2 Lectures in Neutrino Theory & Phenomenology

André de Gouvêa

Northwestern University

Tentative Outline for The Two Hours

[I will try to avoid repetition with many of the excellent lectures you will hear this and next week, including those by Guenette, Dodelson, Winslow, Ahmed, and Tait. I will also allude to them when necessary. Finally, when in doubt, they – as opposed to I – are always right.]

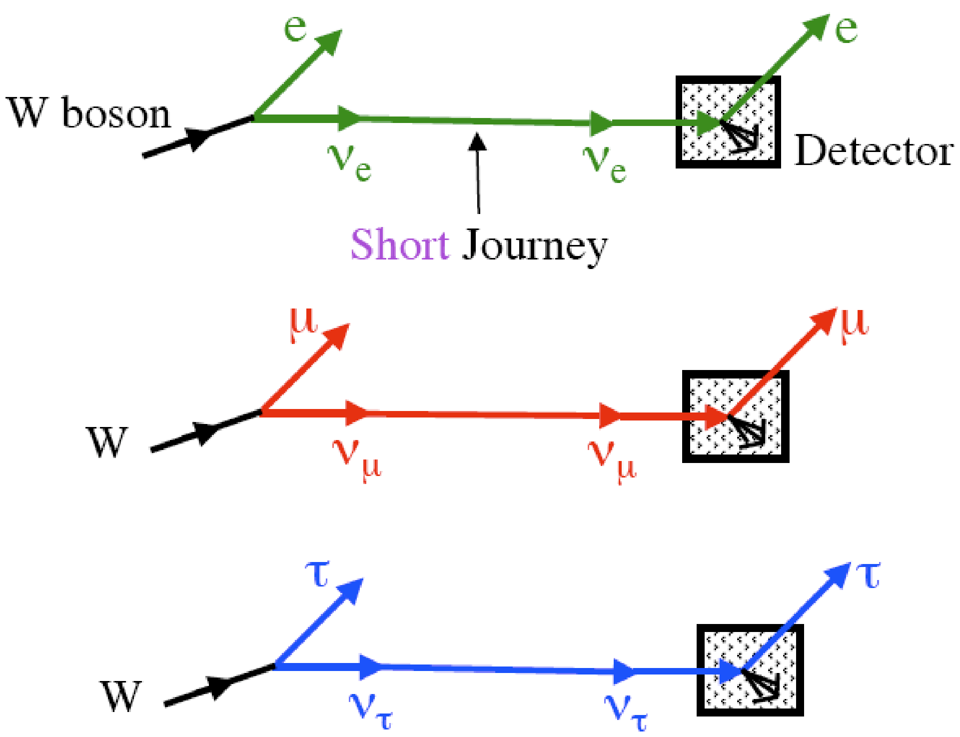
- Neutrino Oscillations;
- What We Know We Don't Know;
- Neutrino Masses As Physics Beyond the Standard Model;
- Some Ideas for Tiny Neutrino Masses, and Some Consequences.

[note: Questions/Suggestions/Complaints are ALWAYS welcome]

Some Neutrino references (WARNING: Biased Sample)

- "Are There Really Neutrinos? – An Evidential History," Allan Franklin, Perseus Books, 2001. Good discussion of neutrino history.
- A. de Gouvêa, "TASI lectures on neutrino physics," hep-ph/0411274;
- R. N. Mohapatra, A. Yu. Smirnov,"Neutrino Mass and New Physics," Ann. Rev. Nucl. Part. Sci. **56**, 569 (2006) [hep-ph/0603118];
- M. C. Gonzalez-Garcia, M. Maltoni, "Phenomenology with Massive Neutrinos," Phys. Rept. **460**, 1 (2008) [arXiv:0704.1800 [hep-ph]];
- C. Giunti and C.W. Kim,"Fundamentals of Neutrino Physics and Astrophysics," Oxford University Press (2007);
- "The Physics of Neutrinos,"V. Barger, D. Marfatia, K. Whisnant, Princeton University Press (2012);
- A. de Gouvêa *et al.*, "Working Group Report: Neutrinos," arXiv:1310:4340;
- A. de Gouvêa, "Neutrino Mass Models," Ann. Rev. Nucl. Part. Sci. **66**, 197 (2016).
- Several lectures at TASI 2020; soon in an arXiv near you.

In the 20th Century, this is how we pictured neutrinos:



- come in three flavors (see figure);
- interact only via weak interactions ($W^\pm, Z^0$);
- have ZERO mass – helicity good quantum number;
- ν_L field describes 2 degrees of freedom:
  - left-handed state ν,
  - right-handed state $\bar{\nu}$ (CPT conjugate);
- neutrinos carry lepton number (conserved):
  - $L(\nu) = L(\ell) + 1$,
  - $L(\bar{\nu}) = L(\bar{\ell}) = -1$.

Something Funny Happened on the Way to the 21st Century

ν Flavor Oscillations

Neutrino oscillation experiments have revealed that neutrinos change flavor after propagating a finite distance. The rate of change depends on the neutrino energy E_ν and the baseline L. The evidence is overwhelming.

- $\nu_\mu \to \nu_\tau$ and $\bar{\nu}_\mu \to \bar{\nu}_\tau$ — atmospheric and accelerator experiments;
- $\nu_e \to \nu_{\mu,\tau}$ — solar experiments;
- $\bar{\nu}_e \to \bar{\nu}_{\rm other}$ — reactor experiments;
- $\nu_\mu \to \nu_{\rm other}$ and $\bar{\nu}_\mu \to \bar{\nu}_{\rm other}$— atmospheric and accelerator expts;
- $\nu_\mu \to \nu_e$ — accelerator experiments.

The simplest and **only satisfactory** explanation of **all** this data is that neutrinos have distinct masses, and mix.

Mass-Induced Neutrino Flavor Oscillations

Neutrino Flavor change can arise out of several different mechanisms. The simplest one is to appreciate that, once **neutrinos have mass**, **leptons can mix**. This turns out to be the correct mechanism (certainly the dominant one), and **only** explanation that successfully explains **all** long-baseline data consistently.

Neutrinos with a well defined mass:

$$\nu_1, \nu_2, \nu_3, \ldots \qquad \text{with masses} \quad m_1, m_2, m_3, \ldots$$

How do these states (neutrino mass eigenstates) relate to the neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$?

$$\nu_\alpha = U_{\alpha i}\nu_i \qquad \alpha = e, \mu, \tau, \quad i = 1, 2, 3$$

U is a unitary mixing matrix. I'll talk more about it later.

The Propagation of Massive Neutrinos

Neutrino mass eigenstates are eigenstates of the free-particle Hamiltonian:

$$|\nu_i\rangle = e^{-iE_it}|\nu_i\rangle, \qquad E_i^2 - |\vec{p}_i|^2 = m_i^2$$

The neutrino flavor eigenstates are linear combinations of ν_i's, say:

$$\begin{aligned} |\nu_e\rangle &= \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle. \\ |\nu_\mu\rangle &= -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle. \end{aligned}$$

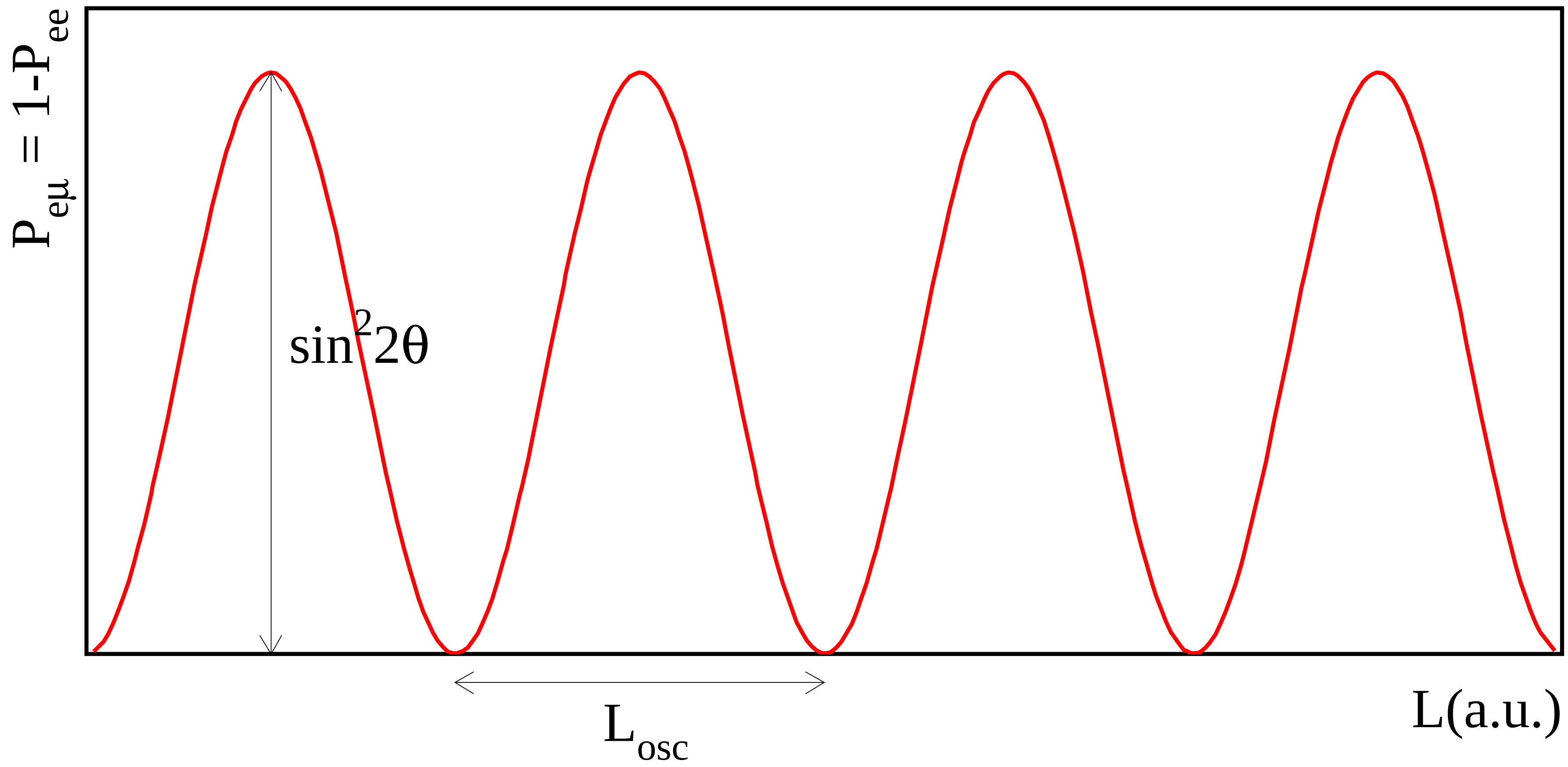
If this is the case, a state produced as a ν_e evolves in vacuum into

$$|\nu(t,\vec{x})\rangle = \cos\theta e^{-ip_1x}|\nu_1\rangle + \sin\theta e^{-ip_2x}|\nu_2\rangle.$$

It is trivial to compute $P_{e\mu}(L) \equiv |\langle\nu_\mu|\nu(t,z=L)\rangle|^2$. It is just like a two-level system from basic undergraduate quantum mechanics! In the ultrarelativistic limit (always a good bet), $t \simeq L$, $E_i - p_{z,i} \simeq (m_i^2)/2E_i$, and

$$\boxed{P_{e\mu}(L) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)}$$

oscillation parameters: $\begin{cases} \pi \frac{L}{L_{\rm osc}} \equiv \frac{\Delta m^2 L}{4E} = 1.267 \left(\frac{L}{\rm km}\right) \left(\frac{\Delta m^2}{\rm eV^2}\right) \left(\frac{\rm GeV}{E}\right) \\ \text{amplitude } \sin^2 2\theta \end{cases}$



There is a long (and oftentimes confused and confusing) history behind this derivation and several others. A comprehensive discussion can be found, for example, in

E.K. Akhmedov, A. Yu. Smirnov, 0905.1903 [hep-ph]

In a nutshell, neutrino oscillations as described above occur whenever

- Neutrino Production and Detection are Coherent → cannot "tell" ν_1 from ν_2 from ν_3 but "see" ν_e or ν_μ or ν_τ.
- Decoherence effects due to wave-packet separation are negligible → baseline not too long that different "velocity" components of the neutrino wave-packet have time to physically separate.
- The energy released in production and detection is large compared to the neutrino mass → so we can assign all of the effect to the neutrino propagation, independent from the production process. Also assures ultra-relativistic approximation good.

$P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$ Works great for $\sin^2 2\theta \sim 1$ and $\Delta m^2 \sim 10^{-3}$ eV^2

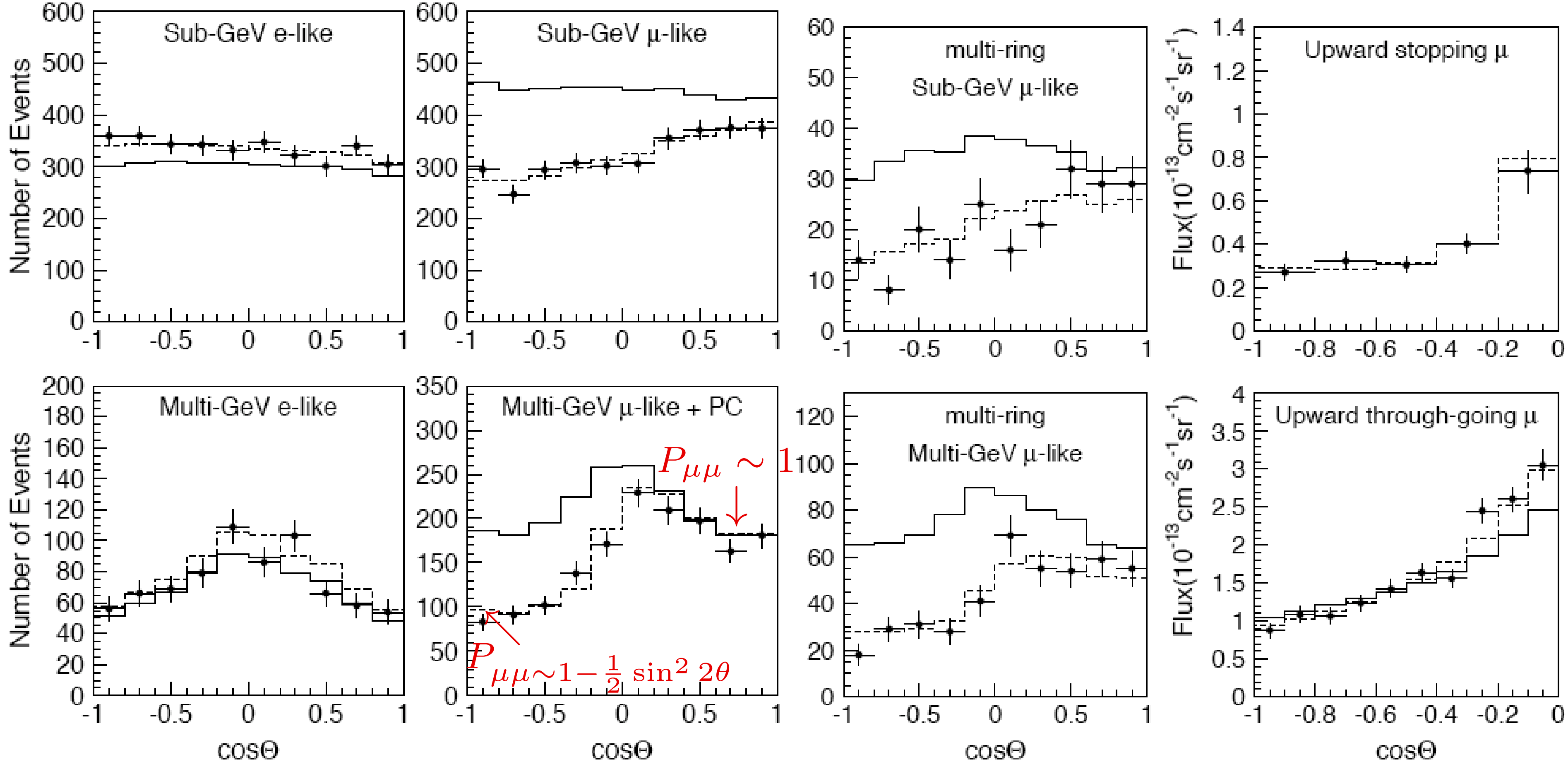
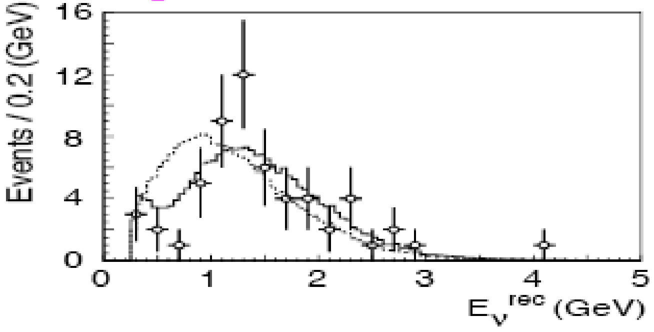


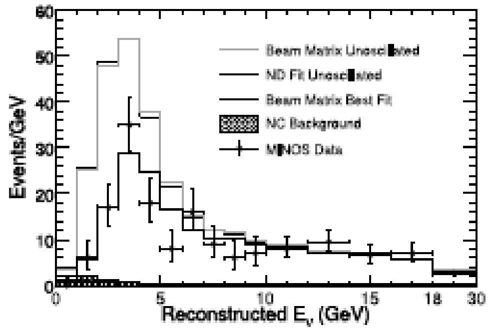
Figure 4. Zenith angle distribution for fully-contained single-ring e-like and μ-like events, multi-ring μ-like events, partially contained events and upward-going muons. The points show the data and the solid lines show the Monte Carlo events without neutrino oscillation. The dashed lines show the best-fit expectations for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. From M. Ishitsuka [Super-Kamiokande Collaboration], hep-ex/0406076.

K2K	ν_μ at KEK	SK	L=250 km
MINOS	ν_μ at Fermilab	Soundan	L=735 km
Opera/Icarus	ν_μ at CERN	Gran Sasso	L=740 km

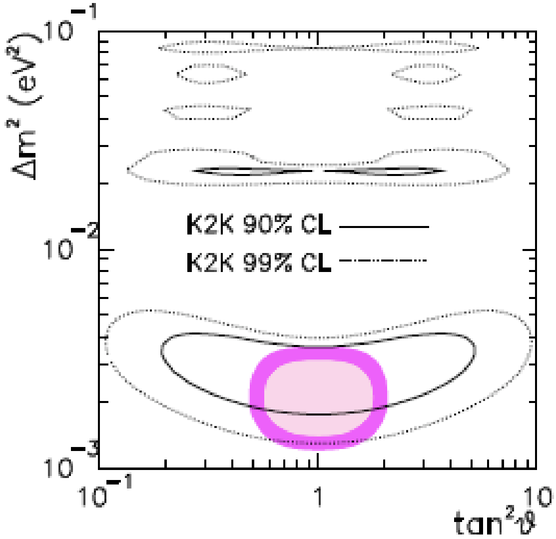
K2K 2004: spectral distortion



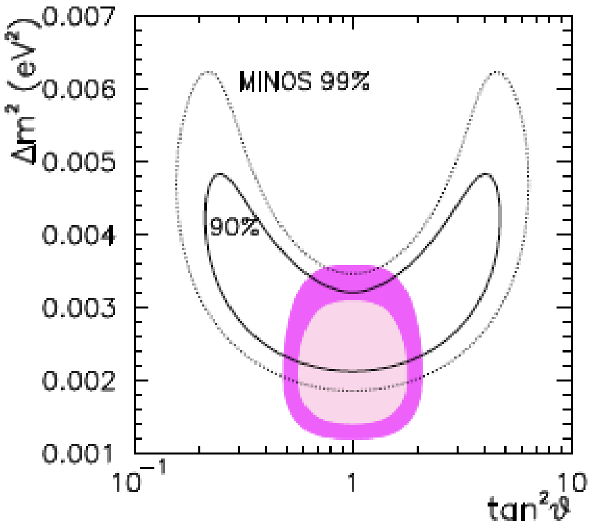
MINOS 2006: spectral distortion



Confirmation of ATM oscillations



Confirmation of ATM oscillations



[Gonzalez-Garcia, PASI 2006]

Matter Effects

The neutrino propagation equation, in the ultra-relativistic approximation, can be re-expressed in the form of a Shrödinger-like equation. In the mass basis:

$$i\frac{\mathrm{d}}{\mathrm{d}L}|\nu_i\rangle = \frac{m_i^2}{2E}|\nu_i\rangle,$$

up to a term proportional to the identity. In the weak/flavor basis

$$i\frac{\mathrm{d}}{\mathrm{d}L}|\nu_\beta\rangle = U_{\beta i}\frac{m_i^2}{2E}U^\dagger_{i\alpha}|\nu_\alpha\rangle.$$

In the 2×2 case, [general state is $|\nu(L)\rangle = a_e|\nu_e\rangle + a_\mu|\nu_\mu\rangle$]

$$i\frac{\mathrm{d}}{\mathrm{d}L}\left(\begin{array}{c} a_e \\ a_\mu \end{array}\right) = \frac{\Delta m^2}{2E}\left(\begin{array}{cc} \sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \cos^2\theta \end{array}\right)\left(\begin{array}{c} a_e \\ a_\mu \end{array}\right),$$

(again, up to additional terms proportional to the 2×2 identity matrix).

Fermi Lagrangian, after a Fiertz rearrangement of the charged-current terms:

$$\mathrm{L} \supset \bar{\nu}_{eL} i\partial_\mu \gamma^\mu \nu_{eL} - 2\sqrt{2} G_F \left(\bar{\nu}_{eL}\gamma^\mu \nu_{eL}\right)\left(\bar{e}_L \gamma_\mu e_L\right) + \ldots$$

Equation of motion for one electron neutrino state in the presence of a non-relativistic electron background, in the rest frame of the electrons:

$$\langle \bar{e}_L \gamma_\mu e_L \rangle = \delta_{\mu 0} \frac{N_e}{2}$$

where $N_e \equiv e^\dagger e$ is the average electron number density (at rest, hence $\delta_{\mu 0}$ term). Factor of 1/2 from the "left-handed" half.

Dirac equation for a one neutrino state inside a cold electron "gas" is (ignore neutrino mass)

$$(i\partial^\mu \gamma_\mu - \sqrt{2} G_F N_e \gamma_0)|\nu_e\rangle = 0.$$

In the ultrarelativistic limit, (plus $\sqrt{2} G_F N_e \ll E$), dispersion relation is

$$E \simeq |\vec{p}| \pm \sqrt{2} G_F N_e, \qquad + \text{ for } \nu, \quad - \text{ for } \bar{\nu}$$

$$i\frac{\mathrm{d}}{\mathrm{d}L}\left(\begin{array}{c} a_e \\ a_\mu \end{array}\right) = \left[\frac{\Delta m^2}{2E}\left(\begin{array}{cc} \sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \cos^2\theta \end{array}\right) + \left(\begin{array}{cc} A & 0 \\ 0 & 0 \end{array}\right)\right]\left(\begin{array}{c} a_e \\ a_\mu \end{array}\right),$$

$A = \pm\sqrt{2}G_F N_e$ (+ for neutrinos, − for antineutrinos).

Note: Similar effect from neutral current interactions common to all (active) neutrino species → proportional to the identity.

In general, this is hard to solve, as A is a function of L: two-level non-relativistc quantum mechanical system in the presence of time dependent potential.

In some cases, however, the solution is rather simple.

Constant A: good approximation for neutrinos propagating through matter inside the Earth [exception: neutrinos that see Earth's internal structure (the crust, the mantle, the outer core, the inner core)]

$$i\frac{\mathrm{d}}{\mathrm{d}L}\left(\begin{array}{c} a_e \\ a_\mu \end{array}\right) = \left(\begin{array}{cc} A & \Delta/2\sin 2\theta \\ \Delta/2\sin 2\theta & \Delta\cos 2\theta \end{array}\right)\left(\begin{array}{c} a_e \\ a_\mu \end{array}\right), \quad \Delta \equiv \Delta m^2/2E.$$

$$P_{e\mu} = \sin^2 2\theta_M \sin^2\left(\frac{\Delta_M L}{2}\right),$$

where

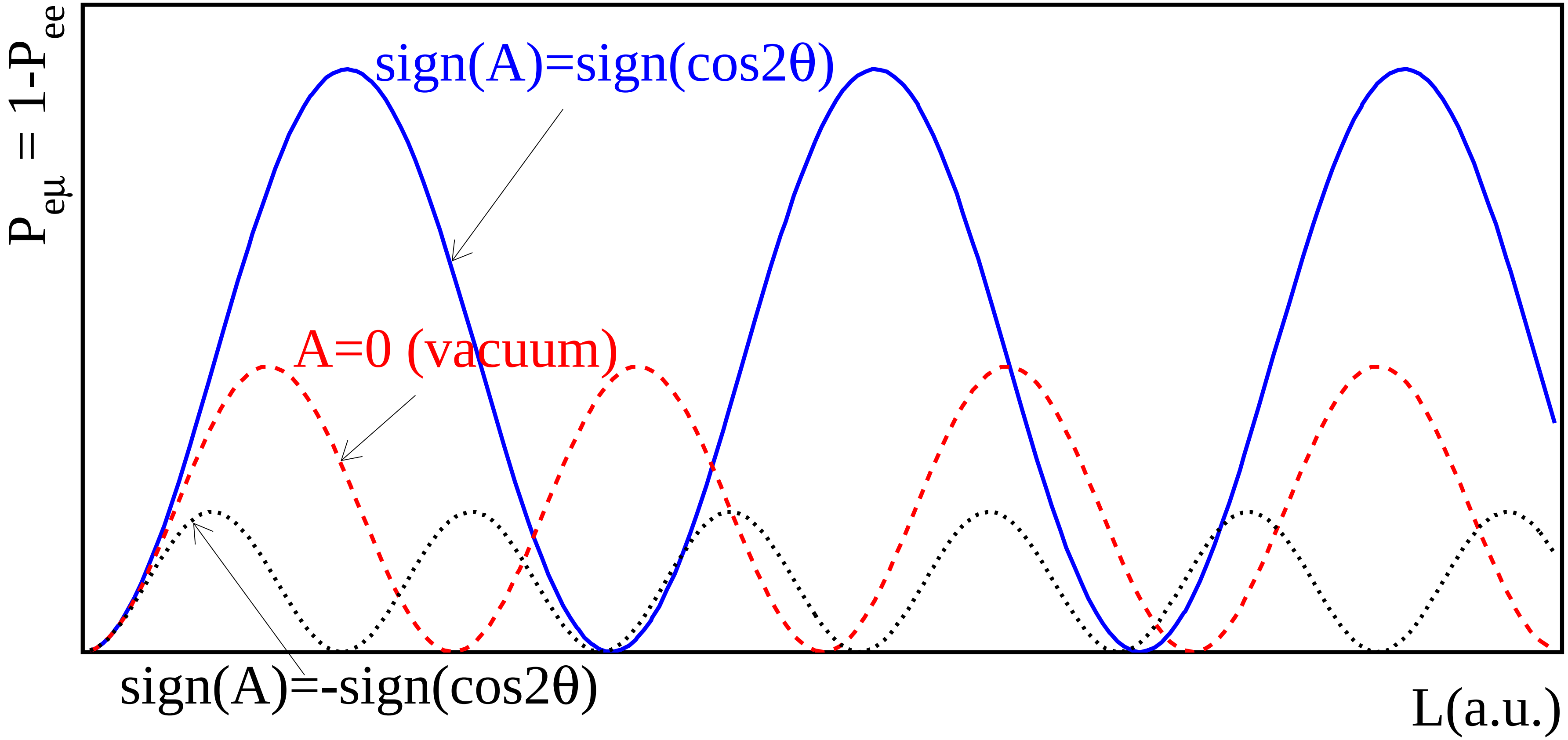
$$\begin{aligned} \Delta_M &= \sqrt{(A - \Delta\cos 2\theta)^2 + \Delta^2\sin^2 2\theta}, \\ \Delta_M \sin 2\theta_M &= \Delta\sin 2\theta, \\ \Delta_M \cos 2\theta_M &= A - \Delta\cos 2\theta. \end{aligned}$$

The presence of matter affects neutrino and antineutrino oscillation differently.

Nothing wrong with this: CPT-theorem relates the propagation of neutrinos in an electron background to the propagation of antineutrinos in a positron background.

Enlarged parameter space in the presence of matter effects.

For example, can tell whether $\cos 2\theta$ is positive or negative.



The MSW Effect

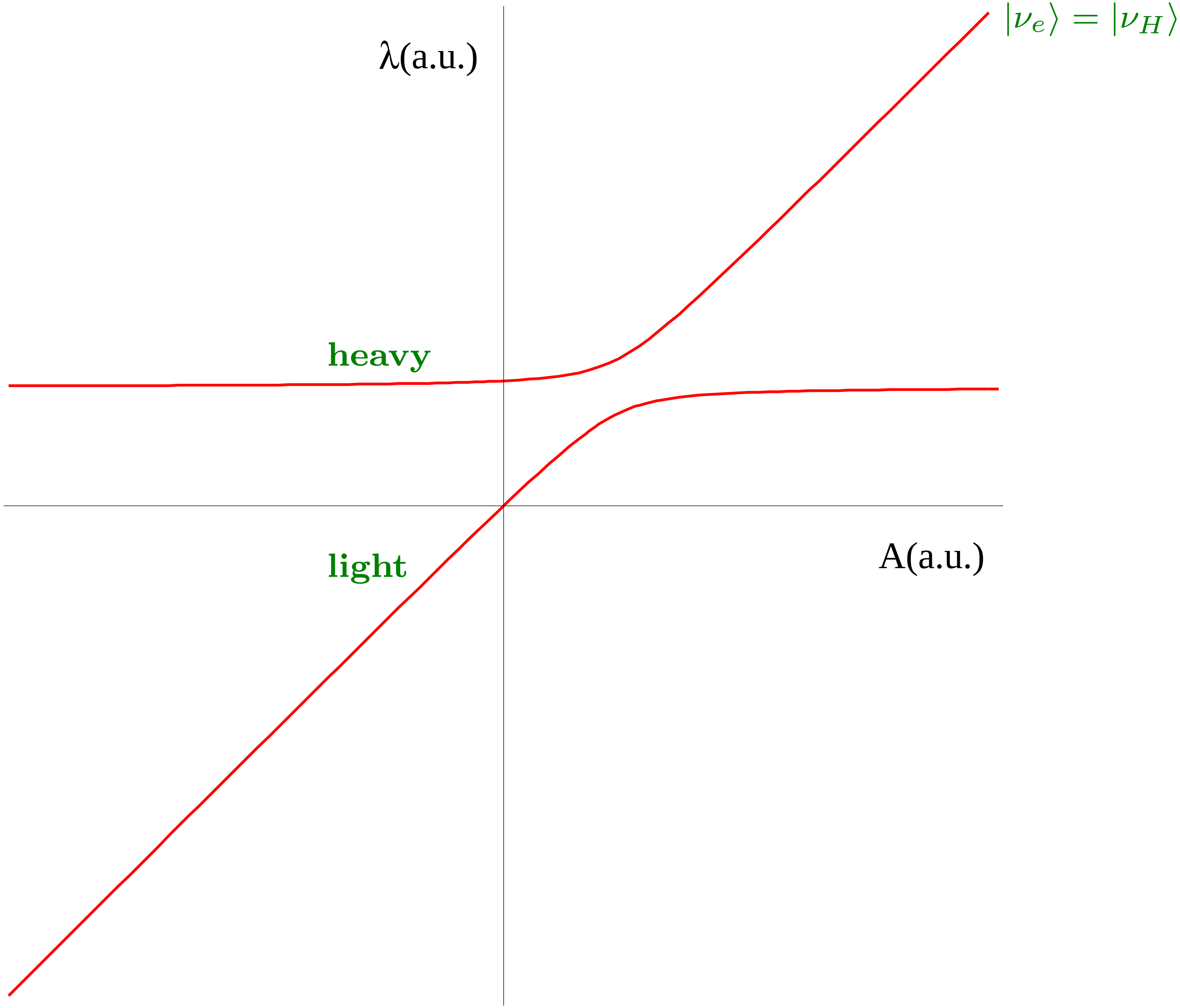
The oldest neutrino puzzle (1960s) is the one posed by solar neutrinos. It is also the one that is most subtle to explain. This is because solar neutrinos traverse a strongly varying matter density on their way from the center of the Sun to the surface of the Earth.

For the Hamiltonian

$$\left[\Delta\left(\begin{array}{cc}\sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \cos^2\theta\end{array}\right)+A\left(\begin{array}{cc}1 & 0 \\ 0 & 0\end{array}\right)\right],$$

it is easy to compute the eigenvalues as a function of A:

(remember, $\Delta=\Delta m^2/2E$)


λ(a.u.)
$|\nu_e\rangle = |\nu_H\rangle$
heavy
light
A(a.u.)

A decreases "slowly" as a function of L $\Rightarrow$ system evolves adiabatically.

$$|\nu_e\rangle = |\nu_{2M}\rangle \text{ at the core } \rightarrow |\nu_2\rangle \text{ in vacuum,}$$
$$P_{ee}^{\rm Earth} = |\langle\nu_e|\nu_2\rangle|^2 = \sin^2\theta.$$

Note that $P_{ee} \simeq \sin^2\theta$ applies in a wide range of energies and baselines, as long as the approximations mentioned above apply —ideal to explain the energy independent suppression of the ^{8}B solar neutrino flux!

Furthermore, large average suppressions of the neutrino flux are allowed if $\sin^2\theta \ll 1$. Compare with $\bar{P}_{ee}^{\rm vac} = 1 - 1/2\sin^2 2\theta > 1/2$.

One can expand on the result above by loosening some of the assumptions. $|\nu_e\rangle$ state is produced in the Sun's core as an *incoherent* mixture of $|\nu_{1M}\rangle$ and $|\nu_{2M}\rangle$. Introduce adiabaticity parameter P_c, which measures the probability that a $|\nu_{iM}\rangle$ matter Hamiltonian state will *not* exit the Sun as a $|\nu_i\rangle$ mass-eigenstate.

$$\begin{aligned} |\nu_e\rangle &\to |\nu_{1M}\rangle, \text{ with probability } \cos^2\theta_M, \\ &\to |\nu_{2M}\rangle, \text{ with probability } \sin^2\theta_M, \end{aligned}$$

where θ_M is the matter angle at the neutrino **production point**.

$$\begin{aligned} |\nu_{1M}\rangle &\to |\nu_1\rangle, \text{ with probability } (1-P_c), \\ &\to |\nu_2\rangle, \text{ with probability } P_c, \\ |\nu_{2M}\rangle &\to |\nu_1\rangle \text{ with probability } P_c, \\ &\to |\nu_2\rangle \text{ with probability } (1-P_c). \end{aligned}$$

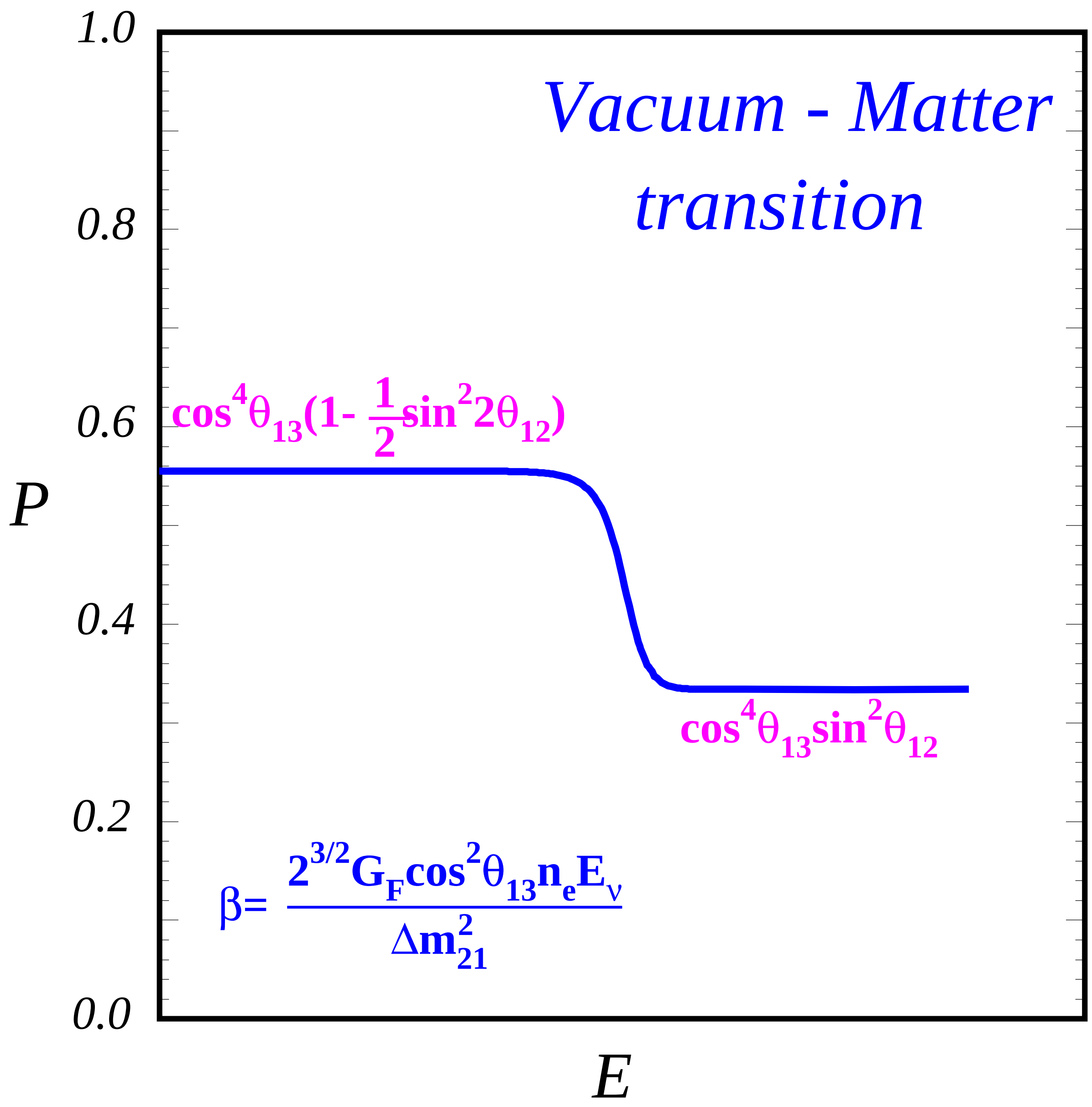
$P_{1e} = \cos^2\theta$ and $P_{2e} = \sin^2\theta$ so

$$\begin{aligned} P_{ee}^{\rm Sun} = &\cos^2\theta_M\left[(1-P_c)\cos^2\theta + P_c\sin^2\theta\right] \\ &+\sin^2\theta_M\left[P_c\cos^2\theta + (1-P_c)\sin^2\theta\right]. \end{aligned}$$

For $N_e = N_{e0}e^{-L/r_0}$, P_c, (crossing probability), is exactly calculable

$$P_c = \frac{e^{-\gamma\sin^2\theta} - e^{-\gamma}}{1-e^{-\gamma}}, \quad \gamma = 2\pi r_0\Delta. \tag{1}$$

Adiabatic condition: $\gamma \gg 1$, when $P_c \to 0$.



We need:

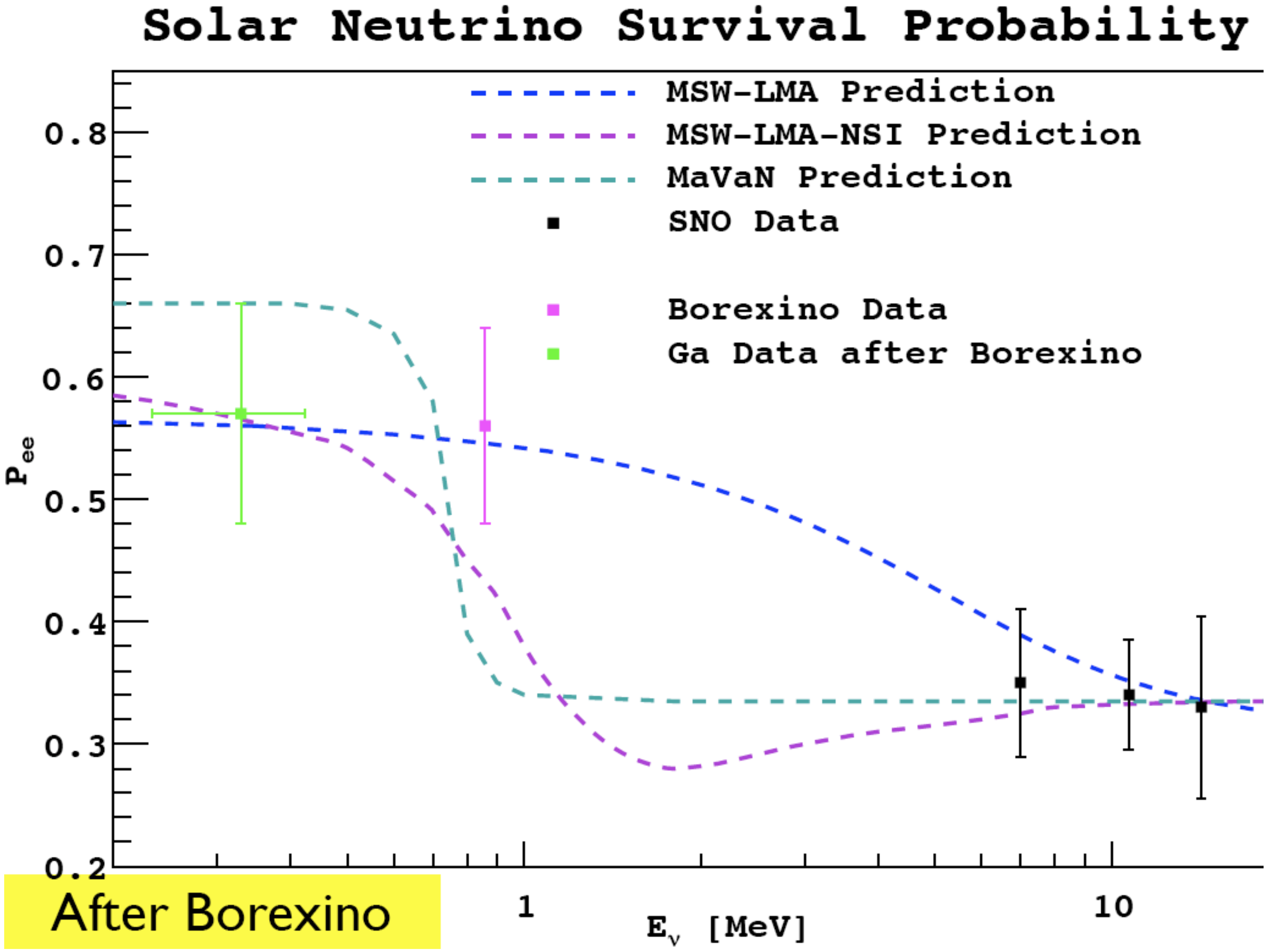
- $P_{ee} \sim 0.3$ (^{8}B neutrinos)
- $P_{ee} \sim 0.6$ (^{7}Be, pp neutrinos)

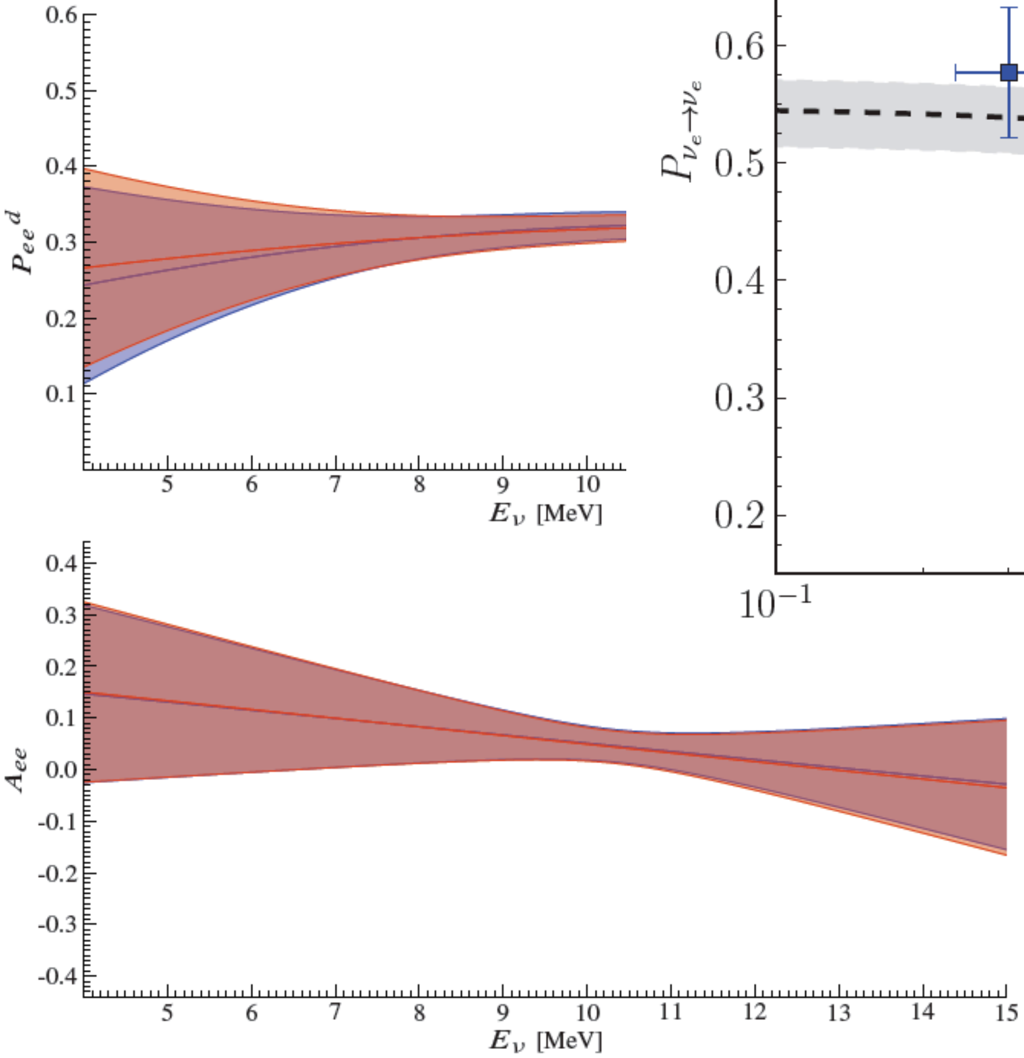
$\Rightarrow \quad \sin^2\theta \sim 0.3$

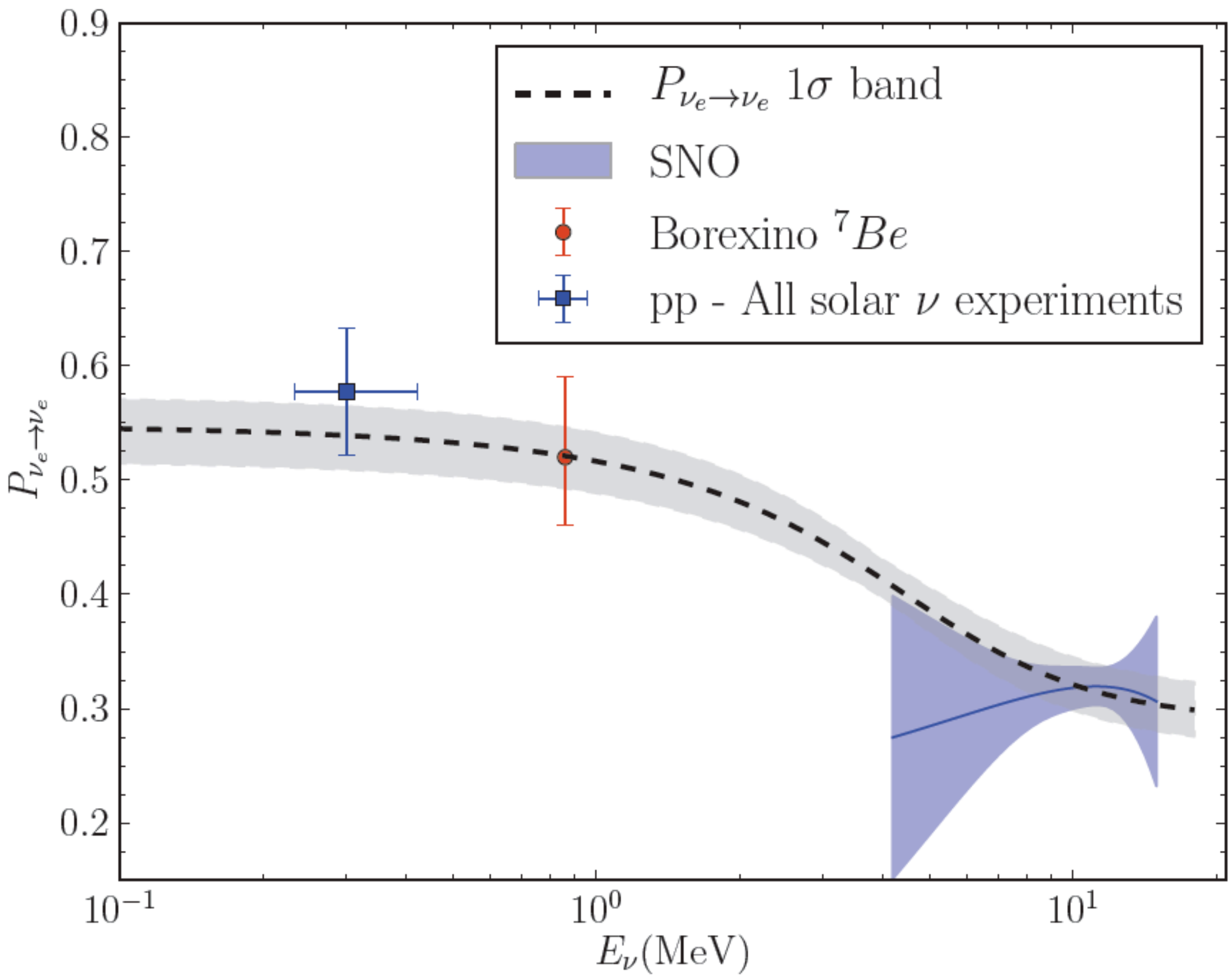
$\Rightarrow \quad \Delta m^2 \sim 10^{-(5\text{ to }4)}\ \text{eV}^2$

for a long time, there were many other options!

(LMA, LOW, SMA, VAC)


Solar Neutrino Survival Probability
MSW-LMA Prediction
MSW-LMA-NSI Prediction
MaVaN Prediction
SNO Data
Borexino Data
Ga Data after Borexino
0.8
0.7
0.6
0.5
0.4
0.3
0.2
P_ee
1
10
E_ν [MeV]
After Borexino



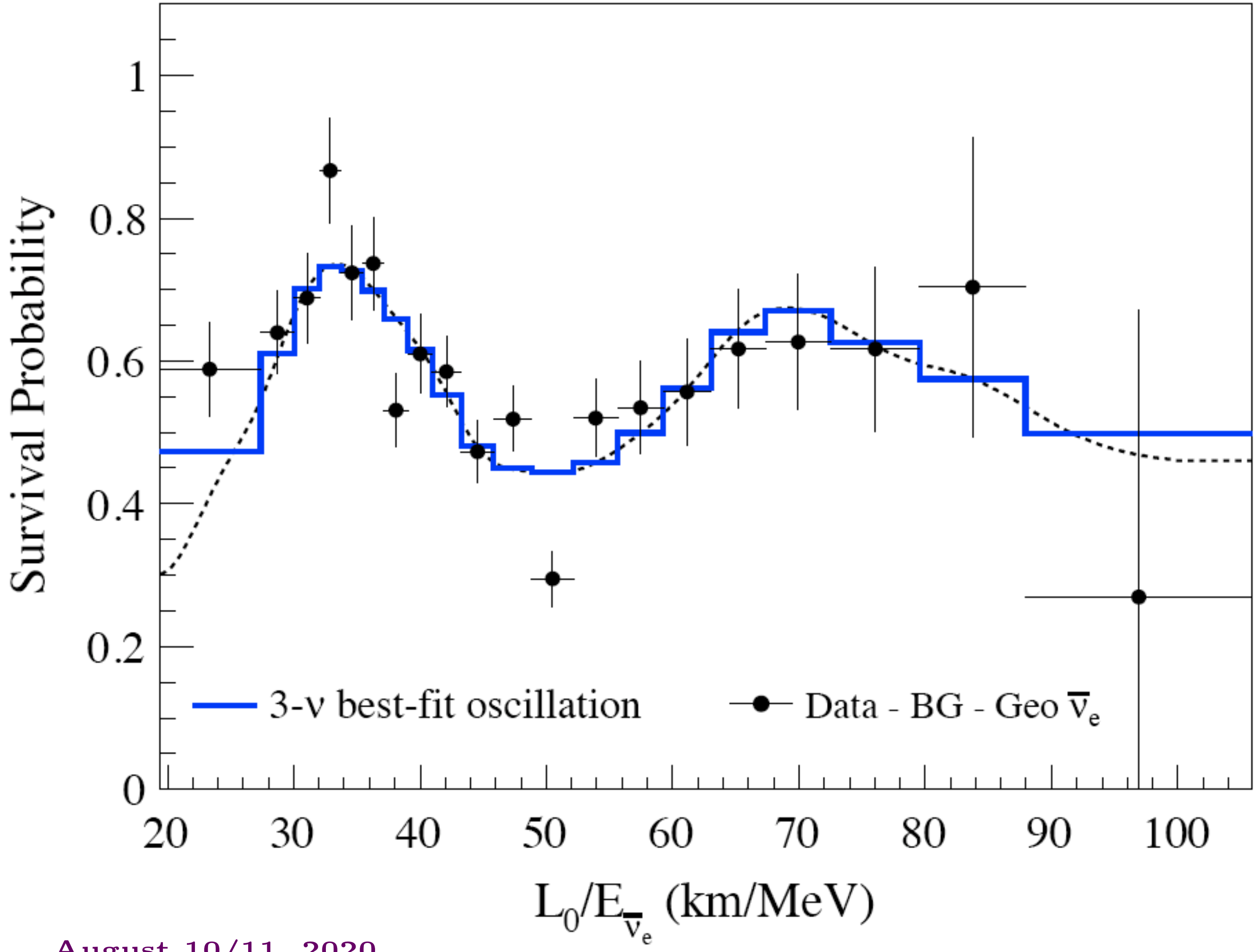


"Final" SNO results, 1109.0763

Solar oscillations confirmed by Reactor experiment: KamLAND

[arXiv:1303.4667]

$$\text{phase}= 1.27 \left(\frac{\Delta m^2}{5\times10^{-5}~\text{eV}^2}\right)\left(\frac{5~\text{MeV}}{E}\right)\left(\frac{L}{100~\text{km}}\right)$$

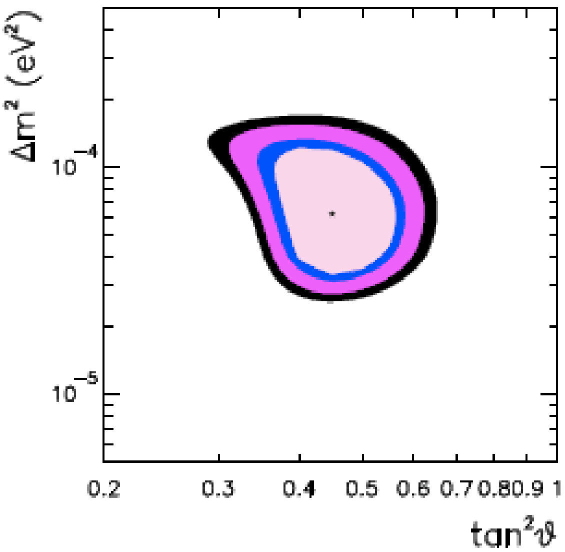


$$P_{ee} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

oscillatory behavior!

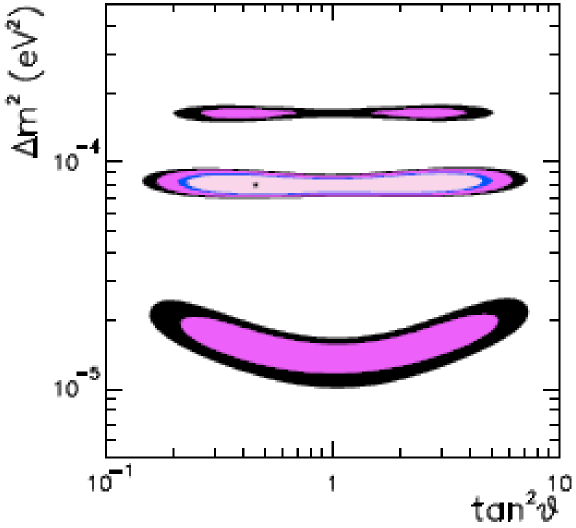
Solar

$\nu_e \rightarrow \nu_{\rm active}$

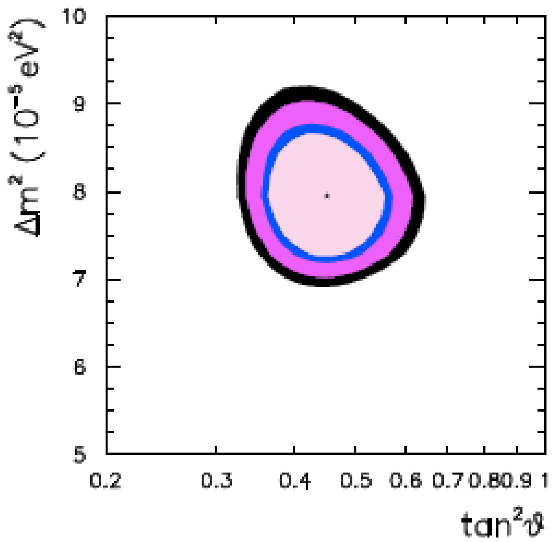


+ KamLAND

$\bar{\nu}_e \not\to \bar{\nu}_e$



ν_e oscillation parameters compatible with $\overline{\nu}_e$: *Sensible* to assume CPT: $P_{ee} = P_{\bar{e}\bar{e}}$



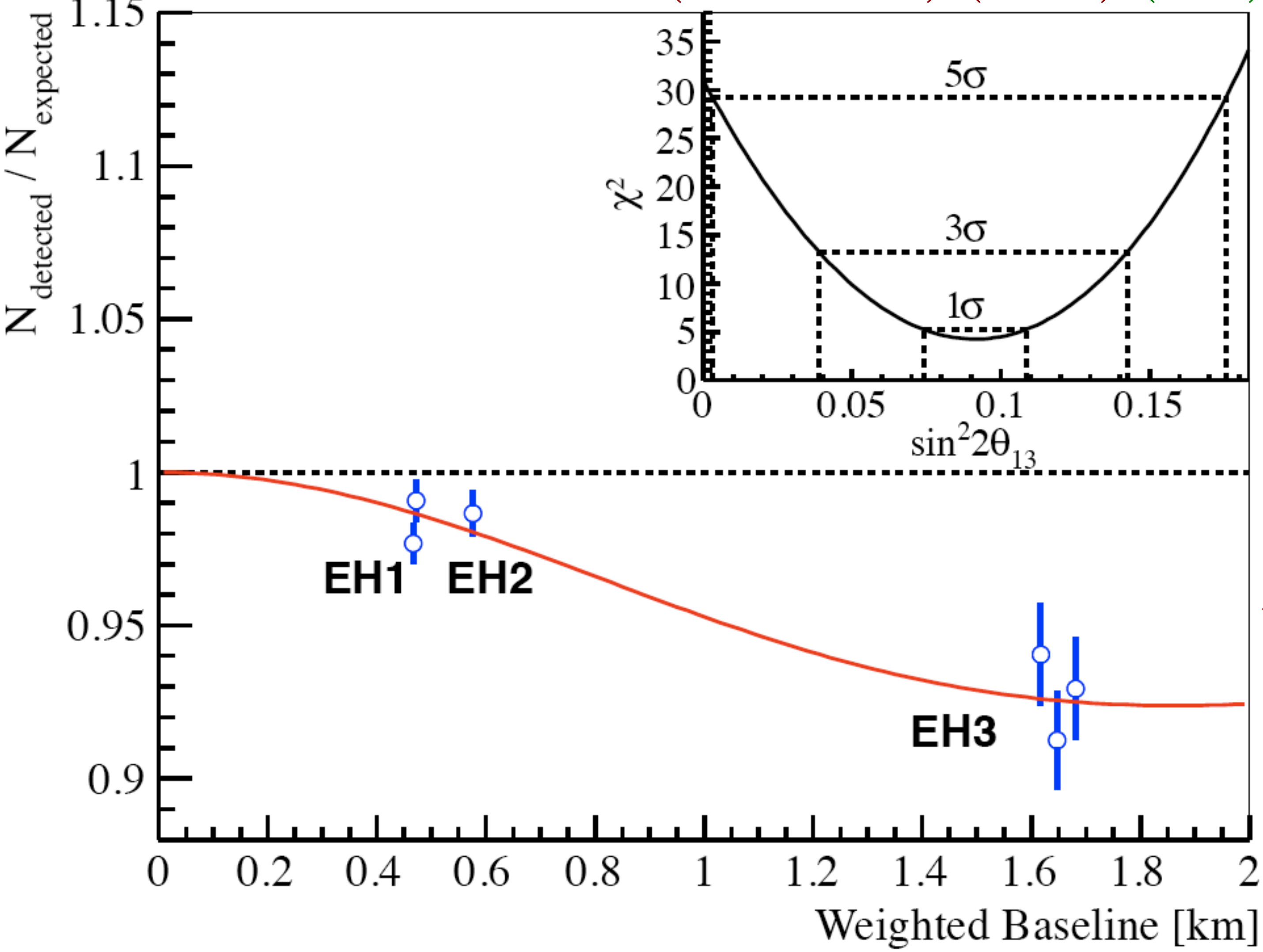
$$\Delta m^2_\odot = \left(8^{+0.4}_{-0.5}\right) \times 10^{-5}\ \text{eV}^2 \quad (1\sigma)$$

$$\tan^2\theta_\odot = 0.45^{+0.05}_{-0.05}$$

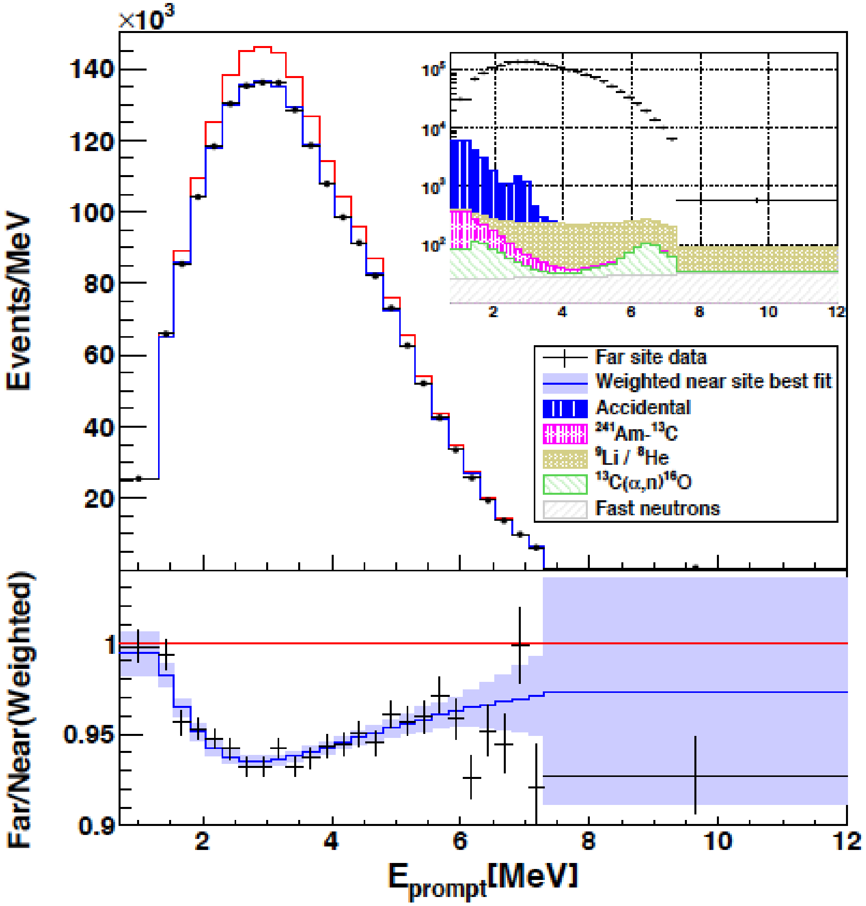
[Gonzalez-Garcia, PASI 2006]

Atmospheric Oscillations in the Electron Sector: Daya Bay, RENO, Double Chooz

$$\text{phase} = 0.64 \left(\frac{\Delta m^2}{2.5\times 10^{-3}\ \text{eV}^2}\right)\left(\frac{5\ \text{MeV}}{E}\right)\left(\frac{L}{1\ \text{km}}\right)$$



$$P_{ee} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$



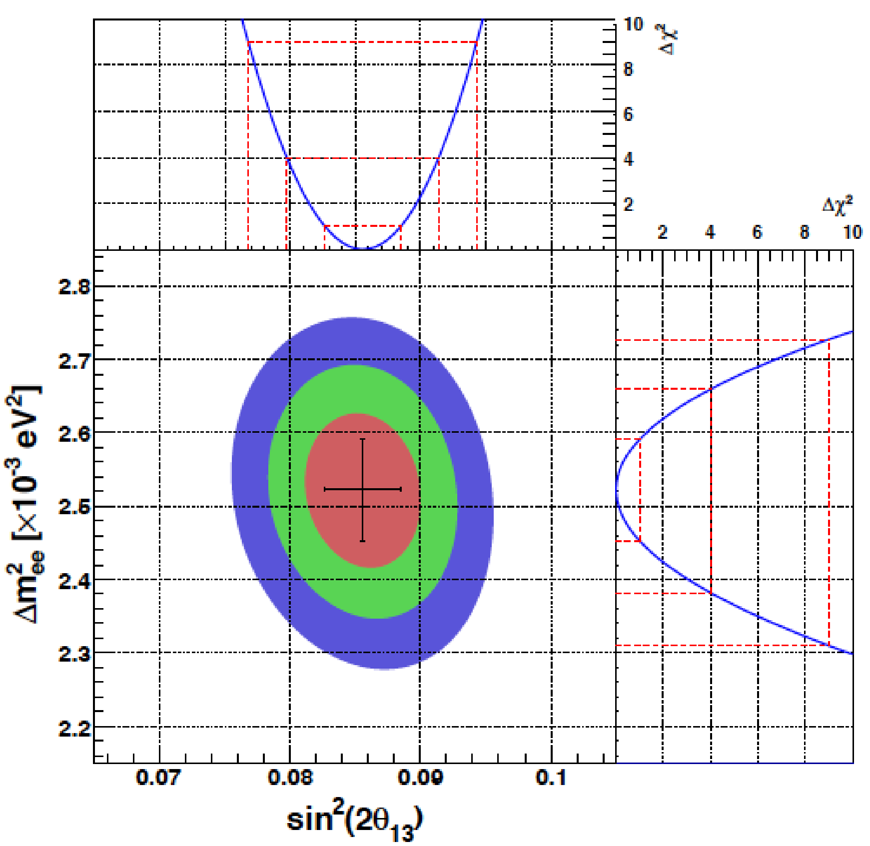


FIG. 3. The background-subtracted spectrum at the far site (black points) and the expectation derived from near-site measurements excluding (red line) or including (blue line) the best-fit oscillation. The bottom panel shows the ratios of data over predictions with no oscillation. The shaded area is the total uncertainty from near-site measurements and the extrapolation model. The error bars represent the statistical uncertainty of the far-site data. The inset shows the background components on a logarithmic scale. Detailed spectra data are provided as Supplemental Material [14].

FIG. 4. The 68.3%, 95.5% and 99.7% C.L. allowed regions in the Δm^2_{ee}-$\sin^2 2\theta_{13}$ plane. The one-dimensional $\Delta\chi^2$ for $\sin^2 2\theta_{13}$ and Δm^2_{ee} are shown in the top and right panels, respectively. The best-fit point and one-dimensional uncertainties are given by the black cross.

Summarizing:

Both the solar and atmospheric puzzles can be properly explained in terms of two-flavor neutrino oscillations:

- **solar**: $\nu_e \leftrightarrow \nu_a$ (linear combination of ν_μ and ν_τ): $\Delta m^2 \sim 10^{-4}$ eV^2, $\sin^2\theta \sim 0.3$.
- **atmospheric**: $\nu_\mu \leftrightarrow \nu_\tau$: $\Delta m^2 \sim 10^{-3}$ eV^2, $\sin^2\theta \sim 0.5$ ("maximal mixing").
- **short-baseline reactors:** $\nu_e \leftrightarrow \nu_a$ (linear combination of ν_μ and ν_τ): $\Delta m^2 \sim 10^{-3}$ eV^2, $\sin^2\theta \sim 0.02$.

Putting it all together – 3 flavor mixing:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{e\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Definition of neutrino mass eigenstates (who are ν_1, ν_2, ν_3?):

- $m_1^2 < m_2^2$ — $\Delta m^2_{13} < 0$ – Inverted Mass Hierarchy
- $m_2^2 - m_1^2 \ll |m_3^2 - m_{1,2}^2|$ — $\Delta m^2_{13} > 0$ – Normal Mass Hierarchy

$$\tan^2\theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2}; \quad \tan^2\theta_{23} \equiv \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2}; \quad U_{e3} \equiv \sin\theta_{13} e^{-i\delta}$$

[For a detailed discussion see AdG, Jenkins, PRD78, 053003 (2008)]

Three Flavor Mixing Hypothesis Fits All* Data Really Well.

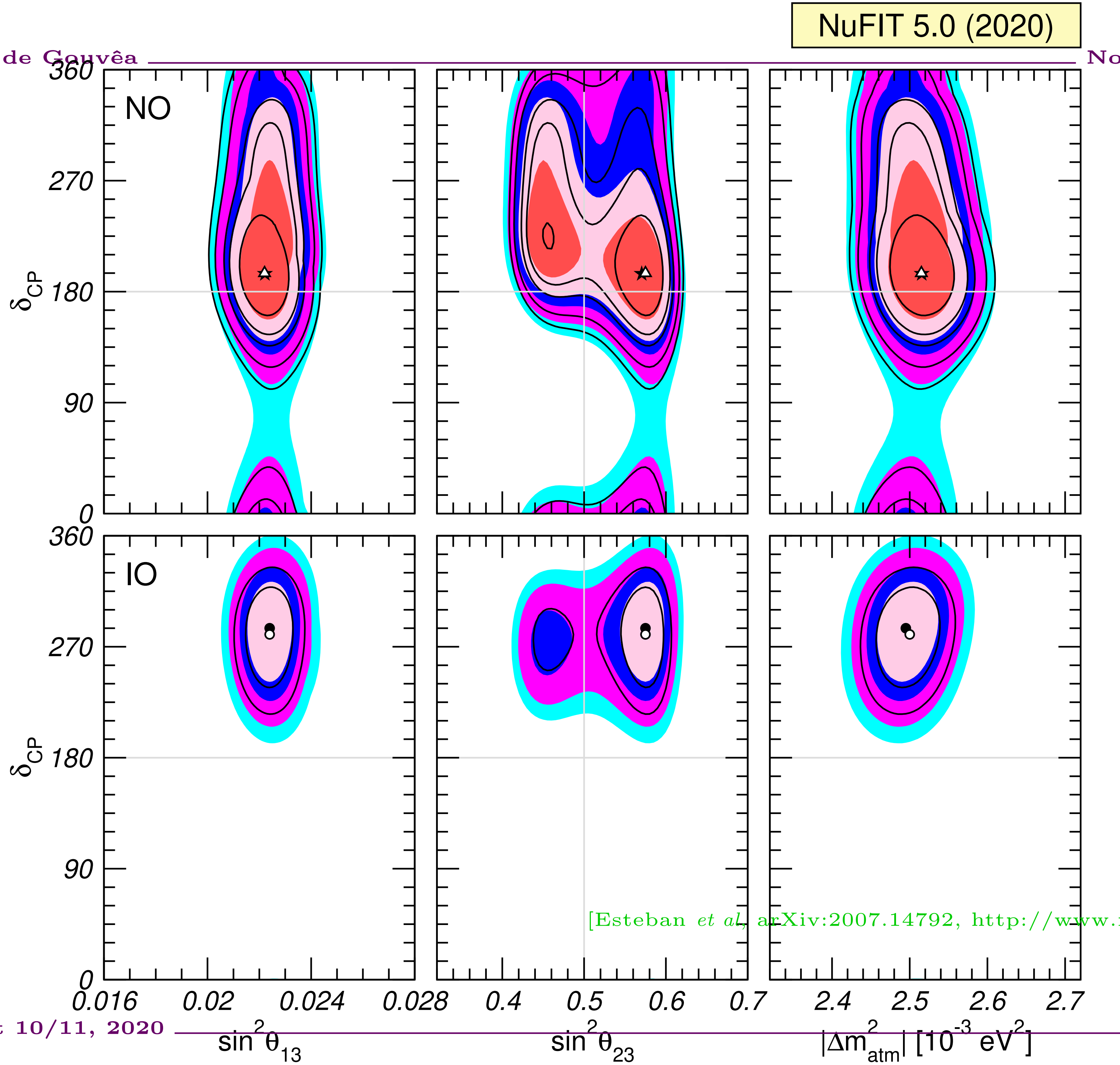
* Modulo short-baseline anomalies.

NuFIT 5.0 (2020)

without SK atmospheric data

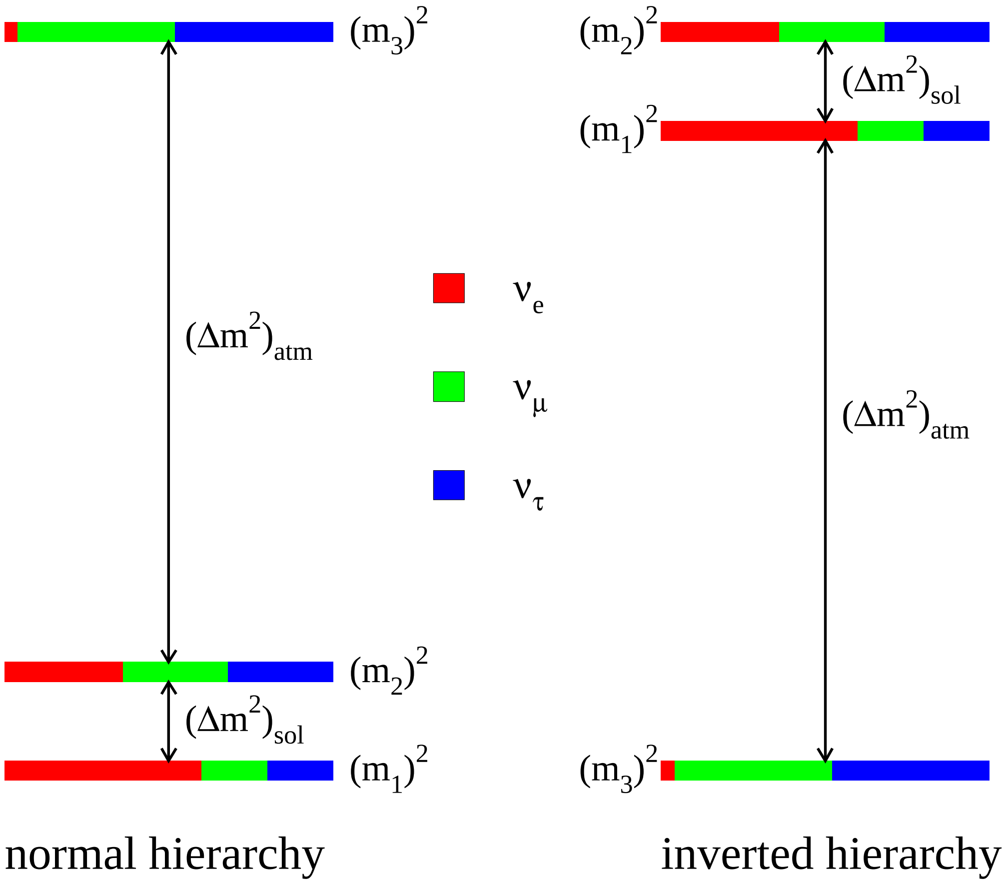
	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.7$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.407 \to 0.618$	$0.575^{+0.017}_{-0.021}$	$0.411 \to 0.621$
$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	$39.6 \to 51.8$	$49.3^{+1.0}_{-1.2}$	$39.9 \to 52.0$
$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02240^{+0.00062}_{-0.00062}$	$0.02053 \to 0.02436$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.61^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{CP}/^\circ$	195^{+51}_{-25}	$107 \to 403$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.514^{+0.028}_{-0.027}$	$+2.431 \to +2.598$	$-2.497^{+0.028}_{-0.028}$	$-2.583 \to -2.412$

[Esteban *et al*, arXiv:2007.14792, http://www.nu-fit.org]



[Esteban *et al*, arXiv:2007.14792, http://www.nu-fit.org]

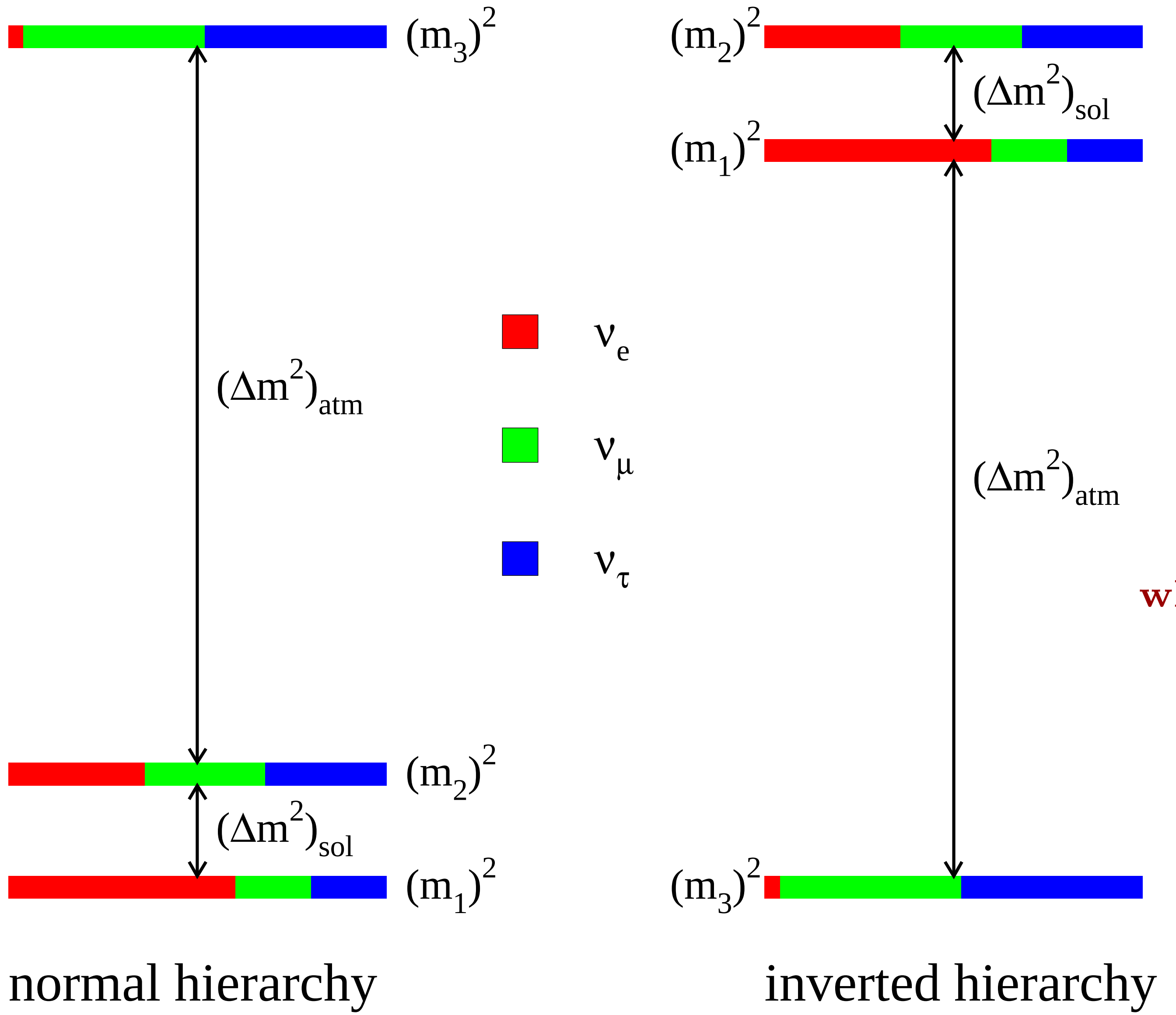
Understanding Neutrino Oscillations: What is Left to Do?



- ~~What is the ν_e component of ν_3?~~ $(\theta_{13} \neq 0!)$
- Is CP-invariance violated in neutrino oscillations? $(\delta \neq 0, \pi?)$ ['yes' hint]
- Is ν_3 mostly ν_μ or ν_τ? $[\theta_{23} \neq \pi/4$ hint$]$
- What is the neutrino mass hierarchy? $(\Delta m^2_{13} > 0?)$ [NH weak hint]

$\Rightarrow$ All of the above can "only" be addressed with new neutrino oscillation experiments

Ultimate Goal: Not Measure Parameters but Test the Formalism (Over-Constrain Parameter Space)



The Neutrino Mass Hierarchy

which is the right picture?

Why Don't We Know the Neutrino Mass Hierarchy?

Most of the information we have regarding θ_{23} and Δm^2_{13} comes from ν_μ disppearance (e.g., SuperK, NOνA, T2K). Roughly speaking, they measure

$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2_{13} L}{4E}\right) + \text{ subleading.}$$

It is easy to see from the expression above that the leading term is simply not sensitive to the sign of Δm^2_{13}.

On the other hand, because $|U_{e3}|^2 \sim 0.02$ and $\frac{\Delta m^2_{12}}{\Delta m^2_{13}} \sim 0.03$ are both small, we are yet to observe the subleading effects.

Determining the Mass Hierarchy via Oscillations – matter effects

Again, necessary to probe $\nu_\mu \to \nu_e$ oscillations (or vice-versa) governed by Δm^2_{13}. This is the oscillation channel that (almost) all next-generation, accelerator-based experiments are concentrating on, including the ongoing experiments T2K and NOνA.

In vaccum

$$P_{\mu e} = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{13}L}{4E}\right) + \text{ “subleading”},$$

so that, again, this is insensitive to the sign of Δm^2_{13} at leading order. However, in this case, matter effects may come to the rescue.

As I discussed already, neutrino oscillations get modified when these propagate in the presence of matter. Matter effects are sensitive to the neutrino mass ordering (in a way that I will describe shortly) and different for neutrinos and antineutrinos.

If $\Delta_{12} \equiv \frac{\Delta m^2_{12}}{2E}$ terms are ignored, the $\nu_\mu \to \nu_e$ oscillation probability is described, in constant matter density, by

$$P_{\mu e} \simeq P_{e\mu} \simeq \sin^2\theta_{23}\sin^2 2\theta_{13}^{\rm eff}\sin^2\left(\frac{\Delta_{13}^{\rm eff}L}{2}\right),$$

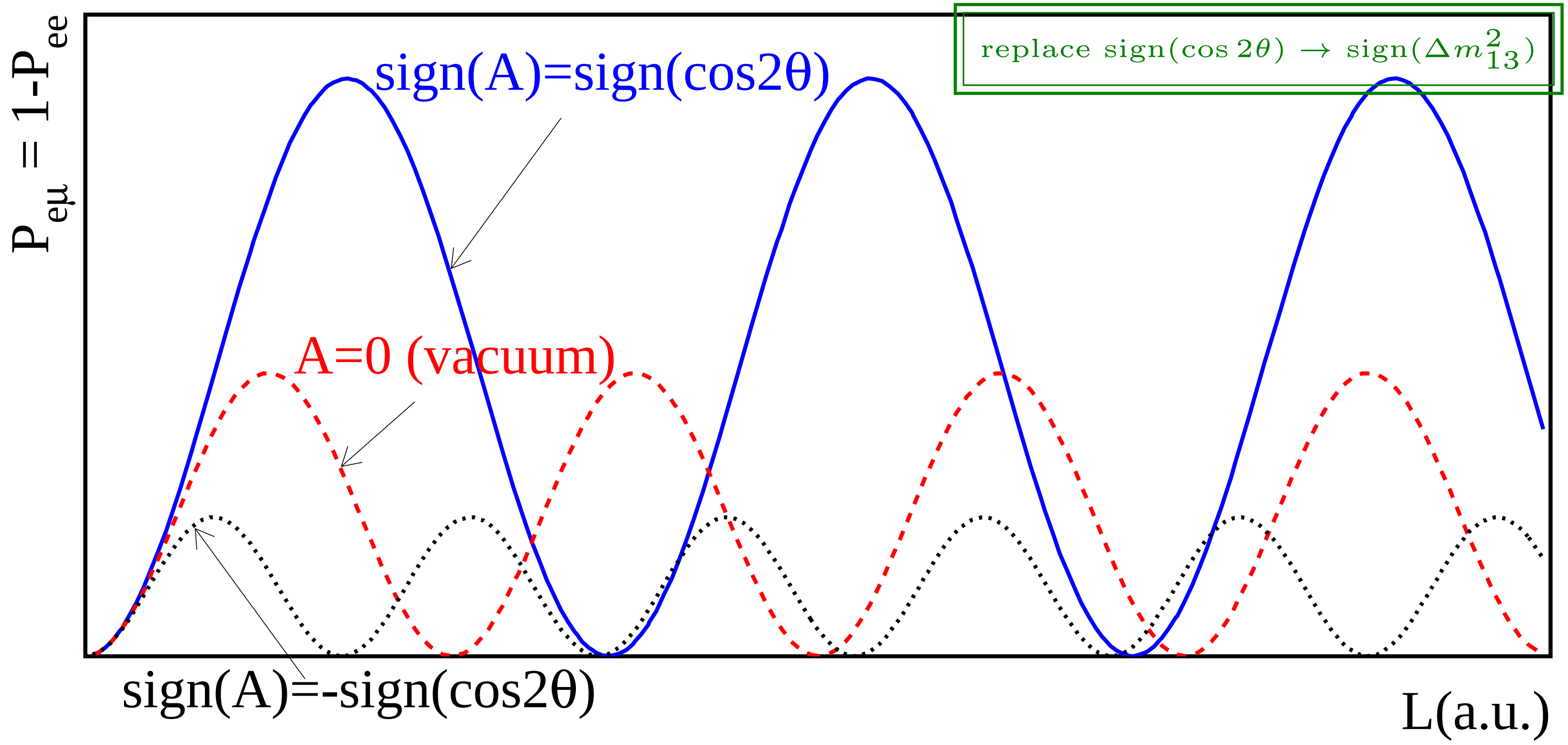
$$\sin^2 2\theta_{13}^{\rm eff} = \frac{\Delta_{13}^2\sin^2 2\theta_{13}}{(\Delta_{13}^{\rm eff})^2},$$

$$\Delta_{13}^{\rm eff} = \sqrt{(\Delta_{13}\cos 2\theta_{13} - A)^2 + \Delta_{13}^2\sin^2 2\theta_{13}},$$

$$\Delta_{13} = \frac{\Delta m^2_{13}}{2E},$$

$A \equiv \pm\sqrt{2}G_F N_e$ is the matter potential. It is positive for neutrinos and negative for antineutrinos.

$P_{\mu e}$ depends on the relative sign between Δ_{13} and A. It is different for the two different mass hierarchies, and different for neutrinos and antineutrinos.



Requirements:

- $\sin^2 2\theta_{13}$ large enough – otherwise there is nothing to see!
- $|\Delta_{13}| \sim |A|$ – matter potential must be significant but not overwhelming.
- $\Delta_{13}^{\rm eff} L$ large enough – matter effects are absent near the origin.

The "Holy Graill" of Neutrino Oscillations – CP Violation

In the old Standard Model, there is only one[a] source of CP-invariance violation:

⇒ The complex phase in V_{CKM}, the quark mixing matrix.

Indeed, as far as we have been able to test, all CP-invariance violating phenomena agree with the CKM paradigm:

- ϵ_K;
- ϵ'_K;
- $\sin 2\beta$;
- etc.

Recent experimental developments, however, provide strong reason to believe that this is not the case: neutrinos have mass, and leptons mix!

[a]modulo the QCD θ-parameter . . .

Golden Opportunity to Understand Matter versus Antimatter?

The SM with massive Majorana neutrinos accommodates **five** irreducible CP-invariance violating phases.

- One is the phase in the CKM phase. We have measured it, it is large, and we don't understand its value. At all.
- One is θ_{QCD} term ($\theta G\tilde{G}$). We don't know its value but it is only constrained to be very small. We don't know why (there are some good ideas, however).
- Three are in the neutrino sector. One can be measured via neutrino oscillations. 50% increase on the amount of information.

We don't know much about CP-invariance violation. Is it really fair to presume that CP-invariance is generically violated in the neutrino sector solely based on the fact that it is violated in the quark sector? Why? Cautionary tale: "Mixing angles are small"

CP-invariance Violation in Neutrino Oscillations

The most promising approach to studying CP-violation in the leptonic sector seems to be to compare $P(\nu_\mu \to \nu_e)$ versus $P(\bar{\nu}_\mu \to \bar{\nu}_e)$.

The amplitude for $\nu_\mu \to \nu_e$ transitions can be written as

$$A_{\mu e} = U_{e2}^* U_{\mu 2} \left(e^{i\Delta_{12}} - 1\right) + U_{e3}^* U_{\mu 3} \left(e^{i\Delta_{13}} - 1\right)$$

where $\Delta_{1i} = \frac{\Delta m^2_{1i} L}{2E}$, $i = 2, 3$.

The amplitude for the CP-conjugate process can be written as

$$\bar{A}_{\mu e} = U_{e2} U_{\mu 2}^* \left(e^{i\Delta_{12}} - 1\right) + U_{e3} U_{\mu 3}^* \left(e^{i\Delta_{13}} - 1\right).$$

[remember: according to unitarty, $U_{e1}U_{\mu 1}^* = -U_{e2}U_{\mu 2}^* - U_{e3}U_{\mu 3}^*$]

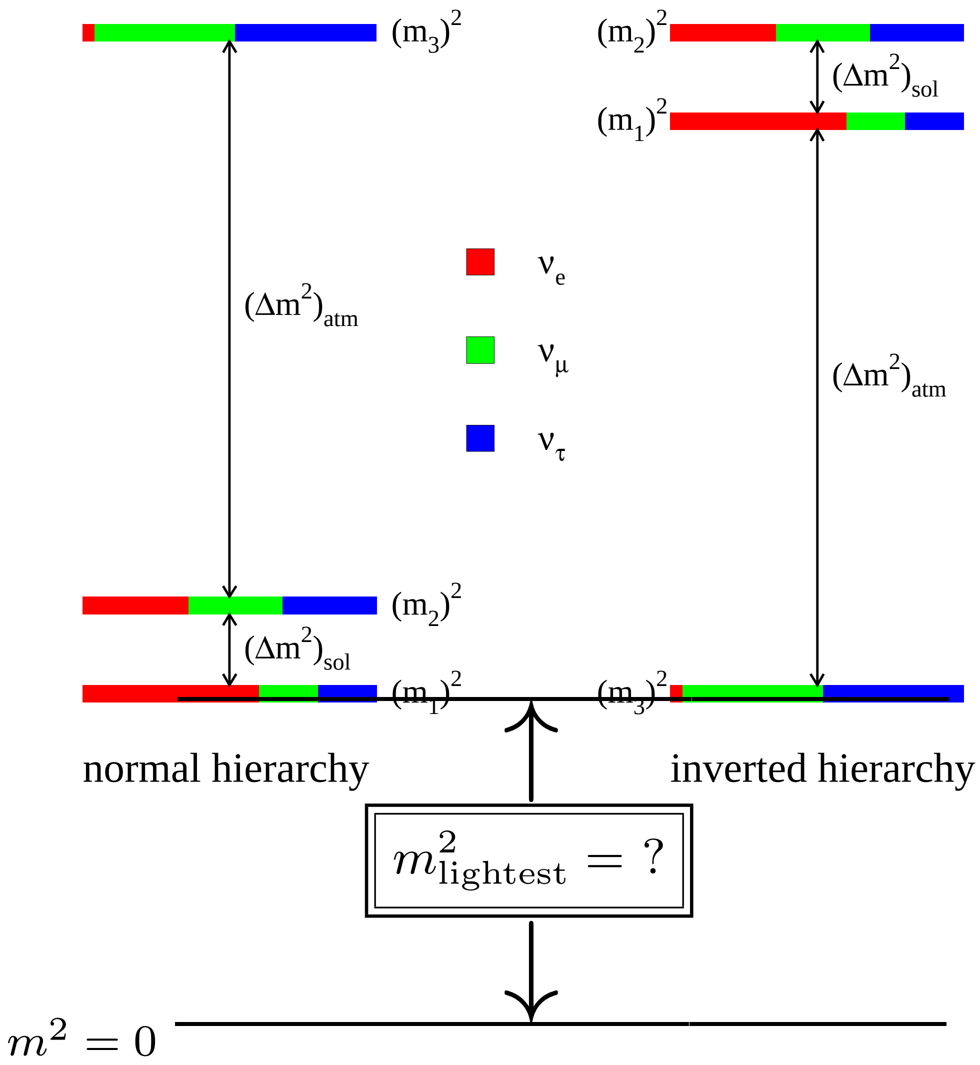
In general, $|A|^2 \neq |\bar{A}|^2$ (CP-invariance violated) as long as:

- Nontrivial "Weak" Phases: $\arg(U_{ei}^* U_{\mu i}) \to \delta \neq 0, \pi$;
- Nontrivial "Strong" Phases: Δ_{12}, $\Delta_{13} \to L \neq 0$;
- Because of Unitarity, we need all $|U_{\alpha i}| \neq 0 \to$ three generations.

All of these can be satisfied, with a little luck: given that two of the three mixing angles are known to be large, **we need $|U_{e3}| \neq 0$. (✓)**

The goal of next-generation neutrino experiments is to determine the magnitude of $|U_{e3}|$. We need to know this in order to understand how to study CP-invariance violation in neutrino oscillations!

What We Know We Don't Know: How Light is the Lightest Neutrino?



So far, we've only been able to measure neutrino mass-squared differences.

The lightest neutrino mass is only poorly constrained: $m^2_{\rm lightest} < 1~{\rm eV}^2$

qualitatively different scenarios allowed:

- $m^2_{\rm lightest} \equiv 0$;
- $m^2_{\rm lightest} \ll \Delta m^2_{12,13}$;
- $m^2_{\rm lightest} \gg \Delta m^2_{12,13}$.

Need information outside of neutrino oscillations:

$\rightarrow$ cosmology, β-decay, $0\nu\beta\beta$

The most direct probe of the lightest neutrino mass – precision measurements of β-decay

Observation of the effect of non-zero neutrino masses **kinematically**.

When a neutrino is produced, some of the energy exchanged in the process should be spent by the non-zero neutrino mass.

Typical effects are very, very small – we've never seen them! The most sensitive observable is the electron energy spectrum from tritium decay.

$$^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}$$

Why tritium? Small Q value, reasonable abundances. Required sensitivity proportional to m^2/Q^2.

In practice, this decay is sensitive to an effective "electron neutrino mass":

$$m^2_{\nu_e} \equiv \sum_i |U_{ei}|^2 m_i^2$$

Experiments measure the **shape** of the end-point of the spectrum, not the value of the end point. This is done by counting events as a function of a low-energy cut-off.

note: LOTS of Statistics Needed!

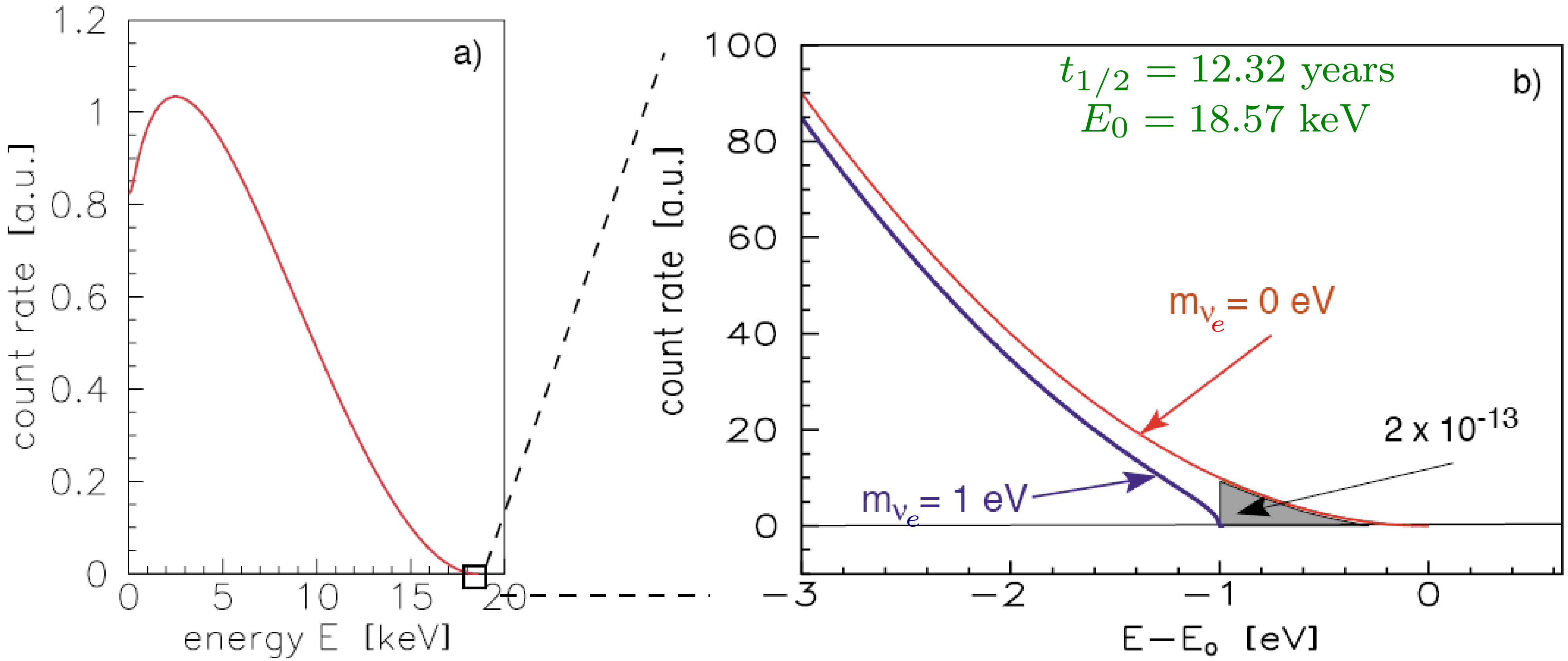


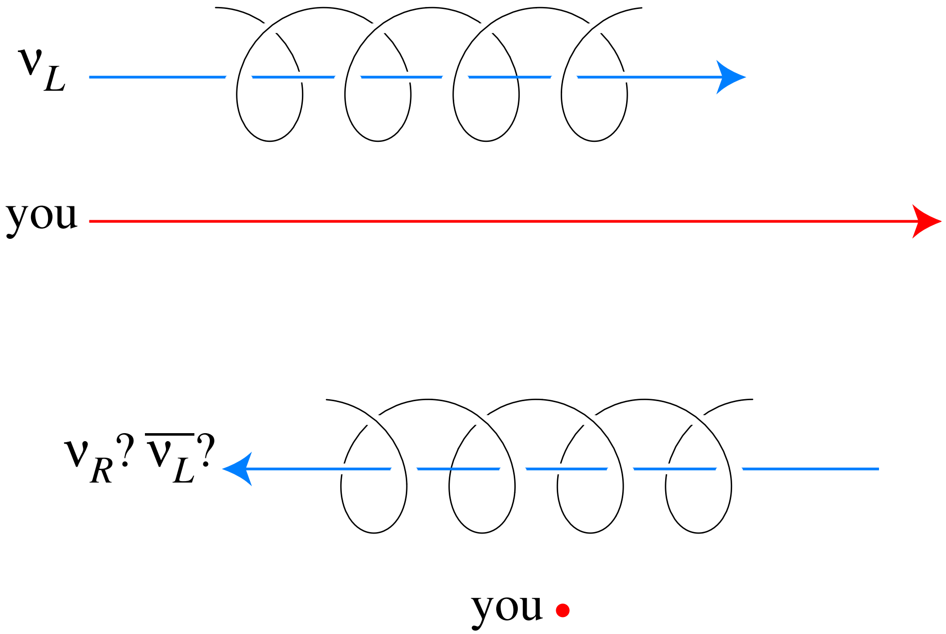
Figure 2: The electron energy spectrum of tritium β decay: (a) complete and (b) narrow region around endpoint E_0. The β spectrum is shown for neutrino masses of 0 and 1 eV.

NEXT GENERATION: The Karlsruhe Tritium Neutrino (KATRIN) Experiment:

(not your grandmother's table top experiment!)

sensitivity $m^2_{\nu_e} > (0.2 \text{ eV})^2$

What We Know We Don't Know (iii) – Are Neutrinos Majorana Fermions?



A massive charged fermion (s=1/2) is described by 4 degrees of freedom:

$$(e^-_L \leftarrow \text{CPT} \rightarrow e^+_R)$$
$$\updownarrow \text{Lorentz}$$
$$(e^-_R \leftarrow \text{CPT} \rightarrow e^+_L)$$

A massive neutral fermion (s=1/2) is described by 4 or 2 degrees of freedom:

$$(\nu_L \leftarrow \text{CPT} \rightarrow \bar{\nu}_R)$$
$$\updownarrow \text{Lorentz} \qquad \text{“DIRAC”}$$
$$(\nu_R \leftarrow \text{CPT} \rightarrow \bar{\nu}_L)$$

“MAJORANA”

$$(\nu_L \leftarrow \text{CPT} \rightarrow \bar{\nu}_R)$$
$$\updownarrow \text{Lorentz}$$
$$(\bar{\nu}_R \leftarrow \text{CPT} \rightarrow \nu_L)$$

How many degrees of freedom are required to describe massive neutrinos?

Why Don't We Know the Answer (Yet)?

If neutrino masses were indeed zero, this is a nonquestion: there is no distinction between a massless Dirac and Majorana fermion.

Processes that are proportional to the Majorana nature of the neutrino vanish in the limit $m_\nu \to 0$. Since neutrinos masses are very small, the probability for these to happen is very, very small: $A \propto m_\nu/E$.

The "smoking gun" signature is the observation of LEPTON NUMBER violation. This is easy to understand: Majorana neutrinos are their own antiparticles and, therefore, cannot carry any quantum numbers — including lepton number.

The deepest probes are searches for Neutrinoless Double-Beta Decay.

Weak Interactions are Purely Left-Handed (Chirality):

For example, in the scattering process $e^- + X \to \nu_e + X$, the electron neutrino is, in a reference frame where $m \ll E$,

$$|\nu_e\rangle \sim |L\rangle + \left(\frac{m}{E}\right)|R\rangle.$$

If the neutrino is a Majorana fermion, $|R\rangle$ behaves mostly like a "$\bar{\nu}_e$," (and $|L\rangle$ mostly like a "ν_e,") such that the following process could happen:

$$e^- + X \to \nu_e + X, \text{ followed by } \nu_e + X \to e^+ + X, \quad P \simeq \left(\frac{m}{E}\right)^2$$

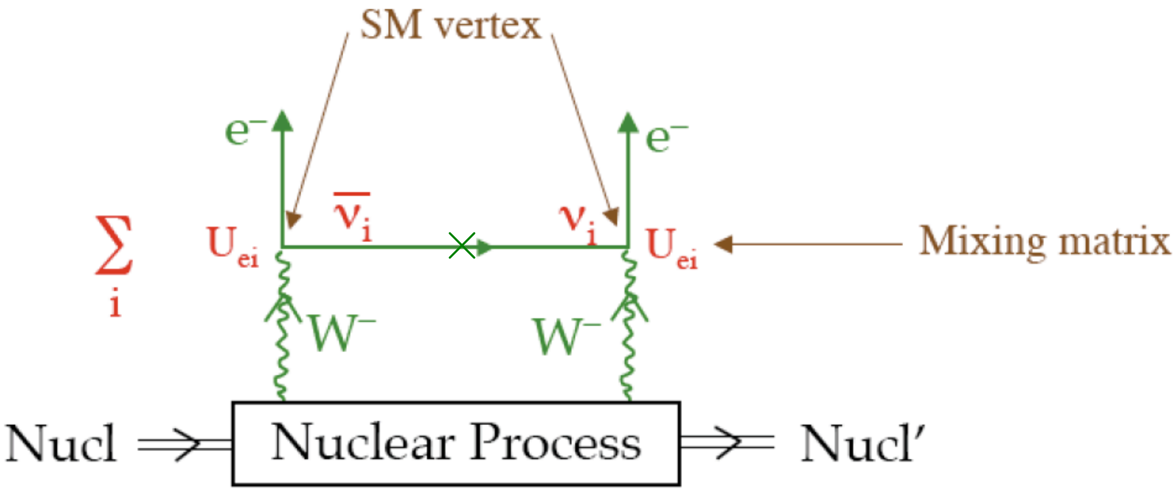
Lepton number can be violated by 2 units with small probability. Typical numbers: $P \simeq (0.1 \text{ eV}/100 \text{ MeV})^2 = 10^{-18}$. VERY Challenging!

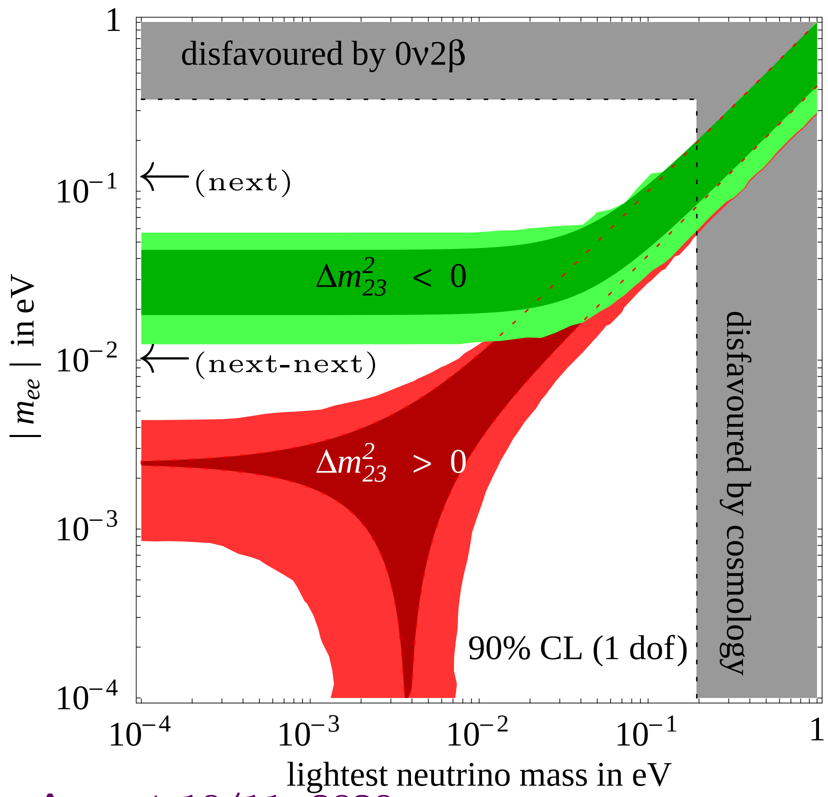
Search for the Violation of Lepton Number (or $B-L$)

Best Bet: search for

Neutrinoless Double-Beta

Decay: $Z \to (Z+2)e^- e^-$





Helicity Suppressed Amplitude $\propto \frac{m_{ee}}{E}$

Observable: $m_{ee} \equiv \sum_i U_{ei}^2 m_i$

$\Leftarrow$ **no longer lamp-post physics!**

$$\mathcal{L} \supset \bar{e}_L U W^\mu \gamma_\mu \nu_L - \bar{e}_L (M_e) e_R - \overline{\nu^c_L}(M_\nu)\nu_L + H.c.$$

Write $U = E^{-i\xi/2} U' E^{i\alpha/2}$, where $E^{i\beta/2} \equiv \mathrm{diag}(e^{i\beta_1/2}, e^{i\beta_2/2}, e^{i\beta_3/2})$, $\beta = \alpha, \xi$

$$\mathcal{L} \supset \bar{e}_L U' W^\mu \gamma_\mu \nu_L - \bar{e}_L E^{i\xi/2} (M_e) e_R - \overline{\nu^c_L}(M_\nu) E^{-i\alpha}\nu_L + H.c.$$

ξ phases can be "absorbed" by e_R,

α phases cannot go away!

on the other hand

Dirac Case:

$$\mathcal{L} \supset \bar{e}_L U W^\mu \gamma_\mu \nu_L - \bar{e}_L (M_e) e_R - \bar{\nu}_R(M_\nu)\nu_L + H.c.$$

$$\mathcal{L} \supset \bar{e}_L U' W^\mu \gamma_\mu \nu_L - \bar{e}_L E^{i\xi/2} (M_e) e_R - \bar{\nu}_R(M_\nu) E^{-i\alpha/2}\nu_L + H.c.$$

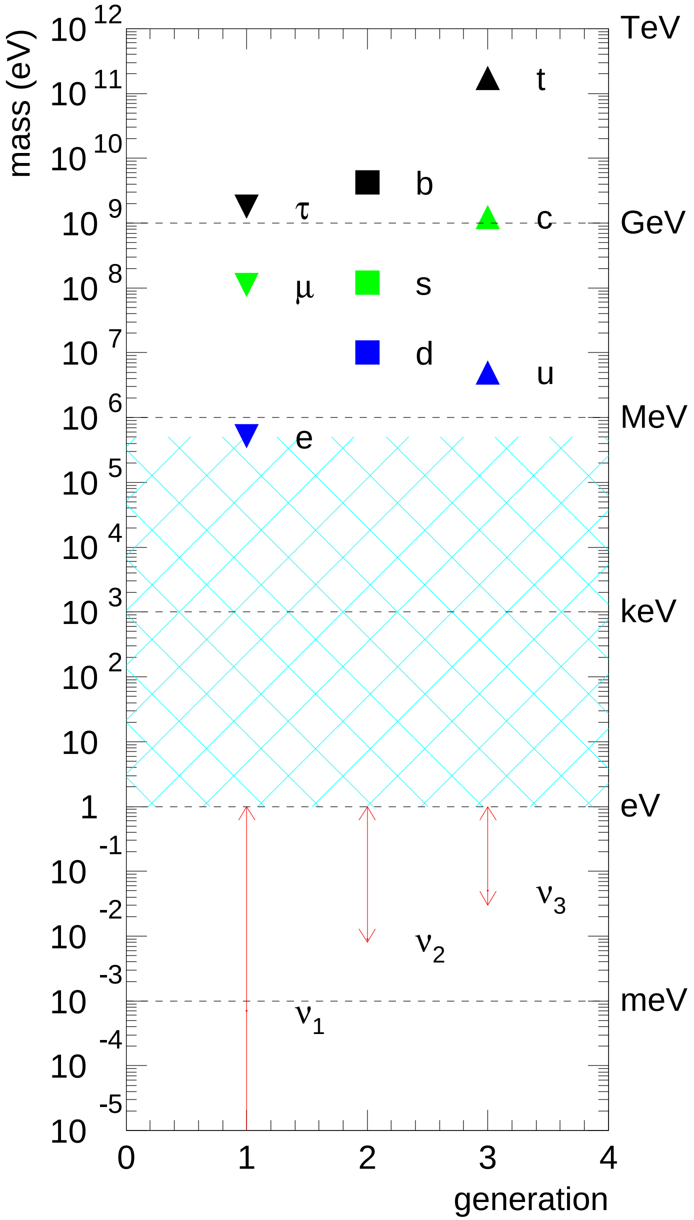
ξ phases can be "absorbed" by e_R, α phases can be "absorbed" by ν_R,

$$V_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{e\tau 2} & U_{\tau 3} \end{pmatrix}' \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & e^{i\alpha_3/2} \end{pmatrix}.$$

It is easy to see that the Majorana phases never show up in neutrino oscillations ($A \propto U_{\alpha i} U^*_{\beta_i}$).

Furthermore, they only manifest themselves in phenomena that vanish in the limit $m_i \to 0$ – after all they are only physical if we "know" that lepton number is broken.

$$A(\alpha_i) \propto m_i/E \quad \to \quad \text{tiny!}$$



NEUTRINOS HAVE MASS

albeit very tiny ones…

SO WHAT?

Only* "Palpable" Evidence of Physics Beyond the Standard Model

The SM we all learned in school predicts that neutrinos are strictly massless. Hence, massive neutrinos imply that the the SM is incomplete and needs to be replaced/modified.

Furthermore, the SM has to be replaced by something qualitatively different.

---

* There is only a handful of questions our model for fundamental physics cannot explain (these are personal. Feel free to complain).

- What is the physics behind electroweak symmetry breaking? (Higgs ✓).
- What is the dark matter? (not in SM).
- Why is there more matter than antimatter? (Not in SM).
- Why does the Universe appear to be accelerating? Why does it appear that the Universe underwent rapid acceleration in the past? (not in SM).

Standard Model in One Slide, No Equations

The SM is a quantum field theory with the following defining characteristics:

- Gauge Group $(SU(3)_c \times SU(2)_{\rm L} \times U(1)_{\rm Y})$;
- Particle Content (fermions: Q, u, d, L, e, scalars: H).

Once this is specified, the SM is unambiguously determined:

- Most General Renormalizable Lagrangian;
- Measure All Free Parameters, and You Are Done! (after several decades of hard experimental work. . .)

If you follow these rules, neutrinos have no mass. Something has to give.

What is the New Standard Model? [νSM]

The short answer is – WE DON'T KNOW. Not enough available info!

$\Updownarrow$

Equivalently, there are several completely different ways of addressing neutrino masses. The key issue is to understand what else the νSM candidates can do. [are they falsifiable?, are they "simple"?, do they address other outstanding problems in physics?, etc]

We need more experimental input, and it looks like it may be coming in the near/intermediate future!

Neutrino Masses, EWSB, and a New Mass Scale of Nature

The LHC has revealed that the minimum SM prescription for electroweak symmetry breaking — the one Higgs double model — is at least approximately correct. What does that have to do with neutrinos?

The tiny neutrino masses point to three different possibilities.

1. Neutrinos talk to the Higgs boson very, very **weakly** (Dirac neutrinos);
2. Neutrinos talk to a **different Higgs** boson – there is a new source of electroweak symmetry breaking! (Majorana neutrinos);
3. Neutrino masses are small because there is **another source of mass** out there — a new energy scale indirectly responsible for the tiny neutrino masses, a la the seesaw mechanism (Majorana neutrinos).

Searches for $0\nu\beta\beta$ help tell (1) from (2) and (3), the LHC, charged-lepton flavor violation, *et al* may provide more information.

νSM – One Path

SM as an effective field theory – non-renormalizable operators

$$\mathcal{L}_{\nu\text{SM}} \supset -y_{ij}\frac{L^iHL^jH}{2\Lambda} + \mathcal{O}\left(\frac{1}{\Lambda^2}\right) + H.c.$$

There is only one dimension five operator [Weinberg, 1979]. If $\Lambda \gg 1$ TeV, it leads to only one observable consequence...

$$\text{after EWSB } \mathcal{L}_{\nu\text{SM}} \supset \frac{m_{ij}}{2}\nu^i\nu^j; \quad m_{ij} = y_{ij}\frac{v^2}{\Lambda}.$$

- Neutrino masses are small: $\Lambda \gg v \to m_\nu \ll m_f$ ($f = e, \mu, u, d$, etc)
- Neutrinos are Majorana fermions – Lepton number is violated!
- νSM effective theory – not valid for energies above at most Λ.
- What is Λ? First naive guess is that Λ is the Planck scale – does not work. Data require $\Lambda \sim 10^{14}$ GeV (related to GUT scale?) [note $y^{\max} \equiv 1$]

What else is this "good for"? Depends on the ultraviolet completion!

The Seesaw Lagrangian

A simple[a], renormalizable Lagrangian that allows for neutrino masses is

$$\mathcal{L}_\nu = \mathcal{L}_{\rm old} - \lambda_{\alpha i} L^\alpha H N^i - \sum_{i=1}^{3} \frac{M_i}{2} N^i N^i + H.c.,$$

where N^i ($i = 1, 2, 3,$ for concreteness) are SM gauge singlet fermions.

$\mathcal{L}_\nu$ is the most general, renormalizable Lagrangian consistent with the SM gauge group and particle content, plus the addition of the N_i fields.

After electroweak symmetry breaking, $\mathcal{L}_\nu$ describes, besides all other SM degrees of freedom, six Majorana fermions: **six neutrinos.**

[a]Only requires the introduction of three fermionic degrees of freedom, no new interactions or symmetries.

To be determined from data: λ and M.

The data can be summarized as follows: there is evidence for three neutrinos, mostly "active" (linear combinations of ν_e, ν_μ, and ν_τ). At least two of them are massive and, if there are other neutrinos, they have to be "sterile."

This provides very little information concerning the magnitude of M_i (assume $M_1 \sim M_2 \sim M_3$).

Theoretically, there is prejudice in favor of very large M: $M \gg v$. Popular examples include $M \sim M_{\rm GUT}$ (GUT scale), or $M \sim 1$ TeV (EWSB scale).

Furthermore, $\lambda \sim 1$ translates into $M \sim 10^{14}$ GeV, while thermal leptogenesis requires the lightest M_i to be around 10^{10} GeV.

we can impose very, very few experimental constraints on M

Tree-Level Realization of the Weinberg Operator

If $\mu = \lambda v \ll M$, below the mass scale M,

$$\mathcal{L}_5 = \frac{LHLH}{\Lambda}.$$

Neutrino masses are small if $\Lambda \gg \langle H \rangle$. Data require $\Lambda \sim 10^{14}$ GeV.

In the case of the seesaw,

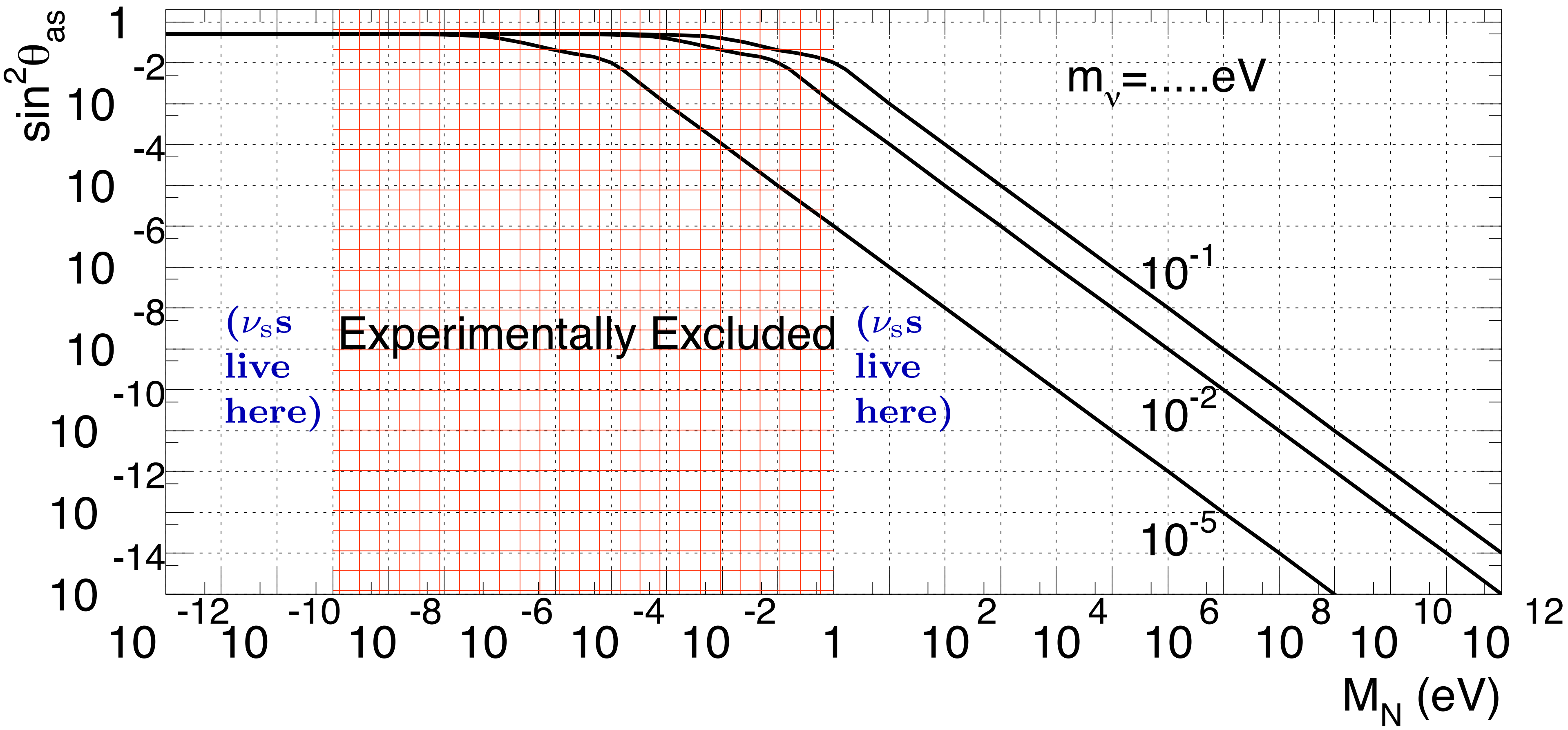
$$\Lambda \sim \frac{M}{\lambda^2},$$

so neutrino masses are small if either

- they are generated by physics at a very high energy scale $M \gg v$ (high-energy seesaw); or
- they arise out of a very weak coupling between the SM and a new, hidden sector (low-energy seesaw); or
- cancellations among different contributions render neutrino masses accidentally small ("fine-tuning").

Constraining the Seesaw Lagrangian

[AdG, Huang, Jenkins, arXiv:0906.1611]



Theoretical upper bound: $M_N < 7.6 \times 10^{24}$ eV $\times \left(\frac{0.1 \text{ eV}}{m_\nu}\right) \Rightarrow\Rightarrow\Rightarrow$

Higher Order Neutrino Masses from $\Delta L = 2$ Physics – Other Paths

Imagine that there is new physics that breaks lepton number by 2 units at some energy scale Λ, but that it does not, in general, lead to neutrino masses at the tree level.

We know that neutrinos will get a mass at some order in perturbation theory – which order is model dependent!

For example:

- SUSY with trilinear R-parity violation – neutrino masses at one-loop;
- Zee models – neutrino masses at one-loop;
- Babu and Ma – neutrino masses at two loops;
- Chen et al, 0706.1964 – neutrino masses at two loops;
- Angel et al, 1308.0463 – neutrino masses at two loops;
- etc.

One Approach Aimed at Phenomenology

- Only consider $\Delta L = 2$ operators;
- Operators made up of only standard model fermions and the Higgs doublet (no gauge bosons);
- Electroweak symmetry breaking characterized as prescribed in SM;
- Effective operator couplings assumed to be "flavor indifferent", unless otherwise noted;
- Operators "turned on" one at a time, assumed to be leading order (tree-level) contribution of new lepton number violating physics.
- We can use the effective operator to estimate the coefficient of all other lepton-number violating lower-dimensional effective operators (loop effects, computed with a hard cutoff).

Results presented are order of magnitude *estimates*, not precise quantitative results. Q: Does this really make sense? A: Sometimes...

$\mathcal{O}$	Operator	Λ [TeV]
$\mathcal{O}_1$	$(LH)(LH)$	$6 \times 10^{10-11}$
$\mathcal{O}_2$	$(LL)(LH)e^c$	$4 \times 10^{6-7}$
$\mathcal{O}_{3_a}$	$(LL)(QH)d^c$	$2 \times 10^{4-5}$
$\mathcal{O}_{3_b}$	$(LQ)(LH)d^c$	$1 \times 10^{7-8}$
$\mathcal{O}_{4_a}$	$(L\overline{Q})(LH)\overline{u^c}$	$4 \times 10^{8-9}$
$\mathcal{O}_{4_b}$	$(LL)(\overline{Q}H)\overline{u^c}$	$2-7$
$\mathcal{O}_8$	$(LH)\overline{e^c}\overline{u^c}d^c$	$6 \times 10^{2-3}$

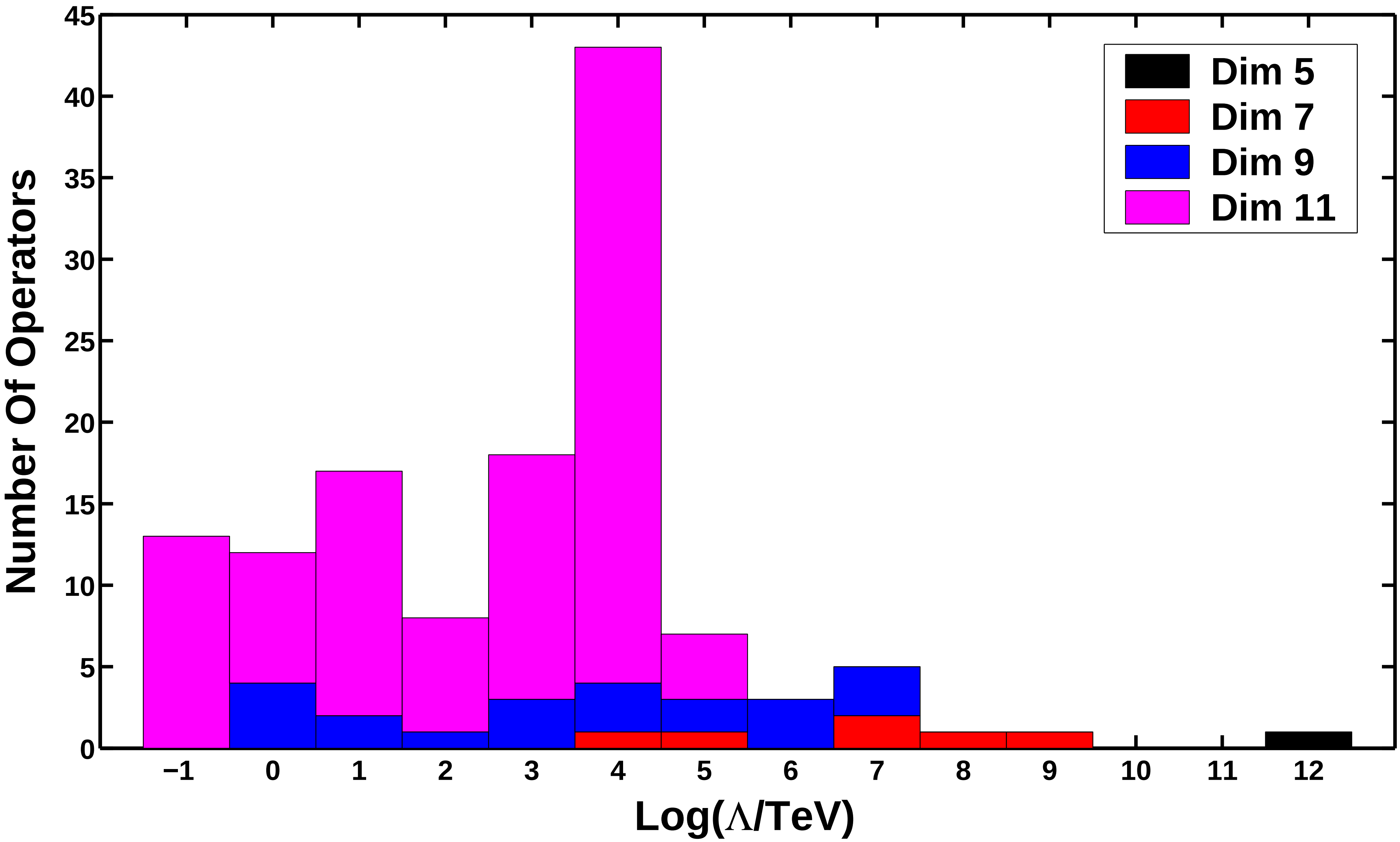
$\mathcal{O}$	Operator	Λ [TeV]
$\mathcal{O}_5$	$(L\overline{H})(LH)(QH)d^c$	$6 \times 10^{4-5}$
$\mathcal{O}_6$	$(LH)(L\overline{H})(\overline{Q}H)\overline{u^c}$	$2 \times 10^{6-7}$
$\mathcal{O}_7$	$(LH)(QH)(\overline{Q}H)\overline{e^c}$	$4 \times 10^{1-2}$
$\mathcal{O}_9$	$(LL)(LL)e^c e^c$	$3 \times 10^{2-3}$
$\mathcal{O}_{10}$	$(LL)(LQ)e^c d^c$	$6 \times 10^{2-3}$
$\mathcal{O}_{11_a}$	$(LL)(QQ)d^c d^c$	$3-30$
$\mathcal{O}_{11_b}$	$(LQ)(LQ)d^c d^c$	$2 \times 10^{3-4}$

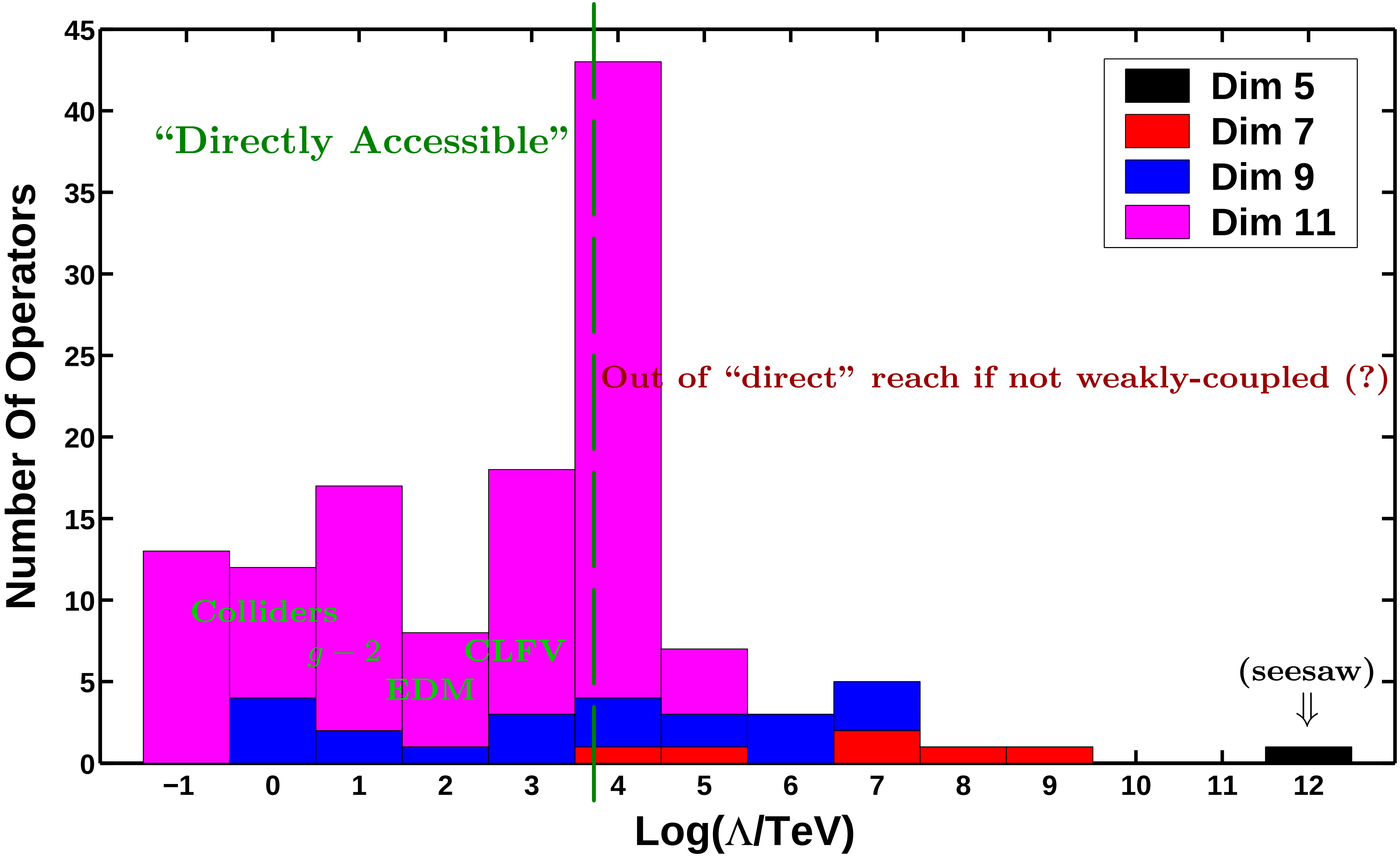
$\mathcal{O}_{12_a}$	$(L\overline{Q})(L\overline{Q})\overline{u^c}\overline{u^c}$	$2 \times 10^{6-7}$
$\mathcal{O}_{12_b}$	$(LL)(\overline{Q}\overline{Q})\overline{u^c}\overline{u^c}$	$0.3-0.6$
$\mathcal{O}_{13}$	$(L\overline{Q})(LL)\overline{u^c}e^c$	$2 \times 10^{4-5}$
$\mathcal{O}_{14_a}$	$(LL)(Q\overline{Q})\overline{u^c}d^c$	10^{2-3}
$\mathcal{O}_{14_b}$	$(L\overline{Q})(LQ)\overline{u^c}d^c$	$6 \times 10^{4-5}$
$\mathcal{O}_{15}$	$(LL)(L\overline{L})d^c\overline{u^c}$	10^{2-3}
$\mathcal{O}_{16}$	$(LL)e^c d^c\overline{e^c}\overline{u^c}$	$0.2-2$
$\mathcal{O}_{17}$	$(LL)d^c d^c\overline{d^c}\overline{u^c}$	$0.2-2$

$\mathcal{O}_{18}$	$(LL)d^c u^c\overline{u^c}\overline{u^c}$	$0.2-2$
$\mathcal{O}_{19}$	$(LQ)d^c d^c\overline{e^c}\overline{u^c}$	$0.1-1$
$\mathcal{O}_{20}$	$(L\overline{Q})d^c\overline{u^c}\overline{e^c}\overline{u^c}$	$4-40$
$\mathcal{O}_s$	$e^c e^c u^c u^c\overline{d^c}\overline{d^c}$	10^{-3}

- Ignore Lorentz, $SU(3)_C$ structure
- $SU(2)_L$ contractions denoted with parentheses
- Λ indicates range in which $m_\nu \in [0.05\text{ eV}, 0.5\text{ eV}]$

hep-ph/0106054; K.S. Babu & C.N. Leung
arXiv:0708.1344; A. de Gouvêa & J. Jenkins
arXiv:1212.6111; P.W. Angel, et al.
arXiv:1404.4057; A. de Gouvêa, at al.


Dim 5
Dim 7
Dim 9
Dim 11
Number Of Operators
Log(Λ/TeV)
45
40
35
30
25
20
15
10
5
0
−1
0
1
2
3
4
5
6
7
8
9
10
11
12



Dim 5
Dim 7
Dim 9
Dim 11
"Directly Accessible"
Out of "direct" reach if not weakly-coupled (?)
Colliders
g − 2
EDM
CLFV
(seesaw)
Number Of Operators
Log(Λ/TeV)
45
40
35
30
25
20
15
10
5
0
−1
0
1
2
3
4
5
6
7
8
9
10
11
12

Dirac Neutrinos – Enhanced Symmetry!(Symmetries?)

Back to

$$\mathcal{L}_\nu = \mathcal{L}_{\rm old} - \lambda_{\alpha i} L^\alpha H N^i - \sum_{i=1}^{3} \frac{M_i}{2} N^i N^i + H.c.,$$

where N_i ($i = 1, 2, 3,$ for concreteness) are SM gauge singlet fermions.

Dirac Neutrinos – Enhanced Symmetry!(Symmetries?)

If all $M_i \equiv 0$, the neutrinos are Dirac fermions.

$$\mathcal{L}_\nu = \mathcal{L}_{\rm old} - \lambda_{\alpha i} L^\alpha H N^i + H.c.,$$

where N_i ($i = 1, 2, 3$, for concreteness) are SM gauge singlet fermions. In this case, the νSM global symmetry structure is enhanced. For example, $U(1)_{B-L}$ is an exactly conserved, global symmetry. This is new!

Downside: The neutrino Yukawa couplings λ are tiny, less than 10^{-12}. What is wrong with that? We don't like tiny numbers, but Nature seems to not care very much about what we like...

More to the point, the failure here is that it turns out that the neutrino masses are not, trivially, qualitatively different. This seems to be a "missed opportunity."

There are lots of ideas that lead to very small Dirac neutrino masses.

Maybe right-handed neutrinos exist, but neutrino Yukawa couplings are forbidden – hence neutrino masses are tiny.

One possibility is that the N fields are charged under some new symmetry (gauged or global) that is spontaneously broken.

$$\lambda_{\alpha i} L^\alpha H N^i \to \frac{\kappa_{\alpha i}}{\Lambda}(L^\alpha H)(N^i \Phi),$$

where Φ (spontaneously) breaks the new symmetry at some energy scale v_Φ. Hence, $\lambda = \kappa v_\Phi/\Lambda$. How do we test this?

E.g., AdG and D. Hernández, arXiv:1507.00916

Gauged chiral new symmetry for the right-handed neutrinos, no Majorana masses allowed, plus a heavy messenger sector. Predictions: new stable massive states (mass around v_Φ) which look like (i) dark matter, (ii) (Dirac) sterile neutrinos are required. Furthermore, there is a new heavy Z'-like gauge boson.

⇒ Natural Conections to Dark Matter, Sterile Neutrinos, Dark Photons!

Understanding Fermion Mixing

The other puzzling phenomenon uncovered by the neutrino data is the fact that Neutrino Mixing is Strange. What does this mean?

It means that lepton mixing is very different from quark mixing:

$$V_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & \mathbf{0.2} \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \qquad V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix}$$

WHY?

$[|(V_{MNS})_{e3}| < 0.2]$

They certainly look VERY different, but which one would you label as "strange"?

Piecing the Neutrino Mass Puzzle

Understanding the origin of neutrino masses and exploring the new physics in the lepton sector will require unique **theoretical** and **experimental** efforts, including ...

- understanding the fate of lepton-number. Neutrinoless double beta decay!
- a comprehensive long baseline neutrino program, towards precision oscillation physics.
- other probes of neutrino properties, including neutrino scattering.
- precision studies of charged-lepton properties ($g-2$, edm), and searches for rare processes ($\mu \to e$-conversion the best bet at the moment).
- collider experiments. The LHC and beyond may end up revealing the new physics behind small neutrino masses.
- cosmic surveys. Neutrino properties affect, in a significant way, the history of the universe. Will we learn about neutrinos from cosmology, or about cosmology from neutrinos?
- searches for baryon-number violating processes.

Concluding Remarks

The venerable Standard Model has finally sprung a leak – neutrinos are not massless!

1. we have a very successful parametrization of the neutrino sector, and we have identified what we know we don't know.
2. neutrino masses are very small – we don't know why, but we think it means something important.
3. lepton mixing is very different from quark mixing – we don't know why, but we think it means something important.
4. we need a minimal νSM Lagrangian. In order to decide which one is "correct" (required in order to attack 2. and 3. above) we must uncover the faith of baryon number minus lepton number ($0\nu\beta\beta$ is the best [only?] bet).

5. We need more experimental input – and more seems to be on the way (this is a truly data driven field right now). We only started to figure out what is going on.

6. The fact that neutrinos have mass may be intimately connected to the fact that there are more baryons than antibaryons in the Universe. How do we test whether this is correct?

7. There is plenty of room for surprises, as neutrinos are very narrow but deep probes of all sorts of physical phenomena. Remember that neutrino oscillations are "quantum interference devices" – potentially very sensitive to whatever else may be out there (e.g., $M_{\rm seesaw} \simeq 10^{14}$ GeV).

8. Finally, we need to resolve the short baseline anomalies. Life could be much more interesting!