From axial form factor to neutrino cross sections

Supported by NSF PHY-1714253

In Collaboration with
Alexander Friedland (SLAC), Sergi Gonzalez-Solis, Kevin Quirion (IU) and
Clayton Ristow (Maryland)

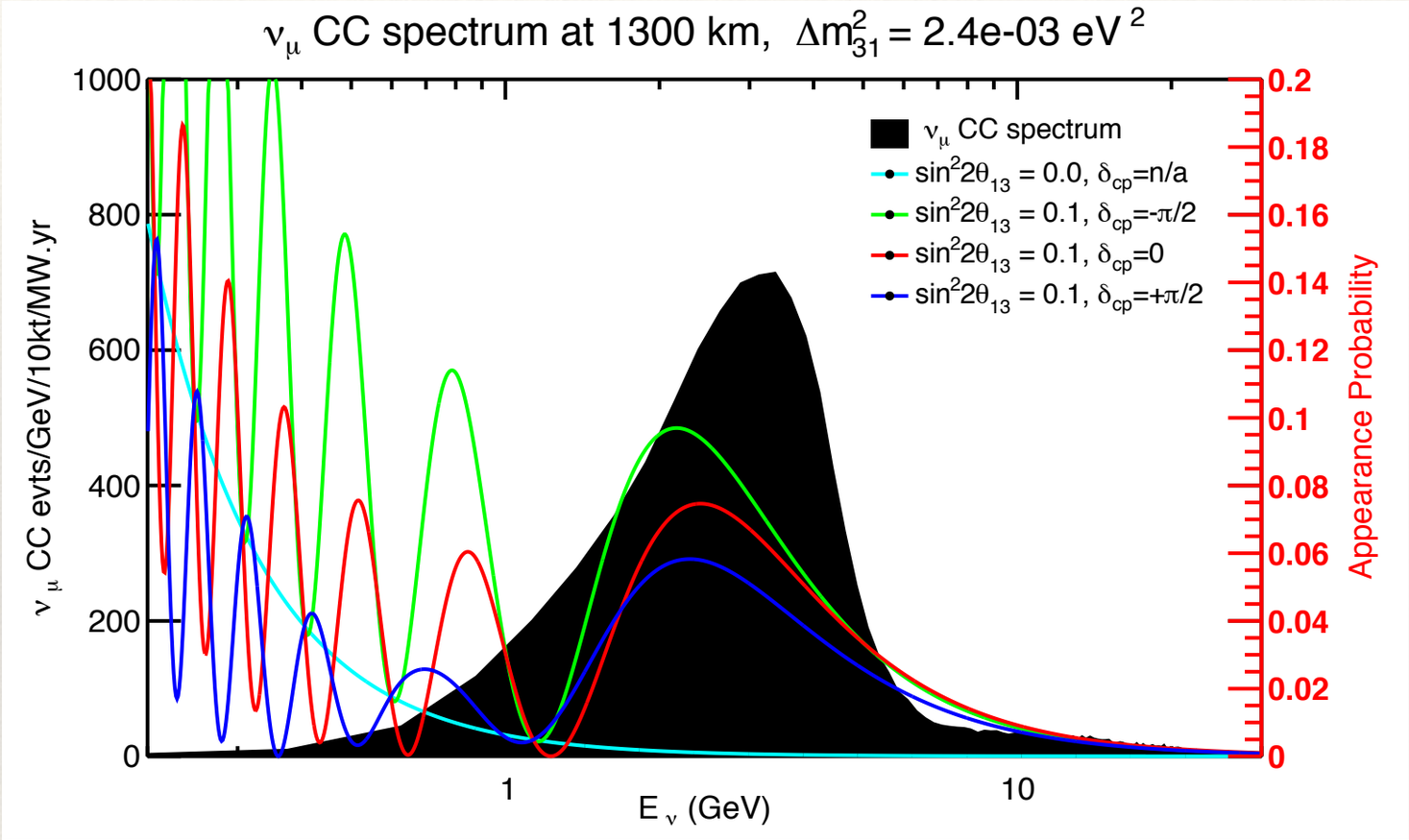
Precision Investigations of the Neutrino Sector,
SLAC Neutrino Workshop
Palo Alto, July 15, 2019

Outline

1. Introduction and Motivation
2. Bridging Lattice QCD and neutrino measurements
3. Improving the Axial Form Factor parametrisation
4. Conclusion and Outlook

Introduction and Motivation

- Accurate neutrino measurements: *LBNE, Science report'13*
  - Mass hierarchy
  - Oscillations
  - CP violation
  - Beyond 3 flavours?
- Precise knowledge of ν numbers



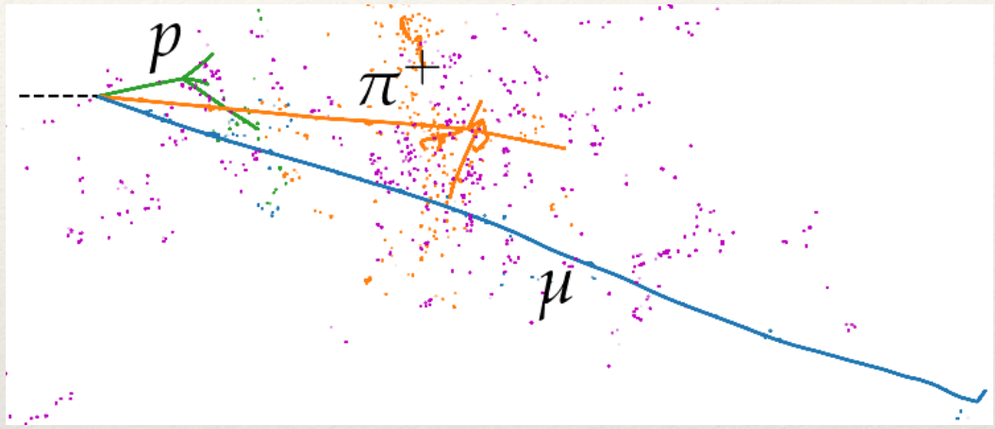
➡ *Neutrino Cross Section*

- Need precise E_ν reconstruction

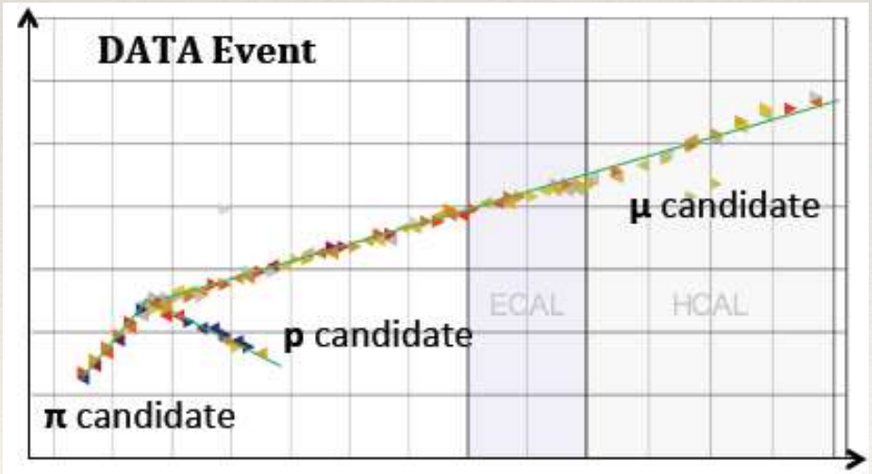
Energy Reconstruction

- To reconstruct E_ν:
  - Measure energy of all final state particles and add them up "Calorimetric method"
  - In a realistic experiment not all energies can be accurately measured
    see ***S. Li*** and ***U. Mosel***'s talks
  - Cross section model needed to improve energy measurement by filling in the missing information
    - e.g., to go from total ionisation charge in LAr in DUNE or light in NOvA, need to predict its composition

Friedland&Li'19



Talk by D.Ruterbories, MINERvA



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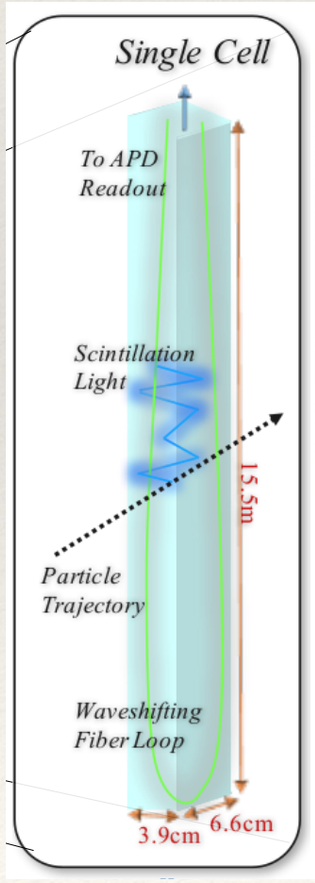
Talk by J. Wolcott, NOvA



Physical Processes Involved

As discussed before

- Quasi elastic scattering
- One pion production through resonances
- Non-resonant pion production
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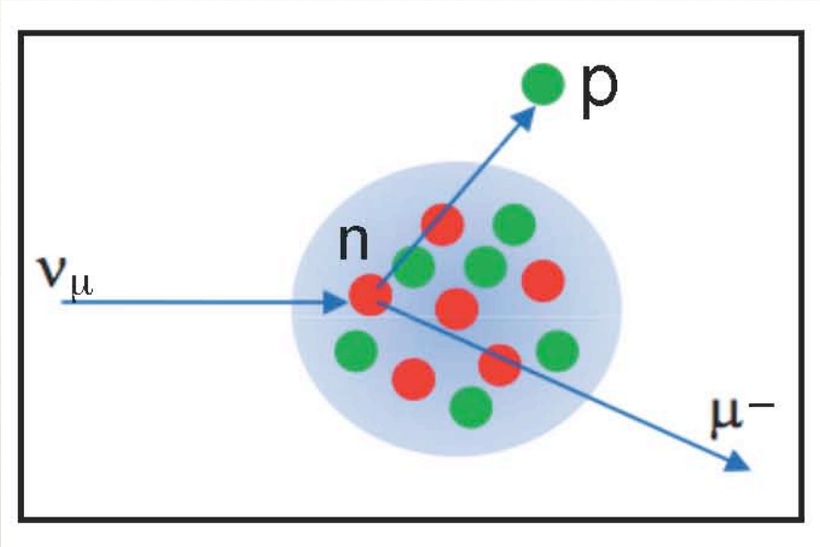

P. Vogel

Quasi-elastic (QE) scattering

Hadronic matrix element involved: $q^2 = (p' - p)^2$; $Q^2 \equiv -q^2$

$$\langle p(p')|J_W^{+\mu}|n(p)\rangle \propto \bar{u}^p(p')\left\{\gamma^\mu F_1^V(q^2) + \frac{i}{2m_N}\sigma^{\mu\nu}q_\nu F_2^V(q^2) + \gamma^\mu\gamma_5 F_A(q^2) + \frac{1}{m_N}q^\mu\gamma_5 F_P(q^2)\right\}u^{(n)}(p)$$

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➡ In the following focus on $F_A(q^2)$

What is know on the Axial Form Factor?

- Old problem
- Traditionally it was assumed to follow a simplistic parametrisation

$$F_A(q^2) = \frac{F_A(0)}{\left(1 - \frac{q^2}{M_A^2}\right)^2}$$ the dipole parametrisation

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    ➡ determined using a combination of processes

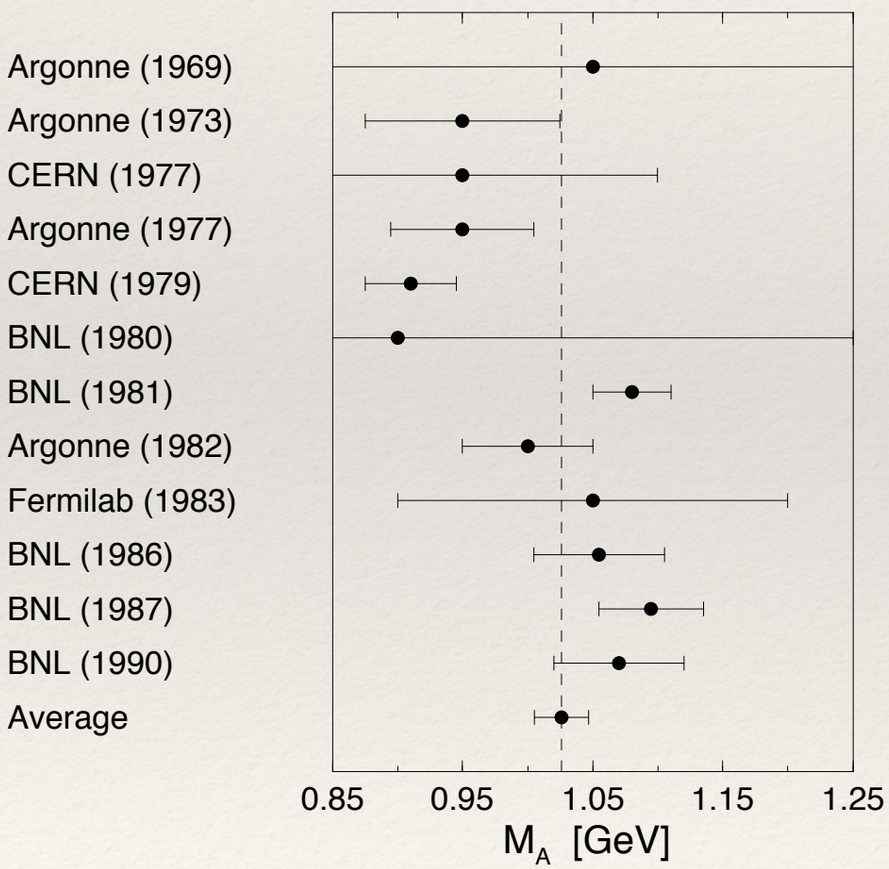
    - Neutrino nucleon cross section: $\sigma\left(\nu N \to \ell N\right)$
    - Pion electroproduction $\gamma^\star(k^2) + N_1(p_1) \to \pi^a(q) + N_2(p_2)$

What is know on the Axial Form Factor?

- Up to recently good agreement between all determination of F_A

A. Liesenfeld et al, MAMI'99

$$\nu N \to \ell N$$



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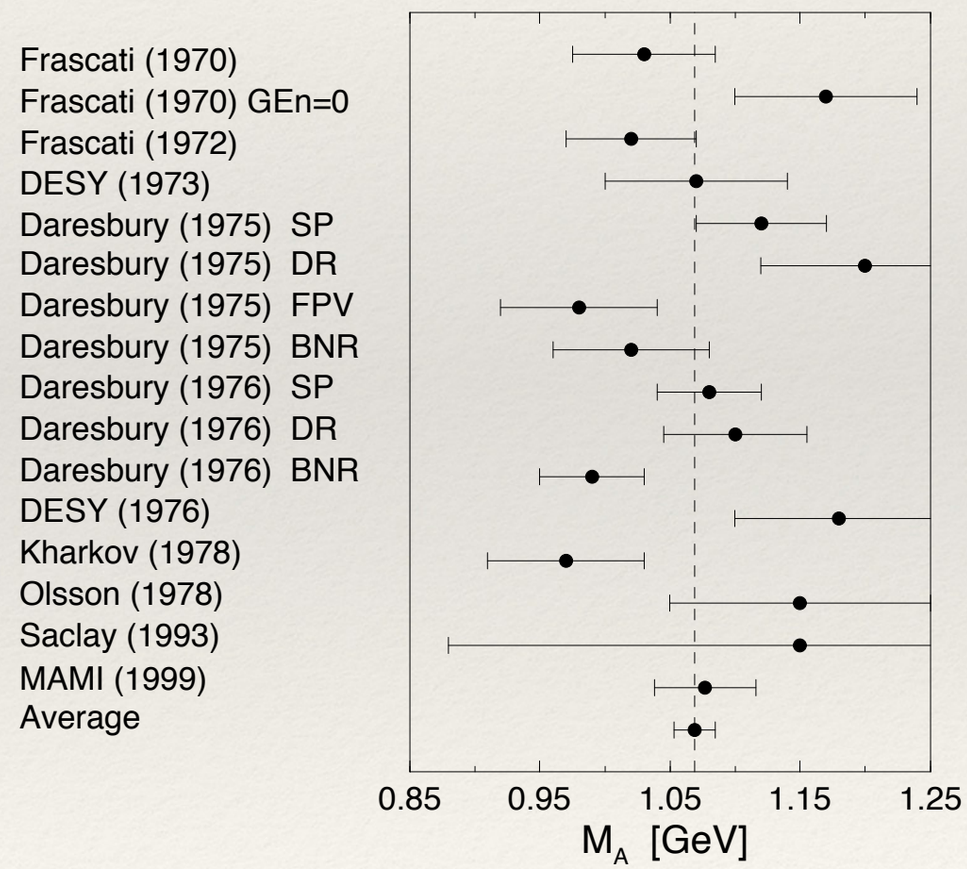
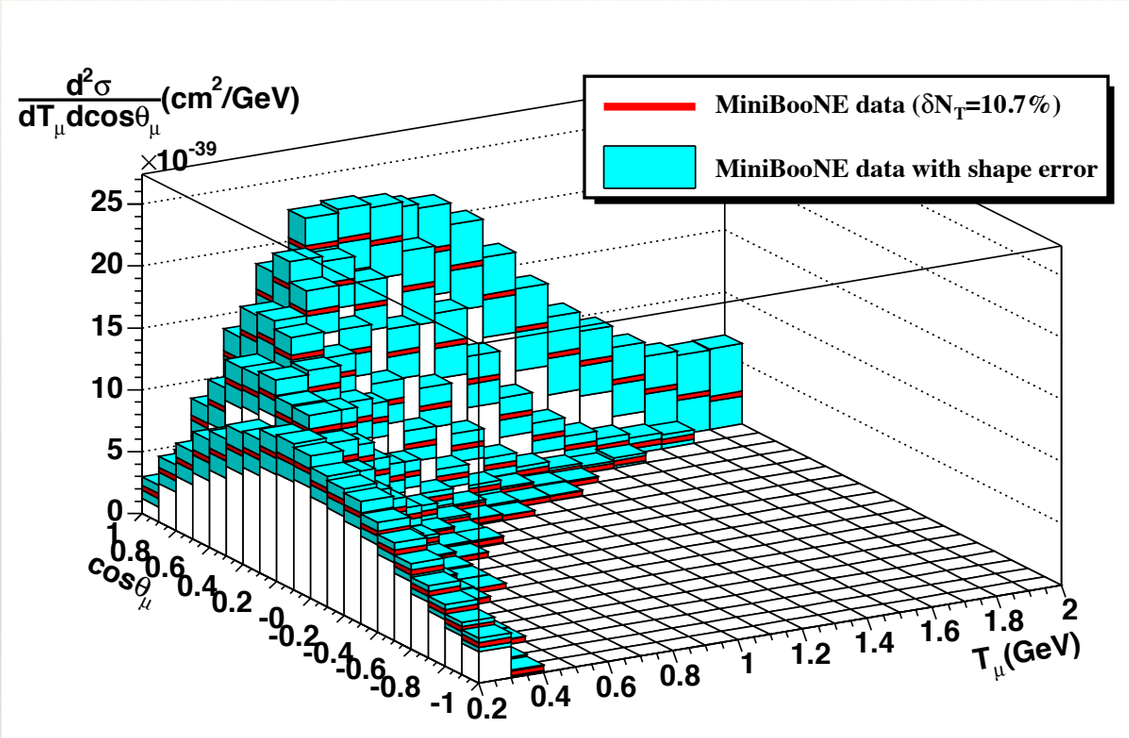
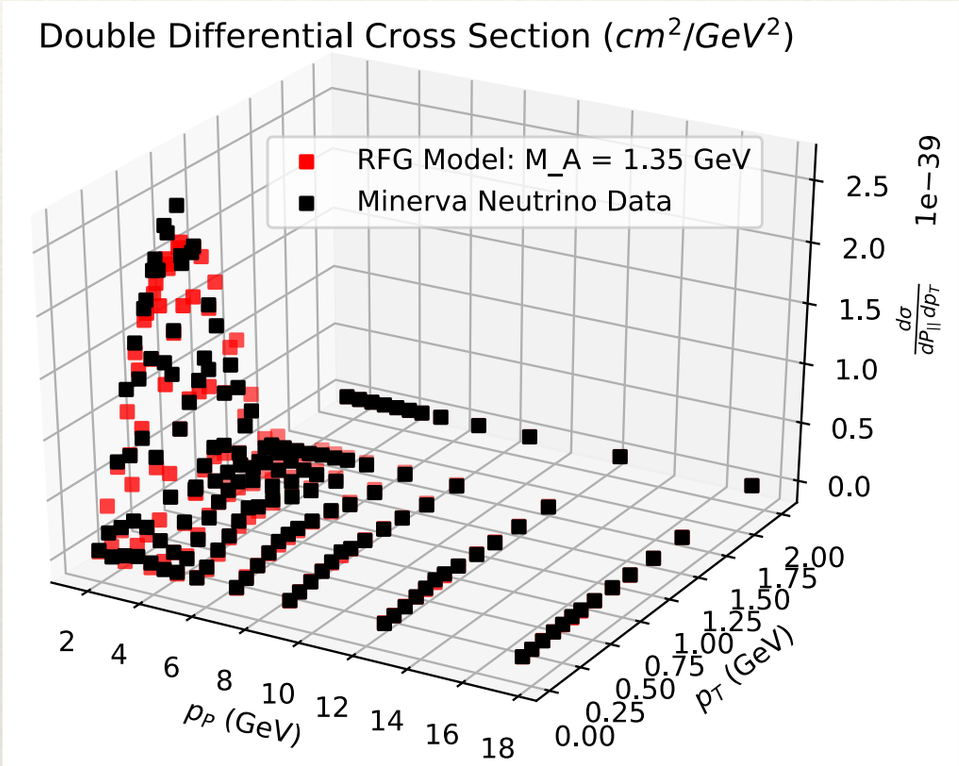


$$M_A = 1.069 \pm 0.016 \text{ GeV}$$

What is know on the Axial Form Factor?

- Recently very significant progress on two fronts:
  - Experimentally many new measurements: *MiniBooNE, K2K, MINERvA, NOMAD*





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K2K [10]	1.20 ± 0.12	0.32 ± 0.06
NOMAD [11]	1.05 ± 0.06	0.42 ± 0.05
MiniBoonNE [12]	1.35 ± 0.17	0.26 ± 0.06
MINERvA [13]	0.99	0.48
MINOS [14]	$1.23^{+0.13}_{-0.09}$	$0.31^{+0.07}_{-0.05}$

$$F_A(q^2) = F(0)\left(1 + \frac{1}{6}\langle r_A^2 \rangle q^2 + \mathcal{O}(q^4)\right)$$

$$\langle r_A^2 \rangle = \frac{12}{M_A^2}$$

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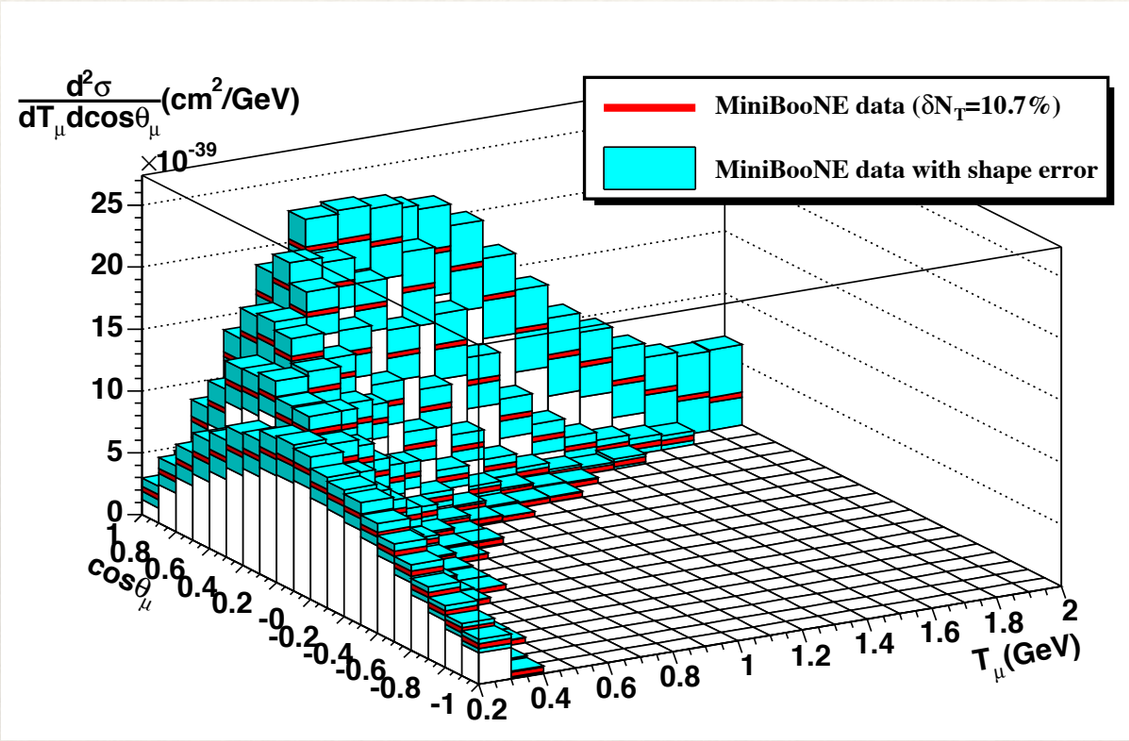

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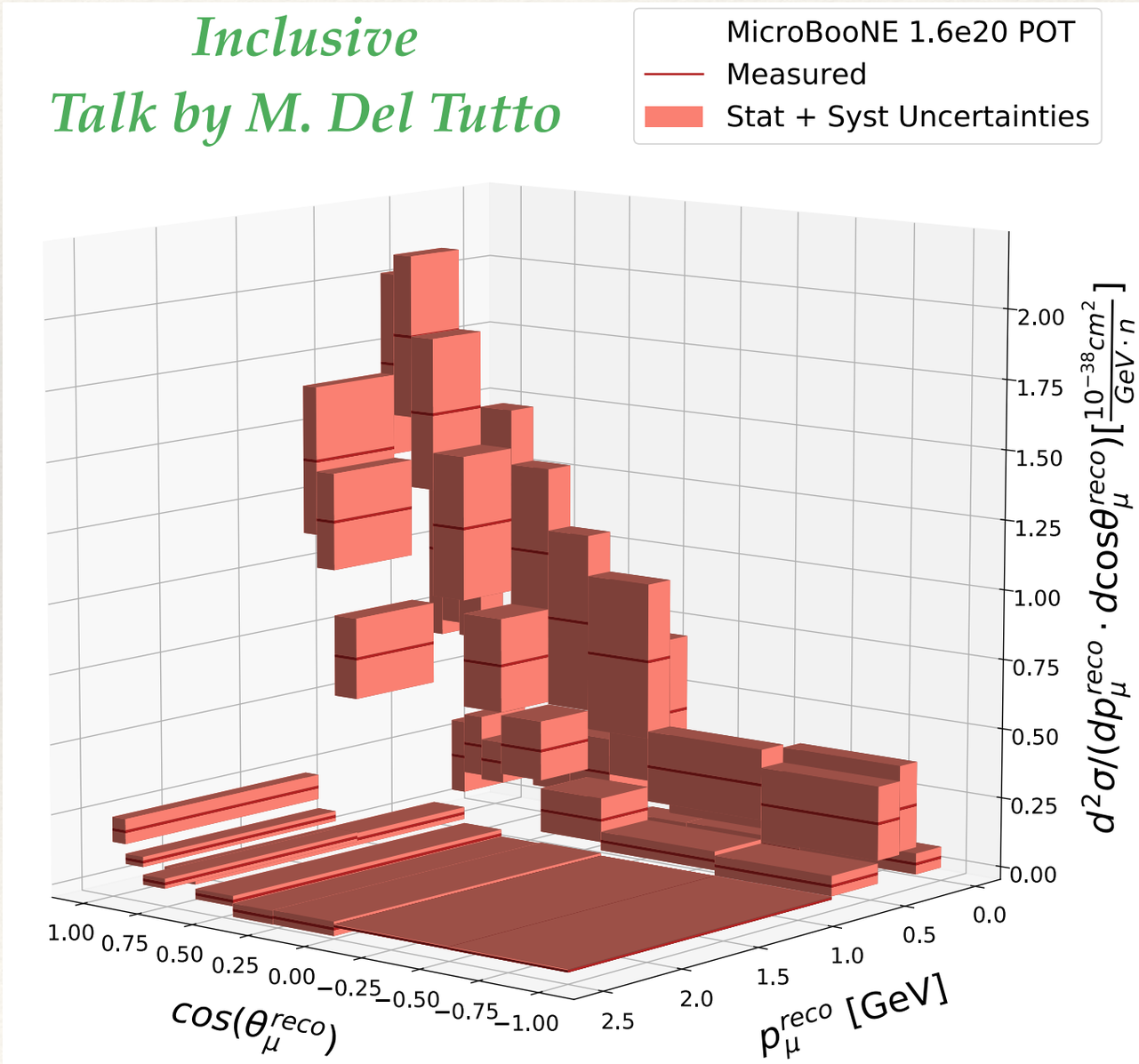
Talk by M. Del Tutto



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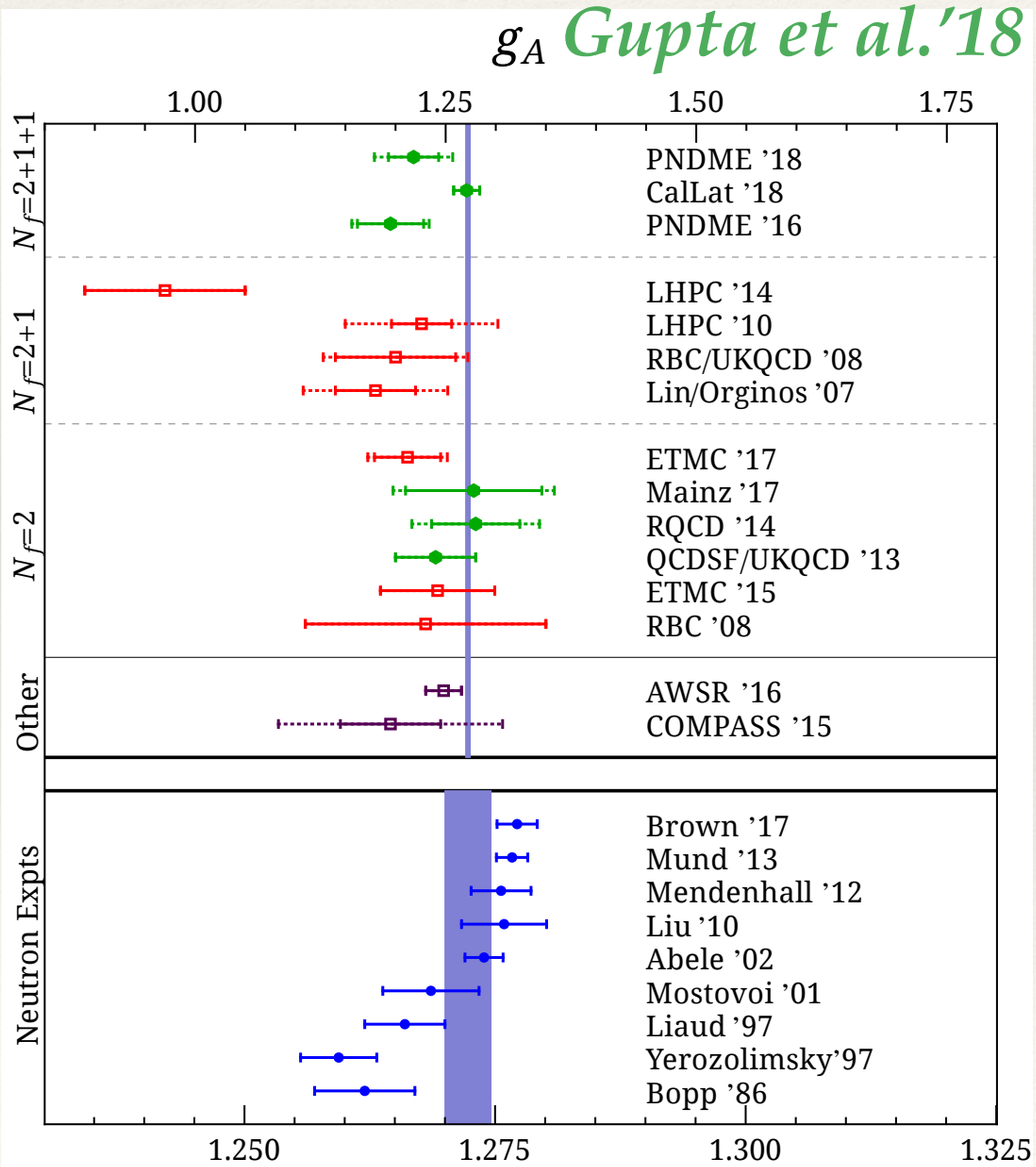
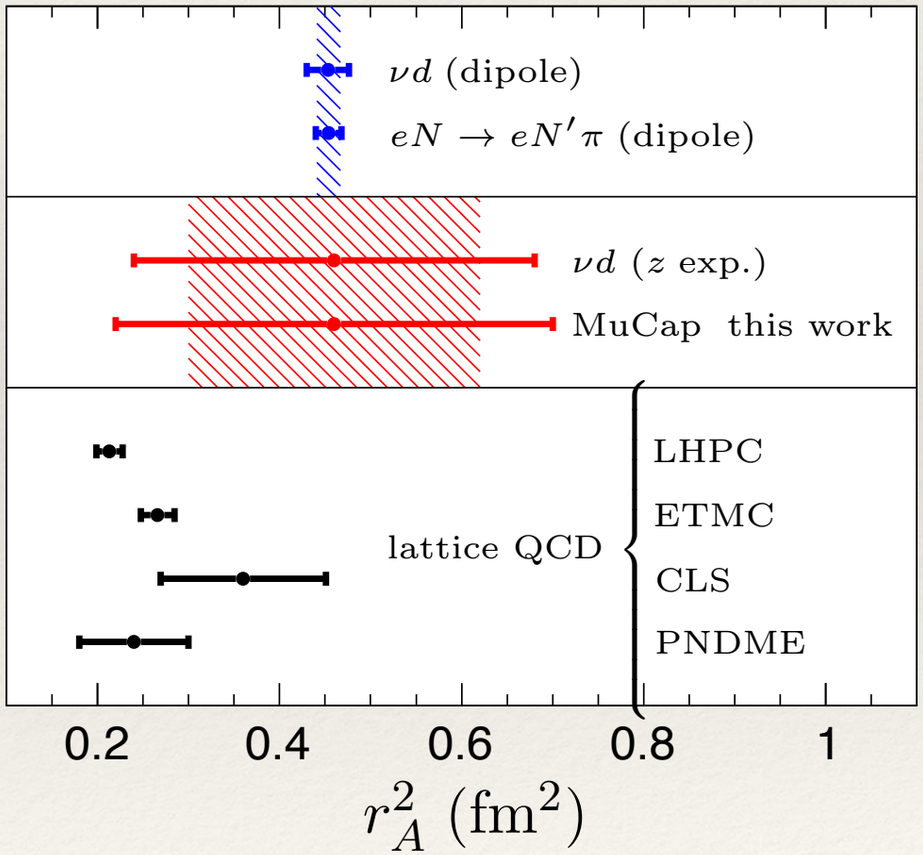


Gupta et al.'18

Hill, Kammel, Marciano, and Sirlin'18



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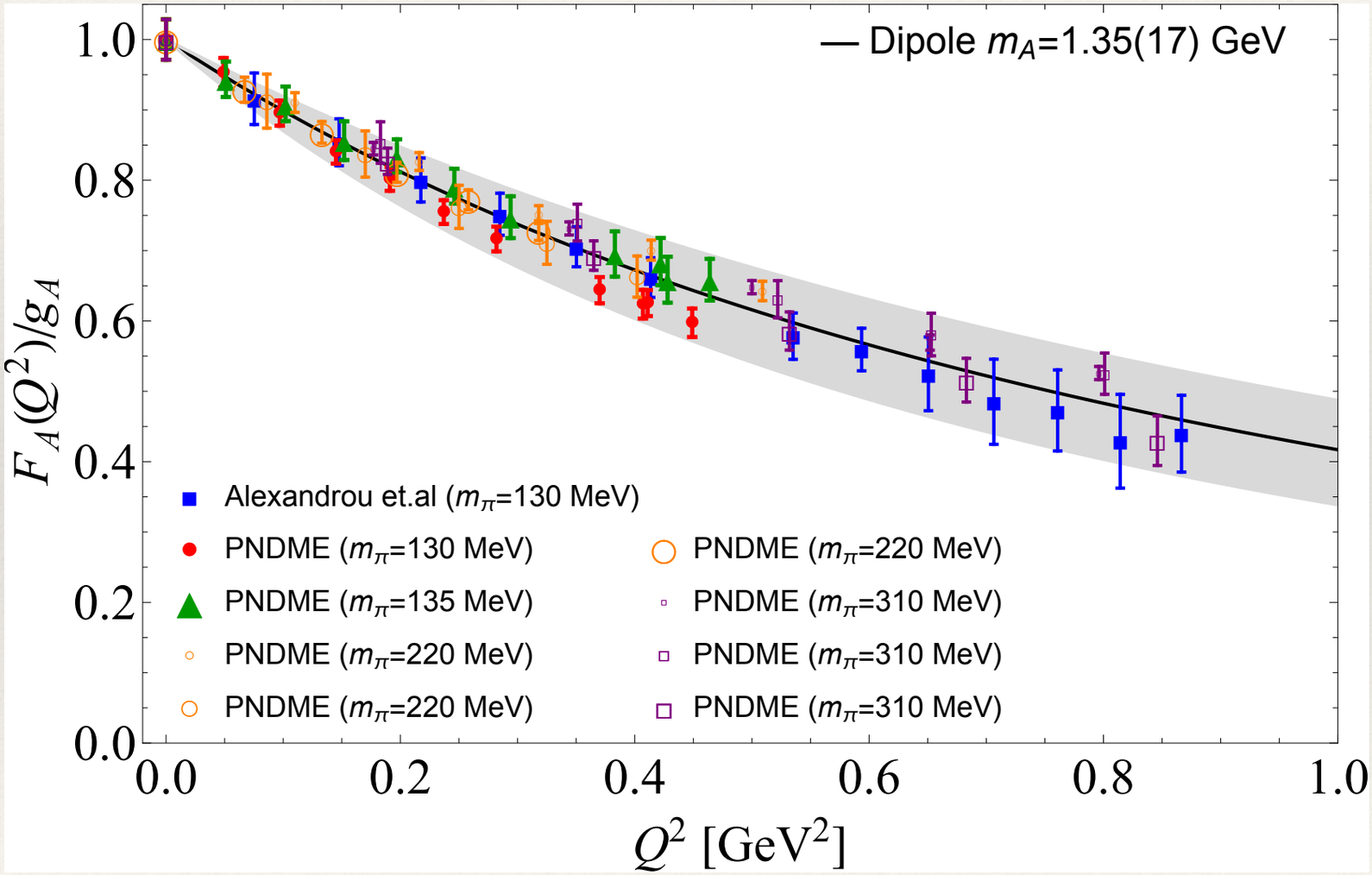

$Q^2 \equiv -q^2$

$N_F = 2$ *Alexandrou et al., ETMC'17*

$N_F = 2+1+1$ *Gupta et al., PNDME collab.'17*

see also Green et al'17, Capitani et al'17

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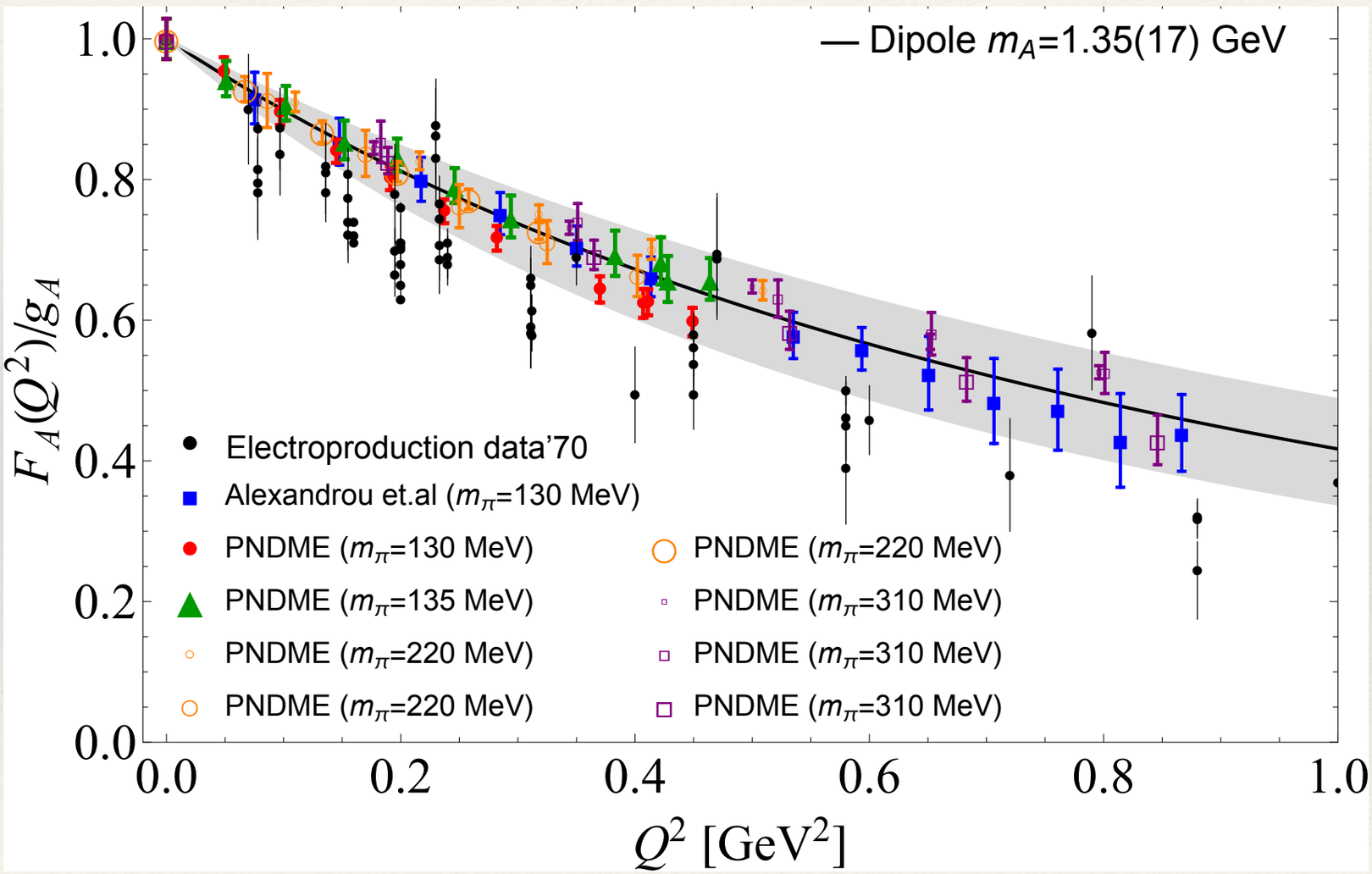

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Bridging Lattice QCD and neutrino measurements

- Connecting predicted $F_A(q^2)$ to measured total and differential cross sections

- Creating a physically motivated analytical parametrisation that can be used to assist and complement the lattice simulations (beyond the dipole)

Connecting predicted $F_A(q^2)$ to cross sections

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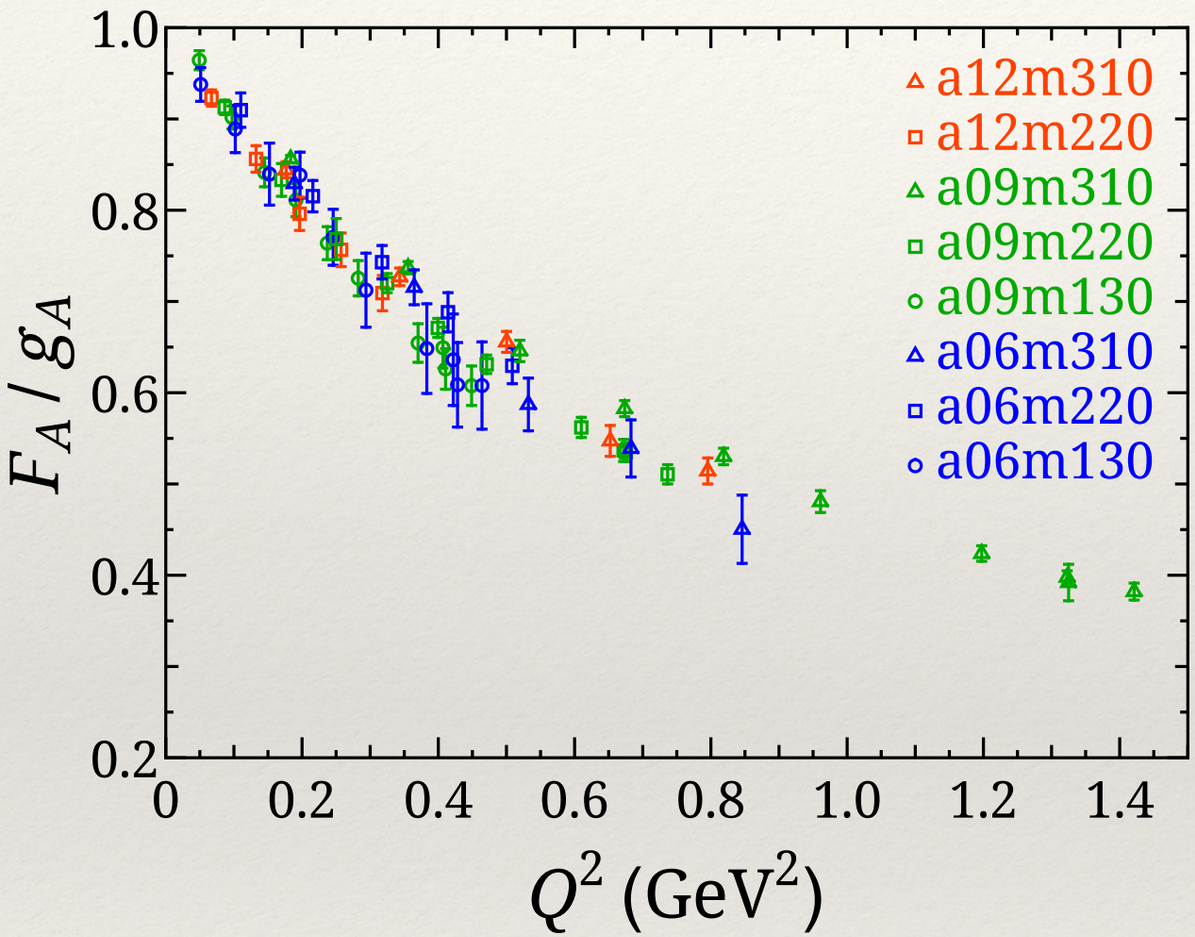
  - Axial form factor
  - Absorption in the medium, FSI
- In general combinations of these effects and one needs to disentangle these contributions
- Recent lattice calculations seem to favour large values of $M_A \sim 1.35$ GeV
  3 groups using different methods and systematics

Which values of Q^2 impact neutrino data?

Jang et al., PNDME collab.'19



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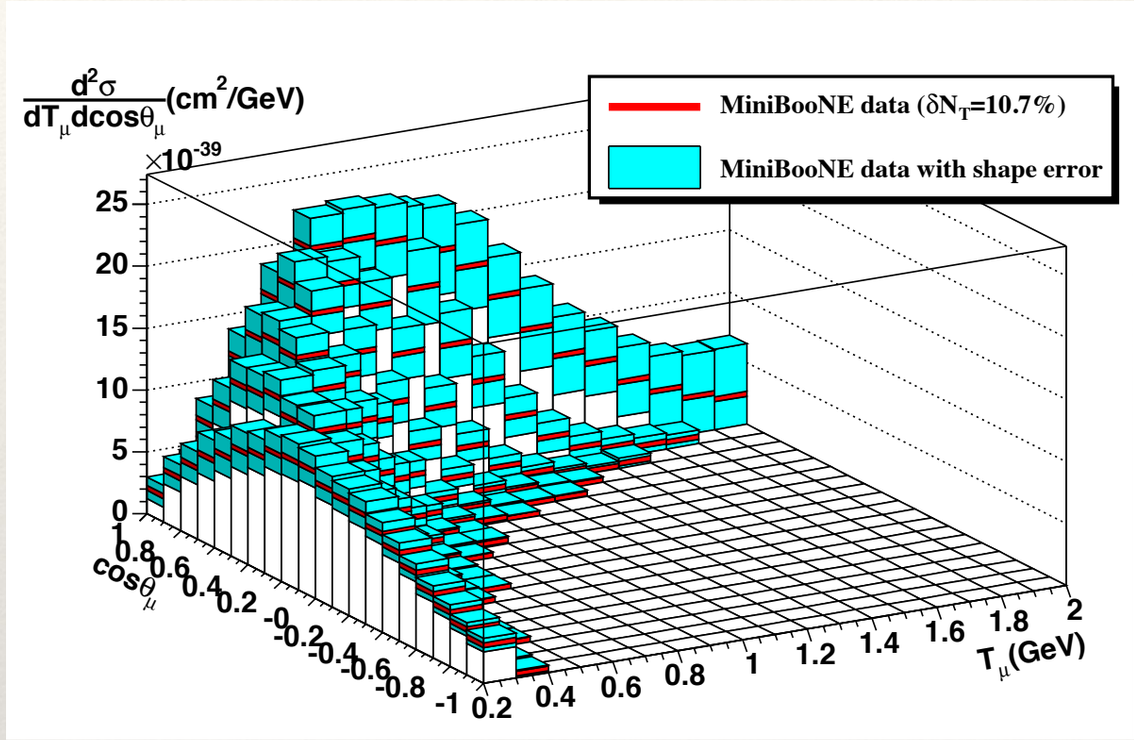
MiniBooNE'10



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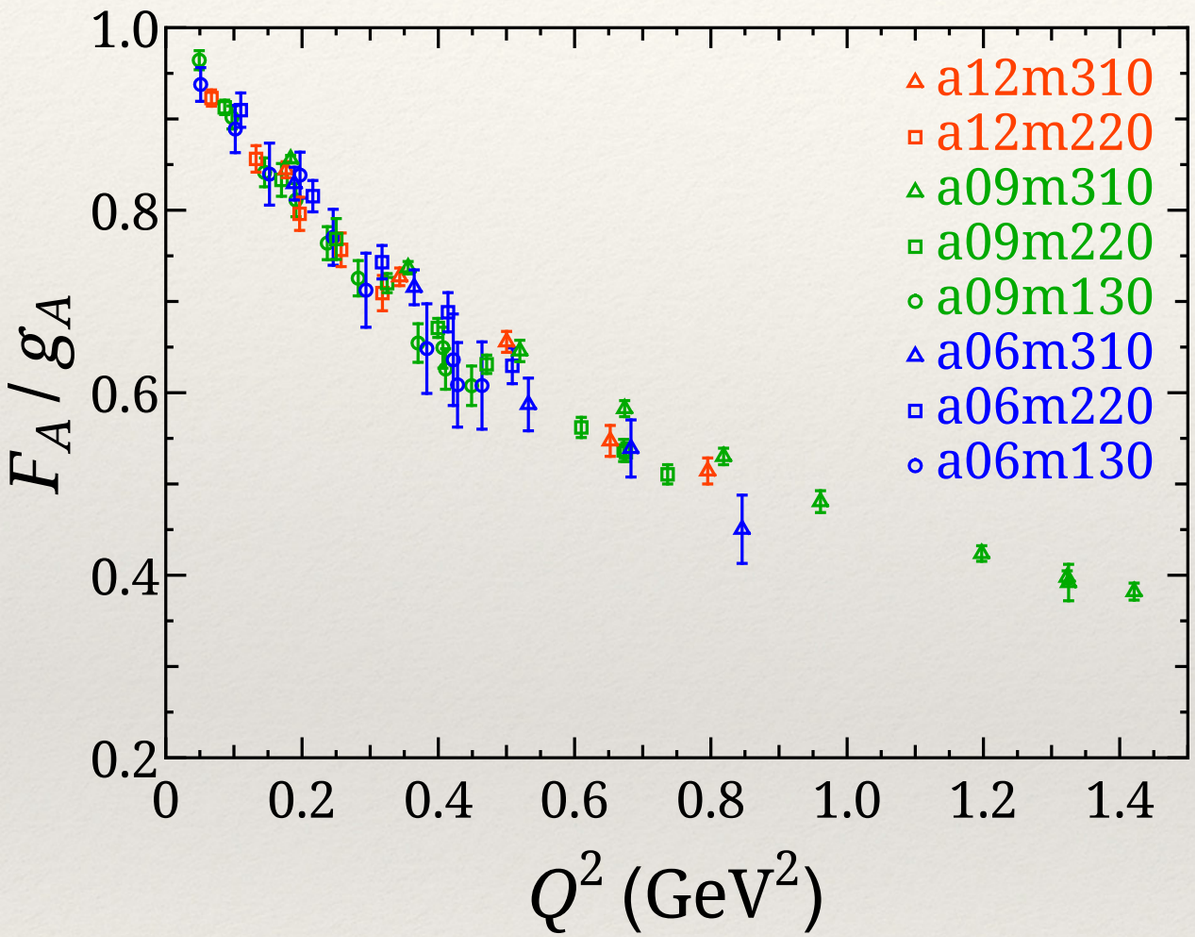

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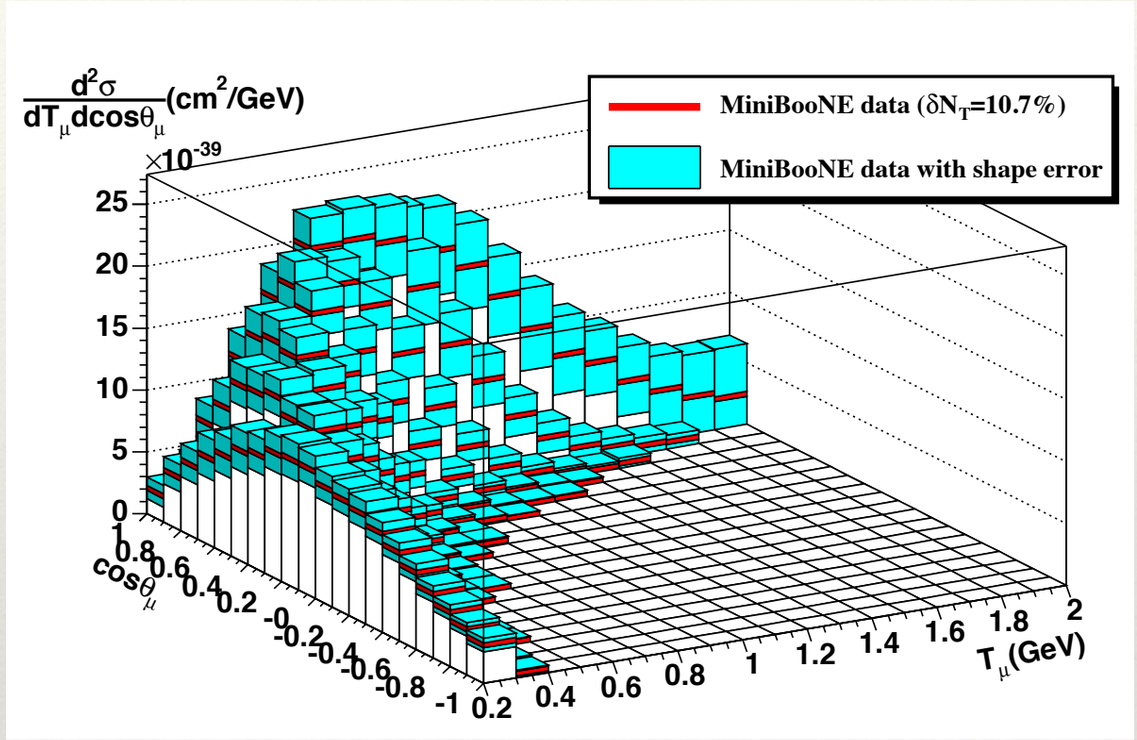


- At the very minimum, since nucleons in ^{12}C one must include their Fermi motion : simplest model

Llewellyn-Smith'72

→ ***Relativistic Fermi Gas*** model

Smith & Moniz'72'75

Quasi-elastic Neutrino Cross Section

- Using ***Relativistic Fermi Gas*** model *Smith&Moniz'72*

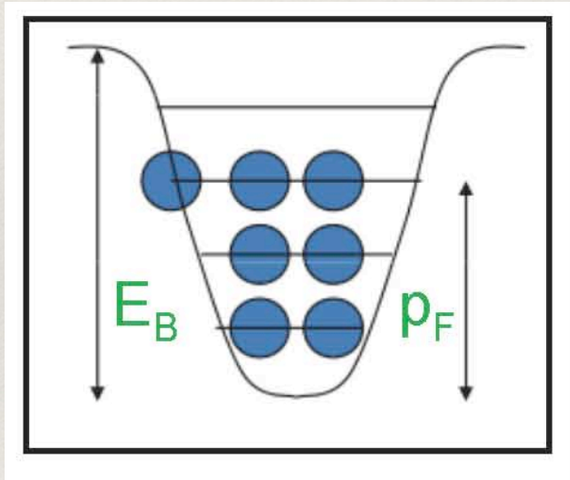
$$\sigma_{\text{nuclear}} = n_i(\boldsymbol{p}) \otimes \sigma_{\text{free}}(\boldsymbol{p} \to \boldsymbol{p}') \otimes [1 - n_f(\boldsymbol{p}')]$$

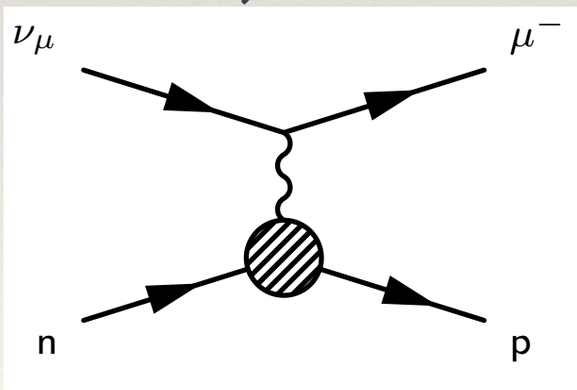
P. Vogel

$$\sigma_{\text{nuclear}} \approx 2V \int \frac{d^3p}{(2\pi)^3} n_i(\boldsymbol{p})$$

Bhattacharya, Hill and Paz'11

$$\left\{ \frac{G_F^2}{16|k \cdot p|} \int \frac{d^3k'}{(2\pi)^3 2E_{\boldsymbol{k}'}} \int \frac{d^3p'}{(2\pi)^3 2E_{\boldsymbol{p}'}} (2\pi)^4 \delta^4(p - p' + q) L^{\mu\nu} H_{\mu\nu} \right\} [1 - n_f(\boldsymbol{p}')]$$





$$\langle p(p') | J_W^{+\mu} | n(p) \rangle \propto$$

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Including effects of the binding energy

$$p^0 \to \epsilon_{\boldsymbol{p}} \equiv E_{\boldsymbol{p}} - \epsilon_b$$

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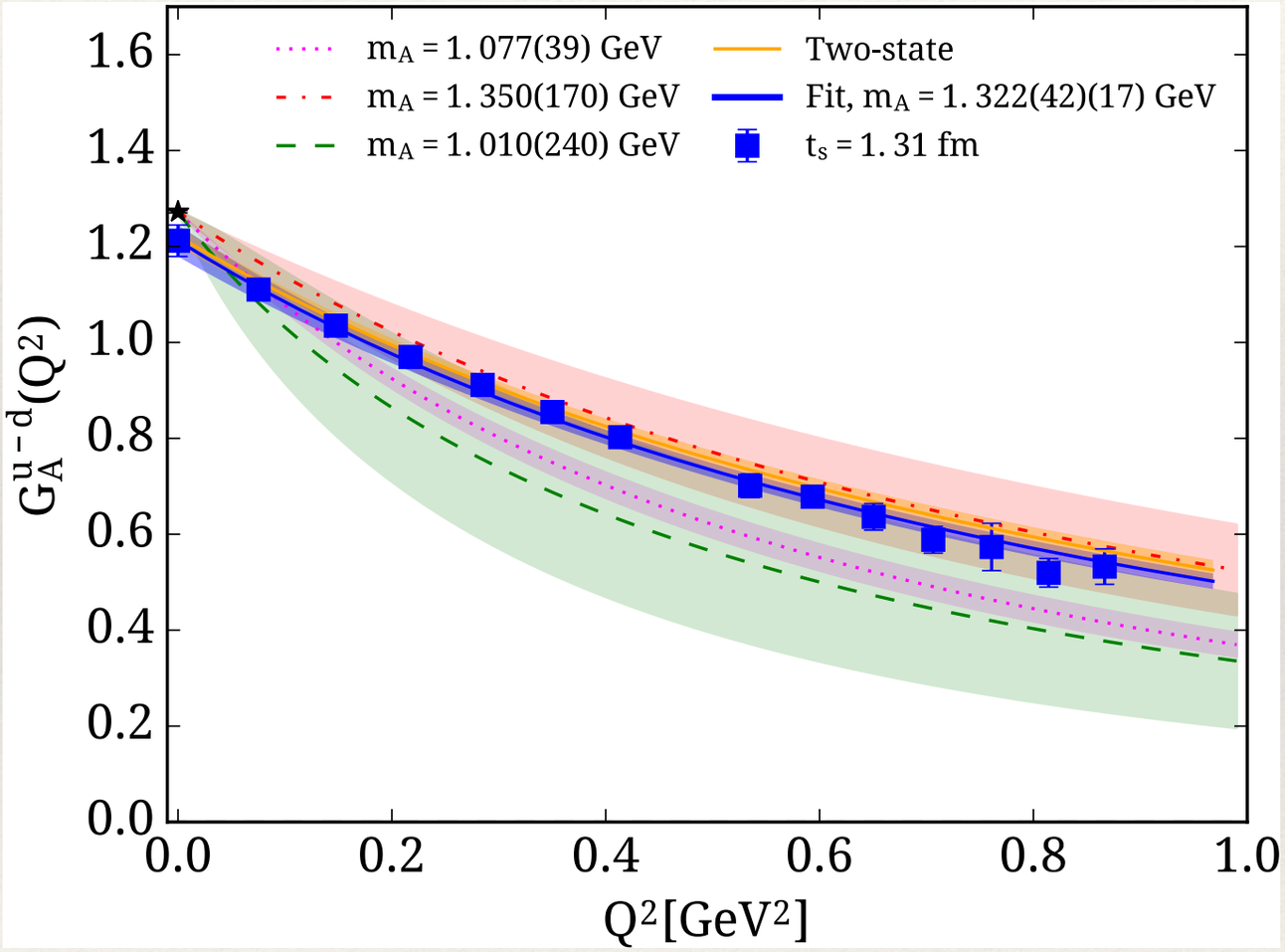
Alexandrou et al., ETMC'17

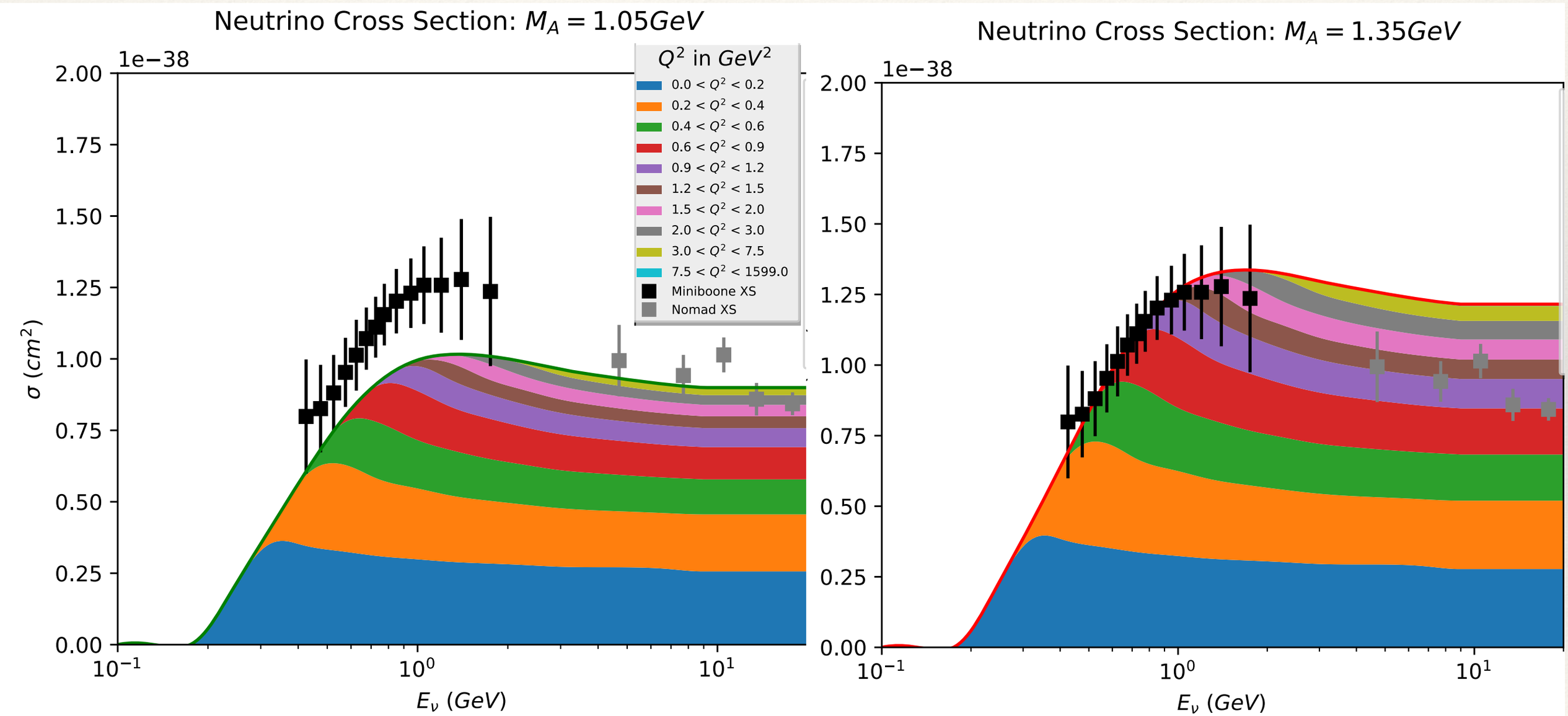


- Which Q^2 range is important for neutrino XS data?
- If one changes the functional form of F_A, how does that impact the XS prediction?

Which values of Q^2 impact neutrino data?

Friedland, Gonzalez-Solis, E.P., Quirion, Ristow in preparation

- Composition of MiniBooNE Cross Section



- At E ~ 0.5 GeV the XS comes from $Q^2 < 0.6\ GeV^2$

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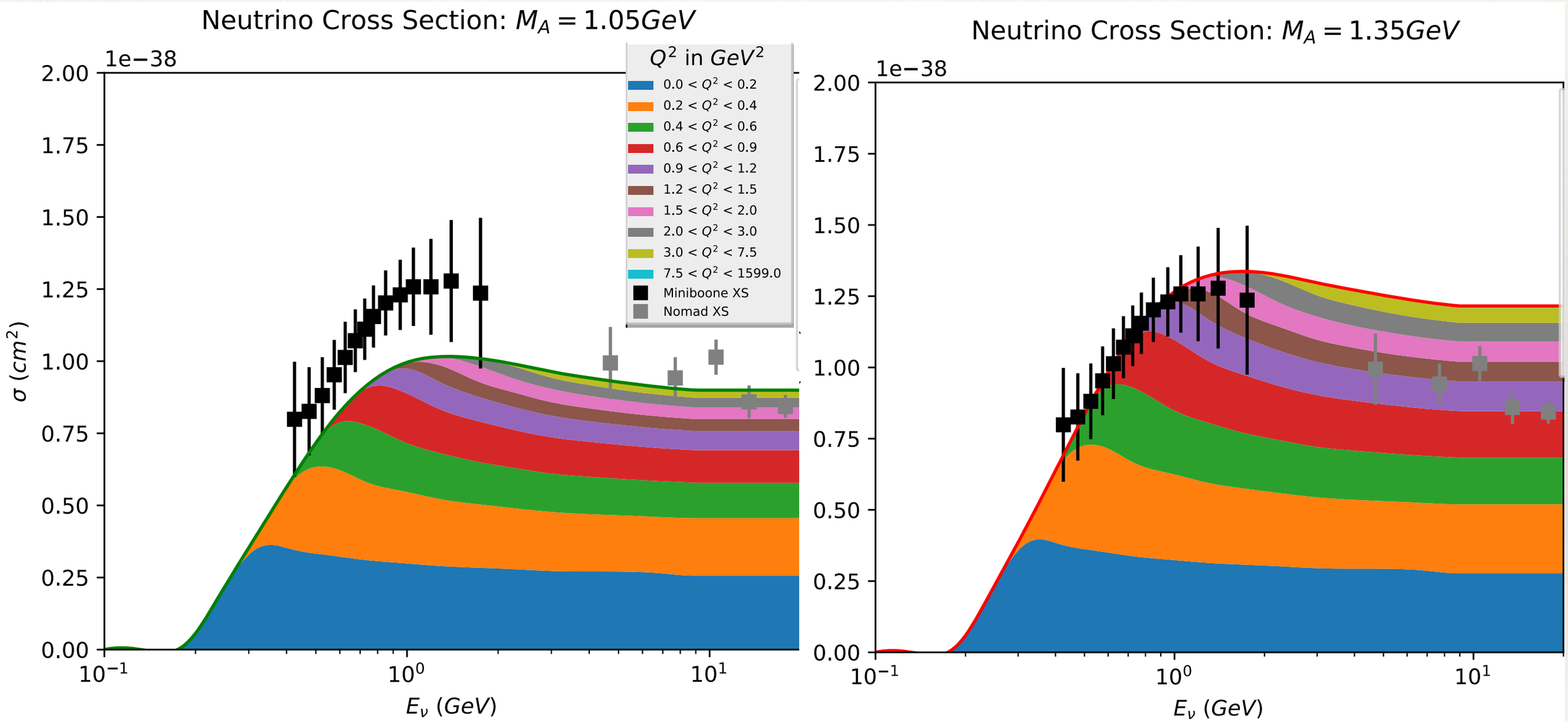
Friedland, Gonzalez-Solis, E.P., Quirion, Ristow in preparation

- Composition of MiniBooNE Cross Section

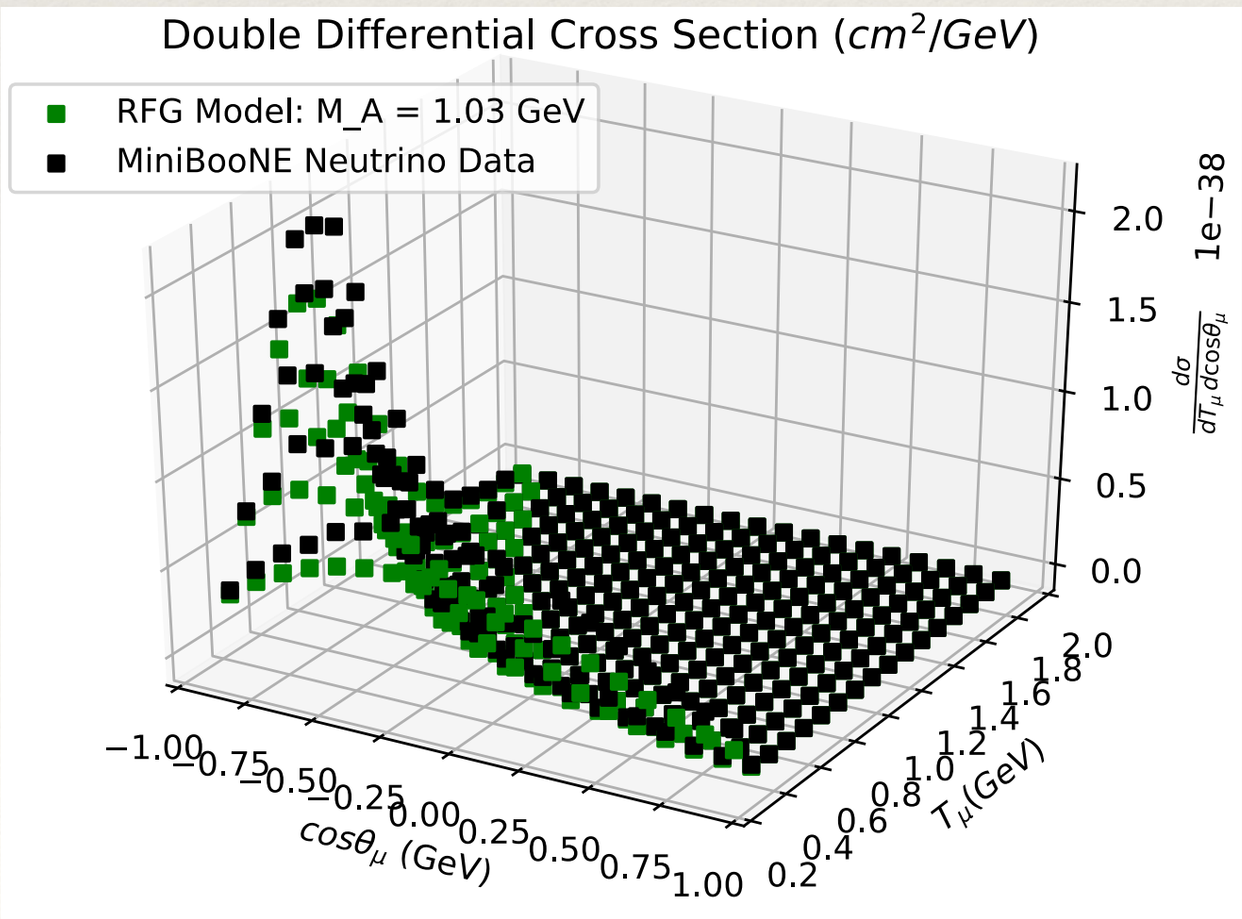


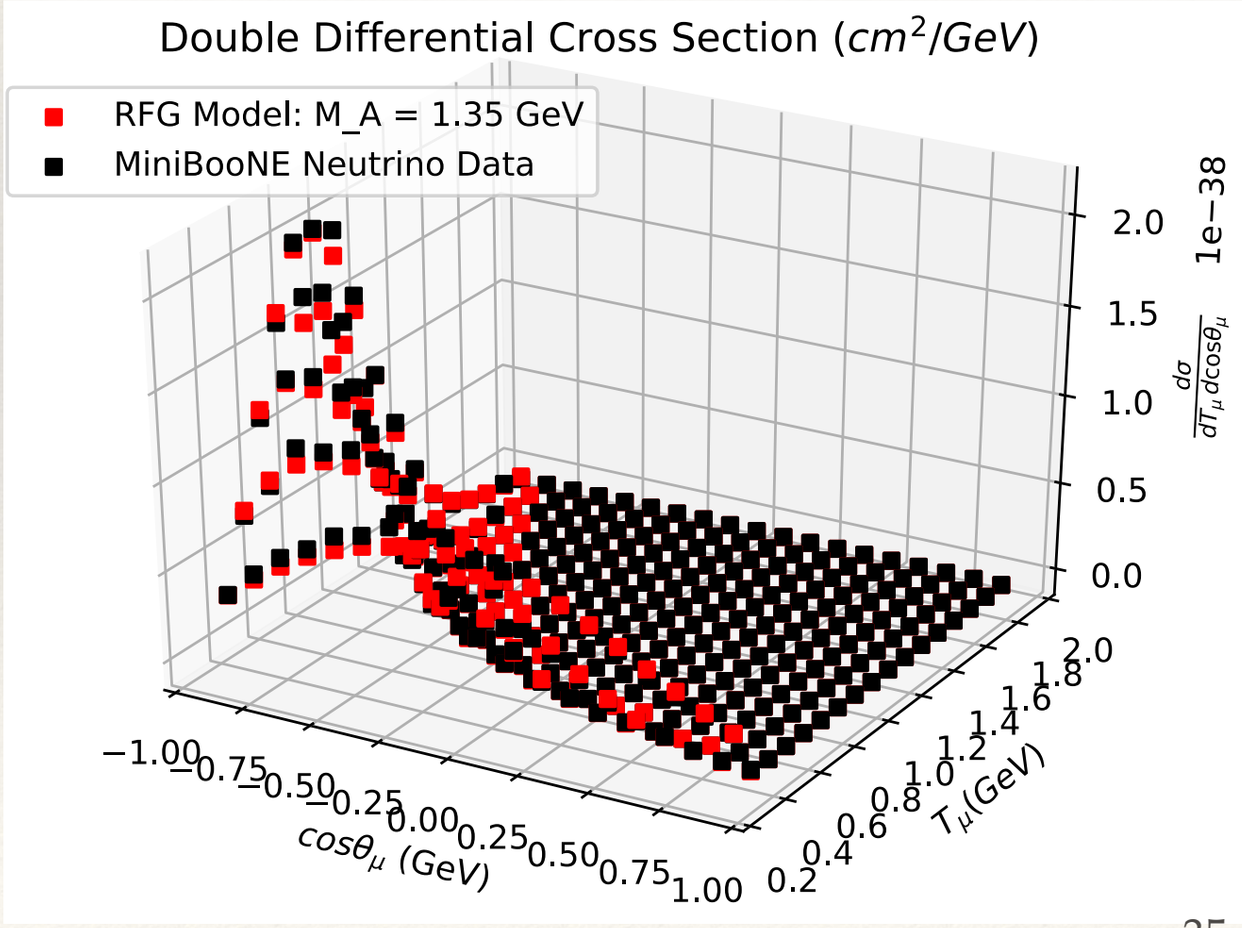
- At E ~ 0.5 GeV the XS comes from $Q^2 < 0.6$ GeV^2
- At E ~ 1 GeV, ~40% contributions from 0.6 GeV^2 < Q^2 < 2 GeV^2

Axial Form Factor Enhancement

Friedland, Gonzalez-Solis, E.P., Quirion, Ristow in preparation

- MiniBooNE cross section measurements seem to favour a 30% enhancement of the axial form factor
- The same enhancement provides a good fit to MiniBooNE differential Cross Section measurements

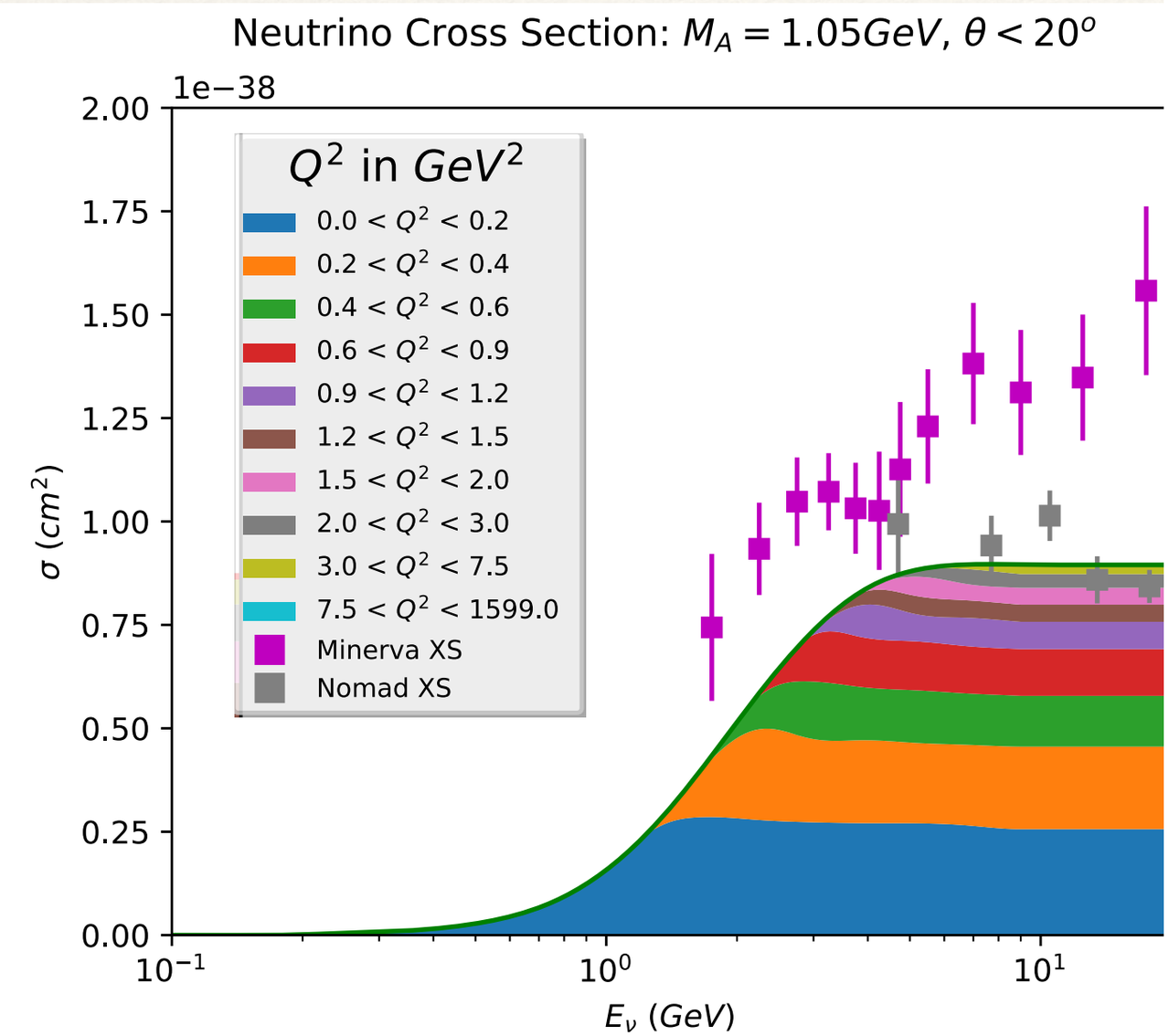


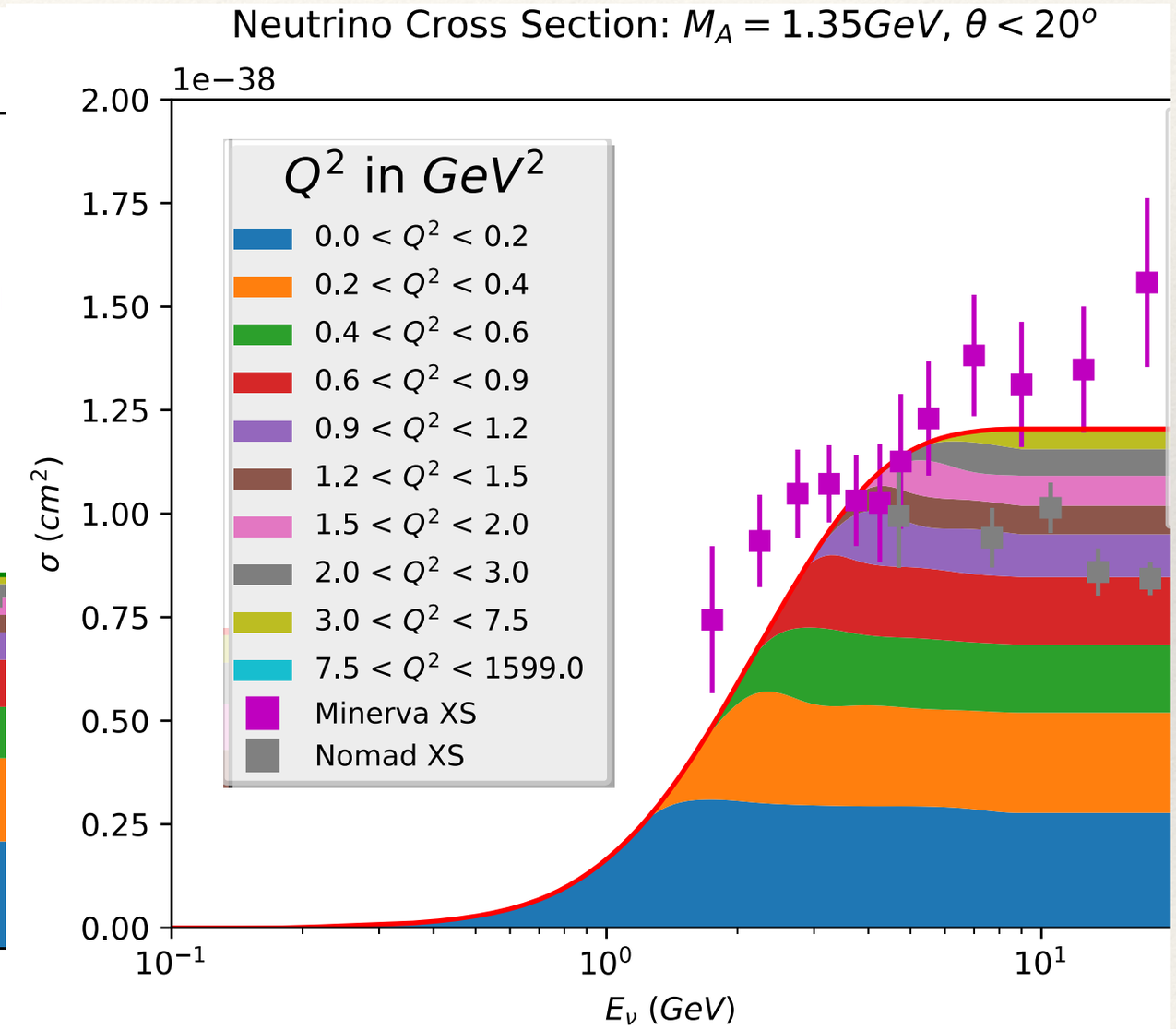


Which values of Q^2 impact neutrino data?

Friedland, Gonzalez-Solis, E.P., Quirion, Ristow in preparation

- Composition of MINERvA Cross Section





- The situation is similar, although E_ν is higher the relevant values are $Q^2 < 2\ GeV^2$

Which values of q^2 impact neutrino data?

- The experimental results point towards a larger value of the axial form factor $M_A \sim 1.35$ GeV
- If true, the value of M_A saturates the cross section leaving little room for multi nucleon effects or others

Which values of q^2 impact neutrino data?

- The experimental results point towards a larger value of the axial form factor $M_A \sim 1.35$ GeV
- If true, the value of M_A saturates the cross section leaving little room for multi nucleon effects
- Is the dipole physically motivated?

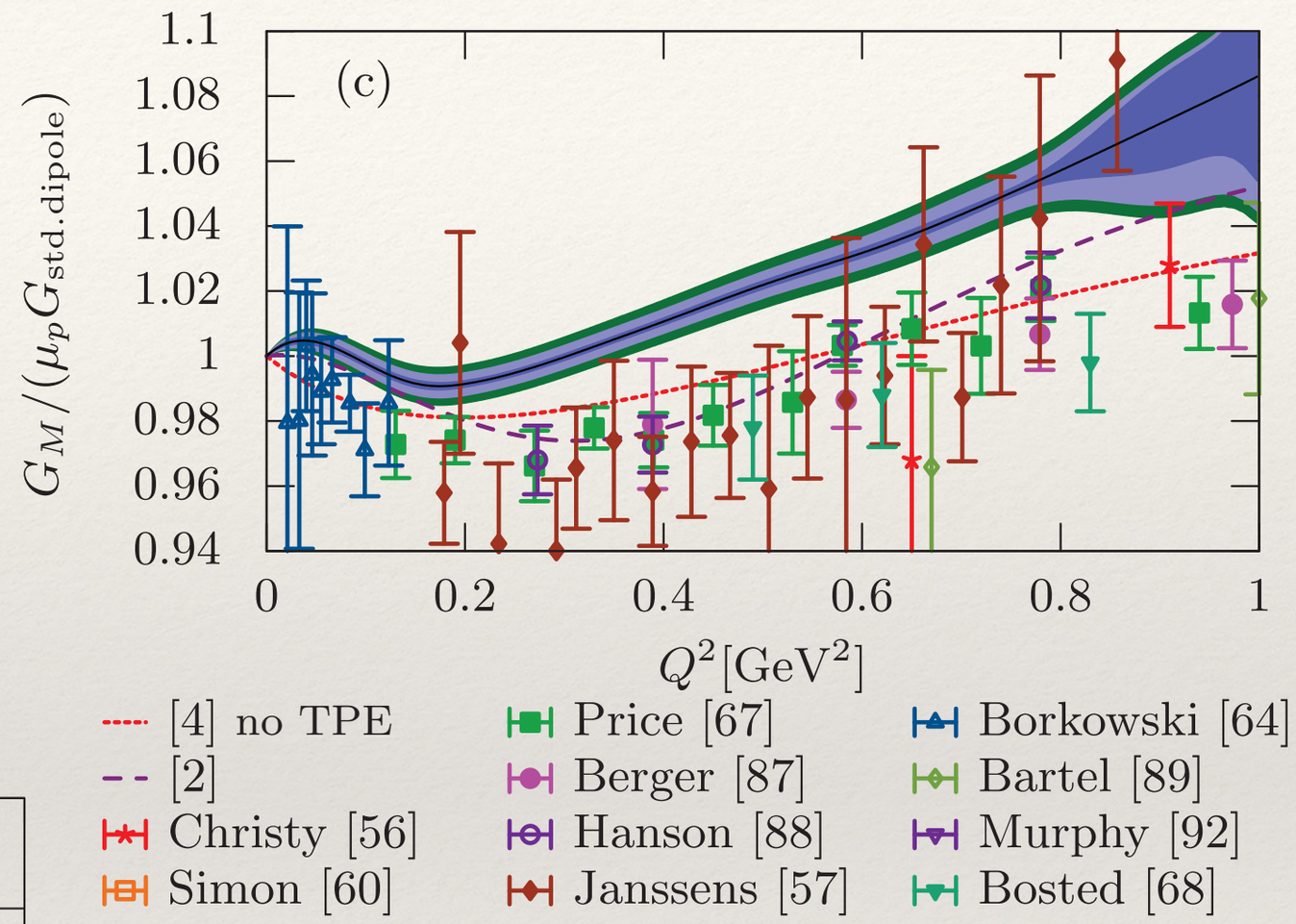
$$F_A(q^2) = \frac{F_A(0)}{\left(1 - \frac{q^2}{M_A^2}\right)^2}$$

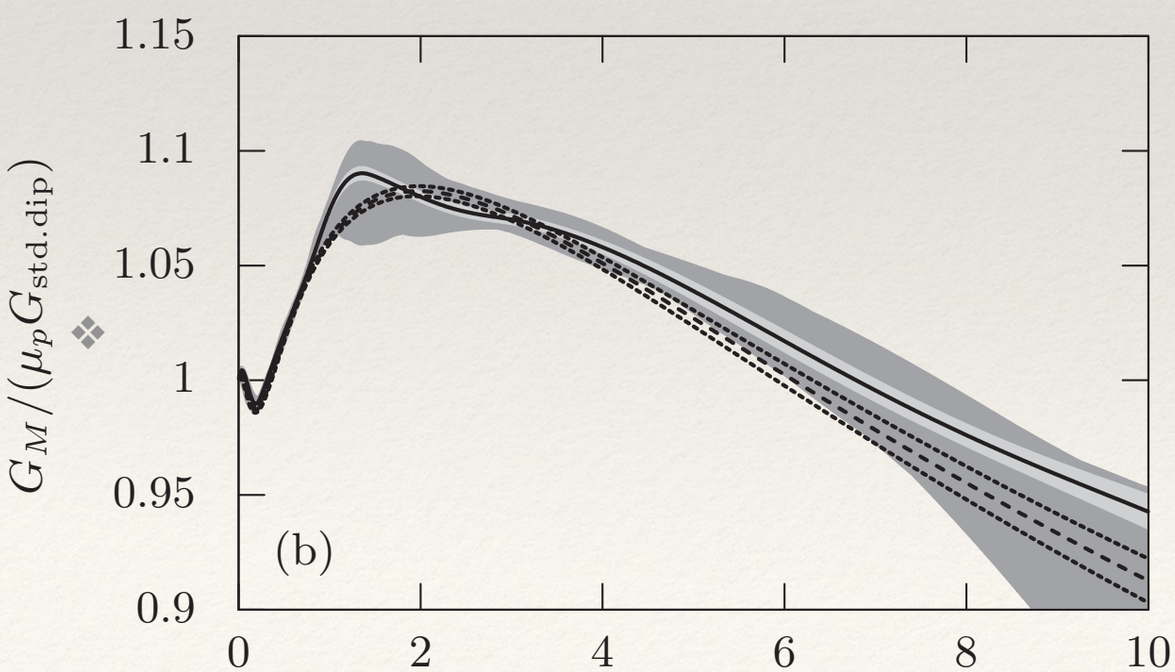
The parametrisation has an impact on different q^2 dependence ranges on the neutrino data

Dipole Parametrization

- With very precise data the dipole parametrisation fails for EM form factors

Bernauer et al, A1 coll.'06





$$G_M^V(t=q^2) \equiv F_1(q^2) + F_2(q^2)$$

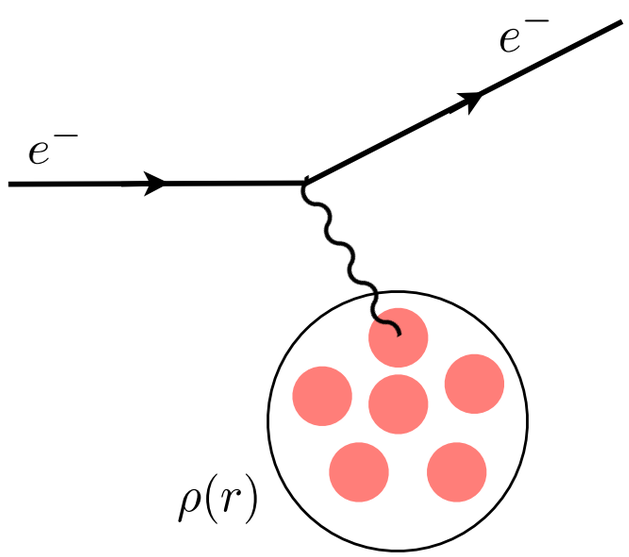
$$G_E^V(t=q^2) \equiv F_1(q^2) + \frac{q^2}{4M_P^2} F_2(q^2)$$

What do we know on the Axial Form Factor?

- Their low energy behaviour: given by ***ChPT***
- They are defined as the Fourier transform of the charge distribution: they can be computed within ChPT at $\mathcal{O}\left(p^4\right)$

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{pointlike}} |F(q^2)|^2$$

$$\begin{aligned} F(q^2) &= \int d^3r\, e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \rho(\boldsymbol{r}) \\ &= \int d^3r \left[1 + i\boldsymbol{q}\cdot\boldsymbol{r} - \frac{1}{2}(\boldsymbol{q}\cdot\boldsymbol{r})^2 + \ldots\right]\rho(r) \\ &= 1 - \frac{1}{6}\langle r^2\rangle \boldsymbol{q}^2 + \ldots \end{aligned}$$



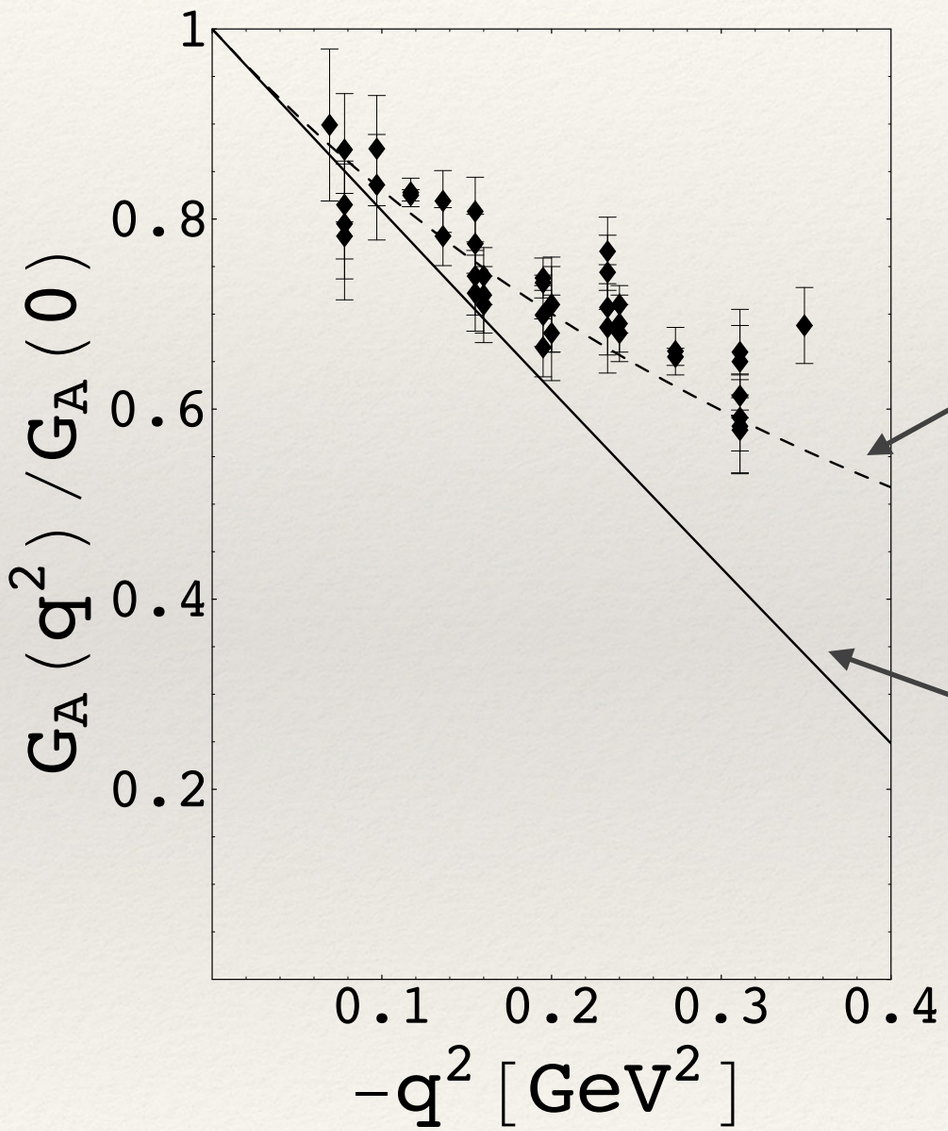
We can perform a Taylor expansion:

$$F(q^2) = F(0)\left(1 + \frac{1}{6}\langle r^2\rangle q^2 + \mathcal{O}(q^4)\right)$$

What do we know on the Axial Form Factor?

- Chiral Perturbation Theory determination of $F_A(q^2) \equiv G_A(q^2)$

Schindler & Scherer'06



Dipole fit

$$G_A(q^2) = \frac{G_A(0)}{\left(1 - \frac{q^2}{M_A^2}\right)^2}$$

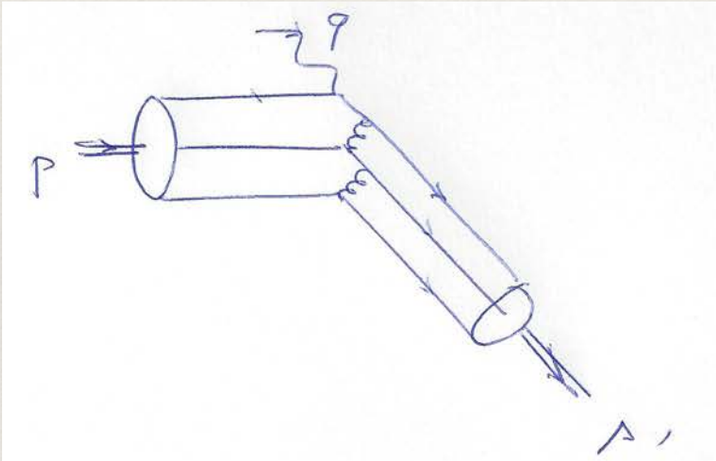
ChPT prediction

What do we know on the Axial Form Factor?

Brodsky & Farrar'75
Brodsky & Lepage'80

- Its high energy behaviour:

  In the limit $-q^2 \equiv Q^2 \to \infty$ configuration with minimum # quarks will dominate

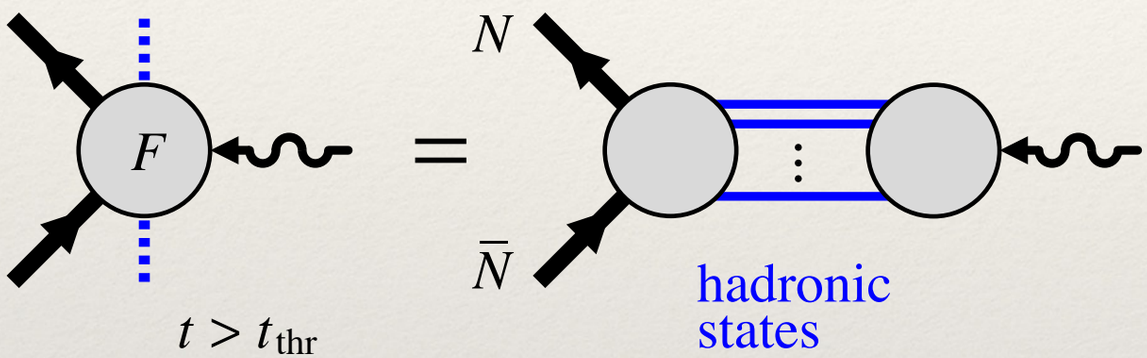


2 hard gluons : $F_A(q^2) \underset{-q^2 \to \infty}{\sim} \dfrac{1}{q^4}$

When does pQCD set in? Experimentally for $Q^2 > 10\ \mathrm{GeV}^2$

Improving the Form Factor parametrisation

- For intermediate energy region: Can try to use ***VMD***
  - ***Analytical structure*** of FF (e.g. F_1 or F_A)

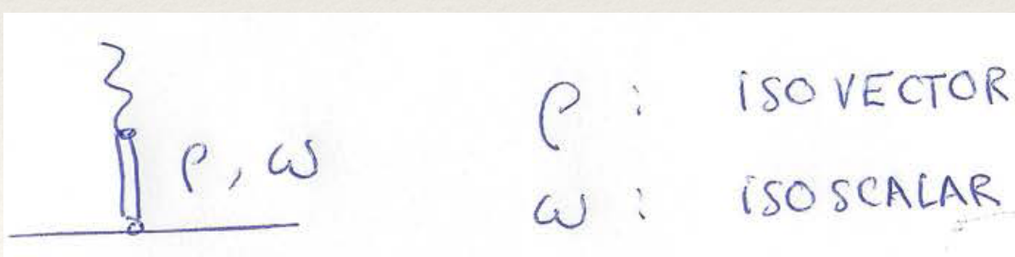


Photon or W sees proton through all hadronic states (with vector or axial-vector Quantum Number)

Isovector: $\pi\pi$ (incl. ρ), 4π, $K\bar{K}$, ...
Isoscalar: 3π (incl. ω), $K\bar{K}$ (incl. ϕ), ...

Processes in unphysical region $t < 4\, m_N^2$

- Resonances (Vector Mesons)



For F_A (Axial Vector Mesons)

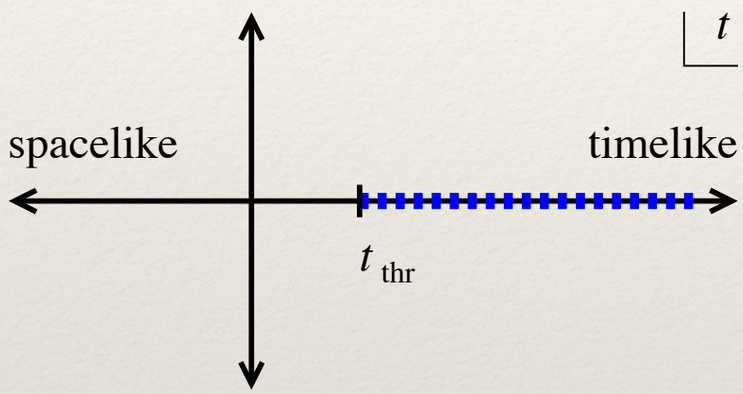
$a_1(1230)$ and $a_1'(1647)$

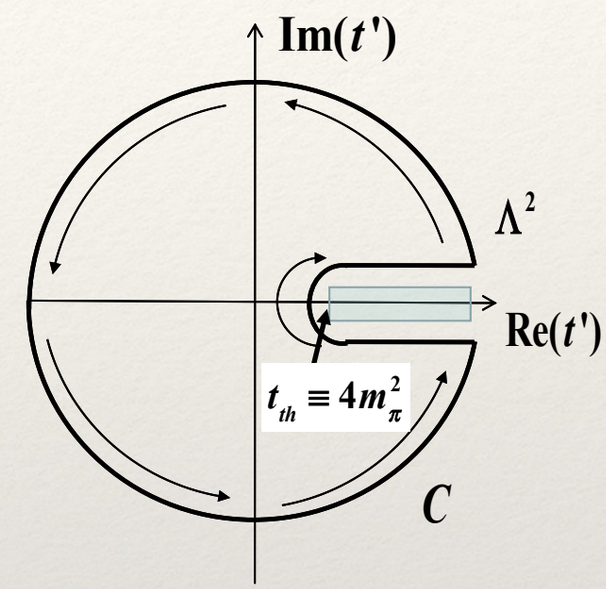
Masjuan et al.'12

$$F_A(t) = g_A \frac{m_{a_1}^2\, m_{a_1'}^2}{(m_{a_1}^2 - t)(m_{a_1'}^2 - t)}$$

Improving the Form Factor parametrisation

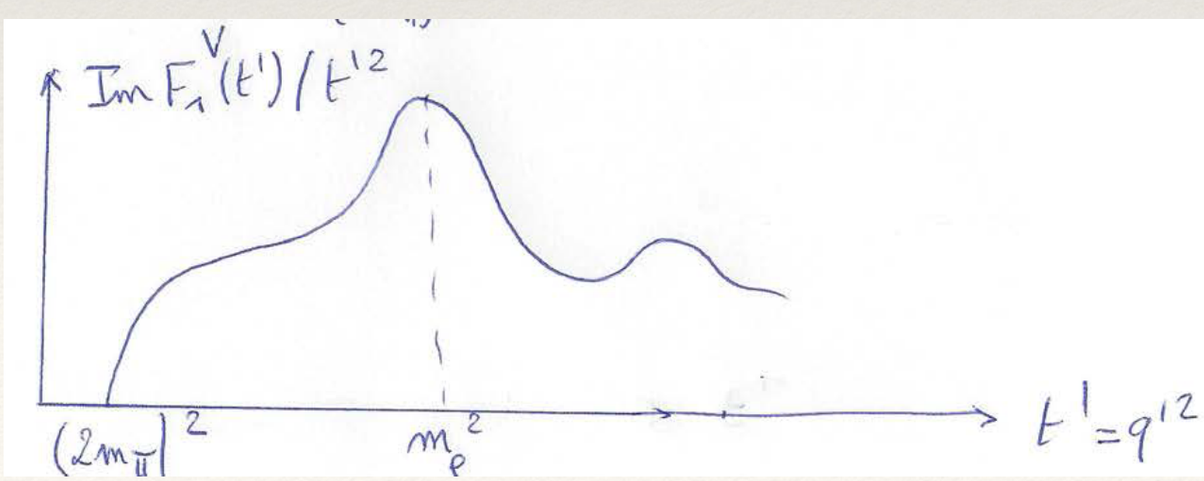
- For intermediate energy region: Can try to use ***VMD***, e.g. EM FF
  - ***Dispersion Relations***





- Use spectral function from theory or from experiment

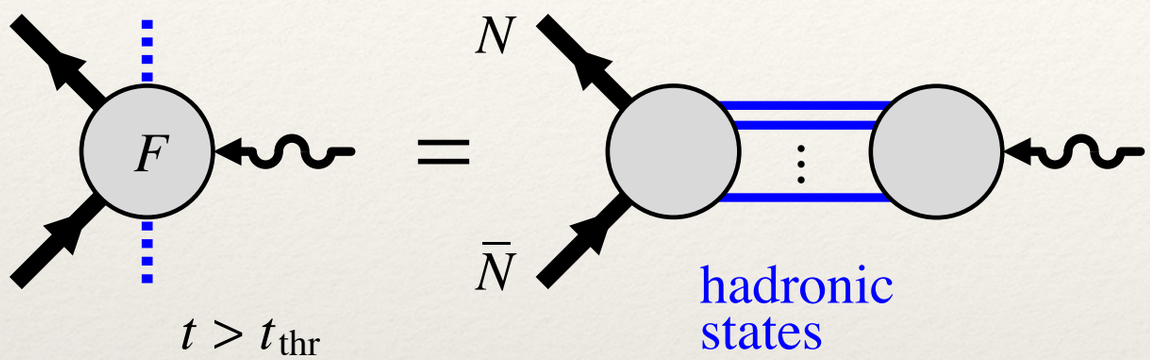
Frazer &Fulco'60, Hohler et al'75



$$F_i(t) = \int_{t_{\rm thr}}^{\infty} \frac{dt'}{\pi} \frac{\mathrm{Im}\, F_i(t')}{t' - t - i0}$$

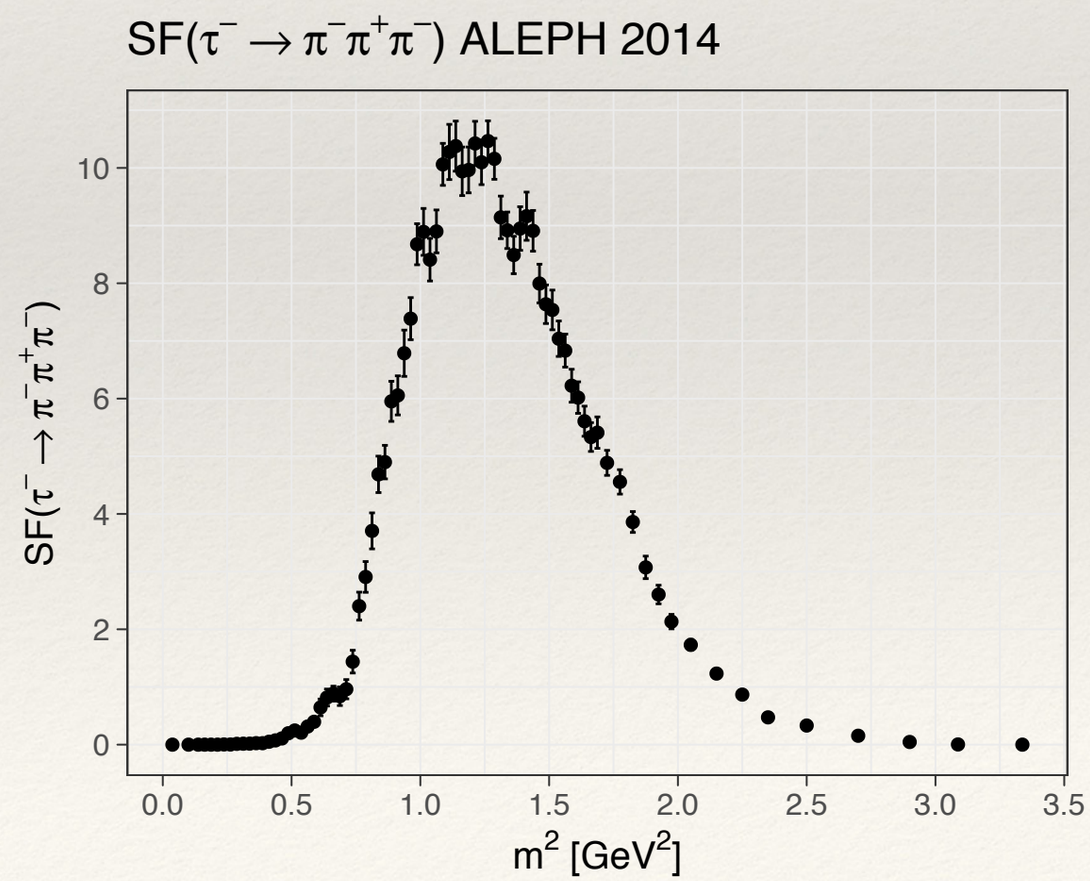
Reconstruct the Axial form factor from Analyticity

- First intermediate state 3π



$$\mathrm{ImF_A(t)} \propto \mathrm{f_{A\to 3\pi}(t)} \times \mathrm{t^*_{3\pi\to N\bar{N}}(t)}$$

- Use the spectral function from $\tau \to 3\pi\nu_\tau$ measured by ALEPH to determine $\mathrm{f_{A\to 3\pi}}$



Axial form factor constrained from tau data and nucleon data

- Build a function that has all good analytical properties and is in agreement with tau data for $q^2 > 0$ and lattice data for $q^2 < 0$

- The dipole parametrisation blows up at $q^2 = M^2_A$

$$F_A(q^2) = \frac{F_A(0)}{\left(1 - \frac{q^2}{M^2}\right)^2}$$

- The two pole parametrisation also blows up

$$\boxed{F_A(t) = g_A \frac{m_{a_1}^2 \, m_{a_1'}^2}{(m_{a_1}^2 - t)(m_{a_1'}^2 - t)}}$$

Masjuan et al.'12

- One need to go out of the real axis: → Use Breit Wigners

Axial form factor constrained from tau data and nucleon data

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- One needs to go out of the real axis: → Use Breit Wigners

$$f_A^{BW}(q^2) = \mathrm{BW}_{a_1(1260)}(q^2)$$

$$\text{with} \quad \mathrm{BW}_R(q^2) = \frac{m_R^2}{m_R^2 - q^2 - iM_R\Gamma_R(q^2)} \quad \text{and} \quad \Gamma_R(q^2) = \Gamma_R \frac{g(q^2)}{g(m_R^2)} \,.$$

Axial form factor constrained from tau data and nucleon data

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- One needs to go out of the real axis: → Use Breit Wigners

$$f_A^{BW}(q^2) = \frac{1}{1+\gamma}\left[\mathrm{BW}_{a_1(1260)}(q^2) + \gamma\, \mathrm{BW}_{a_1(1640)}(q^2)\right]$$

with $\mathrm{BW}_R(q^2) = \dfrac{m_R^2}{m_R^2 - q^2 - iM_R\Gamma_R(q^2)}$ and $\Gamma_R(q^2) = \Gamma_R \dfrac{g(q^2)}{g(m_R^2)}$.

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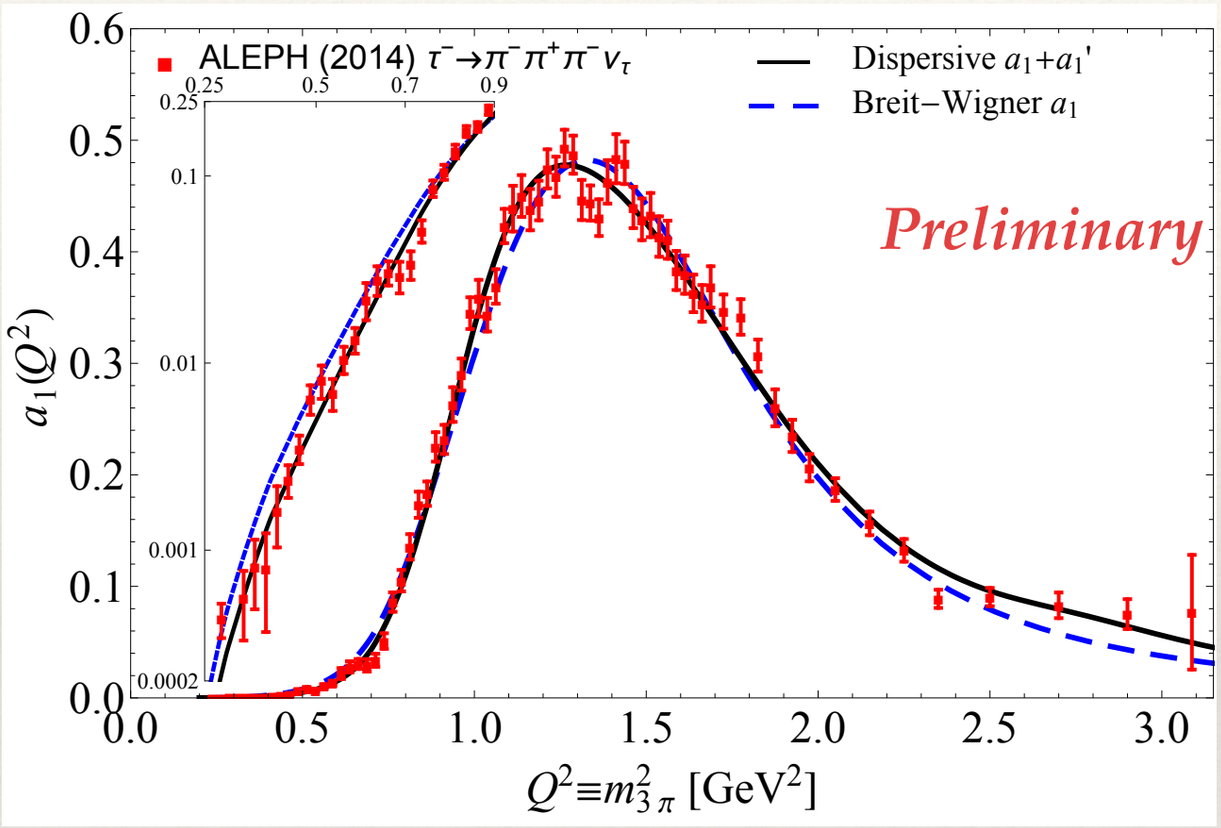
- Use a dispersive parametrisation:

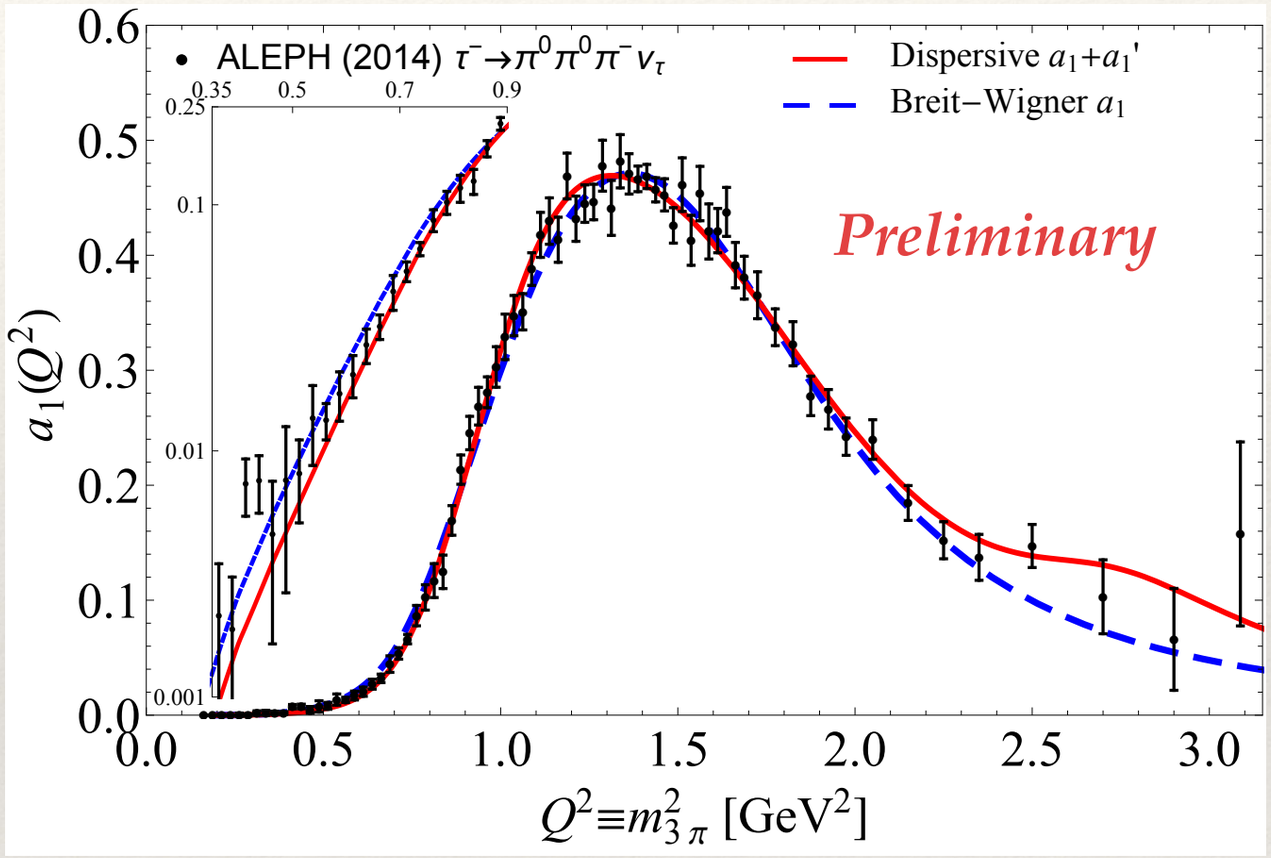
$$f_A^{disp}(q^2) = \exp\left[\alpha_1 q^2 + \frac{q^4}{\pi}\int_{9m_\pi^2}^{\infty} ds' \frac{\delta(s')}{s'^2(s' - q^2 - i\epsilon)}\right]$$

Axial form factor constrained from Tau data and nucleon data

- Results from tau data

Friedland, Gonzalez-Solis E.P., in preparation



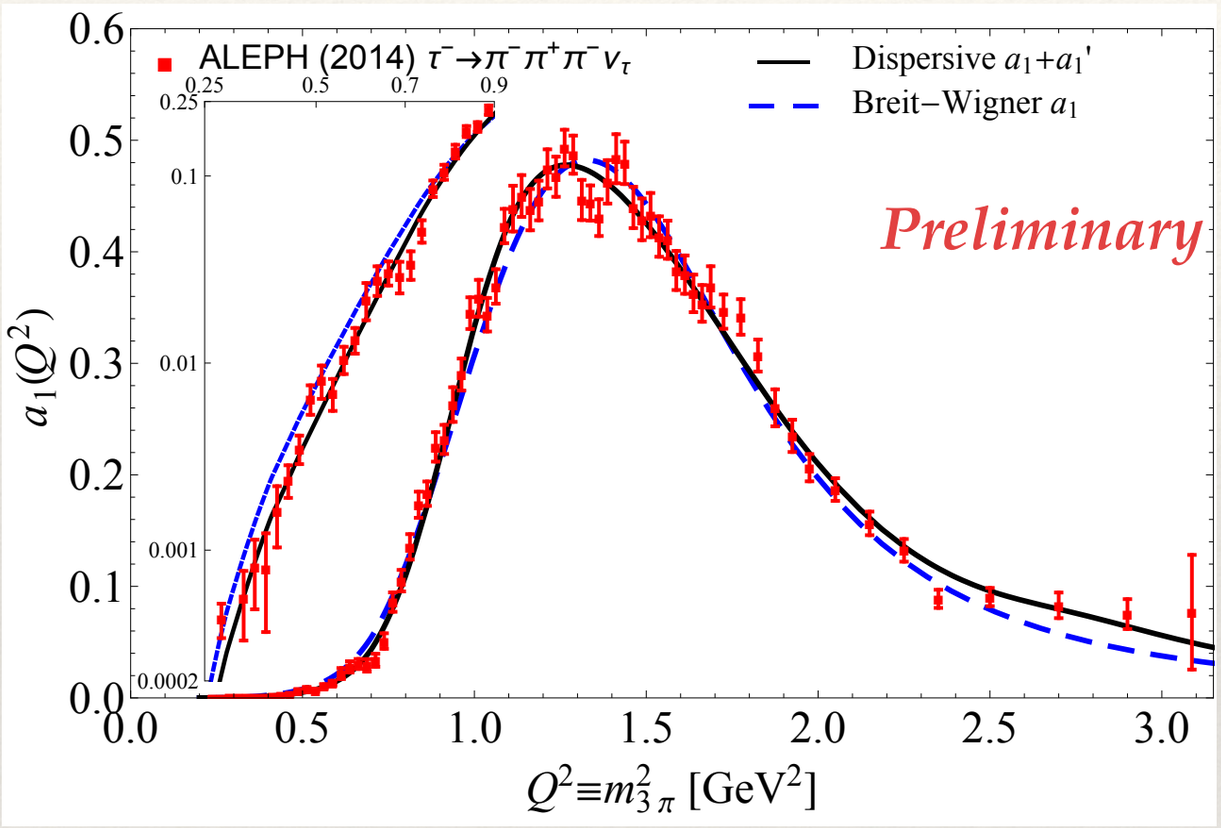


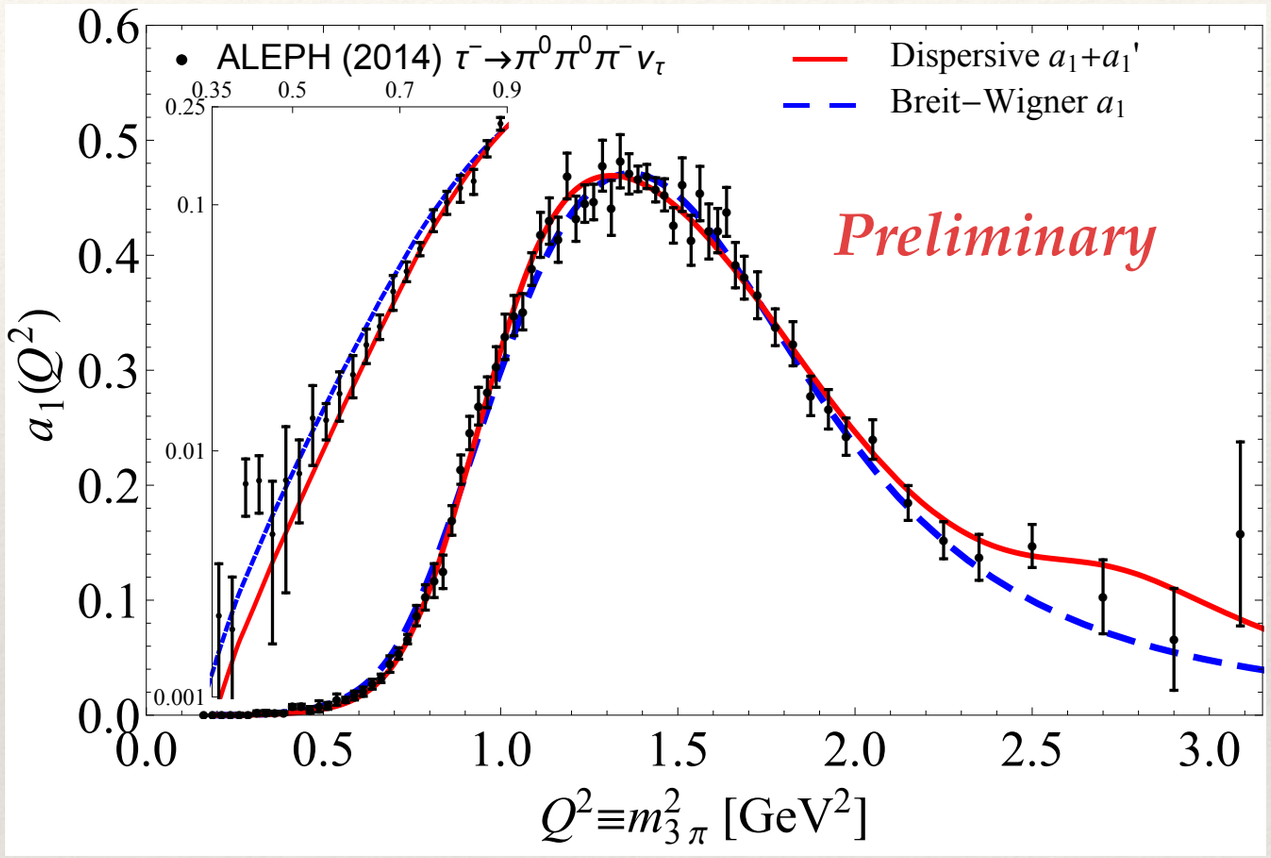
- Notice that the peak is consistent with $M_A \sim 1.2$ - 1.3 GeV, but not with M_A=1.03 GeV

Axial form factor constrained from Tau data and nucleon data

- Results from tau data

Friedland, Gonzalez-Solis E.P., in preparation

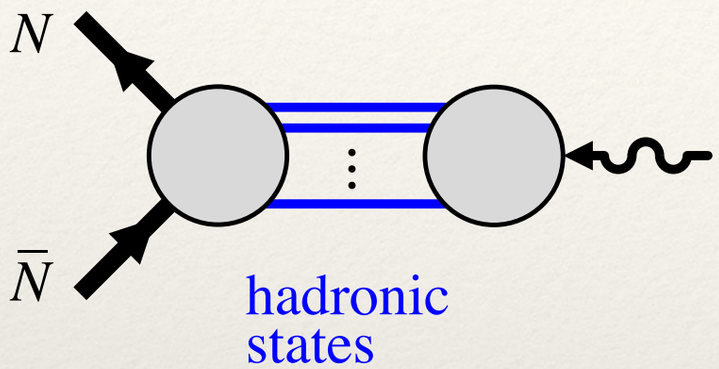




- How to connect to the nucleon?

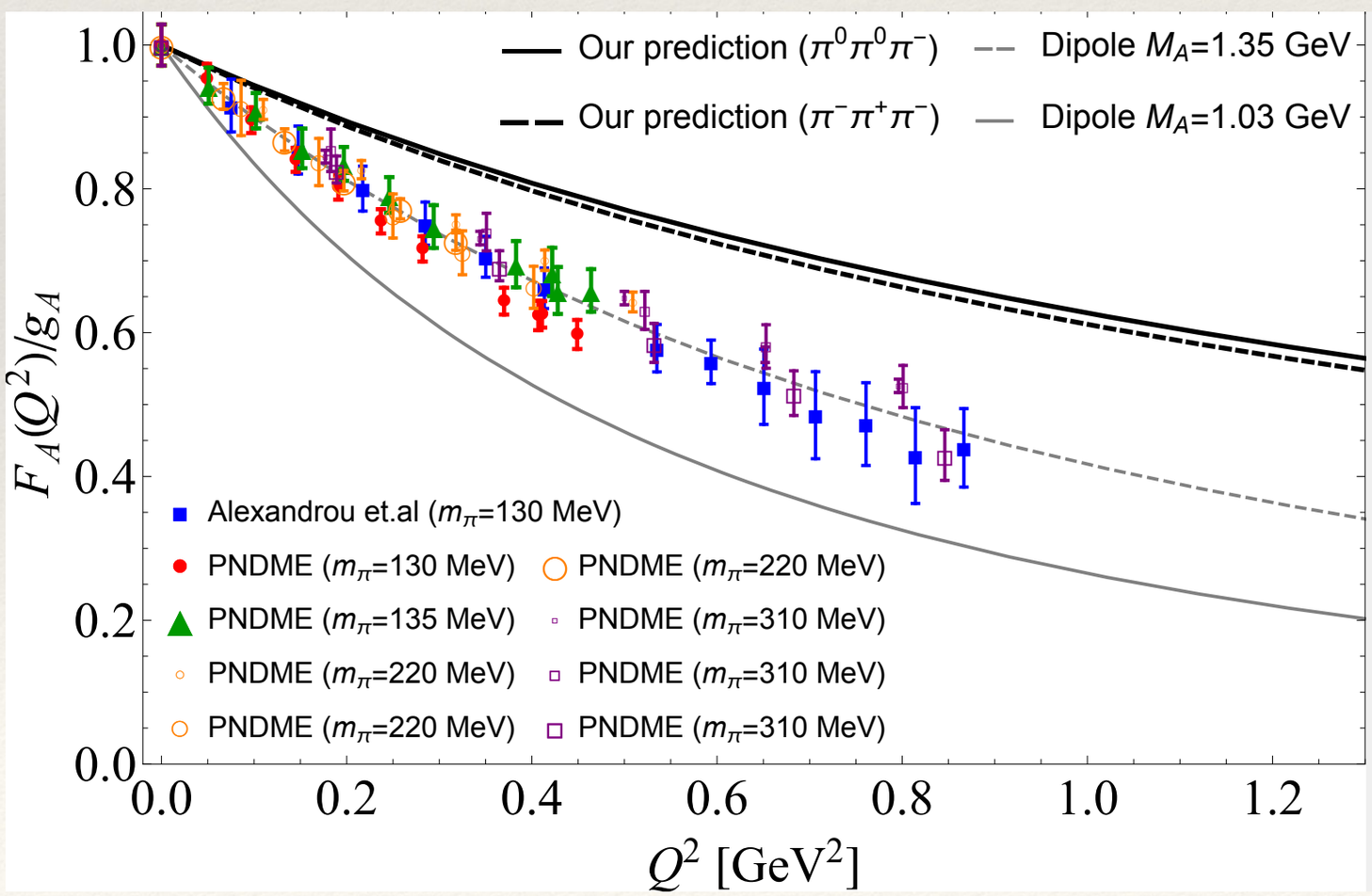
Axial form factor constrained from Tau data and nucleon data

- How to connect to the nucleon?



- Take a constant g_A

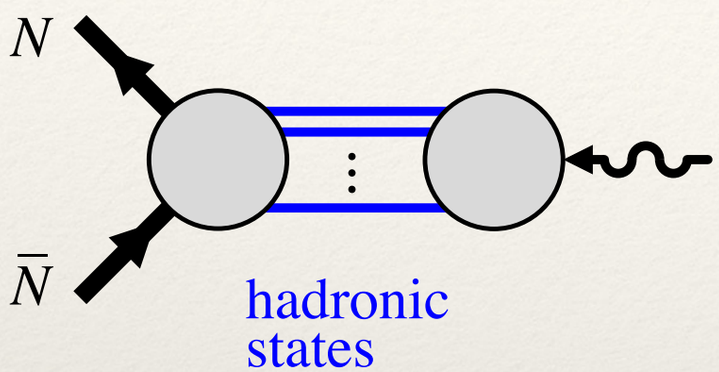
$$F_A(q^2) = g_A \cdot f_{A\to 3\pi}\left(q^2\right)$$



Does not work!

Axial form factor constrained from Tau data and nucleon data

- How to connect to the nucleon?

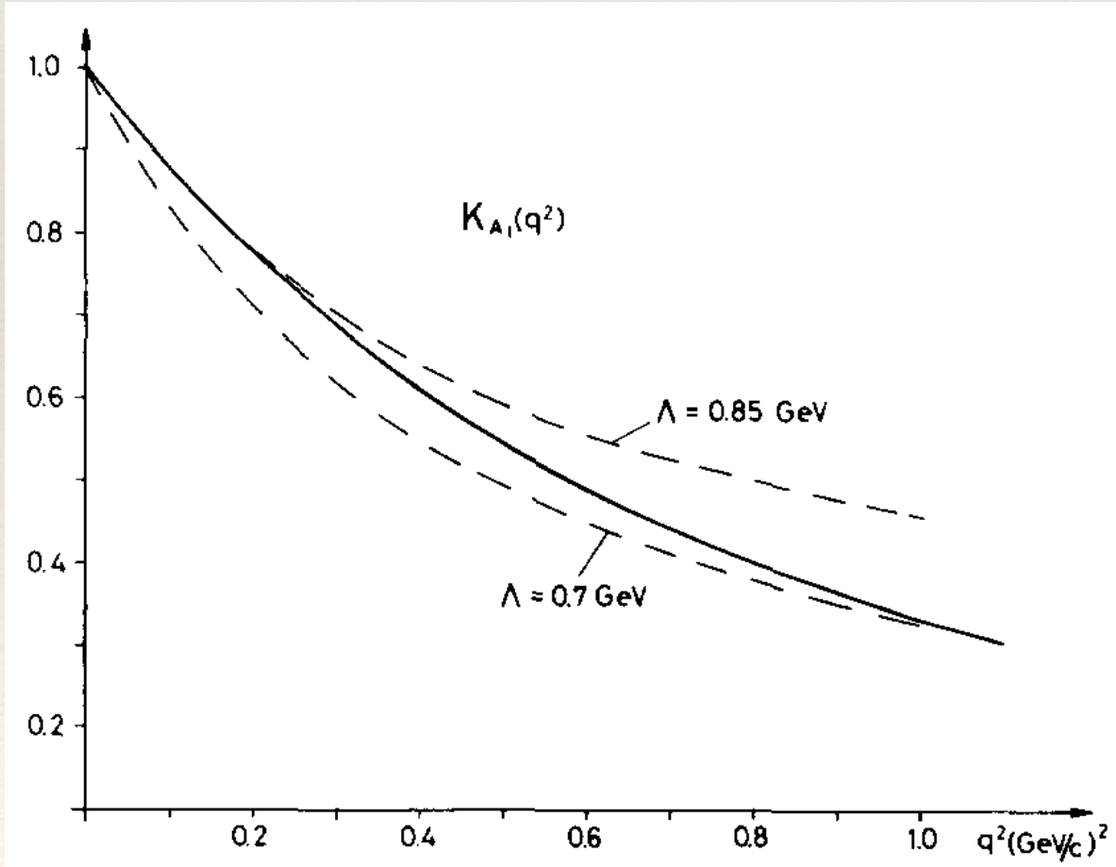


Gari & Kaulfuss'84

- Use Axial Vector Meson Dominance assumption

$$F_A(q^2) = f_{A\to 3\pi}\left(q^2\right) \cdot K_A\left(q^2\right)$$

- Several q^2 regions:
  - Low energy region
    one pole $K_A\left(q^2\right) = g_A \frac{\Lambda_{LE}^2}{\Lambda_{LE}^2 - q^2}$



Axial form factor constrained from Tau data and nucleon data

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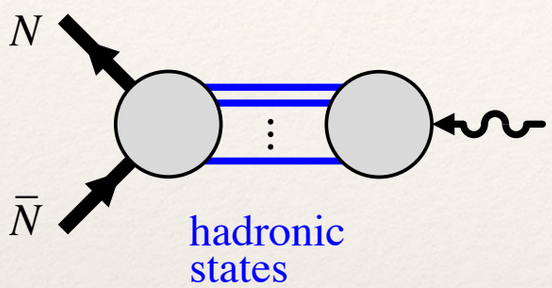

- Several q^2 regions:
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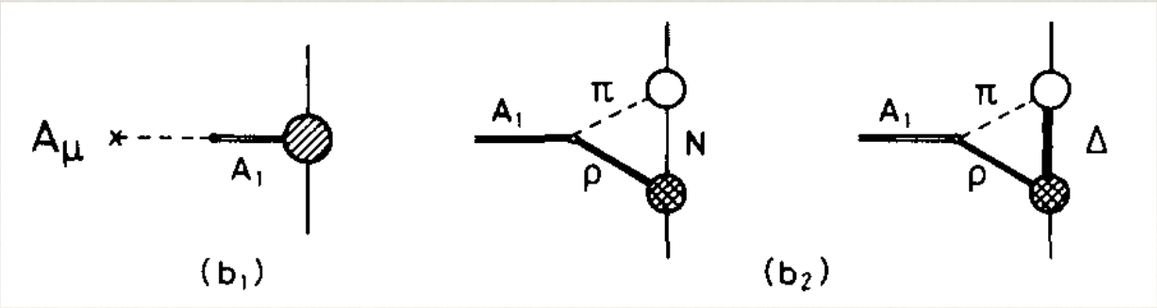
  - Intermediate region:



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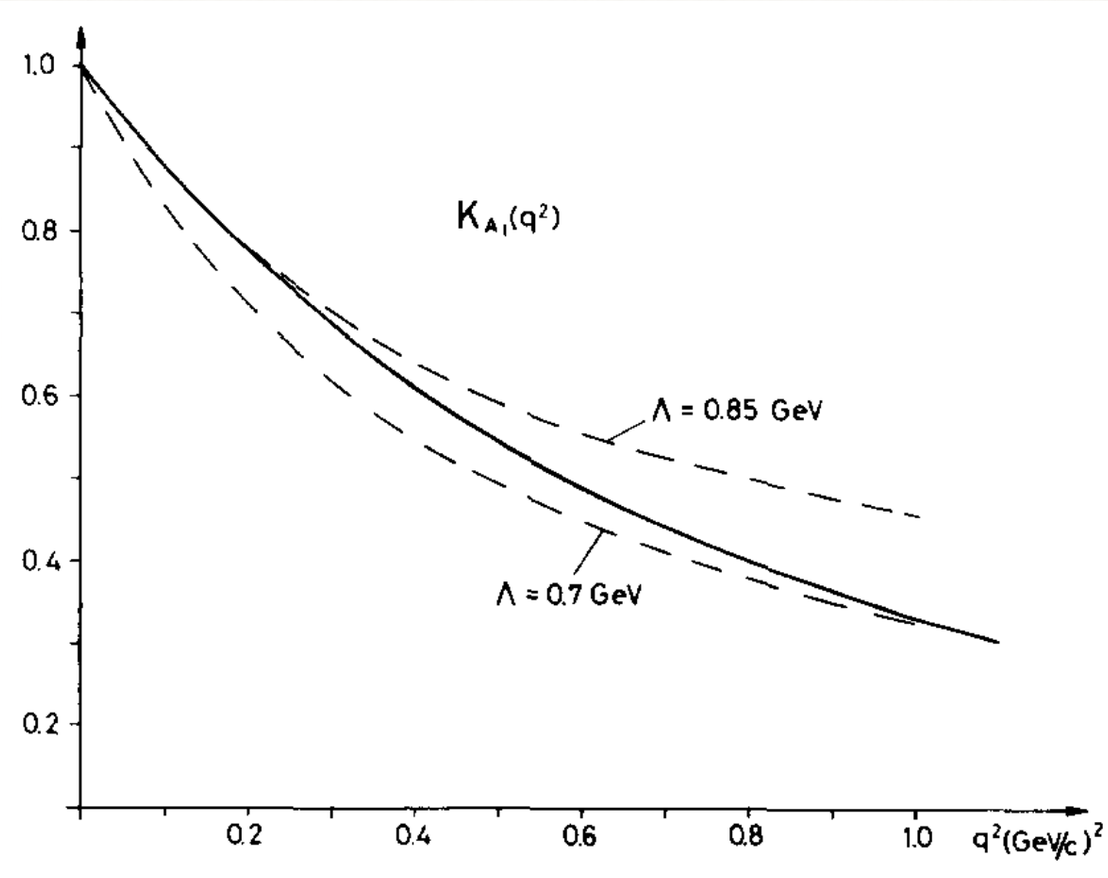

Axial form factor constrained from Tau data and nucleon data

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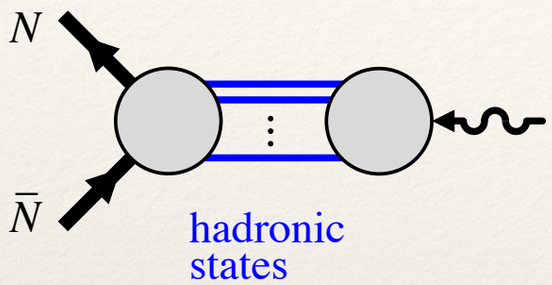

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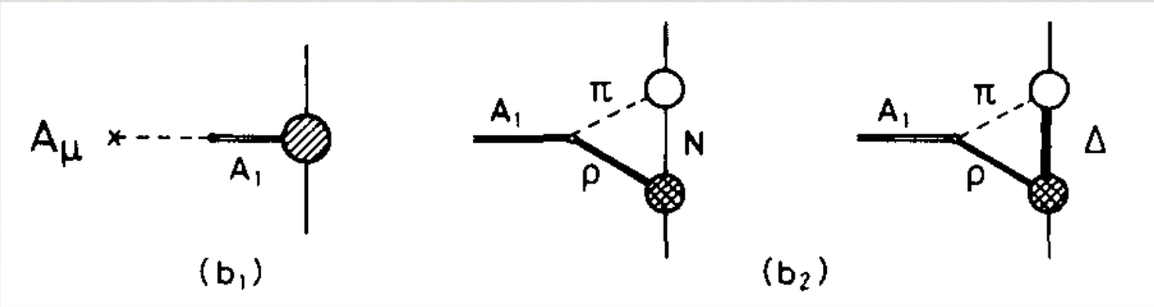

  - High energy region: pQCD

$$K_A\left(q^2\right) = g_A \frac{\Lambda_{HE}^2}{\Lambda_{HE}^2 - q^2}$$

Gari & Kaulfuss'84

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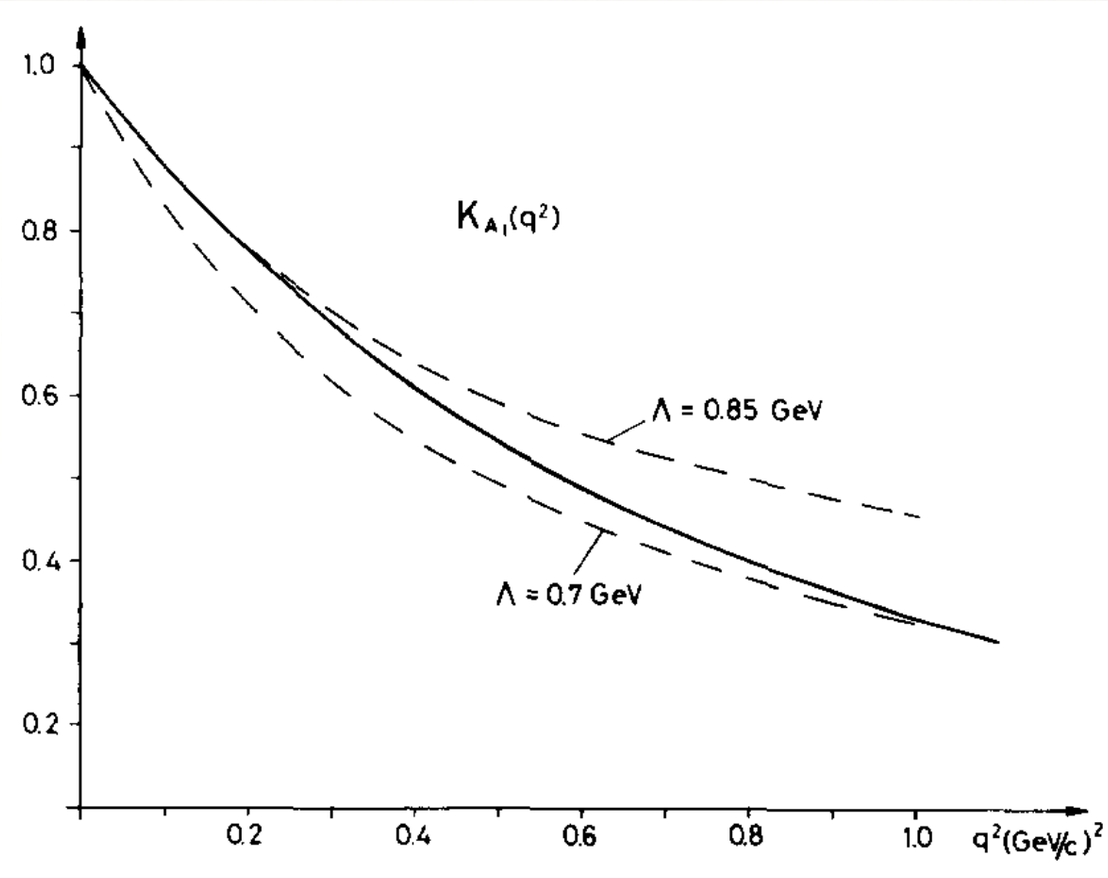

Axial form factor constrained from Tau data and nucleon data

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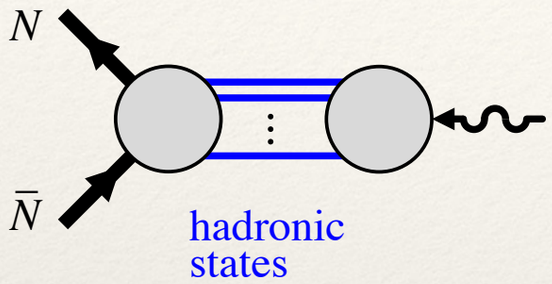

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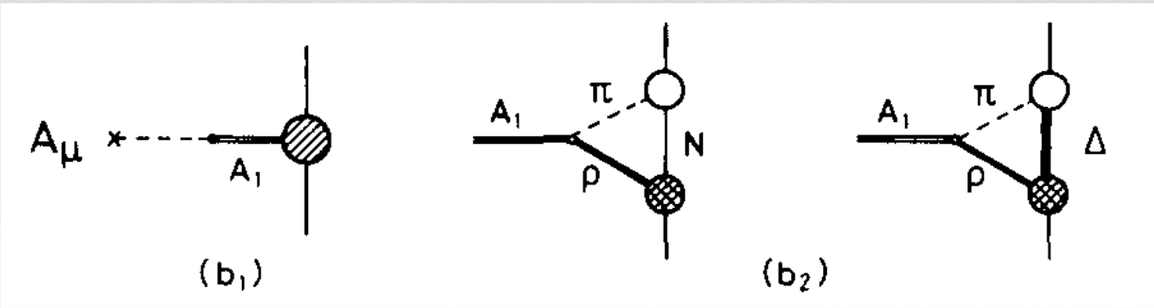

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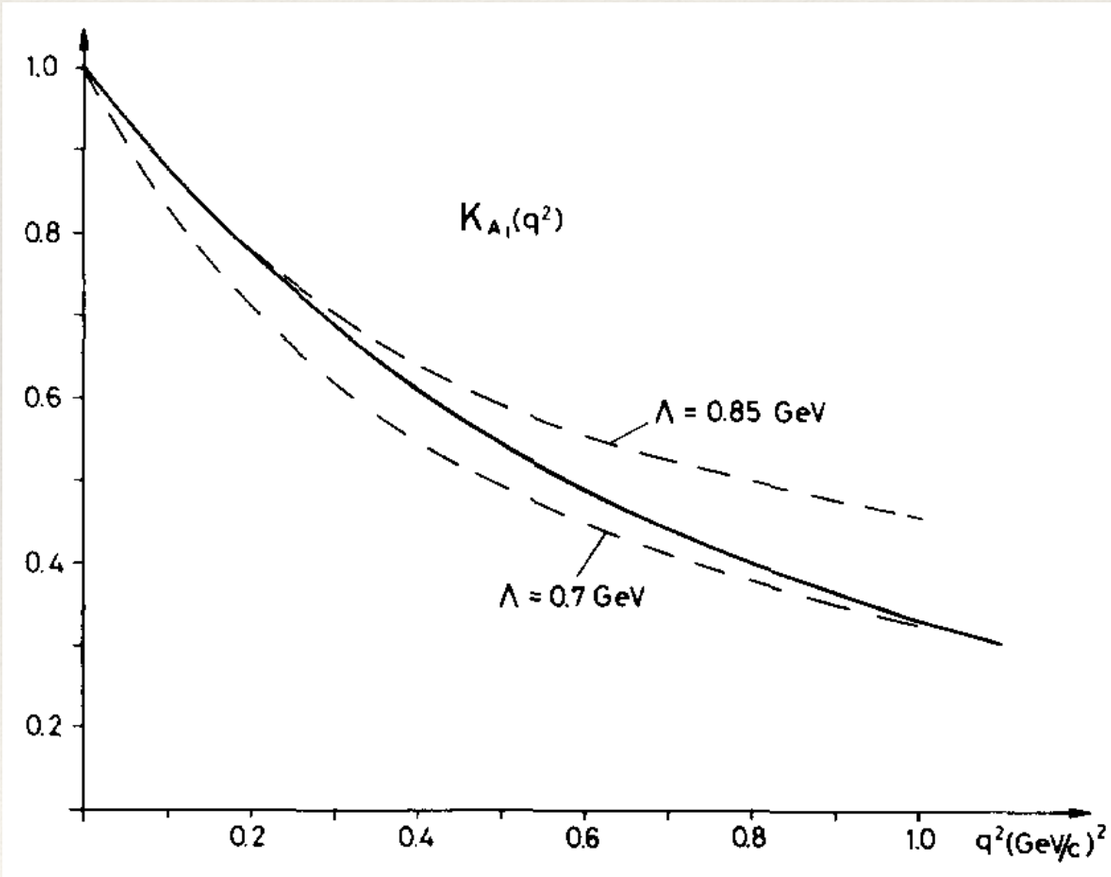
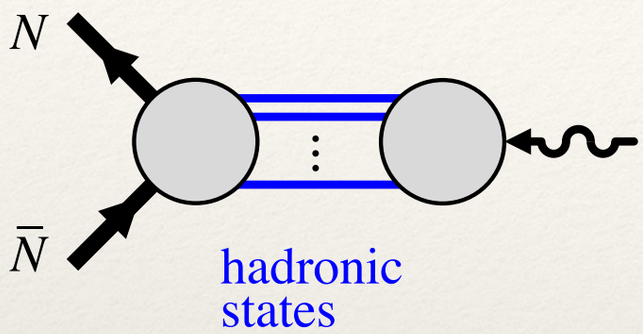


Friedland, Gonzalez-Solis E.P., Quirion in progress

Axial form factor constrained from Tau data and nucleon data

- How to connect to the nucleon?



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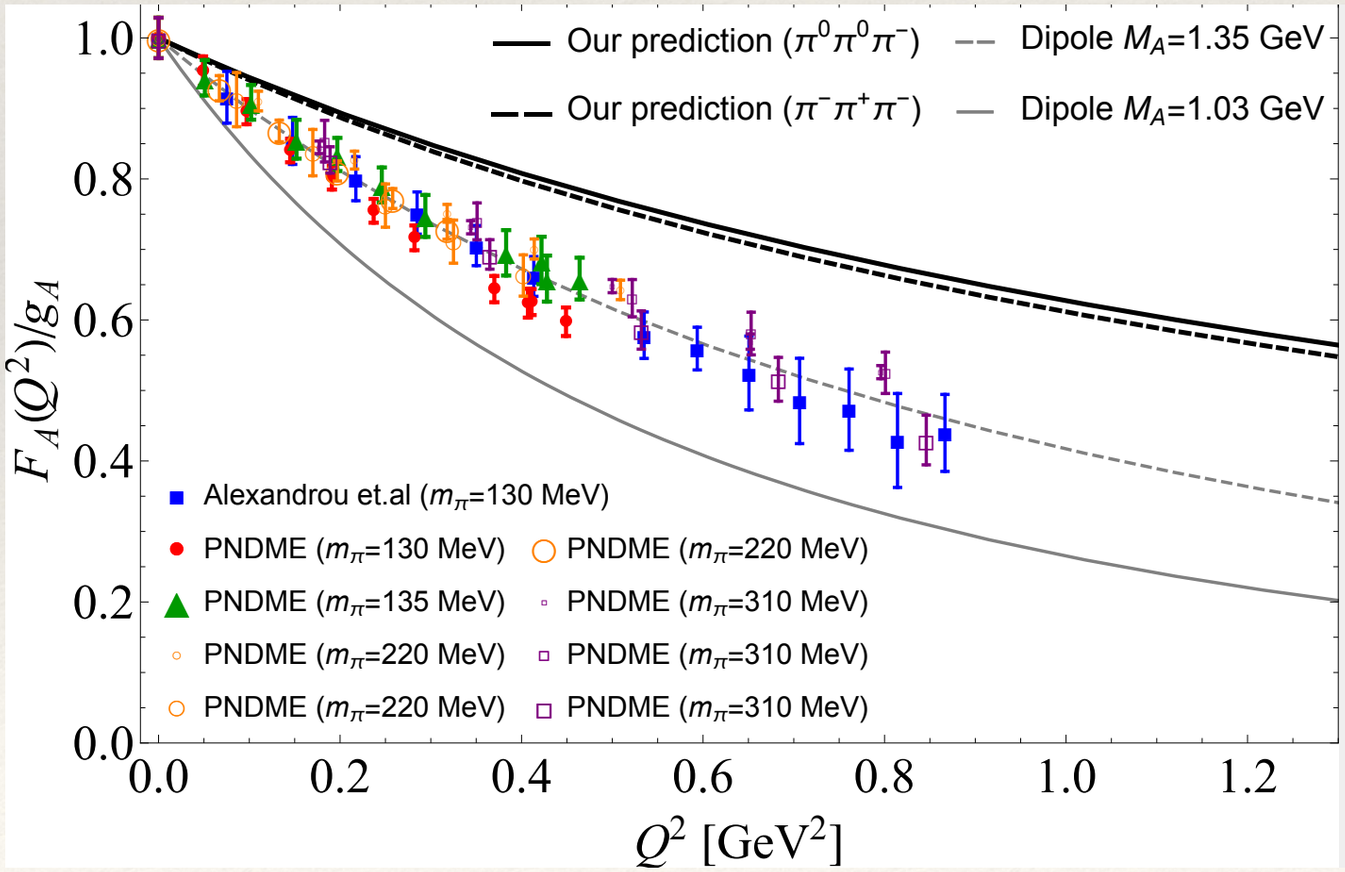


→ Fit $K_A\left(q^2\right)$ to lattice data

Conclusion and Outlook

- With neutrino physics entering precision era, we have new demands for better understanding of neutrino-nucleon interactions
- The problem has many facets:
  - Old, schematic dipole parametrisation of the nucleon form-factors urgently needs to be revisited and improved
  - The results need to be combined with other hadronic effects, e.g., both resonant and non-resonant pion production.
  - Further, one needs to incorporate nuclear effects, such as two-nucleon currents and final-state interactions

Conclusion and Outlook

- The axial form factor is not so well known, but we have several methods to tackle the problem:
  - We have identified which Q^2 are the most relevant for quasi elastic neutrino nucleus new measurements
    This corresponds to $Q^2 < 2\ \mathrm{GeV}^2$

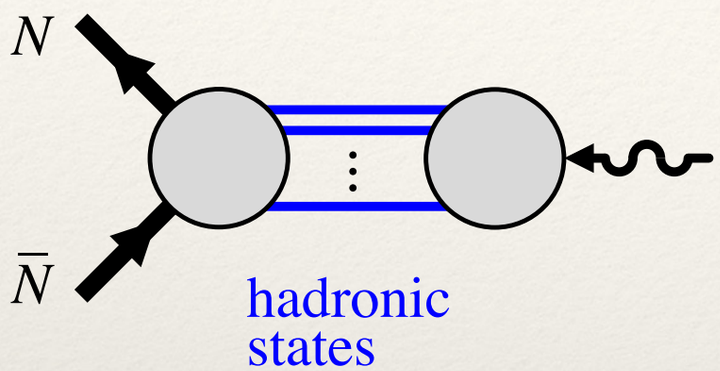
Conclusion and Outlook

- The axial form factor is not so well known, but we have several methods to tackle the problem:
  - At low and high Q^2 we can use ChPT and pQCD
  - The intermediate Q^2 region (~1 GeV^2) is challenging, but LQCD has been making remarkable progress. *See talk by M. Constantinou*
  - z-parametrization is a good method to relax the overly restrictive dipole parametrisation *Bhattacharya, Hill, Paz'11*
  - Analytically: we can use analyticity and dispersive techniques, and take advantage the tau data to learn about the properties of the 3π intermediate state *presently being developed*

Back Up

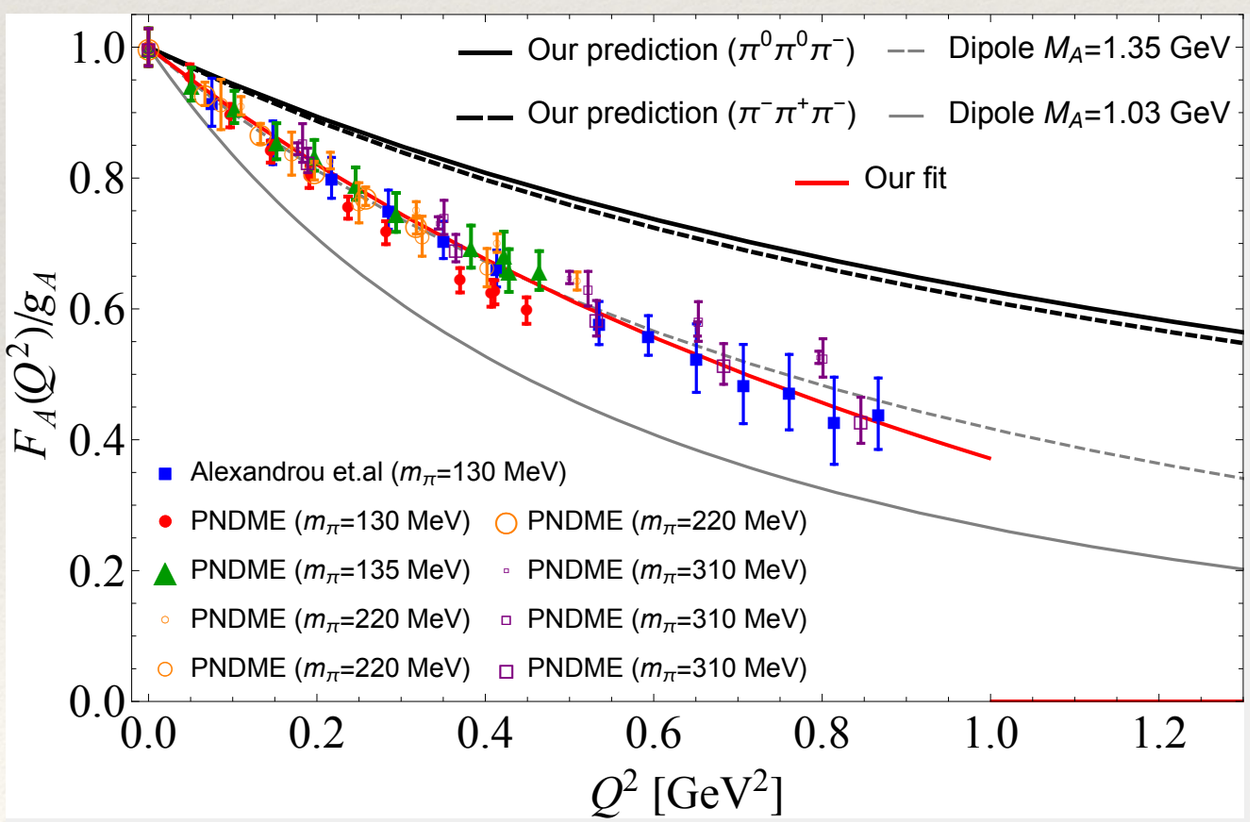
Axial form factor constrained from Tau data and nucleon data

- How to connect to the nucleon?



- Introduce a q^2 dependance

$$F_A(q^2) = f_{A\to 3\pi}(q^2) \cdot \left(1 - \langle r_A^2 \rangle_{a1NN}\right) q^2$$



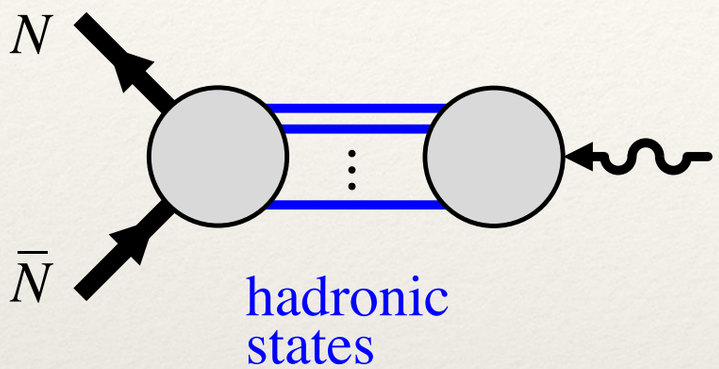
$$\langle r_A^2 \rangle = \langle r_A^2 \rangle_{a1} + \langle r_A^2 \rangle_{a1NN}$$

→ Fit to lattice data

$$\langle r_A^2 \rangle = 0.14(1)[\mathrm{fm}^2] + 0.09(1)[\mathrm{fm}^2]$$
$$= 0.23(1)[\mathrm{fm}^2]$$

Axial form factor constrained from Tau data and nucleon data

- How to connect to the nucleon?



- Introduce a q^2 dependance

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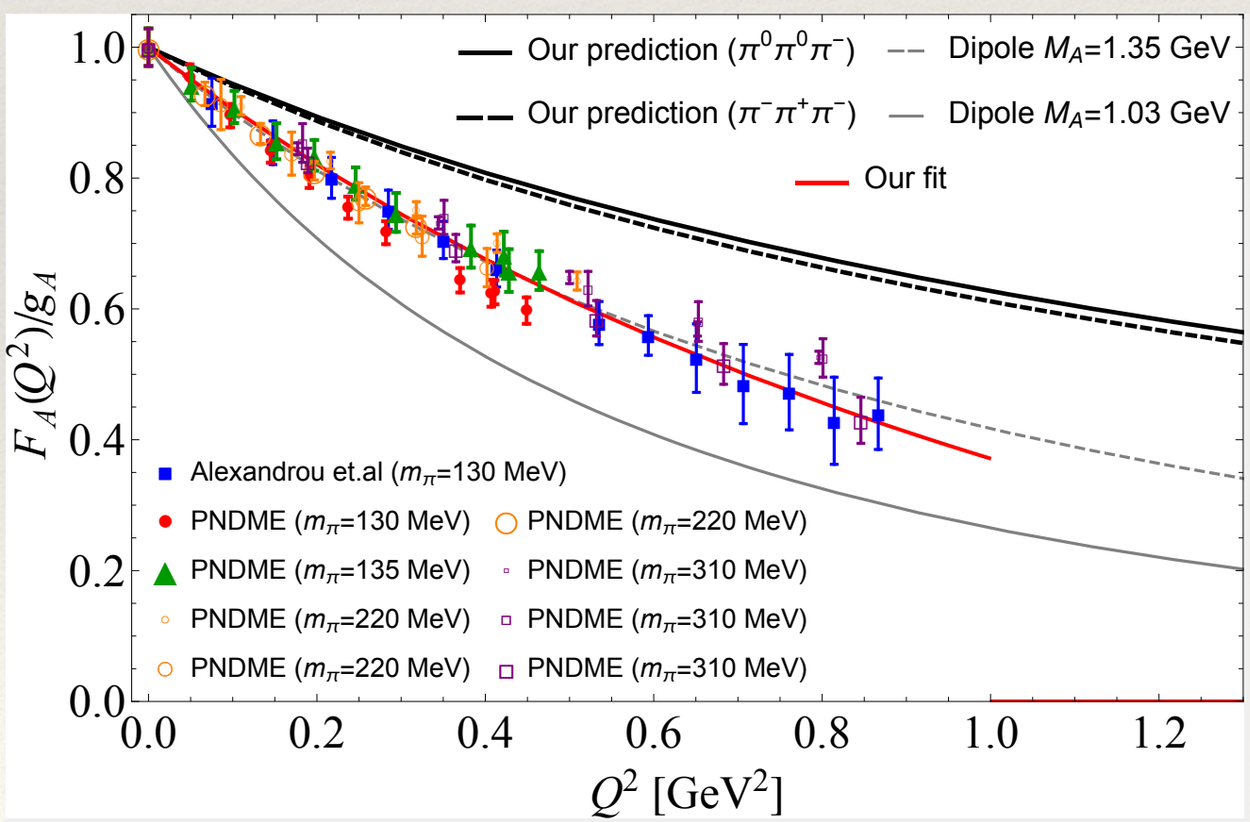
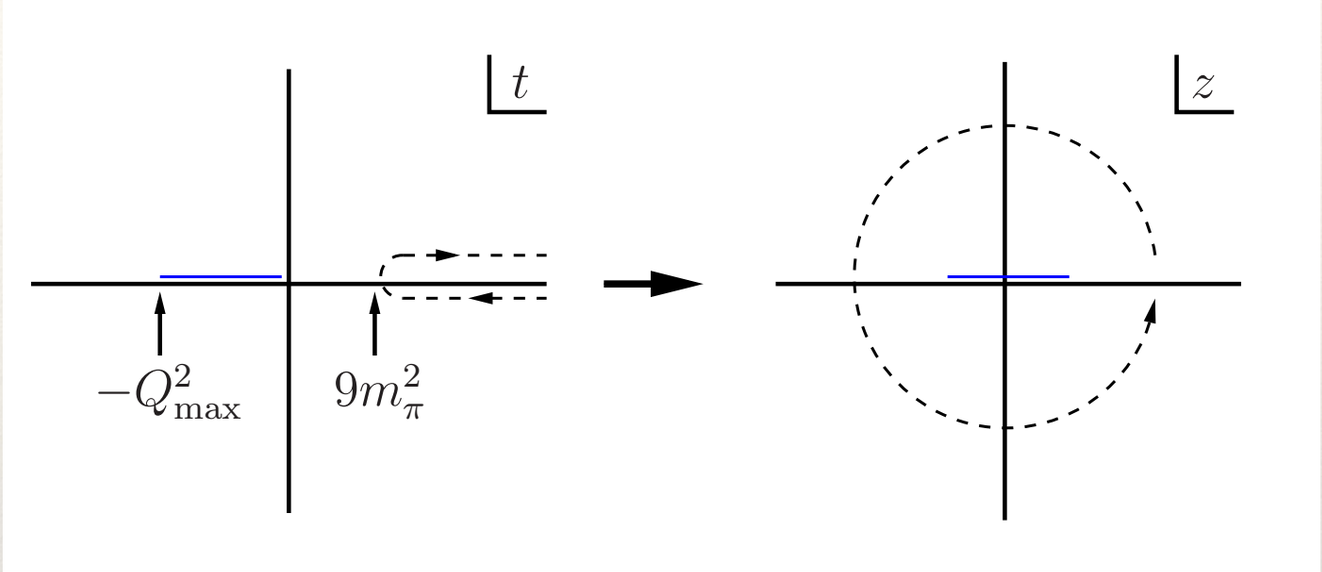


$$F_A(q^2) = g_A\left(1 + \frac{1}{6}\langle r^2\rangle q^2 + \mathcal{O}(q^4)\right)$$

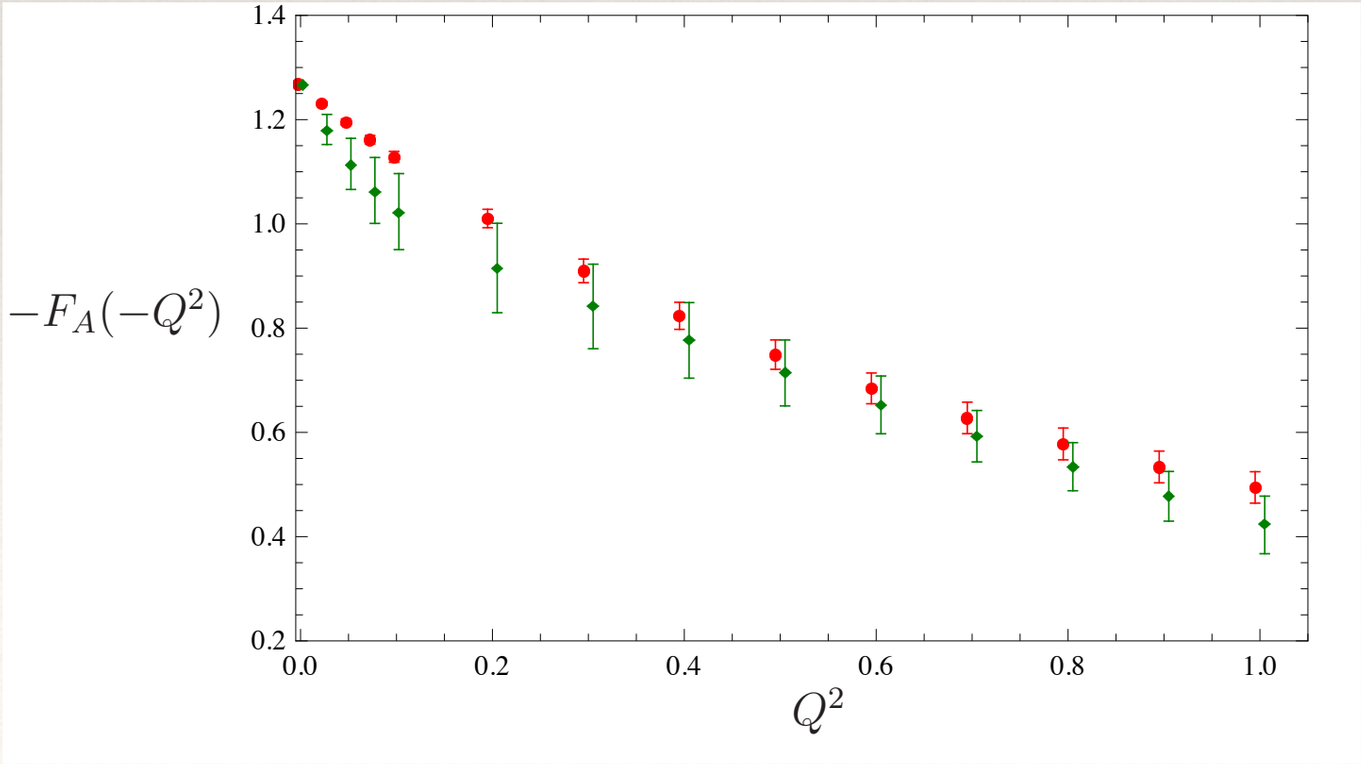
Reference	m_A [GeV]	$\langle r_A^2 \rangle$ [fm^2]
K2K	1.20 ± 0.12	0.32 ± 0.06
NOMAD	1.05 ± 0.06	0.42 ± 0.05
MiniBoonNE	1.35 ± 0.17	0.26 ± 0.06
MINERvA	0.99	0.48
MINOS	$1.23^{+0.13}_{-0.09}$	$0.31^{+0.07}_{-0.05}$
This work (preliminary)	—	0.23(1)

z parametrisation for the FF

$$f_A(q^2) = \sum_{k=0}^{\infty} a_k z(q^2)^k$$



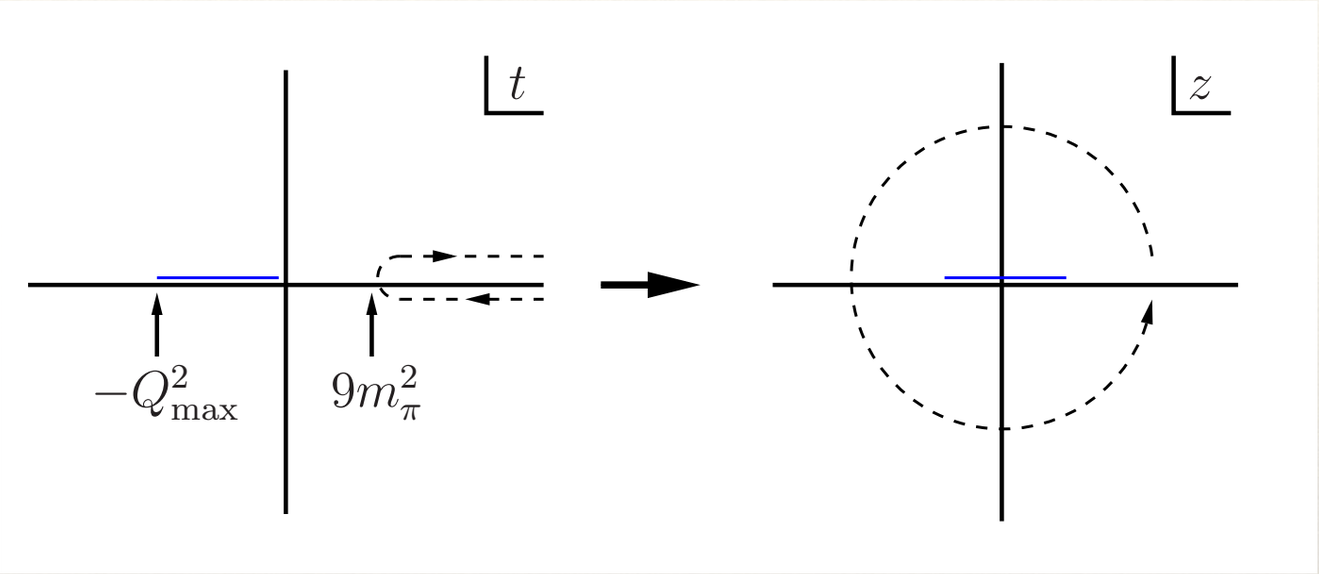
Bhattacharya, Hill and Paz'11



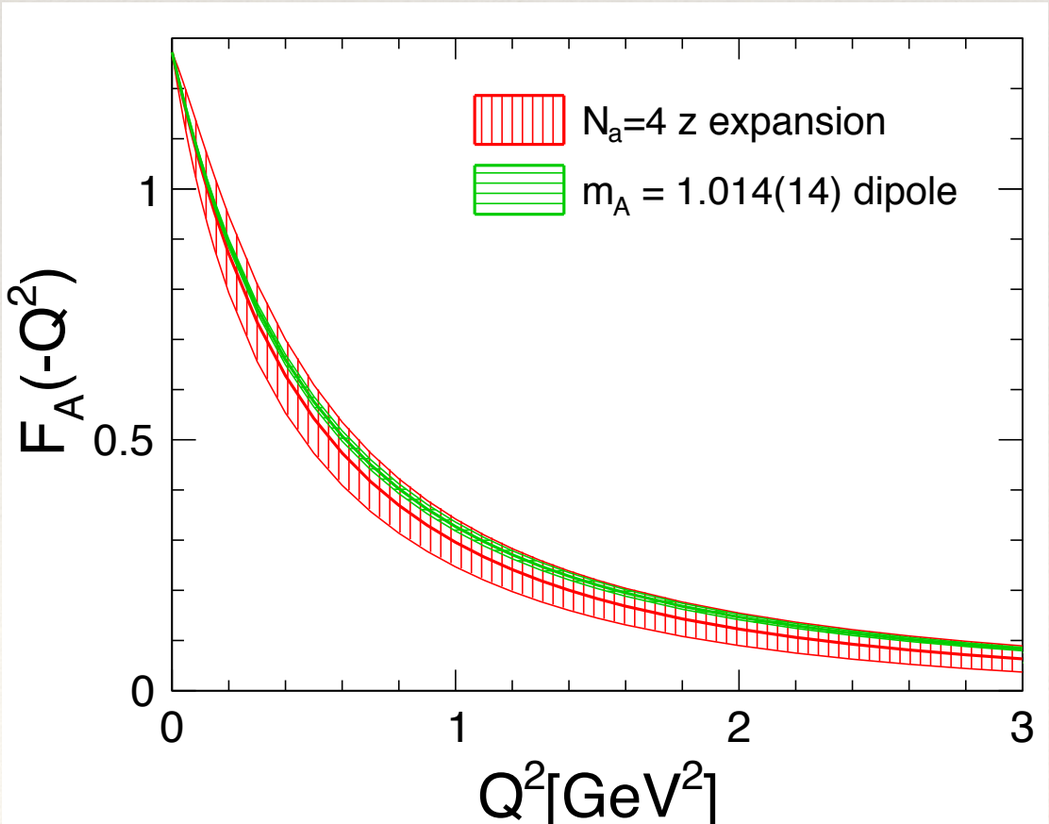
$$z(q^2, s_{\rm th}, q_0) = \frac{\sqrt{s_{\rm th} - q^2} - \sqrt{s_{\rm th} - q_0}}{\sqrt{s_{\rm th} - q^2} + \sqrt{s_{\rm th} - q_0}}$$

z parametrisation for the FF

$$f_A(q^2) = \sum_{k=0}^{\infty} a_k z(q^2)^k \,.$$



Meyer, Betancourt, Gran and Hill'16



$$z(q^2, s_{\rm th}, q_0) \;=\; \frac{\sqrt{s_{\rm th} - q^2} - \sqrt{s_{\rm th} - q_0}}{\sqrt{s_{\rm th} - q^2} + \sqrt{s_{\rm th} - q_0}}$$

$$r_A^2 = 0.46(22)\,{\rm fm}^2$$

Axial and Pseudo-Scalar Form Factors

- π electroproduction using ChPT at threshold

$$e^- p \to e^- n \pi^+$$

S-wave electric dipole amplitude of threshold charged pion electroproduction :

$$E_{0+}^{(-)}\Big|_{m_\pi=0} = \sqrt{1-\frac{q^2}{4m_N^2}}\,\frac{e g_A}{8\pi f_\pi}\left[F_A(q^2)+\frac{q^2}{4m_N^2-2q^2}F_M(q^2)\right]$$

- This low energy theorem is strictly valid in the chiral limit ($m_\pi = 0$)

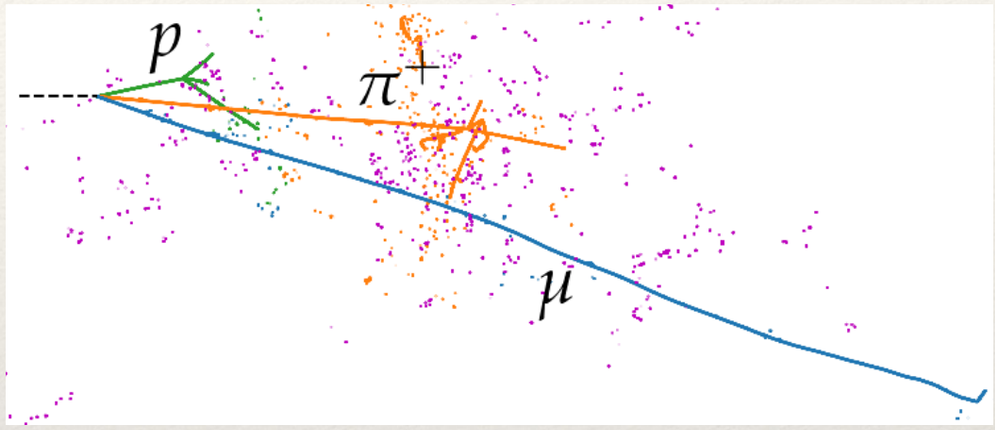
  ➡ Extrapolation using ChPT *Bernard, Kaiser & Meissner'01*

- Problem: most of experimental data outside of ChPT range

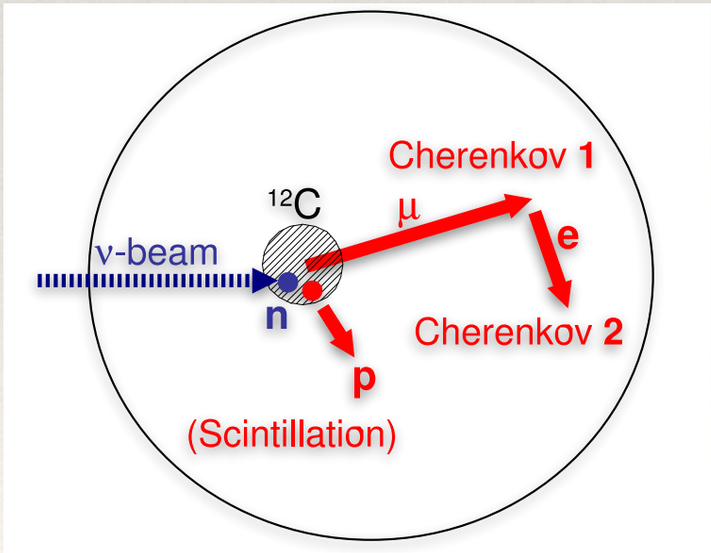
Energy Reconstruction

- To reconstruct E_ν:
  - Measure energy of all final state particles and add them up "Calorimetric method"
  - In a realistic experiment not all energies can be accurately measured see ***S. Li*** and ***U. Mosel***'s talks
  - Cross section model needed to improve energy measurement by filling in the missing information
    - e.g., to go from total ionisation charge in LAr in DUNE or light in NOVA, need to predict its composition

Friedland&Li'19

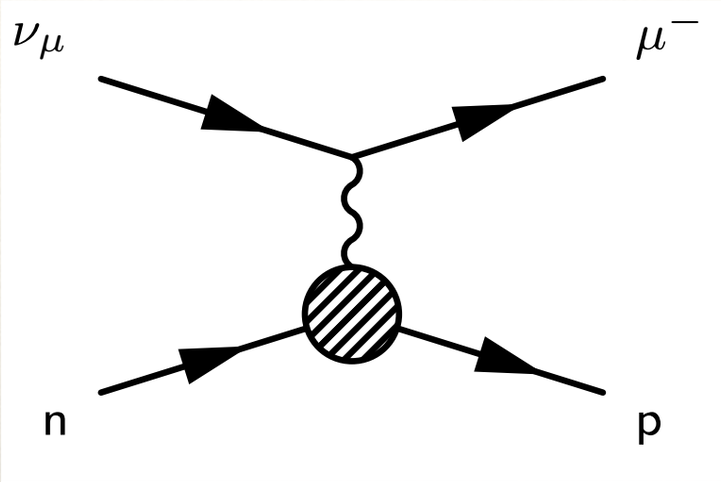


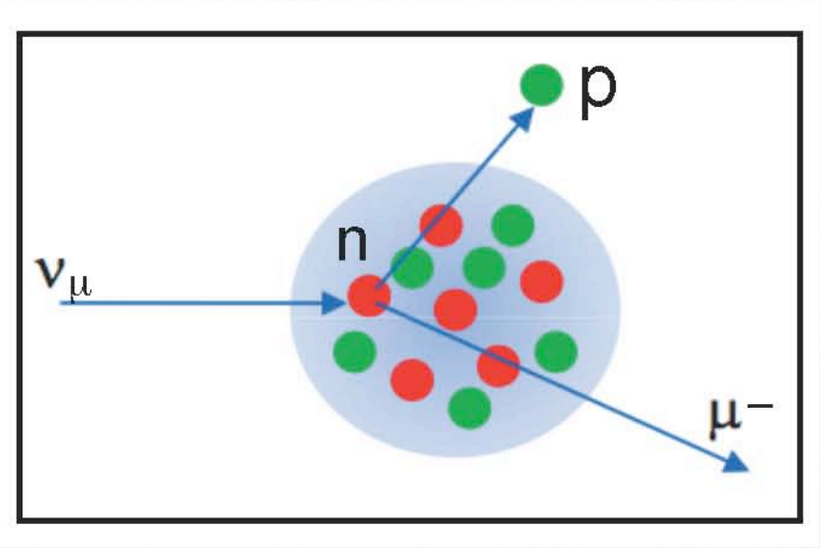
MiniBooNE'10



→ requires ***accurate physical model of interaction***

Quasi-elastic Neutrino Cross Section





P. Vogel

- To compute the neutrino-nucleus cross section in the quasi-elastic regime, 2 parts:
  - From quarks to protons and neutrons → *Form factors*
  - From protons and neutrons to nucleus:
    - Nucleons in nucleus not at rest: at the minimum **Fermi motion** → *Relativistic Fermi Gas* model *Smith&Moniz'72*
    - More elaborated models include spectral functions and nucleon-nucleon interactions

Neutrino Cross Section

$$N_\nu(E) \sim \Phi_\nu(E) \times \sigma_\nu(E) \times \text{target}$$

ν flux

(# neutrinos)

depends on your ν source

$$\Phi_\nu(E) = \Phi_{\nu\text{unosc}}(E) \times P(\nu_\mu \to \nu_e)$$

ν cross section

tiny (~10^{-38} cm^2)

$\sigma_\nu^{tot} \sim E_\nu$

at 1 GeV $\sigma(\nu N) \sim 10^{-38}$ cm^2,
compare to $\sigma(pp) \sim 10^{-26}$ cm^2

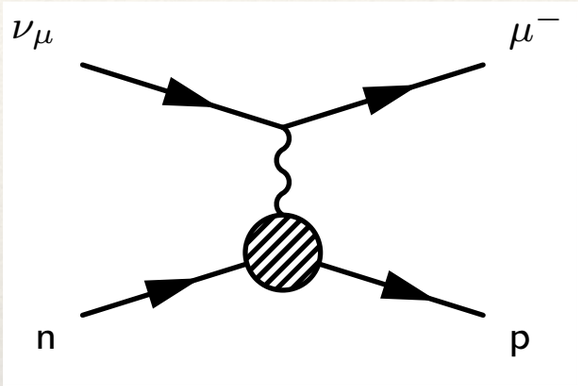
$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

- To determine the oscillation probability $P(\nu_\mu \to \nu_e)$

→ Need to know the ***Cross Section*** very precisely

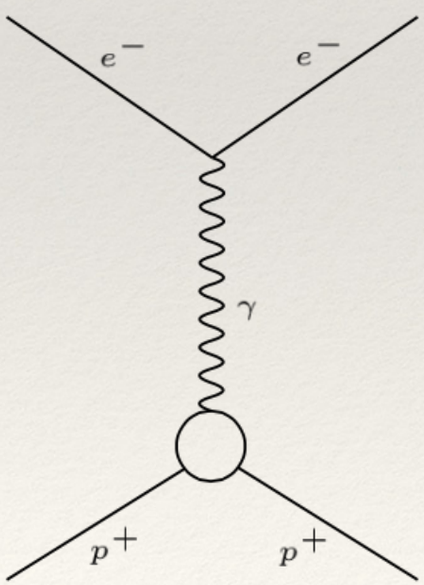
Electromagnetic Form Factors

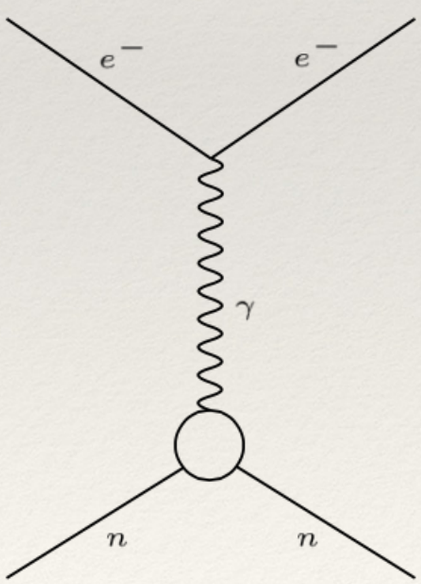
$$\langle p(p')|J_W^{+\mu}|n(p)\rangle \propto \bar{u}^p(p')\left\{\gamma^\mu F_1^V(q^2)+\frac{i}{2m_N}\sigma^{\mu\nu}q_\nu F_2^V(q^2)+\gamma^\mu\gamma_5 F_A(q^2)+\frac{1}{m_N}q^\mu\gamma_5 F_P(q^2)\right\}u^{(n)}(p)$$



- 4 form factors to determine
  - $F_1^V(q^2)$ and $F_2^V(q^2)$:

    ➡ Can be related using CVC to EM processes

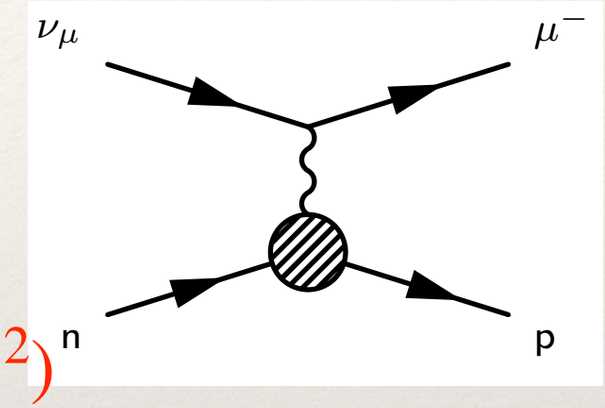




$$F_{1,2}^V(q^2) = F_{1,2}^p(q^2) - F_{1,2}^n(q^2)$$

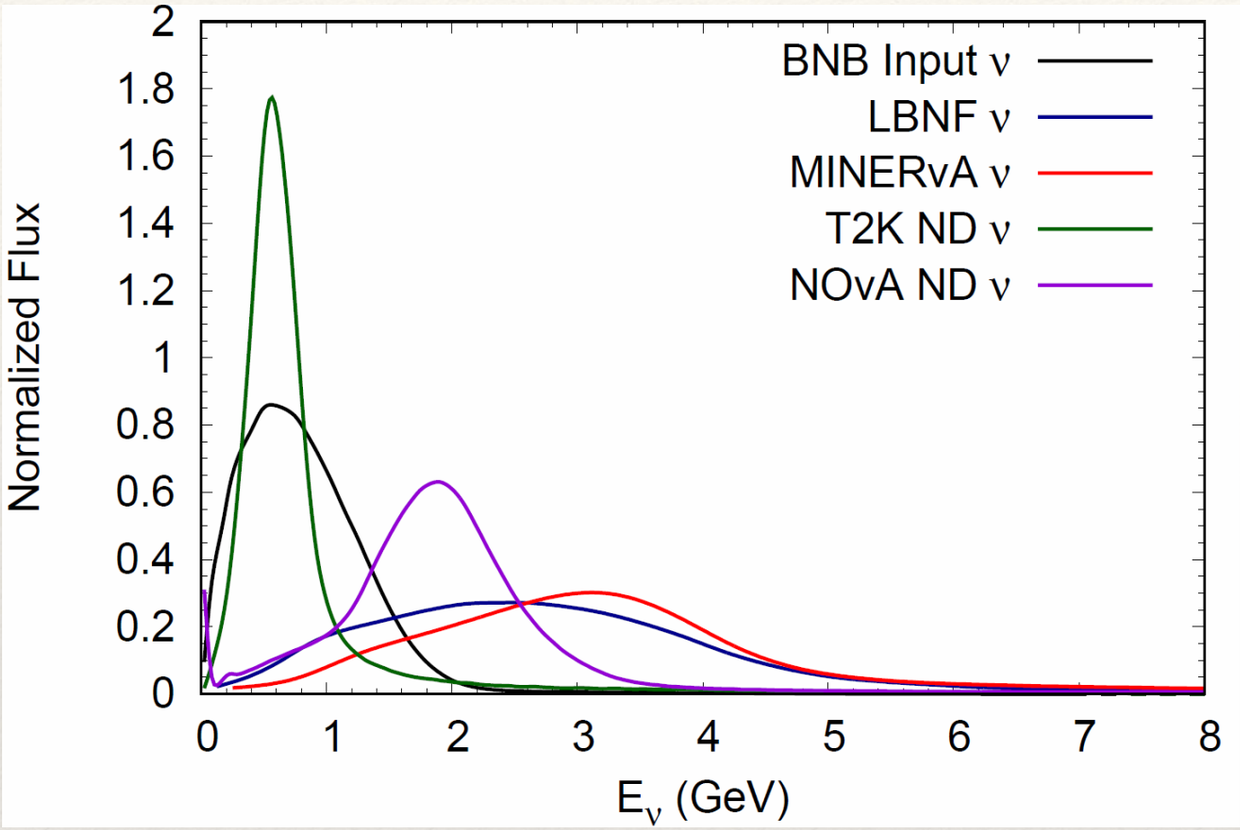
Axial and Pseudo-Scalar Form Factors

$$\langle p(p')|J_W^{+\mu}|n(p)\rangle \propto \bar{u}^p(p')\left\{\gamma^\mu F_1^V(q^2) + \frac{i}{2m_N}\sigma^{\mu\nu}q_\nu F_2^V(q^2) + \gamma^\mu\gamma_5 F_A(q^2) + \frac{1}{m_N}q^\mu\gamma_5 F_P(q^2)\right\}u^{(n)}(p)$$



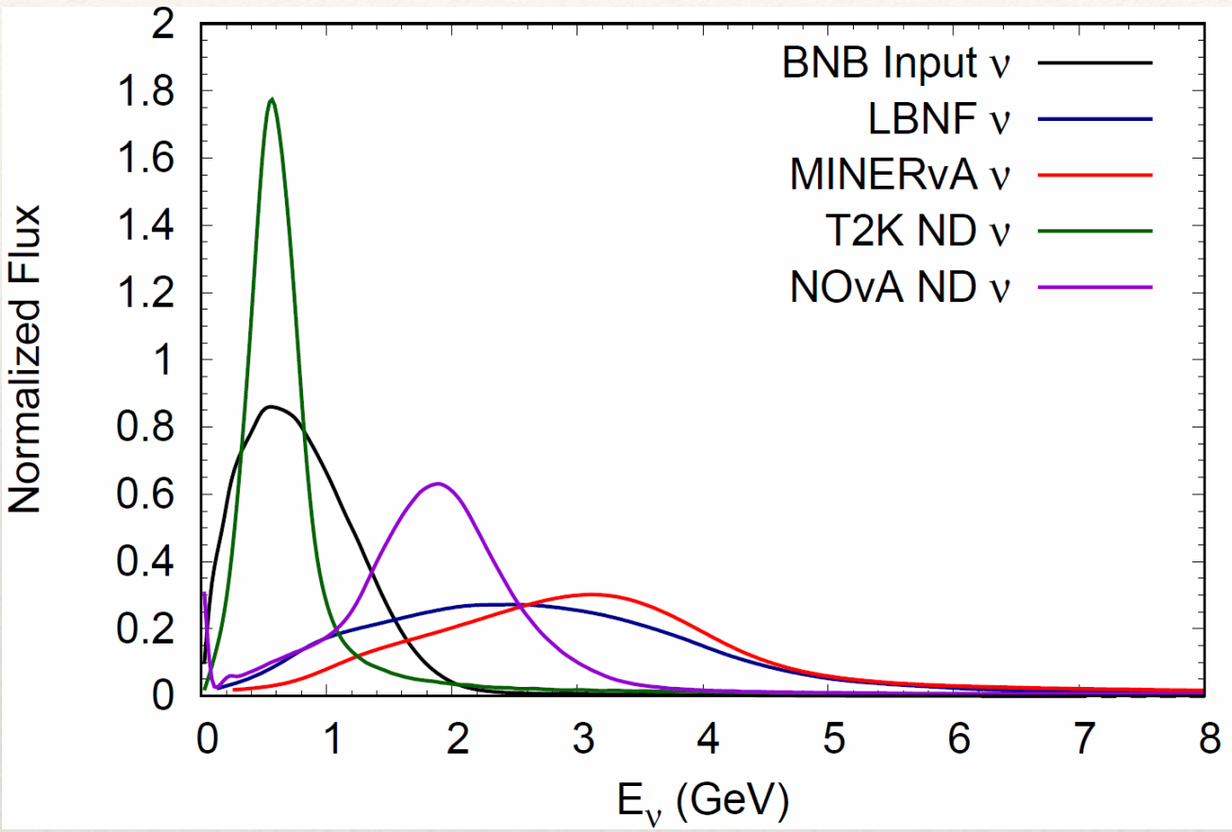
- 4 form factors to determine
  - $F_A(q^2)$: Axial Form Factor
  - $F_P(q^2)$: Pion pole approx.: $F_P(q^2) = \dfrac{2m_N^2}{m_\pi^2 - q^2}F_A(q^2)$
- How to determine $F_A(q^2)$ experimentally?
  - νN scatt.: the best way but old data from 80s: *BNL'81, ANL'82, FNAL'83* ***Reanalysis by*** *Meyer et al'16*
  - π electroproduction using ChPT at threshold
    Also old extraction ***model dependent*** *Bernard, Kaiser & Meissner'01*
  - μ capture: allow to determine $F_P(q^2)$ *Hill, Kammel, Marciano & Sirlin'18*

Energy distribution of neutrino beams

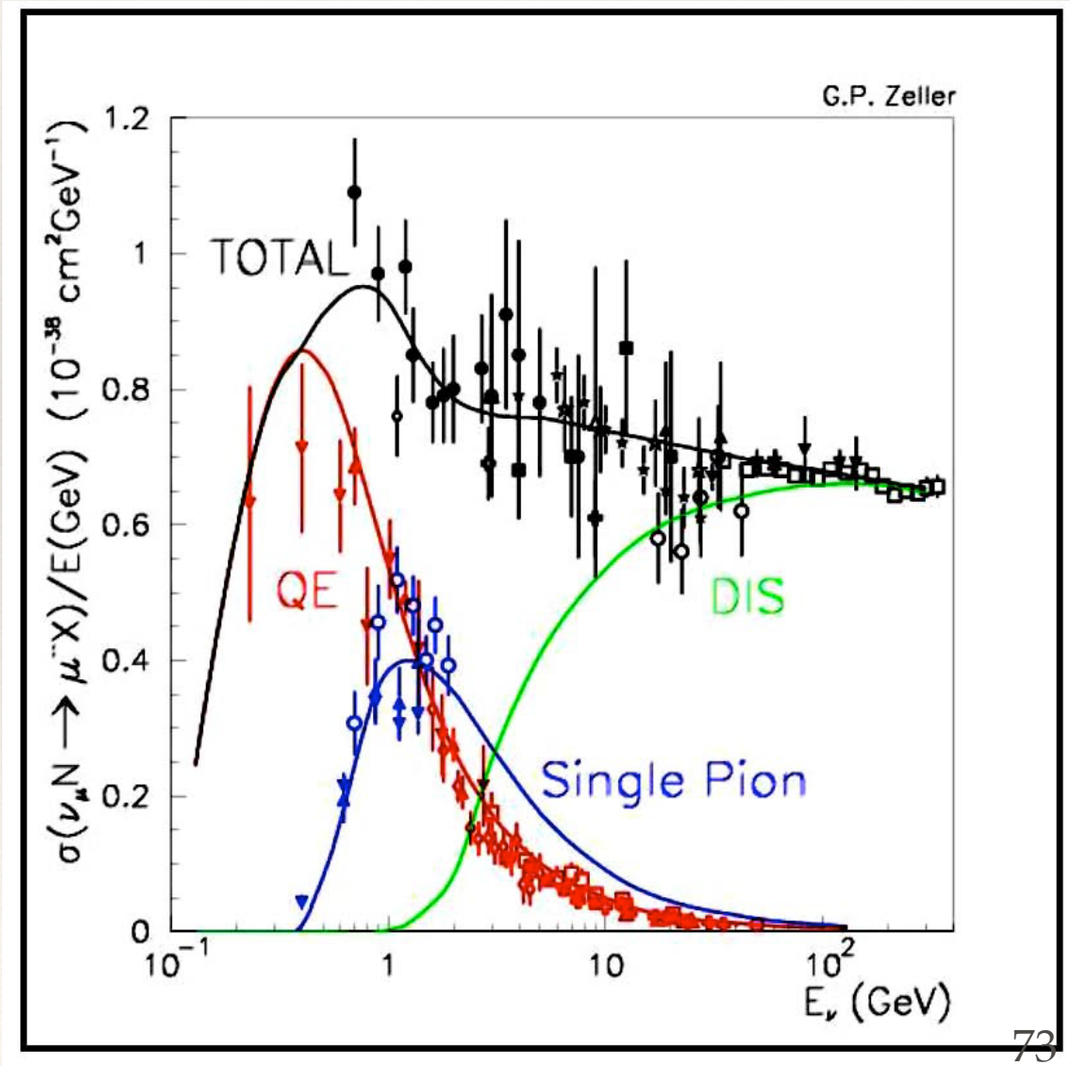


- Eν reconstruction from final state

Energy distribution of neutrino beams



- Eν reconstruction from final state



- Different Energy regions:
  - Quasi-Elastic + FSI
  - Resonance-pion production
  - Deep Inelastic Scattering

What do we know on the Form Factors?

- For intermediate energy region: Can try to use ***VMD***
  - Approximate spectral function with 2 narrow resonances

$$\text{i.e. } \frac{1}{\pi}\,\mathrm{Im}\,F_1^V(t') \simeq a_\rho\,\delta(t'-m_\rho^2) + a_{\rho'}\,\delta(t'-m_{\rho'}^2)$$

$$\Downarrow$$

$$F_1^V(q^2) \simeq \frac{a_\rho}{m_\rho^2 - q^2} + \frac{a_{\rho'}}{m_{\rho'}^2 - q^2}$$

and impose

$$F_1^V(0) = 1$$

$$\Downarrow$$

$$\left(\frac{a_\rho}{m_\rho^2} + \frac{a_{\rho'}}{m_{\rho'}^2}\right) = 1.$$

  - Has approximate dipole behaviour:

$$\text{IF} \quad a_{\rho'} \simeq -a_\rho \simeq -\frac{m_\rho^2\, m_{\rho'}^2}{(m_{\rho'}^2 - m_\rho^2)}$$

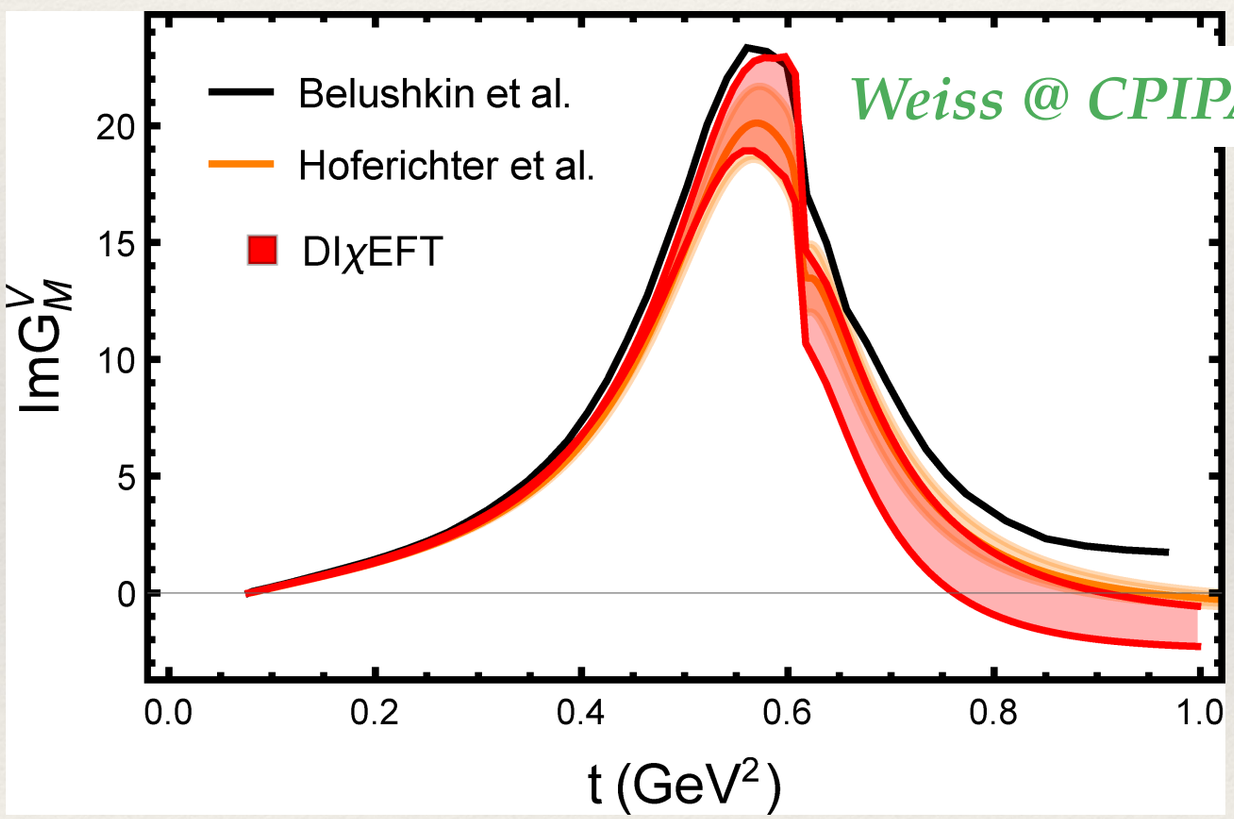
$$F_1^V(q^2) \simeq \frac{a_\rho\,(m_{\rho'}^2 - m_\rho^2)}{(m_\rho^2 - q^2)(m_{\rho'}^2 - q^2)} \simeq \frac{1}{\left(1 - \frac{q^2}{m_\rho^2}\right)\left(1 - \frac{q^2}{m_{\rho'}^2}\right)}$$

Arise from residues of nearby poles with equal magnitude and ***opposite signs***

What do we know on the Form Factors?

- Better treatment of the spectral function

→ *Dispersive parametrizations*



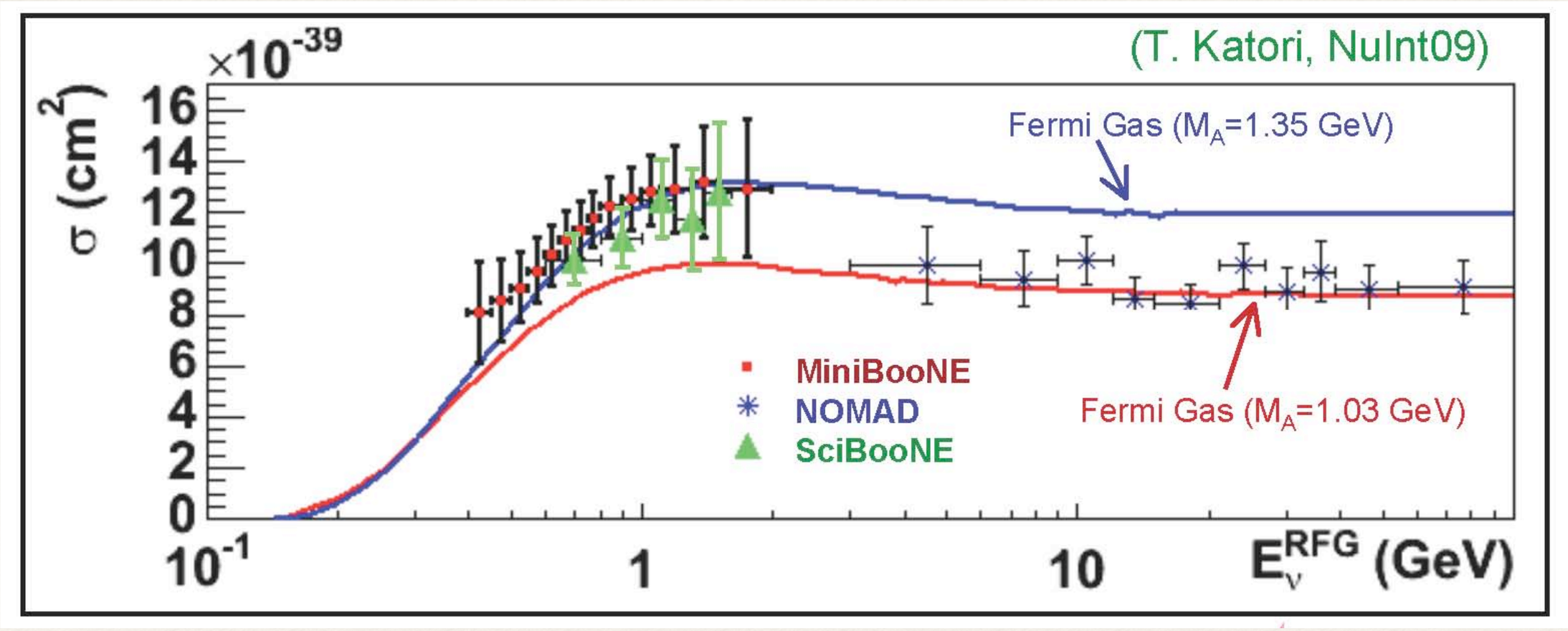
Weiss @ CPIPANP18

Belushkin, Hammer, Meissner'06
Lorenz, Hammer, Meissner'12
Hoferichter et al'16
Leupold'17
Alarcon & Weiss'18
...

$$G_M^V(t = q^2) \equiv F_1(q^2) + F_2(q^2)$$

Dipole Parametrization

- If we could expect such a parametrisation to work at very low energy or High Energy, there is no reason for it to work at ~1 GeV where MiniBooNE took data

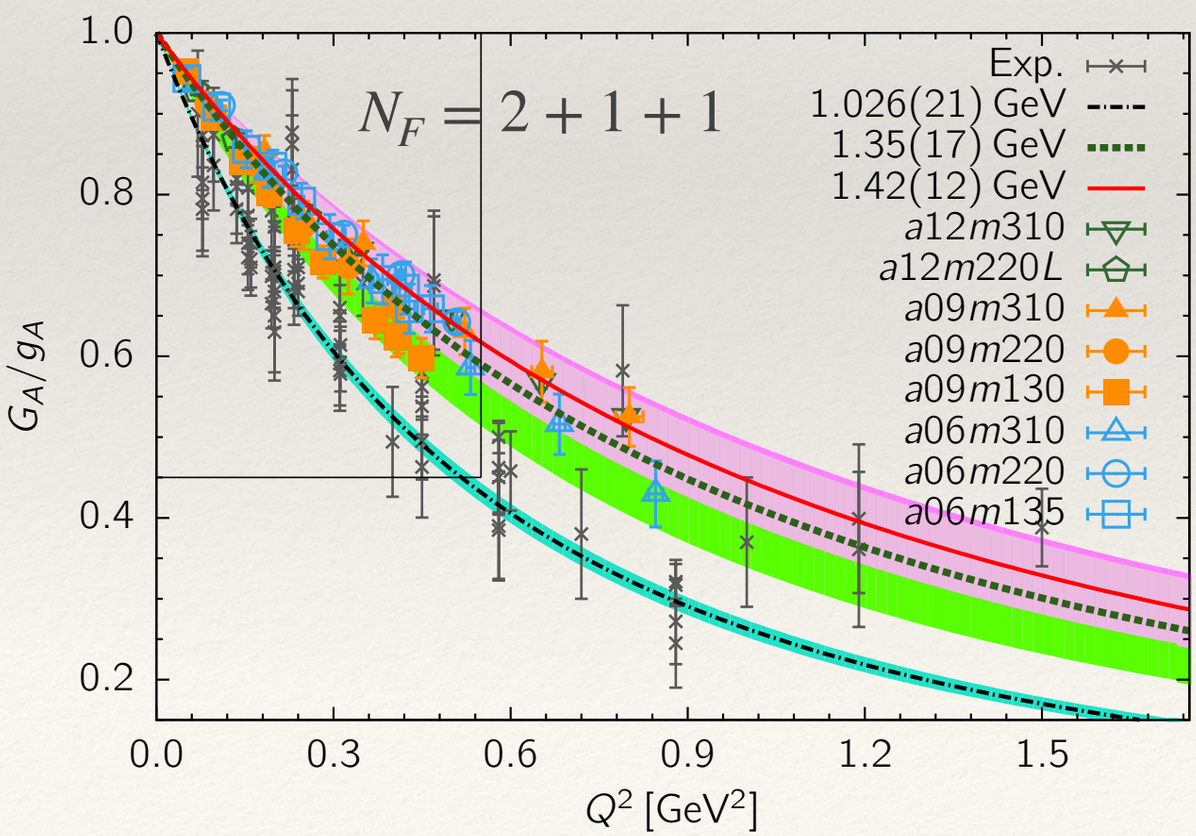


- 30% discrepancy between MiniBooNE and NOMAD

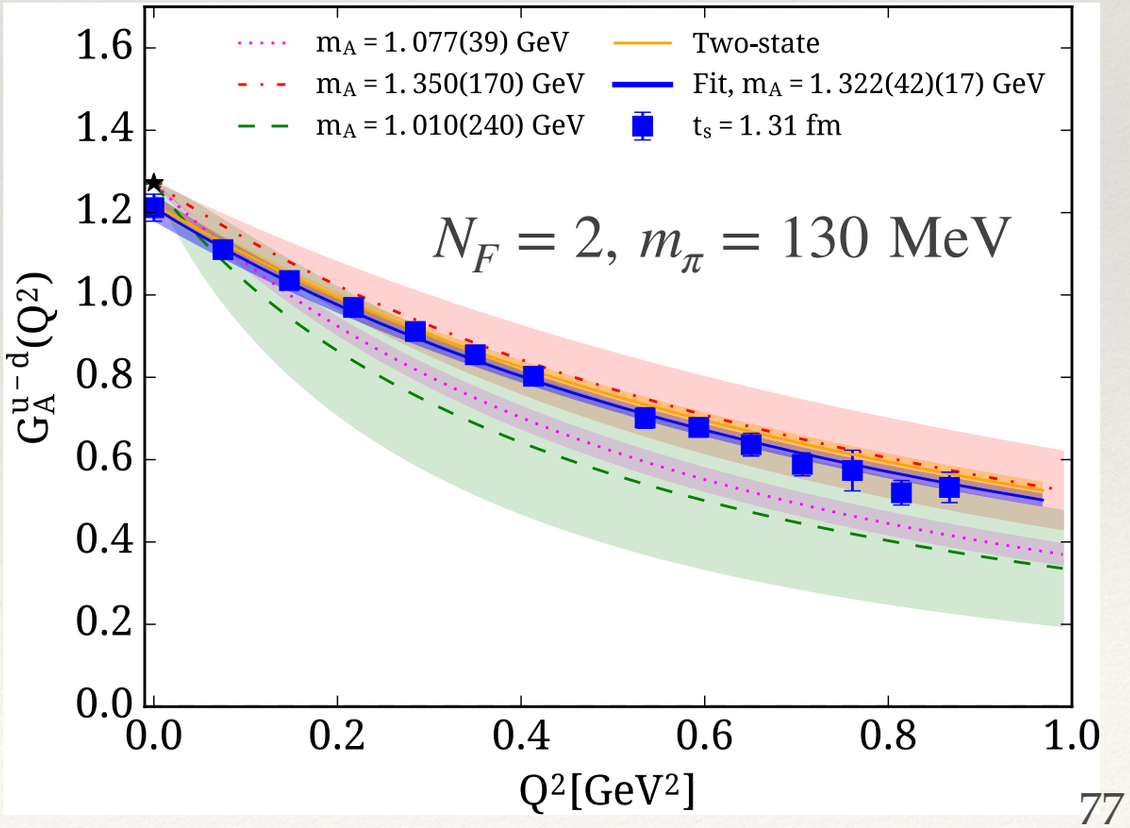
Axial Form Factor puzzle

- A priori not obvious how much of the 30% discrepancy in the MiniBooNE cross section due to axial FF versus nuclear effects.
- However two reasons to consider the axial form factor seriously *supported by recent lattice results, see also *Capitani et al'17*

Gupta et al. PNDME collab.'17



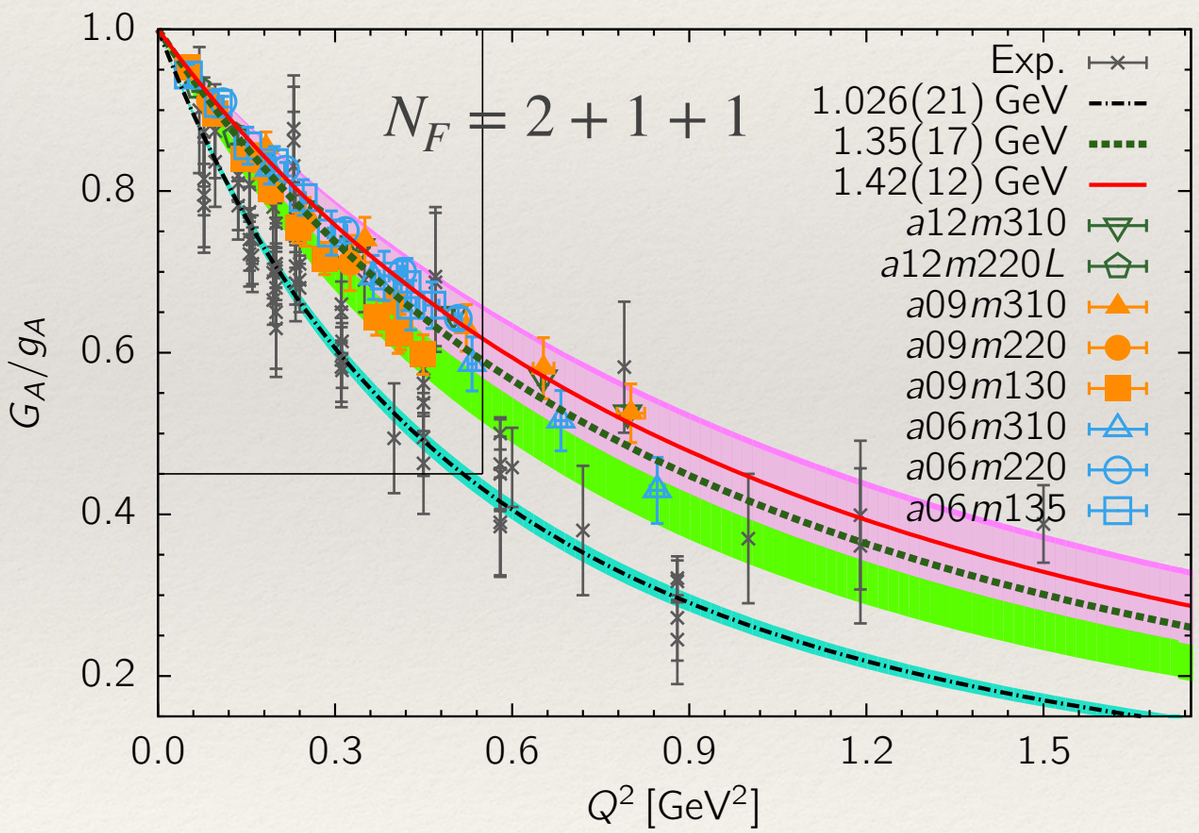
Alexandrou et al.'17



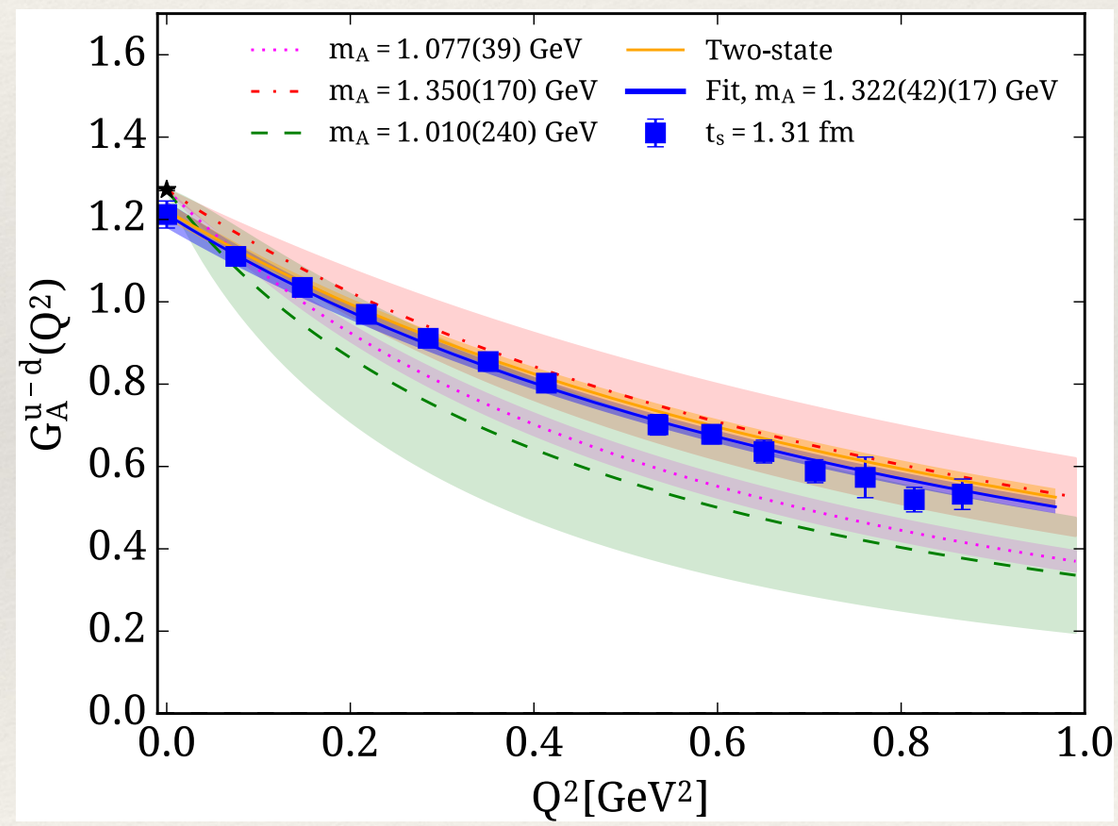
What is know on the Axial Form Factor?

- Recently very significant progress on two fronts:
  - Lattice QCD results on $g_A \equiv F_A(0)$ and $F_A(q^2)$

Gupta et al., PNDME collab.'17



Alexandrou et al., ETMC'17



see talk by M. Constantinou