Almost invisibles in the Universe
Goals of today’s lecture

● Understand the Universe across time, intuitively and as described with equations

● Understand the impact of *almost invisibles* on that system: neutrinos, dark matter, dark energy, even more hypothetical fields

● Get a sense of how we can measure the evolution of the Universe, and what that has told us so far

● Set foundation for lectures this week
Two principles we’ll use

- There is no special place in the Universe - it is homogeneous and isotropic
- Gravity is described by General Relativity
A General Relativity balloon

How could you describe the time evolution of the balloon?
A General Relativity balloon

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By the steps an ant could take!
A General Relativity balloon

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Printing a coordinate system $x$ on the balloon,

$v_{\text{ant}} \ dt = a(t) \ |dx|$
A General Relativity balloon

How could you describe the time evolution of the balloon?

By the steps an ant could take!

Printing a coordinate system $x$ on the balloon,

$$v_{\text{ant}} \ dt = a(t) \ |dx|$$

Printing a coordinate system on the universe and using photons for ants,

$$c \ dt = a(t) \ |dx|$$
Friedman-Lemaître-Robertson-Walker Metric

\[ ds^2 = \sum_{i,j=0}^{3} g_{ij} dx^i dx^j = 0 \] for light, with coordinates \( x^0 = t, x^{1,2,3} \)
Friedman-Lemaître-Robertson-Walker Metric

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for light, with coordinates \( x^0 = t, x^{1,2,3} \)

\[ ds^2 = c^2 dt^2 - a^2(t) \left[ dr^2 + f_K^2(r) (d\theta^2 + \sin^2 \theta d\phi^2) \right] \]

is the most general metric that is homogeneous / isotropic
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\[
f_K(r) = \begin{cases} 
  r, & K = 0 \\
  1/K \sin(Kr), & K > 0 \\
  1/|K| \sinh(|K|r), & K < 0
\end{cases}
\]
FLRW Metric + Einstein Equations: The dynamics of a Universe

\[ ds^2 = c^2 dt^2 - a^2(t) \left[ dr^2 + f_K^2(r) (d\theta^2 + \sin^2 \theta d\phi^2) \right] \]

Einstein Equations

\[ R_{ij} - \frac{1}{2} g_{ij} R = \frac{8\pi G}{c^4} T_{ij} \]

\[ \frac{\ddot{a}}{a} = -\frac{4}{3} \pi G \left( \rho + \frac{3p}{c^2} \right) \]
The dynamics of a Universe with a cosmological constant

- A positive cosmological constant causes a positive acceleration
- A fluid with equation of state $p = -c^2 \rho$ has the same effect

\[ R_{ij} - \frac{1}{2} g_{ij} R - \left( \Lambda g_{ij} \right) = \frac{8\pi G}{c^4} T_{ij} \]

- $\Lambda$ takes over if density and pressure of all else in the universe are small enough

\[ \ddot{a} = -\frac{4}{3} \pi G \left( \rho + \frac{3p}{c^2} \right) + \frac{\Lambda c^2}{3} \]
The density of a Universe with a bunch of stuff

- A general fluid is described by its equation of state $p = w \rho c^2$
- Some fluids we know:
  - (cold) matter: $w=0$
  - radiation, relativistic matter: $w=\frac{1}{3}$
  - cosmological constant: $w=-1$
  - curvature: $w=-\frac{1}{3}$
  - a fluid you have designed: $w=w(t \text{ or } a \text{ or } \rho)$
- Density changes with the expansion of the Universe as
  $$\rho(t) = \rho_0 a^{-3(1+w)}(t)$$
The dynamics of a Universe with a bunch of stuff

- Given the current expansion rate \( H_0 = \frac{\dot{a}}{a} \text{ now} \), there is a “critical density” that makes for a flat universe.

  Critical density = the density of \( \Lambda \) for an otherwise empty universe

- Expressing each component as a fraction of \( \rho_c \) and integrating leads to the useful equation:

\[
H^2(t) = H_0^2 \left( \Omega_{r,0} a^{-4}(t) + \Omega_{m,0} a^{-3}(t) + \Omega_{k,0} a^{-2}(t) + \Omega_{\Lambda,0} \right) = H_0^2 \sum_i \Omega_{i,0} a^{-3(1+w_i)}
\]

\[
\rho_c = \frac{3H_0^2}{8\pi G}.
\]

\[
\begin{align*}
\Omega_m &= \frac{\rho_m}{\rho_c} \\
\Omega_r &= \frac{\rho_r}{\rho_c} \\
\Omega_{\Lambda} &= \frac{\Lambda c^2}{8\pi G \rho_c} \\
\Omega_k &= 1 - \Omega_m - \Omega_r - \Omega_{\Lambda}
\end{align*}
\]
The complete history of the Universe
Understanding the history of the Universe: Inflation

- A phase of exponential growth, increasing the size of the Universe by a factor of $\sim 10^{30}$ within $t \sim 10^{-32}$s
  - How can you make that happen?

$$\frac{\ddot{a}}{a} = -\frac{4}{3} \pi G \left( \rho + \frac{3p}{c^2} \right) + \Lambda \frac{c^2}{3}$$
Understanding the history of the Universe:

**Inflation**

- A phase of exponential growth, increasing the size of the Universe by a factor of $\sim 10^{30}$ within $t \sim 10^{-32}$s
  - Need large vacuum energy density

\[ \frac{\ddot{a}}{a} = \frac{4}{3} \pi G \left( \rho + \frac{3p}{c^2} \right) + \Lambda \frac{c^2}{3} \]

- Completely dilutes all other components of the Universe
- Perfectly nulls the curvature of the Universe
- Causally disconnects parts of the Universe
Understanding the history of the Universe:

The hot universe

- *somehow* the vacuum energy must turn into radiation: reheating

\[ H^2(t) = H_0^2 \left( \Omega_{r,0} a^{-4}(t) + \Omega_{m,0} a^{-3}(t) + \Omega_{k,0} a^{-2}(t) + \Omega_{\Lambda,0} \right) \]

- *somehow* in this primordial plasma, the abundance of matter exceeded the abundance of antimatter

- Neutrinos are initially coupled to this plasma, substantial share of energy density

- Expansion stops particle interactions: freeze out.

  The abundance of light nuclei today implies that the density of Baryons must be small.

- Eventually the Universe becomes diffuse/cold/transparent: relic photons as Cosmic Microwave Background until today

Source: Grupen 2020
Dark Matter

Evidence:

- Low abundance of ‘regular’ matter implied by primordial nucleogenesis + presence of massive structures today = most matter must be non-Baryonic
- Rotation curves / motions of galaxies: most of their gravity not due to stars/gas
- Patterns in the CMB (see next time), expansion, and growth of structure imply that the total density of matter must be ~6x the density of Baryons
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Dark Matter could be:

- One or more stable, massive (now non-relativistic) particles from the primordial plasma that do not affect the formation of elements
- One or more stable, light particles that are present at non-relativistic speeds
- Standard model particles ruled out. Primordial black holes mostly ruled out.
Understanding the history of the Universe: Matter domination

- Gravitation slows the expansion of the Universe

\[
\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G \left( \rho + \frac{3p}{c^2} \right)
\]

- A lot of things that happen here are **totally visible**...
  - Density fluctuations grow from $1/10^5$ to unity
  - Stars, galaxies, galaxy clusters form
  - Supermassive black holes grow at centers of galaxies
  - ...
  - But: all of these are closely related to dark matter clustering
Understanding the history of the Universe: Dark Energy takes over

- \[ \frac{\dot{a}}{a} = -\frac{4}{3} \pi G \left( \rho + \frac{3p}{c^2} \right) + \Lambda \frac{c^2}{3} > 0 \text{ once } \rho \text{ diluted enough,} \]
  - if there is a positive \( \Lambda \)
  - The expansion is then accelerating!

- Requires about 70% or energy at present day to be in the form of vacuum energy density.
  - This is strangely close to the matter density.
  - This is \( \sim \) a hundred orders of magnitude \textit{less} than a naive calculation of vacuum energy suggests.
  - We do not have nearly as many clues to its nature as in the case of dark matter.
How do we know there is Dark Energy?
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\[ \text{redshift of light received} \]
\[ z = \frac{a_0}{a(t)} \]

Matter + Dark Energy

Matter only

look-out distance, look-back time
How do we know there is Dark Energy?

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look-out distance, look-back time

Matter + Dark Energy
longer distance
shorter distance

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redshift of light received

Matter + Dark Energy
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look-out distance, look-back time
How do we know there is Dark Energy?

Measurement of the distance-redshift relation with “standard candles”: Type Ia Supernovae

Redshift of light received
\[ z = \frac{a_0}{a(t)} \]
How do we know there is Dark Energy?

Measurement of the distance-redshift relation with “standard rulers”: scale of the Baryonic Acoustic Oscillation peak

redshift of light received $\zeta = \frac{a_0}{a(t)}$

$\zeta \approx 1000$: CMB

look-out distance, look-back time
The contents of *our* Universe

- Based on all observations, the universe is about 13.8 billion years old and today contains:
  - 70% vacuum energy
  - 25% dark matter
  - 5% baryons
- Time variations of dark energy equation of state are not well constrained
- Light new particles could influence early universe physics and expansion history
Two tensions

- Measurements of local expansion rate $H_0$ disagree with the parameter needed to describe expansion history at $>4\sigma$
  - Could point at additional particle(s) / interactions in early Universe that change size of “standard ruler”
  - Could point at very recent additional acceleration
  - Could point at systematic errors

- Measurement of late-time density fluctuation amplitudes disagree with early-time fluctuation amplitude at $\sim 2$-$3\sigma$
  - Could point at additional particle(s) / interactions
  - Could point at modifications of gravity
  - Could point at statistical fluke or systematic errors
Tomorrow:

Cosmic structure and **Almost invisibles**