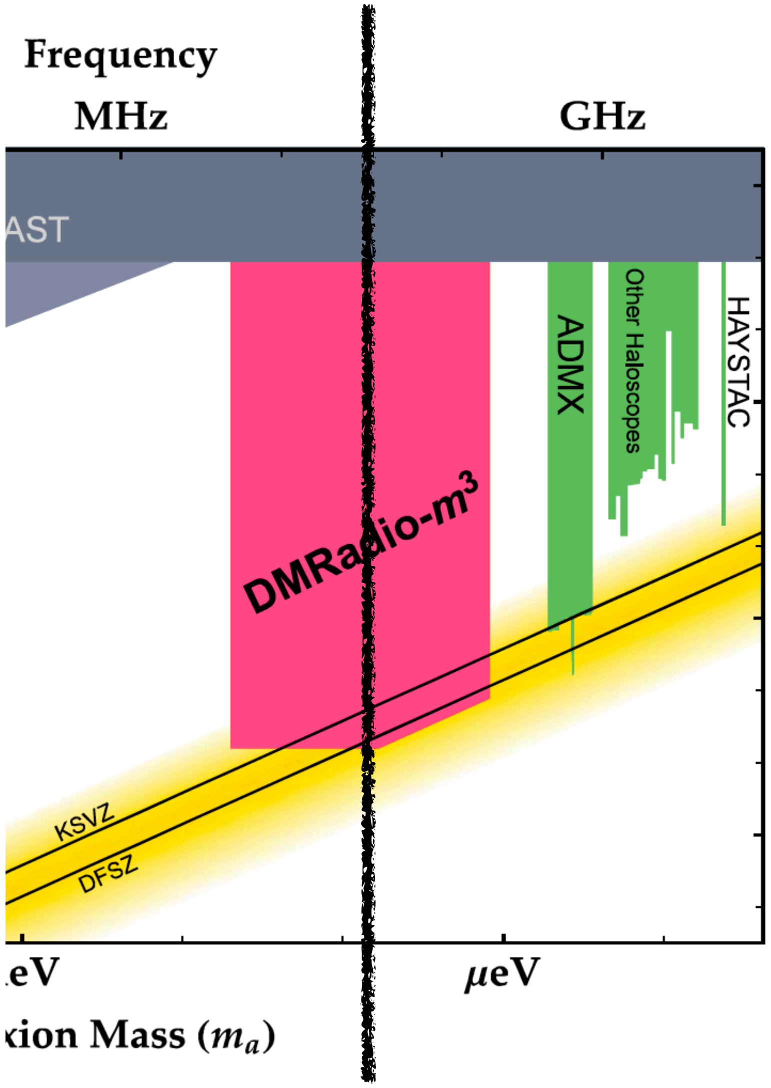
DM Radio M^3 Coaxial Design

DM Radio Collaboration Meeting
J. Singh — Stanford University
13 August 2020

Objectives

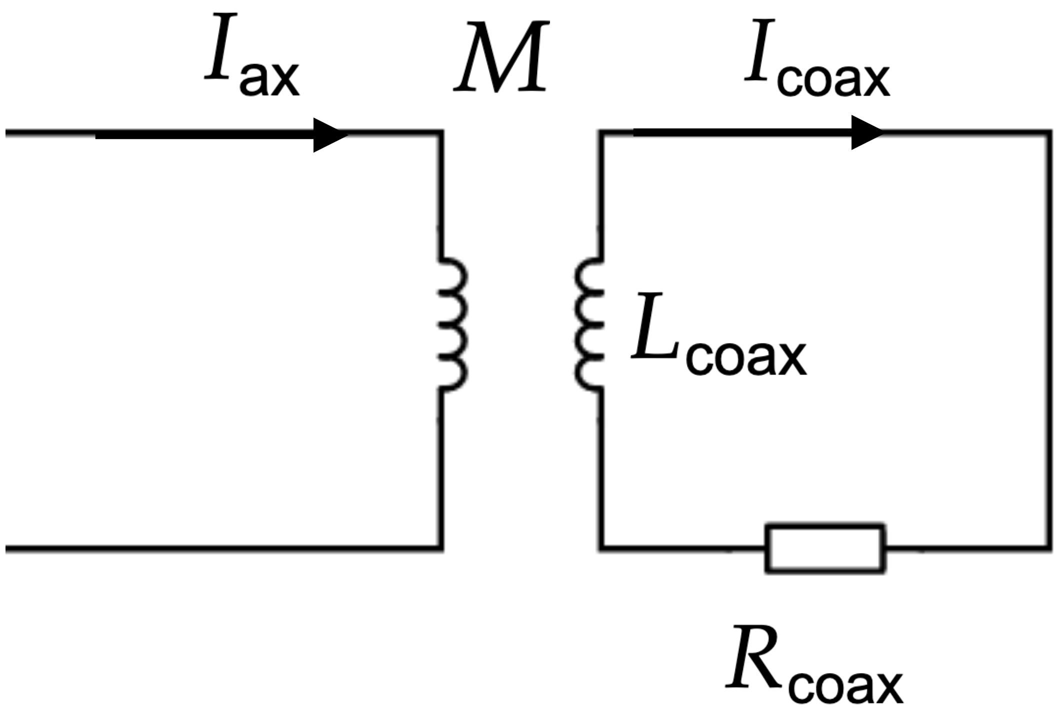
- Strawman design that couples coax to resonator. Assumptions:
  - **Quasistatic (30 MHz)**
  - **Pencil Limit**
  - **Homogenous B field in coaxial pickup**
- Evaluate FOM for coupled system to get sensitivity estimate:
  - Evaluate impact on coupled energy and Q.
  - Compare with performance required in proposal.



30 MHz

Uncoupled Coax Circuit Model

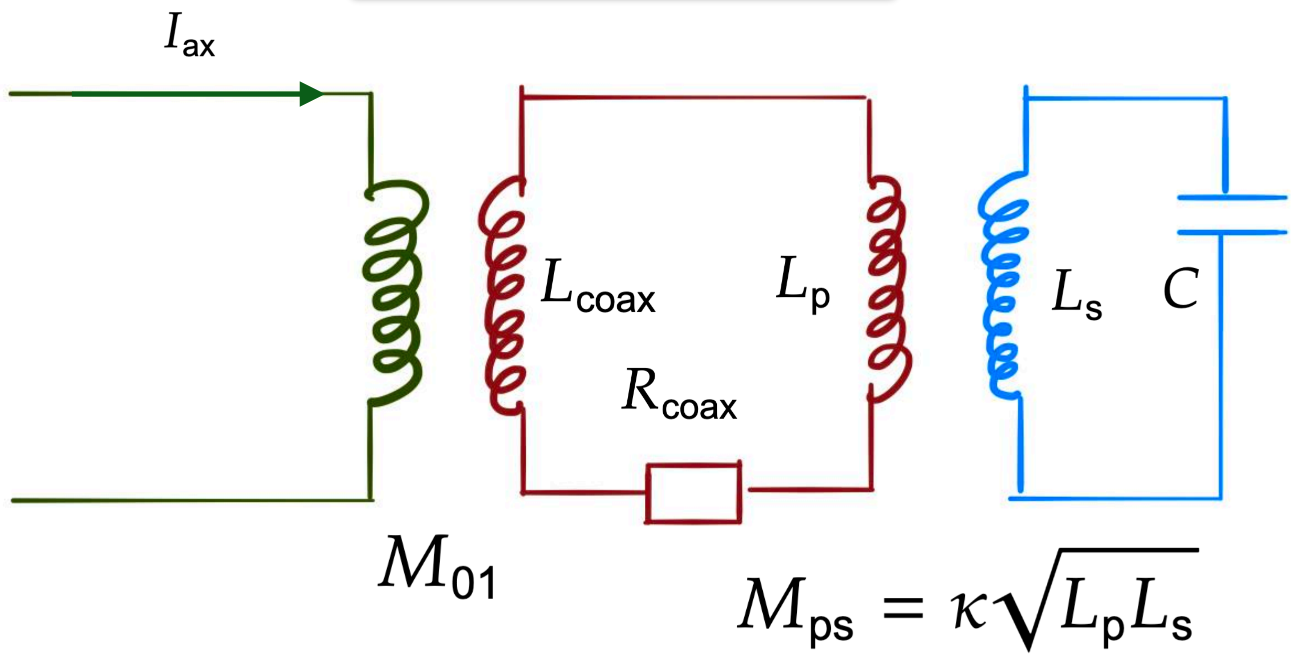
$$U_{\text{coupled}} = \frac{\phi_{\text{axion}}^2}{2L_{\text{coax}}} = \frac{(MI_{\text{ax}})^2}{2L_{\text{coax}}}$$



Coupled Circuit Model

Coax

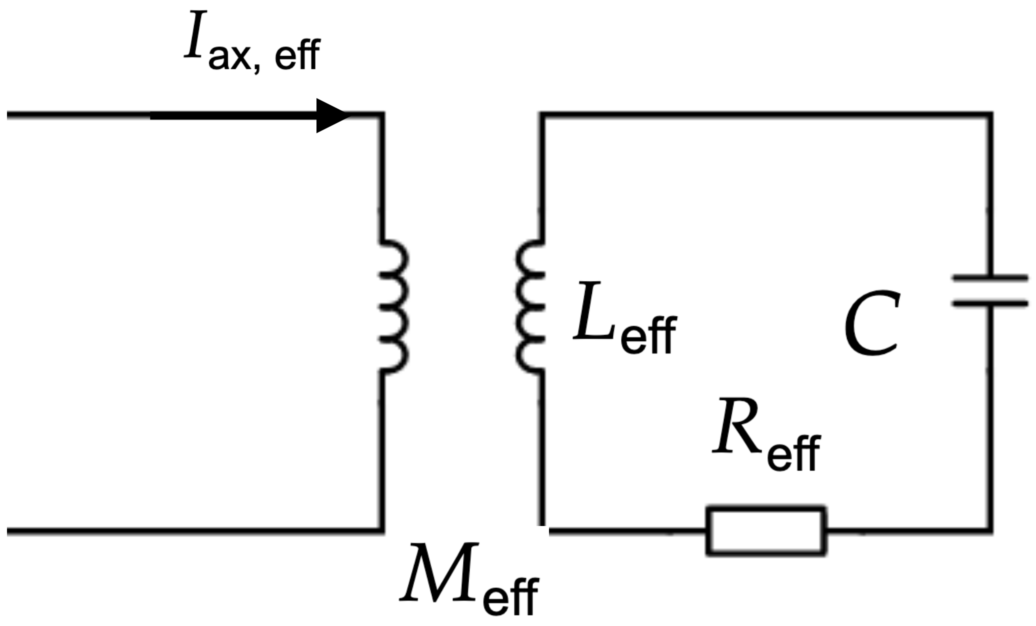
DM Input (Stiff Current)



$$M_{ps} = \kappa\sqrt{L_p L_s}$$

Resonator (outside B field)

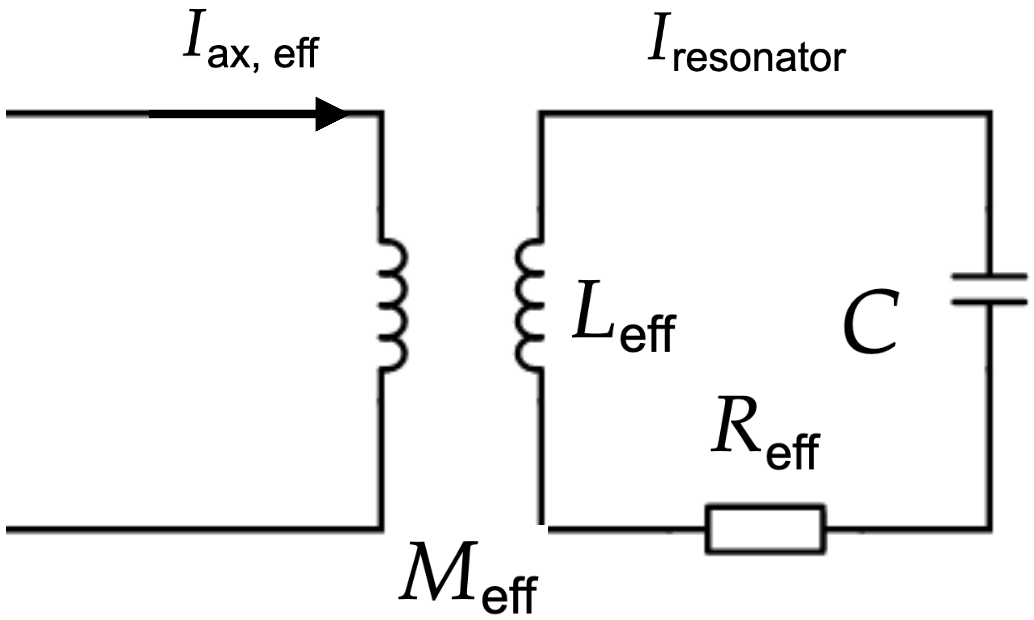
Effective Stiff Input Current $I_{ax,\ eff}$



Effective (Resonator + Coax) Circuit

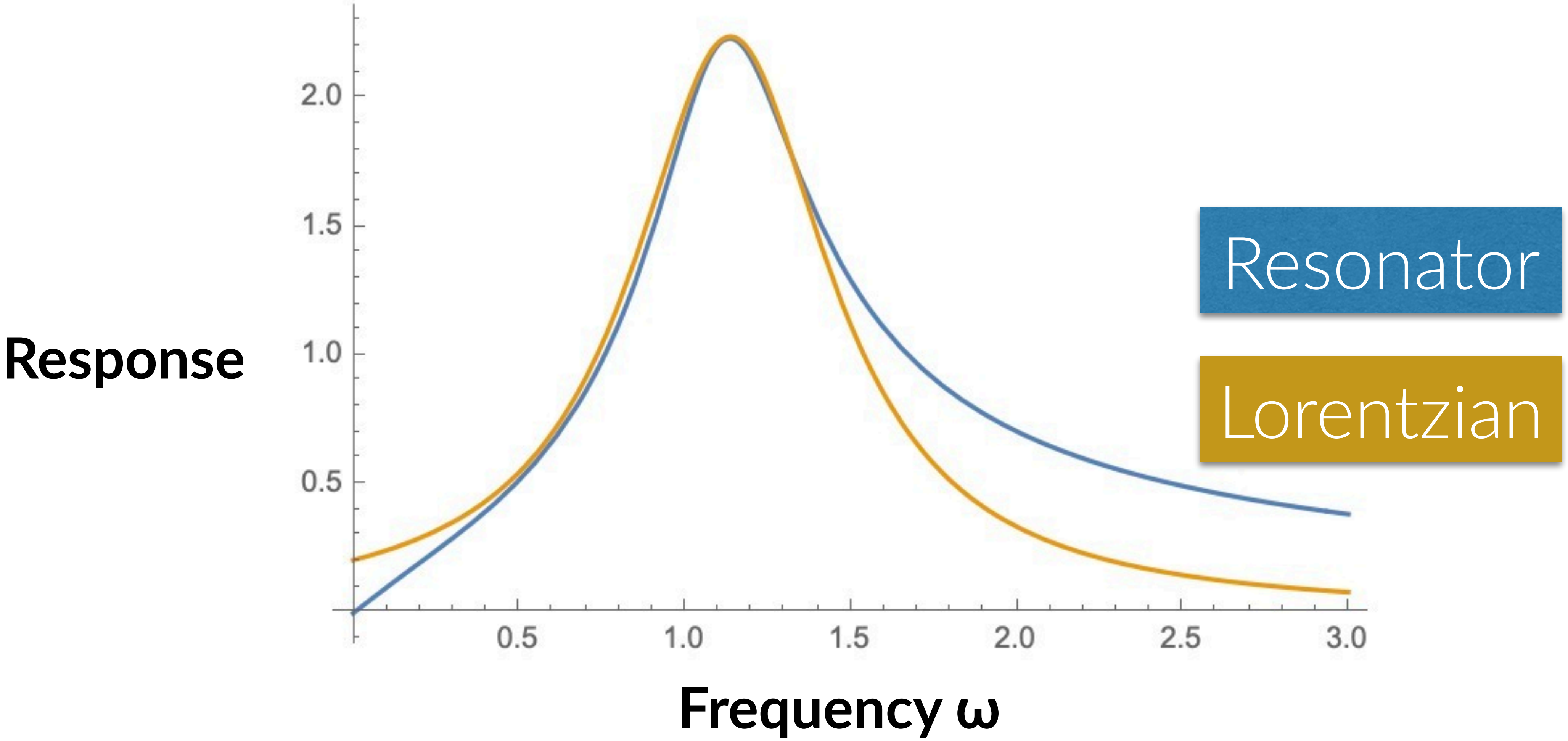
Coupled Circuit Model

$$U_{\text{coupled}} = \frac{\phi_{\text{axion}}^2}{2L_{\text{eff}}} = \frac{(M_{\text{eff}} I_{\text{ax, eff}})^2}{2L_{\text{eff}}}$$



Resonant Frequency & Q

- Effective circuit response with impedance $Z = i\omega L_{eff} - i/\omega C + R$ is Lorentzian near resonance.



- Justifies expanding Z around resonant frequency to first order and approximating effective circuit as RLC resonator.

- Resonant frequency defined by minimum of Im(Z)

$$Im(Z(\omega_0)) = 0$$

- Resonator parameters from Taylor Expansion.

$$Z(\omega) \approx R + 2iL(\omega - \omega_0)$$

- Q from resonant frequency & Taylor Expansion.

$$Q = \frac{\omega_0 L}{R}$$

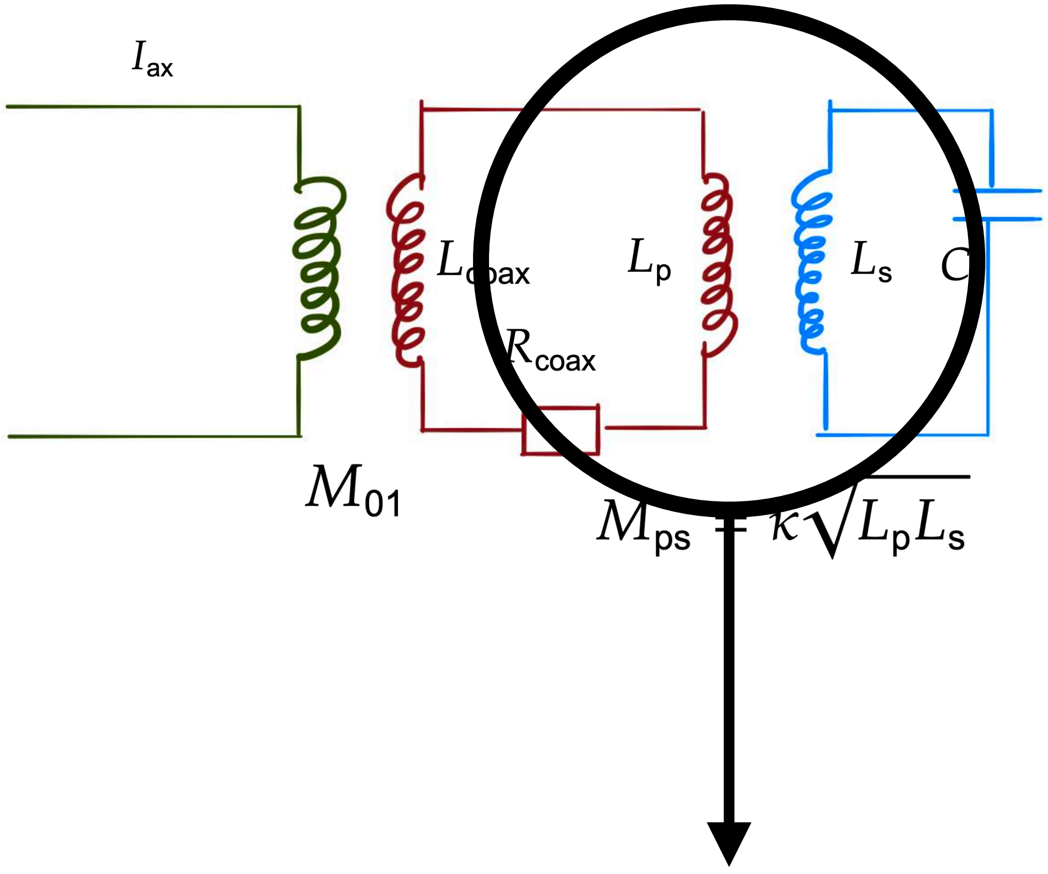
- With Q and ratio of coupled energy, goal is to choose ‘good’ set of parameters which are physically reasonable.

- ‘Good’: degradation of Q & energy coupling that is tolerable vis-à-vis science goals.

- ***WE DID NOT OPTIMISE FOR PARAMETERS***
    - Parameter values picked to give a *reasonable starting point* for further iteration.

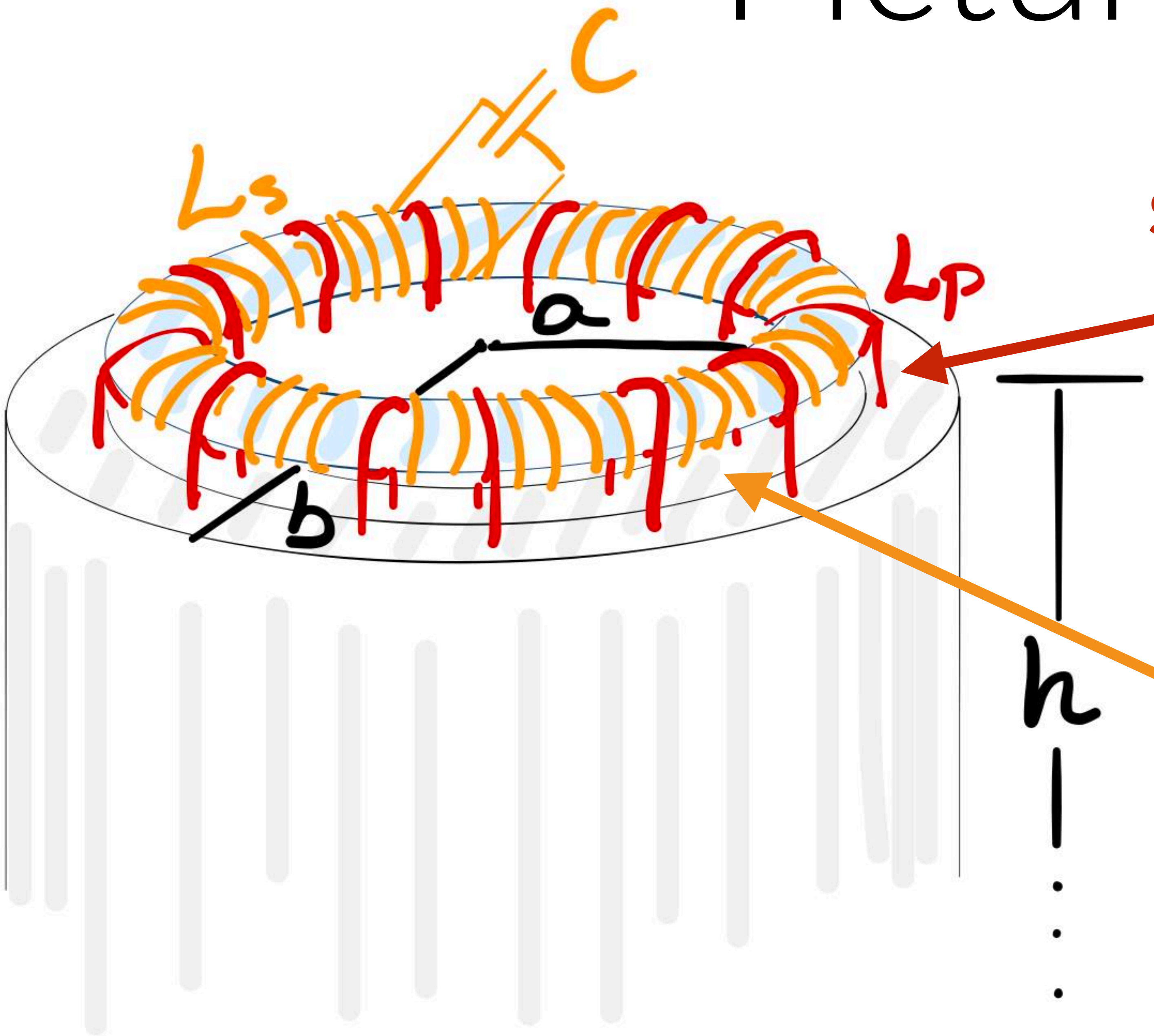
Strawman Parameters

- Coax parameters: **a = 0.3m, b = 0.68m, h = 1.38m**
- Transformer:
  - **L_p = 0.2 μH**
  - **κ = 0.8**
  - Wires in transformer assumed superconducting and lossless
- Resonator Parameters: **L_s = 0.35 μH C = 100 pF**
- **T = 100 mK, B = 4.5 T**
- **f = 32 MHz, Q = 1.6 × 10^6**



Transformer

Picture



Primary Coil:
Series of single wire loops (r=5cm) connected to the coax slit

Secondary Coil:
Toroid circular x-section r = 5cm
Toroid radius = 45cm

Not to scale!

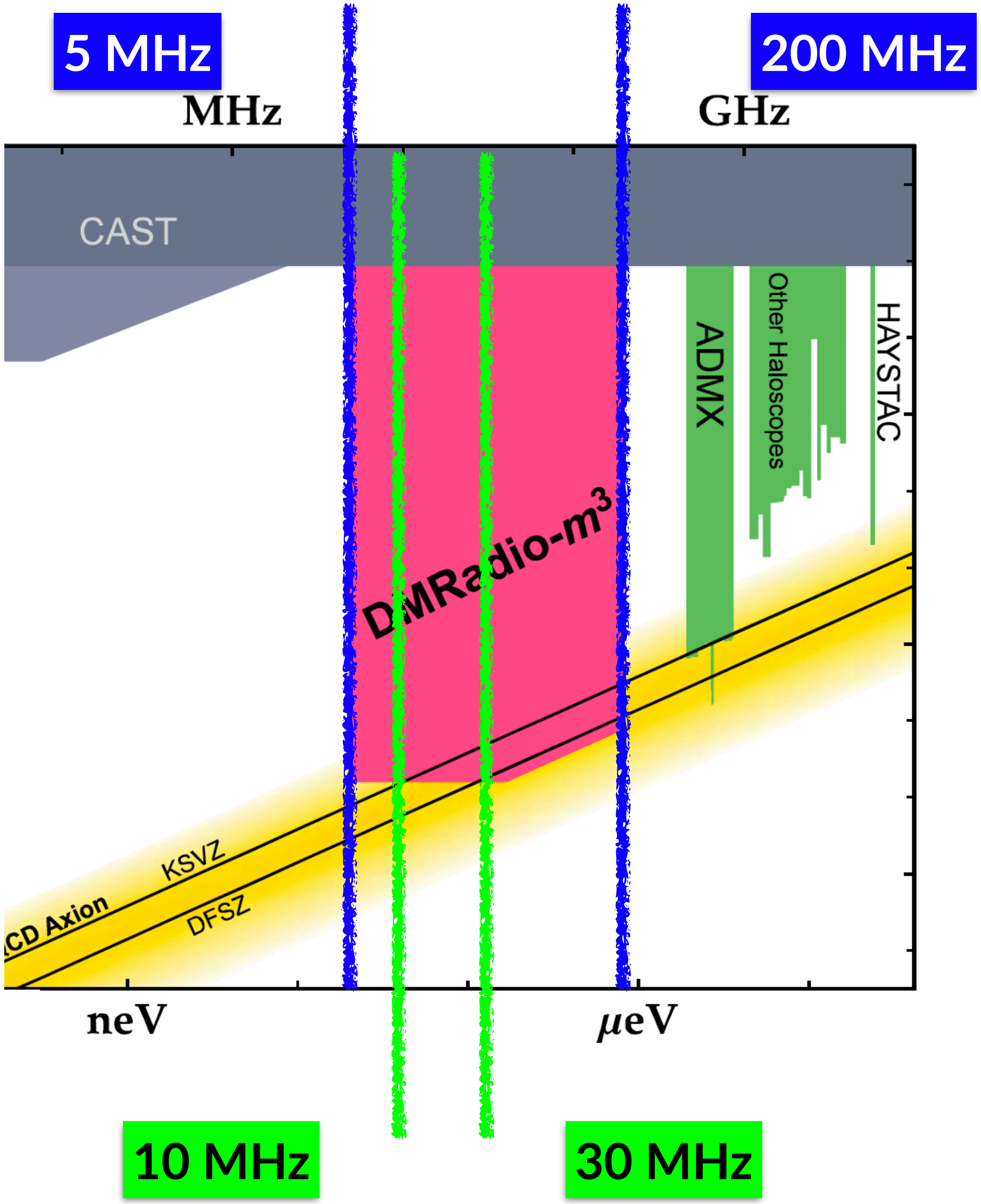
Performance Estimate

- Transformer reduces coupled energy to 0.22× the original.
- Uncoupled coax Q: $3.5 \times 10^5 \rightarrow$ Coupled Resonator Q: 1.6×10^6.
- Performance margin of -11%, i.e. underperforming vs. science goal.
- $B_0 = 4.5\text{T} \rightarrow B_0 = 5\text{T}$: 0% margin, i.e. minimum for science goal, **plausible starting point**.

Next Steps

- This straw man to be used for magnet design, get a real magnetic field profile.
- Refine inductance calculations beyond pencil using COMSOL etc.
- Components of a possible modeling campaign:
  - Coax inductance simulations
  - Transformer – what Lp, Ls optimise energy transfer, what geometric constraints limit κ?
  - Resonator – need as high Q as possible
  - A different coupling mechanism??

Summary



Coax Pickup:

Material: Cu,

$\sigma = 6 \times 10^{7}\ \mathrm{Sm}^{-1}$ (293K)

$a = 0.3\mathrm{m}$

$b = 0.68\mathrm{m}$

$h = 1.38\mathrm{m}$

$V = 2\mathrm{m}^{3}$

$B_0 = 5\mathrm{T}$

Transformer:

$L_{\mathrm{primary}} = 0.2\ \mu\mathrm{H}$

$L_{\mathrm{secondary}} = 0.35\mu\mathrm{H}$

$K = 0.8$

Resonator:

$L_{\mathrm{secondary}} = 0.35\mu\mathrm{H}$

$C = 100\ \mathrm{pF}$

$f = 32\mathrm{MHz}$

$Q = 1.6 \times 10^{6}$