Geant4 10.6 beta

Electromagnetic Physics III

Geant4 Tutorial at Chalk River
Dennis Wright (SLAC)
based on slides of Mihaly Novak (CERN)
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Outline

- Stepping in Geant4
  - G4SteppingManager and its responsibilities
  - Connection to EM physics
- Step size and its limitation
  - in ionization and bremsstrahlung
  - in transportation and multiple scattering
- Summary

General Stepping

- G4Track encapsulates both static (G4ParticleDefinition) and dynamic properties (energy, momentum, direction, etc.) of a particle
- Geant4 propagates these tracks, one track at a time, step by step
  - beginning with first step of a primary or secondary track
  - ending with a particle:
    - left in the World volume with zero kinetic energy
    - dropped below the (user-defined) tracking cut
    - which disappears in a destructive interaction
  - track properties updated after each step
  - any secondaries created pushed onto track stack
- G4SteppingManager is responsible for coordinating one step
  - implements a general stepping algorithm which is independent of particle type or its assigned processes

EM Processes and Stepping

- Charged particle processes ionization, bremsstrahlung and multiple scattering are all continuous-discrete
  - discrete part has step limit (path length to next interaction) determined by restricted cross section
  - continuous part has a step limit due to maximum allowed energy loss along the step
  - because of along-step energy loss, kinetic energy is different at pre- and post-step points

Restricted Stopping Power

- Secondary electrons and gammas produced with energies below E^{cut} are not simulated explicitly
- They are described as continuous energy loss along the step and are based on a mean value
- This mean value is the restricted stopping power (mean energy loss along a unit step length)

$$-\frac{\mathrm{d}E}{\mathrm{d}x}_{\text{rest}}(E; E^{\text{cut}}, \ldots) = \int_0^{E^{\text{cut}}} k \frac{\mathrm{d}\sigma}{\mathrm{d}k}(E, ..)\mathrm{d}k$$

Restricted Cross Section

- Electrons and gammas with energies above E^{cut} are simulated explicitly as discrete interactions
- Discrete interaction probability is determined by the restricted cross section

$$\sigma_{\text{rest}}(E; E^{\text{cut}}, \ldots) = \int_{E^{\text{cut}}}^{E} \frac{\mathrm{d}\sigma}{\mathrm{d}k}(E, ..)\mathrm{d}k$$

- This covers only the interactions in which the secondary has kinetic energy above the production threshold

Discrete Part of Step Limit

- Interactions will propose a step length
  - target atom number density is N and atomic interaction cross section is σ (assumed constant for now)
  - p.d.f. of the interaction length *x* is

$$p(x) = N\sigma \exp(-xN\sigma)$$

  - the mean (expected value) of the interaction length *x* is

$$\mathbb{E}(x) = \frac{1}{N\sigma} \equiv \lambda = \frac{1}{\Sigma}$$

  where λ is the mean free path and $\Sigma = N\sigma = 1/\lambda$ is the macroscopic cross section

- If there are M independent interactions with Σ_i the interaction with the shortest interaction length x_i is chosen by the simulation

Discrete Part of Step Limit

- Typically, Monte Carlo simulations will sample the path length to the next discrete interaction point using the distribution exp(Σ_t) where Σ_t is the total macroscopic cross section

$$\Sigma_t = \sum_{i=1}^{M} \Sigma_i = N \sum_{i=1}^{M} \sigma_i$$

- Then at the post step point, the type *i* of the discrete interaction is sampled according to the discrete probabilities

$$p(\text{proc} = i) = \Sigma_i / \Sigma_t$$

- But in Geant4, each discrete process proposes an interaction length sampled from its own macroscopic cross section: exp(Σ_i)
  - the process with the shortest interaction length is the one that occurs
- In this way Geant4 already selects at the pre-step point what (if anything) will happen at the post-step point

Discrete Part of Step Limit

- For particles that have ionization and bremsstrahlung the corresponding restricted macroscopic cross sections Σ_{rest} (E, E^{cut}) are used to propose the discrete step limit
- Due to the continuous part (along-step energy losses), the energies at the pre-step and post-step points will be different
- The cross section therefore is generally not constant along the step. This is accounted for by:
  - the addition of a fictitious interaction δ with cross section energy dependence such that

$$\Sigma_i^r(E) + \Sigma_i^\delta(E) = \Sigma_i(\not{E}) \equiv \Sigma_i^{\text{const}} \implies \text{constant along the step}$$

and $\Sigma_i^r(E) \leq \Sigma_i^{\text{const}}$ along the step, which implies that $\Sigma_i^{\text{const}} = \max\{\Sigma_i(E)\}$

- The new constant cross section is used to sample, at the pre-step point, the interaction length to the real or fictitious interaction
- At the post-step point, after energy loss has been accounted for, the probability that the fictitious interaction occurs is

$$p(\delta) = 1 - \Sigma_i^r(E^{\text{post}})/\Sigma_i^{\text{const}}$$

Continuous Part of Step Limit

- Up to now, the discrete part has been considered:
  - each process proposed a step length
  - the shortest of these was selected as a candidate step length
  - corresponding process was selected as the candidate process
  - flag set to indicate that current candidate step length was proposed by discrete part of candidate process
  - a possible energy loss along the step was considered
- Now, G4SteppingManager asks the continuous part of each process to propose its own step limits
  - starting with all the previous (discrete) settings and type flag
  - each proposed continuous limit is compared to the current candidate limit
  - if current continuous step limit is shorter than the current candidate, the candidate step length, process and type flag (continuous) are updated accordingly

Continuous Part of Step Limit

- For particles that have ionization and bremsstrahlung, the following continuous step limit function is used:

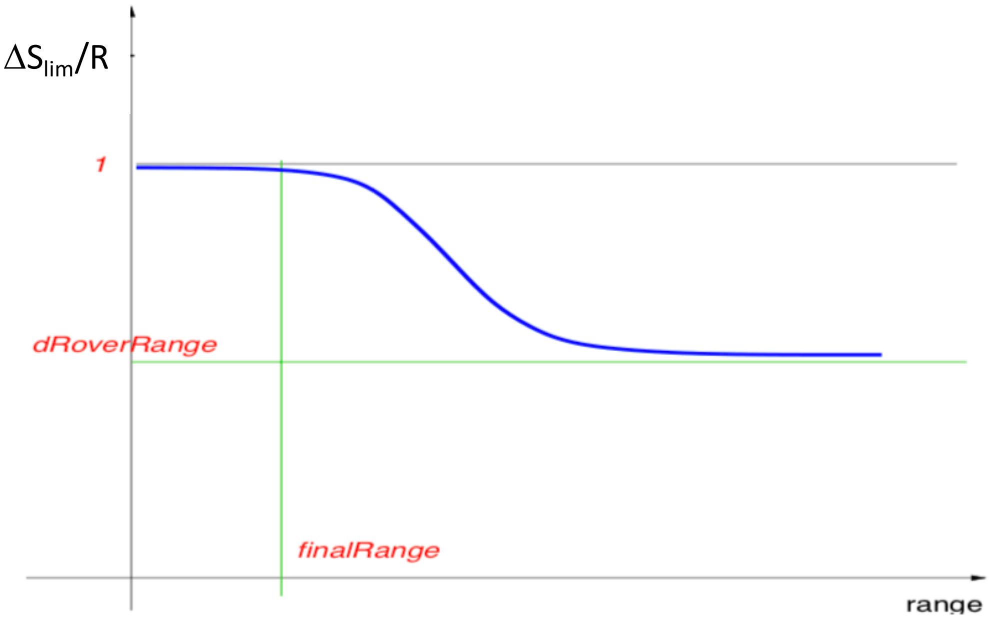
When the particle range $R > \rho_R \equiv finalRange$:

$$\Delta S_{lim} = \alpha_R R + \rho_R (1 - \alpha_R) \left(2 - \frac{\rho_R}{R}\right)$$

- **default value:** $\rho_R = 1.0[mm]$
- $\alpha_R \equiv dRoverRange$
- **default value:** $\alpha_R = 0.2$
- **at high energies:** $\Delta S_{lim} \approx \alpha_R R$

When the particle range $R < \rho_R$:

- **low energies:** $\Delta S_{lim} = R$



- Based on restricted range, computed from restricted stopping power

Multiple Scattering and Transportation

- Up to now, a candidate physics step length has been selected which is
  - the current maximum of the step lengths proposed by all discrete and continuous processes
  - and we have assumed that the particle will travel this length in a straight line in its original direction
    - to the post-step point where the selected discrete interaction takes place
    - to the post-step point where no discrete interaction takes place, if a continuous interaction proposed the shortest step length

- However, there are two special continuous processes left: transportation (which always occurs) and multiple scattering (which may occur). The end-of-step limitation depends on one of three conditions:
  - the particle has no Coulomb scattering (A)
  - it has Coulomb scattering and is described by a single-scattering model (discrete) (B)
  - it has Coulomb scattering and is described by a multiple scattering model (continuous process) (C)

Transportation

- For particles that do not have Coulomb scattering (A):
  - the only remaining continuous process is transportation
  - it is the last process to propose a step length
  - particle is supposed to be transported the selected distance from the pre-step point along its original direction, according to all the foregoing physics
  - but transportation now gets to propose its step limit:
    - if particle can be propagated to its selected distance without crossing a volume boundary, the transportation process accepts the proposed length
    - otherwise, particle is transported to volume boundary and proposed step length is shortened accordingly

Single Coulomb Scattering

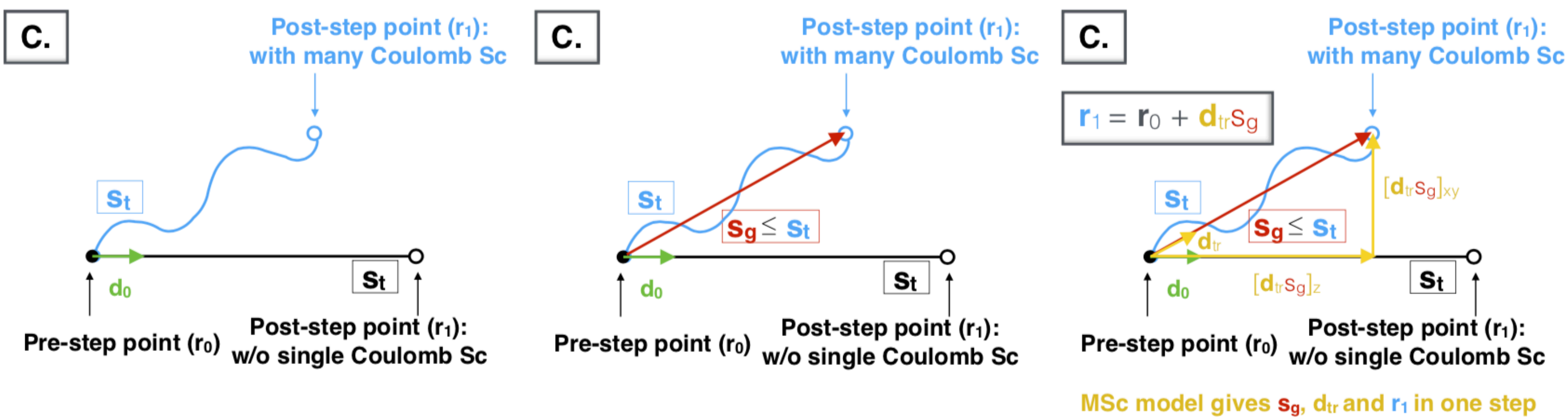
- For particles that have single Coulomb scattering (B)
  - elastic scattering was already accounted for in the step limit since it is included in the list of discrete processes
  - so everything is the same as in case A, since the only remaining continuous process is transportation

A. and B.

$$\mathbf{r}_1 = \mathbf{r}_0 + \mathbf{d}_0 s_t$$

Multiple Coulomb Scattering

- For particles that have the continuous process multiple Coulomb scattering (C)
  - elastic scattering is not included in the list of discrete interactions → cannot propose an elastic step size
  - with elastic scattering there would be many scatterings and changes of direction along (s_t) the proposed step length → zig-zag trajectory instead of straight line
  - the multiple scattering model provides the real step length (s_g) and final direction of travel d_{tr}



Multiple Coulomb Scattering

- So, with transportation the last, multiple scattering is next to last to provide its step limitation
  - multiple scattering can further limit the current candidate path length s_t
  - after its own step limitation, multiple scattering will change the current true step length s_t to the geometrical step length s_g by computing the corresponding transport distance and transport direction d_{tr}

- After multiple scattering, transportation invokes its step limitation by providing the transport distance s_g instead of the true step length s_t
- From this point on, everything is identical to cases A and B

Summary

- G4SteppingManager is responsible for coordinating all the physics processes that can occur during a step
  - discrete and continuous compete

- Ionization, bremsstrahlung, multiple scattering and transportation couple in a complicated way to produce the final step length and interaction at the post-step point, with process sampling and step limit proposals in the following order
  - bremsstrahlung and ionization
  - multiple scattering
  - transportation

- It is not mandatory to use production cuts or multiple scattering in a Geant4 simulation
  - some models allow this, in fact give an exact solution to the transport problem
  - but this is usually never practical above a few 100 keV