Vibration Analysis

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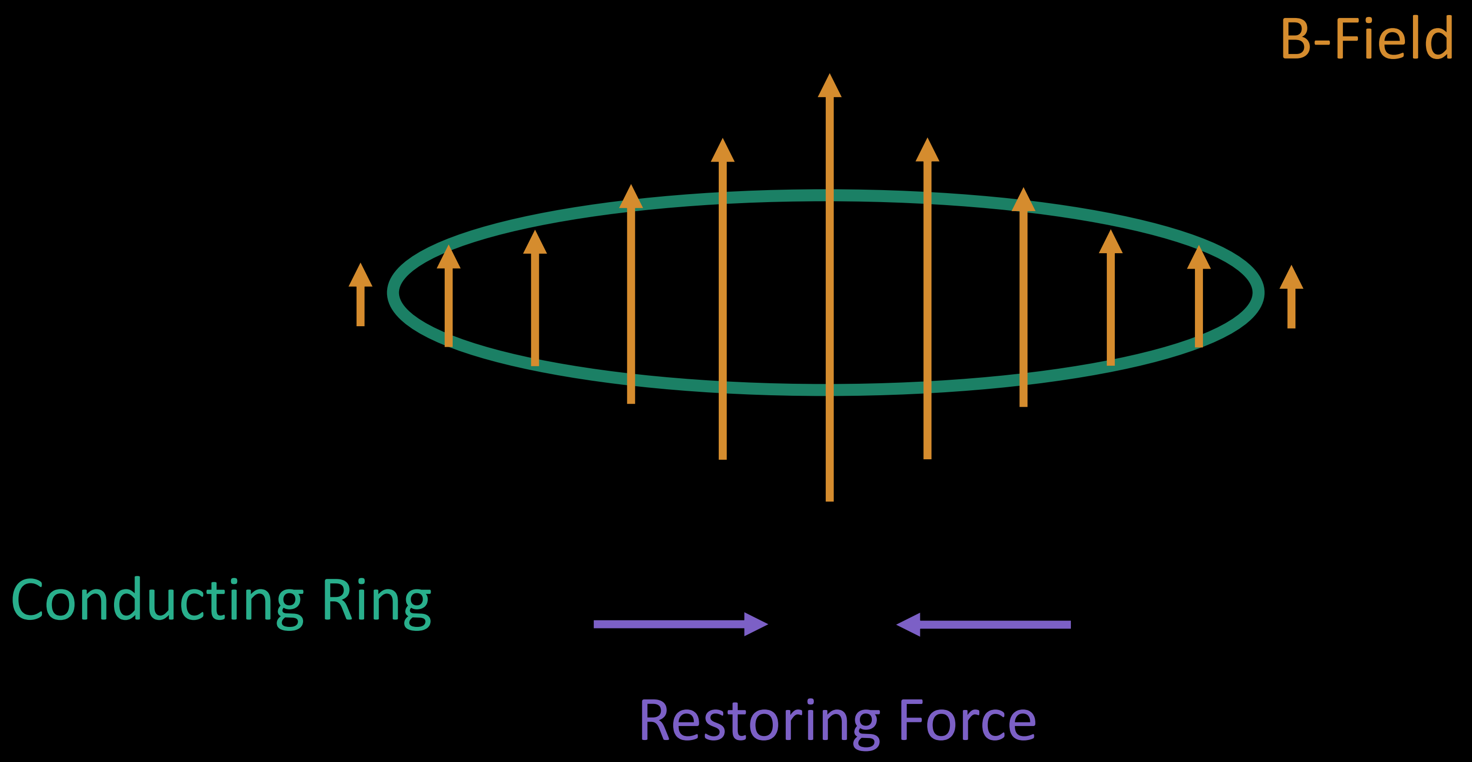
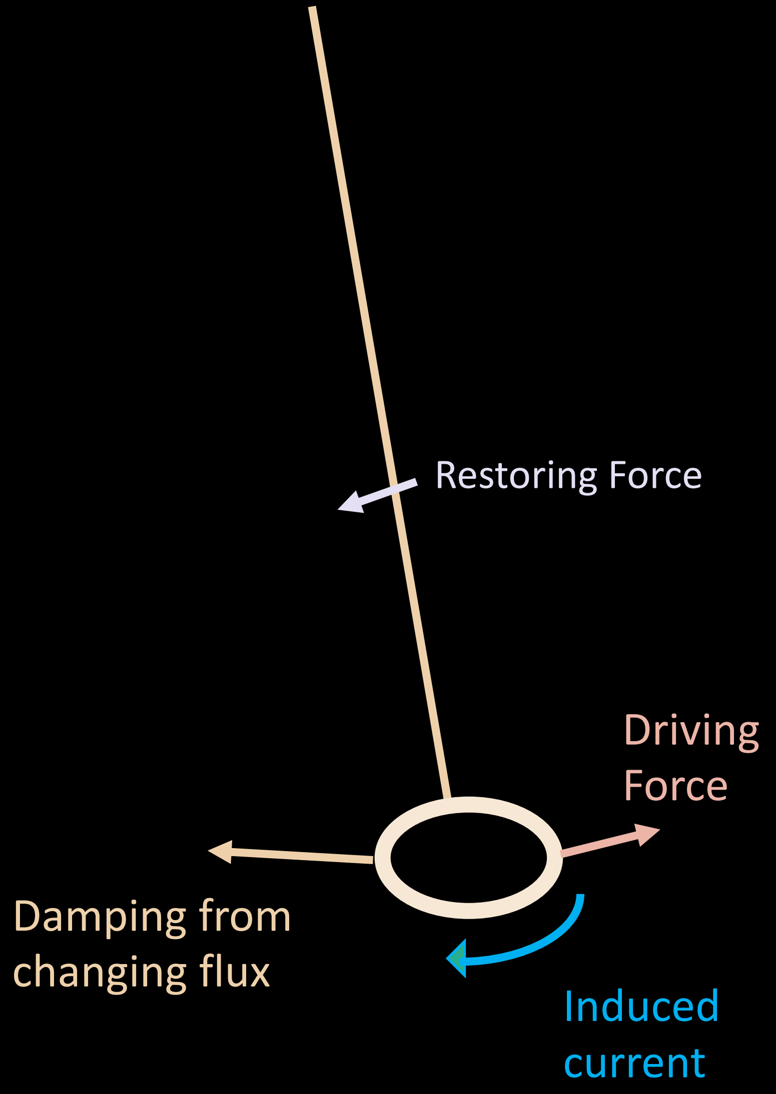
Overview

- I explored different methods of vibration analysis using ABRACADABRA as a template
  - I used the most basic model of the ABRA setup to analyze vibrations
    - Ring in a magnetic field
      - Analytic approach
      - Numeric approach
    - Pendulum attached to a spring
      - Simulation (COMSOL Random Vibration Analysis)

Ring in a Magnetic Field

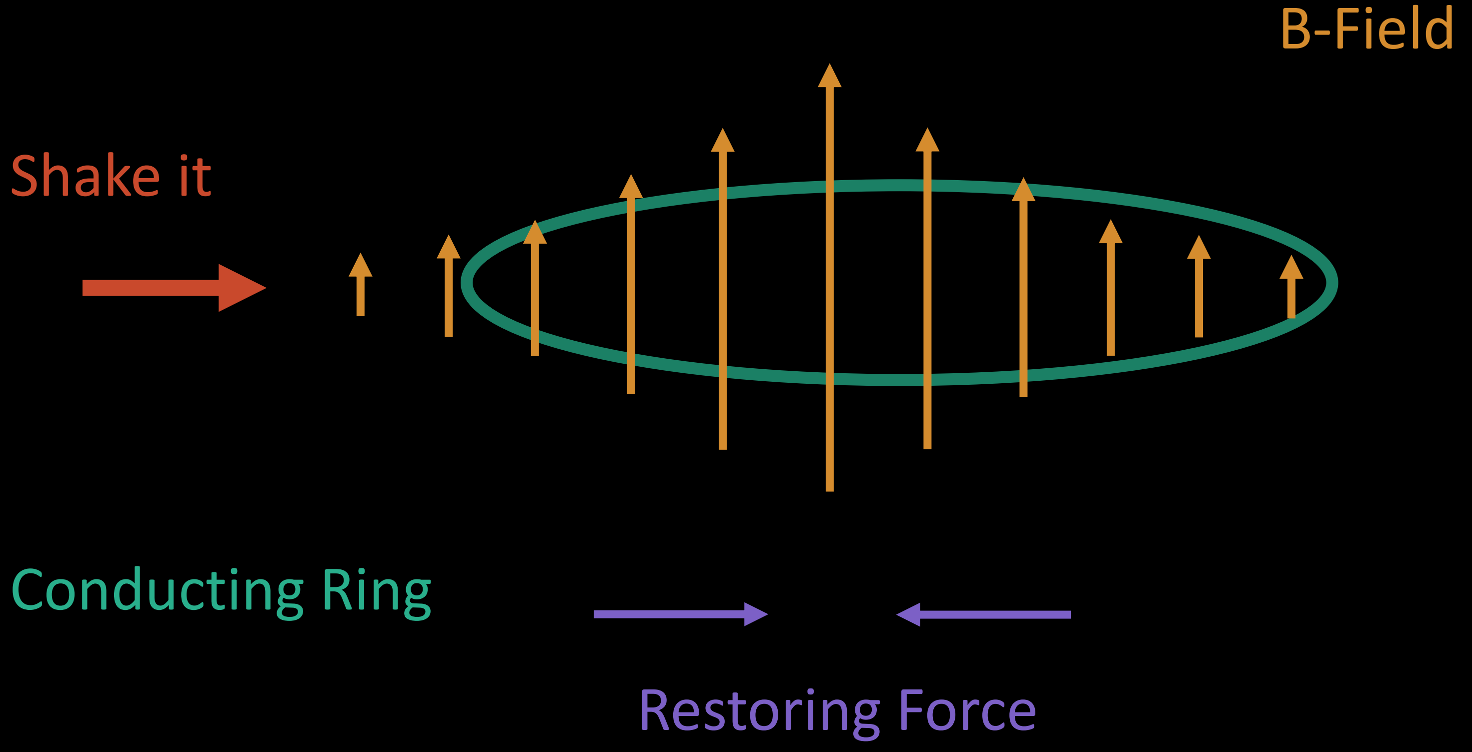
Working in 2D

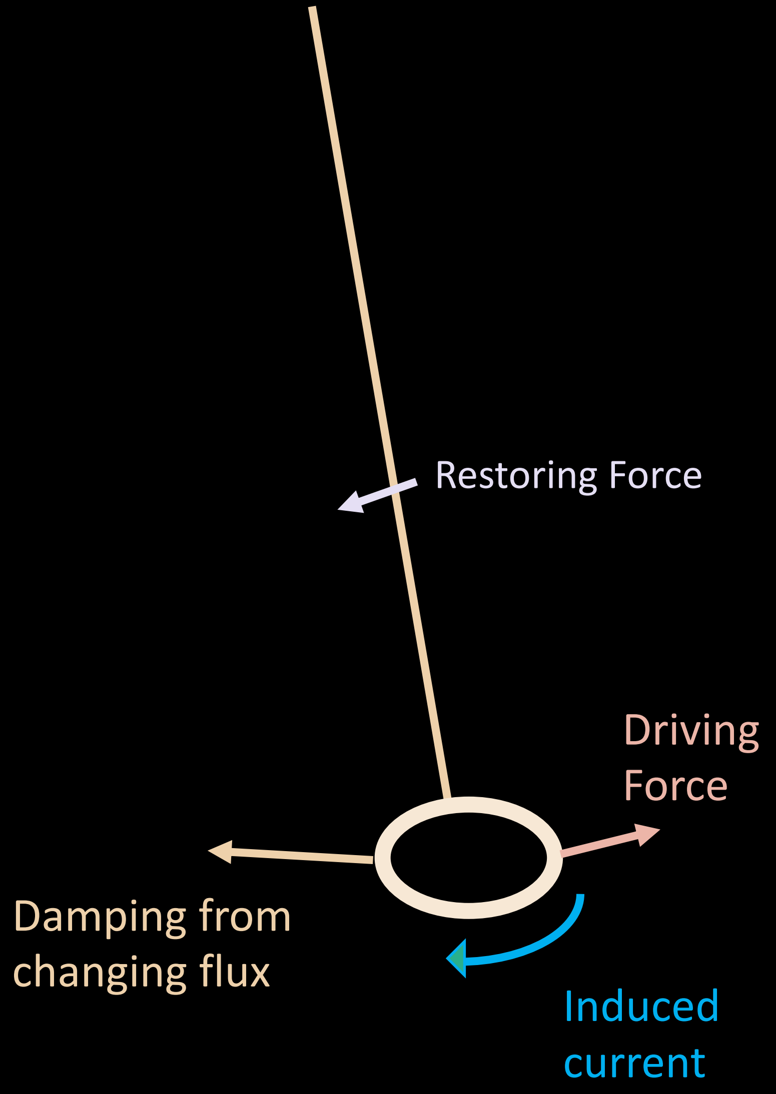




Ring in a Magnetic Field

Working in 2D





Analytic Approach

Starting from the fundamental equations

Laws of Motion

$$m\ddot{\boldsymbol{x}} = \int \boldsymbol{J}\times\boldsymbol{B}\,dV - \kappa\boldsymbol{x} + F_{Driving}$$

Lorentz Force — Restoring Force — Driving Force

Electromagnetic Induction

$$\nabla\times\rho J = -\frac{\partial B}{\partial t}$$

Analytic Approach

Taking a linear expansion of the magnetic field assuming a constant gradient taking (x_0,y_0) as the center point of the circle

$$B \approx B(x_0, y_0) + \frac{dB(x_0,y_0)}{dx}(x - x_0) + \frac{dB(x_0,y_0)}{dy}(y - y_0)$$

The equations become

$$m\ddot{\overrightarrow{x_0}} = \pi R^2 I(t)\frac{dB(x_0,y_0)}{d\vec{x}} - \kappa\overrightarrow{x_0} + F_{Driving}$$

$$-\pi R^2\left(\frac{dB(x_0,y_0)}{dx}\dot{x_0} + \frac{dB(x_0,y_0)}{dy}\dot{y_0}\right) = I(t)\mathcal{R} + L\frac{dI}{dt}$$

Analytic Approach

From solving the simplified equations:

$$|I(\omega)| = \frac{1}{\mathcal{R}m}\frac{\pi\omega R^2\left(\frac{dB(x_0,y_0)}{dx}F_{\mathbf{0}x}+\frac{dB(x_0,y_0)}{dy}F_{\mathbf{0}y}\right)}{\sqrt{|(1+\omega^2\tau^2)({\omega_0}^2-\omega^2)^2+\omega^2\pi^2R^8\xi^2+2\omega^2\pi^2R^4\tau\xi({\omega_0}^2-\omega^2)|}}$$

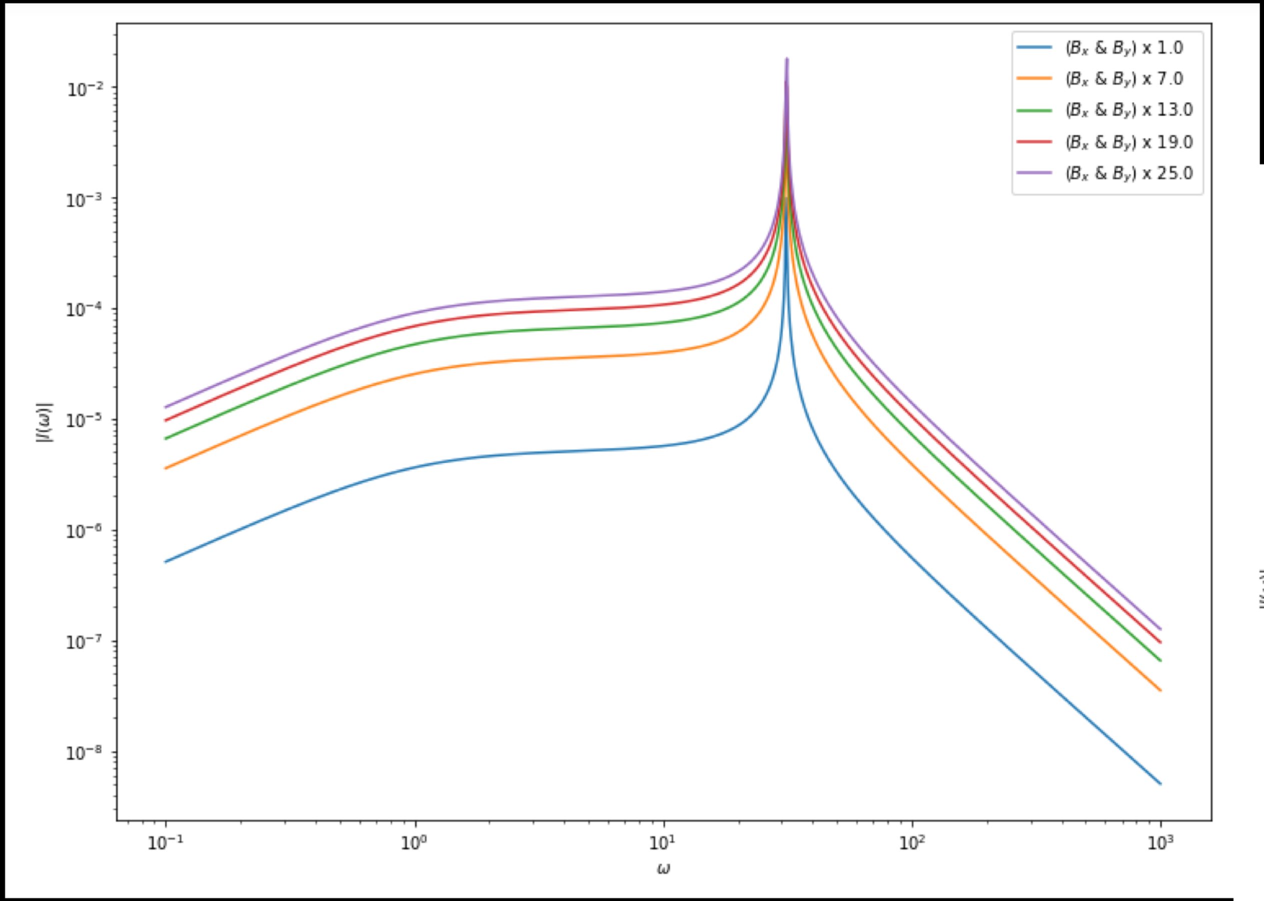
Where

$\tau = L/\mathcal{R}$ (time constant)

${\omega_0}^2 = \kappa/m$ (resonant frequency)

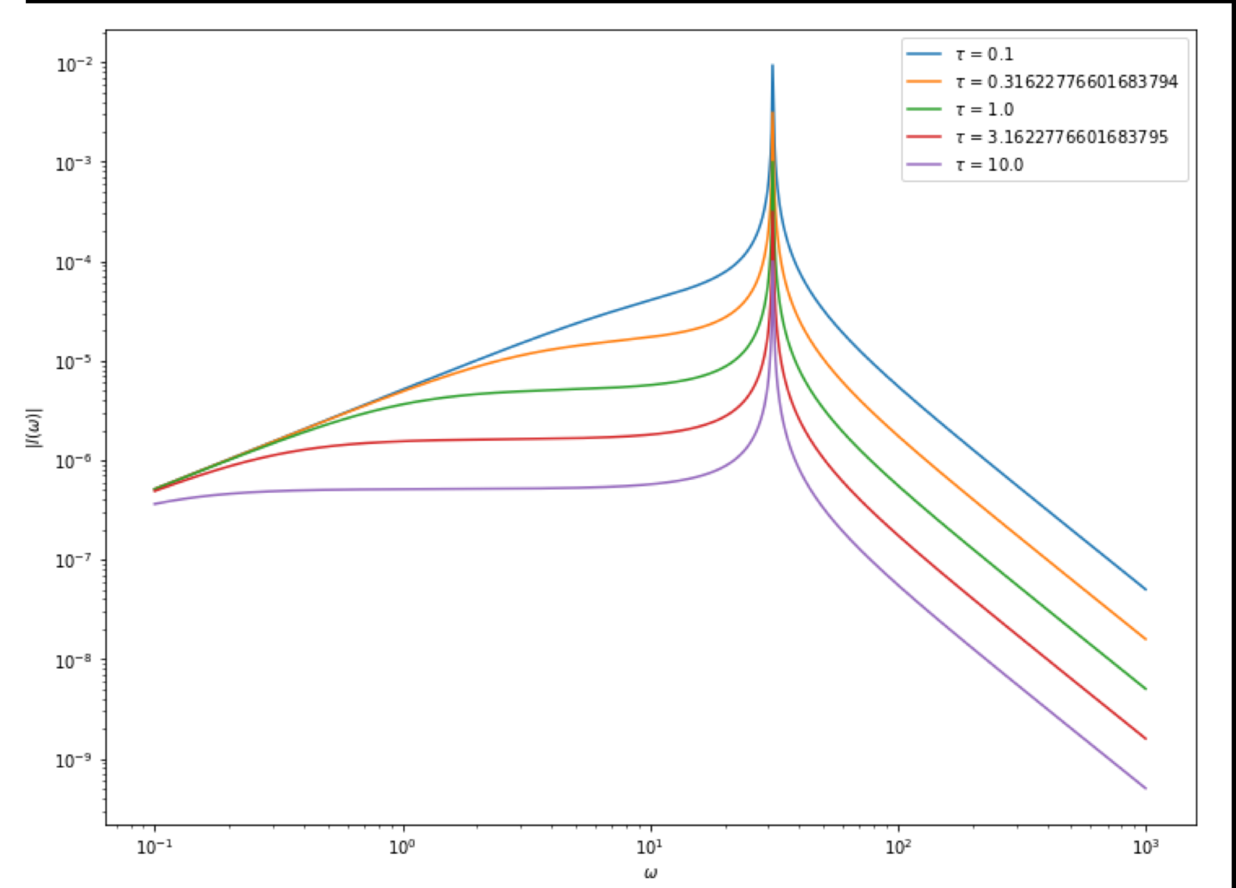
$\xi = \left(\left(\frac{dB(x_0,y_0)}{dx}\right)^2 + \left(\frac{dB(x_0,y_0)}{dy}\right)^2\right)/(\mathcal{R}m)$ (convenient term)

Analytic Approach Varying time constant and B-gradient



Magnitude increases with an increased magnetic field gradient

Magnitude rolls off at lower frequencies for high values of the time constant



Numeric Approach

To use an ODE solver in a non-linear B-field regime, I used a numeric approach to find the Lorentz force and induction

$$m\ddot{\boldsymbol{x}} = \int \boldsymbol{J}\times\boldsymbol{B}\,dV - \kappa\boldsymbol{x} + F_{Driving} \quad \& \quad \nabla\times\rho J = -\frac{\partial B}{\partial t}$$

For the numeric approach I have taken the flowing steps

1. Find the flux through the ring over a grid of different center points
2. Move the ring on the flux grid
3. Find the induced voltage and power
4. Find the damping coefficient
5. Solve for motion of the ring

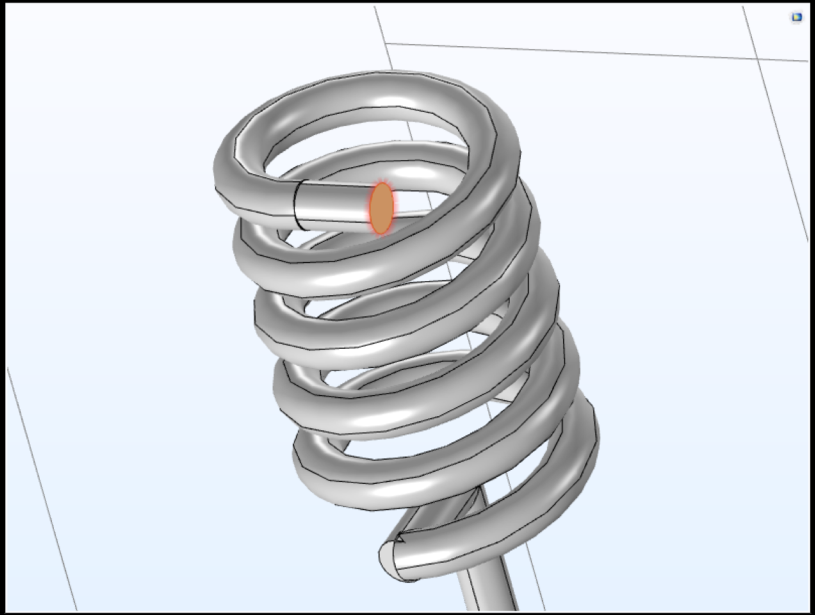
COMSOL Random Vibration Analysis

- Relies on load-based PSD input and output as a function of frequency only
  - Linear transfer functions are assumed
  - Correlation of multiple PSDs can be specified
- Based on Reduced order model framework
  - Black box with inputs and outputs, using the COMSOL Frequency Domain, Model
- Runs three different studies
  - Eigenfrequency
  - Frequency Domain
  - Model Reduction
    - The first two studies are run through the last study

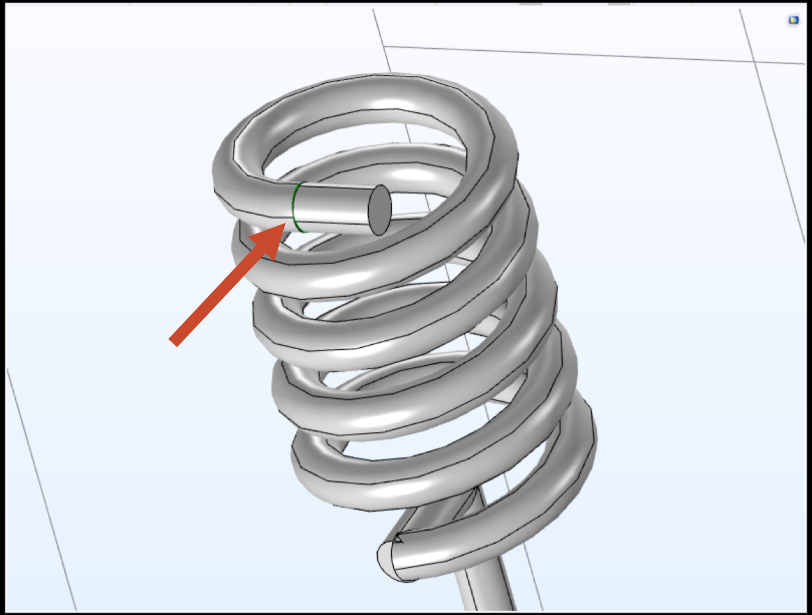
COMSOL Simulation of a Pendulum attached to a spring

Basic simplified ABRACADABRA setup

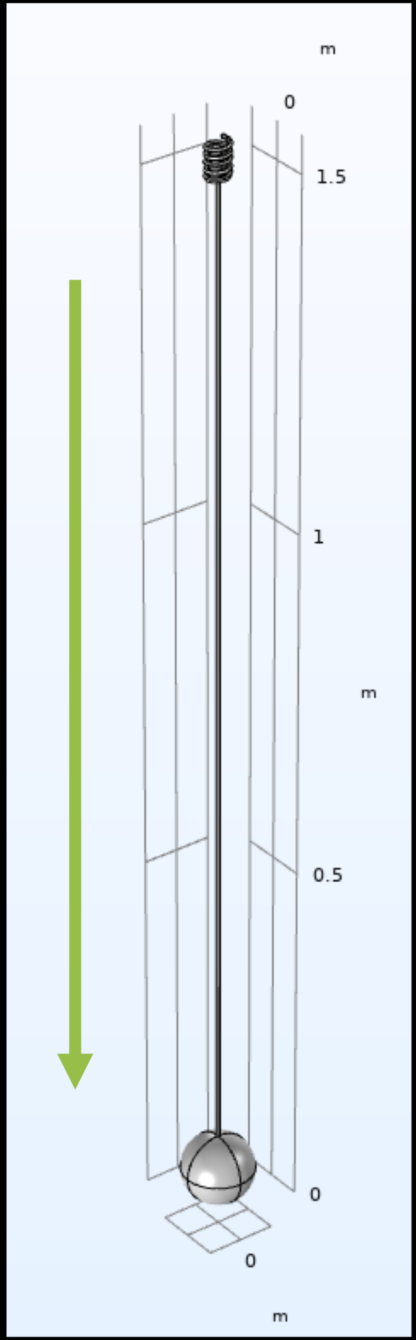
- Spring, cable, ball



Fixed at the end point

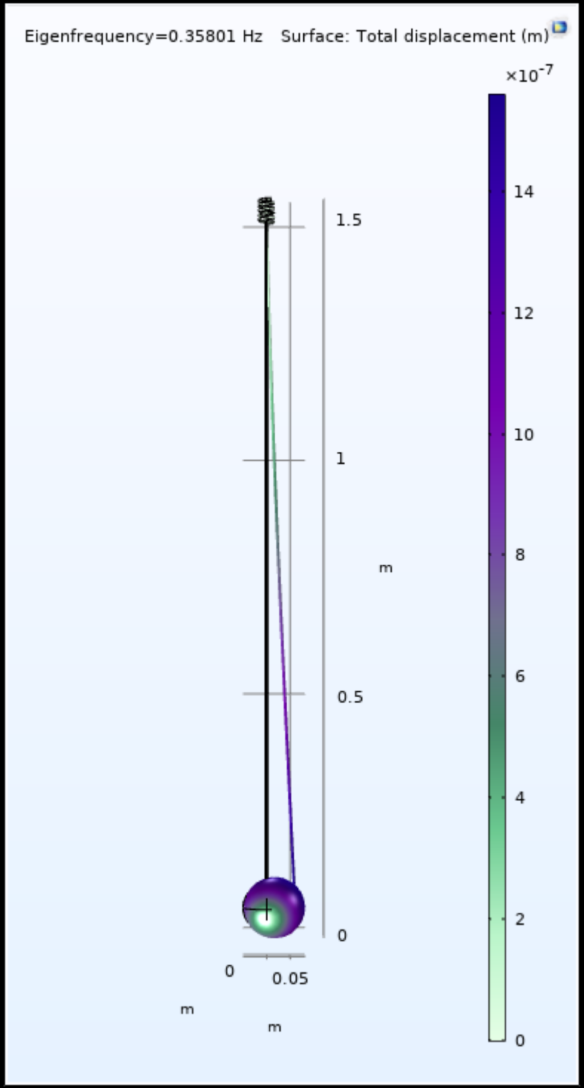


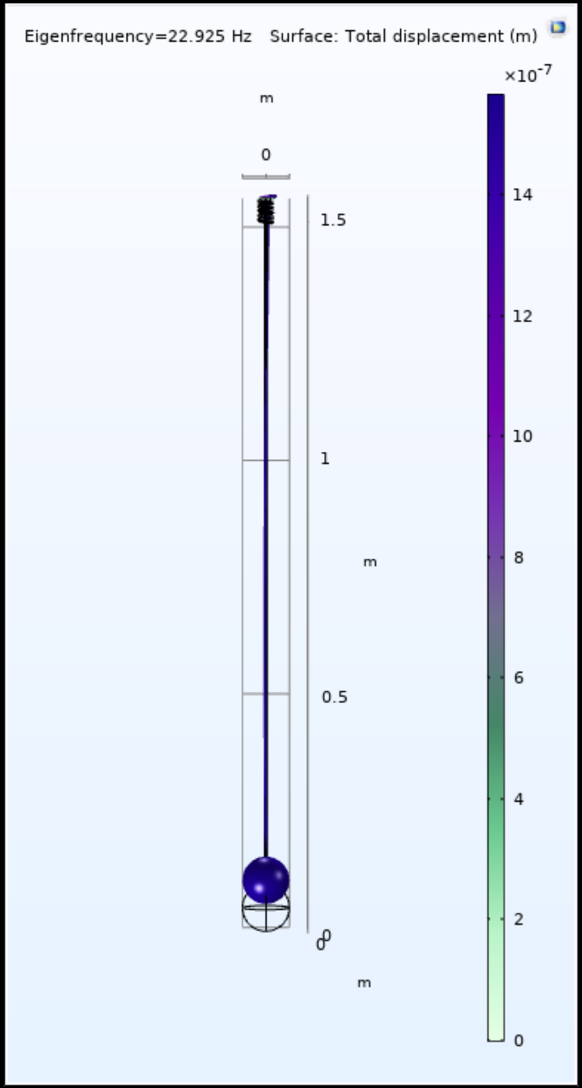
Load

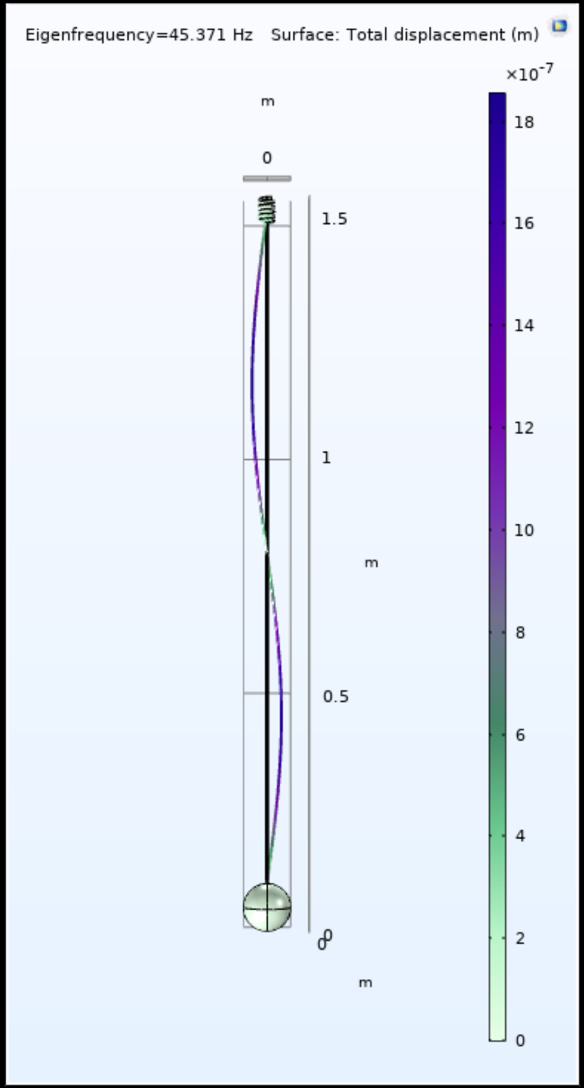


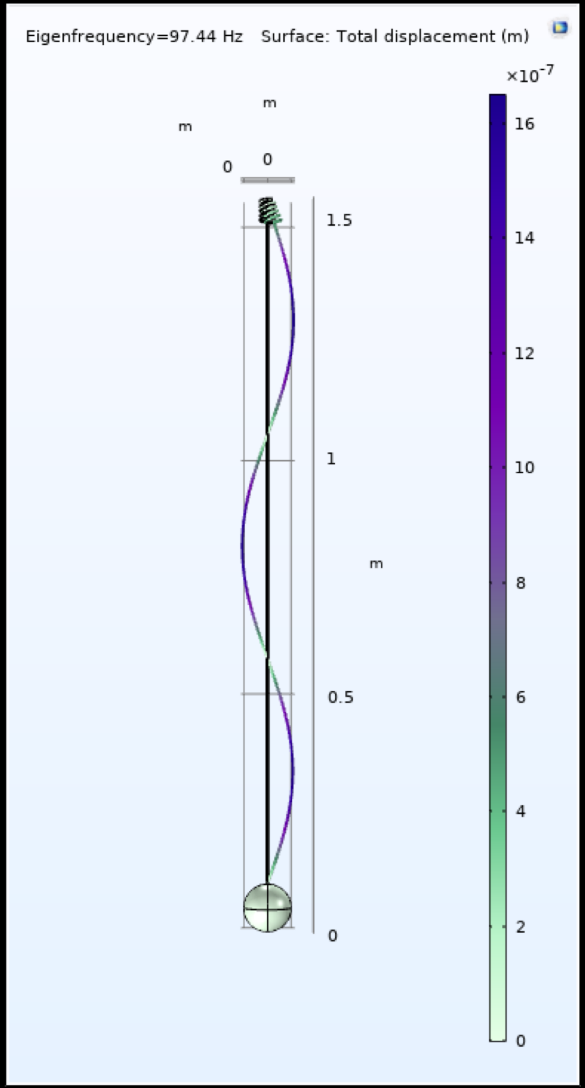
Gravity

After running the Eigenfrequency study



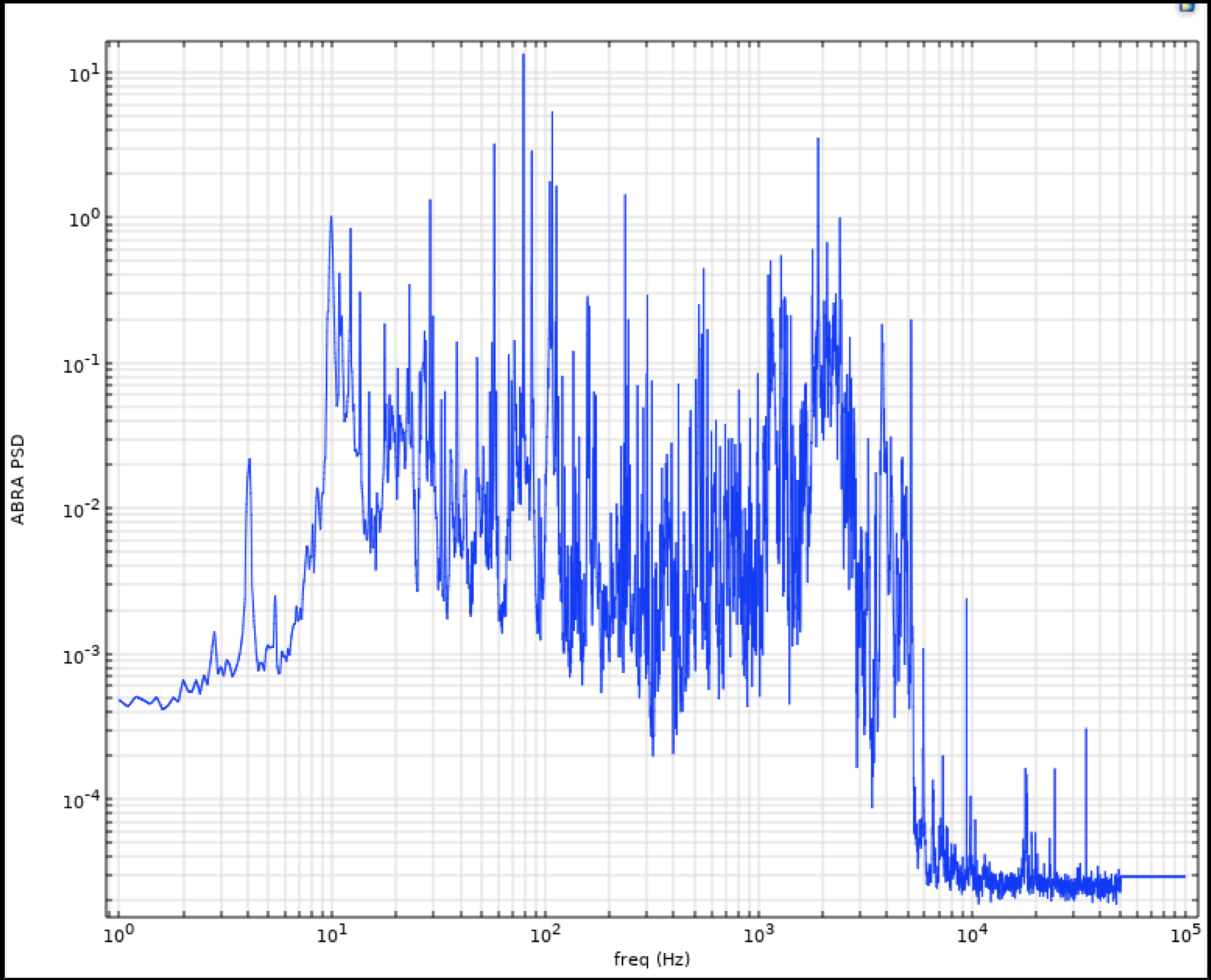






COMSOL ABRA Random Vibration Analysis

Inputted PSD taken from accelerometer at the top of the cryostat

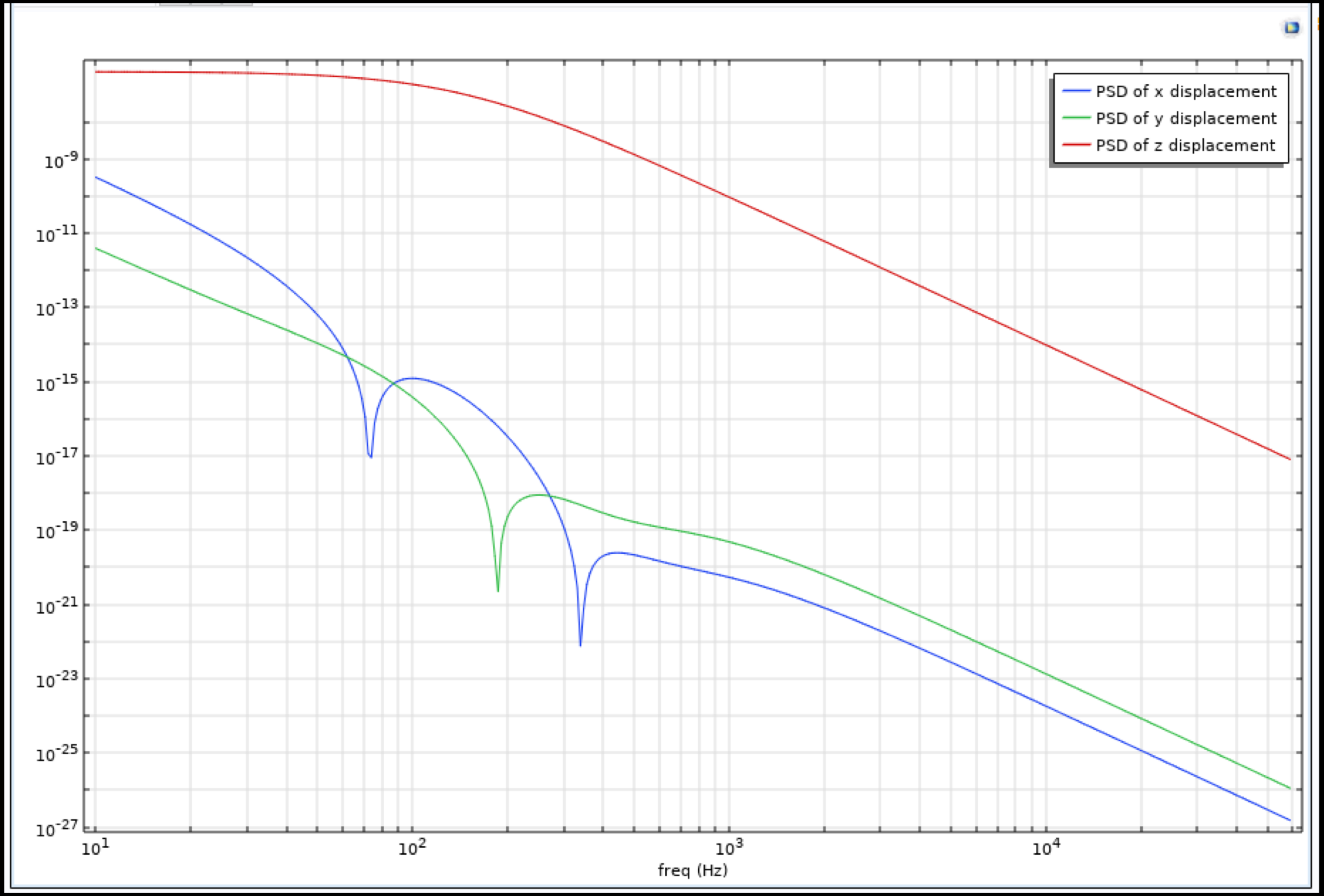


After running the Random Vibration Analysis

Here are the results of the PSD data on displacement indifferent directions

There are many different options of PSD data that COMSOL can calculate

This PSD data could then be inputted to another random vibration analysis



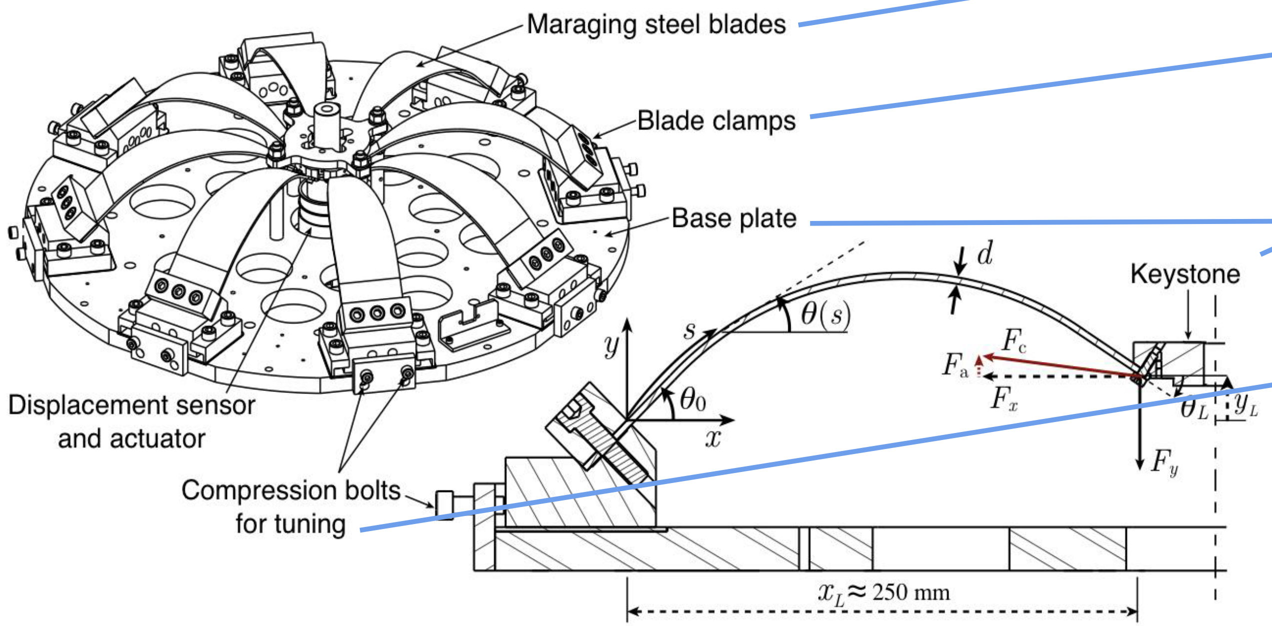
Designing a Vibration Isolation System for ABRA

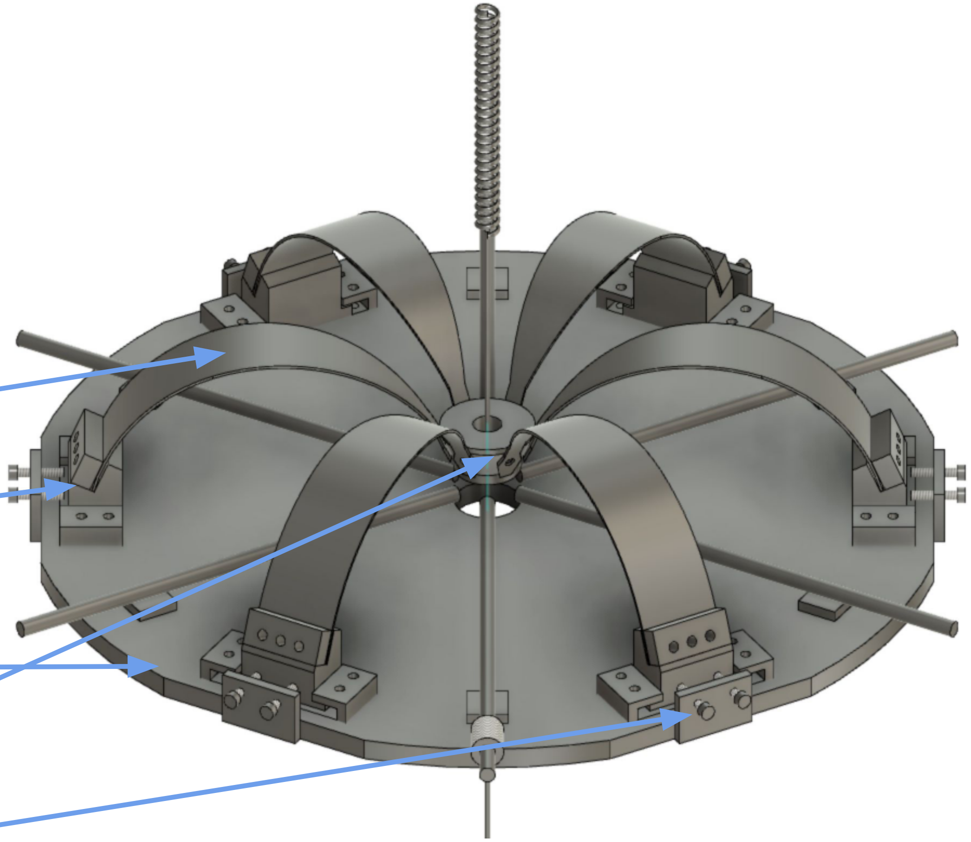
An MIT undergrad, Sabrina Cheng, has done a lot of work recently to design a vibration isolation system for the current ABRA structure

The following slide was made by Sabrina

Geometric Anti-spring (GAS) Filters

- Tune horizontal compression x_L → more compression = less stiff = lower natural frequency
- Tune center of percussion → change mass distribution to achieve a $\frac{1}{f^2}$ attenuation at high frequencies
- Takes advantage of horizontal space





Fusion 360 drawing by Sabrina Cheng (MIT '21)

https://research.vu.nl/ws/portalfiles/portal/42110552/chapter+5.pdf

Summary

All of these analyses are works in progress, suggestions are welcome

The methods shown can be adapted to suit DM Radio 50L or m^3

Recap:

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