47th SLAC SUMMER INSTITUTE

The leptonic g-2 puzzle

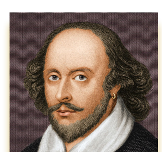
Team 14

Arushi Bodas[a], Rupert Coy[b], Elisabeth Maria Niel[c], Felix Kress[d], Peilong Wang[e], Jennifer Rittenhouse West[f]

(a) University of Maryland, College Park, USA
(b) CNRS Laboratoire Charles Coulomb, Montpellier, France
(c) CNRS, Laboratoire de l'Accélérateur Linéaire, Orsay, France
(d) Imperial College London, London, UK
(e) Southern Methodist University, USA
(f) SLAC National Accelerator Laboratory, USA

The question

The g-2 of both the electron and muon appear to differ from the predictions of the SM but with opposite signs. This is indicative of the possibility of some unusual new physics, since most new physics models would suggest a same-sign shift. Construct a model that leads to such predictions and examine its corresponding predictions for the tau lepton whose value of g-2 is presently quite poorly known. What can current or planned future facilities do to improve our knowledge of the tau g-2 ?



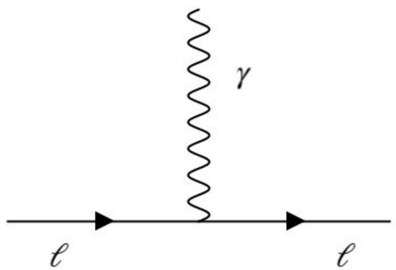
Outline

1. Intro to g-2
2. Our model
3. Predictions and Experiments for Tau measurement

Anomalous Magnetic Moment

Leptons have an intrinsic magnetic moment defined as: $$\vec{\mu} = g\frac{e}{2m}\vec{S}$$

Dirac equation predicts g = 2, but loop diagrams give a small (“anomalous”) correction to g



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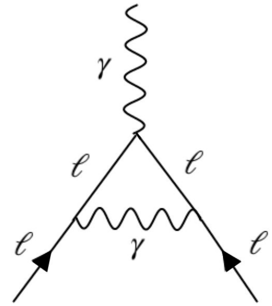

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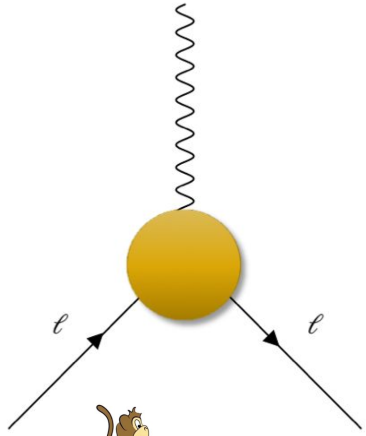
Californian loop diagrams

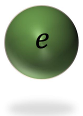


Anomalous magnetic moment :

$$a = \frac{g-2}{2} \neq 0$$

SM predictions vs measurements

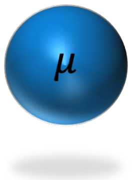
Electron: Hanneke et al. (0801.1134)



$$\Delta a_e = a_e^{exp} - a_e^{SM} = (-8.7 \pm 3.6) \times 10^{-13}$$

2.4 σ tension

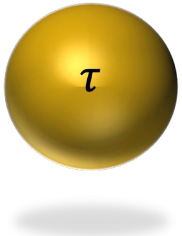
Muon: Brookhaven g-2 exp (0401008)



$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (2.74 \pm 0.73) \times 10^{-9}$$

3.6 σ tension

Tau:



$$-0.052 < a_\tau < 0.013$$

DELPHI, LEP2 (0406010)

$$a_\tau = 0.00117721(5)$$

No tension (yet!)

Challenges in making a new model

1. Opposite sign for *e* and μ anomalies

$$\Delta a_e = (-8.7 \pm 3.6) \times 10^{-13} \qquad \Delta a_\mu = (2.74 \pm 0.73) \times 10^{-9}$$

2. If BSM physics respects lepton flavour universality (e.g. 1402.7065):

$$\text{Expected}: \frac{\Delta a_e}{\Delta a_\mu} = \frac{m_e^2}{m_\mu^2} \sim 10^{-5} \quad \text{Observed}: \frac{\Delta a_e}{\Delta a_\mu} \approx 3 \times 10^{-4}$$

Possible solutions

- Dominant contribution to *e* and μ anomalies from different BSM processes (new scalar: one-loop and two-loop Barr-Zee diagrams, *1808.10252*)
- Lepton flavour violation (*1811.10209, 1908.00008*)
- BSM does not respect lepton flavour universality (scalar LQ models with specific couplings: 1807.11484,1810.11028, 1812.06851, 1906.01870)

3. Lepton flavour violation highly constrained from LEP $\mu \to e\gamma$ process.
So we will focus on a model with lepton universality violation

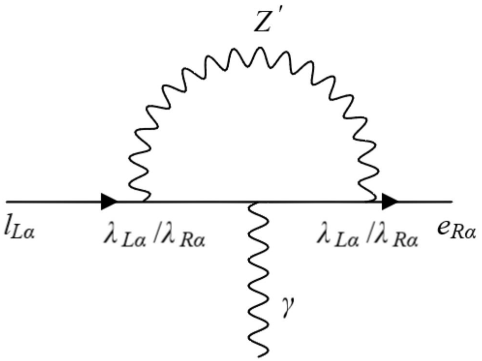
The LFUV Z' model

Introduce new gauge **U(1)'** mediated by **MeV-scale Z'**

Other ingredients:

➢ Different lepton flavours should have different charges (LFUV)

➢ If LH and RH couplings to Z' are equal, contribution to g-2 is always positive. So need chiral couplings!



Consequence:
Higgs is charged under the U(1)' to make Yukawa interactions invariant

Intriguingly, lots of other anomalies have possible light Z' solutions, e.g. ^{8}Be anomaly (1905.05031), reactor anomaly (1803.08506), proton radius puzzle (1301.0905), cosmic neutrino spectrum (1409.4180)

The model in detail

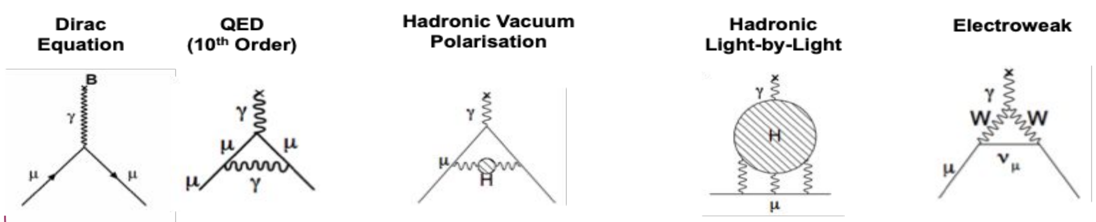
Z’ is the gauge boson of the new symmetry U’(1). The covariant derivative of e.g. the lepton doublet changes to:

$$D_\mu \to D_\mu^{SM} + ig'\lambda_{L\alpha} Z'_\mu$$

The lepton kinetic terms gives the interaction vertices,

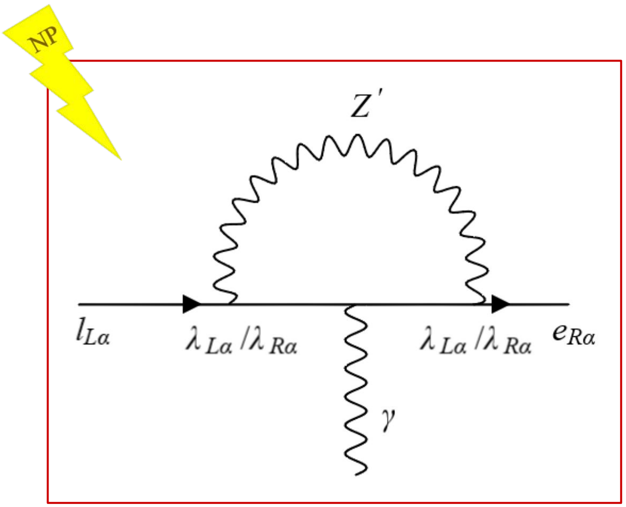
$$\mathcal{L} \supset \sum_\alpha g' \left(\lambda_{L\alpha} \bar{l}_{L\alpha} \gamma^\mu Z'_\mu l_{L\alpha} + \lambda_{R\alpha} \bar{e}_{R\alpha} \gamma^\mu Z'_\mu e_{R\alpha} \right)$$

New diagram contributing to g-2:



SM

+



Anomaly cancellation

- ➢ Gauge symmetry requires anomaly cancellation:
  $U(1)'\,SU(3)^2$, $U(1)'\,SU(2)^2$, $U(1)'\,U(1)^2$, $U(1)'^2\,U(1)$, $U(1)'^3$ and $U(1)'\text{grav}^2$

Need to **introduce new fermions** to cancel these anomalies

- ➢ Economical case involves a vector-like quark, vector-like SU(2) doublet, and U(1)'-charged SM singlet
  - This singlet makes $U(1)'^3$ and $U(1)'\text{grav}^2$ anomaly-free simultaneously
  - The new fermions charged under the SM need to be **heavy** to avoid collider bounds
- ➢ New SU(4) gauge group **confines** at a scale >> v, giving dynamical mass to the exotic fermions

Representations

NEW

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)'$	$SU(4)$
H	$\mathbf{1}$	$\mathbf{2}$	$1/2$	1	$\mathbf{1}$
l_{Le}	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	λ_{Le}	$\mathbf{1}$
$l_{L\mu}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$\lambda_{L\mu}$	$\mathbf{1}$
$l_{L\tau}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$4-\lambda_{Le}-\lambda_{L\mu}$	$\mathbf{1}$
$e_{R\alpha}$	$\mathbf{1}$	$\mathbf{1}$	-1	$\lambda_{L\alpha}-1$	$\mathbf{1}$
q_L	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$4/3$	$\mathbf{1}$
u_R	$\mathbf{3}$	$\mathbf{1}$	$2/3$	$1/3$	$\mathbf{1}$
d_R	$\mathbf{3}$	$\mathbf{1}$	$-1/3$	$1/3$	$\mathbf{1}$
ψ_L	$\mathbf{3}$	$\mathbf{1}$	0	$-1/2$	$\mathbf{4}$
ψ_R	$\mathbf{3}$	$\mathbf{1}$	0	0	$\mathbf{4}$
ψ'_L	$\mathbf{1}$	$\mathbf{1}$	0	$3/2$	$\mathbf{4}$
ψ'_R	$\mathbf{1}$	$\mathbf{1}$	0	0	$\mathbf{4}$
χ_L	$\mathbf{1}$	$\mathbf{2}$	0	-4	$\mathbf{4}$
χ_R	$\mathbf{1}$	$\mathbf{2}$	0	0	$\mathbf{4}$
χ'_L	$\mathbf{1}$	$\mathbf{2}$	0	8	$\mathbf{4}$
χ'_R	$\mathbf{1}$	$\mathbf{2}$	0	0	$\mathbf{4}$
f_L	$\mathbf{1}$	$\mathbf{1}$	0	13	$\mathbf{1}$
f_R	$\mathbf{1}$	$\mathbf{1}$	0	0	$\mathbf{1}$

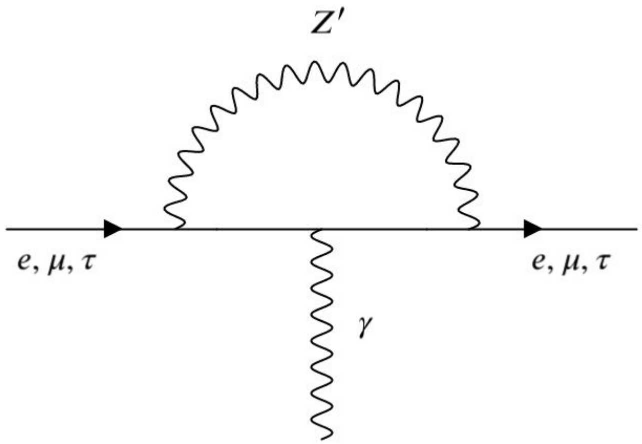
The LFUV Z' model

- Diagram gives for the electrons

$$\Delta a_e = \frac{g'^2(\lambda_{Le}^2 - \lambda_{Le} - 1)m_e^2}{12\pi^2 m_{Z'}^2}$$

which is negative for

$$\frac{1}{2}\left(1-\sqrt{5}\right) < \lambda_{Le} < \frac{1}{2}\left(1+\sqrt{5}\right)$$



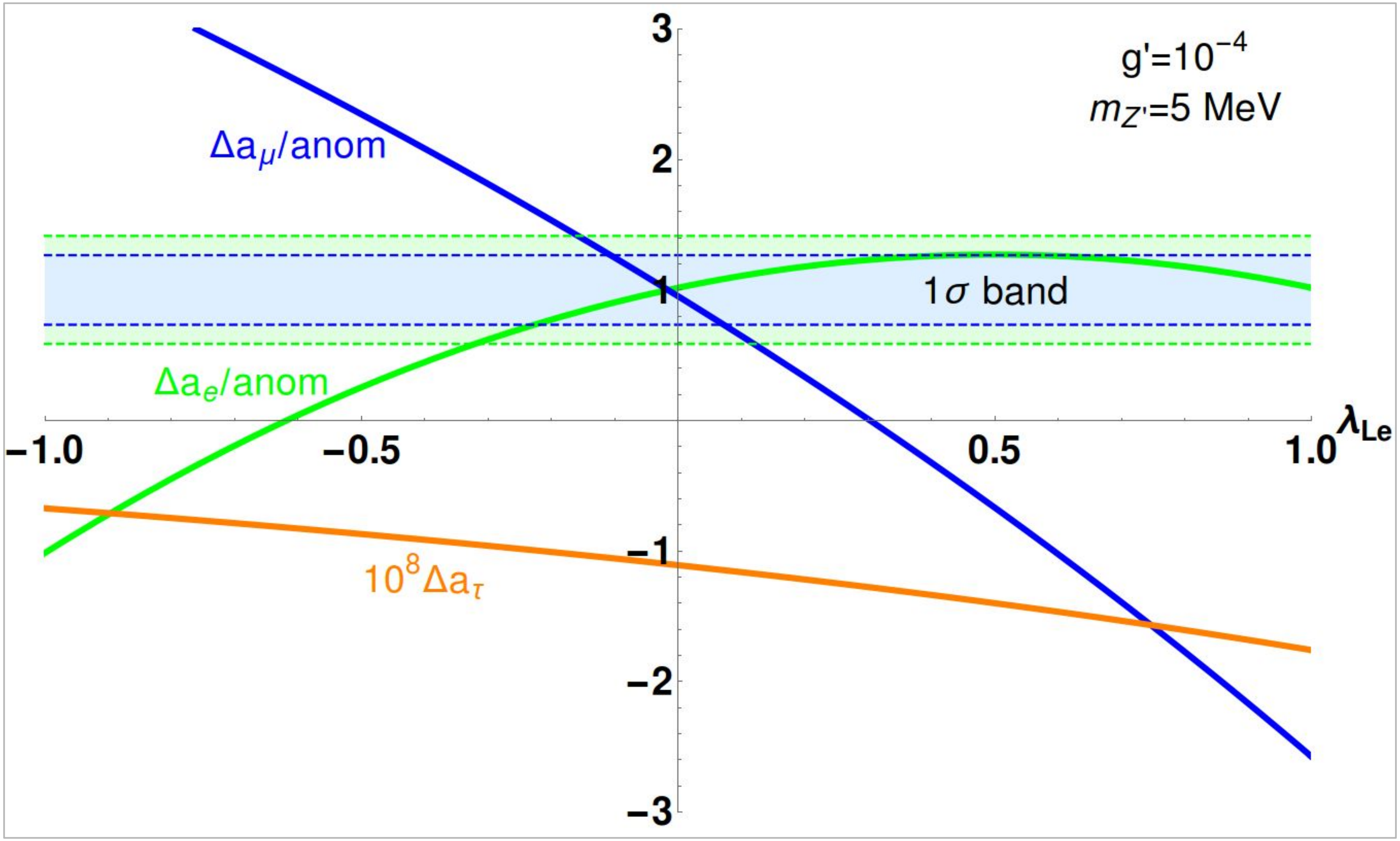
- And for muons, we found

$$\Delta a_\mu = \frac{g'^2 m_\mu^2}{8\pi^2}\int_0^1 dx\, dy\, dz\, \delta(x+y+z-1)\frac{z\left[1+z-2\lambda_{L\mu}(z-1)+2\lambda_{L\mu}^2(z-1)\right]}{(1-z)^2 m_\mu^2 - xy m_{Z'}^2}$$

Charges of the lepton flavours are related by

$$\boxed{\lambda_{L\mu,\tau} = \frac{-24+10\lambda_{Le}-\lambda_{Le}^2 \pm \sqrt{47568-7736\lambda_{Le}-92\lambda_{Le}^2+4\lambda_{Le}^3+\lambda_{Le}^4}}{2\lambda_{Le}-12}}$$

Results



To fit both anomalies exactly: $\lambda_{Le} = -0.016$ which leads to $\boxed{\Delta a_\tau = -1.1 \times 10^{-8}}$

Constraints on the model

Experimental constraints:

- Various LHC searches, e.g. vector-like quarks, heavy charged leptons
- Neutrino Trident, which probes $\nu_\mu + N \to \nu_\mu \mu^+ \mu^- + N$
- Will induce tiny shifts in most low-energy processes $\propto g'^2 \ll 1 \text{ or } g'^4 \lll 1$

In our model:

➢ Z' field has a Stueckelberg mass to avoid BBN bounds

➢ All the extra fermionic fields are made heavy to avoid collider bounds.

tau g-2 measurement on e+e- collider

Our model predicts that a_τ is **smaller** than Standard Model at the order of **10^{-8}**.

The SM prediction is a_τ is 0.00117721(5), so our correction is small.

On e+e- collider, the previous measurement is made by using ee -> $\tau\tau\gamma$ channel to measure the form factor.

Generalised form: $$-ie\bar{u}(p')\{F_1(q^2)\gamma^\mu + i\boxed{F_2(q^2)}\sigma^{\mu\nu}\frac{q_\nu}{2m_\tau} + F_3(q^2)\gamma^5\sigma^{\mu\nu}\frac{q_\nu}{2m_\tau}\}u(p)\epsilon_\mu(q)$$

$$a_\tau \equiv \frac{g_\tau - 2}{2} = \boxed{F_2(0)}$$

We could improve event selection, background modeling, systematic error, etc in the analysis of this channel.

To have a high precision (**10^{-5}** order or more), this way (by measuring the form factor with the data from colliders) is probably not an easy one to go.

Maybe we need a <u>dedicated precision experiment</u> to achieve this goal.

An experiment for tau's g-2?

The current experimental bounds: $-0.052 < a_\tau^\gamma < 0.013, \quad 95\% \text{ CL}$

Problem: Tau's lifetime is too short $\tau = 290.6 \pm 1\, fs$

One would need $B \sim 10^4\, T$

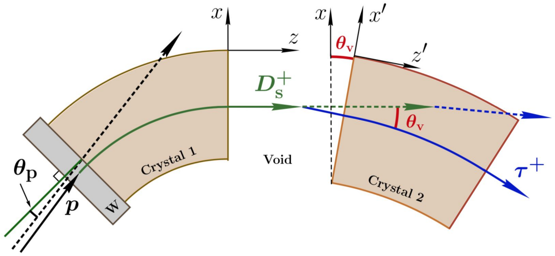
Solution: use a bending crystal producing an intense effective magnetic field

arXiv:1810.06699 [hep-ph]

$$p\,p \to D_s^+ X$$

$$D_s^+ \to \tau^+ \nu_\tau$$

The polarisation vector rotates by an angle which is proportional to *g*.



$$\angle\left(\vec{\mathbb{P}}_i \vec{\mathbb{P}}_f\right) = (1 + \gamma_\tau\, a_\tau)\,\Theta$$

τ^+ flight in the crystal

A detector

$$\frac{1}{N}\frac{dN}{d\Omega} = \frac{1}{4\pi}\left(1 + \alpha\,\vec{\mathbb{P}} \cdot \vec{n}\right)$$

$$\tau^+ \to \pi^+\pi^+\pi^-\bar{\nu}_\tau$$

Conclusion

“This model is crazy” (Arushi)

“It’s not good. Actually, it’s good” (Felix)

“Ummm….yeh, I agree” (Peilong)

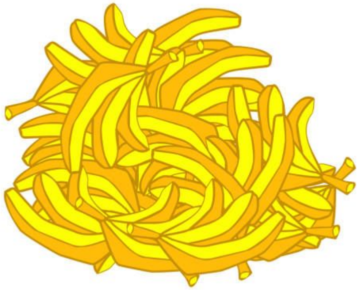
“Nature is tricksy, and I am the red triangle” (Jennifer)

“I see new fields everywhere my friend” (Elisabeth)

“Well, I like it…” (Rupert)

Backup

(More bananas!!)



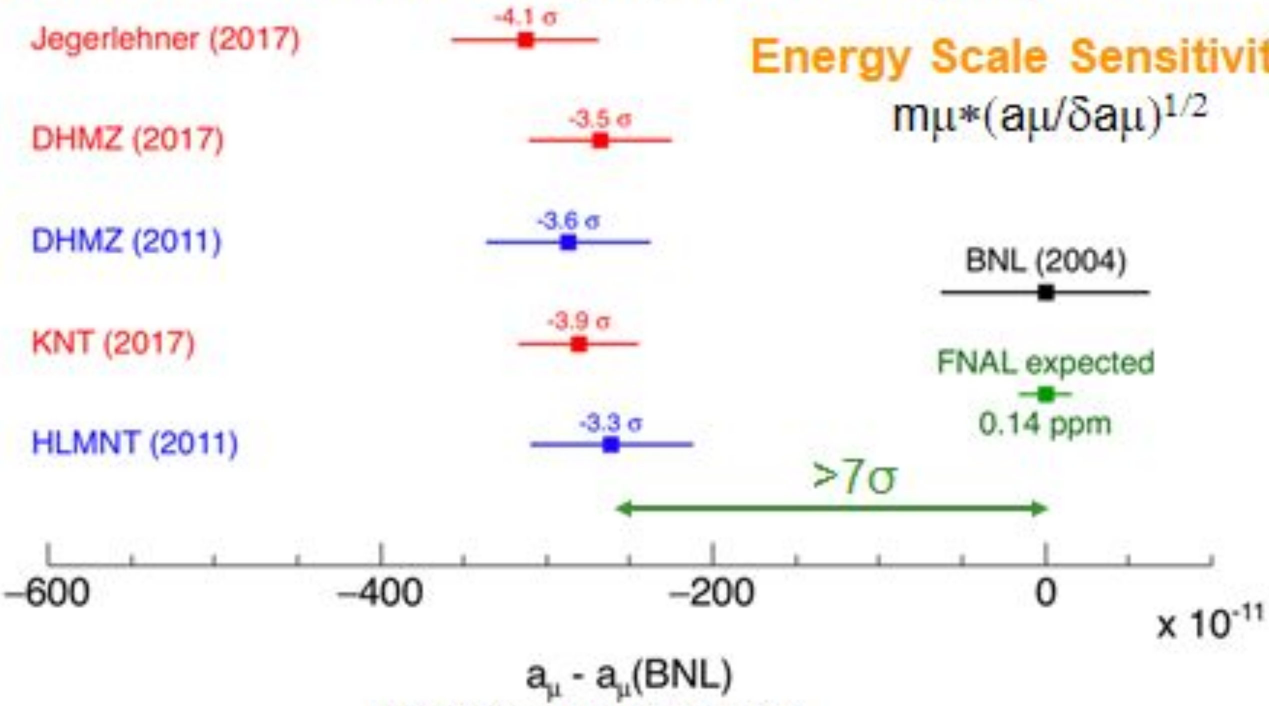
Uncertainty Goal for FNAL Muon g-2

- Last result (BNL) was already a precise measurement (550 ppb)

$$a_\mu^{exp} = 116\,592\,089\,(54)_{st}\,(33)_{sy}\,(63)_{tot} \times 10^{-11}$$

- E989 is trying to do 4 times better: **140 ppb**

- If the central value remain unchanged, then discrepancy > 7σ:



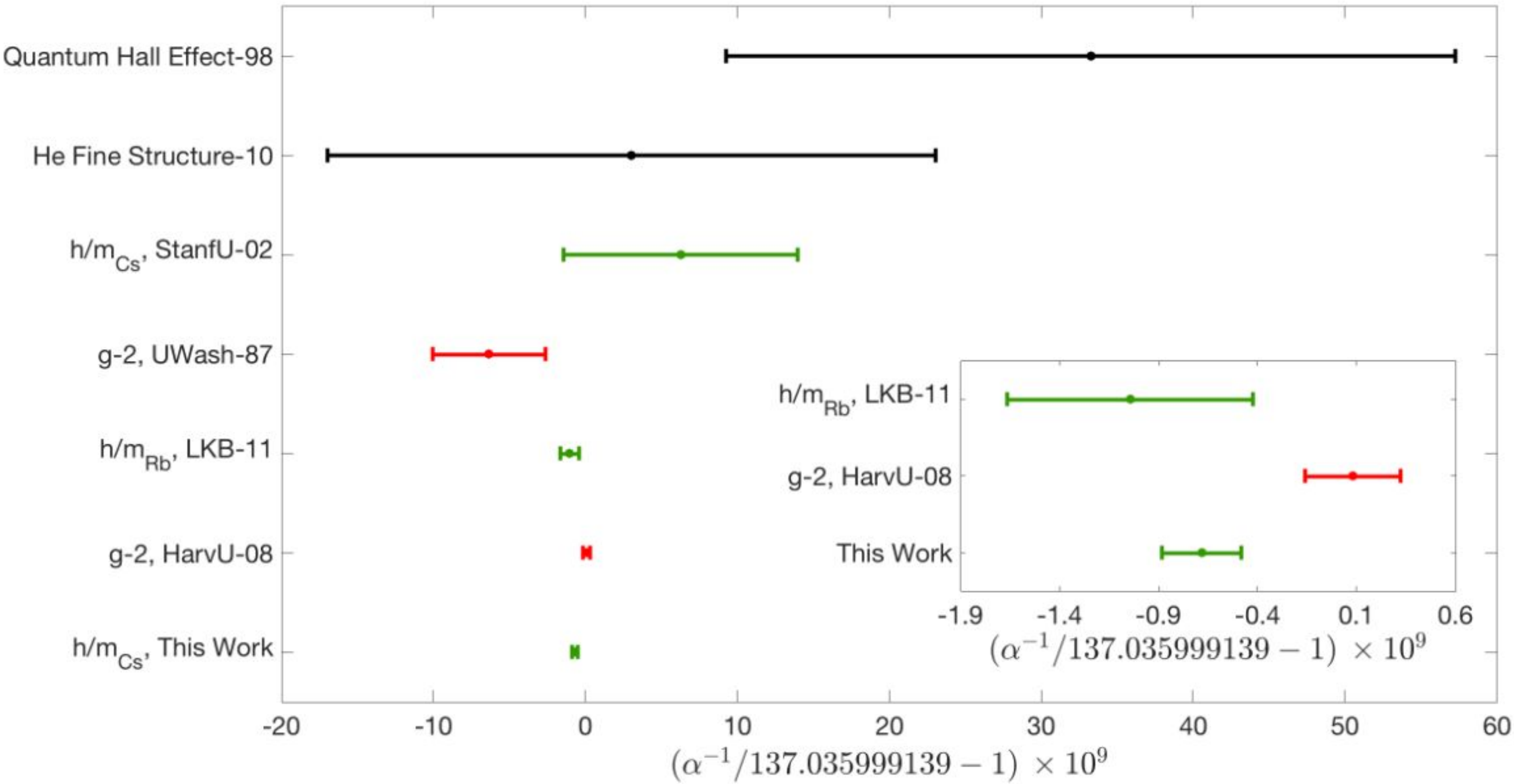
Energy Scale Sensitivity ~ $m_\mu * (a_\mu/\delta a_\mu)^{1/2}$

Introduction: muon g-2

SM contribution	10^{11} × (value ± error)		Refs and notes
QED (5 loops)	116584718.951	± 0.080	[Ayoma et al, 2012, Laporta'17]
EW (2 loops)	153.6	± 1.0	[Gnendiger et al, 2013]
HVP (LO)	6923	± 42	➡ ± (25-34) [KNT18, DHMZ'17]
HVP (NLO)	-98.4	± 1.0	[Hagiwara et al, 2011]
HVP (NNLO)	12.4	± 0.1	[Kurz et al, 2014]
HLbL	105	± 26	[Prades et al, 2014] ``Glasgow consensus"
HLbL (NLO)	3	± 2	[Colangelo et al, 2014]
Total	116591803	± 49	[Davier et al, 2011]
Experiment	116592089	± 63	[Bennet at al, 2006]
Diff (Exp. - SM):	286	± 80	

The difference is large: ~ 2 × (EW contribution)

g-2 electron



Quantum Hall Effect-98
He Fine Structure-10
h/m$_{Cs}$, StanfU-02
g-2, UWash-87
h/m$_{Rb}$, LKB-11
g-2, HarvU-08
h/m$_{Cs}$, This Work
-20
-10
0
10
20
30
40
50
60
$(\alpha^{-1}/137.035999139 - 1) \times 10^9$
h/m$_{Rb}$, LKB-11
g-2, HarvU-08
This Work
-1.9
-1.4
-0.9
-0.4
0.1
0.6
$(\alpha^{-1}/137.035999139 - 1) \times 10^9$

Alternative Representations

$$\mathcal{L} = y_\psi \overline{\psi_L} \Phi \psi_R$$

$$+ y_\chi \overline{\chi_L} \varphi \chi_R$$

$$+ y_f \overline{f_L} S f_R + h.c. \,,$$

$$\langle \Phi \rangle \,, \langle \varphi \rangle \,, \langle S \rangle \gg v$$

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)'$
H	$\mathbf{1}$	$\mathbf{2}$	$1/2$	1
l_{Le}	$\mathbf{1}$	$\mathbf{2}$	-1/2	q_{Le}
$l_{L\mu}$	$\mathbf{1}$	$\mathbf{2}$	-1/2	$q_{L\mu}$
$l_{L\tau}$	$\mathbf{1}$	$\mathbf{2}$	-1/2	$4 - q_{Le} - q_{L\mu}$
$\ell_{R\alpha}$	$\mathbf{1}$	$\mathbf{1}$	-1	$q_{L\alpha} - 1$
q_L	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$4/3$
u_R	$\mathbf{3}$	$\mathbf{1}$	$2/3$	$1/3$
d_R	$\mathbf{3}$	$\mathbf{1}$	-1/3	$1/3$
ψ_L	$\mathbf{3}$	$\mathbf{1}$	0	-2
ψ_R	$\mathbf{3}$	$\mathbf{1}$	0	0
Φ	$\mathbf{1}$	$\mathbf{1}$	0	2
χ_L	$\mathbf{1}$	$\mathbf{2}$	0	-16
χ_R	$\mathbf{1}$	$\mathbf{2}$	0	0
φ	$\mathbf{1}$	$\mathbf{1}$	0	-16
f_L	$\mathbf{1}$	$\mathbf{1}$	0	13
f_R	$\mathbf{1}$	$\mathbf{1}$	0	0
S	$\mathbf{1}$	$\mathbf{1}$	0	13

Some previous solutions

New scalar: one loop plus Barr-Zee

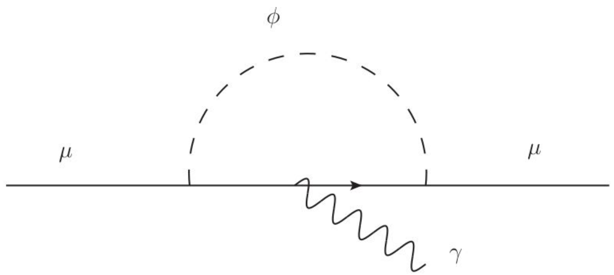


FIG. 1: One-loop ϕ contribution to $g_\mu - 2$.

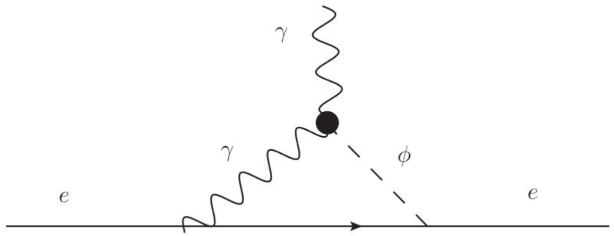


FIG. 2: Effective two-loop Barr-Zee diagram contribution to $g_e - 2$, with heavy fermion loops integrated out.

LQ one-loop diagrams