m^3 resonances: parasitic resonance in toroid & mode resonances in solenoid

Nicholas Rapidis
Stanford
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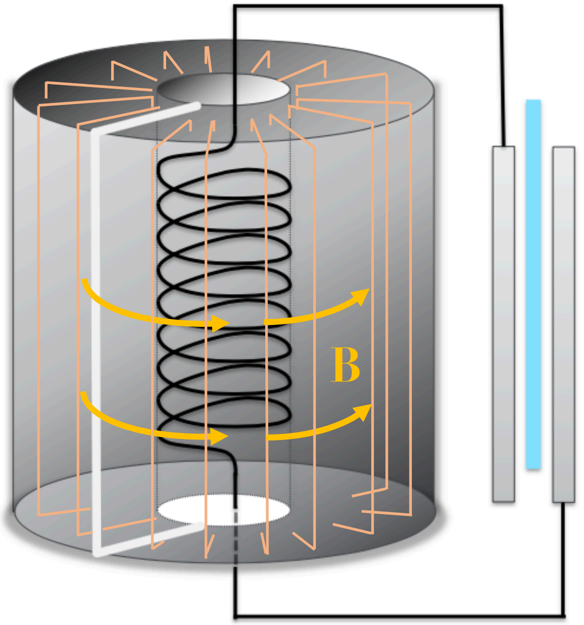
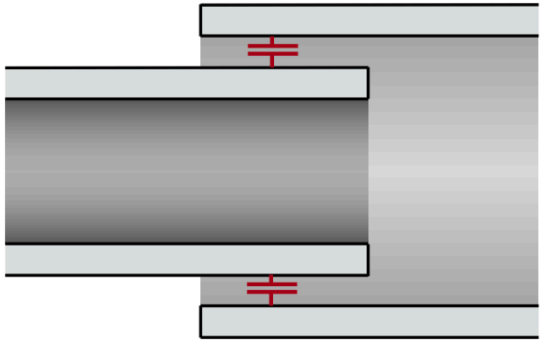
Parasitic resonance

50 L geometry

- Vertical cut allows currents to flow on outer portion of sheath → induced magnetic field in center picked up by resonator



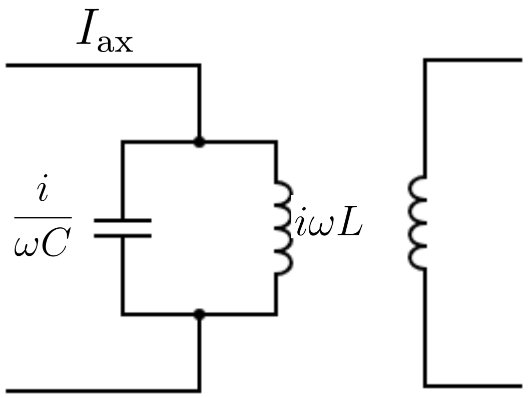
- To shield resonator from losses → make "snake swallowing its own tail" geometry



50 L geometry

- Capacitance in circuit causes resonance.

- At frequencies higher than resonant frequency, current flows through C, not L → no coupled energy to pickup resonator

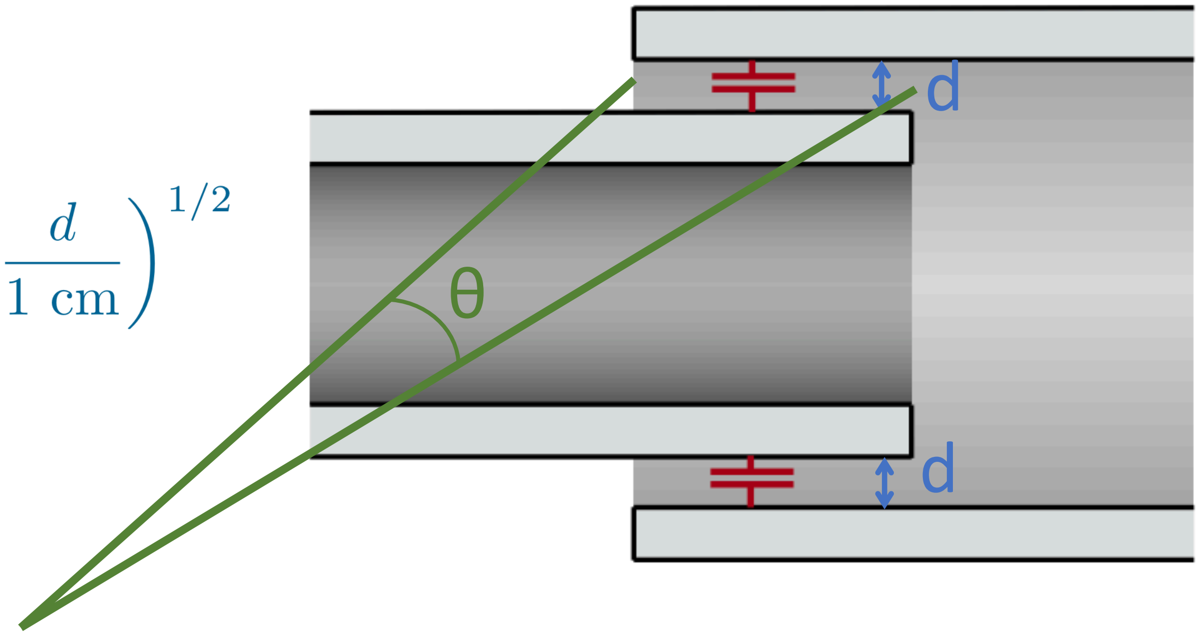


- Resonant frequency can only be tuned down, not up

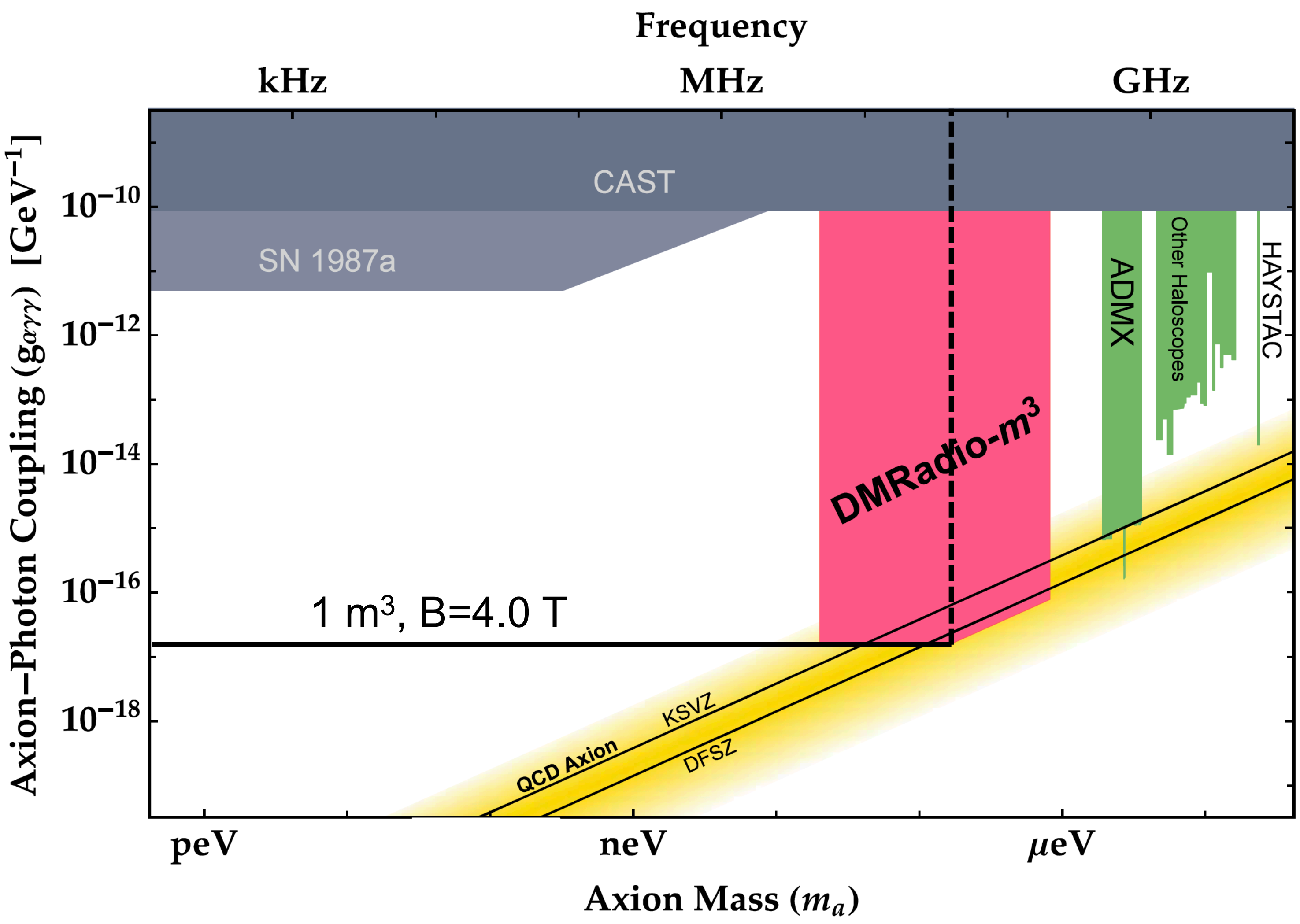
50 L geometry

- 50 L resonance ~ 50 MHz. For m^3 will be even **lower**. Kills DFSZ sensitive region

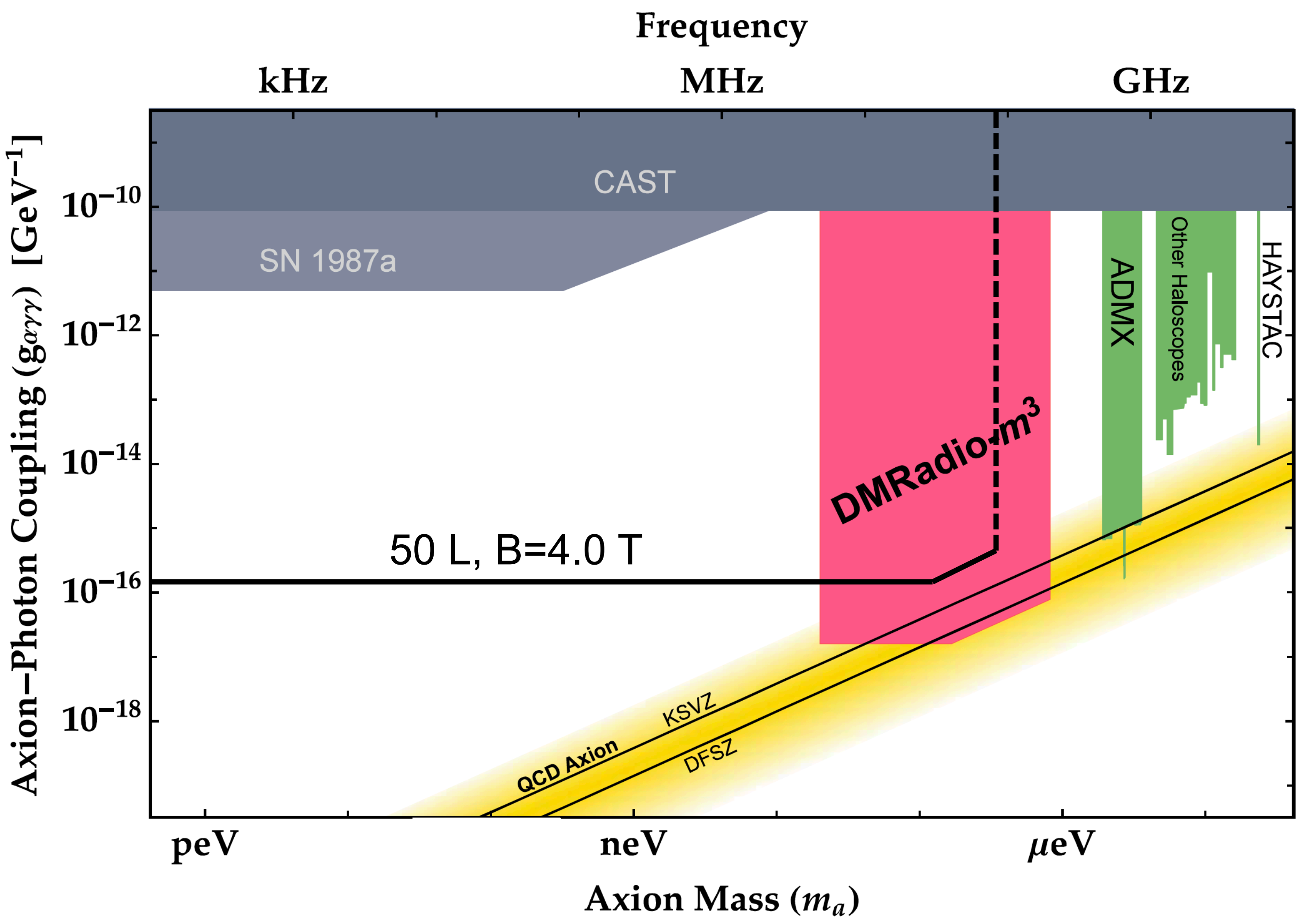
$$f_p \approx 48.5\ \text{MHz} \left(\frac{0.05\ \text{m}^3}{V_{\text{tor}}}\right)^{1/2} \left(\frac{1\ \text{rad}}{\theta}\right)^{1/2} \left(\frac{d}{1\ \text{cm}}\right)^{1/2}$$



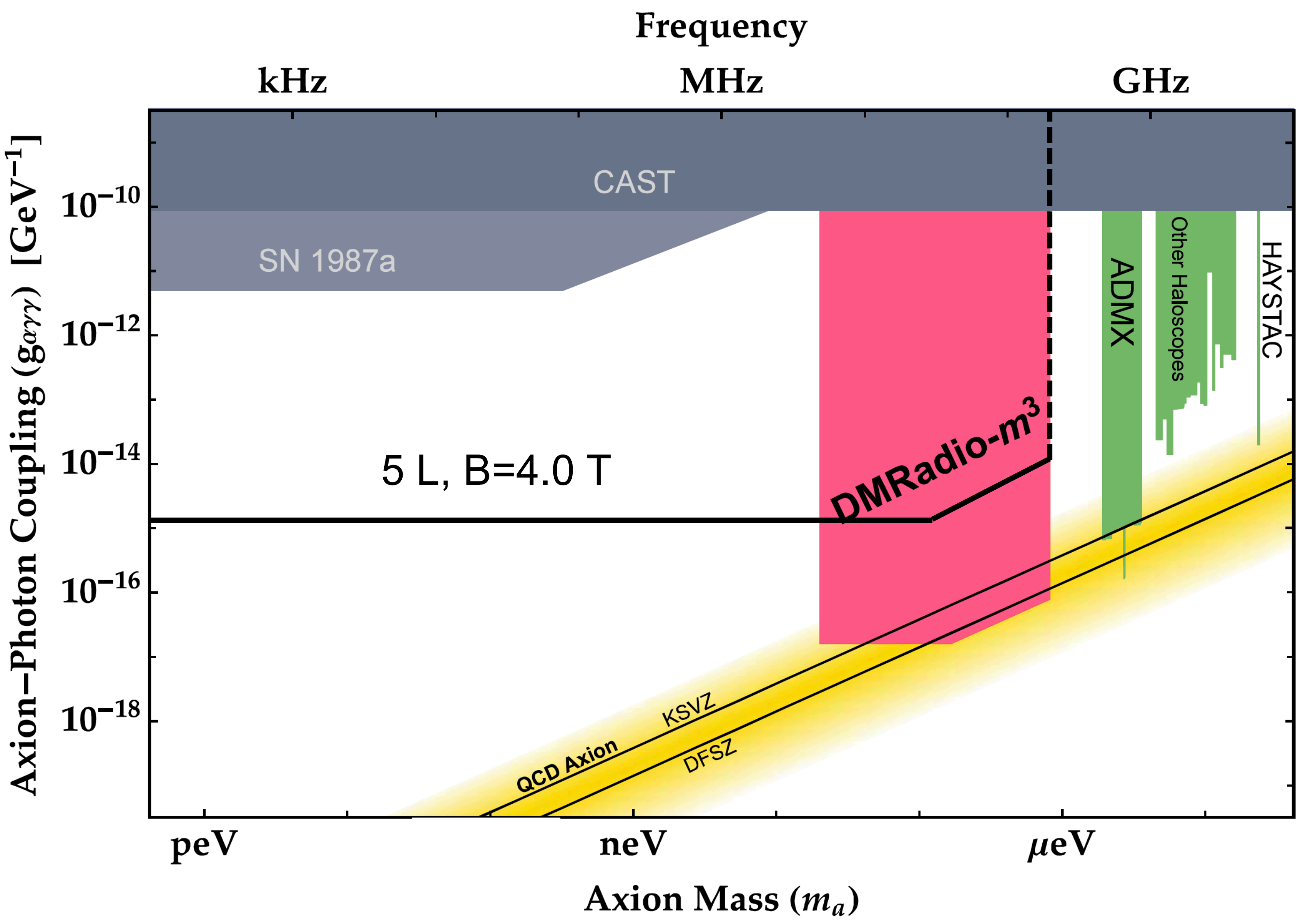
Limitation of parametric resonance



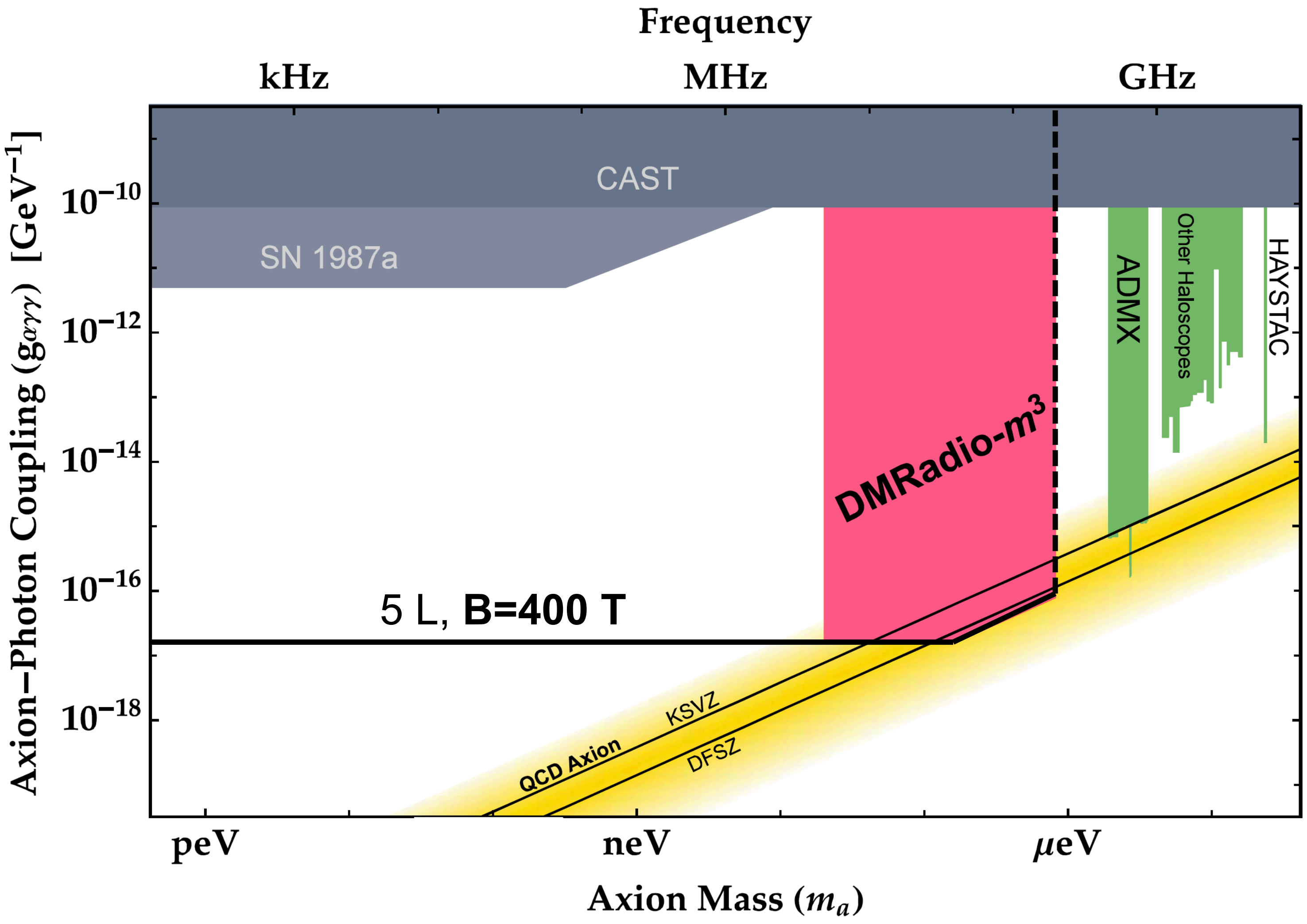
Limitation of parametric resonance



Limitation of parametric resonance



Limitation of parametric resonance



Abandoning Toroid

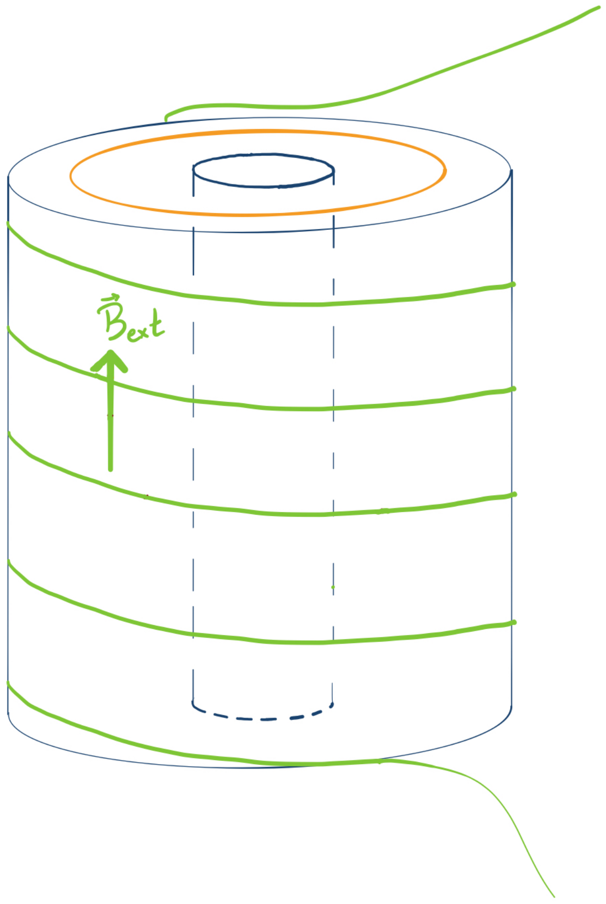
- Parasitic capacitance is fatal and cannot be reasonably avoided

- Only workaround is ~5L toroid with 400 T magnet

- Must change geometry

Toroid → Solenoid

- Change magnet to solenoidal geometry
- Cut *emmunctory* (aka slit) at top of sheath and place inductors there to pick up signal
- Avoids parasitic capacitance issues



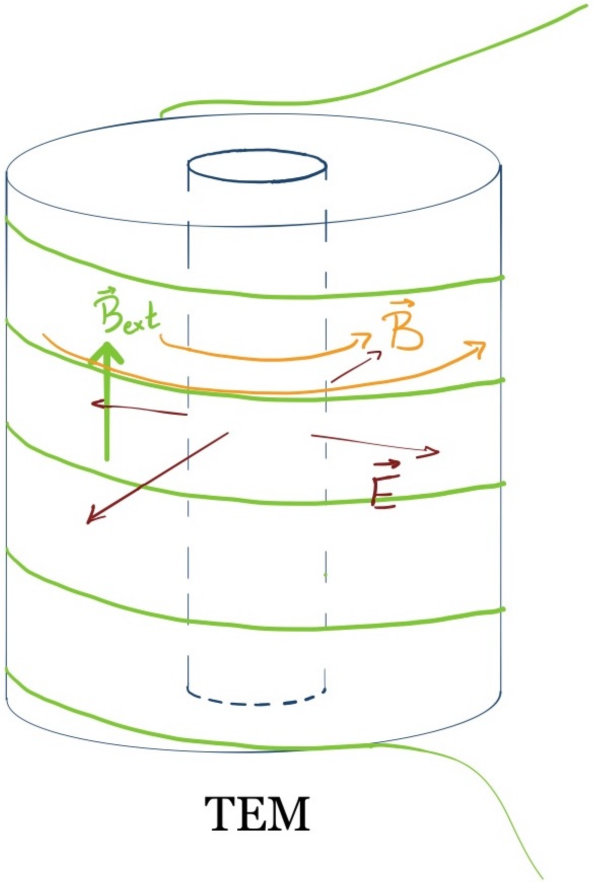
Cavity resonances in m^3

High frequency limit

$$\lambda_{DM} \lesssim \text{size of detector}$$

- Sheath exhibits resonances (modes)
- Power is reflected at non-resonant frequencies
- Which modes are useful and which detrimental?

Mode types



TEM TM TE

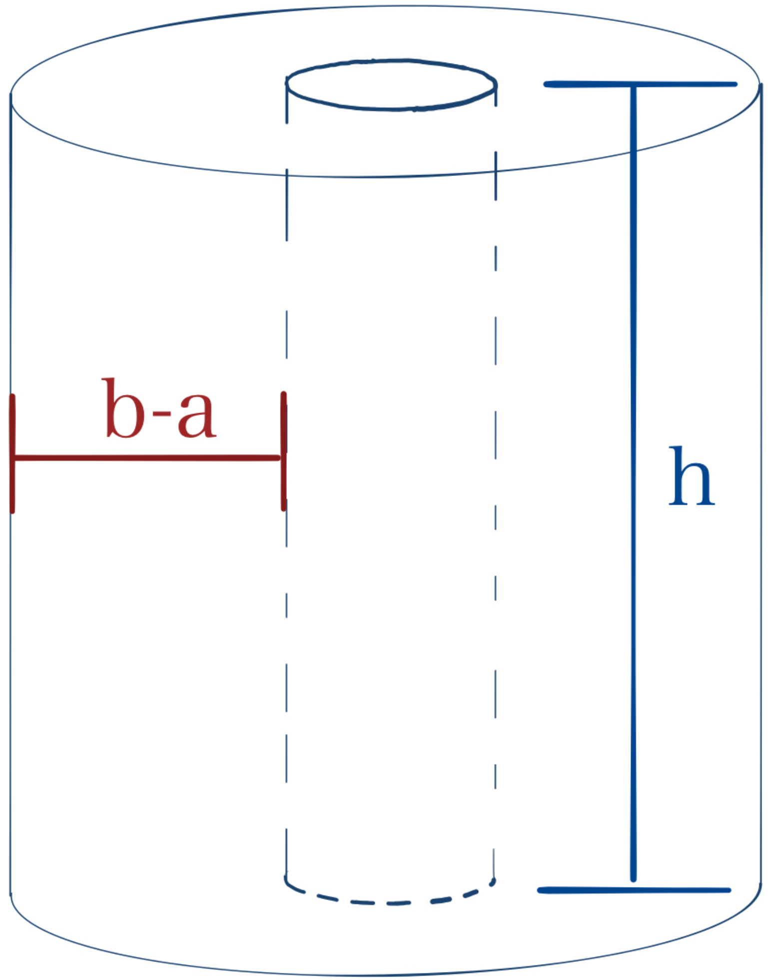
Mode frequencies

Solve Laplace equation in sheath:

$$\omega_{nmp} = c\sqrt{\alpha_{nm}(a,b)^2 + \frac{p\pi}{h}^2}$$

radial wavenumber
(TE & TM modes only)

vertical wavenumber
(all modes except TM_{mn0})

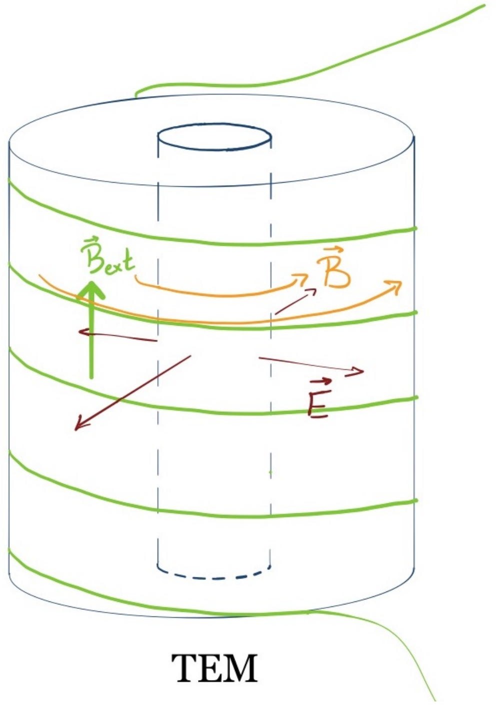


Fully general solution for any a, b, h

Mode frequencies

Lowest frequency mode in cavity where b-a < h is TEM_1:

$$\nu_{\mathrm{TEM}_1} = \frac{c}{2h} = 200\ \mathrm{MHz} \left(\frac{0.75\ \mathrm{m}}{h} \right)$$



TEM

- J_{axion} does not excite TEM_1 → no energy coupled

$$\mathrm{Power} \sim \int E \cdot B \sim \int E \cdot J_{ax}$$

- f_{TEM1} gives upper bound on h

Mode frequencies

- Any modes with J_{axion} coupling (TM modes) would mix with other "intruder" modes when tuning

- Frequency parameter space would not be entirely accessible. This is an issue that plagues haloscopes (e.g. HAYSTAC, ADMX)

➢ Avoid mode region

Conclusions

Toroid	Solenoid
Parasitic resonances sets upper limit on frequency.	Can avoid parasitic resonance altogether
Magnet inside sheath	Magnet outside sheath: can swap in different sized sheaths/include multiple in the magnet
Modes are far above parasitic resonance → can be ignored	Modes cannot be ignored but are easy to understand and their frequencies can be predicted analytically

Next steps: simulate region below f_{TEM1} to understand where coupled energy is sufficiently high. This will set upper frequency cutoff on m^3