CPV and Leptogenesis

1. mixing matrix phases + leptonic unitarity triangle

2. leptogenesis in the type 1 seesaw
leptonic mixing matrix (lives in generation space; rotates from charged lepton $\alpha$ to neutrino $i$) with three angles (index order reversed wrt quarks):

$$U_{\alpha i} = \begin{bmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{bmatrix}
\begin{bmatrix}
c_{13} & s_{13}e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{i\delta} & 0 & c_{13}
\end{bmatrix}
\begin{bmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{bmatrix}P$$

$$= \begin{bmatrix}
c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\
-c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\
s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23}
\end{bmatrix}P$$

$$P = \text{diag}\{e^{-i\phi_1/2}, e^{-i\phi_2/2}, 1\}$$ for Majorana, identity for Dirac
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P = \text{diag}\{e^{-i\phi_1/2}, e^{-i\phi_2/2}, 1\} \text{ for Majorana, identity for Dirac}
\]

\[
\theta_{23} \simeq \pi/4 \quad \theta_{12} \simeq \pi/6 \quad \theta_{13} \simeq 0.15, 8^\circ \quad \delta \sim 1.4\pi
\]

(global fits of www.nu-fit.org)

for comparison, in CKM:

\[
\theta_{23} \simeq V_{cb} \simeq 0.04 \quad \theta_{12} \simeq V_{us} \simeq 0.225 \quad \theta_{13} \simeq V_{ub} \simeq 0.004
\]
The drunken Unitarity triangle

Not hear much about “leptonic unitarity triangle”
1. not measure elements at tree in CC
2. Also, it drinks.
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Amplitude to oscillate from flavour $\alpha$ to $\beta$ over distance $L$:

$$A_{\alpha\beta}(L) = U_{\alpha 1} U_{\beta 1}^* + U_{\alpha 2} U_{\beta 2}^* e^{-i(m_2^2 - m_1^2)L/(2E)} + U_{\alpha 3} U_{\beta 3}^* e^{-i(m_3^2 - m_1^2)L/(2E)}$$
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at $L = 0$ unitarity : $\Rightarrow A_{\alpha\beta} = 1$ for $\alpha = \beta$
$A_{\alpha\beta} = 0$ for $\alpha \neq \beta$

$\iff$ unitarity triangle (in complex plane)
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$\Leftrightarrow$ unitarity triangle(in complex plane)

At $L = t \neq 0$, two of the vectors rotate in the complex plane, with frequencies
$$(m_j^2 - m_1^2)/2E$$
oscillations $\Leftrightarrow$ time-dependent non-unitarity
“drunken unitarity” $\Rightarrow$ intuition when can use 2-flavour approx
About two-flavour analyses: atm/LBL $\nu_\mu$ disappearance

Amplitude to oscillate from flavour $\mu$ to $\tau$ over distance $L$:

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At $L \sim (m_3^2 - m_1^2)/E$, vector “3” rotates, at frequency $(m_3^2 - m_1^2)/2E$
About two-flavour analyses: atm/LBL $\nu_\mu$ disappearance

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At $L \sim (m_3^2 - m_1^2)/E$, vector “3” rotates, at frequency $(m_3^2 - m_1^2)/2E$.

$\Rightarrow$ “Atmospheric” neutrinos, also LBL ($\nu_\mu$ disappearance via $\Delta m^2_{31}$ oscillations):

$$A_{\mu\tau}(L) \sim U_{\mu 1} U^*_{\tau 1} + U_{\mu 2} U^*_{\tau 2} + U_{\mu 3} U^*_{\tau 3} e^{-i(m_3^2-m_1^2) L/(2E)}$$

$U_{\mu 3} U^*_{\tau 3}$ oscillates on timescale $t = L \sim (m_3^2 - m_1^2)/E$.
$U_{\mu 2} U^*_{\tau 2}$ $\sim$ stationary, measure $\theta_{23}$.
Three flavour probability required for CPV $\propto$ triangle area

$$
\mathcal{P}_{\alpha\beta}(L) = \delta_{\alpha\beta} - 4 \sum_{i<j} \text{Re}\{U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}\} \sin^2 \frac{x_{ji}}{2} + 2 \sum_{i<j} \text{Im}\{U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}\} \sin x_{ji}
$$

Last term violates CP $\Leftrightarrow$ opposite sign for $\nu_\alpha \rightarrow \nu_\beta$.
(Also matter effect opposite sign for $\bar{\nu}$).

$\propto$ area of triangle $\propto \tilde{J} = 8 c_{13}^2 s_{13} c_{23} s_{23} c_{12} s_{12}$:

Suppose triangle base $\in \text{Re} = U_{\mu 1} U_{\tau 1}^*$. Then base*height $\propto \text{Im}\{U_{\mu 1} U_{\tau 1}^* U_{\mu j}^* U_{\tau j}\}$

$\propto \tilde{J} \sin \delta$
Leptogenesis

*a class of recipes, that use (majorana) neutrino mass models to generate the matter excess*

► what matter excess?
► required ingredients?
► a simple seesaw model
► how it works...
Preambule

1. about “What the stars (and us) are made of” (5% of U)
   \[ \approx H \approx \text{baryons} \]
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2. I am made of baryons(defn) ... observation... all matter we see is made of baryons (not anti-baryons)

3. quantify as \((s_0 \approx 7n_{\gamma,0})\)

\[
Y_B \equiv \left. \frac{n_B - n_{\bar{B}}}{s} \right|_0 = 3.86 \times 10^{-9} \Omega_B h^2 \approx (8.53 \pm 0.11) \times 10^{-11}
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PLANCK
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PLANCK

⇒ Question : where did that excess come from?
Where did the matter excess come from?

1. the U(niverse) is matter-anti-matter symmetric?
   = islands of particles and anti-particles
   ✗ no! not see $\gamma$s from annihilation
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2. U was born that way...
   \[ \times \text{no! After birth of U, there was “inflation”} \]
   \[ \text{▷ (only theory explaining coherent temperature fluctuations in microwave background that arrive from causally disconnected regions today...)} \]
   \[ \text{▷ “60 e-folds” inflation } \equiv V_U \rightarrow 10^{90} V_U \]

\[ (n_B - \overline{n}_B) \rightarrow 10^{-90} (n_B - \overline{n}_B), \text{ s from } \rho \text{ of inflation...} \]
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3. created/generated/cooked after inflation...
Three ingredients to prepare in the early U (old russian recipe)

1. B violation: if Universe starts in state of $n_B - n_{\bar{B}} = 0$, need $\mathcal{B}$ to evolve to $n_B - n_{\bar{B}} \neq 0$
Three ingredients to prepare in the early Universe (old Russian recipe)

1. B violation: if Universe starts in state of $n_B - n_{\bar{B}} = 0$, need $B$ to evolve to $n_B - n_{\bar{B}} \neq 0$

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   Present in the SM quarks, observed in Kaons and Bs, searched for in leptons (...T2K,future expts)
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3. out-of-thermal-equilibrium ...equilibrium = static. “generation” = dynamical process
No asym.s in un-conserved quantum #s in equilibrium
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1. **B violation**: if Universe starts in state of \( n_B - n_{\bar{B}} = 0 \), need to evolve to \( n_B - n_{\bar{B}} \neq 0 \).

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3. **Out-of-thermal-equilibrium**: equilibrium = static. “generation” = dynamical process. No asym.s in un-conserved quantum #s in equilibrium. From end inflation \( \rightarrow \) BBN, Universe is an expanding, cooling thermal bath, so non-equilibrium from:
   - slow interactions: \( \tau_{int} \gg \tau_U = \text{age of Universe} \) (\( \Gamma_{int} \ll H \))
   - phase transitions:
ingredient 1: Does the SM conserve $B$?

$B, L$ are global symmetries of the SM Lagrangian ($q, \ell$ doublets, $e, u, d$ singlets)

$$\mathcal{L}_{SM} \supset \bar{q} D q, \bar{\ell} D \ell, \bar{\ell} H e, \bar{q} \tilde{H} u, \bar{q} H d$$

so, classically, there are conserved currents, and $B$ and $L$ are conserved. (So $B + L$ and $B - L$ are conserved.)
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Good—proton appears stable: $\tau_p \gtrsim 10^{33}$ yrs ($\tau_U \sim 10^{10}$ yrs).
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Good—proton appears stable: $\tau_p \gtrsim 10^{33}$ yrs ($\tau_U \sim 10^{10}$ yrs).

But the SM does not conserve $B + L$...

In QFT, there is the axial anomaly...

...anomalously, the fermion current associated to a classical symmetry is not conserved.

see Polyakov, “Gauge Fields + Strings,” 6.3=qualitative effects of instantons
ingredient 1: the SM does not conserve $B + L$

$B + L$ is anomalous. Formally, for one generation ($\alpha$ colour):

$$\sum_{SU(2)\text{ singlets}} \partial^\mu (\bar{\psi} \gamma_\mu \psi) + \partial^\mu (\bar{\ell} \gamma_\mu \ell) + \partial^\mu (\bar{q}^\alpha \gamma_\mu q_\alpha) \propto \frac{1}{64\pi^2} W^A_{\mu\nu} \tilde{W}^{\mu\nu A}.$$

where integrating the RHS over space-time counts “winding number” of the SU(2) gauge field configuration.

⇒ Field configurations of non-zero winding number are sources of a doublet lepton and three (for colour) doublet quarks for each generation.
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thanks to V Rubakov
SM B+L violation: rates

At $T = 0$ is tunneling process (from winding # to next, “instanton”): $\Gamma \propto e^{-8\pi/g^2}$

At $0 < T < m_W$, can climb over the barrier: $\Gamma_{B+L} \sim e^{-m_W/T}$  
\hspace{1in} $T < m_W$  
\hspace{1in} $T > m_W$

$\Rightarrow$ fast SM B+L at $T > m_W$

$\Gamma_{B+L} > H$ for $m_W < T < 10^{12}$ GeV

SM B+L called “sphalerons”

$\Rightarrow$ if produce a lepton asym, “sphalerons” partially transform to a baryon asym. !!

$\star \star \star$ SM B+L is $\Delta B = \Delta L = 3 \ (= N_f)$. No proton decay! $\star \star \star$
Summary of preliminaries: A Baryon excess today:

- Want to make a baryon excess $\equiv Y_B$ after inflation, that corresponds today to $\sim 1$ baryon per $10^{10}$ $\gamma$s.
- Three required ingredients: $B$, $CP$, $TE$.
  Present in SM, but hard to combine to give big enough asym $Y_B$

  Cold EW baryogen? Tranberg et al ...

  $\Rightarrow$ evidence for physics Beyond the Standard Model (BSM)
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One observation to fit, many new parameters...

$\Rightarrow$ prefer BSM motivated by other data $\Leftrightarrow m_\nu \Leftrightarrow$ seesaw! (uses non-pert. SM $B\!L$)
• add 3 singlet $N$ to the SM in charged lepton and $N$ mass bases, at scale $> M_i$:

$$\mathcal{L} = \mathcal{L}_{SM} + \lambda_{\alpha J} \overline{N}_J \ell_\alpha \cdot \phi - \frac{1}{2} \overline{N}_J M_J N_J^c$$

$M_i$ unknown ($\phi v = \langle \phi^0 \rangle$), and Majorana ($\mathcal{V}$). $\mathcal{CP}$ in $\lambda_{\alpha J} \in \mathcal{C}$. 

add 18 parameters: $M_1, M_2, M_3$

$18 - 3 (\ell$ phases) in $\lambda$
The type I seesaw

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$M_i$ unknown ($\phi \nu = \langle \phi^0 \rangle$), and Majorana ($\nu^\dagger \nu$). $\mathcal{CP}$ in $\lambda_{\alpha J} \in \mathbb{C}$.

- at low scale, for $M \gg m_D = \lambda \nu$, light $\nu$ mass matrix

\[ m_{\nu} = \lambda M^{-1} \lambda^T \nu^2 \]

for $\lambda \sim h_t$, $M \sim 10^{15}$ GeV

$\lambda \sim 10^{-7}$, $M \sim 10$ GeV $\sim .05$ eV

“natural” $m_{\nu} \ll m_f$ : $m_{\nu} \propto \lambda^2$, and $M > \nu$ allowed.

\[ m_{\nu} = \lambda M^{-1} \lambda^T \nu^2 \]
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- at low scale, Higgs mass contribution

$$\delta m_{\phi}^2 \approx - \sum_{l} [\lambda^\dagger \lambda]_{ll} M_i^2 \sim \frac{m_{\nu} M_i^3}{8\pi^2 v^4} v^2$$

for $M \gtrsim 10^7$ GeV $> v^2$ tuning problem

(? adding particles to cancel 1 loop...but higher loop? Need symmetry to cancel $\geq 2$ loop?)

$\Rightarrow$ do seesaw with $M_i \lesssim 10^8$ GeV?

(NB, in this talk, $\phi = $ Higgs, $H =$ Hubble)
Leptogenesis in the type 1 seesaw: usually a Fairy Tale

Once upon a time, a Universe was born.
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At the christening of the Universe, the fairies give the Standard Model and the Seesaw (heavy sterile $N_j$ with $\mathcal{L}$ masses and $\mathcal{C}\mathcal{P}$ interactions) to the Universe.
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1. Assuming it's hot enough, a population of $N$s appear—they like the heat.

2. As the temperature drops below $M$, the $N$ population decays away.

3. In the $\mathcal{CP}, \mathcal{L}$ interactions of the $N$, an asym. in SM leptons is created.
Leptogenesis in the type 1 seesaw: usually a Fairy Tale

If this asymmetry can escape the big bad wolf of thermal equilibrium...
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4. If this asymmetry can escape the big bad wolf of thermal equilibrium...
5. the lepton asym gets partially reprocessed to a baryon asym by non-perturbative $B + L$ -violating SM processes (“sphalerons”)

And the Universe lived happily ever after, containing many photons. And for every $10^{10}$ photons, there were 6 extra baryons (wrt anti-baryons).
Estimate something: TE + dynamics

Suppose $M_1 \ll M_{2,3}, \ T_{\text{reheat}} > M_1 \sim 10^9\text{GeV}$

Recipe: calculate suppression factor for each Sakharov condition, multiply together to get $Y_B$:

$$
\frac{n_B - n_{\bar{B}}}{s} \sim \frac{1}{3g_*} \epsilon L, CP \eta_{TE} \sim 10^{-3} \epsilon \eta \quad \text{(want } 10^{-10})
$$

$s \sim g_\ast n_\gamma$, $\epsilon = \text{lepton asym in decay}, \ \eta \sim \text{TE process}$
Estimate something: $\mathcal{T} \Phi + \text{dynamics}$

Suppose $M_1 \ll M_{2,3}$, $T_{\text{reheat}} > M_1 \sim 10^9 \text{GeV}$

1 produce a population of $N_1$s, via e.g. $(q\ell_\alpha \rightarrow Nt_R)$

Get thermal density $n_N \simeq n_\gamma$ if $M_1 \lesssim T$, and $\tau_{\text{prod}} < \tau_U$:
Estimate something: $\mathcal{E} + \text{dynamics}$

Suppose $M_1 \ll M_{2,3}$, $T_{\text{reheat}} > M_1 \sim 10^9\text{GeV}$

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$$\Gamma_{\text{prod}} \sim \sigma v n \sim \frac{h^2 \lambda^2}{T^2} \frac{T^3}{\pi^2} \sim \frac{h^2 \lambda^2}{\pi^2} T > H \simeq \frac{10 T^2}{m_{\text{pl}}}, \quad \Rightarrow \quad \frac{\lambda^2}{\pi^2} > \frac{10 T}{m_{\text{pl}}} \bigg|_{T=M_1}$$

Suppose satisfied...
Estimate something: \( T E + \) dynamics

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\]

Suppose satisfied...

2 Lepton asym is produced in \( N \) int. (eg decays) if there is \( CP \); can survive after inverse processes (eg decays) = “washout” become rare enough

\[
\Gamma_{ID}(\phi \ell \rightarrow N) \sim \Gamma_{\text{decay}} e^{-M_1/T} = \left[ \lambda \lambda^\dagger \right]_{11} M_1 \frac{1}{8\pi} e^{-M_1/T} < \frac{10 T^2}{m_{pl}}
\]
Estimate something: $\mathbb{T}E + \text{dynamics}$

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$$\Gamma_{\text{prod}} \sim \sigma v n \sim \frac{h_t^2 \lambda^2}{T^2} \frac{T^3}{\pi^2} \sim \frac{h_t^2 \lambda^2}{\pi^2} T > H \simeq \frac{10 T^2}{m_{pl}}, \quad \Rightarrow \quad \frac{\lambda^2}{\pi^2} > \left. \frac{10 T}{m_{pl}} \right|_{T=M_1}$$

Suppose satisfied...

2 Lepton asym is produced in $N$ int. (eg decays) if there is $\mathbb{C}\mathbb{P}$; can survive after inverse processes (eg decays) = “washout” become rare enough

$$\Gamma_{ID}(\phi\ell \rightarrow N) \simeq \Gamma_{\text{decay}} e^{-M_1/T} = \frac{[\lambda \lambda^\dagger]_{11} M_1}{8\pi} e^{-M_1/T} < \frac{10 T^2}{m_{pl}}$$

At temperature $T_\alpha$ when Inverse Decays turn off,

$$\frac{n_N}{n_\gamma}(T_\alpha) \simeq e^{-M_1/T_\alpha} \simeq \frac{H}{\Gamma(N \rightarrow \ell_\alpha \phi)}$$

can calculate this
Estimate something: $\mathbb{T}E + \text{dynamics}$

Suppose $M_1 \ll M_{2,3}, T_{\text{reheat}} > M_1 \sim 10^9 \text{GeV}$

1. produce a population of $N_1$s, via e.g. $(q\ell_\alpha \rightarrow Nt_R)$

Get thermal density $n_N \simeq n_\gamma$ if $M_1 \lesssim T$, and $\tau_{\text{prod}} < \tau_U$:

$$\Gamma_{\text{prod}} \sim \sigma v n \sim \frac{h_t^2 \lambda^2}{T^2} \frac{T^3}{\pi^2} \sim \frac{h_t^2 \lambda^2}{\pi^2} T > H \simeq \frac{10 T^2}{m_{\text{pl}}}, \quad \Rightarrow \frac{\lambda^2}{\pi^2} > \left. \frac{10 T}{m_{\text{pl}}} \right|_{T=M_1}$$

Suppose satisfied...

2. Lepton asym is produced in $N$ int. (eg decays) if there is $CP$; can survive after inverse processes (eg decays) = “washout” become rare enough

$$\Gamma_{\text{ID}}(\phi \ell \rightarrow N) \simeq \Gamma_{\text{decay}} e^{-M_1/T} = \left[ \frac{\lambda \lambda^\dagger}{8\pi} \right]_{11} \frac{M_1}{H} e^{-M_1/T} < \frac{10 T^2}{m_{\text{pl}}}$$

At temperature $T_\alpha$ when Inverse Decays turn off,

$$\frac{n_N}{n_\gamma}(T_\alpha) \simeq e^{-M_1/T_\alpha} \simeq \frac{H}{\Gamma(N \rightarrow \ell_\alpha \phi)}$$

can calculate this

so (1/3 is from SM $B+\bar{L}$, $s \sim g_\ast n_\gamma$, $\epsilon_\alpha$ is lepton asym in decay)

$$\frac{n_B - n_{\bar{B}}}{s} \sim \frac{1}{3} \sum_\alpha \epsilon_\alpha \frac{n_N(T_\alpha)}{g_\ast n_\gamma} \sim 10^{-3} \epsilon \frac{H}{\Gamma}$$
(want $10^{-10}$)
Estimate $\epsilon$, the CP and L asymmetry in decays

Recall (in $S$-matrix) \( CP : \langle \phi \ell | S | N \rangle \rightarrow \langle \phi \bar{\ell} | S | \bar{N} \rangle = \langle \phi \ell | S | N \rangle \), (\( \bar{\eta} \) = anti-$\eta$)
Estimate $\epsilon$, the CP and L asymmetry in decays

Recall (in $S$-matrix) $\text{CP}$:

$$\langle \phi \ell | S | N \rangle \rightarrow \langle \phi \ell | S | \bar{N} \rangle = \langle \phi \ell | S | N \rangle, \ (\bar{\eta} = \text{anti-}\eta)$$

In leptogenesis, need $\mathcal{CP}$, $\mathcal{L}$ interactions of $N_i$...for instance:

finite temp : Beneke et al 10

\[
\begin{align*}
\epsilon_l^\alpha &= \frac{\Gamma(N_i \rightarrow \phi \ell_\alpha) - \Gamma(\bar{N}_i \rightarrow \phi \bar{\ell}_\alpha)}{\Gamma(N_i \rightarrow \phi \ell) + \Gamma(\bar{N}_i \rightarrow \phi \bar{\ell})} \quad \text{(recall } N_i = \bar{N}_i) \\
&\sim \text{fraction } N \text{ decays producing excess lepton}
\end{align*}
\]
Estimate $\epsilon$, the CP and L asymmetry in decays

Recall (in $S$-matrix) $CP : \langle \phi \ell | S | N \rangle \rightarrow \langle \bar{\phi} \ell | S | \bar{N} \rangle = \langle \bar{\phi} \ell | S | N \rangle$, ($\bar{\eta} =$anti-$\eta$)

In leptogenesis, need $CP , \mathcal{L}$ interactions of $N_i$...for instance:

\[ \epsilon_l^\alpha = \frac{\Gamma(N_i \rightarrow \phi \ell_\alpha) - \Gamma(\bar{N}_i \rightarrow \bar{\phi} \ell_\alpha)}{\Gamma(N_i \rightarrow \phi \ell) + \Gamma(\bar{N}_i \rightarrow \bar{\phi} \ell)} \]  
(recall $N_i = \bar{N}_i$)

\[ \sim \text{ fraction } N \text{ decays producing excess lepton} \]

\[
\begin{align*}
N_i & \quad \times \quad \phi \\
\times \quad \lambda & \quad \times \quad \ell_\alpha
\end{align*}
\]

Just try to calculate $\epsilon_1$?

- no asym at tree
- asym at tree $\times$ loop, if $CP$ from complex cpling and on-shell particles in the loop (divergences cancel in diff, need Im part of Feynman param integrtn)
\( \mathbb{CP} \), complex couplings, loops unitarity and all that...(estimate \( \epsilon \), no loop cahn)

1 the S-matrix \( S \equiv 1 + iT \) is CPT invariant

\[
\langle \phi \ell | S | N \rangle = \langle N | S | \phi \ell \rangle \quad (= \langle \phi \ell | S^\dagger | N \rangle^*)
\]
\( \Phi \), complex couplings, loops unitarity and all that...(estimate \( \epsilon \), no loop calc)

1. The S-matrix \( S \equiv 1 + iT \) is CPT invariant

\[ \langle \phi \ell | S | N \rangle = \langle N | S | \phi \ell \rangle \quad (= \langle \phi \ell | S^\dagger | N \rangle^* ) \]

And unitary: \( SS^\dagger = 1 = (1 + iT)(1 - iT^\dagger) \)

\[ \Rightarrow iT - iT^\dagger + TT^\dagger = 0 \]
$\mathcal{CP}$, complex couplings, loops unitarity and all that...(estimate $\epsilon$, no loop caln)

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$$\Rightarrow i\langle \phi_\ell | T | N \rangle - i\langle \phi_\ell | T^\dagger | N \rangle + \langle \phi_\ell | TT^\dagger | N \rangle = 0$$

---

Kolb+Wolfram, NPB '80, Appendix
The S-matrix $S \equiv 1 + i T$ is CPT invariant:

$$\langle \phi \ell | S | N \rangle = \langle N | S | \phi \ell \rangle \quad (= \langle \phi \ell | S^\dagger | N \rangle^* )$$

and unitary: $SS^\dagger = 1 = (1 + i T)(1 - i T^\dagger)$

$$\Rightarrow i T - i T^\dagger + TT^\dagger = 0$$

$$\Rightarrow i \langle \phi \ell | T | N \rangle - i \langle \phi \ell | T^\dagger | N \rangle + \langle \phi \ell | TT^\dagger | N \rangle = 0$$

$$|\langle \phi \ell | T | N \rangle|^2 = |\langle \phi \ell | T^\dagger | N \rangle|^2 - i \langle \phi \ell | T^\dagger | N \rangle \langle N | TT^\dagger | \phi \ell \rangle + i \langle N | T | \phi \ell \rangle \langle \phi \ell | TT^\dagger | N \rangle + ...$$
\( \mathcal{CP} \), complex couplings, loops unitarity and all that...

(estimate \( \epsilon \), no loop caln)

1 the S-matrix \( S \equiv 1 + iT \) is CPT invariant

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\langle \phi \ell | S | N \rangle = \langle N | S | \phi \ell \rangle = (\langle \phi \ell | S^\dagger | N \rangle^*)
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\]

\[
|\langle \phi \ell | T | N \rangle|^2 = |\langle \phi \ell | T^\dagger | N \rangle|^2 - i\langle \phi \ell | T^\dagger | N \rangle \langle N | TT^\dagger | \phi \ell \rangle + i\langle N | T | \phi \ell \rangle \langle \phi \ell | TT^\dagger | N \rangle + ...
\]

2 We are interested in a \( \mathcal{CP} \) asymmetry:

\[
\epsilon \propto \int d\Pi \left( |\langle \phi \ell | T | N \rangle|^2 - \langle \overline{\phi \ell} | T | N \rangle|^2 \right)
\]

SO (this formula exact, if I kept 2s and sums; \( \int d\Pi = \) phase space)

\[
\epsilon \propto \int d\Pi \text{Im} \left\{ \langle \phi \ell | T^\dagger | N \rangle \langle N | TT^\dagger | \phi \ell \rangle \right\}
\]

\( \Rightarrow \) need complex cplings, and on-shell particles in a loop
Estimating $\epsilon$ for hierarchical $N_I$

Consider simple case: $M_1 \ll M_{2,3}$. Suppose lepton asym generated in $\mathcal{CP}$, $\mathcal{L}$ decays of $N_1$:

$$
\epsilon_1^\alpha = \frac{\Gamma(N_1 \to \phi \ell_\alpha) - \Gamma(\tilde{N}_1 \to \tilde{\phi} \tilde{\ell}_\alpha)}{\Gamma(N_1 \to \phi \ell) + \Gamma(\tilde{N}_I \to \tilde{\phi} \tilde{\ell})} 
$$

(recall $N_1 = \tilde{N}_1$)

(NB, no intermediate $N_1$ because cplg combo real)

\[ N_I \xrightarrow{\chi} \phi \times \ell_\alpha \quad N_I \xrightarrow{\lambda^* \phi} \lambda \quad N_I \xrightarrow{\lambda \phi} \lambda \]

\[ + N_I \xrightarrow{\lambda^* \phi} \lambda \quad N_I \xrightarrow{\lambda \phi} \lambda \]
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$$[\kappa]_{\alpha\beta} \sim \frac{[m_\nu]_{\alpha\beta}}{v^2}$$
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Recall

$$\Gamma \epsilon \sim \text{Im}\left\{ \langle \phi \bar{\ell} | T | N \rangle^* \langle N | T | \phi \bar{\ell} \rangle \langle \phi \bar{\ell} | T^\dagger | \phi \ell \rangle \right\}$$
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Recall

$$\Gamma\epsilon \sim \text{Im}\left\{ \langle \bar{\phi} \ell | T | N \rangle^* \langle N | T | \phi \ell \rangle \langle \phi \ell | T^\dagger | \phi \ell \rangle \right\}$$

$$\Gamma\epsilon \propto \int d\mathcal{P} \text{Im}\left\{ |M(N \to \phi \ell)|M(N \to \phi \ell)M(\phi \ell \to \phi \ell) \right\}$$

$$\epsilon_1 \sim M \frac{\text{Im}\{\lambda \lambda \kappa^*\}}{8\pi |\lambda|^2}$$
Estimating $\epsilon$ for hierarchical $N_i$

Consider simple case: $M_1 \ll M_{2,3}$. Suppose lepton asym generated in CP, $L$ decays of $N_1$:

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$$[\kappa]_{\alpha \beta} \sim \frac{[m_\nu]_{\alpha \beta}}{v^2}$$

Recall

$$\Gamma_\epsilon \sim \text{Im} \left\{ \langle \phi \ell | T | N \rangle^* \langle N | T | \phi \ell \rangle \langle \phi \ell | T^\dagger | \phi \ell \rangle \right\}$$

$$\Gamma_\epsilon \propto \int d\Pi \text{Im} \left\{ M^* (N \to \bar{\phi} \ell) M(N \to \phi \ell) M(\bar{\phi} \ell \to \phi \ell) \right\}$$

$$\epsilon_1 \sim M \frac{\text{Im} \{\lambda \lambda \kappa^*\}}{8\pi |\lambda|^2} < \frac{3}{8\pi} \frac{m_\nu^{\text{max}} M_1}{v^2} \sim 10^{-6} \frac{M_1}{10^9 \text{GeV}} \gtrsim 10^{-6}$$

so for $M_1 \ll M_{2,3}$, need $M_1 \gtrsim 10^9$ GeV to obtain sufficient $\epsilon$. 
Maybe want $M_K < 10^9$ GeV? Leptogenesis with lighter $N_I$...

1. $M_I \sim M_J \Leftrightarrow$ resonantly enhance $\epsilon$ ... up to $\epsilon \lesssim 1$.
2. at lower $T$, age of Universe is longer, takes more care to get out-of-equilibrium...
3. need to generate $L$ asym before Electroweak PT (to profit from sphalerons)...

$\Rightarrow$ leptogenesis via $N_I$ decay/scattering works for degen $N_I$ down to $M_I \sim$ TeV

For even lighter $M_I$, can make asym by coherent oscillations and scatterings of $N_I$...
**νMSM** : type 1 seesaw below 100 GeV gives BAU and DM

Asaka + Shaposhnikov

thesis Canetti

**ingredients** : SM +

\[ N_{2,3} : 100 \text{ MeV} \lesssim M_{2,3} \lesssim 10 \text{ GeV}, \Delta M \approx \begin{cases} 10^{-6} \text{ eV} & Y_B, \Omega_{DM} \\ \text{keV} & Y_B, \text{NOT } \Omega_{DM} \end{cases} \]

Yukawas \( \equiv \) give 2 light SM neutrinos via seesaw

\( N_1 : M_1 \sim \text{keV} \). WDM candidate.

feeble coupled (negligeable contribution \( m_{\nu,SM} \))

**scenario** :

Population of \( N_{2,3} \) produced via Yukawas before EPT

Produce \( \Delta L \rightarrow Y_B \), before EPT, via oscillations of \( N_{2,3} \) coherent with Yukawa scattering to \( \nu_{SM} \)

Produce \( \Delta L \gtrsim 10^{-5} \) via osc. and decay of \( N_{2,3} \) after EPT

Can produce sufficient distribution of \( N_1 \) via osc.

**tests** :

\( N_{2,3} \) : beam dump, SHIP

\( N_1 \) as DM : X-rays from DM decay, WDM bounds (depend on momentum distribution)
How does asym generation work? (very simplified!)

1 at $T \lesssim \text{TeV}$ (recall $\lambda \lesssim 10^{-7}$), produce $N_2, N_3$ via Yukawa interaction $\lambda \tilde{N} \ell \cdot \phi$. 
How does asym generation work? (very simplified!)

1 at $T \lesssim \text{TeV}$ (recall $\lambda \lesssim 10^{-7}$), produce $N_2, N_3$ via Yukawa interaction $\lambda \overline{N}_{\ell} \cdot \phi$

2 $N_2, N_3$ oscillate (almost degenerate)

3 back to $\nu_L$ via $\lambda$
How does asym generation work? (very simplified!)

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at $\tau_U \sim \tau_{osc}$, 1,2,3 are coherent, so CPV from $\lambda$-$\Delta M^2$-$\lambda$ gives flavour asyms in $\nu_{L\alpha}$ (to small)

*lepton number in $\ell_L + N_R$ is conserved* (actually, $L_{SM}$+ helicity of $N_i$)

from $\tau_{osc} \rightarrow \tau_{EWPT}$, asyms in $\nu_{L\alpha}$ seed asyms in $N \rightarrow$ asyms in $\nu_{L\alpha}$ (enough asym)

...works also in detailed calculations with all available technology...

(eg also include lepton number violating interactions)

Teresi Hambye
Eijima + Shaposhnikov
Ghiglieri+ Laine
\[ U^2 = \text{Tr}[\lambda M^{-2} \lambda^\dagger] \]
Summary

*Leptogenesis* is a class of recipes, that use (majorana) neutrino mass models to generate the matter excess. The model generates a lepton asymmetry (before the Electroweak Phase Transition), and the non-perturbative SM $B+L$ violn reprocesses it to a baryon excess.

* efficient, to use the BSM for $m_\nu$ to generate the Baryon Asym.
* using SM $B+L$ violn ($\Delta B = \Delta L = 3$) avoids proton lifetime bound
* *it works* ...rather well, for a wide range of parameters
more CPV in \( \{ U, m_\nu \} \) if Majorana

- Suppose that all parameters in \( \mathcal{L} \) that can be complex (\( U \) and \( m_\nu \)), are complex
- 3 angles and 6 phases in generic unitary matrix \( U \) (18 real parameters in arbitrary \( 3 \times 3 \) complex matrix. Unitarity \( UU^\dagger = 1 \) reduces this to 9.)
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- five relative phases between the fields \( e_L, \mu_L, \tau_L, \nu_1, \nu_2, \nu_3 \) ...so can choose the 5 relative phases among LH fermions, to remove all but one phase in the mixing matrix.
more CPV in \( \{ U, m_\nu \} \) if Majorana

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• 3 angles and 6 phases in generic unitary matrix \( U \) (18 real parameters in arbitrary \( 3 \times 3 \) complex matrix. Unitarity \( UU^\dagger = 1 \) reduces this to 9.)
• five relative phases between the fields \( e_L, \mu_L, \tau_L, \nu_1, \nu_2, \nu_3 \) ...so can choose the 5 relative phases among LH fermions, to remove all but one phase in the mixing matrix.
• now check if can make the masses real : if dirac masses, absorb phase of mass with \( \nu_RI \). If \( \nu_L3 \) has Majorana mass, between self and anti-self, choose absolute phase of \( \nu_L3 \) to make the mass real. Now all LH fermion phases are fixed, and cannot remove phases from \( m_\nu1, m_\nu2 \).

\[ \Rightarrow \] extra CPV in processes where Majorana mass appears linearly (not as \( mm^* \), not in kinematics = not in oscillations)