M3 Coaxial design and estimated figure of merit

Cady van Assendelft, Stanford University
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Frequency ranges

Low frequency: quasistatic treatment

$$a, b, h \ll \lambda_{\mathrm{ax}}$$

This analysis:

Treat pickup as a lumped-element inductor

Intermediate regime

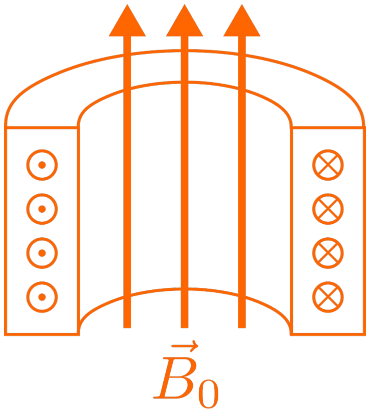
Will require simulations for accurate analysis

High frequency: cavity limit

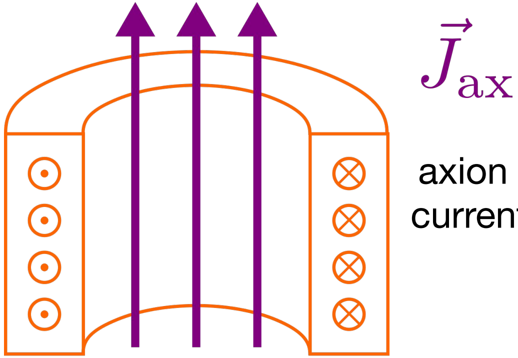
$$a, b, h \sim \lambda_{\mathrm{ax}}$$

Mode analysis of pickup structure (Nicholas' slides)

Solenoidal magnet and coaxial pickup

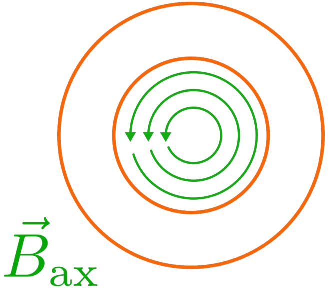


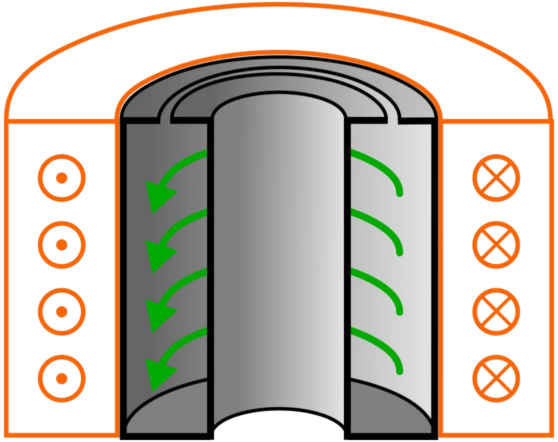
Solenoidal magnet with example field lines



axion effective current density

Top view of and axion-induced magnetic field

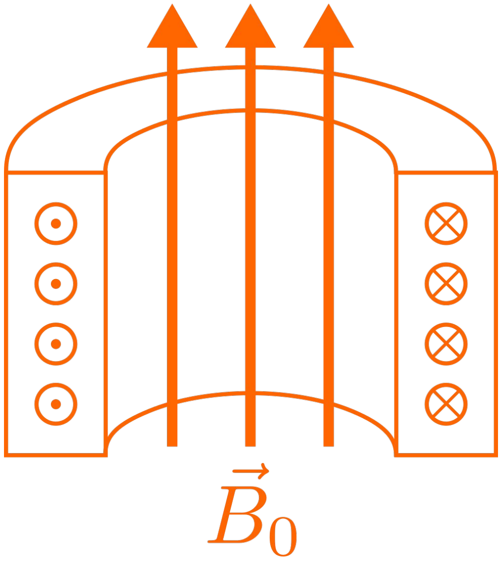




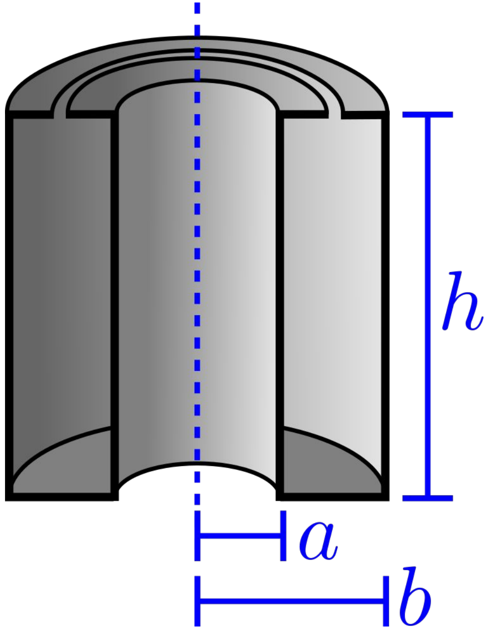
Coaxial pickup structure inside solenoidal magnet captures flux from axion-induced magnetic field

Simplifying assumptions

1) Homogeneous magnetic field in pickup region

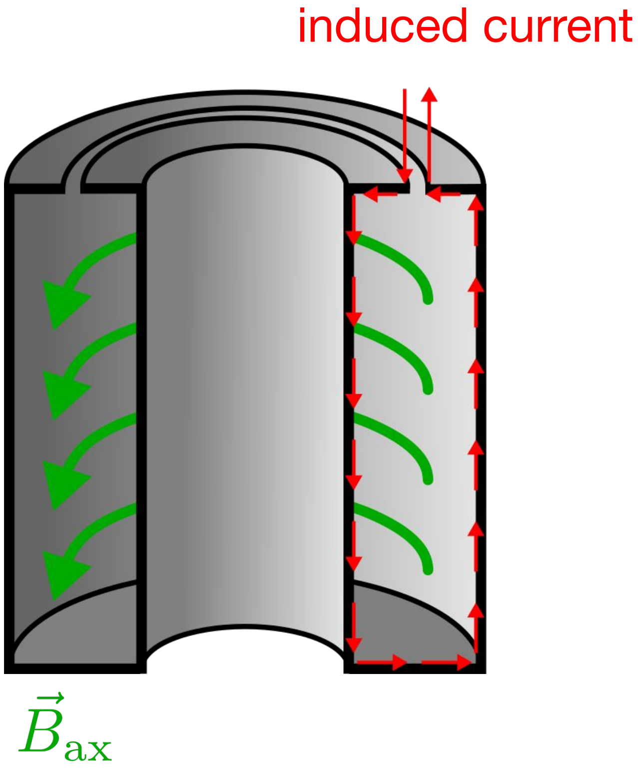


2) Pencil limit for pickup inductance (h ≫ b)



both of these assumptions break down as we move towards a more "squat" geometry

Coupled energy



Coupled energy is calculated from the flux through the pickup structure.

$$\Phi = \int B_{\mathrm{ax}} \cdot \mathrm{d}A$$

$$E_{\mathrm{coupled}} = \frac{\Phi^2}{2L}$$

Design process overview

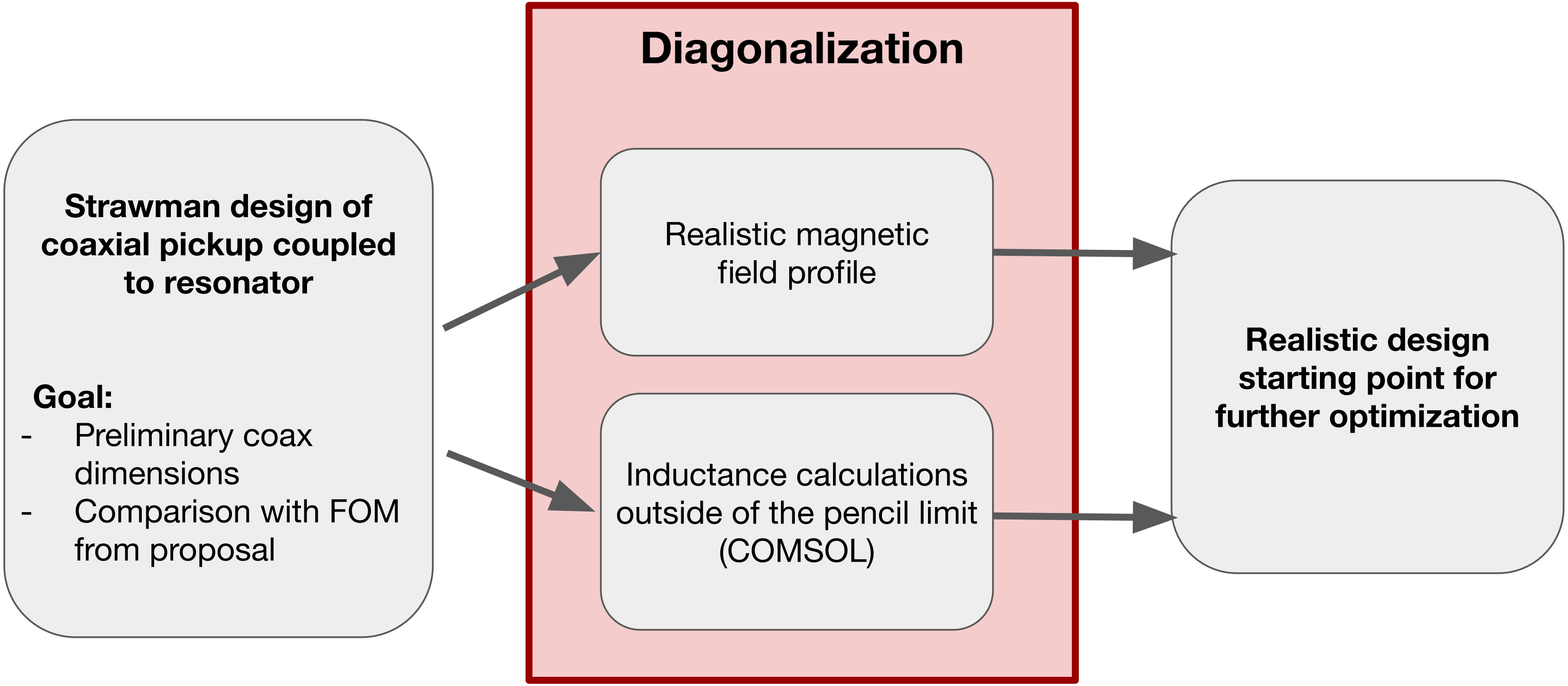


Figure of merit

$$\mathrm{FOM} \propto \underbrace{\frac{c_{\mathrm{PU}} B_0 V^{5/6} Q^{1/4}}{\eta^{1/4} T^{1/4}}}_{\text{valid in the quasi-static limit}} \propto \frac{E_{\mathrm{coupled}}^{1/2} Q^{1/4}}{\eta^{1/4} T^{1/4}}$$

1) How do we map the coaxial design onto this figure of merit?
2) Can we match the target science reach with a preliminary coax design?

Parameters from proposal:

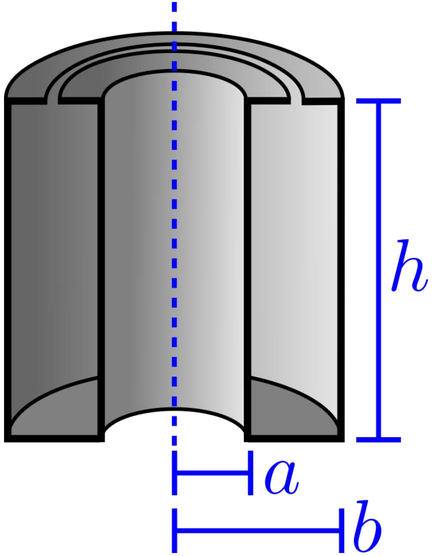
Geometric factor	c_{PU}	0.20
Magnetic field	B_0	4T
Pickup volume	V	$1\mathrm{m}^3$
Quality factor	Q	10^6
Temperature	T	20 mK
SQUID noise parameter	η	20
Performance margin	–	42%

Coax figure of merit (not coupled to resonator)

$$\mathrm{FOM} \propto \frac{c_{\mathrm{PU}} B_0 V^{5/6} Q^{1/4}}{\eta^{1/4} T^{1/4}} \propto \frac{E_{\mathrm{coupled}}^{1/2} Q^{1/4}}{\eta^{1/4} T^{1/4}}$$

$$V = \pi b^2 h$$

$$Q = \omega L / R_{\mathrm{eff}}$$



Free parameters

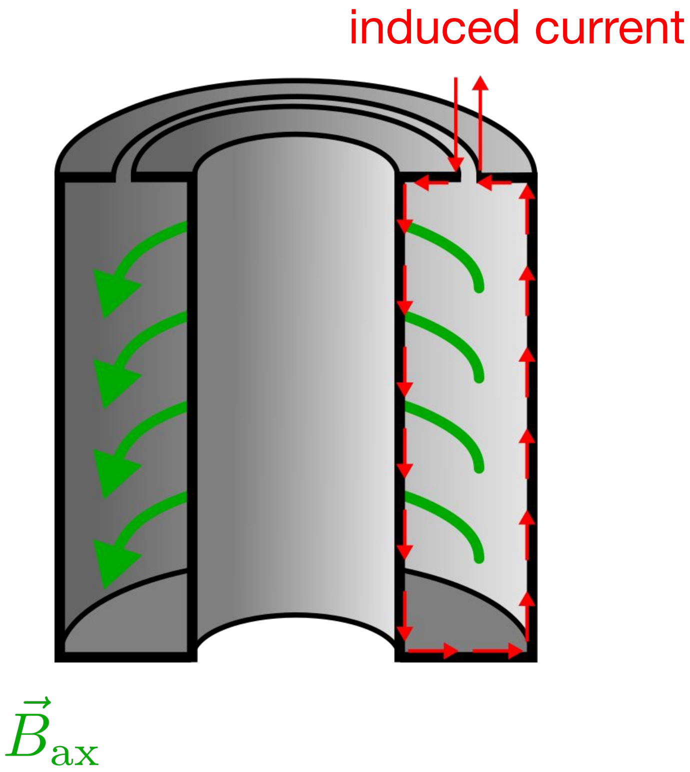
determines Q { a: inner radius

determines V { b: outer radius, h: height

a, b, h } determines c_{PU}

B_0: magnetic field

Copper physics and R_{eff}



The pickup structure will be made of **copper.**

At **room temperature,** a 1 m^3 coaxial pickup has
Q ~ 35,000 at the $\lambda/4$ frequency

However, the **skin effect** is a function of frequency as well as **conductivity**, which is dependent on temperature.

At cryogenic temperatures, scale Q by replacing R with a frequency-dependent R_{eff}

The same coaxial pickup at cryogenic temperatures has quality factor Q ~ 375,000 at the same frequency

Preliminary parameters

Constrained h/b ≥ 2

Analysis done at 30 MHz

BAD ASSUMPTIONS:

- Homogeneous magnetic field
- Pencil limit inductance

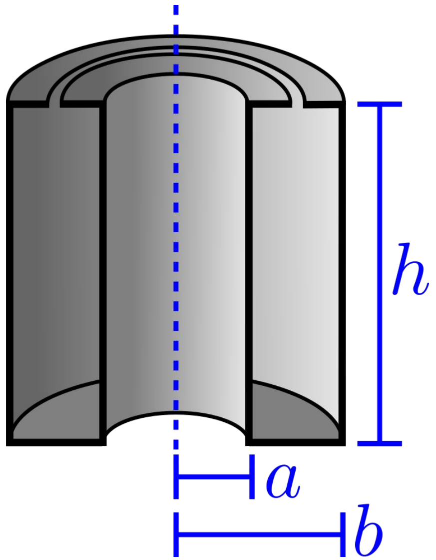
	$1\ \mathrm{m}^3$	$1.5\ \mathrm{m}^3$	$2\ \mathrm{m}^3$
a	24 cm	27 cm	30 cm
b	55 cm	62 cm	68 cm
h	1.1 m	1.25 m	1.38 m
B_0	4.0 T	4.5 T	4.5 T
c_{PU}	0.12	0.12	0.12
Q	2.8×10^5	$3.2\ \times10^5$	$3.5\ \times10^5$
Performance margin	-35%	2%	32%

Largest performance margin

** the figure of merit will be degraded by coupling to the resonator **

Key takeaways

- Can achieve ballpark figure of merit with a **copper, coaxial** pickup structure
- Optimization pushes towards higher volume, squatter geometry



Given this preliminary analysis, **plausible** to achieve DM Radio M3 science goals with a solenoidal magnet.

Once we couple to a resonator — are these science goals still plausible?