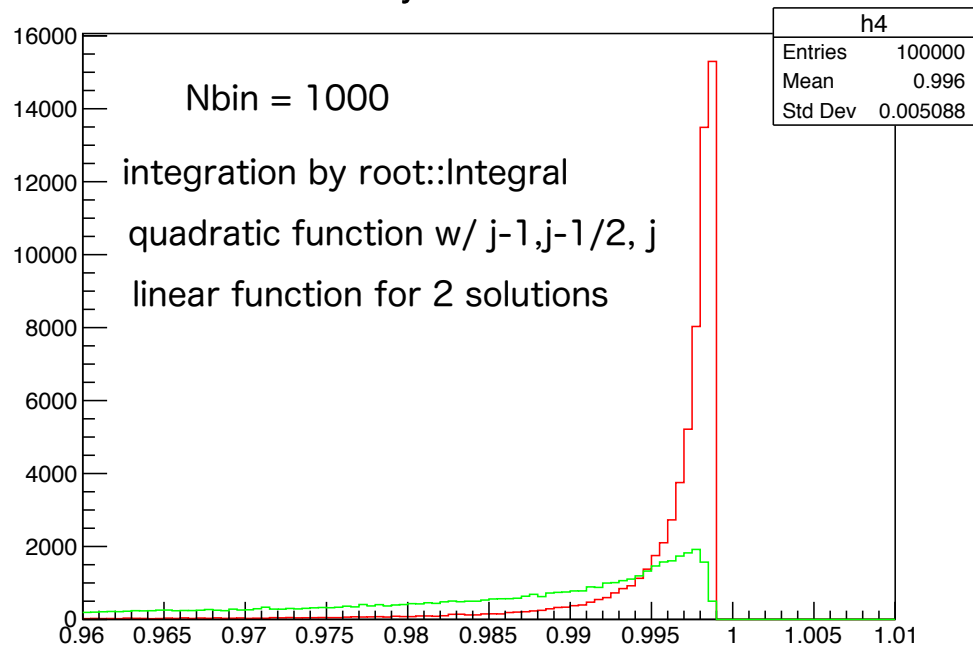
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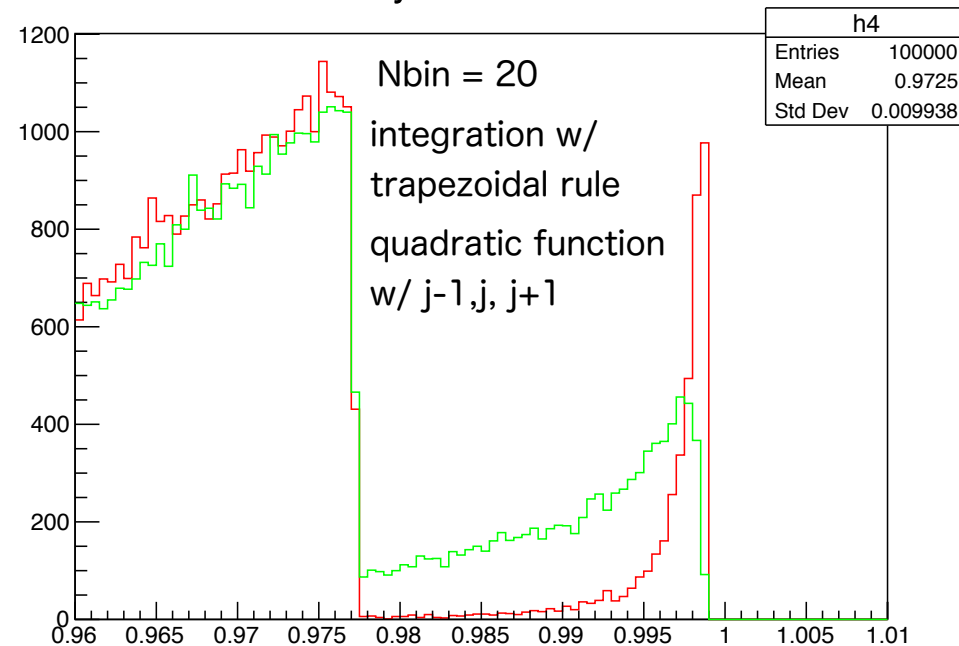
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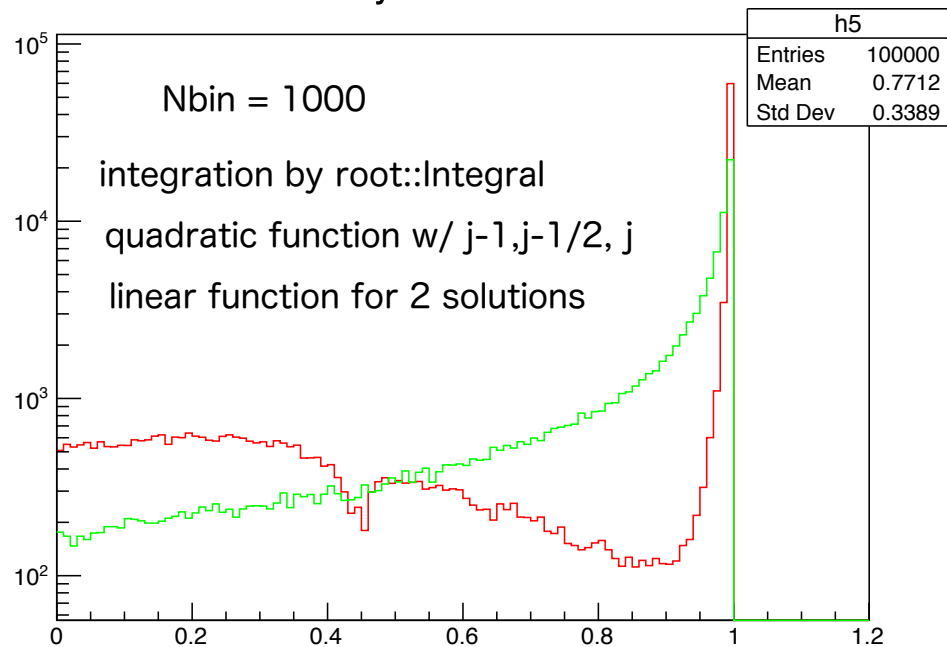
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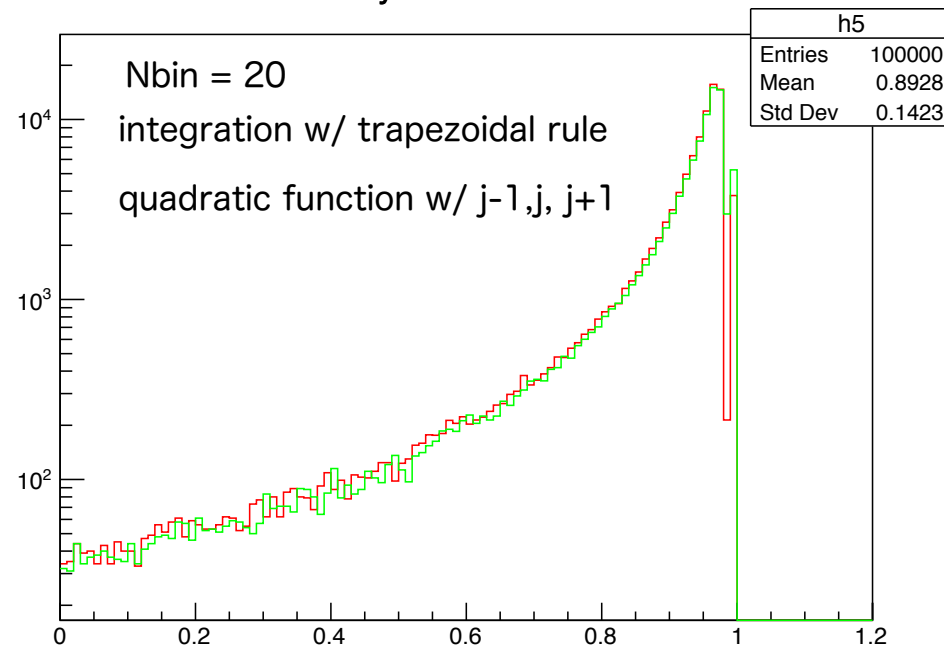
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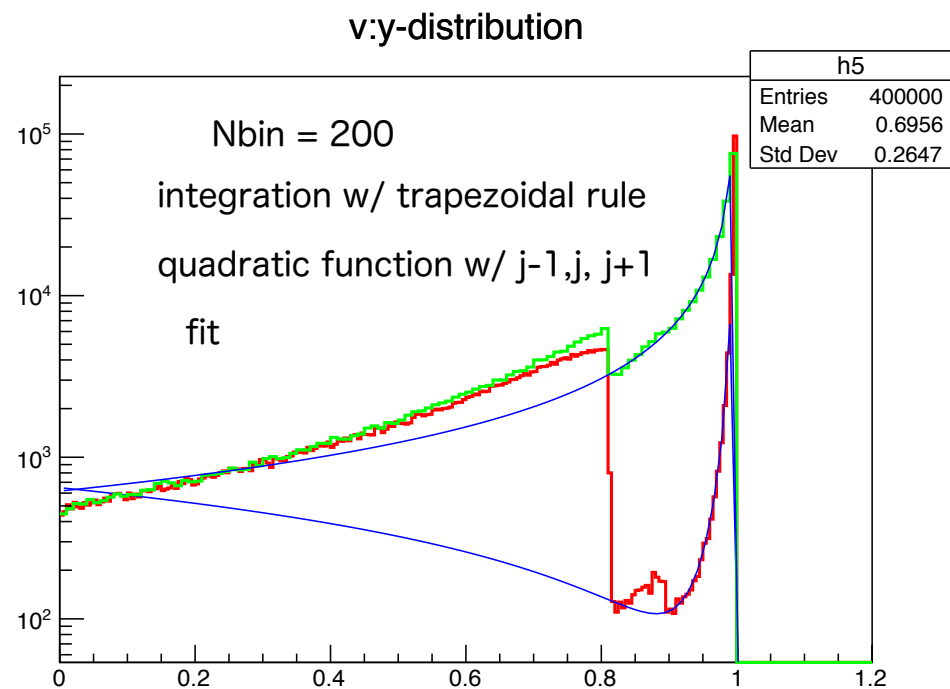
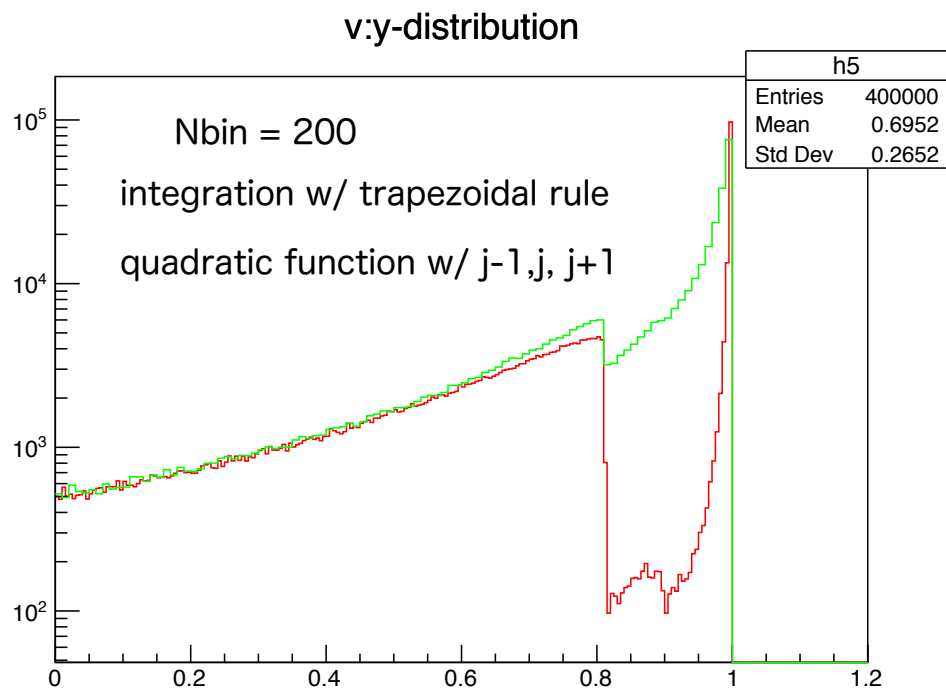
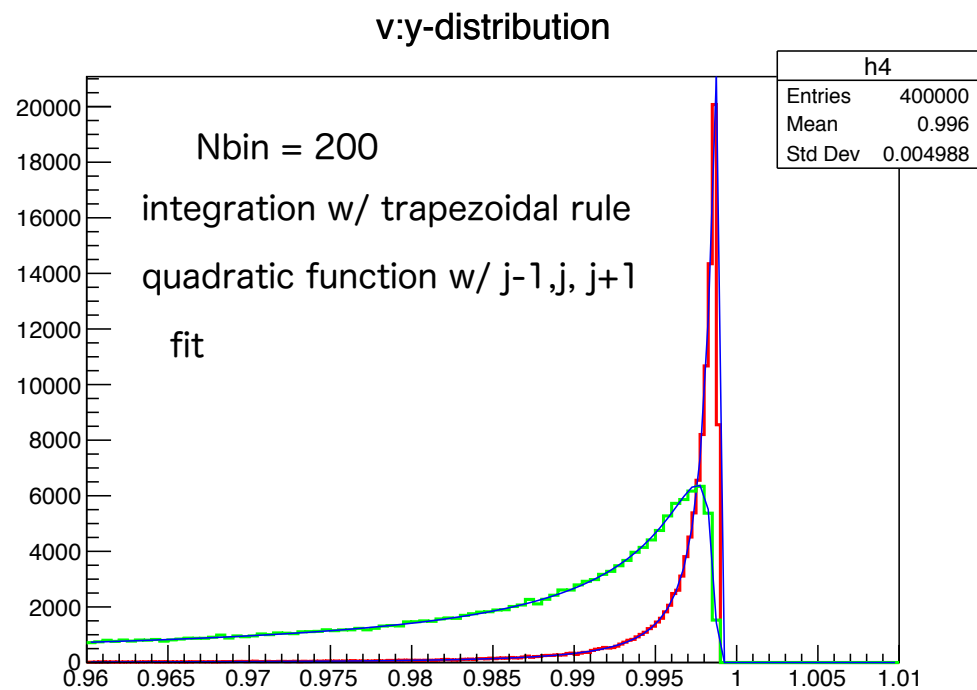
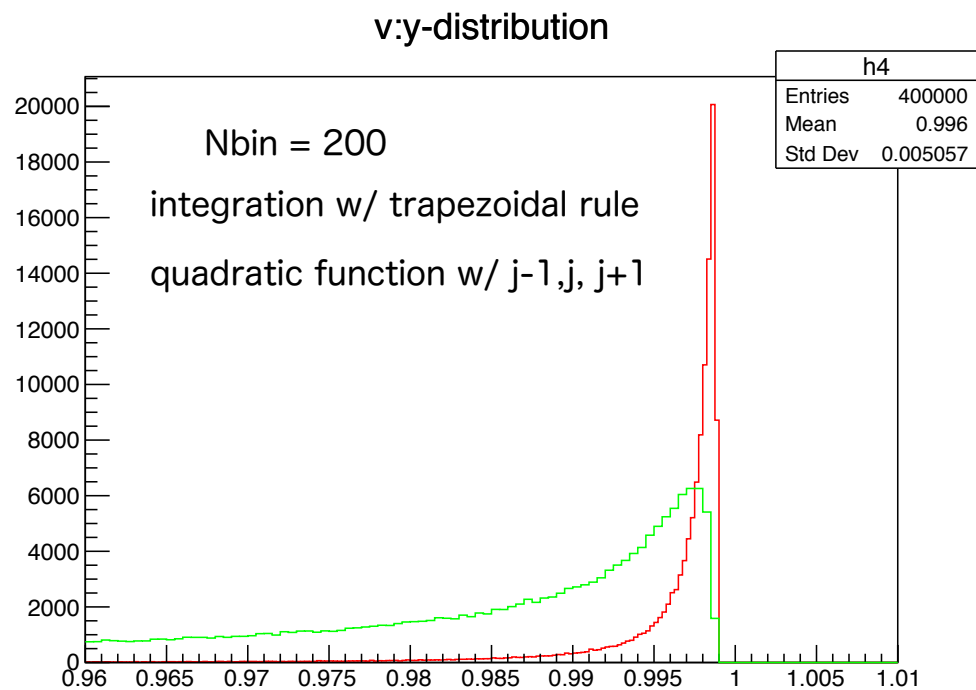


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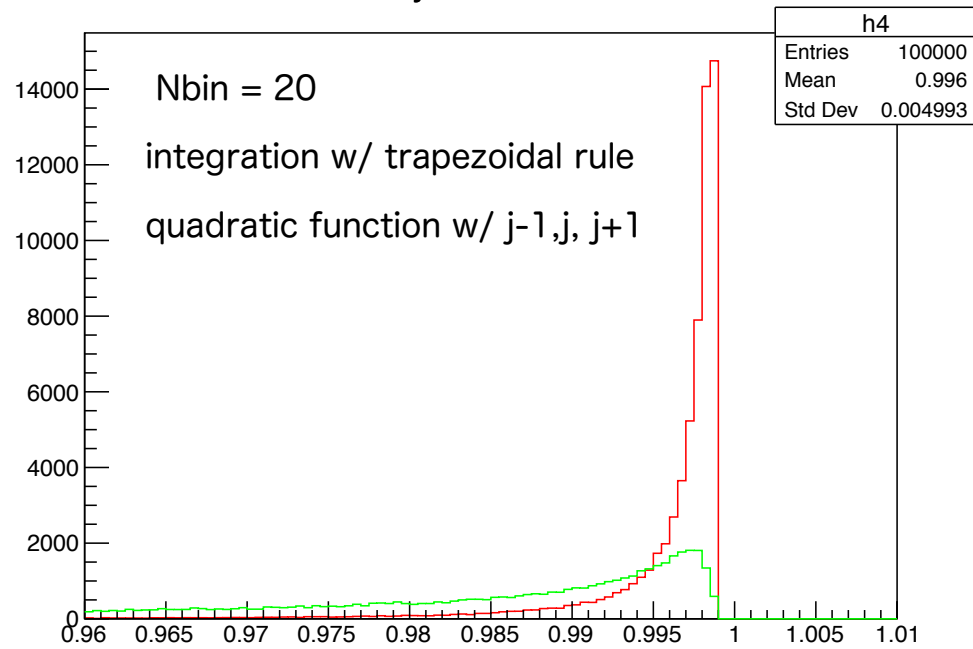


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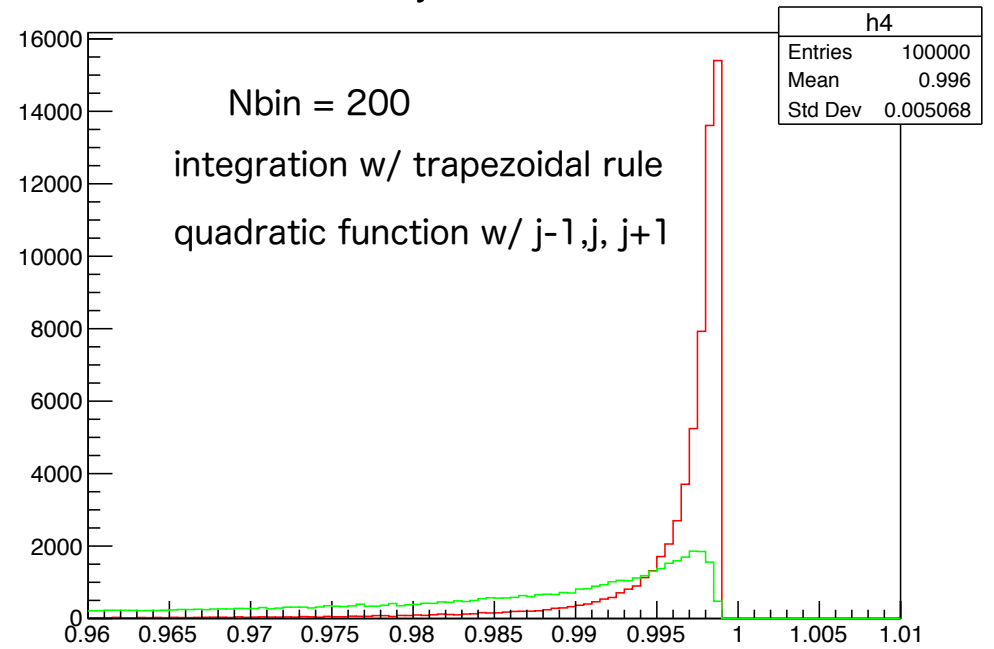




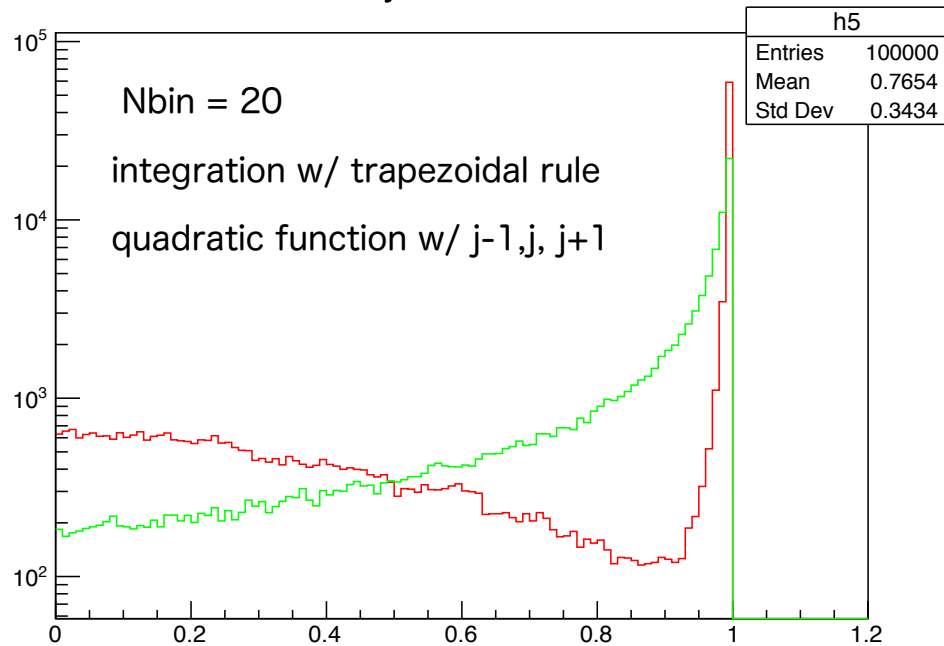
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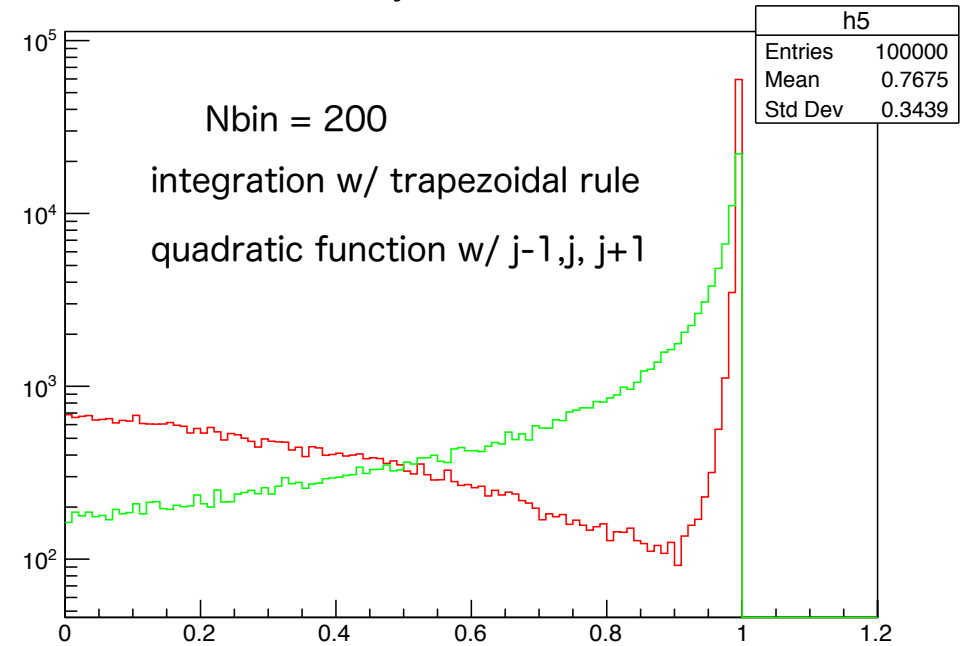
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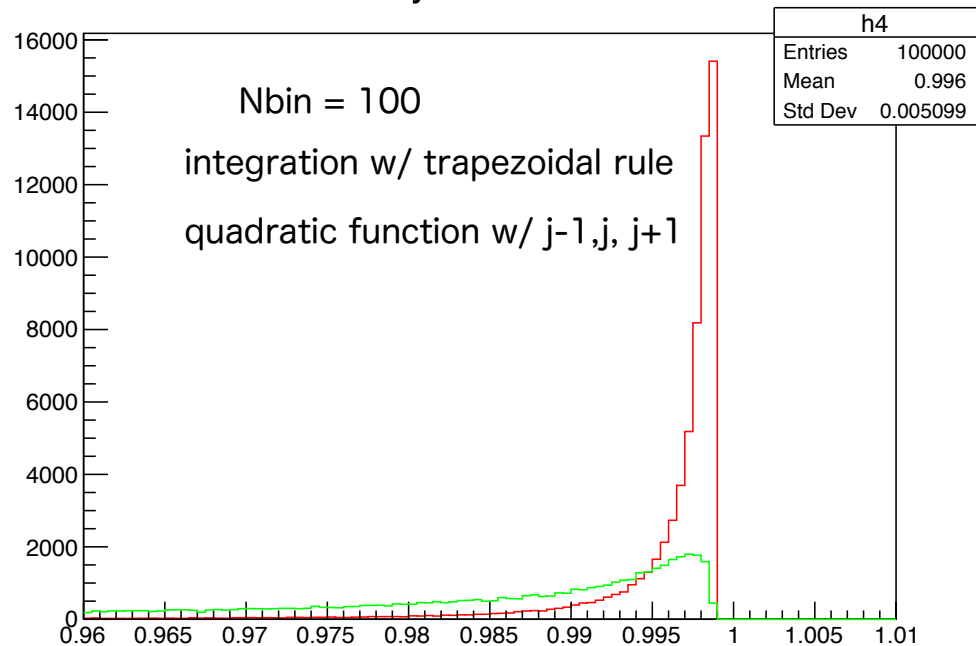
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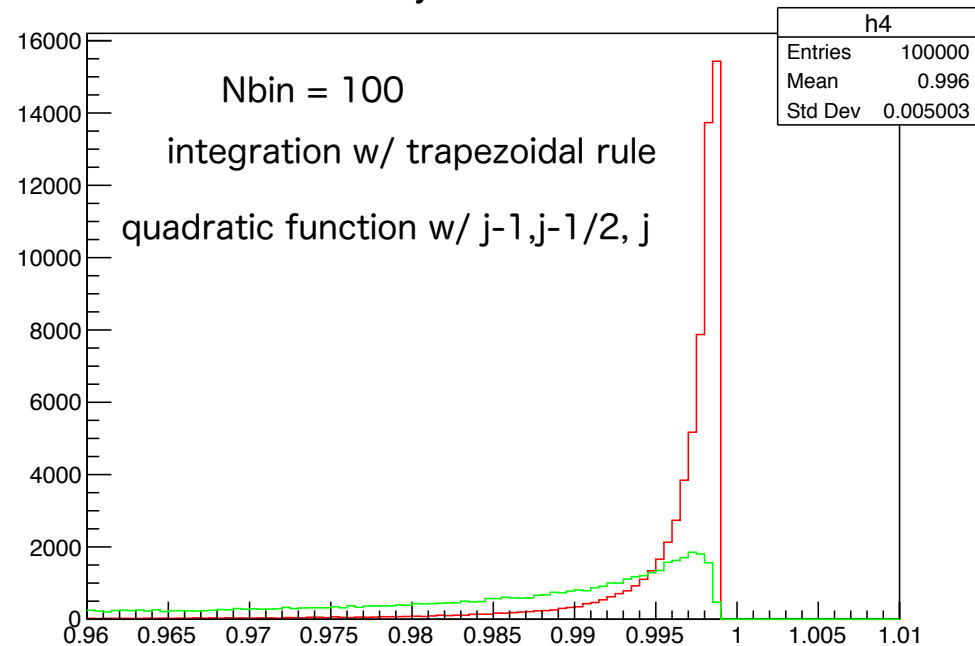
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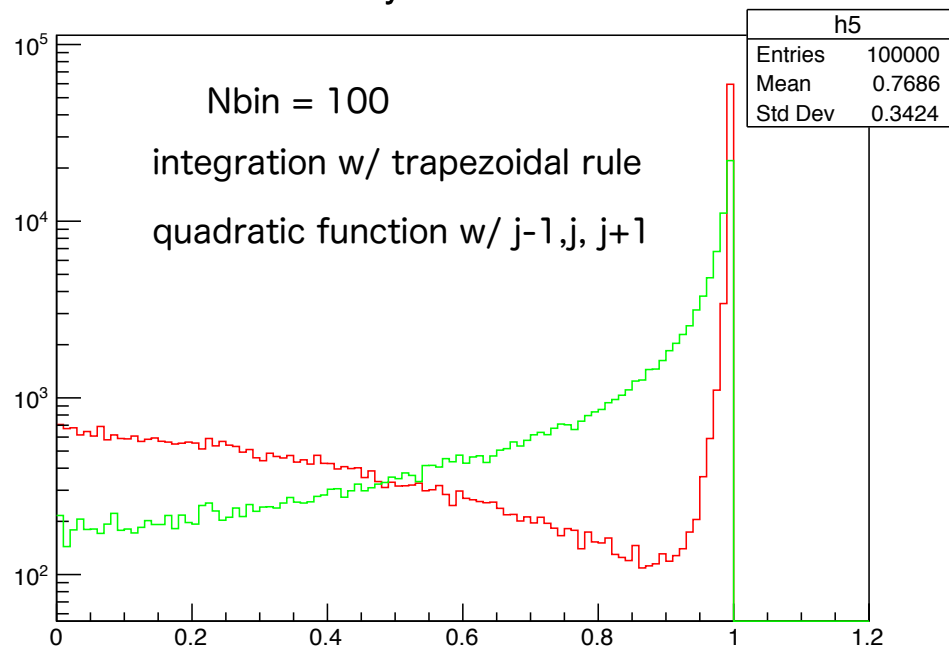
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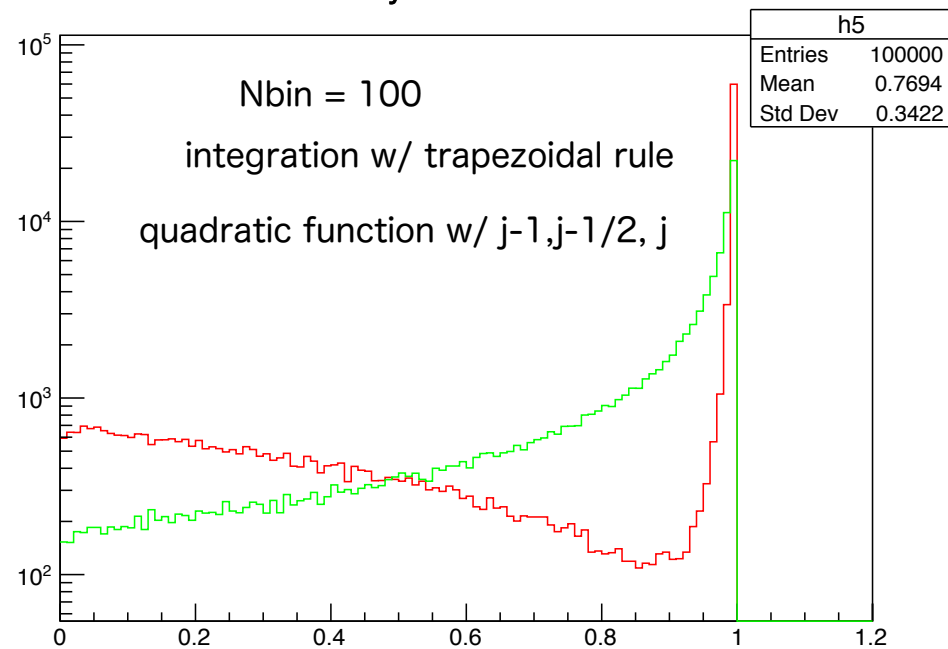
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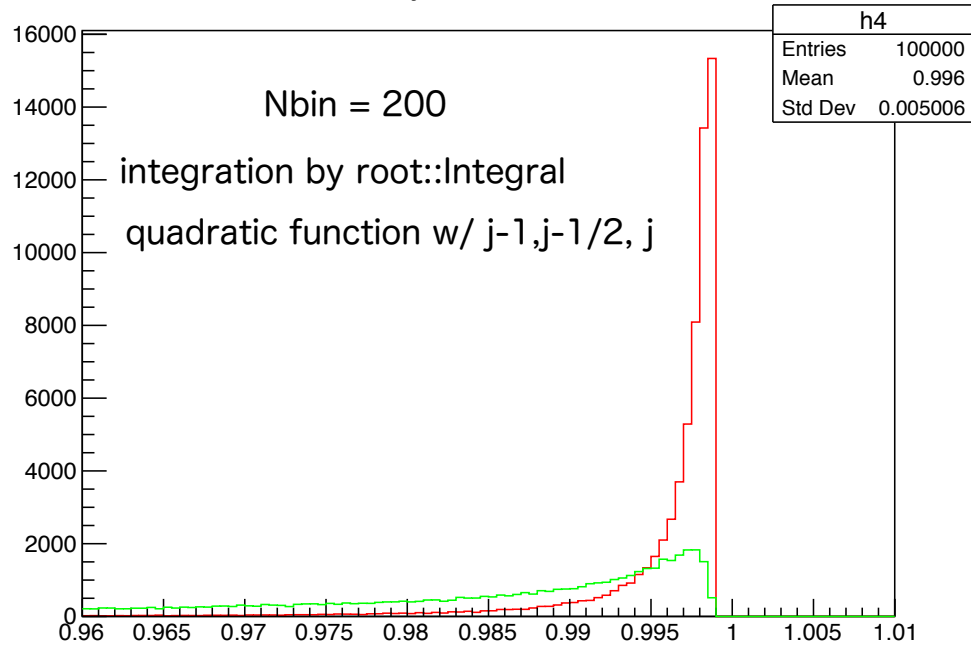
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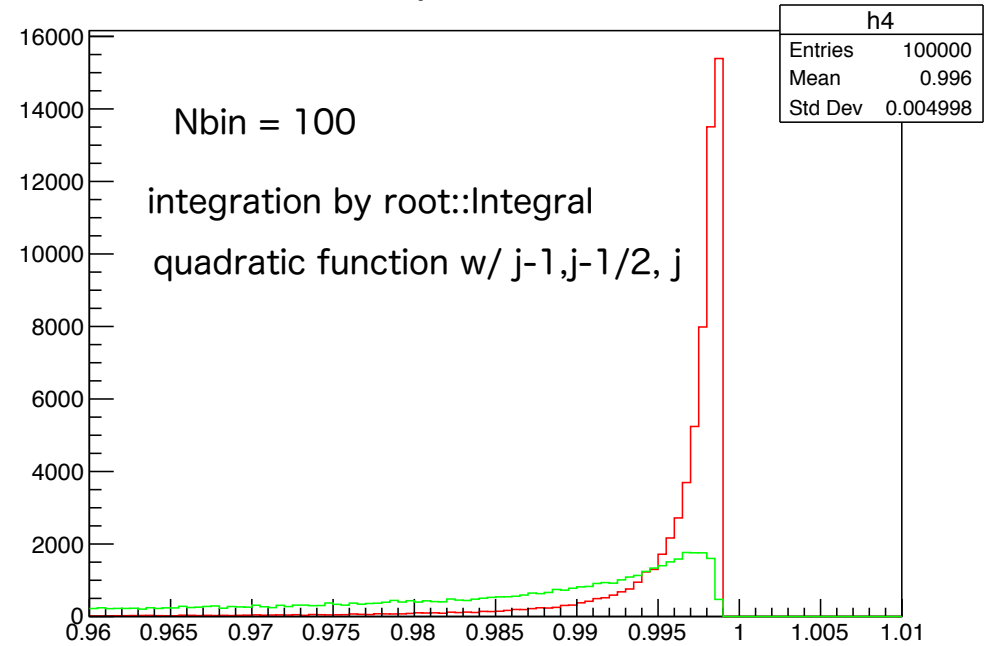
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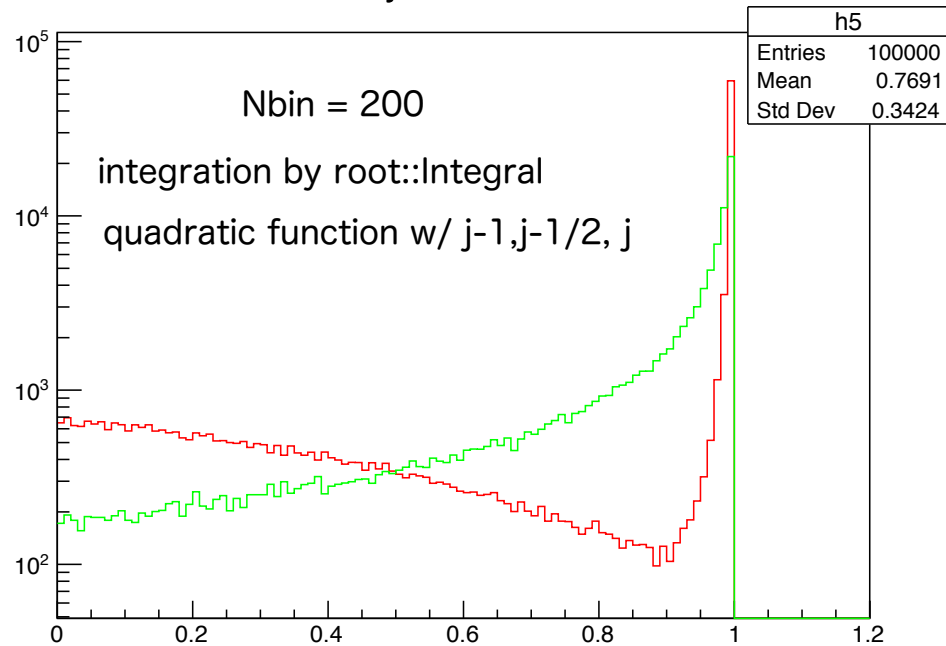
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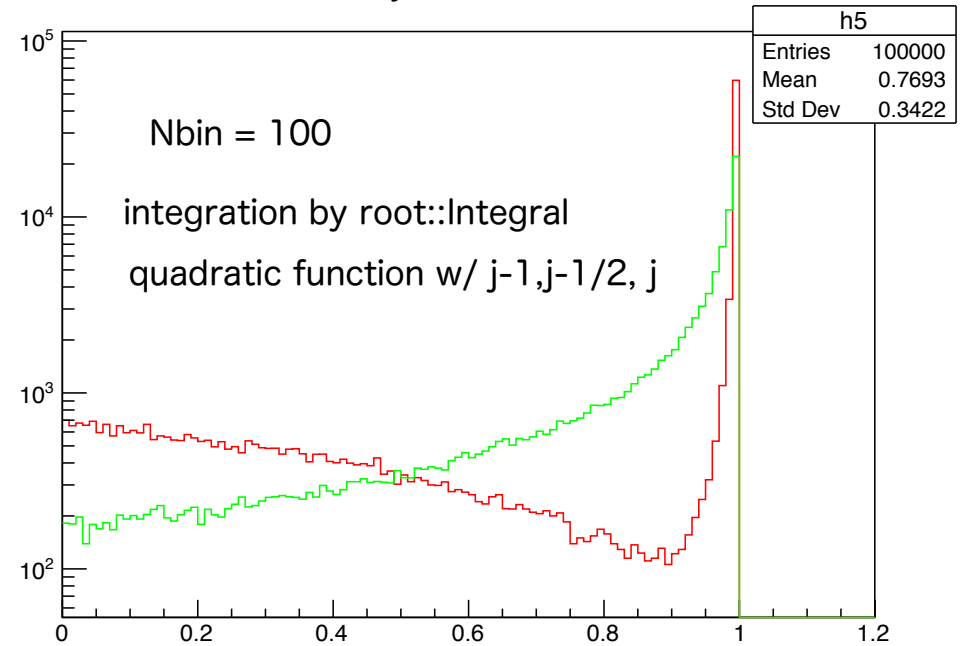
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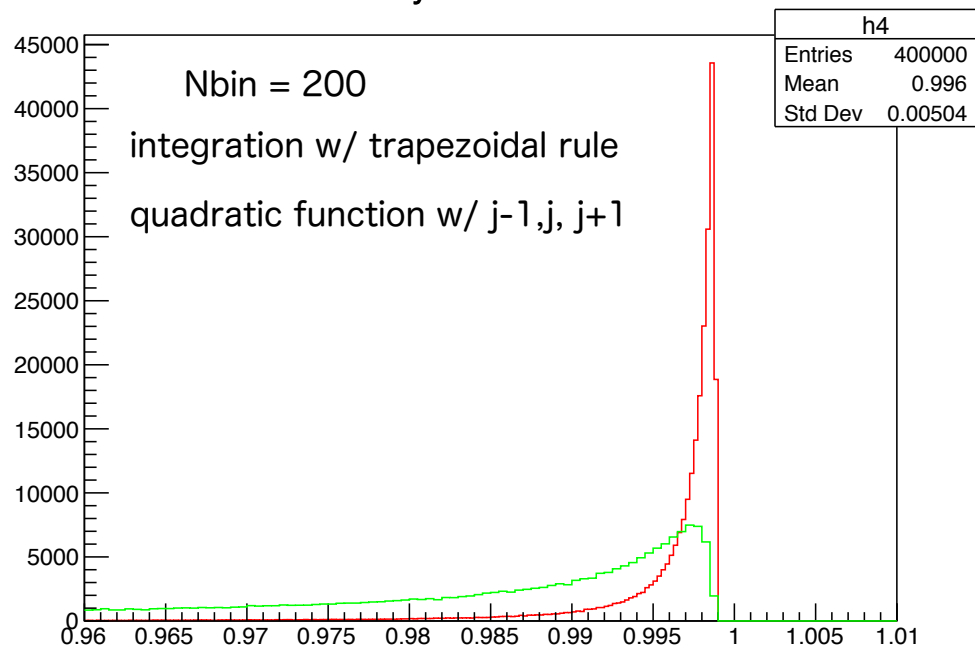
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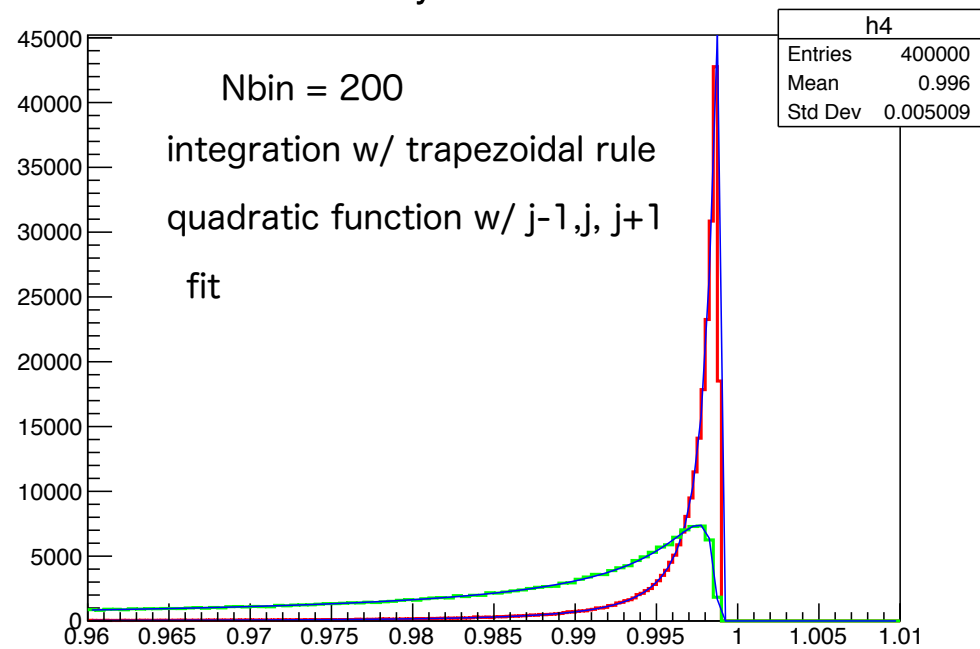
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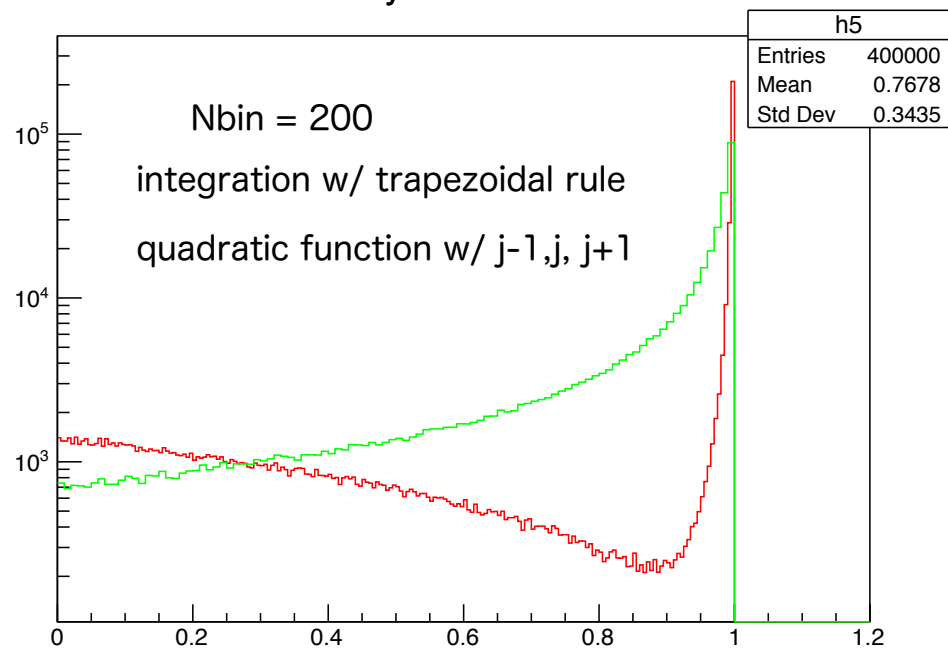
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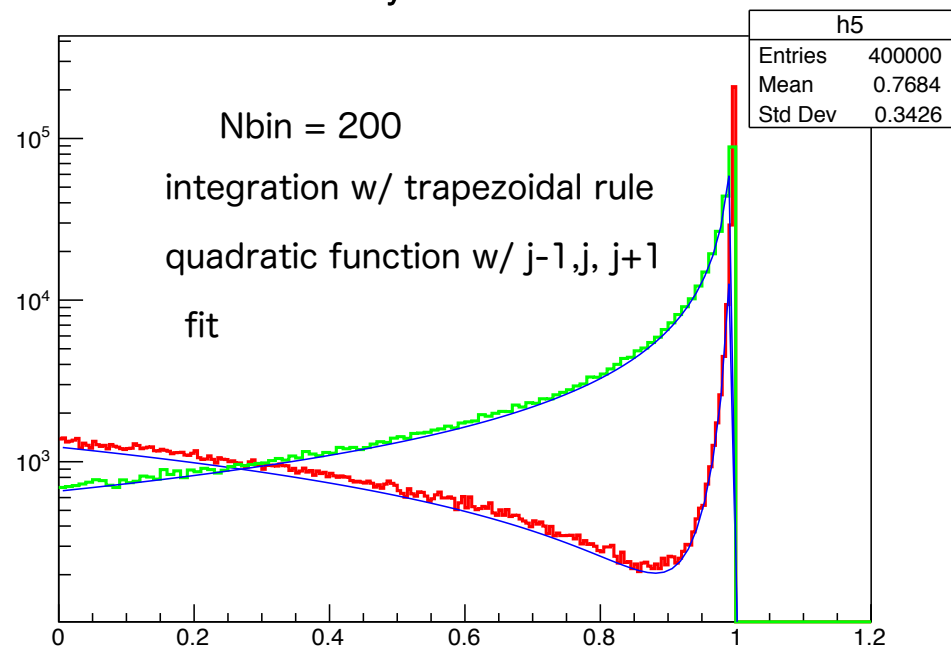
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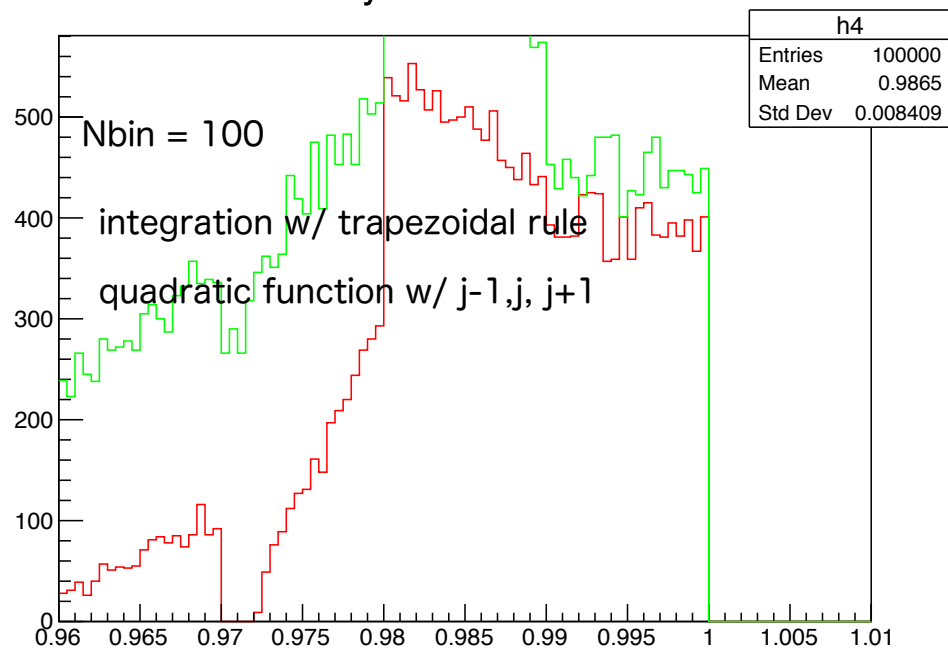
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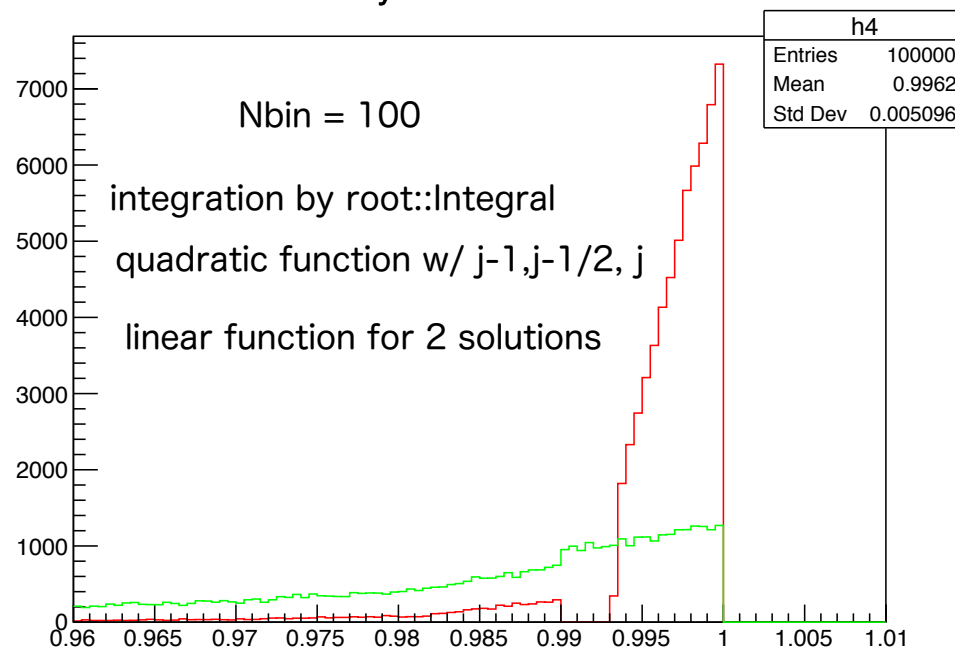
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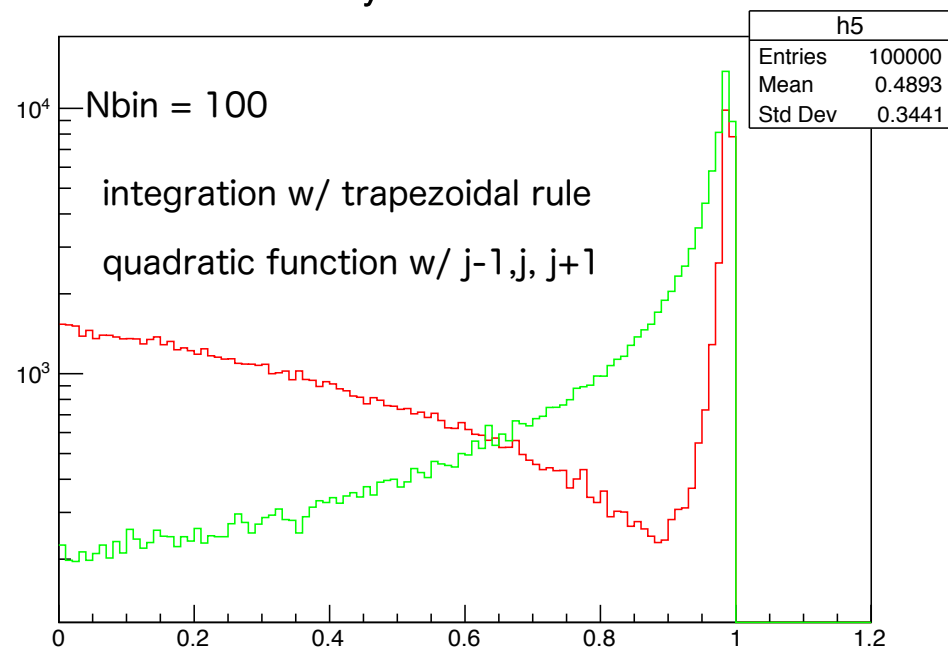
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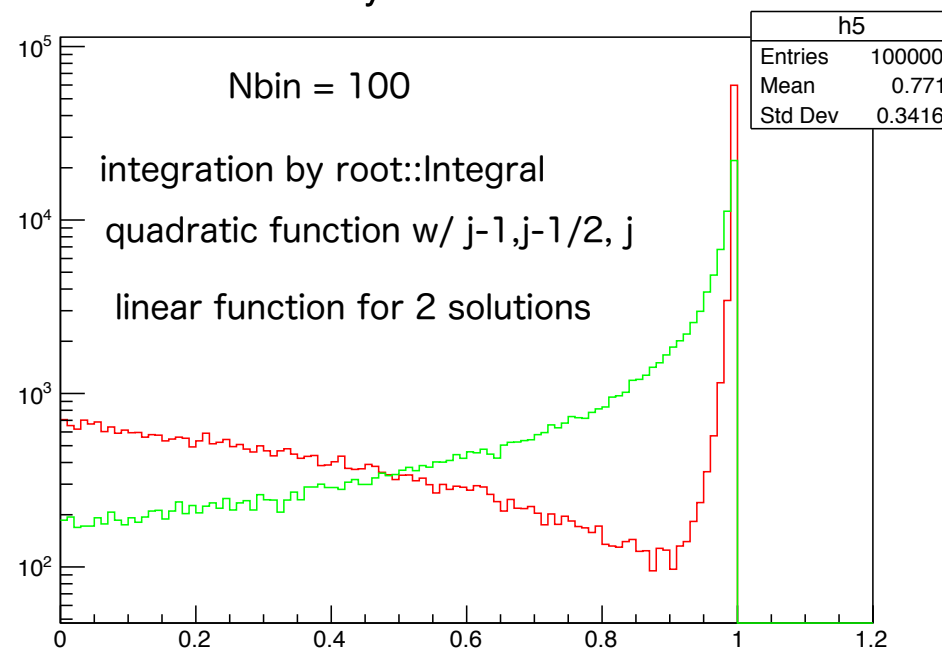
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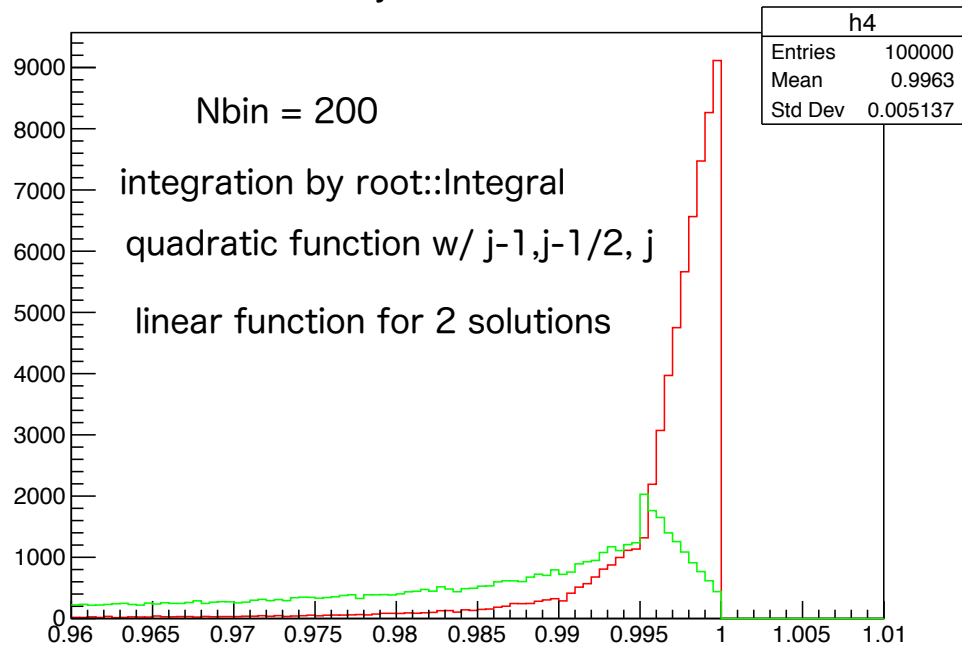
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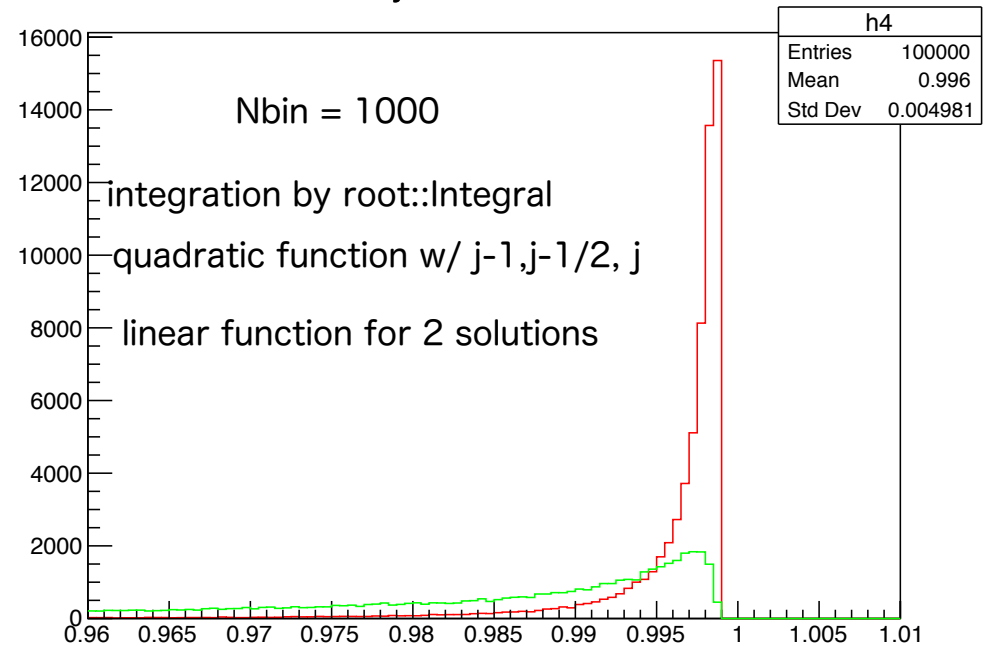
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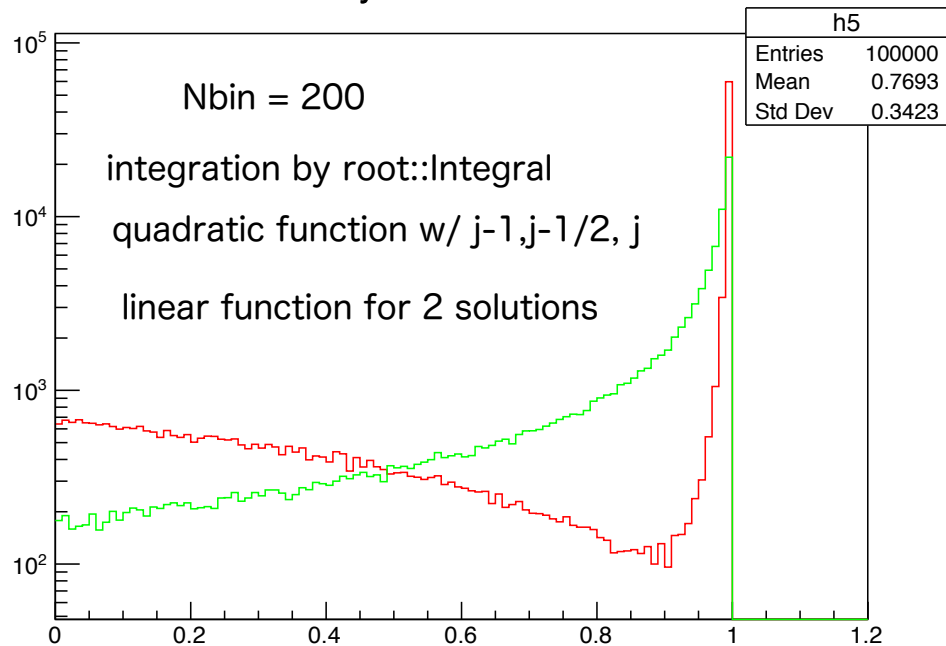
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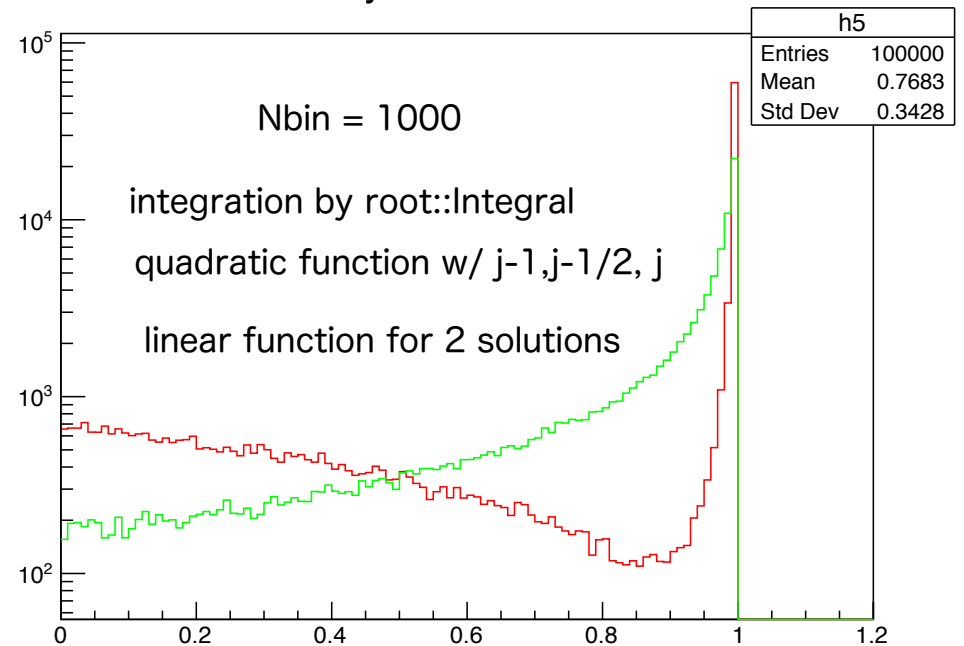
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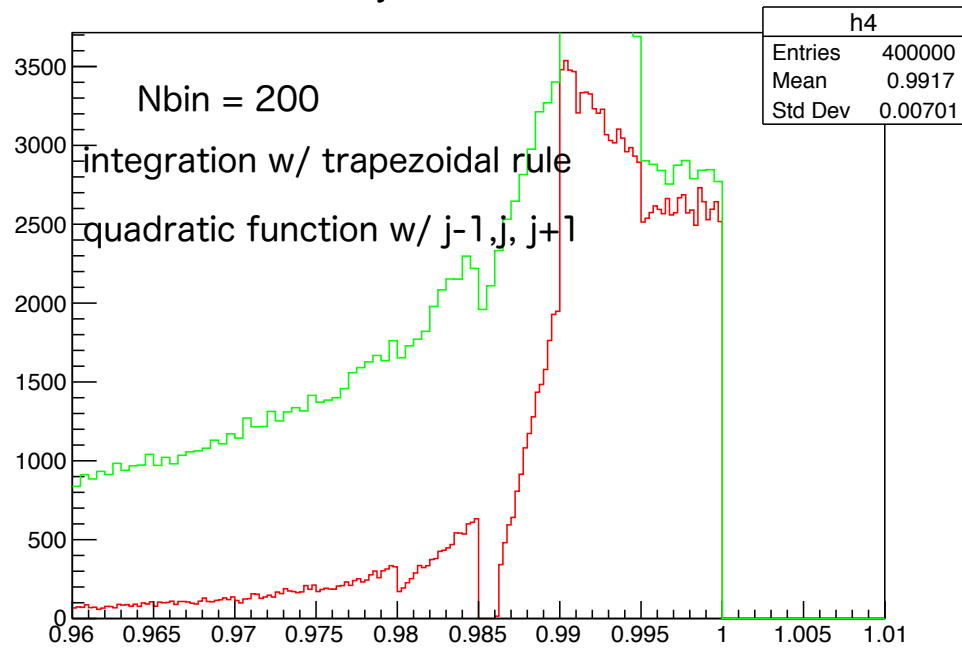
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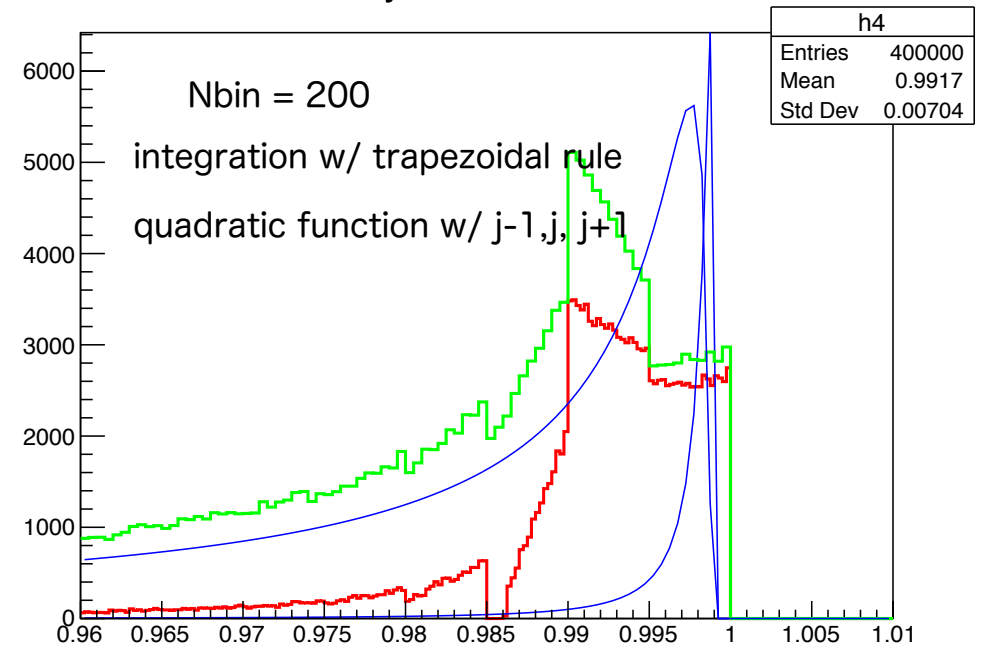
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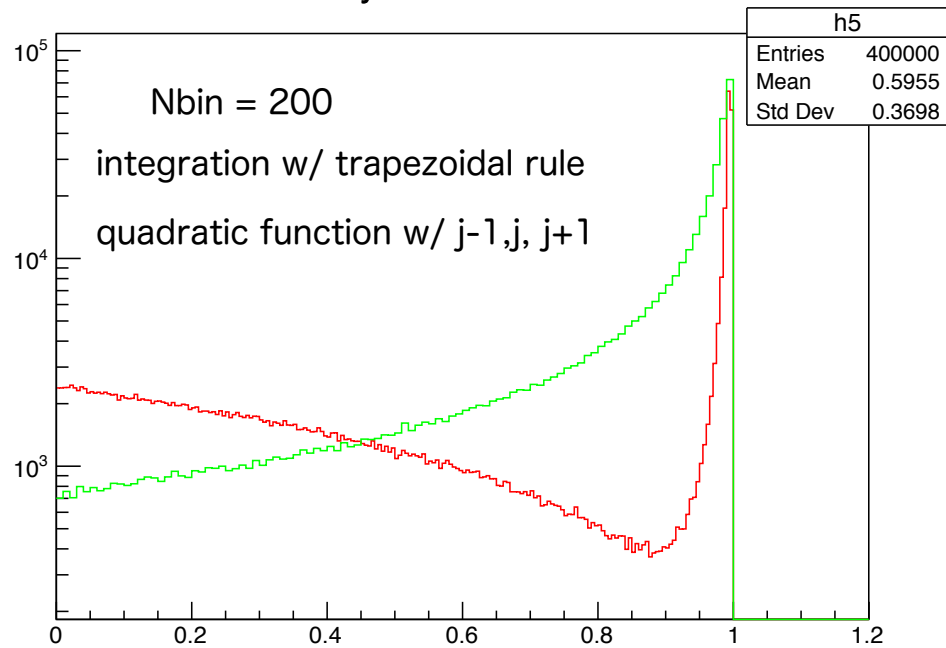
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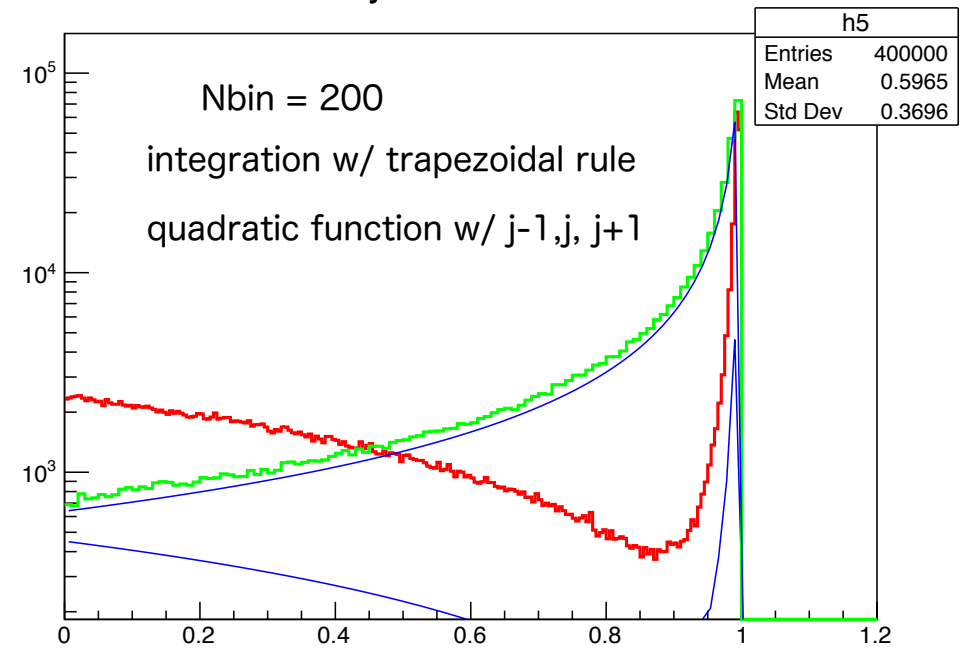
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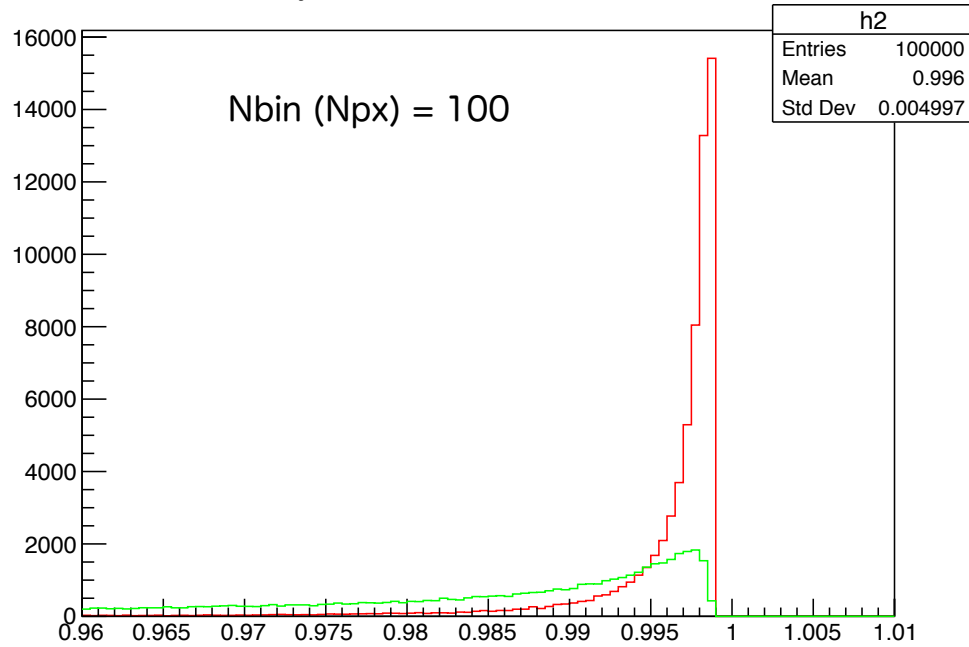


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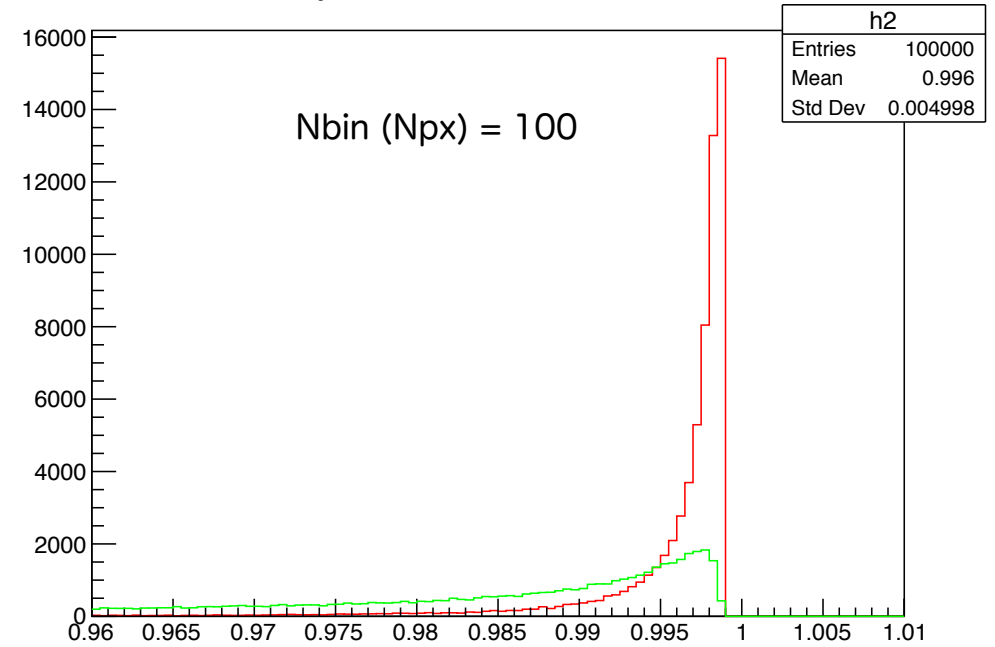


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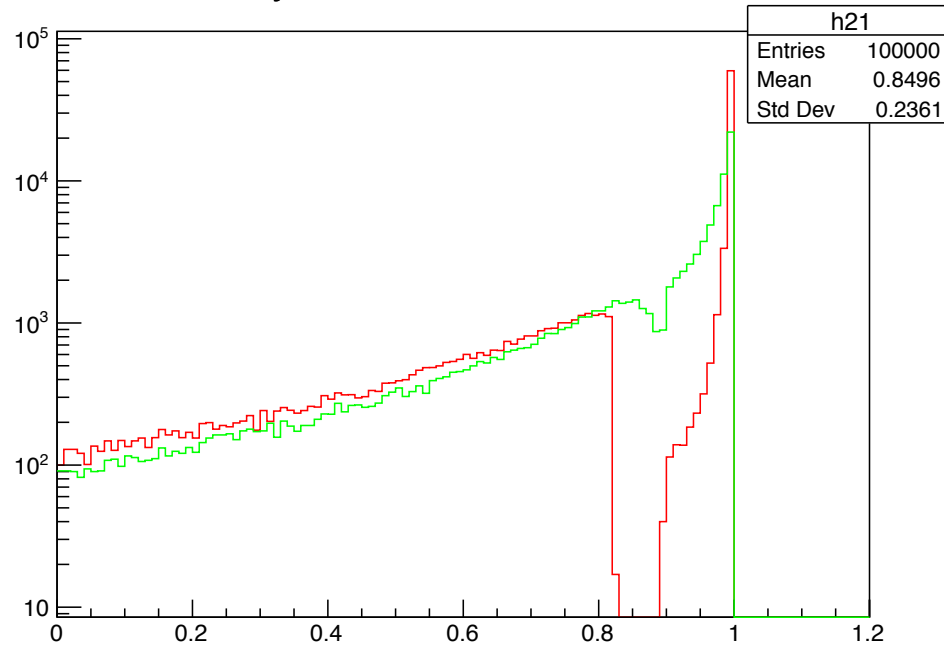
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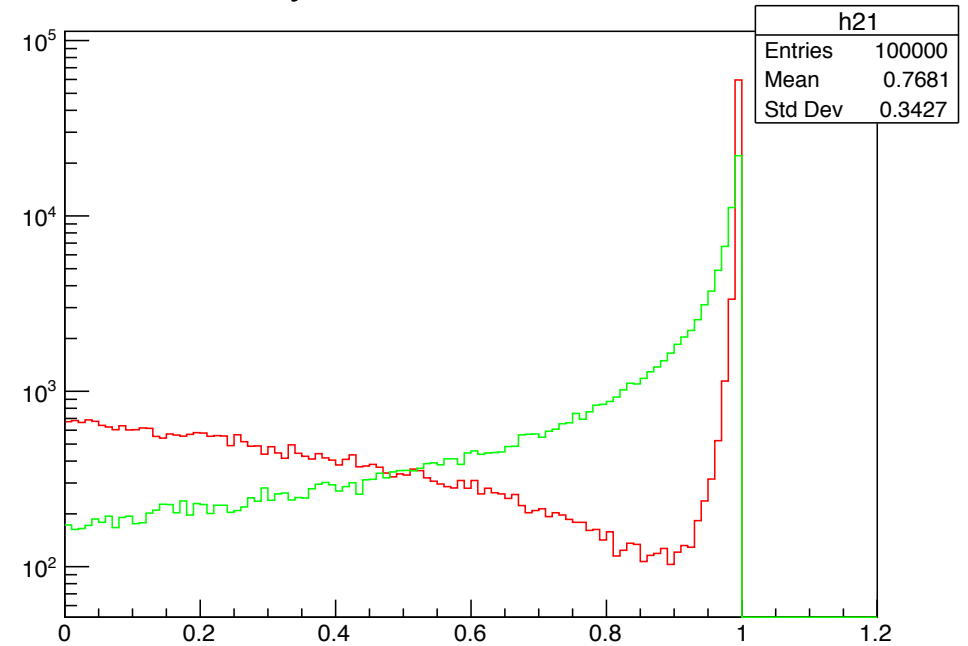
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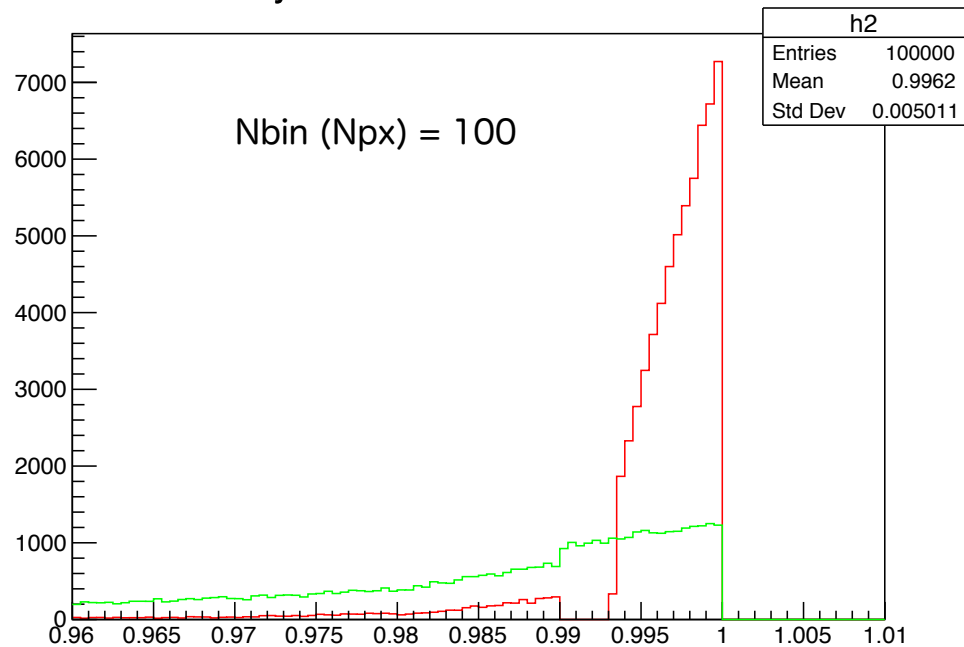


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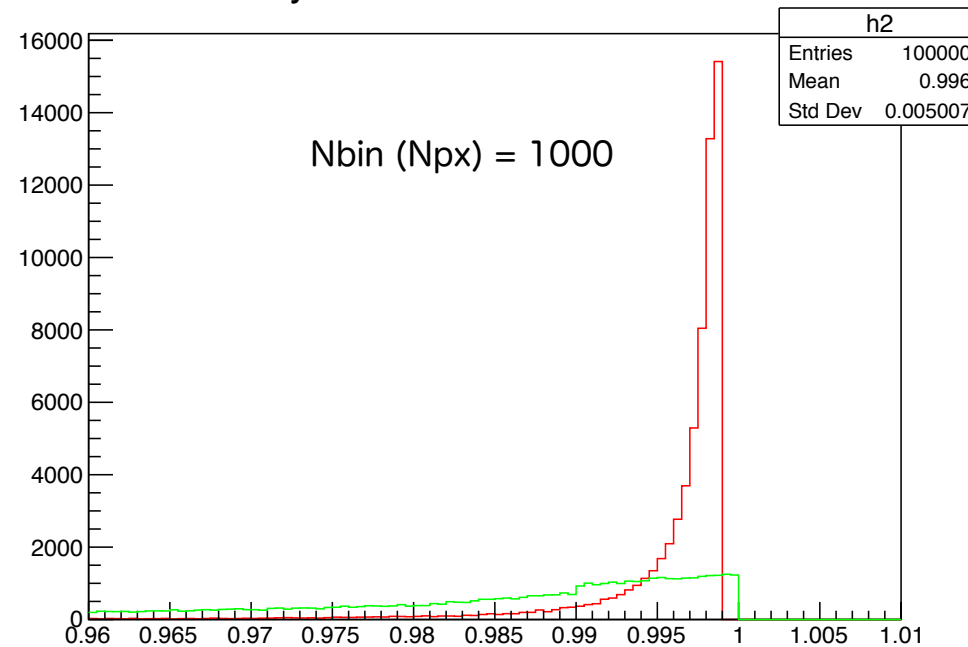


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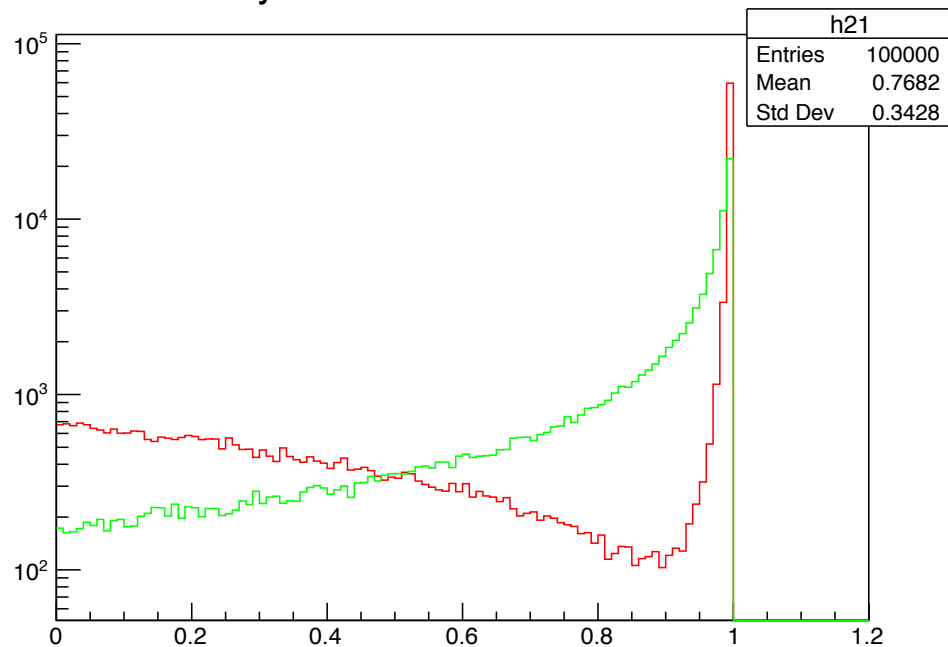
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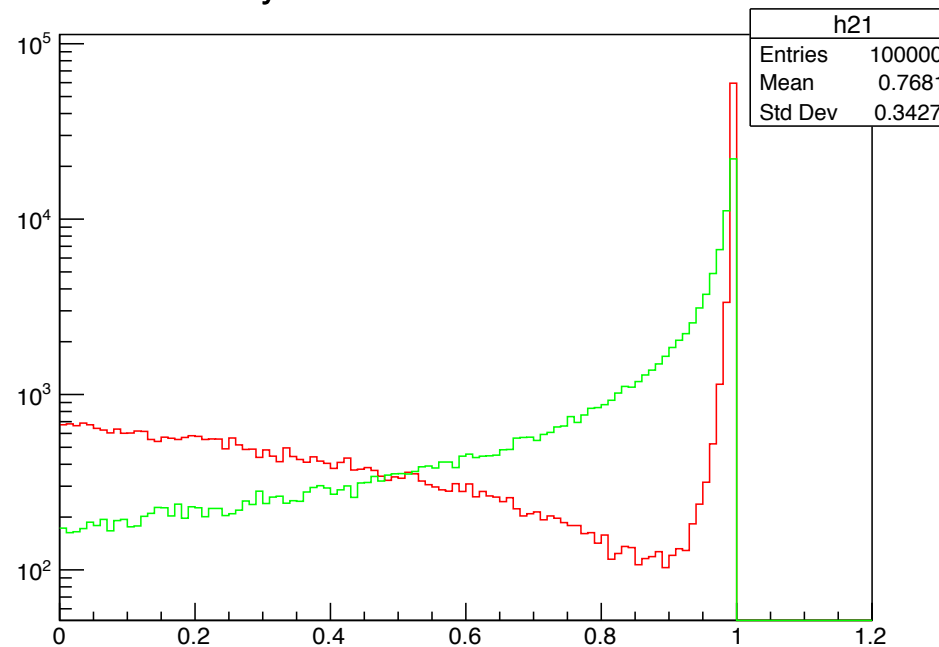
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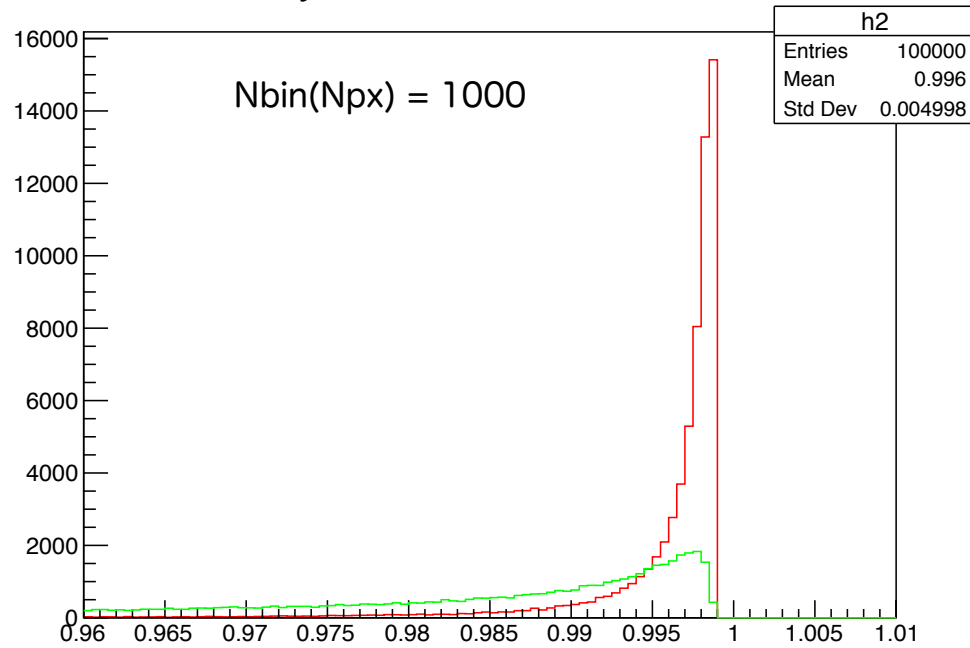


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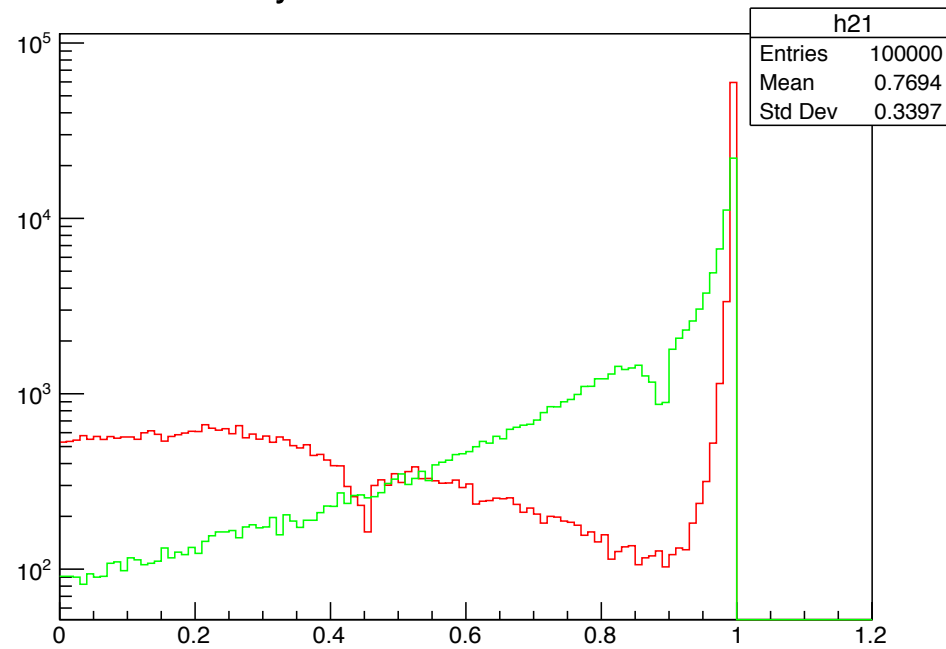


GetRandom()

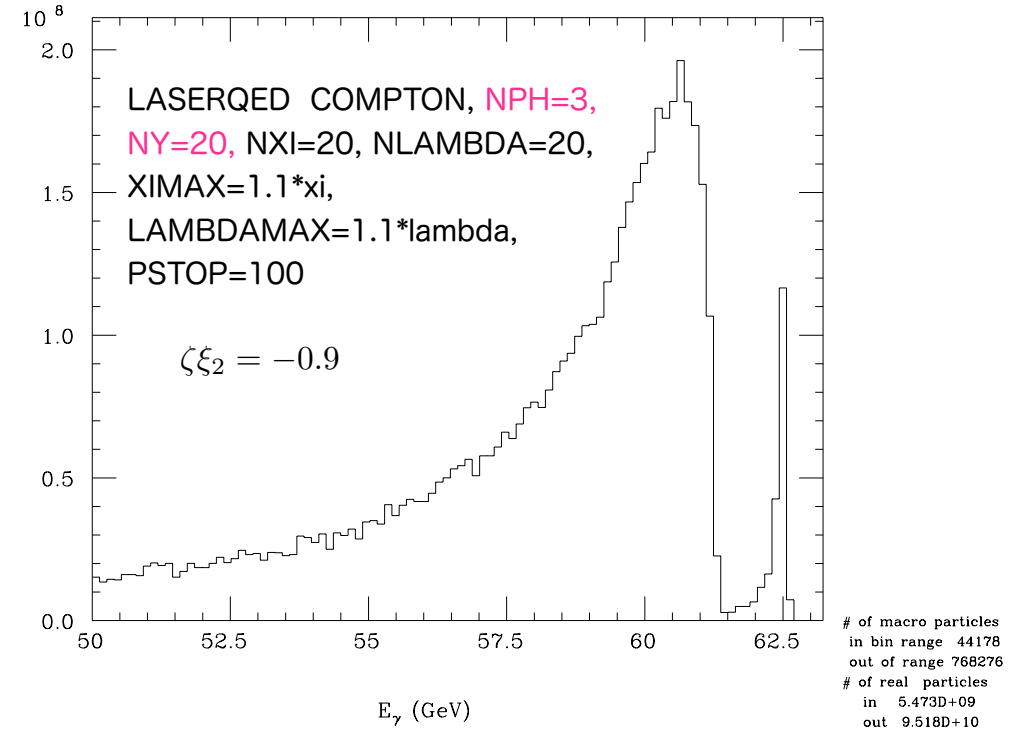
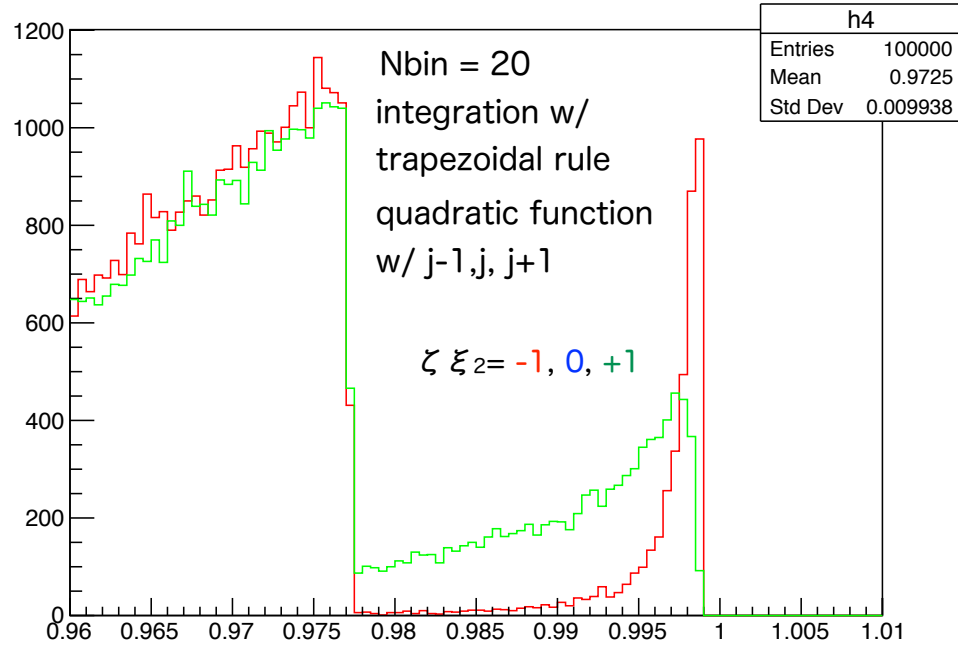
v:y-distribution:GetRandom



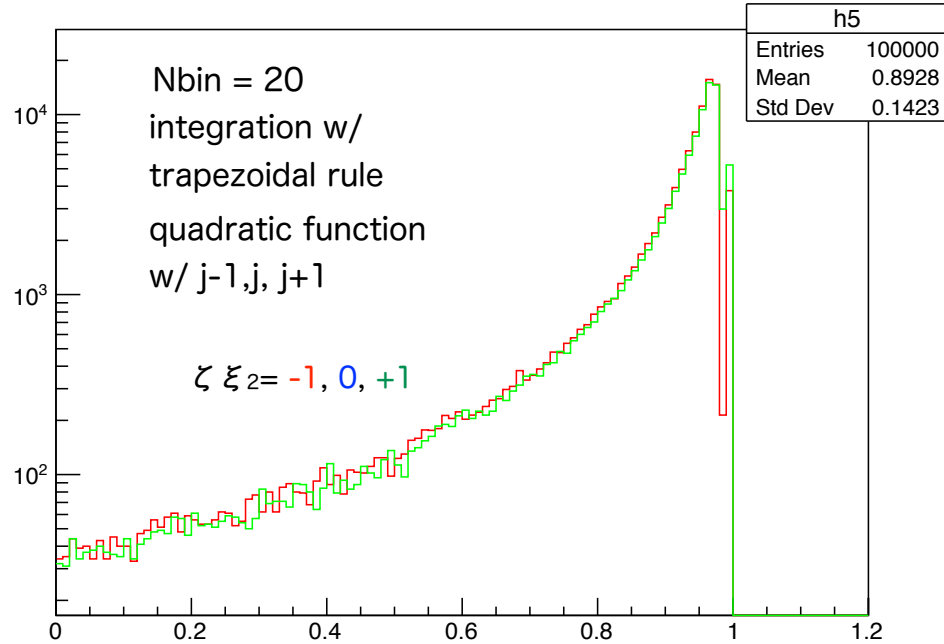
v:y-distribution:GetRandom



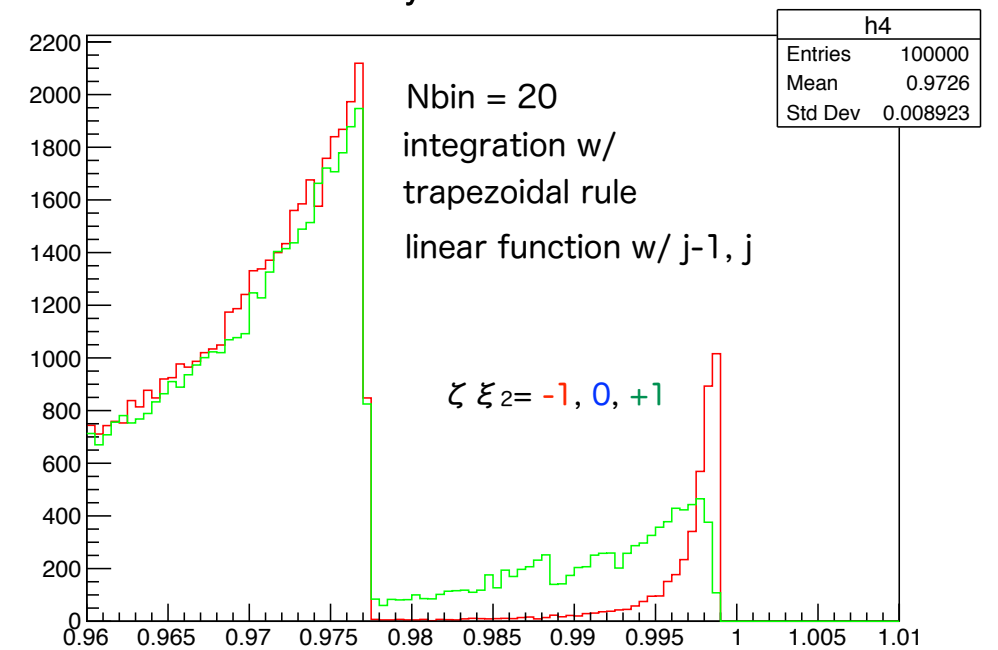
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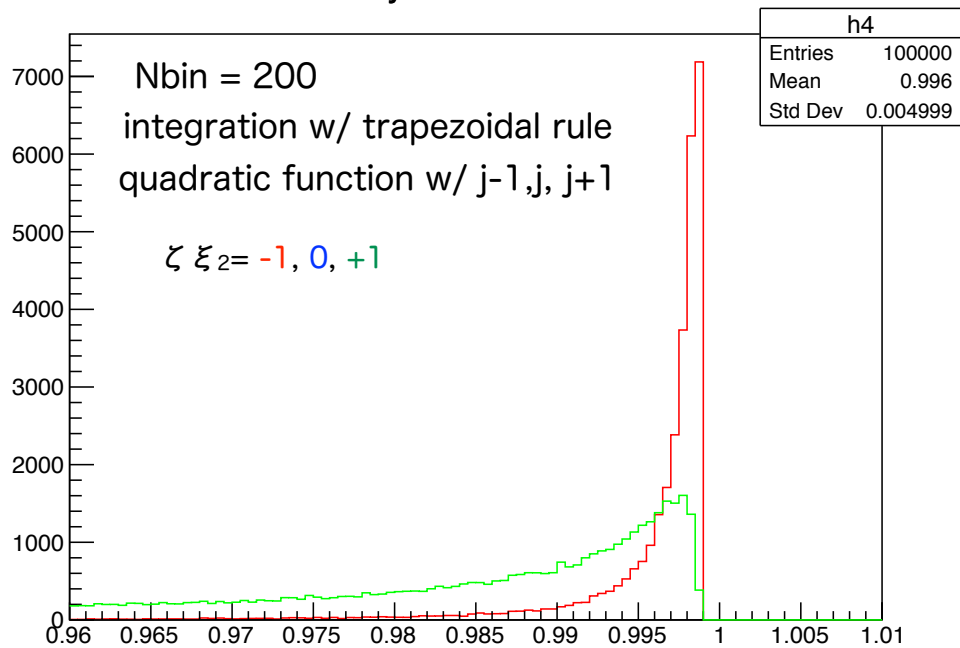
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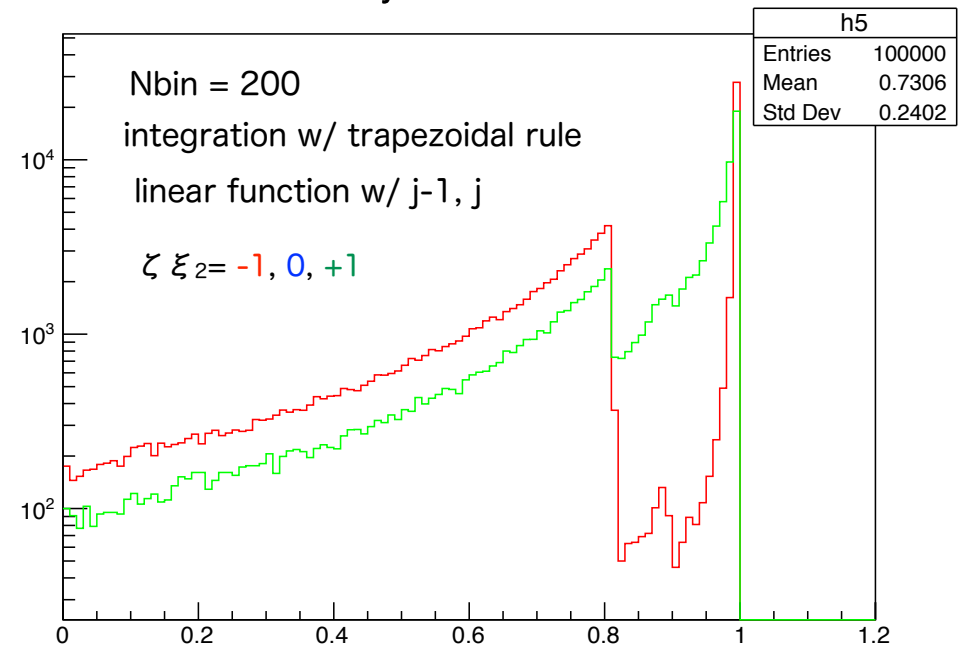
v:y-distribution



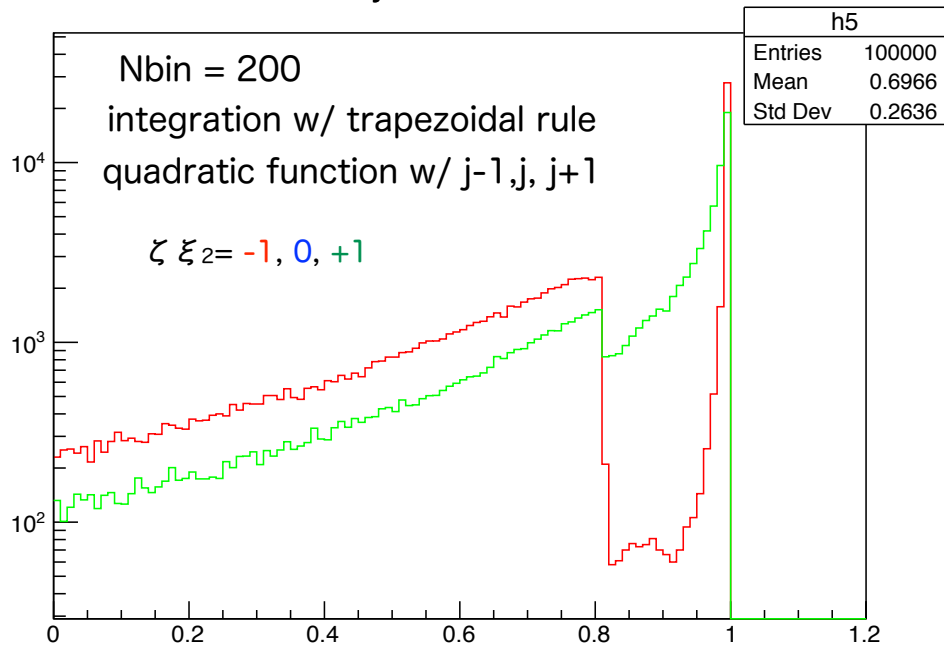
v:y-distribution



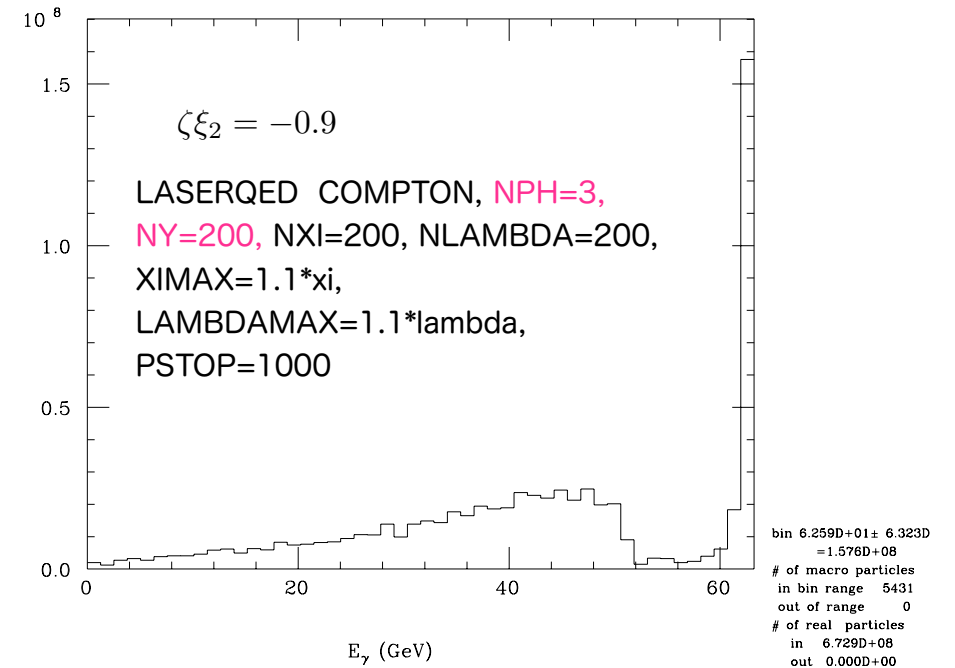
v:y-distribution



v:y-distribution

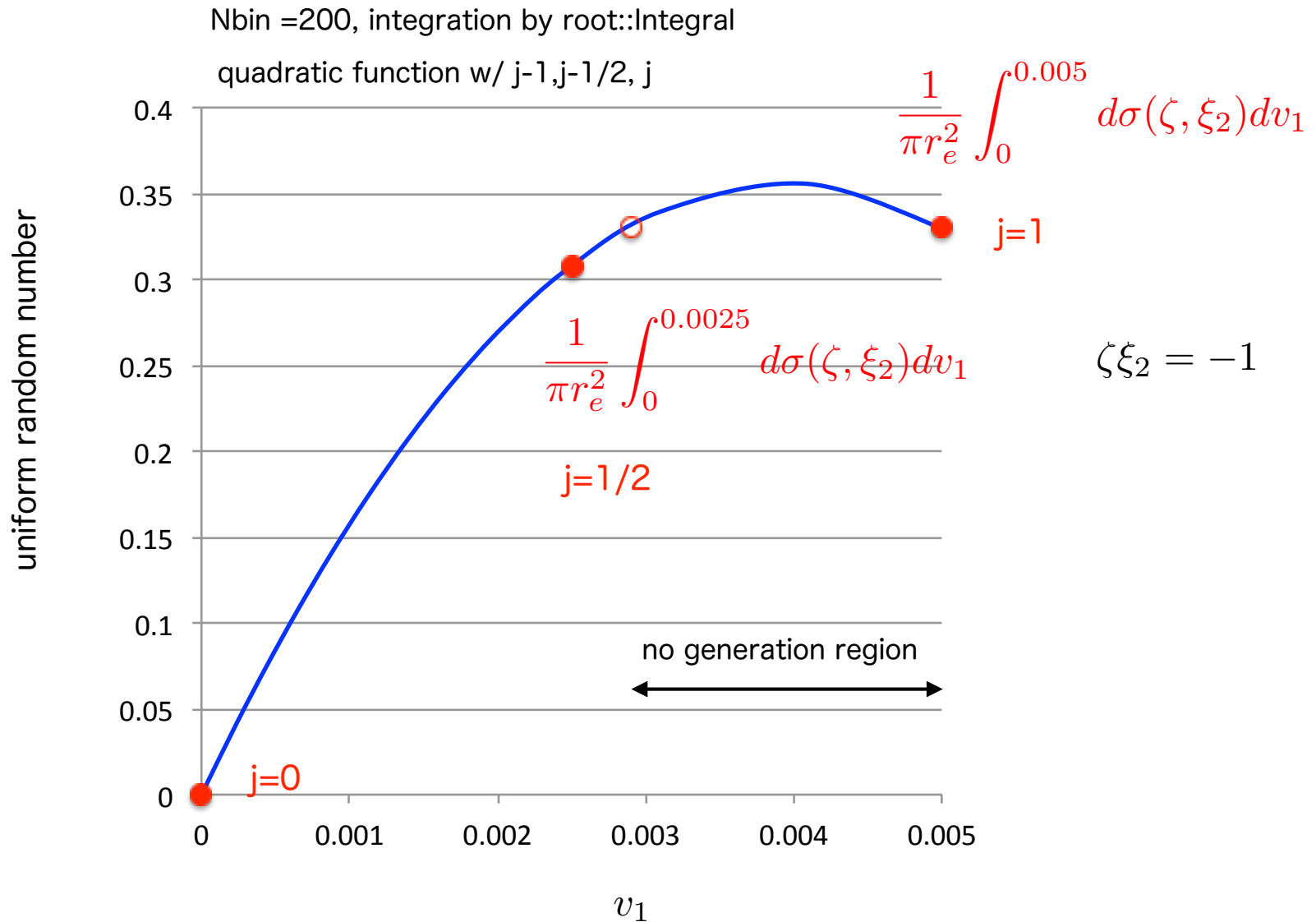
ILC $\gamma\text{-}\gamma$ for a Higgs Factory 3
Right-Going Primary Photon Energy Spectrum after CP

20201228(162821) CAIN2.42



In case of v_n , a serious issue in the interpolation by quadratic function

$$\frac{1}{\pi r_e^2} \frac{d\sigma}{dv_n}(\zeta, \xi_2) = \frac{2}{x\xi^2} (F_{1n} + \zeta\xi_2 F_{2n}) \frac{u_n}{(1+u)^2}$$





(1) $NY=200$ (LASERQED) is not enough. $NY > 1000$ must be used for the correct energy distribution at least.

(2) The variable of z_n , where $n=nph$, must be used as in the case of the linear Compton scattering. With this choice, $NY=20$ should be enough.

I explain my results in each page of appended PDF.

Page 1 : It shows definitions of variables used in the linear Compton scattering, where there are two Lorentz frames to calculate the cross section.

Page 2 : continued. It also shows z -variable which is used in the CAIN linear Compton scattering.

Page 3 : It shows definitions of variables proposed for the non-linear Compton scattering, where n photons collide with a slightly massive electron (m^*) in the strong laser field. The cross section is a sum of n . The cross sections are expressed as a function of y or z_n . It also shows relations to the linear Compton scattering case with the z variable, which are straight forward relations.

reference : M.Galynskii, E.Kuraev, M.Levchuk, V.Telnov, NIM A472(2001) 267-279

Page 4 : It shows the variables (u, u_n, v_n) used for the non-linear Compton scattering in CAIN. The cross section is shown as a function of v_n , which is compared with the two expressions in the page-3. Direct relation of our variables to ones in CAIN is also shown.

Page 5 : It shows the ComptonFunc's of the linear and non-linear Compton scattering cross section as a function of y and helicities of the incident electron and photon. The ComptonFunc is usually the cross section normalized by πr_e^2 , where r_e is the classical electron radius except for the one with z_n which is further normalized by $u_n/(4 L_{un})$ as shown in the page-6. ($n=1$)

Page 6 : It shows the ComptonFunc's as a function of y and z_n as well as the two helicities, where the bottom figures are shown with expanded scales (y and z_n). ($n=1$)

Page 7 : It shows the ComptonFunc's as a function of v_n as well as the two helicities, where the bottom figures are shown with expanded scale (v_n). ($n=1$)

Following pages show results by simulations with ROOT 6.14/06 (built for macosx64, <http://root.cern.ch>). The three variables, i.e. v_n , z_n and y , have been used for the calculations as the random number in 0 to 1. Events have been generated with the corresponding Compfunc with $n=1$, where the ComptonFunc is integrated over the channel contents in each bin ($j= 1$ to N_{bin} and $N_{bin}=N_Y$ in CAIN) then it is normalized to 1. Two methods are used for the integration, which are one with trapezoidal rule same as the CAIN and root::Integral of TF1 in ROOT. The random number is calculated following this ComptonFunc. For each bin the integral is approximately by a quadratic (parabola) or a linear functions. For the case of parabola, two methods are used to calculated the parabola coefficients which are $(j-1, j, j+1)$ and $(j-1, j-1/2, j)$ where the uniform random number is located between $(j-1)$ and (j) . After the variable is transformed to y , all the results are shown as a function of y .

Page 8 : In case of v_n : it shows the photon energy(y) distributions with $N_{bin}=200$, both the integrations and the parabola defined by $(j-1, j, j+1)$, where red and green colors correspond to the helicities in the same way as previous pages.

Page 9 : In case of v_n : it shows the photon energy(y) distributions with $N_{bin}=200$ and the parabola defined by $(j-1, j-1/2, j)$ and ones with linear function in case of allowed double solutions along the parabola in order to study a depleted region of $0.7 < y < 0.8$.

Page 10 : In case of v_n : it shows the photon energy(y) distributions with $N_{bin}=1000$, root::Integral and the parabolas defined by $(j-1, j, j+1)$ and $(j-1, j-1/2, j)$. Both the shapes are close to the analytic formulas (correct distributions) in Page-5. However, there is a small depleted region in both.

Page 11 : In case of v_n : it shows ones with linear function to see any improvement. It also shows the distribution with $N_{bin}=20$, integration with trapezoidal rule and the parabola by $(j-1, j, j+1)$ which must be same conditions as in the CAIN-default parameters.

Page 12 : In case of v_n : it shows the distribution with $N_{bin}=200$, integration with trapezoidal rule and the parabola by $(j-1, j, j+1)$ which must be same as the CAIN with $N_Y=200$. Right figures show fitted lines by the "correct" function, where this fit failed for $y < 0.9$.

Page 13 : In case of z_n : it shows the distributions with $N_{bin}=20$ and $N_{bin}=200$, the integration with trapezoidal rule and the parabola by $(j-1, j, j+1)$. Both are very close to the analytic formulas (correct distributions) in Page-5.

Page 14 : In case of z_n : it shows the distributions with $N_{bin}=100$, the integration with trapezoidal rule and the parabolas by $(j-1, j, j+1)$ and $(j-1, j-1/2, j)$. There is no difference and both are very close to the analytic formulas (correct distributions) in Page-5.

Page 15 : In case of z_n : it shows the distributions with $N_{bin}=200$ and $N_{bin}=100$, the root::integration and the parabolas by $(j-1, j-1/2, j)$. Both are very close to the analytic formulas (correct distributions) in Page-5.

Page 16 : In case of z_n : it shows the distribution with $N_{bin}=200$, integration with trapezoidal rule and the parabola by $(j-1, j, j+1)$ Right figures show nicely fitted lines by the "correct" function.

Page 17 : In case of y : it shows the distribution with $N_{bin}=100$, integration with trapezoidal rule, the parabola by $(j-1, j, j+1)$ and the root::integral, the parabola by $(j-1, j-1/2, j)$ Both have wrong distributions close to $y=1$.

Page 18 : In case of y : it shows the distribution with $N_{bin}=200$ and $N_{bin}=1000$, the `root::integral`, the parabola by $(j-1, j-1/2, j)$ with linear function in case of allowed double solutions along the parabola. The distribution with $N_{bin}=1000$ is close to the correct one.

Page 19 : In case of y : it shows the distribution with $N_{bin}=200$, integration with trapezoidal rule and the parabola by $(j-1, j, j+1)$ Right figures show fitted lines with failure by the "correct" function.

Page 20 : In cases of v_n and z_n : It shows distributions calculated by `Root::GetRandom()` with the default bin number of 100. The z_n distribution shows the correct one,, while the v_n shows the depleted region in the same way as our calculations. Actually, the `Root::GetRandom()` calculations are very similar to ours with $N_{bin}=N_{px}$, `root::Integral` and the parabola by $(j-1, j-1/2, j)$.

Page 21 : In cases of y : It shows distributions calculated by `Root::GetRandom()` with $N_{px}=100$ and $N_{px}=1000$. It also show that one with $N_{px}=1000$ reproduces the correct distribution.

Page 22 : In cases of v_n : It shows distributions calculated by `Root::GetRandom()` with $N_{px}=1000$. It is very similar to right figures in page-10.

Page 23 : The CAIN default distribution (top-right figure) is compared with the v_n - y distributions with $N_{bin}=20$, the integration with trapezoidal rule, the parabola by $(j-1, j, j+1)$ in order to see reproducibility of our calculations with the same conditions. It seems to be a good reproducibility .

Page 24 : The CAIN distribution with $N_Y=200$ (bottom-right figure) is compared with the v_n - y distributions with $N_{bin}=200$, the integration with trapezoidal rule, the parabola by $(j-1, j, j+1)$ in order to see reproducibility of our calculations with the same conditions. It seems to be a good reproducibility .

Page 25 : In case of v_n , there is a serious issue in the interpolation by quadratic function as shown in a figure which is an example with $N_{bin}=200$, the `root::Integral`, the parabola by $(j-1, j-1/2, j)$, $j=1$. As the `CompFunc` has a very sharp peak at $v_1=0$ (page-7), the parabola is not good approximation in this bin with $j=1$ where there is two allowed solutions for $0.005 > v_1 > 0.0029$. Since the first solution is chosen to determine v_1 , values in this region are not generated so that it shows a depleted region in the distribution. In such case, two linear functions are defined for $(j-1) - (j-1/2)$ and $(j-1/2) - (j)$. However, no good distributions were generated as shown in previous pages unless $N_{bin}>1000$.
