Lepton Flavour Violation

(and Leptogenesis)

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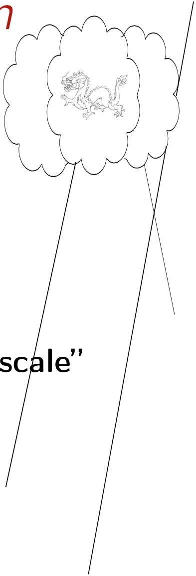
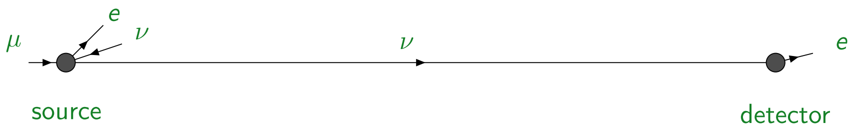


1. **leptons in SM,LFV,...**
2. **EFT= parametrising LFV at low energy with contact interactions** **Kuno Okada**
   = path up the mountain ⇔ travels in "scale"
3. CPV in the leptons
4. leptogenesis

Lepton Flavour Violation

- three lepton flavours in the Standard Model : e, μ, τ (flavour $\equiv$ mass eigenstate)
  different from quarks, where 6 flavours
- LFV= CLFV = FCNC$|_{quarks}$
  $\equiv$ charged lepton flavour change, at a point = ν oscillations don't count.



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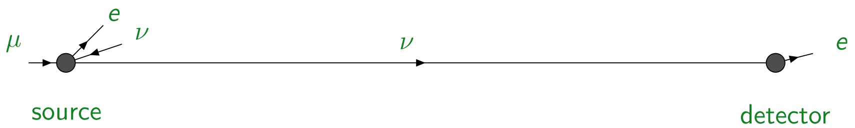


- Lepton Flavour Change is BSM that exists, just don't know rate :
  - none in the Standard Model with $m_\nu = 0$
  - occurs with m_ν and mixing matrix U

  m_ν renormalisable Dirac : LFV amplitudes GIM-suppressed (like quarks)

$$\mathcal{A} \propto \frac{m_\nu^2}{m_W^2} \Rightarrow BR \lesssim 10^{-48}$$

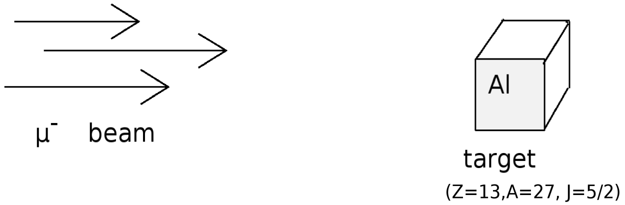
$\Rightarrow$ if see LFV, what can we learn about the new leptonic physics ?
suppose : m_ν NOT renormalisable-Dirac, + New Physics heavy$\Rightarrow$EFT

Some current constraints on Branching Ratios

some processes	current constraints on BR	future sensitivities
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$	6×10^{-14} (MEG)
$\mu \to e\bar{e}e$	$< 1.0 \times 10^{-12}$(SINDRUM)	10^{-16} (2018, Mu3e)
$\mu A \to eA$	$< 7 \times 10^{-13}$ Au, (SINDRUM)	10^{-16} (Mu2e,COMET)
		10^{-18} (PRISM/PRIME)
$\overline{K_L^0} \to \mu\bar{e}$	$< 4.7 \times 10^{-12}$ (BNL)	
$K^+ \to \pi^+\bar{\mu}e$	$< 1.3 \times 10^{-11}$ (E865)	10^{-12} (NA62)

$\mu A \to eA \equiv$ bound μ in $1s$ of nucleus A becomes e

(What is $\mu \to e$ conversion ? (not a nuclear boogyman)



- μ^- captured by Al nucleus, tumbles down to $1s$. $(r \sim Z\alpha/m_\mu \gtrsim r_{Al})$
- in SM : muon capture $\mu + p \to \nu + n$

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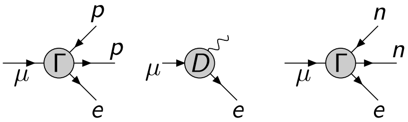

$$\Gamma = \{I, \gamma_5, \gamma^\alpha, \gamma^\alpha\gamma_5, \sigma\}$$
$$\Gamma = \{S, P, V, A, T\}$$

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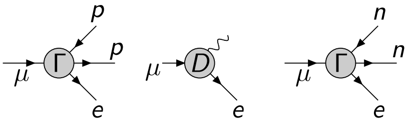

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$\approx$ WIMP scattering on nuclei

1) "Spin Independent" rate $\propto A^2$ (amplitude $\propto \sum_N \propto A$)
2) "Spin Dependent" rate $\sim \Gamma_{SI}/A^2$ (Σ nucleons $\propto$ spin of only unpaired nucleon)

Why is $\mu \to e$ conversion interesting?

1. maybe where $\mu \to e$ could be discovered?
   - experimental sensitivity to BR will improve : $10^{-12} \to 10^{-16}$ (expts under construction) $\to 10^{-18}$ (planned expts)
     (future sensitivity to $BR(\mu \to e\bar{e}e) \sim 10^{-16}$)
   - $\Gamma_{SI}(\mu A \to eA) \propto A^2$, whereas $\Gamma(\mu A \to \nu A')$ is not coherently enhanced. So

$$BR_{SI}(\mu A \to eA) \equiv \frac{\Gamma_{SI}(\mu A \to eA)}{\Gamma(\mu A \to \nu A')} \propto A^2 |\sum C|^2$$

$\Rightarrow$ sensitivity $C \sim \sqrt{BR}/A$?

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2. ...its coherent $\Leftrightarrow$ many SI operators interfere

$$BR_{SI}(\mu A \to e_R A) = \frac{32 G_F^2 m_\mu^5}{\Gamma_{capt}} \Big| \widetilde{C}_{V,R}^{pp} V_A^{(p)} + \widetilde{C}_{S,L}^{\prime\, pp} S_A^{(p)} + \widetilde{C}_{V,R}^{nn} V_A^{(n)} + \widetilde{C}_{S,L}^{\prime\, nn} S_A^{(n)} + \frac{C_{D,L}}{4} D \Big|^2$$

if 'tis observed, how to know which operators contribute ?

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$\tau \to \ell\gamma$	$< 3.3, 4.4 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to 3\ell$	$< 1.5 - 2.7 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II, LHCb ?)
$\tau \to e\phi$	$< 3.1 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$h \to \tau^\pm e^\mp$	$< 6.9 \times 10^{-3}$	
$Z \to e^\pm \mu^\mp$	$< 7.5 \times 10^{-7}$	

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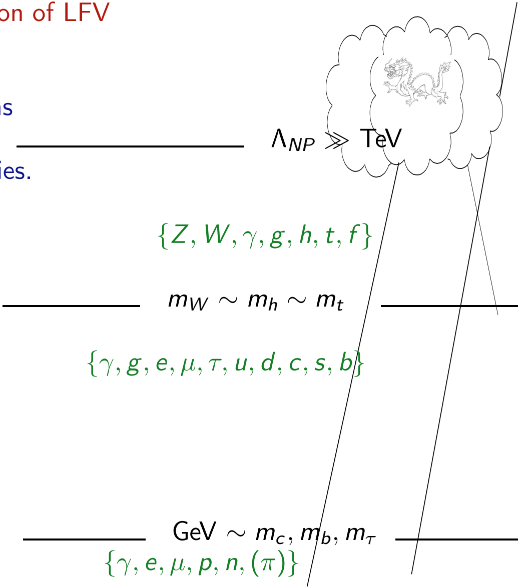
EFT as a Lagrangian parametrisation of LFV

construct LFV contact interactions
out of dynamical fields,
impose relevant (gauge) symmetries.



(Maybe, or not, select
by operator dimension)

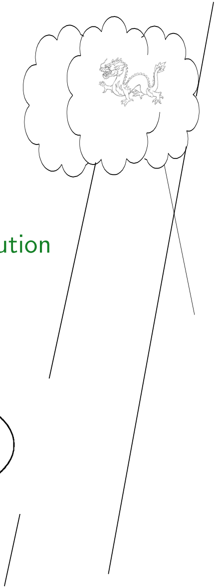
$$\delta\mathcal{L} = \sum_\zeta \sum_O \frac{C_O^\zeta}{v^n} O^\zeta + h.c.$$

$(v = 174 \text{ GeV})$

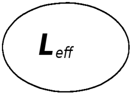
$$\mathcal{O}_{V,LL}^{e\mu ee} = (\overline{e}\gamma^\alpha P_L \mu)(\overline{e}\gamma_\alpha P_L e)$$

$\Rightarrow$ theoretical parametrisation of the data= express LFV rates in terms of $\{C_O^\zeta\}$.

EFT to transport coefficients in scale

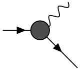


1. "scale" $\equiv \mu$ of dim.reg.
   Miracle of EFT : can interpret "à la Wilson" !
   see Georgi, ARNPS,1993
2. But loops are small : surely negligeable ?
   $\nearrow$ data is sensitive to loop effects
   $\searrow$ tree level is not always the best-constrained contribution
3. in practise — how to calculate loops in EFT ?
4. results : constraints, flat directions and sensitivities



But surely loops effects in LFV are negligeable ?

Two dipole operators contribute to $\mu \to e\gamma$:



$$\begin{aligned} \delta\mathcal{L}_{meg} &= -\frac{4G_F}{\sqrt{2}} m_\mu \left(C_{D,R}\overline{\mu_R}\sigma^{\alpha\beta} e_L F_{\alpha\beta} + C_{D,L}\overline{\mu_L}\sigma^{\alpha\beta} e_R F_{\alpha\beta} \right) \\ BR(\mu \to e\gamma) &= 384\pi^2(|C_{D,R}|^2 + |C_{D,L}|^2) < 5.7 \times 10^{-13} \\ &\Rightarrow |C_X^D| \lesssim 10^{-8} \end{aligned}$$

MEG expt, PSI

How big does one expect $C_{D,X}$ to be ?

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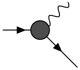

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How big does one expect $C_{D,X}$ to be ?Suppose operator coefficient

			$n = 1$	$n = 2$
$\frac{m_\mu}{v^2} C_{D,X} \sim \frac{e m_\mu}{(16\pi^2)^n \Lambda^2}$	$\Rightarrow$	probes	$\Lambda \lesssim 100$ TeV	10 TeV
$\frac{m_\mu}{v^2} C_{D,X} \sim \frac{e v}{(16\pi^2)^n \Lambda^2}$	$\Rightarrow$	probes	$\Lambda \lesssim 3000$ TeV	300 TeV

$\Rightarrow \mu \to e$ expts probe multi-loop effects in NP theories with $\Lambda_{NP} \gg$ reach of LHC

But QED loops are $\mathcal{O}(\alpha/4\pi)$... surely negligeable correction to tree ?

Work top-down = suppose a model that gives only tensor operator at m_W :

$$2\sqrt{2}G_F \; C_T(\overline{u}\sigma u)(\overline{e}\sigma P_Y \mu)$$

1 :forget RGEs Match to nucleons $N \in \{n, p\}$: $\widetilde{C}_T^{NN} \simeq \langle N|\bar{u}\sigma u|N\rangle C_T^{uu} \lesssim \frac{3}{4} C_T^{uu}$

nucl. matrix ele. : EngelRTO KlosMGS

$$\Rightarrow \boldsymbol{BR(\mu A \to eA) \approx BR_{SD} \approx \frac{1}{2}|C_T|^2}$$

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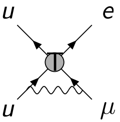
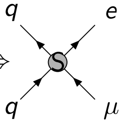
$C_T^{uu}(\overline{u}\sigma u)(\overline{e}\sigma P_Y\mu)$ 

$+\ldots \quad \Rightarrow$ 

$$64\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}\ C_T^{uu}\ (\overline{u}u)(\overline{e}P_Y\mu)$$

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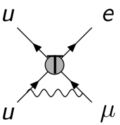
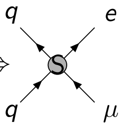
$C_T^{uu}(\overline{u}\sigma u)(\overline{e}\sigma P_Y \mu)$ 

$+ \ldots \quad \Rightarrow$ 

$$64\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}\;\; C_T^{uu}\;\; (\overline{u}u)(\overline{e}P_Y\mu)$$

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Then match to nucleons : $\widetilde{C}_S^{NN} = \langle N|\bar{u}u|N\rangle \Delta C_S^{uu} \sim C_T^{uu}$ so $\widetilde{C}_S^{pp} \gtrsim \widetilde{C}_T^{pp}$,

$$\boldsymbol{BR(\mu A \to eA) \approx BR_{SI} \sim Z^2|2C_T^{uu}|^2 \sim 10^3 BR_{SD}}$$

loops can change Lorentz structure/external legs $\Rightarrow$ different operator whose coefficient better constrained

To calculate loops in EFT

regularise with dim.reg., renormalise with $\overline{MS}$, + Fierz and γ_5 in 4-d because only ever looking for $1/\epsilon$ poles.

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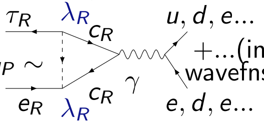
- Suppose renormalisable model, mediates LFV. *eg* New leptoquark of mass Λ_{NP}.

$\mathcal{M}_{NP} \sim$



+...(include wavefns)

- Allows to calculate $\mathcal{M}_{NP}(\tau \to e\rho)$ (also $\tau \to 3e$), which is finite

$$\langle e(p_e, s_e), \rho(p_\rho)|\boldsymbol{S}|\tau(\vec{P} = 0, S_\tau)\rangle = i\tilde{\delta}^4(P - p_e - p_\rho)\mathcal{M}_{NP}$$

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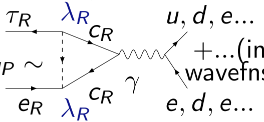

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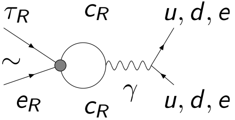
- But $\Lambda_{NP} \gg m_\tau$?
  1 : just calculate *relevant* dynamics = SM.
  2 : $\mathcal{O}(m_\tau^2/\Lambda^2)$ part of $\mathcal{M}_{NP}$ accurate enough ?

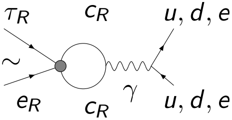
At tree level, in $\mathcal{L}$, replace LQ $\to$ $\underbrace{\frac{\lambda_R\lambda_R}{2\Lambda_{NP}^2}}_{\text{coeff. } C_{LQ}} \underbrace{(\overline{e}\gamma P_R\tau)(\overline{c}\gamma P_R c)}_{\text{operator } O_{LQ}}$

- And calculate $\mathcal{M}_{eff}$

$\mathcal{M}_{eff} \sim$



To calculate, ctd

$\mathcal{M}_{eff} \sim$ 

- *Yeeks !* $\mathcal{M}_{eff}$ diverges...
  ...regularise+add counterterms (for all operators generated)
  ($\Rightarrow \delta\mathcal{L} : C_{LQ}\mathcal{O}_{LQ} \to \mu^{2\epsilon}\sum_N C_{LQ}Z_{LQ,N}Z_\psi^{n/2}\mathcal{O}_N$)

- With renormalised $\mathcal{L}$, obtain finite $\mathcal{M}_{eff,ren}$ ...except, still logμ in $\mathcal{M}_{eff,ren}$.

- cancel μ-dependence of time-ordered-product-of-fields in usual way :
  require coupling constants (= operator coefficients) to be μ-dependent
  $\Rightarrow$ RGEs : $\mu\frac{d}{d\mu}\vec{C} = \vec{C}\left(\mu\frac{d}{d\mu}[Z]\right)Z^{-1} \equiv \vec{C}\cdot[\Gamma]$

- *Eureeka !* $\mathcal{M}_{eff,ren}$ with running coefficients $C(\mu)$, is finite + μ-indep.
  But :had to renormalise operators—result *cannot* depend on associated μ, or on scheme (no operators in renormalisable models),*so only can calculate* scheme-indep, μ-indep quantities !
  $\Rightarrow$ *at one loop, coeff of* $(1/\epsilon + \log)$ is scheme-indep
  $\Rightarrow$ allows to obtain *all* $(\frac{\log}{16\pi^2})^n$ terms ! (see Barr-Zee comments in a few slides)

But what is that scheme-indep log-term in $\mathcal{M}_{eff,ren}$?

- to calculate $C(\mu)$:

  1. match Greens fns in model, to tree level Greens fns in EFT... gives coeff $C(\Lambda_{NP}) \sim \lambda^2/\Lambda^2$

  2. scale evolution of $\vec{C}$ from RGEs (soln is "scale-ordered" exponential) ; run coeffs from $\Lambda_{NP} \to m_\tau$

- evaluate operator matrix element at m_τ : (more difficult for 4q)

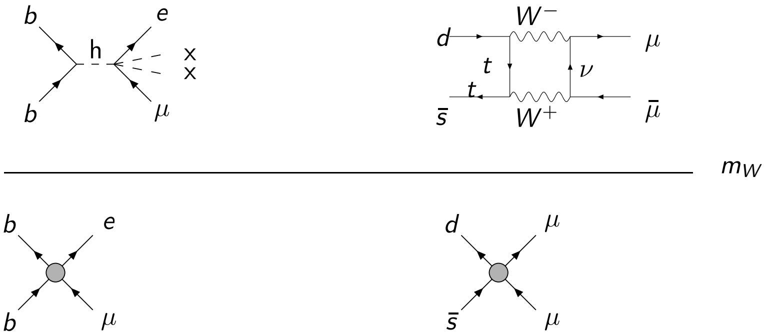
  $\langle e, \rho|(\overline{e}\gamma P_R \tau)(\overline{q}\gamma q)|\tau\rangle \propto \overline{u}_e \gamma P_R u_\tau \langle\rho|(\overline{q}\gamma q)|0\rangle$

  $\langle e, \bar{e}, e|(\overline{e}\gamma P_R \tau)[(\overline{e}\gamma e)]|\tau\rangle \propto (\overline{u}_e \gamma P_R u_\tau)(\overline{u}_e \gamma P_R u_e)$

  (troubles with quarks below m_τ where QCD becomes strong...)

- then $\mathcal{M}_{eff,ren} \simeq C_j(m_\tau)\langle f|\mathcal{O}_j|i\rangle(m_\tau)$

What means “matching at tree-level in the EFT” : examples



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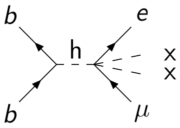
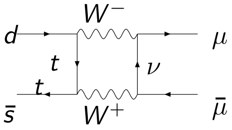
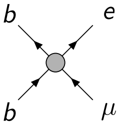
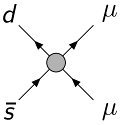


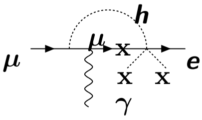


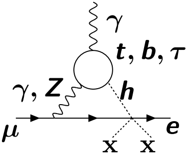
m_W

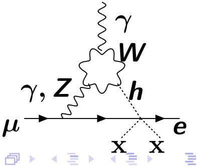




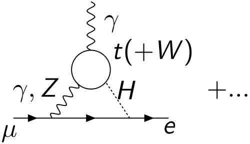
At "tree" in the EFT, do these match at m_W onto the dipole ? (h the SM Higgs)







Why do EFT ? Surely models are easier ?

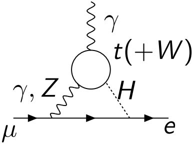


⋆ SM loop calns hard — do calculation once

⋆ just need coeffs of $1/\epsilon \Rightarrow$ all $\log^n / (16\pi^2)^n$ terms

Ex : suppose want to calculate Barr-Zee diagrams for $m_H \gg m_t$:

Why do EFT ? Surely models are easier ?



$+ \ldots$

⋆ SM loop calns hard — do calculation once
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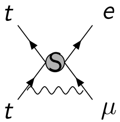
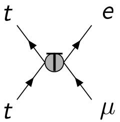
Ex : suppose want to calculate Barr-Zee diagrams for $m_H \gg m_t$:

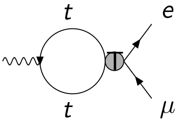
In one-loop EFT :



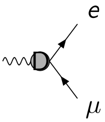
$+ \ldots \quad \Rightarrow$



$$\Delta C_T = C_S \frac{\alpha}{4\pi}(2Q_t) \ln \frac{\Lambda}{m_W}$$



$\Rightarrow$



$$m_\mu \Delta C_D = \Delta C_T \frac{\alpha}{4\pi}\Big(\frac{8N_c Q_t}{e}\boldsymbol{m_t}\Big) \ln \frac{\Lambda}{m_W}$$
$$= C_S \left(\frac{\alpha}{4\pi}\right)^2 \Big(\frac{16 N_c Q_t^2}{e}\boldsymbol{m_t}\Big) \ln^2 \frac{\Lambda}{m_W}$$

$\mathcal{O}([\alpha \ln]^2/\Lambda^2)$ part of Barr-Zee diagrams (t, heavy Higgs) in 1-loop RGES !
Can check QCD corrections ~cancel at $\mathcal{O}([\alpha \ln]^3/\Lambda^2)$
(running with QED above m_W because simple)

What do we learn about NP in lepton sector ?

$\Lambda_{NP} \gg$ TeV

$\{Z, W, \gamma, g, h, t, f\}$

$\vec{C}_{above} = \vec{C}_{below}\,\boldsymbol{V}$ $\quad m_W \sim m_h \sim m_t$

$\{\gamma, g, e\mu, \tau, u, d, c, s, b\}$

$$\text{RGEs} : \mu\frac{\partial}{\partial\mu}\vec{C} = \vec{C}\boldsymbol{\Gamma}$$
$$\Rightarrow \vec{C}(m_\tau) \sim \vec{C}(m_W)\exp\{-\boldsymbol{\Gamma}\log\}$$

GeV $\sim m_c, m_b, m_\tau$

$\{\gamma, e, \mu, p, n, (\pi)\}$ data

Including the loops in EFT

line up all operator coefficients in row vector $\vec{C}$, satifies $\mu \frac{\partial}{\partial \mu} \vec{C} = \vec{C}\mathbf{\Gamma}$. Solution :

$$\vec{C}(m_\mu) = \vec{C}(m_W)\boldsymbol{G}$$

Including the loops in EFT

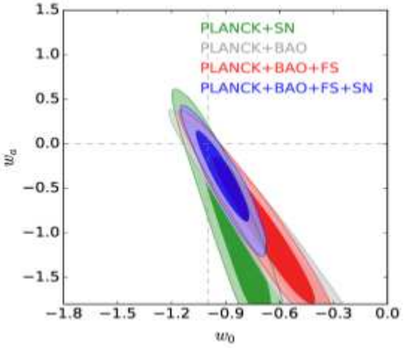
line up all operator coefficients in row vector $\vec{C}$, satifies $\mu\frac{\partial}{\partial\mu}\vec{C} = \vec{C}\mathbf{\Gamma}$. Solution :

$$\vec{C}(m_\mu) = \vec{C}(m_W)\boldsymbol{G}$$

$$\begin{aligned}
C_{D,X}(m_\mu) = \; & C_{D,X}(m_W)\left(1 - 16\frac{\alpha_e}{4\pi}\ln\frac{m_W}{m_\mu}\right) \\
& -\frac{\alpha_e}{4\pi e}\ln\frac{m_W}{m_\mu}\left(-8\frac{m_\tau}{m_\mu}C^{\tau\tau}_{T,XX} + C^{\mu\mu}_{S,XX} + C_{2loop}\right) \\
& +16\frac{\alpha_e^2}{2e(4\pi)^2}\ln^2\frac{m_W}{m_\mu}\left(\frac{m_\tau}{m_\mu}C^{\tau\tau}_{S,XX}\right) \\
& -8\lambda^{a_T}\frac{\alpha_e}{4\pi e}\ln\frac{m_W}{2\text{ GeV}}\left(-\frac{m_s}{m_\mu}C^{ss}_{T,XX} + 2\frac{m_c}{m_\mu}C^{cc}_{T,XX} - \frac{m_b}{m_\mu}C^{bb}_{T,XX}\right) f_{TD} \\
& +16\frac{\alpha_e^2}{3e(4\pi)^2}\ln^2\frac{m_W}{2\text{ GeV}}\left(\sum_{u,c}4\frac{m_q}{m_\mu}C^{qq}_{S,XX} + \sum_{d,s,b}\frac{m_q}{m_\mu}C^{qq}_{S,XX}\right)
\end{aligned}$$

$\lambda = \alpha_s(m_W)/\alpha_s(2\text{GeV}) \simeq 0.44$, $f_{TS} \simeq 1.45$, $a_S = 12/23$, $a_T = -4/23$.

Do we get ellipses in parameter space— or are there flat directions ?



Ideally : more constraints than parameters, build models that sit in overlap of ellipses

$\Rightarrow$ **How many constraints on how many coefficients from $\mu \to e\gamma$, $\mu \to e\bar{e}e$ and $\mu - e$ conversion ?**

Kuno Okada

μ interaction with nucleon $N \in \{n, p\}$ parametrised by 20 4-f operators :

$$\begin{array}{rrll} S,\ V & \overline{e}P_X\mu\overline{N}N & \overline{e}\gamma^\alpha P_X\mu\overline{N}\gamma_\alpha N & X \in \{L, R\} \\ A,\ T & \overline{e}\gamma^\alpha P_X\mu\overline{N}\gamma_\alpha\gamma_5 N & \overline{e}\sigma^{\alpha\beta}P_X\mu\overline{N}\sigma_{\alpha\beta}N & \\ P,\ Der & \overline{e}P_X\mu\overline{N}\gamma_5 N & \overline{e}\gamma^\alpha P_X\mu(\overline{N}i\overleftrightarrow{\partial}_\alpha\gamma_5 N) & \end{array}$$

Matching in χPT gives Derivative. But absorb in matching into $G_O^{N,q}$= quark matrix elements in nucleons. and 2 dipoles

$$D \qquad \overline{e}\sigma^{\alpha\beta}P_X\mu F_{\alpha\beta}$$

which also contribute in $\mu \to e\gamma$, $\mu \to e\bar{e}e$. For $\mu \to e\bar{e}e$

$$\begin{array}{lll} V & (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e) & (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e) \\ S & (\overline{e}P_Y\mu)(\overline{e}P_Y e) & \end{array}$$

28 operators

chiral basis for the lepton current (relativistic e), but not for the non-rel. nucleons.

$\mu \leftrightarrow e$ operators (at scale $m_W \leftrightarrow m_\tau$) ; otherwise flav.diag.

$m_\mu(\overline{e}\sigma^{\alpha\beta}P_Y\mu)F_{\alpha\beta}$ *dim* 5 Kuno-Okada

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e)$$
$$(\overline{e}P_Y\mu)(\overline{e}P_Y e) \qquad (\overline{e}P_Y\mu)(\overline{\mu}P_Y\mu)$$
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$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_Y f) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_X f)$$
$$(\overline{e}P_Y\mu)(\overline{f}P_Y f) \qquad (\overline{e}P_Y\mu)(\overline{f}P_X f)$$
$$(\overline{e}\sigma P_Y\mu)(\overline{f}\sigma P_Y f)$$

dim 6

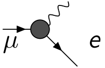
$$\frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}G^{\alpha\beta} \qquad \frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}\widetilde{G}^{\alpha\beta}$$
$$\frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}F^{\alpha\beta} \qquad \frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}\widetilde{F}^{\alpha\beta}$$

dim 7

...zzz...

$f \in \{\tau, u, d, c, s, b\}$, $P_X \neq P_Y = (1 \pm \gamma_5)/2$
82 operators. + 80 more if allow quark flavour-changing.$\times$3 to account for $\mu \leftrightarrow e$, $\tau \leftrightarrow e$, and $\tau \leftrightarrow \mu$. + 48 $\Delta L_i = 2$.

Counting constraints : $\boldsymbol{\mu \to e\gamma + \mu \to e\bar{e}e}$



Two dipole operators contribute to $\mu \to e\gamma$:

$$\delta\mathcal{L}_{meg} = -\frac{4G_F}{\sqrt{2}} m_\mu \left(C_{D,R}\overline{\mu_R}\sigma^{\alpha\beta} e_L F_{\alpha\beta} + C_{D,L}\overline{\mu_L}\sigma^{\alpha\beta} e_R F_{\alpha\beta} \right)$$

$$BR(\mu \to e\gamma) = 384\pi^2(|C_{D,R}|^2 + |C_{D,L}|^2) < 5.7 \times 10^{-13} \Rightarrow |C_X^D| \lesssim 10^{-8}$$

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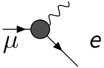

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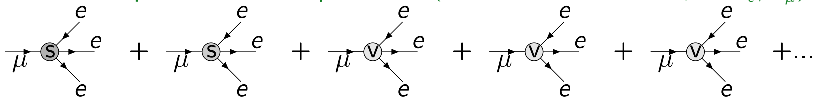
$$BR(\mu \to e\gamma) = 384\pi^2 (|C_{D,R}|^2 + |C_{D,L}|^2) < 5.7 \times 10^{-13} \Rightarrow |C_X^D| \lesssim 10^{-8}$$

2dipoles + 6 4-f-ops contribute to $\mu \to e\bar{e}e$, (most interference between ops $\propto m_e^2/m_\mu^2$)



$(\bar{e}P_L\mu)(\bar{e}P_Le)$ $(\bar{e}P_R\mu)(\bar{e}P_Re)$ $(\bar{e}\gamma P_L\mu)(\bar{e}\gamma P_Le)$ $(\bar{e}\gamma P_R\mu)(\bar{e}\gamma P_Re)$ $(\bar{e}\gamma P_L\mu)(\bar{e}\gamma P_Re)$

$$BR(\mu \to e\bar{e}e) = \frac{|C_{S,LL}|^2 + |C_{S,RR}|^2}{8} + 2|C_{V,RR}|^2 + 2|C_{V,LL}|^2 + |C_{V,LR}|^2 + |C_{V,RL}|^2$$

$$\leq 10^{-12} \quad \Rightarrow \quad \boldsymbol{|C_X| \lesssim 10^{-6} \sqrt{BR/10^{-12}}}$$

see nothing in $\mu \to e\gamma$, $\mu \to e\bar{e}e$, $\Rightarrow$ all 8 Cs small

$(BR(\mu \to e\gamma) < 6 \cdot 10^{-\mathbf{14}}, BR(\mu \to e\bar{e}e) < 10^{-\mathbf{16}} \Rightarrow C_D \leq 2 \cdot 10^{-\mathbf{9}})$

see something $\Rightarrow$ distinguish operator via angular distributions?

$\mu \to e$ conversion, again

$$\mathrm{BR}_{SI}(A\mu \to Ae) \;\propto\; \left|\widetilde{C}^{pp}_{V,R} V^{(p)}_A + \widetilde{C}'^{pp}_{S,L} S^{(p)}_A + \widetilde{C}^{nn}_{V,R} V^{(n)}_A + \widetilde{C}'^{nn}_{S,L} S^{(n)}_A + C_{D,L} D\right|^2 + \{L \leftrightarrow R$$

$$\sim\; Z^2 \left|\vec{C}_R \cdot \hat{v}_A\right|^2 + Z^2 \left|\vec{C}_L \cdot \hat{v}_A\right|^2 \qquad \vec{v}_A \equiv \left(V^{(p)}_A, S^{(p)}_A, V^{(n)}_A, S^{(n)}_A, D_A\right)$$

$$\mathrm{BR}_{SD}(A\mu \to Ae) \;\sim\; \left|\widetilde{C}^{NN}_{A,L} + 2\widetilde{C}^{NN}_{T,R}\right|^2 + \left|\widetilde{C}^{NN}_{A,R} + 2\widetilde{C}^{NN}_{T,L}\right|^2 \qquad (N \text{ odd})$$

Can distinguish SD vs SI, L vs R. But if observe SI conversion, how to know if is due to scalar/vector operator on n or p?

$\mu \to e$ conversion, again

$$\mathrm{BR}_{SI}(A\mu \to Ae) \propto \left|\widetilde{C}^{pp}_{V,R} V^{(p)}_A + \widetilde{C}'^{pp}_{S,L} S^{(p)}_A + \widetilde{C}^{nn}_{V,R} V^{(n)}_A + \widetilde{C}'^{nn}_{S,L} S^{(n)}_A + C_{D,L} D\right|^2 + \{L \leftrightarrow R$$

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Can distinguish SD vs SI, L vs R. But if observe SI conversion, how to know if is due to scalar/vector operator on n or p ?Change nucleus ?

KitanoKoikeOkada

$$S^{(p)}_A, V^{(p)}_A \sim \int d^3x \widetilde{\psi}^{1s}_\mu |f_p(x)|^2 \widetilde{\psi}^*_e (\bar{p}\{1, \gamma_0\} p)$$

$\mu \rightarrow e$ conversion, again

$$\mathrm{BR}_{SI}(A\mu \to Ae) \;\propto\; \left|\widetilde{C}^{pp}_{V,R} V^{(p)}_A + \widetilde{C}'^{pp}_{S,L} S^{(p)}_A + \widetilde{C}^{nn}_{V,R} V^{(n)}_A + \widetilde{C}'^{nn}_{S,L} S^{(n)}_A + C_{D,L} D\right|^2 + \{L \leftrightarrow R\}$$

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$$S^{(p)}_A, V^{(p)}_A \sim \int d^3x \widetilde{\psi}^{1s}_\mu |f_p(x)|^2 \widetilde{\psi}^*_e (\bar{p}\{1, \gamma_0\} p)$$

different "target vectors" $\vec{v}_A$ for different nuclear targets

target vectors "live" in coefficient space, like $\vec{C} = (\widetilde{C}^{pp}_V, \widetilde{C}^{pp}_S, \widetilde{C}^{nn}_V, \widetilde{C}^{nn}_S, (D))$

1. 1st exptal search (eg Gold) probes $\vec{C} \parallel \vec{v}_{Au}$

2. next target, suff large component $\perp$ Gold

$\Rightarrow$ three (suitable) nuclear targets (+improve theory caln) could probe 3 combinations of $\{\widetilde{C}^{pp}_V, \widetilde{C}^{pp}_S, \widetilde{C}^{nn}_V, \widetilde{C}^{nn}_S\}$

DKunoSaporta
DKunoYamanaka

parametrise with 20 nucleon ops (8 SI : S,V) + (12 SD : P,A,T)
+2 dipole operators
+6 four-lepton operators

1. constrain 2 dipoles +6 4ℓ coeffs with $\mu \to e\gamma + \mu \to e\bar{e}e$

2. Spin Indep now : constrain 4 combinations of 8 $\{S, V\}$ coefficients
SI future : constrain 6 combinations of 8 $\{S, V\}$ coefficients

3. Spin-Dependent, now : (?) 2 counstraints ? (Ti ?)
future : $4 \to 8$ constraints ?
n vs p by comparing odd-p, A vs T vs P $\Leftrightarrow$ dedicated nucl.caln.)

$$\Rightarrow 28 \text{ coefficients}, \left\{ \begin{array}{cc} \text{now} & 12 \to 14 \\ \text{future} & 18 \to 22 \end{array} \right\} \text{constraints}$$

...so what to do ?
(no ellipse in coeff space even at exptal scale)

Can still calculate sensitivities...

sensitivity : "one at a time bound" = below, parameter is to small to see in expt. (But larger possible, if cancelled by another contribution.)

coefficient	$\mu \to e\gamma$	$\mu \to e\bar{e}e$	$\mu - e$ conv.
$\lvert C_{D,X}\rvert$	1.12×10^{-8}	4.30×10^{-7}	2.35×10^{-7}
$\lvert C^{ee}_{V,XX}\rvert$	1.10×10^{-4}	7.80×10^{-7}	1.86×10^{-5}
$\lvert C^{ee}_{V,XY}\rvert$	2.55×10^{-4}	9.34×10^{-7}	3.77×10^{-5}
$\lvert C^{ee}_{S,XX}\rvert$	1.73×10^{-4}	2.8×10^{-6}	(3.64×10^{-3})
$\lvert C^{\mu\mu}_{V,XX}\rvert$	1.10×10^{-4}	5.60×10^{-5}	1.85×10^{-5}
$\lvert C^{\mu\mu}_{V,XY}\rvert$	2.56×10^{-4}	1.12×10^{-4}	3.77×10^{-5}
$\lvert C^{\mu\mu}_{S,XX}\rvert$	8.24×10^{-7}	(1.58×10^{-5})	(1.73×10^{-5})
$\lvert C^{\tau\tau}_{V,XX}\rvert$	3.80×10^{-4}	1.95×10^{-4}	1.24×10^{-5}
$\lvert C^{\tau\tau}_{V,XY}\rvert$	4.40×10^{-4}	1.91×10^{-4}	1.25×10^{-5}
$\lvert C^{\tau\tau}_{S,XX}\rvert$	5.33×10^{-6}	1.02×10^{-4}	1.12×10^{-4}
$\lvert C^{\tau\tau}_{S,XY}\rvert$	—	—	—
$\lvert C^{\tau\tau}_{T,XX}\rvert$	1.10×10^{-8}	(4.20×10^{-7})	(2.30×10^{-7})

Table – Current sensitivities of $\mu \to e\gamma$, $\mu \to e\bar{e}e$, and $\mu - e$ conv. to the coefficients, at m_W, of QCD×QED-invariant 2- and 4-lepton operators. $X, Y \in \{L, R\}, X \neq Y$.

Summary

Lepton Flavour Violation is BSM that exists, and experimental sensitivities set to improve by orders of magnitude in coming years ($\rightarrow BR \sim 10^{-16} \rightarrow 10^{-18}$ for $\mu \leftrightarrow e$).

Current experimental constraints are restrictive $\Rightarrow$ sensitivity to loop-induced LFV at scales beyond the LHC

If assume LFV induced by heavy New Physics, can parametrise LFV interactions with non-renormalisable operators, and include loop effects via RGEs for operator coefficients. Find that (in $\mu \leftrightarrow e$ sector)

- (many) more operators than constraints $\Rightarrow$ many "flat directions"
-
-
- *question for you :what else can we learn ?*

I use Dirac spinors, with 4 degrees of freedom(dof) labelled by $\{\pm E, \pm s\}$, in *chiral* decomposition

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad , \; \{\gamma^\alpha\} = \left\{ \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix} \right\}$$

$$\{\sigma_i\} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\psi_L = \; P_L \, \psi \quad \text{avec} \quad P_L = \frac{(1-\gamma_5)}{2} \quad , \; \psi_R = \; P_R \, \psi$$

chirality is *not* an observable ($\rightarrow$ helicity $= \pm \hat{s} \cdot \hat{k} = \pm 1/2$ in relativistic limit), but $P_{L,R}$ simple to calculate with :)

notation : $\overline{(\psi_R)} = (P_R \psi)^\dagger \gamma_0 = \psi^\dagger P_R \gamma_0 = \psi^\dagger \gamma_0 P_L = (\overline{\psi})_L$

$(\psi^c)_L = P_L(-i\gamma_0\gamma_2\gamma_0\psi^*) = -i\gamma_0\gamma_2\gamma_0\psi^*_R$

- 3 generations of lepton doublets, and charged singlets :

$$\ell_{\alpha L} \in \left\{ \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} , \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} , \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \right\} \qquad e_{\alpha R} \in \{e_R,\ \mu_R,\ \tau_R\}$$

in charged lepton mass basis $\equiv$ **flavour** basis (greek index, eg α).

Leptons in the Standard Model

- 3 generations of lepton doublets, and charged singlets :

$$\ell_{\alpha L} \in \left\{ \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} , \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} , \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \right\} \qquad e_{\alpha R} \in \{e_R,\ \mu_R,\ \tau_R\}$$

in charged lepton mass basis ≡ **flavour** basis (greek index, eg α).

- (only) three lepton flavours in the Standard Model : e, μ, τ (*different* from quarks, where 6 flavours)
  so LFV ≡ CLFV ≡ $\text{FCNC}|_{quarks}$

- 3 generations of lepton doublets, and charged singlets :

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in charged lepton mass basis $\equiv$ **flavour** basis (greek index, eg α).

- (only) three lepton flavours in the Standard Model : e, μ, τ (*different* from quarks, where 6 flavours)
  so LFV $\equiv$ CLFV $\equiv$ FCNC$|_{quarks}$
- No ν_R in SM because
  1. data did not require m_ν when SM was defined (ν are shy in the lab...)
  2. ν_R an SU(2) singlet $\Leftrightarrow$ no gauge interactions
     $\Rightarrow$ not need ν_R for anomaly cancellation
     $\Rightarrow$ if its there, its hard to see

Lagrangian that reproduces leptons interactions(but not ν masses)

(also most general renormalisable, $SU(2) \times U(1)$-invariant $\mathcal{L}$)

(Exercise : why not write $\overline{\ell_\mu} \not{D} \, \ell_e$?)

$$\mathcal{L} = i\overline{\ell_{L}}^{T}_{\alpha} \gamma^\mu \mathbf{D}_\mu \, \ell_{L\alpha} + i\overline{e_R}_\alpha \gamma^\mu D_\mu \, e_{R\alpha}$$
$$- \left[(\overline{\nu_{\alpha L}}, \overline{e_{\alpha L}}) y_\alpha \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} e_{\alpha R} + \text{h.c.} \right]$$

$$\mathbf{D}_\mu = \partial_\mu + i\frac{g}{2}\sigma^a W^a_\mu + ig' Y(\ell_L) B_\mu \ , \ D_\mu = \partial_\mu + ig' Y(e_R) B_\mu$$

B^μ hypercharge gauge boson, $Y(f) = T_3 + Q_{em}$

$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$, $m_\alpha = y_\alpha \langle H^0 \rangle$, $\langle H^0 \rangle \equiv v \simeq m_t$

Why are the LFV bounds so good ?

Current $\mu \to e$ Branching Ratios $\lesssim 10^{-12}$. Normalised to *weak* muon decay

$$BR(\mu \to e\bar{e}e) \equiv \frac{\Gamma(\mu \to e\bar{e}e)}{\Gamma(\mu \to e\bar{\nu}\nu)} \quad , \quad \Gamma(\mu \to e\bar{\nu}\nu) = \frac{G_F^2 m_\mu^5}{192\pi^3} = \frac{m_\mu^5}{1536\pi^3 v^4} \simeq \frac{1}{2 \times 10^{-6}\mathrm{s}}$$

...so if $\Gamma(\mu \to e\bar{e}e) \simeq \frac{m_\mu^5}{1536\pi^3 \Lambda_{LFV}^4}$ then $BR \lesssim \begin{cases} 10^{-12} \Rightarrow \Lambda_{LFV} \sim 10^3 v \simeq 200\ \mathrm{T} \\ 10^{-16} \Rightarrow \Lambda_{LFV} \sim 10^3 v \simeq 2000 \end{cases}$

Compare to $\frac{(g-2)_\mu}{2} \equiv a \simeq \alpha_{em}/\pi$ (measure Eqns o Motion : QED *amplitude*) :
torque $\vec{\tau} = \vec{\mu} \times \vec{B}$; $\vec{\mu} = g \frac{e}{2m} \vec{S}$

$$\begin{aligned} \Delta a &\equiv a^{SM} - a^{exp} \simeq 3 \times 10^{-9} \\ &\sim \frac{m_\mu^2}{16\pi^2 \Lambda_{NP}^2} \end{aligned}$$

$\Rightarrow \Lambda_{NP} \sim m_t$.

because BR is ratio to weak decays

SMEFT above m_W : $2\ell 2q$

$$\mathcal{O}_{\ell q}^{(1)\alpha\beta nm} = \frac{1}{2}(\overline{\ell}_\alpha \gamma^\mu \ell_\beta)(\overline{q}_n \gamma_\mu q_m) \qquad \mathcal{O}_{\ell q}^{(3)\alpha\beta nm} = \frac{1}{2}(\overline{\ell}_\alpha \gamma^\mu \tau^a \ell_\beta)(\overline{q}_n \gamma_\mu \tau^a q_m)$$

$$\mathcal{O}_{eq}^{\alpha\beta nm} = \frac{1}{2}(\overline{e}_\alpha \gamma^\mu e_\beta)(\overline{q}_n \gamma_\mu q_m)$$

$$\mathcal{O}_{\ell u}^{\alpha\beta nm} = \frac{1}{2}(\overline{\ell}_\alpha \gamma^\mu \ell_\beta)(\overline{u}_n \gamma_\mu u_m) \qquad \mathcal{O}_{\ell d}^{\alpha\beta nm} = \frac{1}{2}(\overline{\ell}_\alpha \gamma^\mu \ell_\beta)(\overline{d}_n \gamma_\mu d_m)$$

$$\mathcal{O}_{eu}^{\alpha\beta nm} = \frac{1}{2}(\overline{e}_\alpha \gamma^\mu e_\beta)(\overline{u}_n \gamma_\mu u_m) \qquad \mathcal{O}_{ed}^{\alpha\beta nm} = \frac{1}{2}(\overline{e}_\alpha \gamma^\mu e_\beta)(\overline{d}_n \gamma_\mu d_m)$$

$$\mathcal{O}_{\ell equ}^{\alpha\beta nm} = (\overline{\ell}_\alpha^A e_\beta)\varepsilon_{AB}(\overline{q}_n^B u_m) \qquad \mathcal{O}_{\ell edq}^{\alpha\beta nm} = (\overline{\ell}_\alpha e_\beta)(\overline{d}_n q_m)$$

$$\mathcal{O}_{T,\ell equ}^{\alpha\beta nm} = (\overline{\ell}_\alpha^A \sigma^{\beta\nu} e_\beta)\varepsilon_{AB}(\overline{q}_n^B \sigma_{\beta\nu} u_m)$$

where ℓ, q are doublets and e, u are singlets, n, m are quark family indices, taken equal, and A, B are SU(2) indices.

SMEFT ops, ctd :4-lepton+ penguins/dipoles

$$\mathcal{O}_{\ell\ell}^{\alpha\beta\rho\sigma} = \frac{1}{2}(\overline{\ell}_\alpha\gamma^\mu\ell_\beta)(\overline{\ell}_\rho\gamma_\mu\ell_\sigma)$$

$$\mathcal{O}_{\ell e}^{\alpha\beta\rho\sigma} = \frac{1}{2}(\overline{\ell}_\alpha\gamma^\mu\ell_\beta)(\overline{e}_\rho\gamma_\mu e_\sigma) \qquad \mathcal{O}_{ee}^{\alpha\beta\rho\sigma} = \frac{1}{2}(\overline{e}_\alpha\gamma^\mu e_\beta)(\overline{e}_\rho\gamma_\mu e_\sigma)$$

$$\mathcal{O}_{eH}^{\alpha\beta} = (H^\dagger H)(\overline{\ell}_\alpha H e_\beta) \qquad \mathcal{O}_{He}^{\alpha\beta} = \frac{i}{2}(H^\dagger \overleftrightarrow{D}_\mu H)(\overline{e}_\alpha\gamma^\mu e_\beta)$$

$$\mathcal{O}_{H\ell(1)}^{\alpha\beta} = \frac{i}{2}(H^\dagger \overleftrightarrow{D}_\mu H)(\overline{\ell}_\alpha\gamma^\mu\ell_\beta) \qquad \mathcal{O}_{H\ell(3)}^{\alpha\beta} = \frac{i}{2}(H^\dagger \overleftrightarrow{D}^a_\mu H)(\overline{\ell}_\alpha\gamma^\mu\tau^a\ell_\beta)$$

$$\mathcal{O}_{eW}^{\alpha\beta} = y_\beta(\overline{\ell}_\alpha\tau^a H\sigma^{\mu\nu}e_\beta)W^a_{\mu\nu} \qquad \mathcal{O}_{eB}^{\alpha\beta} = y_\beta(\overline{\ell}_\alpha H\sigma^{\mu\nu}e_\beta)B_{\mu\nu}$$

where y_β = charged lepton Yukawa, and
$i(H^\dagger \overleftrightarrow{D}^a_\mu H) \equiv i(H^\dagger\tau^a D_\mu H) - i(D_\mu H)^\dagger\tau^a H$.

For 4-lepton ops, can have $\Delta L_i = 1, 2$. Also, Warsaw basis supposes that flavour change across the bilinears is allowed ; if impose one unit of flavour change in first bilinear, should add triplet operator $\mathcal{O}_{\ell\ell}^{\mathbf{3},e\mu ee}$. If in $\mathcal{L}$ sum all indices over all flavours, there are 2s for $\mathcal{O}_{ee}$ and $\mathcal{O}_{\ell\ell}$ ($(\overline{e}\gamma^\mu\mu)(\overline{\tau}\gamma_\mu\tau) = (\overline{\tau}\gamma^\mu\tau)(\overline{e}\gamma_\mu\mu)$) :

To calculate SI $\mu - e$ conversion (at LO in χPT...)

differs from WIMP scattering in that μ and nucleus charged

1. suppose start with $\mu \leftrightarrow e$ operators involving gluons, γ, u, d, s, c, b
2. match quark/gluon operators onto nucleon ($N \in \{n, p\}$) operators :

$$\bar{q}(x)\Gamma_O q(x) \to G_O^{N,q} \bar{N}(x)\Gamma_O N(x)$$

Gs in Appendix

eg, $\langle N|\bar{q}(x)q(x)|N\rangle = G_O^{N,q}\langle N|\bar{N}(x)N(x)|N\rangle = G_O^{N,q}\overline{u_N}(P_f)u_N(P_i)e^{-i(P_f-P_i)x}$

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3. *imagine* to build the atom as a bound state of nucleus and muon in $1s$ state

in P+S

$$|\mu A(\vec{P}_i = 0)\rangle = \sqrt{\frac{2(M_A + m_\mu)}{4M_A m_\mu}} \sum_s \int \frac{d^3k}{(2\pi)^3} \widetilde{\psi}_\mu(\vec{k})\, |A(-\vec{k})\rangle \otimes |\mu(\vec{k}, s)\rangle$$

then build nucleus as bd state of nucleons (app. B of 1203.3542),gives :

SD overlap int : guess from SD DM targets

$$\langle e, A|\widetilde{C}_O O|\mu A\rangle \propto \widetilde{C}_O(\bar{u}_e\Gamma_O u_\mu) \int d^3x \psi_\mu^{1s}|f_N(x)|^2\psi_e(\bar{N}\Gamma_O N)$$

where "overlap integral" over nucleus, of muon wavefn ($\widetilde{\psi}_\mu^{1s}$), nucleon density ($|f_N(x)|^2$), e wavefn ($\psi_e \sim e^{-iqx}$) and operator performed in KitanoKoikeOkada.

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3. look up rate in KitanoKoikeOkada, PRD (2002), eqn 14, check your operator normalisation against KKO eqn 1, read numerical value of overlap integrals from table I, and divide by capture rate in table VIII of KKO.

Shortcut to calculate $\mu - e$ conversion

shortcut for current bounds (Gold and Titanium) :write

$BR_{SI}(\mu A \to eA) = B_A \Big[|\hat{v}_A \cdot \vec{C}_L|^2 + |\hat{v}_A \cdot \vec{C}_R|^2 \Big]$

where

$$\vec{C}_L = (\widetilde{C}_{D,R}, \widetilde{C}^{pp}_{S,R}, \widetilde{C}^{pp}_{V,L}, \widetilde{C}^{nn}_{S,R}, \widetilde{C}^{nn}_{V,L})$$

$$B_A \equiv \frac{32 G_F^2 m_\mu^5 |\vec{v}_A|^2}{\Gamma_{cap}(A)} = \begin{cases} 250 & Ti \\ 300 & Au \\ 142 & Al \end{cases}$$

and normalised overlap integrals of KKO are lined up in target vectors

$$\hat{v}_{Ti} = (0.250, 0.426, 0.458, 0.503, 0.541)$$
$$\hat{v}_{Au} = (0.222, 0.289, 0.458, 0.432, 0.686)$$

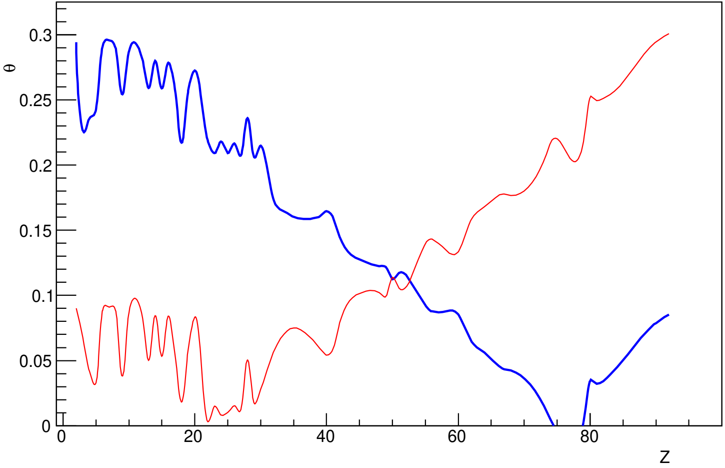
(Spin Dep : ?likely in noise of SI signal—RGEs of QED mix T,A$\to$ S, V.
To calculate, need nuclear caln, see discussion inCiriglianoDKuno, DKunoSaporta)

Gs

$G_V^{p,u} = G_V^{n,d} = 2$	$G_V^{p,d} = G_V^{n,u} = 1$	$G_V^{p,s} = G_V^{n,s} = 0$
$G_A^{p,u} = G_A^{n,d} = 0.84$ $G_A^{p,u} = G_A^{n,d} = 0.863$	$G_A^{p,d} = G_A^{n,u} = -0.43$ $G_A^{p,d} = G_A^{n,u} = -0.345$	$G_A^{p,s} = G_A^{n,s} = -.085$ $G_A^{p,s} = G_A^{n,s} = -.0240$
$G_S^{p,u} = 5.9$ $(G_S^{p,u} = 9.0)$ $G_S^{n,u} = 5.0$ $(G_S^{n,u} = 8.1)$ $G_S^{N,c} = \frac{2m_N}{17m_c}$	$G_S^{p,d} = 5.0$ $(G_S^{p,d} = 8.2)$ $G_S^{n,d} = 6.0$ $(G_S^{n,d} = 9.0)$ $(G_S^{N,b} = \frac{2m_N}{17m_b}$	$G_S^{p,s} = 0.42$ $G_S^{n,s} = 0.42$ $(G_S^{n,s} = 0.42)$
$G_P^{p,u} = 144 = G_P^{n,d}$	$G_P^{p,d} = -150 = G_P^{n,u}$	$G_P^{p,s} = -4.9 = G_P^{n,s}$
$G_T^{p,u} = G_T^{n,d} = 0.77(7)$	$G_T^{p,d} = G_T^{n,u} = -0.23(3)$	$G_T^{p,s} = G_T^{n,s} = .008(9)$
$G_S^{N,gg} = -8\pi m_N/(9\alpha_s v)$		

Table – Matching coefficients between nucleon and operators and gluon or light-quark(flavour-diagonal) operators. References in DKunoSaporta. Scalar Gs in parentheses are EFT caln, otherwise from lattice. In all cases, the $\overline{\mathrm{MS}}$ quark masses at $\mu = 2$ GeV are taken as $m_u = 2.2$ MeV, $m_d = 4.7$ MeV, and $m_s = 96$ MeV. The nucleon masses are $m_p = 938$ MeV and $m_n = 939.6$ MeV.

Current data+ theory uncertainty $\sim 10\%$: two targets give $\Delta\theta > 0.2$



$$\vec{v}_A = (V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)}), \text{ and } BR \propto |\vec{v}_A \cdot \vec{C}|^2$$

$\vec{v}_{Au} \cdot \vec{v}_Z \equiv |\vec{v}_{Au}||\vec{v}_Z| \cos\theta$ ...plot θ on vertical axis