Quasistatic sheath simulations/c_{PU}

DM Radio collaboration meeting

August 13, 2020

Chiara P. Salemi

Goal: know our experimental sensitivity

$$\mathscr{P}_{FOM} = c_{PU} \frac{B_0 V^{5/6} Q^{1/4}}{\eta^{1/4} T^{1/4}}$$

- Sheath simulations give c_{PU}^2
  - Fraction of available energy that can be coupled into experiment and measured
- Dependent on
  - Total volume
  - Form factor
  - Fill factor

$$c_{PU} = f c_{sheath}$$

- c_{sheath} from axion energy coupling into sheath
- f from transfer from sheath onto pickup cylinder

Optimize simulations for coupled energy

axion physics

$$U_{sheath} = \frac{\hbar c}{4\mu_0^2} g_{a\gamma\gamma}^2 \rho_{DM} L_{sheath} \left(\int \vec{B} \cdot d\vec{A} \right)^2$$

from sheath form factor and size

from sheath form factor, fill factor, and size

optimize this!

$$c_{sheath}^2 = \frac{L_{sheath} \left(\int \vec{B} \cdot d\vec{A} \right)^2}{4\mu_0 B_{rms}^2 V_{tor}^{5/3}}$$

this cancels with the numerator in the original FOM expression

*note that I normalize this "U_{proxy}" by B_0

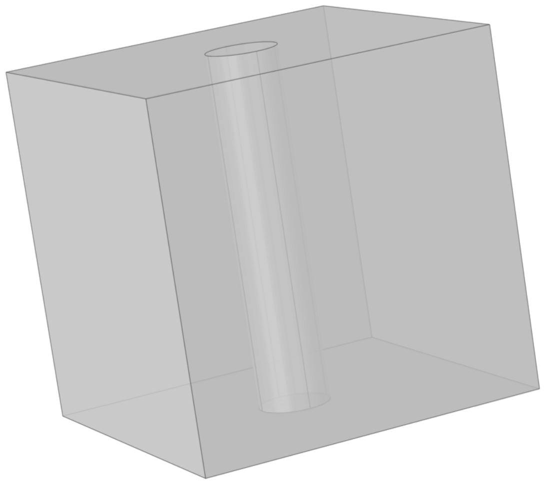
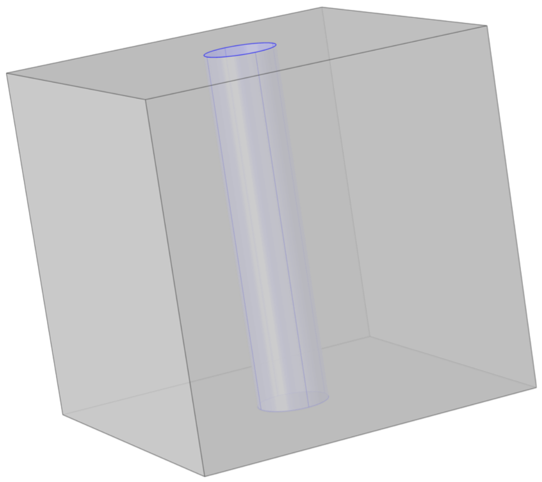
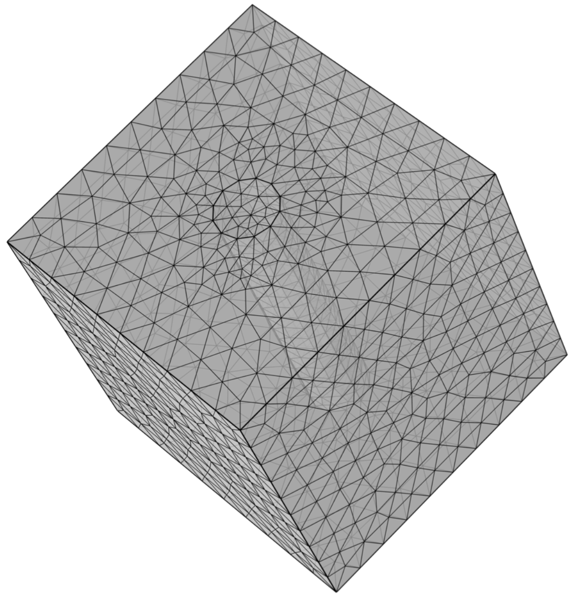
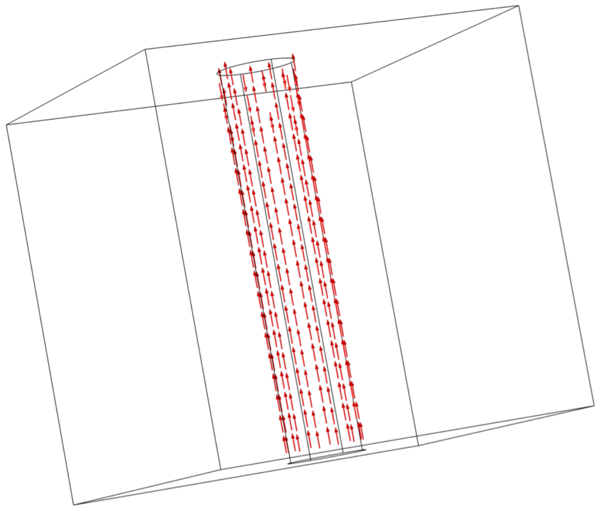
Agenda

- COMSOL 2D-axisymmetric method
- Sheath-only simulations and results (c_{sheath})
- Sheath + pickup simulations and results (f)
- Putting it all together (c_{PU})

method overview

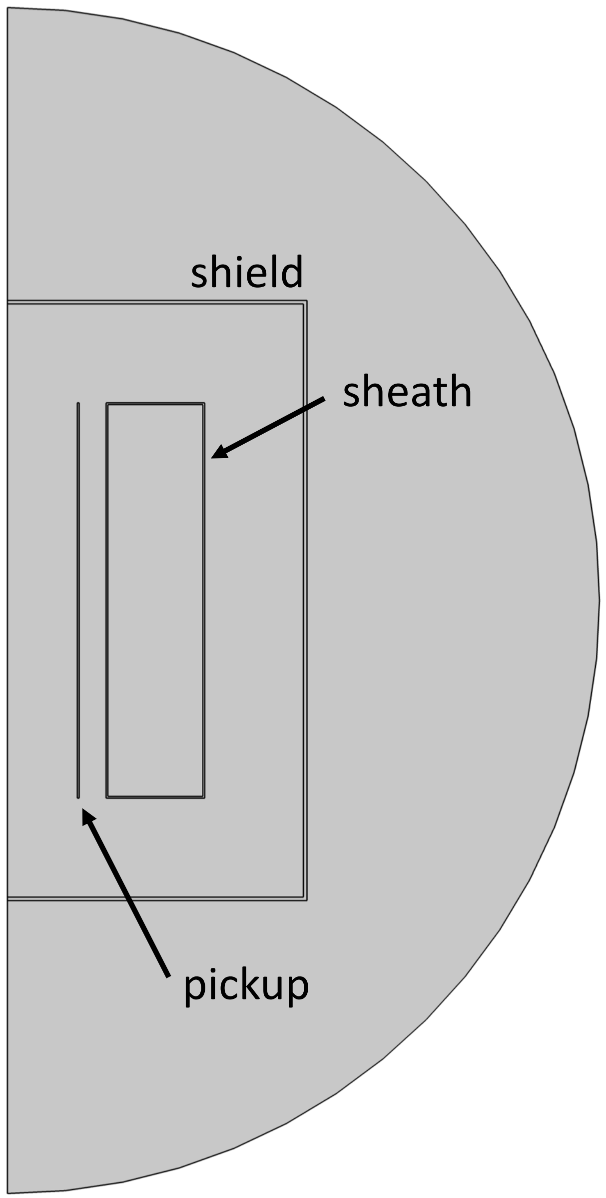
EM modeling with COMSOL Multiphysics

- Finite element
- Can combine different types of physics (see talk by Kaliroë Pappas)
- Here, use quasistatic simulations that solve for magnetic fields

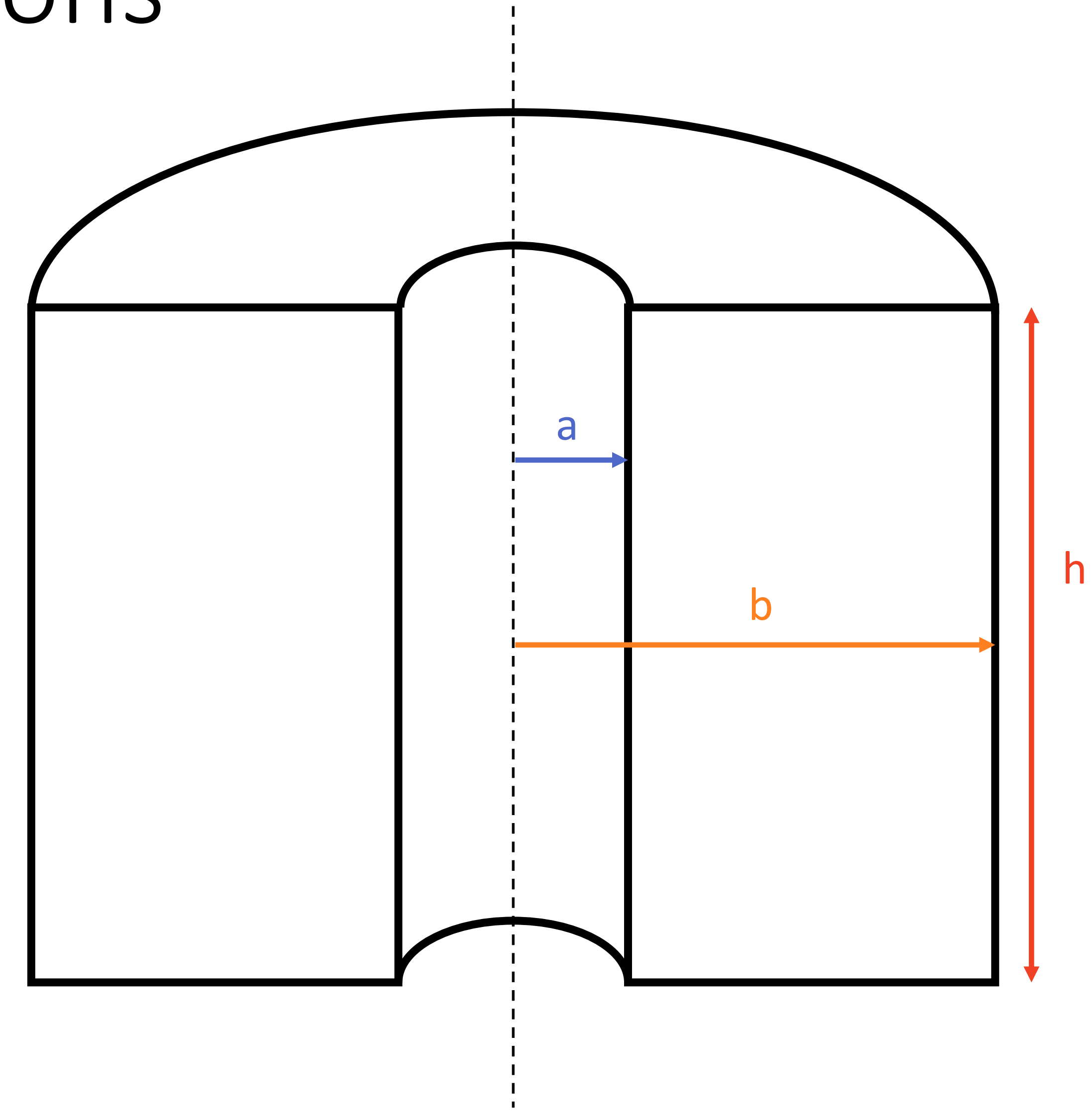





Geometry for these simulations

- 2D axisymmetry
  - Good approximation
  - Greatly simplifies model
- Do not simulate magnet

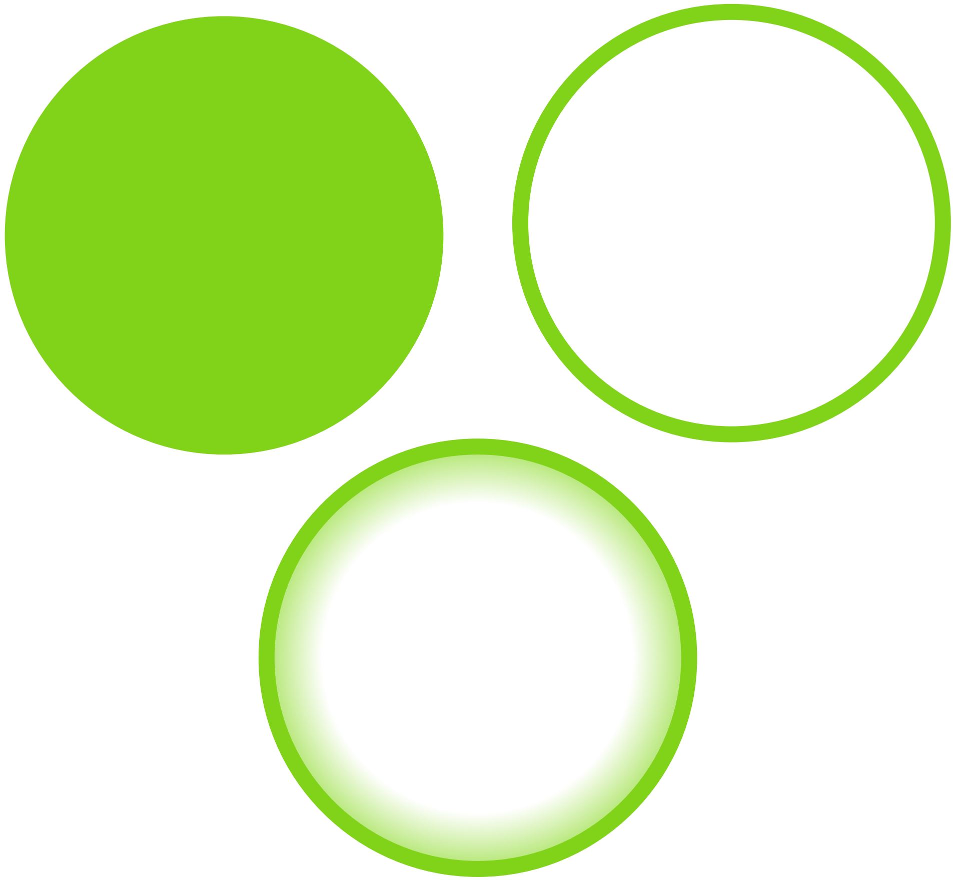


Note on dimensions

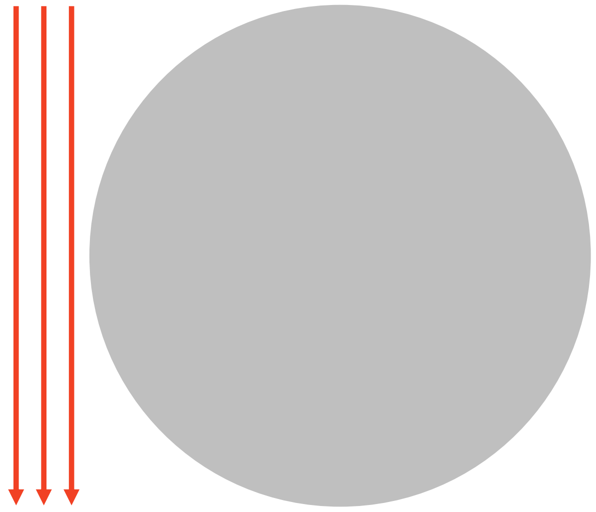


a, b, and h refer to the *magnet* dimensions, not the sheath

Simulating currents multiple ways

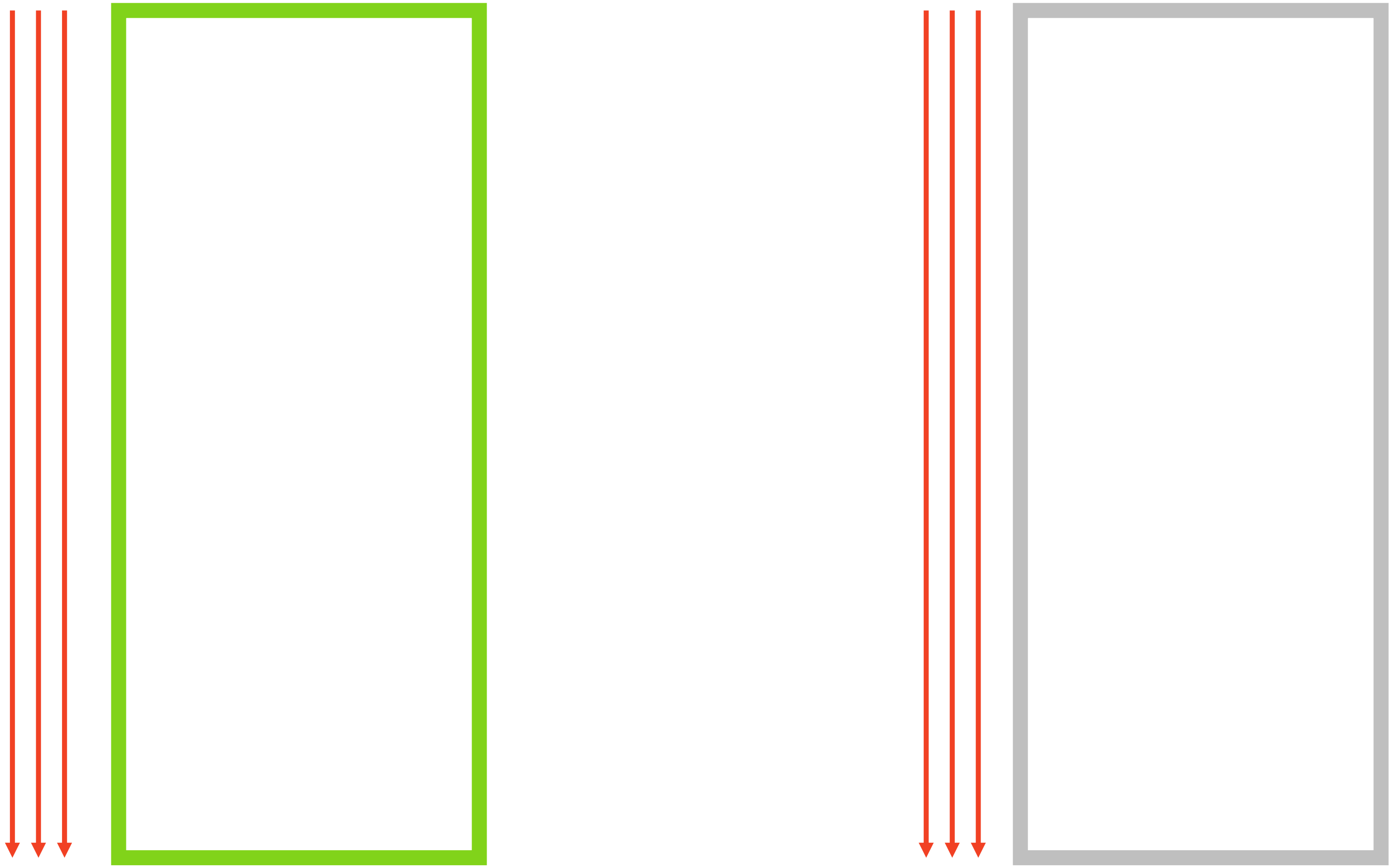


Coil current applied homogeneously,
homogeneously on surface, or self-distributed

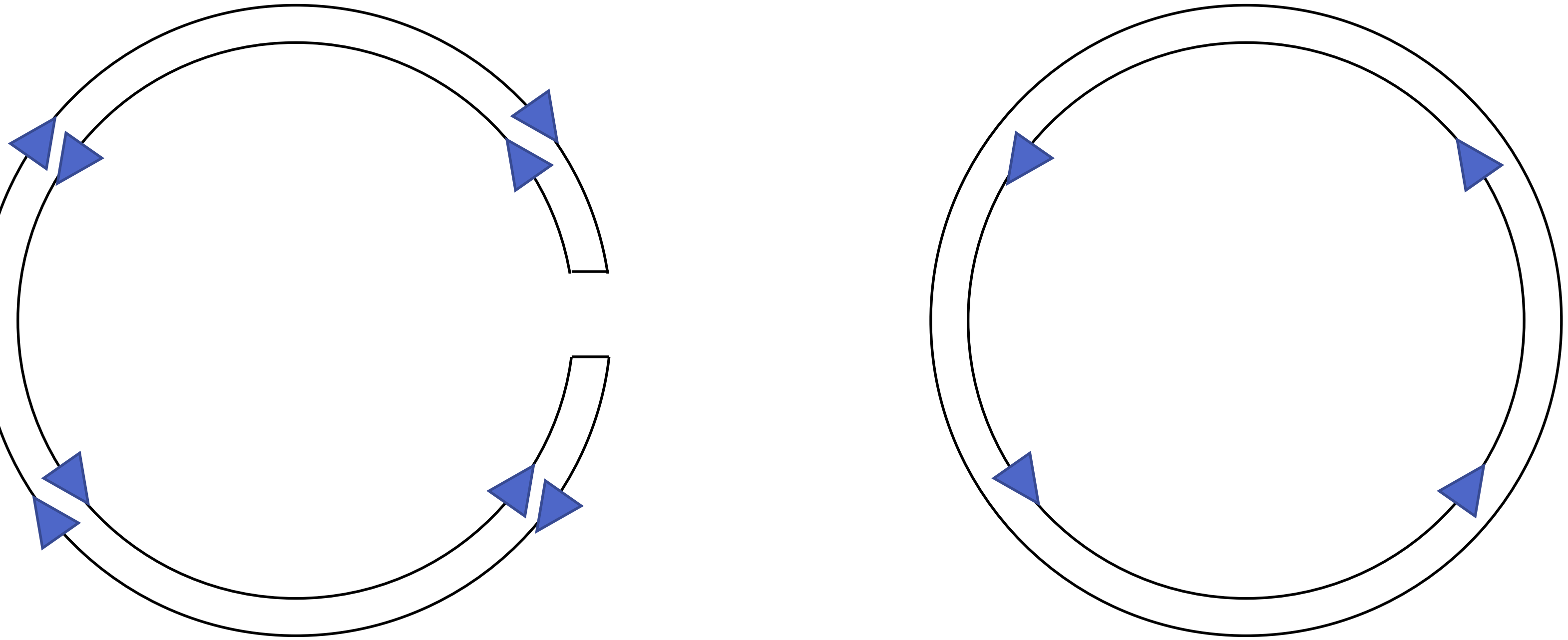


No current, only boundary conditions to
specify magnetic flux

Homogeneous coil results similar to BCs



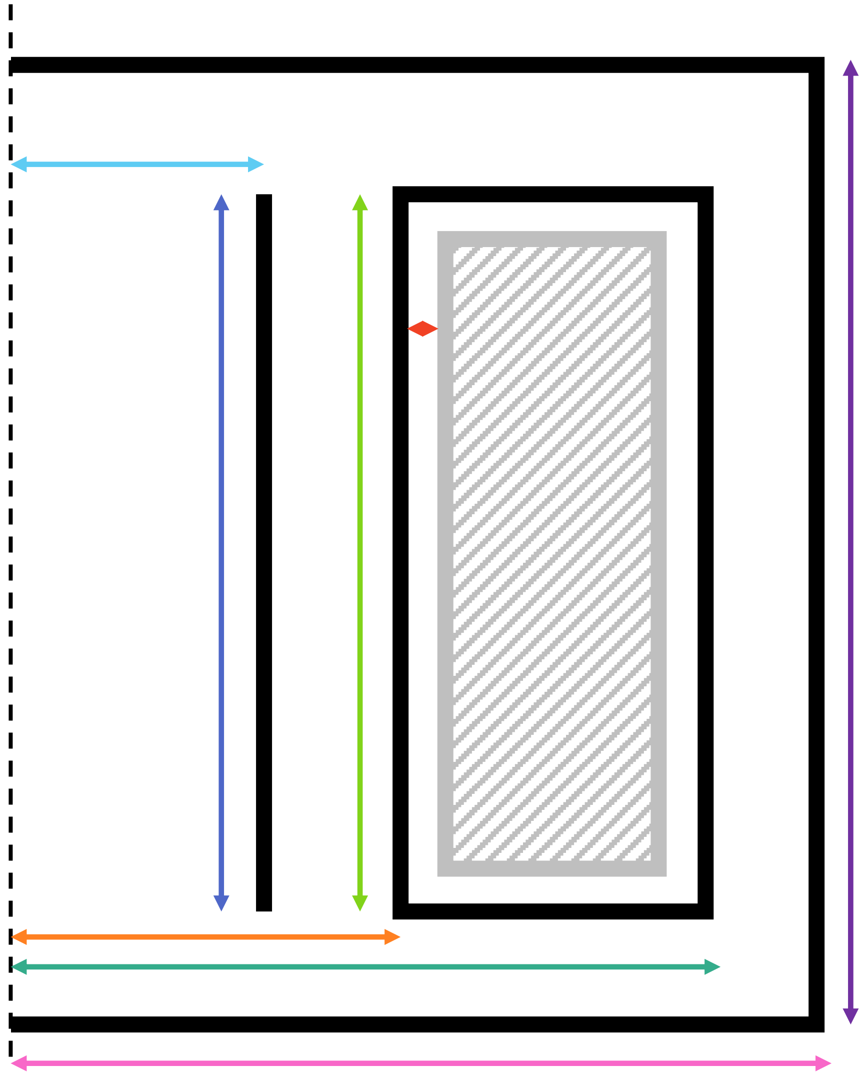
Cannot directly simulate distribution from the axion current—2D axisymmetry means there is no current return path



Goal = maximize coupled energy

Knobs to turn

- Pickup cylinder height
- Pickup radius
- Magnet/sheath offset
- Sheath height
- Sheath inner radius
- Sheath outer radius
- Shield radius
- Shield height



$$c_{sheath}$$

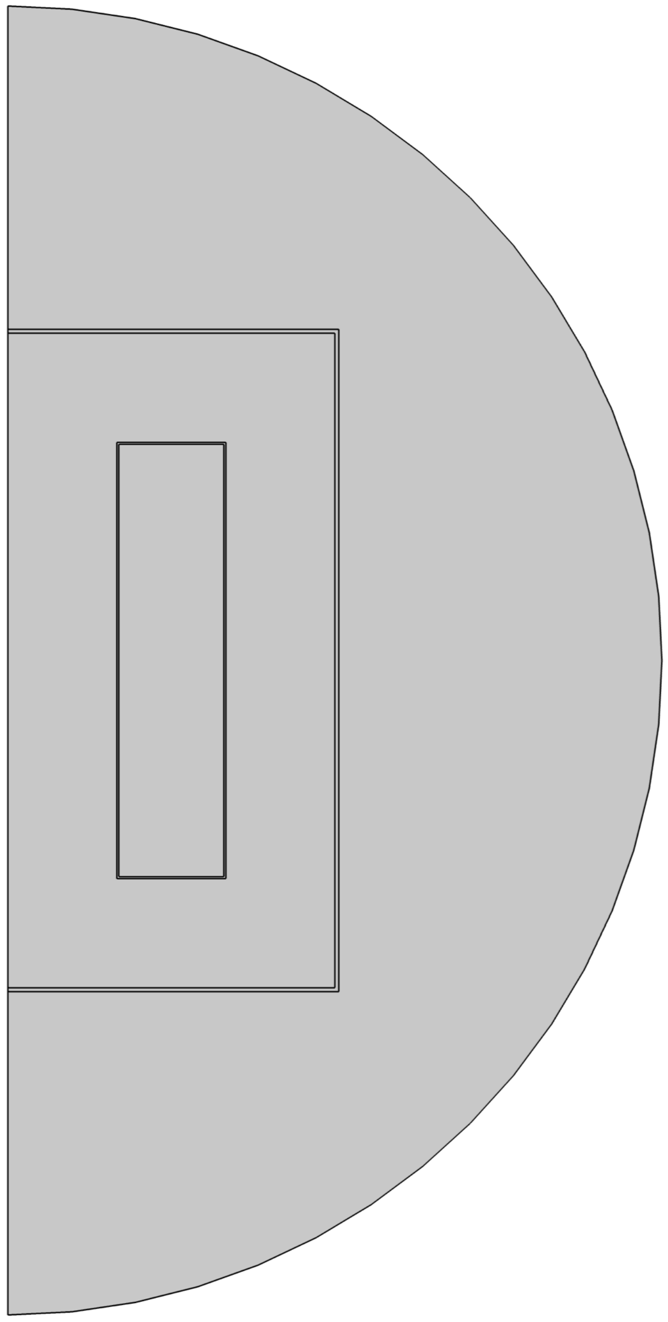
Simulating the sheath

Study steps

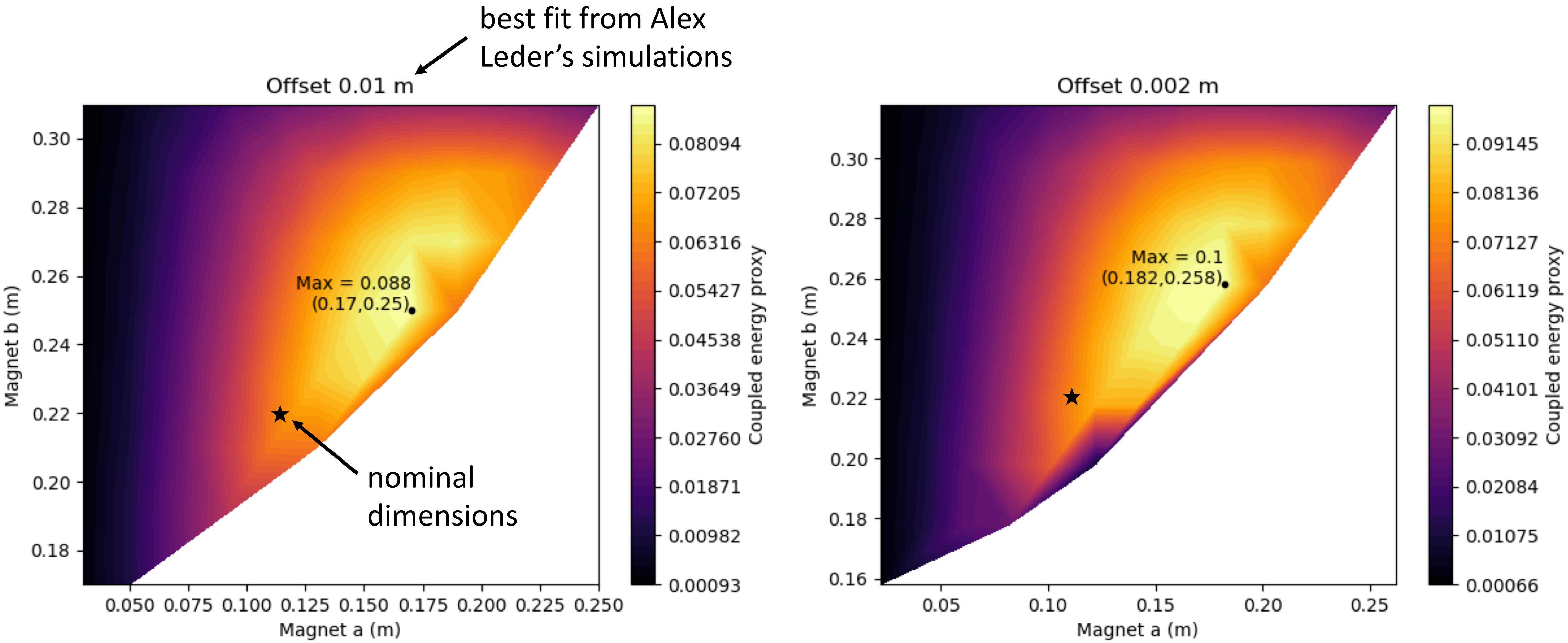
1. Apply coil current/BCs to sheath
2. Measure L_{sheath} with DC simulation
3. Calculate U_{proxy}

Specifics

- Fix magnet volume
- Scan over magnet dimensions
- Scan over magnet/sheath offset



Coupled energy from BCs



Other ingredients for FOM

$$c_{sheath}^2 = \frac{L_{sheath}\left(\int \vec{B}\cdot d\vec{A}\right)^2}{4\mu_0 {B_{rms}}^2 {V_{tor}}^{5/3}}$$

$$B_{rms} = B_0 a\sqrt{\frac{\ln\left(b/a\right)}{b^2 - a^2}}$$

$$V_{tor} = \pi h\left(b^2 - a^2\right)$$

c_{sheath} @ max coupled energy

a = 17 cm

b = 25 cm

h = 47.4 cm

d = 1 cm

$$c_{sheath} = 0.088$$

a = 18.2 cm

b = 25.8 cm

h = 47.6 cm

d = 2 mm

$$c_{sheath} = 0.092$$

c_{sheath} @ 50 L nominal dimensions

a = 11 cm

b = 22 cm

h = 44 cm

d = 1 cm

$\star\, c_{sheath} = 0.088$

a = 11 cm

b = 22 cm

h = 44 cm

d = 2 mm

$c_{sheath} = 0.097$

$$f$$

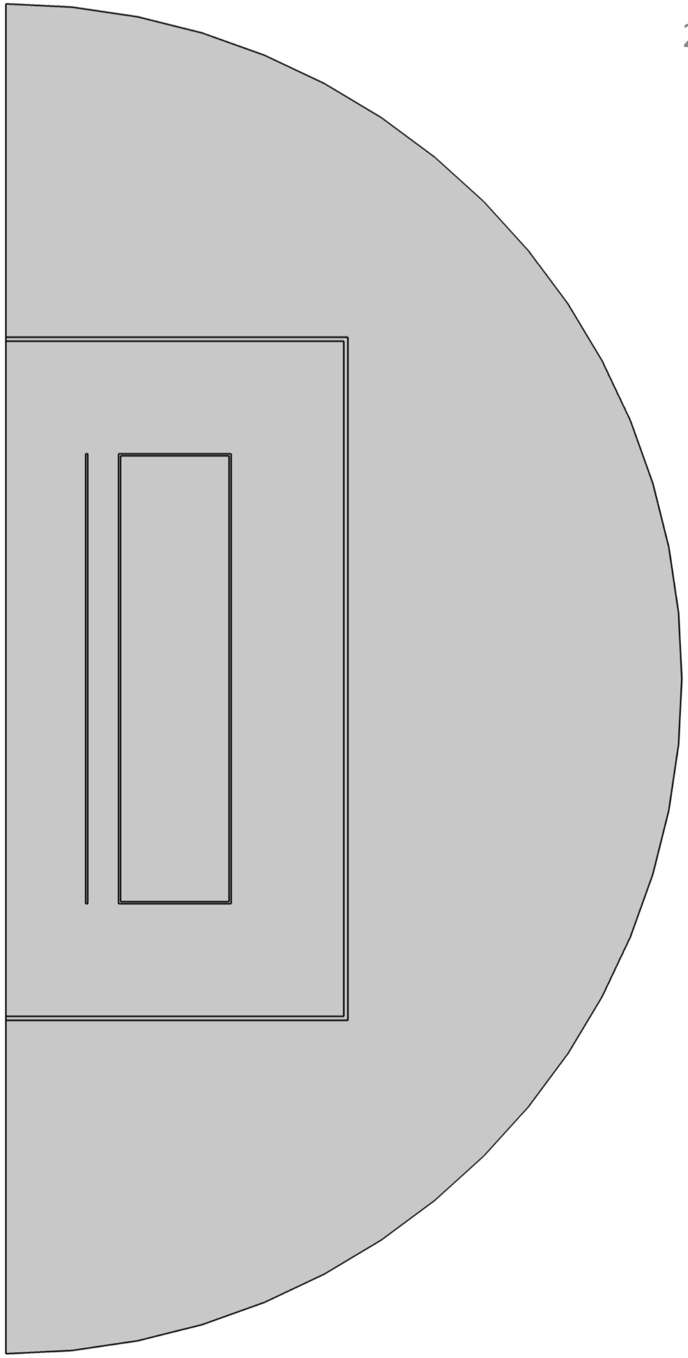
Simulating the transfer

Study steps

1. Apply coil current/BCs to sheath
2. Measure I_{pickup} with AC simulation
3. Calculate M_{ps} and k

Specifics

- Fix sheath dimensions (50 L)
- Scan over pickup dimensions



FOM for transfer

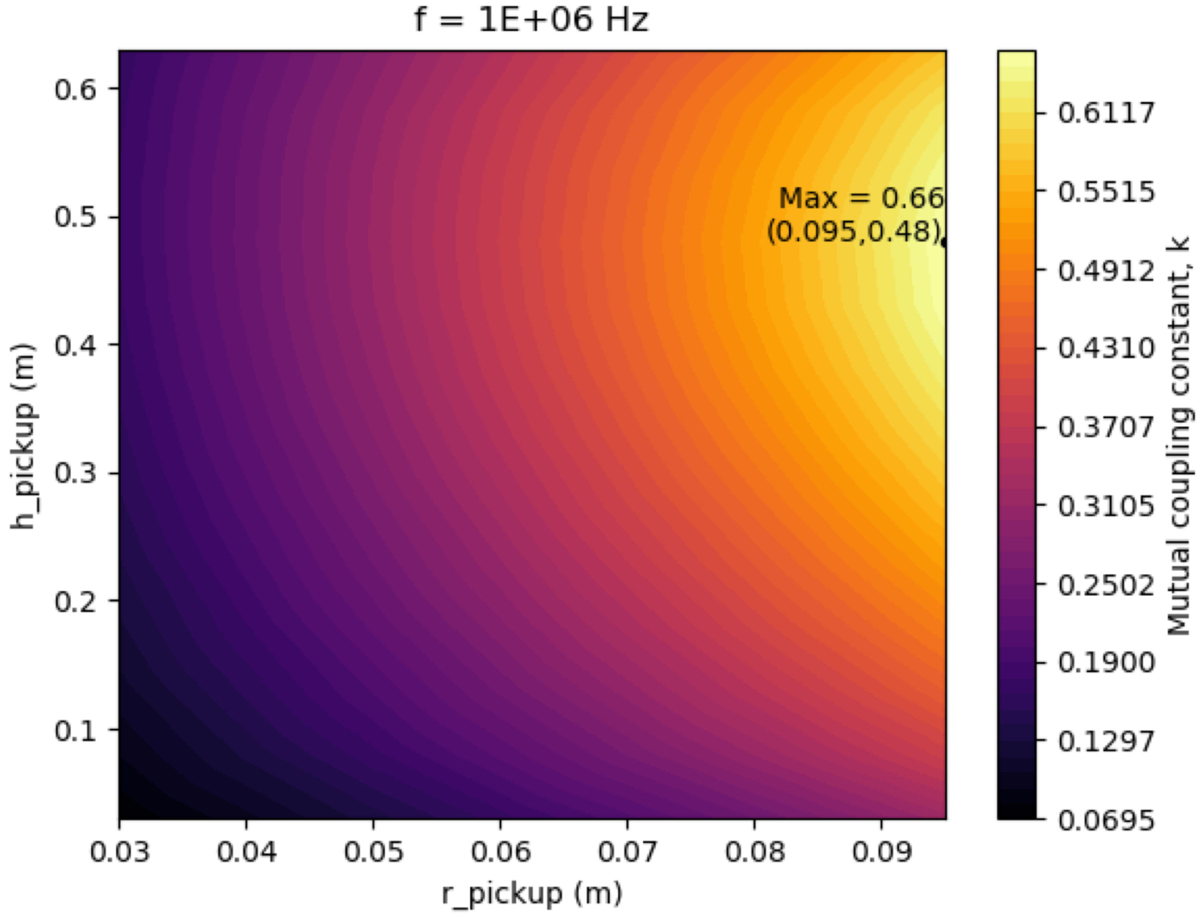
$$\Phi_p = M_{ps} I_s + L_p I_p$$

$$M_{ps} = k\sqrt{L_p L_s}$$

$$k < 1$$

$$k^2 = \frac{U_{pickup}}{U_{sheath}} = f^2$$

Transfer with coil current on sheath



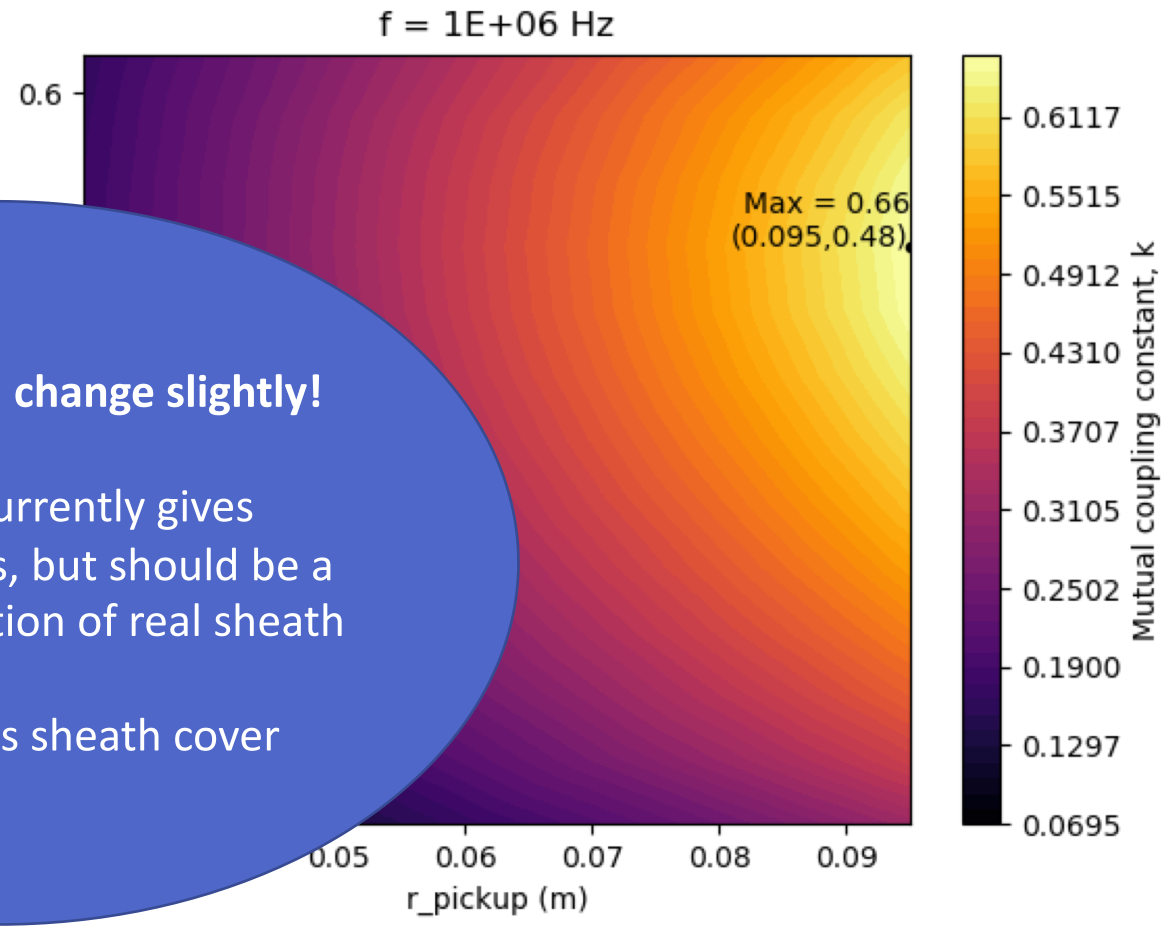
$$k = f = 0.66$$

r_p = 9.5 cm

h_p = 48 cm

PRELIMINARY

Transfer with coil current on sheath



This number will change slightly!

BC method currently gives unphysical results, but should be a better approximation of real sheath

Pickup overlaps sheath cover

$$k = f = 0.66$$

r_p = 9.5 cm

h_p = 48 cm

PRELIMINARY

$$c_{PU}$$

- 50 L nominal magnet dimensions
  - a = 11 cm
  - b = 22 cm
  - h = 44 cm
- Cylindrical shield
  - r_{shield} = 33 cm
  - h_{shield} = 66 cm
- 1 cm magnet/sheath offset
- Cylindrical pickup
  - r_{pickup} = 9.5 cm
  - h_{pickup} = 48 cm

- 50 L nominal magnet dimensions
  - a = 11 cm
  - b = 22 cm
  - h = 44 cm
- Cylindrical shield
  - r_{shield} = 33 cm
  - h_{shield} = 66 cm
- 1 cm magnet/sheath offset
- Cylindrical pickup
  - r_{pickup} = 9.5 cm
  - h_{pickup} = 48 cm

$$c_{PU} = f c_{sheath} = 0.058$$

Backup

derivations
extra plots
toroid parasitic resonance

Derivations

Couple axions to sheath

- Energy coupled onto sheath from axion effective current

$$U_{sheath} = \frac{1}{4} L_{sheath} I_{axion}^2$$

- Axion effective current density

$$\vec{J}_{eff} = \frac{\sqrt{\hbar c}}{\mu_0} g_{a\gamma\gamma} \sqrt{\rho_{DM}} \int \vec{B} \cdot d\vec{A}$$

- Coupled energy with integrated axion current

$$U_{sheath} = \frac{\hbar c}{4\mu_0^2} g_{a\gamma\gamma}^2 \rho_{DM} L_{sheath} \left(\int \vec{B} \cdot d\vec{A} \right)^2$$

Fitting this into the standard equation*

$$U_c = \left(c_{sheath}\kappa_a c B_{rms} V_{tor}^{1/3}\right)^2 \rho_{DM} V_{tor} \qquad \kappa_a = g_{a\gamma\gamma}\sqrt{\hbar c \epsilon_0}$$

$$U_c = c_{sheath}^2 g_{a\gamma\gamma}^2 \hbar c^3 \epsilon_0 \rho_{DM} B_{rms}^2 V_{tor}^{5/3}$$

$$\frac{\hbar c}{4\mu_0^2} g_{a\gamma\gamma}^2 \rho_{DM} L_{sheath} \left(\int \vec{B}\cdot d\vec{A}\right)^2 = c_{sheath}^2 g_{a\gamma\gamma}^2 \hbar c^3 \epsilon_0 \rho_{DM} B_{rms}^2 V_{tor}^{5/3}$$

$$c_{sheath}^2 = \frac{L_{sheath}\left(\int \vec{B}\cdot d\vec{A}\right)^2}{4\mu_0 B_{rms}^2 V_{tor}^{5/3}}$$

*see *Optimization of a Toroidal Experiment…* Eqn. 21

note that I use c_{sheath} rather than c_{PU} here to allow a non-unity coupling to the pickup

Calculating f

$$f^2 = \frac{U_{pickup}}{U_{sheath}} = \frac{L_{pickup} I_{pickup}^2}{L_{sheath} I_{sheath}^2}$$

$$\Phi_p = M_{ps} I_s - L_p I_p = 0$$

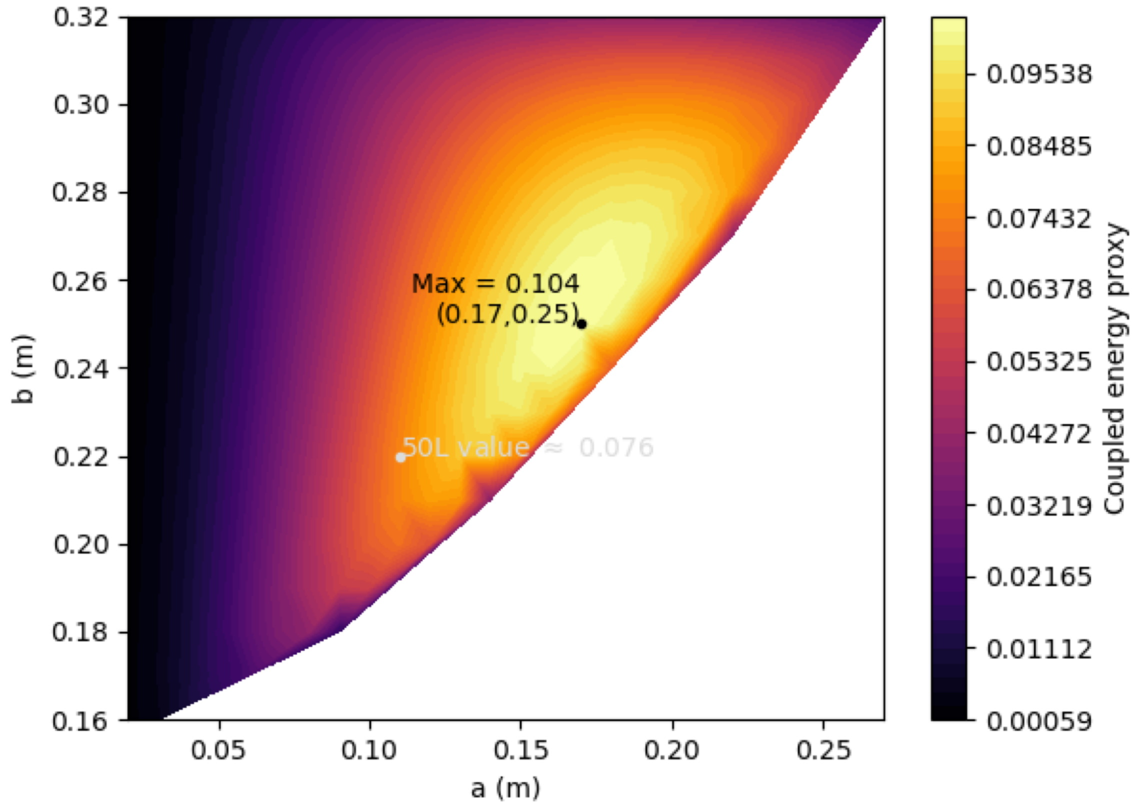
$$\frac{I_p}{I_s} = \frac{M_{ps}}{L_p} = \frac{k\sqrt{L_p L_s}}{L_p} = k\sqrt{\frac{L_s}{L_p}}$$

$$k = \frac{I_p}{I_s}\sqrt{\frac{L_p}{L_s}}$$

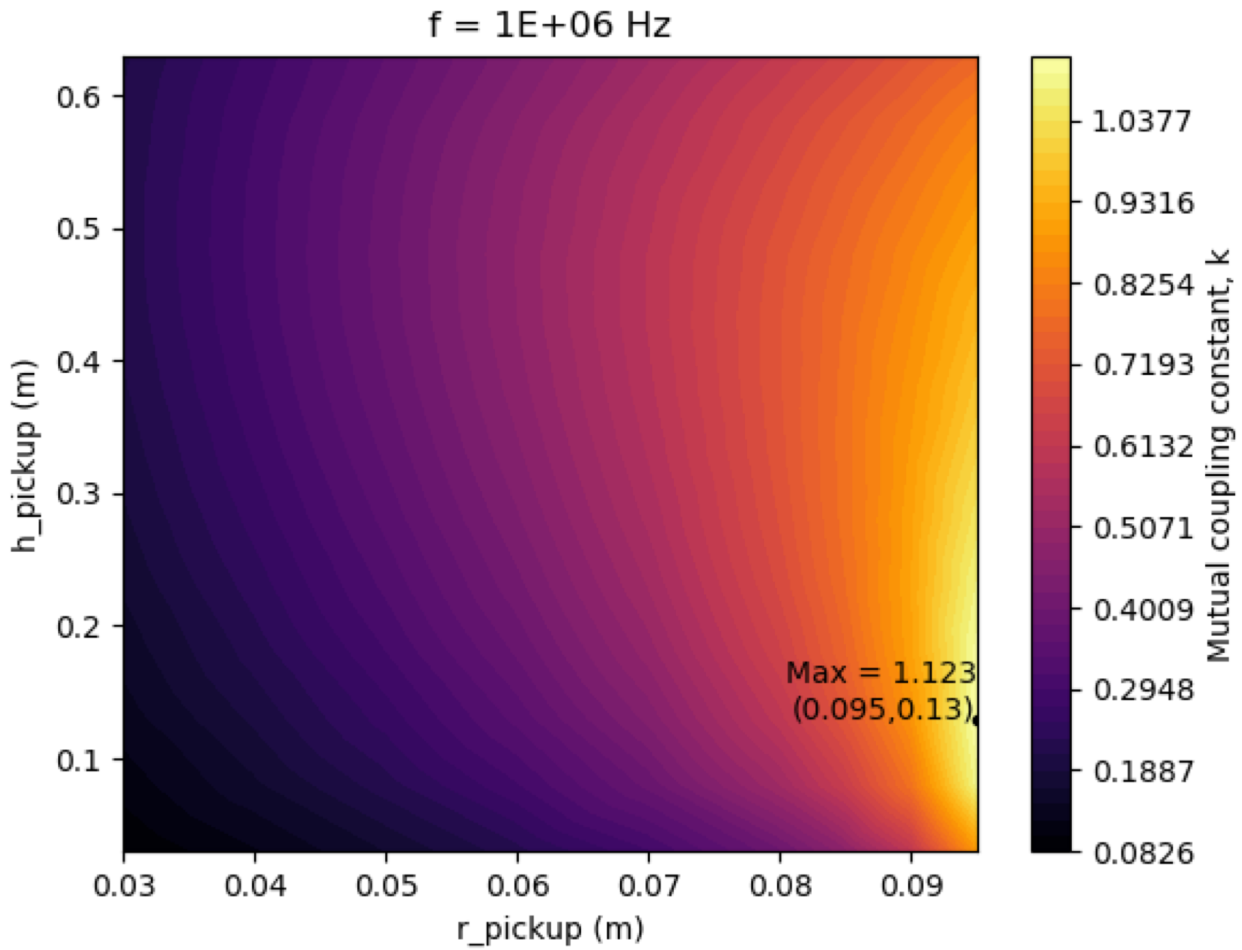
$$k^2 = \frac{L_p I_p^2}{L_s I_s^2}$$

Extra plots

Zero offset, c_{sheath}=0.096



Transfer with BCs on sheath, k>1

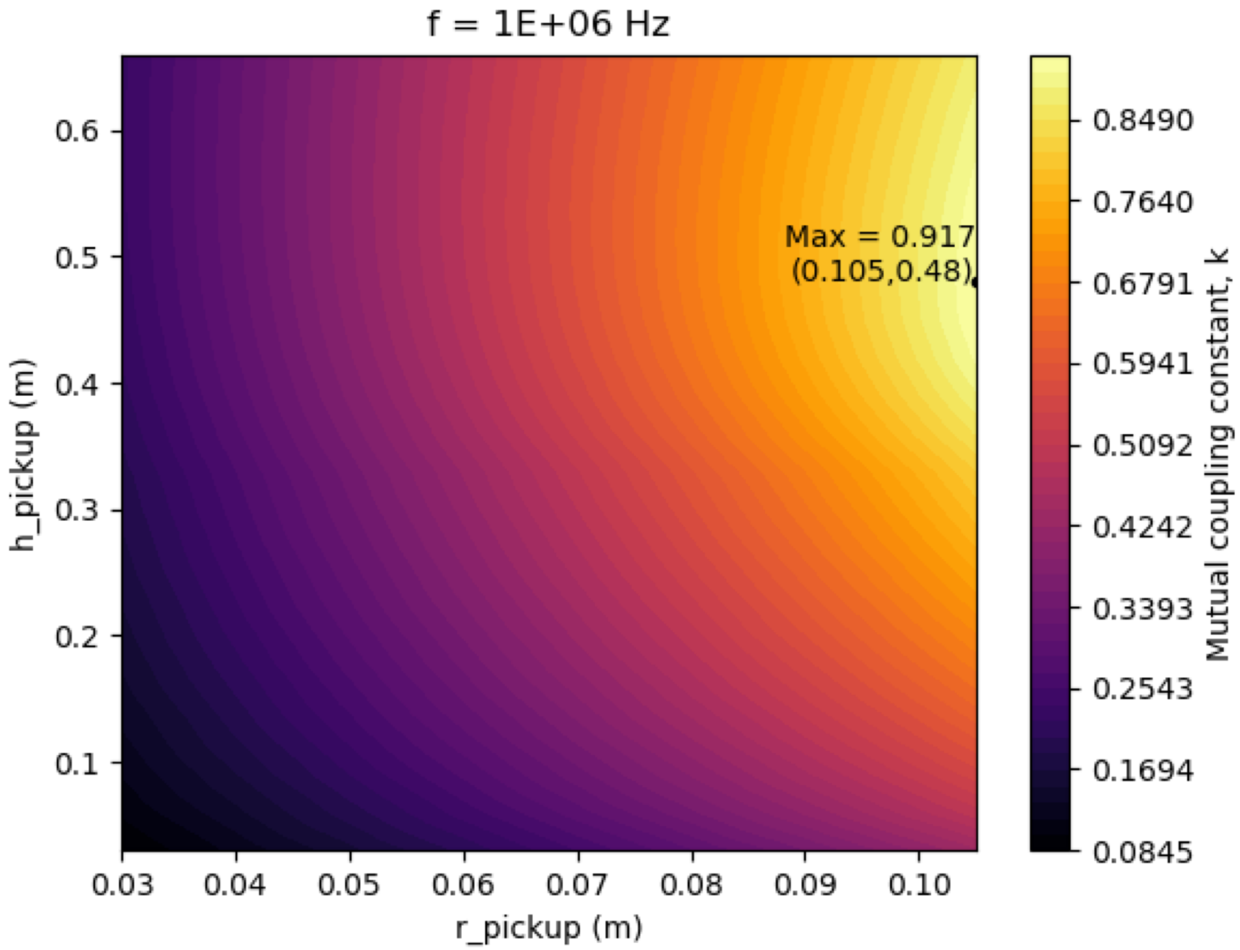


It sure would be nice if by adding a pickup we could increase our sensitivity!

Alas… conservation of energy

Transfer with coil current on sheath

no magnet/sheath offset



$$k = f = 0.917$$

r_p = 10.5 cm

h_p = 48 cm

Parasitic resonance

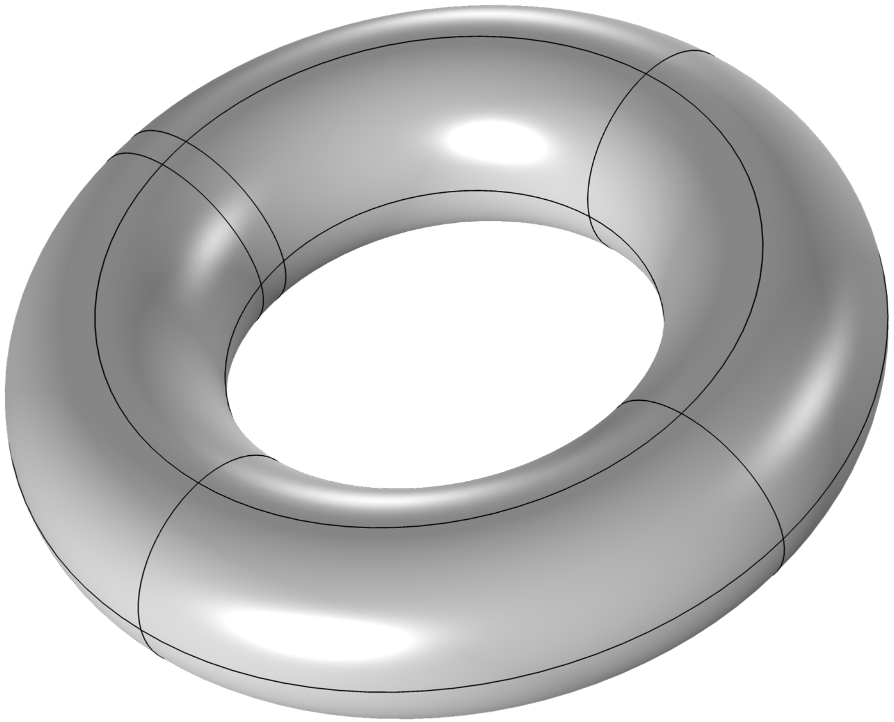
Converge-able method to find toroid resonance

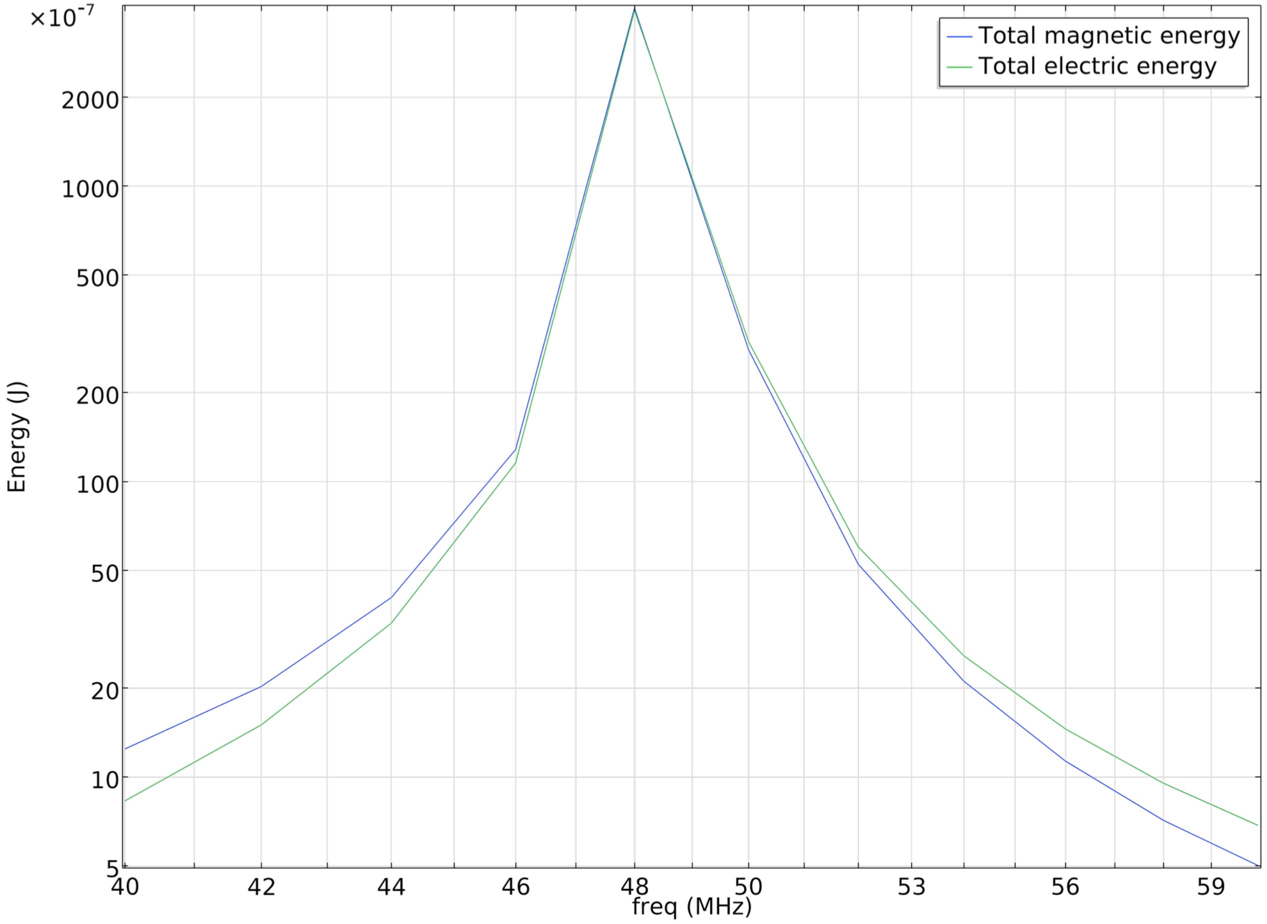
- Magnetic fields module
- Lumped port excitation
- Assume currents are surface currents
  - Metal region not solved for
- Currently, surface is copper
- Enclosed in “SC” shield

Approximate dimensions

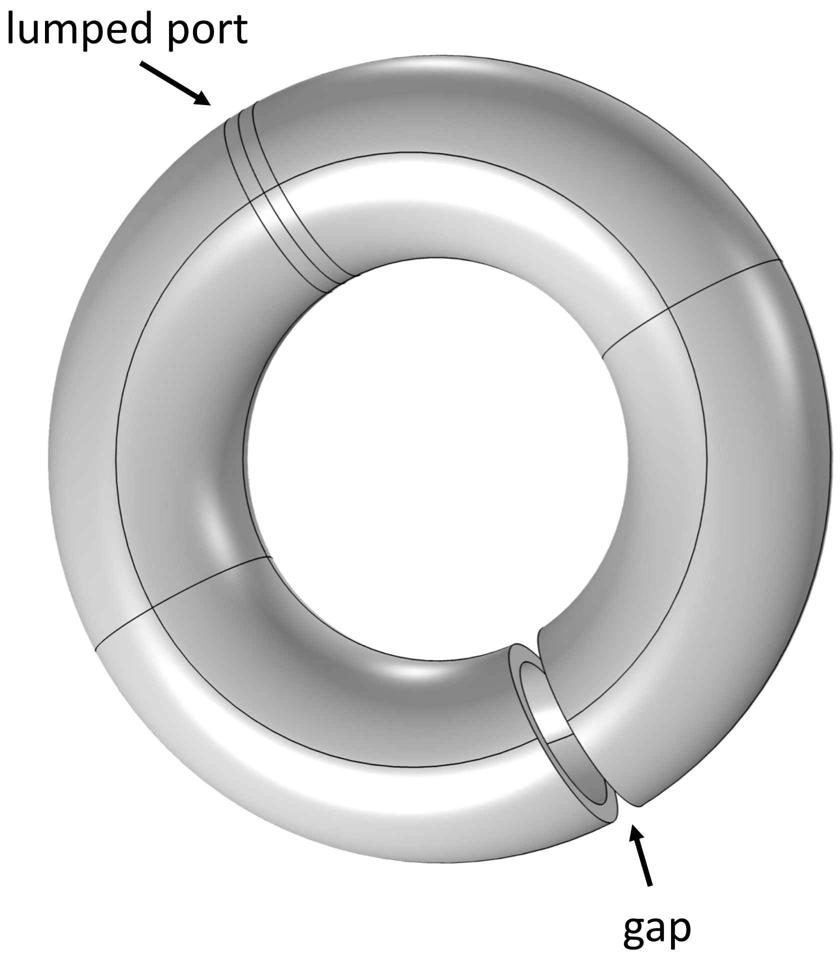
- Scale 50 L dimensions by cube root of 20=2.7
  - a = 0.297 m
  - b = 0.594 m
  - h = 1.188 m
- For now, using circular cross section
  - diameter = b – a

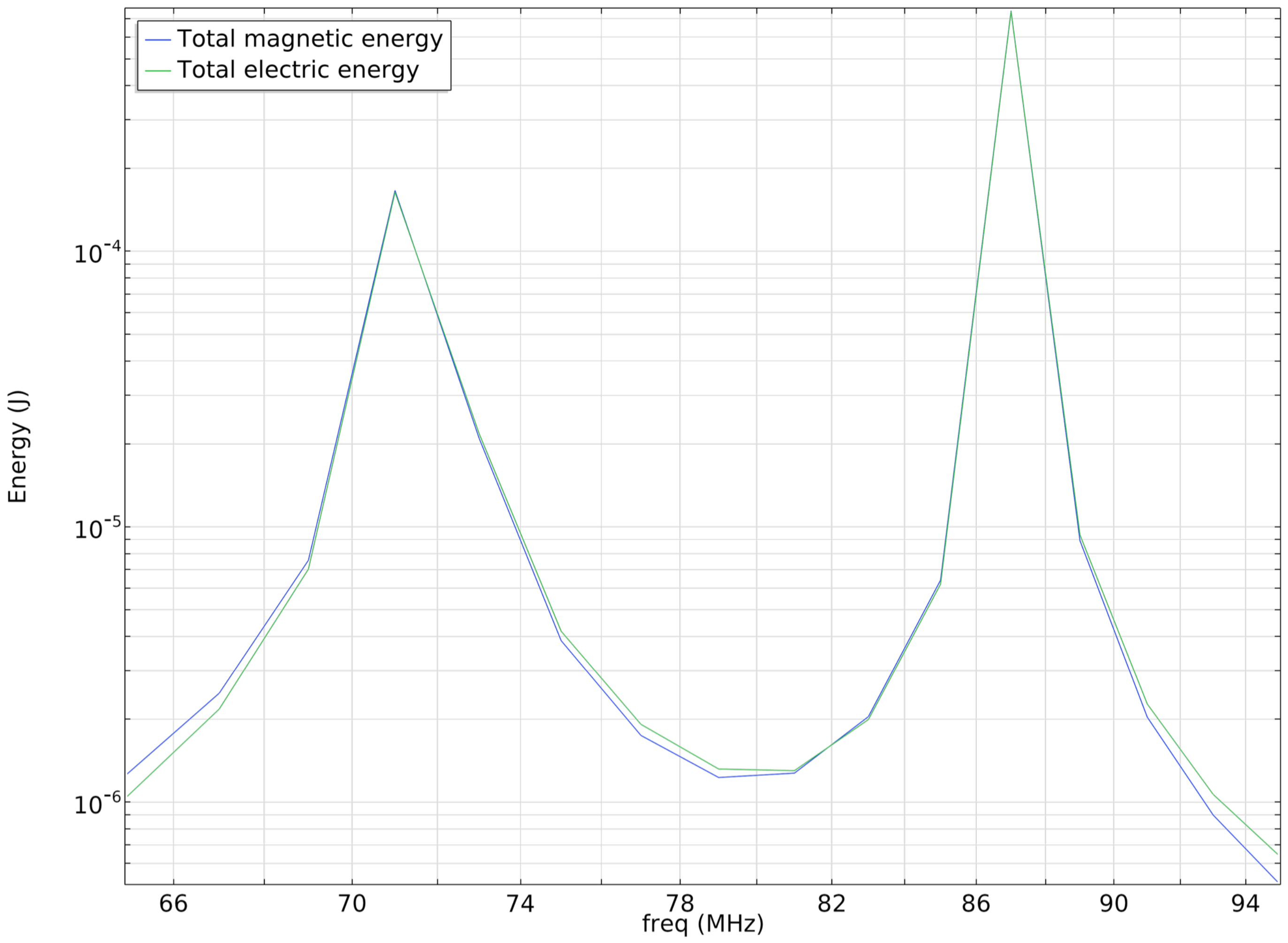
No gap resonance



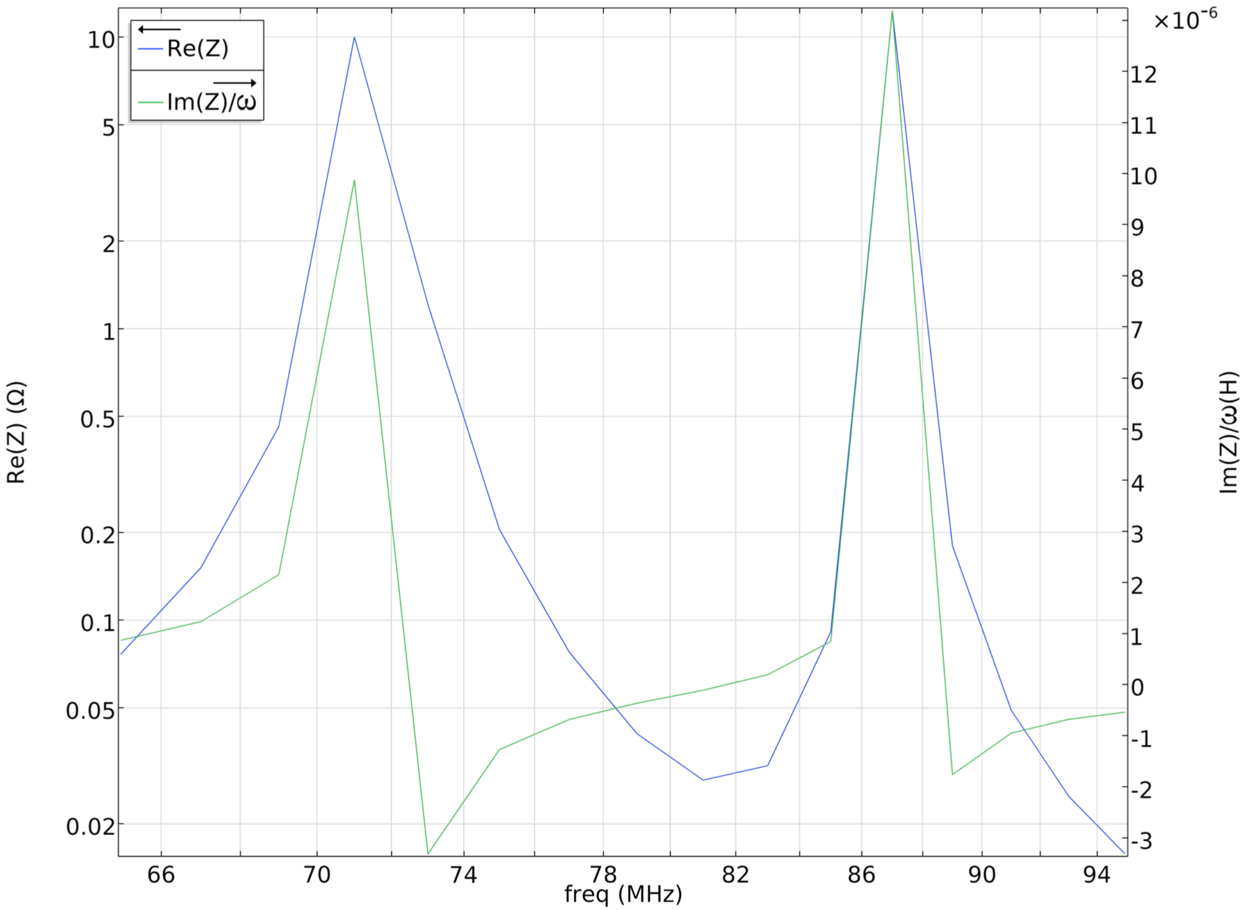


One gap resonance

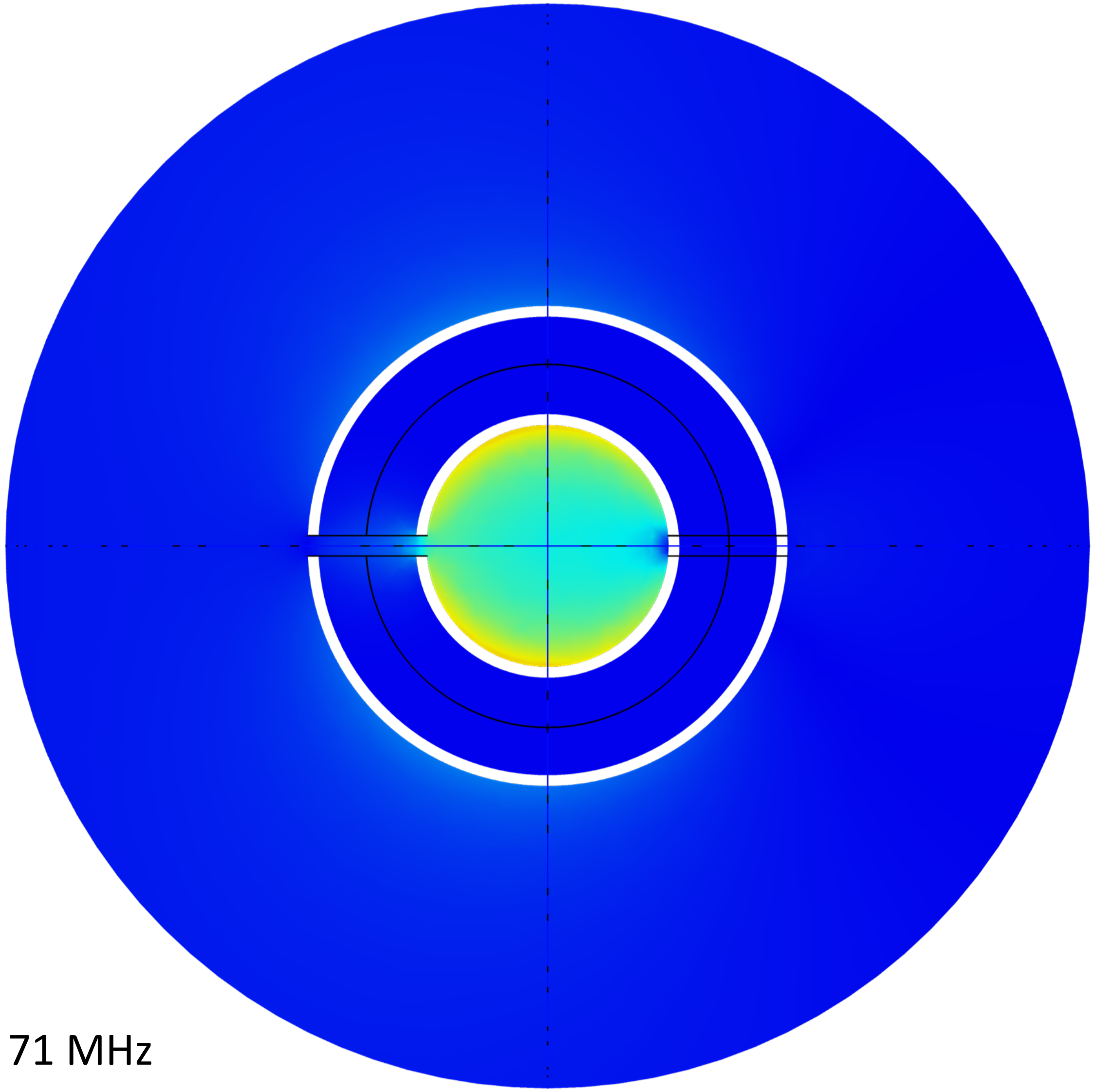


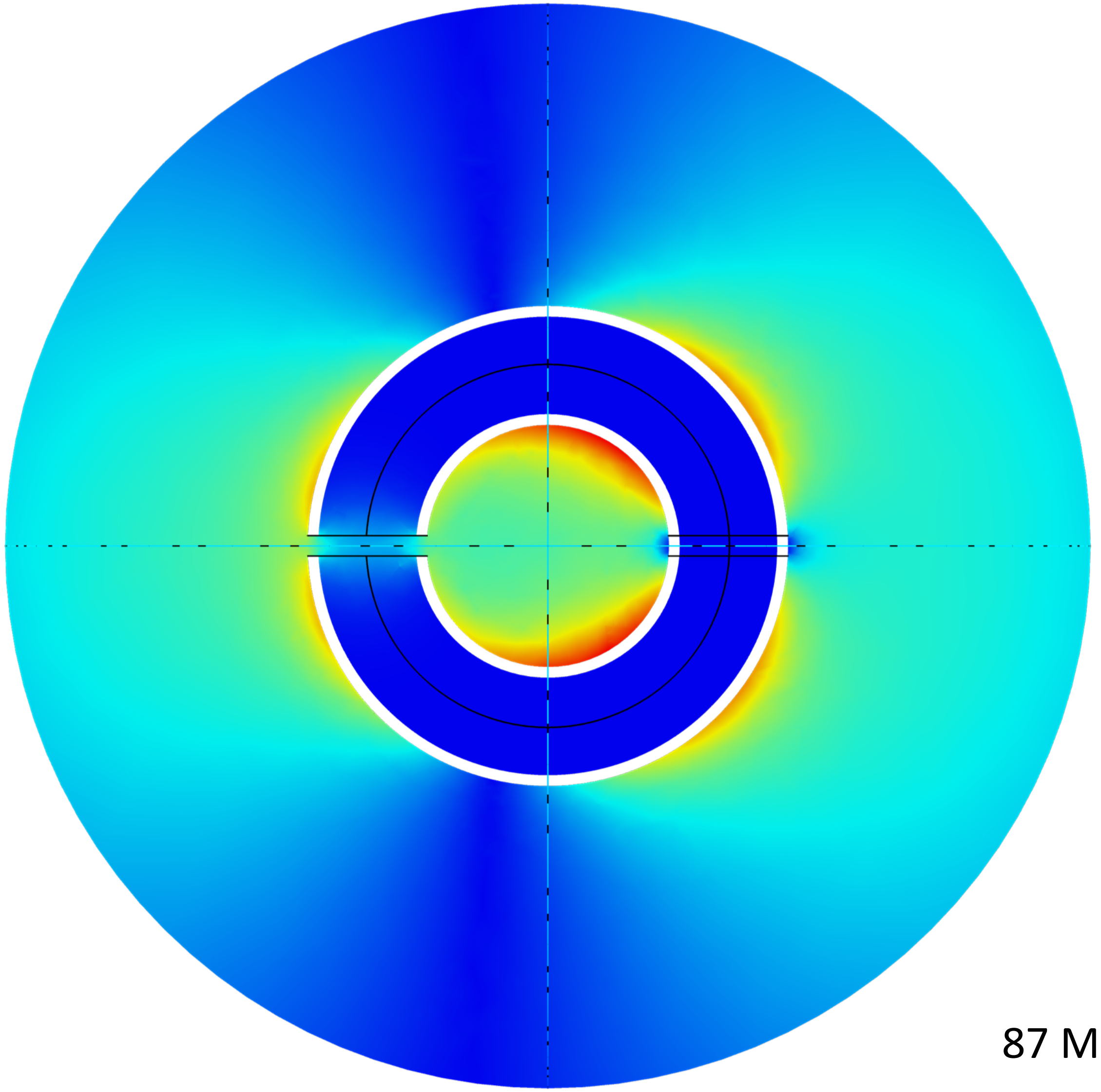


Impedance, one gap



Magnetic flux, one gap





Next

- Method checks
  - compare to analytic approximations
  - vary geometry to see trends (thickness, radii, gap width, …)
  - test different port geometries
- Realistic geometry
- More gaps
- Gaps → overlaps
- Copper → SC