Thoughts on Unitarity Triangle(s) Fits

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Outline

- The CKM Matrix
- Unitarity
- Theory considerations:
  - Inclusive semileptonic decays
  - CP violation
  - Time dependent CP asymmetries
- Fits galore
- New physics effects
The CKM matrix
The Standard Model
fundamental particle zoo
A problem with masses

- SU(2) gauge invariance requires the **gauge bosons** to be exactly massless.

- Mass terms for **fermions** break gauge invariance too:

\[ \mathcal{L}_m = -m\bar{\psi}\psi = -m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R) \]

- Both problems are solved by spontaneously breaking \( SU(2) \times U(1)_Y \rightarrow U(1)_{\text{em}} \)
The SM Lagrangian

- The following Lagrangian describes almost everything that we have ever observed.

\[
\mathcal{L}_{SM} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} F_{\mu\nu}^i F^{i\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \theta_{\text{QCD}} \frac{g_s^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^{a\rho\sigma} \\
+ \bar{L}_L i \mathcal{D} L_L + \bar{Q}_L i \mathcal{D} Q_L + \bar{e}_R i \mathcal{D} e_R + \bar{d}_R i \mathcal{D} d_R + \bar{u}_R i \mathcal{D} u_R \\
+ |D_\mu H|^2 + m_H^2 |H^\dagger H| - \lambda |H^\dagger H|^2 \\
- [\bar{L}_L H Y^e e_R + \bar{Q}_L H Y^d d_R + \bar{Q}_L H^c Y^u u_R + \text{h.c.}]
\]

- Hypercharge:

\[
\begin{array}{cccccc}
\frac{1}{2} & \frac{1}{2} & -1 & \frac{1}{6} & \frac{1}{2} & -\frac{1}{3} \\
\end{array}
\]

\[
H^c = i\sigma_2 H^* 
\]

- After SSB, \( H \rightarrow \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \) and \( H^c \rightarrow \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \) generate fermion masses.

- \( \theta_{\text{QCD}} \) violates CP and is experimentally constrained to be very small (< 10^{-10}). It is possible to ensure a vanishing \( \theta_{\text{QCD}} \) by imposing a certain U(1) symmetry on the theory (Peccei-Quinn). A generic prediction of theories with a U(1)_{PQ} is the existence of an axion.
The CKM matrix

- Let’s focus on the Yukawa interactions:
  \[ \mathcal{L}_{\text{Yukawa}} = -[\bar{L}_L H Y_E e_R + \bar{Q}_L H Y_D d_R + \bar{Q}_L H^c Y_U u_R + \text{h.c.}] \]

- The Yukawa's are complex 3x3 matrices:
  \[ Y_U = U_L Y^\text{diag}_U U_R, \quad Y_D = D_L Y^\text{diag}_D D_R, \quad Y_E = E_L Y^\text{diag}_E E_R \]

- From Gauge to Mass eigenstates:
  - neutral currents: \[ \bar{u}_L^0 \not\!Z u_L^0 \implies \bar{u}_L \not\!Z U_L U_L^\dagger u_L = \bar{u}_L \not\!Z u_L \]
  - charged currents: \[ \bar{u}_L^0 \not\!W d_L^0 \implies \bar{u}_L \not\!W U_L D_L^\dagger d_L = \bar{u}_L \not\!W V_{\text{CKM}} d_L \]

\[ V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]
Flavor Changing Neutral currents (GIM mechanism)

1-loop (down sector):

\[
V_{ub}^* V_{us} f\left(\frac{m_u^2}{m_W^2}\right) + V_{cb}^* V_{cs} f\left(\frac{m_c^2}{m_W^2}\right) + V_{tb}^* V_{ts} f\left(\frac{m_t^2}{m_W^2}\right)
\]

\[
V_{tb}^* V_{ts} \left( f\left(\frac{m_t^2}{m_W^2}\right) - f\left(\frac{m_u^2}{m_W^2}\right) \right) + V_{cb}^* V_{cs} \left( f\left(\frac{m_c^2}{m_W^2}\right) - f\left(\frac{m_u^2}{m_W^2}\right) \right)
\]

\[
\sim V_{tb}^* V_{ts} \left( f\left(\frac{m_t^2}{m_W^2}\right) - f(0) \right) \propto V_{tb}^* V_{ts} \frac{m_t^2}{m_W^2}
\]

1-loop (up sector):

\[
V_{ud}^* V_{cd} f\left(\frac{m_d^2}{m_W^2}\right) + V_{us}^* V_{cs} f\left(\frac{m_s^2}{m_W^2}\right) + V_{ub}^* V_{cb} f\left(\frac{m_b^2}{m_W^2}\right)
\]

\[
V_{ub}^* V_{cb} \left( f\left(\frac{m_b^2}{m_W^2}\right) - f\left(\frac{m_d^2}{m_W^2}\right) \right) + V_{us}^* V_{cs} \left( f\left(\frac{m_s^2}{m_W^2}\right) - f\left(\frac{m_d^2}{m_W^2}\right) \right)
\]

\[
\sim V_{ub}^* V_{cb} \left( f\left(\frac{m_b^2}{m_W^2}\right) - f(0) \right) \propto V_{ub}^* V_{cb} \frac{m_b^2}{m_W^2}
\]

\[
\sim (\text{combination of CKM entries}) \frac{m_b^2}{m_W^2}
\]
Physical parameters in the CKM matrix

- We started with two arbitrary complex matrices \((Y_U, Y_D)\): \(4n^2\) parameters
- We used three unitary matrices: \(-3n^2\) parameters
  \[Y_U = V_{CKM}D_L Y_U^{\text{diag}} U_R, \quad Y_D = D_L Y_D^{\text{diag}} D_R\]
- The matrices \(e^{-i\phi} U_L, e^{i\phi} U_R, e^{i\phi} D_R\) achieve the same structure: +1 parameter
- Total number of parameters: \(4n^2 - 3n^2 + 1 = n + n + (n - 1)^2\)
  \[Y_U^{\text{diag}}, \quad Y_D^{\text{diag}}, \quad V_{CKM}\]
- Physical parameters in the CKM:
  \[(n - 1)^2 = \frac{n(n - 1)}{2} + \frac{(n - 1)(n - 2)}{2}\]
  \[\underbrace{\text{angles}}_{\text{angles}} \quad \underbrace{\text{phases}}_{\text{phases}}\]
  repeat the calculation for real Yukawa’s and using orthogonal matrices: \(2n^2 - 3n(n - 1)/2 = n + n + n(n - 1)/2\)
- For \(n_f = 2\) there are no phases: CP violation requires three generations (Kobayashi-Maskawa mechanism)
Physical parameters in the CKM matrix

- The gauge invariant part of the Lagrangian depends on 4 parameters.
- The Yukawa sector depends on 54 parameters out of which only 13 are observables.

\[ U(3)_{LL} \otimes U(3)_{ER} \otimes U(3)_{QL} \otimes U(3)_{UR} \otimes U(3)_{DR} \]

\[ Y_E, Y_D, Y_U \]

\[ U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau \otimes U(1)_B \]

Lepton numbers baryon number
Parameterizations of the CKM matrix

- **Standard parametrization:**

  \[ V = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
  -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
  s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
  \end{pmatrix} \]

  where \( s_{ij} = \sin \theta_{ij} \) and \( c_{ij} = \cos \theta_{ij} \)

  \[ s_{12} \simeq |V_{us}|, \ s_{23} \simeq |V_{cb}|, \ s_{13} \simeq |V_{ub}| \]

- **Wolfenstein parametrization:**

  \[ \lambda \equiv s_{12}, \ A \equiv s_{23}/\lambda^2, \ \rho + i\eta \equiv s_{13}e^{i\delta}/A\lambda^3 \]

  \[ V = \begin{pmatrix}
  1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\
  -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
  A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
  \end{pmatrix} + O(\lambda^4) \]

  Note that modern accuracy requires to extend the Wolfenstein parametrization to \( O(\lambda^8) \). This is done assuming that (*) holds exactly.
“Direct” determination of CKM entries

- $|V_{ud}| = 0.97420(21)$: $\theta^+ \rightarrow \theta^+$ nuclear $\beta$ decay

- $|V_{us}| = 0.2243(5)$: $K^0 \rightarrow \pi l \nu +$ lattice QCD

- $|V_{cd}| = 0.218(4)$: $\nu_\mu N \rightarrow \mu X$ vs $\nu_\mu N \rightarrow \mu \mu^+ \nu_\mu X$. $D \rightarrow \pi l \nu$ and $D \rightarrow l \nu +$ lattice QCD

- $|V_{cs}| = 0.997(17)$: $D_s \rightarrow \mu \nu$, $D \rightarrow Kl \nu$, $W \rightarrow cs +$ lattice QCD

- $|V_{cb}|_{\text{incl}} = (42.46\pm0.88) \times 10^{-3}$: $B \rightarrow X_c l \nu$
  $|V_{cb}|_{\text{excl}} = (39.08\pm0.91) \times 10^{-3}$: $B \rightarrow D(\*) l \nu$ and $B \rightarrow \tau \nu +$ lattice QCD

- $|V_{ub}|_{\text{incl}} = (4.52\pm0.20) \times 10^{-3}$: $B \rightarrow X_u l \nu$
  $|V_{ub}|_{\text{excl}} = (3.73\pm0.14) \times 10^{-3}$: $B \rightarrow \pi l \nu +$ lattice QCD

- $|V_{tb}| = 1.019(25)$: single top production

- $|V_{td}| = (8.1\pm0.5) \times 10^{-3}$: $\Delta M_{Bd} +$ lattice QCD

- $|V_{ts}| = (39.4\pm2.3) \times 10^{-3}$: $\Delta M_{Bs} +$ lattice QCD
The Jarlskog invariant

- It can be shown that, in the SM, CPV effects are present if and only if

\[ F_u F_d J \neq 0 \]

where:

\[
F_u = (m_u^2 - m_c^2)(m_c^2 - m_t^2)(m_t^2 - m_u^2)
\]

\[
F_d = (m_d^2 - m_s^2)(m_s^2 - m_b^2)(m_b^2 - m_d^2)
\]

\[
J = \text{Im}[V_{us} V_{cd} V_{cs} V_{ub}]
\]

\[
= c_{12} c_{23} c_{13}^2 s_{12} s_{23} s_{13} \sin \delta
\]

\[
= A^2 \lambda^6 \eta
\]

- If there is a degeneracy either in the up or down quark mass matrices we have more freedom to choose the unitary transformations \( D_L \), \( D_R \) and \( U_R \) and we can eliminate the CPV phase \( \delta \)

- All CPV effects are proportional to \( J \), hence they are small even if the phase \( \delta \) is large
Unitarity
Unitarity triangles: $VV^\dagger = V^\dagger V = 1$

- $B_d$: $(V^\dagger V)_{bd} = 0$
  
  \[ V_{td} = |V_{td}| \, e^{-i\beta} \]
  \[ V_{ub} = |V_{ub}| \, e^{-i\gamma} \]

- $B_s$: $(V^\dagger V)_{bs} = 0$
  
  $\beta_s = \arg(V_{ts}) = \eta \lambda^2 + O(\lambda^4)$

- $D$: $(VV^\dagger)_{uc} = 0$
  
  $\alpha_D = -\gamma$
  \[ \beta_D = \gamma + \pi + O(\lambda^4) \]
  \[ \gamma_D = O(\lambda^4) \]
\[
\alpha : \ B \to \pi\pi(\rho\rho) \\
\gamma : \ B \to DK
\]

\[
\begin{align*}
\sin 2\beta &< 0 \\
\Delta m_d, \sin 2\beta
\end{align*}
\]

Two main fitters:

- CKMfitter
- UTfit

\[
\begin{align*}
V_{td} &= |V_{td}|e^{-i\beta} \\
V_{ub} &= |V_{ub}|e^{-i\gamma} \\
V_{ts} &= |V_{ts}|e^{-i\beta_s}
\end{align*}
\]
$\text{UT}_{Bd}$ through the years: 1995 before the B factories
$U_{TBd}$ through the years: 2001 first B factories results
UT\textsubscript{Bd} through the years: 2004
UT_{Bd} through the years: 2006 $\Delta M_{Bs}$ at Tevatron
$UT_{Bd}$ through the years: 2009 end of B-factories
$U_{T_{Bd}}$ through the years: 2016 LHCb and lattice-QCD
$\text{UT}_{\text{Bd}}$ through the years: now
$U_T^{B_s}$: current status
$UT_D$: current status
Semileptonic decays: $b \rightarrow (u, c) \ell \nu$
Effective Hamiltonian

There is a hierarchy between the W/Z/t scales and the energies/masses of the external particles we are interested in:

\[ m_W, m_Z, m_t \leftrightarrow m_b, m_c, m_s \]

This suggests that the physics at these two scales can be treated independently:

The non-perturbative input required are the \( B \to D \) form factors of the relevant current \((\bar{b}\gamma^\mu c)\).
Effective Hamiltonian

- A generic amplitude has the expansion:
  \[ A(i \rightarrow f) = 1 + \alpha_s(1 + L) + \alpha_s^2(1 + L + L^2) + O(\alpha_s^3) \]
  where \( L = \log \frac{m_W^2}{p_{\text{ext}}^2} \) and \( p_{\text{ext}} \) is a generic external momentum.
- At order \( \alpha_s^n \) there are terms proportional to \( \alpha_s^n L^n, \alpha_s^n L^{n-1}, \) etc...
- For \( p_{\text{ext}}^2 \ll m_W^2 \) the convergence of the perturbative expansion is spoiled:
  \[ A(i \rightarrow f) \sim 1 + \alpha_s \log \frac{m_W^2}{p_{\text{ext}}^2} + \cdots \]
  \[ = 1 + \alpha_s \log \frac{m_W^2}{\mu^2} + \alpha_s \log \frac{\mu^2}{p_{\text{ext}}^2} \cdots \]
  in the WC and in the matrix element of the operator
  \[ = C(\mu) \langle f | O(\mu) | i \rangle \]
  \[ = C'(\mu_0) \left( \frac{\alpha_s(\mu_0)}{\alpha_s(\mu_b)} \right)^{\gamma_0 \beta_0} \langle f | O(\mu_b) | i \rangle \]
  anomalous dimension of the operator

- Choosing \( \mu_0 \sim m_W \) and \( \mu_b \sim p_{\text{ext}} \) all large logs have been re-summed.
Effective Theories below $m_b$

- $m_{NP}$
- $m_t$
- $m_W, m_Z$
- $m_b$
- $m_c$
- $\sqrt{\Lambda_{QCD}} E_{\text{external}}$
- $\Lambda_{QCD}$

Effective Weak Hamiltonian

- local RGE

Optical Th./OPE
HQET

- local RGE

SCET
pQCD

- non-local RGE

Lattice QCD
LCSR

Isospin and Dalitz plot Analyses

Naive Factorization
for multi ($\geq 3$) hadron final states
Effective Theories

\[ m_W \quad \text{perturbative} \rightarrow \text{integrate out} \]

\[ A(B \rightarrow X) = \sum C_i(\mu) \langle X \left| O_i(\mu) \right| B \rangle + O \left( \frac{m_b^2}{m_W^2} \right) \]

\[ m_b \quad \text{dependence on external states (possibly non-perturbative)} \]
Effective Theories below $m_b$

$$\sqrt{\Lambda_{QCD} m_b}$$

perturbative $\rightarrow$ integrate out

but $p_{ext} \sim m_b$ !!

$$\langle X | O_i(\mu) | B \rangle = C J * \Phi + O\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

$\Lambda_{QCD}$ non-perturbative
Exclusive decays: \( B \to (\pi, D, D^*)\ell\nu \)

- We “just” need the form factors of the semileptonic V-A current

\[
\frac{d\Gamma(B \to \pi\ell\nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_\pi^2 - m_\pi^2}}{q^4 m_B^2} \left[ \left( 1 + \frac{m_\ell^2}{2q^2} \right) m_B^2 (E_\pi^2 - m_\pi^2) |f_+(q^2)|^2 \right. \\
+ \left. \frac{3m_\ell^2}{8q^2} (m_B^2 - m_\pi^2)^2 |f_0(q^2)|^2 \right]
\]

\( \ell = e, \mu \)

\[
\sim \frac{G_F^2 |V_{ub}|^2}{24\pi^3} (E_\pi^2 - m_\pi^2)^{3/2} |f_+(q^2)|^2
\]

- high-\(q^2\), small \(E_\pi\) easier on the lattice
- low-\(q^2\), large \(E_\pi\) easier experimentally

\[ z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \]
Inclusive decays: $B \rightarrow X_c \ell \nu$

- Using the optical theorem and an Operator Product Expansion (OPE) it is possible to write an expansion in which the leading term is free of hadronic uncertainties!

$$\sum_{X_c} \left| \begin{array}{c} B \\ X_c \\ \ell \\ \nu \end{array} \right|^2 \sim \text{Im} \int d^4x e^{iqx} \langle B | TO(0)O(x) | B \rangle$$

$$= \text{Im} \left[ C_3 \langle B | \bar{b}b | B \rangle + C_5 \langle B | \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b | B \rangle + \cdots \right]$$

[See the PDG review and references therein]
Inclusive decays: \( B \rightarrow X_c \ell \nu \)

- Matching calculation:
Inclusive decays: $B \rightarrow X_c \ell \nu$

Matrix elements:

$$\frac{1}{2M_B} \langle B | \bar{b} b | B \rangle = 1 + \frac{\lambda_1 + 3\lambda_2}{2m_b^2} + \ldots$$

$$\frac{1}{m_b^2} \frac{1}{2M_B} \langle B | \bar{b} \sigma_{\mu \nu} G^{\mu \nu} b | B \rangle = \frac{6\lambda_2}{m_b^2} + \ldots$$

Final result ($\rho = m_c^2/m_b^2$):

$$\Gamma(B \rightarrow X_c e \bar{\nu}) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left\{ \left( 1 + \frac{\lambda_1 + 3\lambda_2}{2m_b^2} \right) \left[ f(\rho) + \frac{\alpha_s}{\pi} g(\rho) \right] - \frac{6\rho_2}{m_b^2} (1 - \rho)^4 + \ldots \right\}$$

Known terms: NNLO at Leading Power (up to $\alpha_s^2$), NLO at subleading power (up to $\alpha_s^2/m_b^2$), LO for the rest ($1/m_b^3, 1/m_b^4$)
Inclusive decays: $B \rightarrow X_u \ell \nu$

- The fully inclusive $B \rightarrow X_u \ell \nu$ rate is the same as $B \rightarrow X_c \ell \nu$ (with $m_c \rightarrow m_u$).
- Experimentally the two processes overlap:
  
  - $\Gamma_c/\Gamma_u \approx 50$
  
  - $M_X$ measures the non-locality of the OPE

[See the PDG review and references therein]
Inclusive decays: $B \to X_u \ell \nu$

- Let’s see what’s going on by injecting a small momentum $(k^2 \approx \Lambda_{QCD}^2)$:

\[
p_B = m_b \nu + (k) \quad \text{with} \quad q = p_e + p_\nu
\]

- Defining $p_X = m_b \nu - q$ the u-quark propagator is:

\[
\frac{1}{(p_X + k)^2} = \frac{1}{p_X^2} + 2 \frac{p_X \cdot k}{p_X^2} + \frac{k^2}{p_X^2 + 2p_X \cdot k + \Lambda_{QCD}^2}
\]

\[
\begin{align*}
\text{cuts!} & \quad m_b \Lambda_{QCD} & \quad \Lambda_{QCD}^2 \\
\end{align*}
\]

- Without cuts ($p_X^2 \approx m_b^2$):

\[
\frac{1}{(p_X + k)^2} = \frac{1}{p_X^2} \left[ 1 - \frac{2p_X \cdot k}{p_X^2} + \cdots \right]
\]

**local** \quad **higher order local operators**

- With cuts ($p_X^2 \approx m_b \Lambda_{QCD}$):

\[
\frac{1}{(p_X + k)^2} = \frac{1}{p_X^2} \left[ 1 - \frac{k^2}{p_X^2 + 2p_X \cdot k} + \cdots \right]
\]

**non-local** \quad **higher order non-local operators**
Inclusive decays: $B \rightarrow X u \ell \nu$

- No cuts: $p_X^2 \approx m_b^2$

The only relevant scales are $m_b \gg \Lambda_{\text{QCD}}$
Inclusive decays: $B \rightarrow X_u \ell \nu$

- **With cuts:** $p_x^2 \approx \Lambda_{QCD} m_b$

\[ S(\omega) = \int \frac{dt}{2\pi} e^{-i\omega t} \langle \bar{B}(v) | \bar{h}_\nu(tn) S_n(tn) S_{n}^\dagger(0) h_\nu(0) | \bar{B}(v) \rangle \]

\[ \Gamma \sim H^2 J \otimes S \]

\[ m_b^2: \text{Hard function} \]

\[ \Lambda_{QCD} m_b: \text{Jet function} \]

\[ \Lambda_{QCD}^2: \text{Shape function} \]
Inclusive decays: universality of the shape function

- With some qualifications related to subleading effects ($\Lambda_{QCD}/m_b$), the shape function is a universal function which appears in multiple processes like $B \to X_s \gamma$ and $B \to X_s \ell^+ \ell^-$.

- The $B \to X_s \gamma$ spectrum at large $E_\gamma$ is essentially controlled by the shape function:

![Diagram showing the spectrum of B to Xs gamma at large Egamma controlled by the shape function.](image)

- $E_\gamma$ cut $\sim 2.5\text{GeV}$
- Measurement $\rightarrow$ OPE prediction
\[ |V_{ub}| = \begin{cases} (4.52 \pm 0.20) \times 10^{-3} & \text{inclusive} \\ (3.73 \pm 0.14) \times 10^{-3} & \text{exclusive} \end{cases} \]
Inclusive vs exclusive tension in the $[V_{ub}, V_{cb}]$ plane

$$\begin{align*}
|V_{cb}| \times 10^3 &= 39.09(68) \\
|V_{ub}| \times 10^3 &= 3.73(14) \quad \text{BGL} \\
p - \text{value} &= 0.32
\end{align*}$$

$$\begin{align*}
|V_{cb}| \times 10^3 &= 39.41(61) \\
|V_{ub}| \times 10^3 &= 3.74(14) \quad \text{CLN} \\
p - \text{value} &= 0.55
\end{align*}$$
CP violation in a nutshell

[Much of this mini-review has been already presented in lectures from R. Coutinho, M. Gersabeck, U. Nierste and T. Gershon]
CP violation in meson decays

1. CPV in mixing

\[ |B^0\rangle \ldots t \ldots \beta |\bar{B}^0\rangle \]
\[ |\bar{B}^0\rangle \ldots t \ldots \bar{\beta} |B^0\rangle \]
if \(|\beta| \neq |\bar{\beta}|\), look at decays to final states accessible only to \(B^0\) or \(\bar{B}^0\)

2. CPV in decay

\[ \Gamma (B \to f) \neq \Gamma (\bar{B} \to \bar{f}) \]
\[ A(B \to f) = \sum A_k e^{i(\delta_k + \phi_k)} \quad \Rightarrow \quad A(\bar{B} \to \bar{f}) = \sum A_k e^{i(\delta_k - \phi_k)} \]
QCD \quad CKM

3. CPV in the interference of decays with and without mixing

\[ B^0 \quad \bar{B}^0 \quad f_{CP} \]
B meson mixing

- Schrödinger equation for two states that at t = 0 are pure $B^0$ and $\bar{B}^0$:

$$i \frac{d}{dt} \left( \begin{array}{c} |B^0(t)\rangle \\ |\bar{B}^0(t)\rangle \end{array} \right) = \left( \begin{array}{cc} \hat{M} - i \frac{\hat{\Gamma}}{2} & \frac{M_{12} - i \frac{\Gamma_{12}}{2}}{2} \\ \frac{M_{12}^* - i \frac{\Gamma_{12}^*}{2}}{2} & \hat{M} - i \frac{\Gamma}{2} \end{array} \right) \left( \begin{array}{c} |B^0(t)\rangle \\ |\bar{B}^0(t)\rangle \end{array} \right)$$

where $\hat{M}$ and $\hat{\Gamma}$ are the mass and decay matrices and come from off-shell (dispersive) and off-shell (absorptive) intermediate states.

- The eigenvalue equation is:

$$\left( M - i \frac{\Gamma}{2} \right) \left( \begin{array}{c} p \\ \pm q \end{array} \right) = \left( M_{L,H} - i \frac{\Gamma_{L,H}}{2} \right) \left( \begin{array}{c} p \\ \pm q \end{array} \right)$$

- The mass eigenstates are ($|p|^2 + |q|^2 = 1$):

$$|B_L\rangle = p |B^0\rangle + q |\bar{B}^0\rangle$$

$$|B_H\rangle = p |B^0\rangle - q |\bar{B}^0\rangle$$

- We define $\Delta M = M_H - M_L$ and $\Delta \Gamma = \Gamma_H - \Gamma_L$
Meson mixing parameters

- Dispersive and absorptive parts of $\Delta B=2$ diagrams yield $M_{12}$ and $\Gamma_{12}$

- Explicit solution of the eigenvalue equation for $M-i\frac{\Gamma}{2}$ are:

\[
(\Delta m)^2 - \frac{1}{4}(\Delta \Gamma)^2 = (4|M_{12}|^2 - |\Gamma_{12}|^2),
\]

\[
\Delta m \Delta \Gamma = 4\Re (M_{12}\Gamma_{12}^*),
\]

\[
\frac{q}{p} = -\frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - \frac{i}{2}\Delta \Gamma} = -\frac{\Delta m - \frac{i}{2}\Delta \Gamma}{2M_{12} - i\Gamma_{12}}.
\]

- These expressions are valid for any kind of meson mixing: $K$, $D$, $B_d$ and $B_s$
The B mixing case

- In the B mixing system, there is empirical evidence for:

\[ |\Gamma_{12}| \ll |M_{12}| \quad \text{and} \quad \Delta \Gamma \ll \Delta m \]

- In general \( |\Gamma_{12}| \ll \Gamma \), because the former stems from final states that are common to \( B^0 \) and \( \bar{B}^0 \).

- \( B_s \) case
  
  Experimental evidence shows that \( \Gamma_{B_s} \ll \Delta m_{B_s} \Rightarrow |\Gamma_{12}^s| \ll \Delta m_{B_s} \)

- \( B_d \) case
  
  Decays to final states common to \( B^0 \) and \( \bar{B}^0 \) have BR in the \( 10^{-3} \) range. We can assume that their combined effect will be in the \( 10^{-2} \) range:

\[ |\Delta \Gamma_{B_d}|/\Gamma_{B_d} = O(10^{-2}) \]

Experimentally \( \Delta m_{B_d} \sim 0.75 \Gamma_{B_d} \); hence we obtain \( |\Gamma_{12}^d| \ll \Delta m_{B_d} \)
The B mixing case

\[(\Delta m)^2 - \frac{1}{4}(\Delta \Gamma)^2 = (4|M_{12}|^2 - |\Gamma_{12}|^2),\]

\[\Delta m \Delta \Gamma = 4 \Re(M_{12}\Gamma_{12}^*),\]

\[\frac{q}{p} = -\frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - \frac{i}{2}\Delta \Gamma} = -\frac{\Delta m - \frac{i}{2}\Delta \Gamma}{2M_{12} - i\Gamma_{12}}.\]

\[\Delta m = 2|M_{12}|\]

\[\frac{q}{p} = -\frac{M_{12}^*}{|M_{12}|} \left[1 - \frac{1}{2} \text{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)\right]\]

\[\Delta \Gamma = 2 \Re\left(\frac{M_{12}}{|M_{12}|} \Gamma_{12}^*\right)\]

\[\lambda_f = \frac{q \tilde{A}_f}{p A_f} = -\frac{M_{12}^* \tilde{A}_f}{|M_{12}| A_f} \left[1 - \frac{1}{2} \text{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)\right]\]

\[|\Gamma_{12}| \ll |M_{12}|\]

\[\Delta \Gamma \ll \Delta m\]
Time evolution

The evolution of pure interaction eigenstates is:

\[ |B^0(t)\rangle = g_+(t)|B^0\rangle + \frac{q}{p} g_-(t)|\bar{B}^0\rangle \]

\[ |\bar{B}^0(t)\rangle = \frac{p}{q} g_-(t)|B^0\rangle + g_+(t)|\bar{B}^0\rangle \]

where

\[ g_+(t) = e^{-imt} e^{-\Gamma t/2} \left[ \cosh \frac{\Delta \Gamma t}{4} \cos \frac{\Delta mt}{2} - i \sinh \frac{\Delta \Gamma t}{4} \sin \frac{\Delta mt}{2} \right] \]

\[ g_-(t) = e^{-imt} e^{-\Gamma t/2} \left[ -\sinh \frac{\Delta \Gamma t}{4} \cos \frac{\Delta mt}{2} + i \cosh \frac{\Delta \Gamma t}{4} \sin \frac{\Delta mt}{2} \right] \]

In the \( \Delta \Gamma = 0 \) limit:

\[ |B^0(t)\rangle = e^{-(im+\Gamma/2)t} \left[ \cos \frac{\Delta mt}{2} |B^0\rangle + i \frac{q}{p} \sin \frac{\Delta mt}{2} |\bar{B}^0\rangle \right] \]
CP violation in mixing

\[ |B^0\rangle \quad \ldots t \ldots . \quad \beta |\bar{B}^0\rangle \quad \Rightarrow \ \beta = (q/p) \, g_-(t) \]
\[ |\bar{B}^0\rangle \quad \ldots t \ldots . \quad \bar{\beta} |B^0\rangle \quad \Rightarrow \ \bar{\beta} = (p/q) \, g_-(t) \]

- if \(|\beta| \neq |\bar{\beta}|\), it is possible to measure CPV in decays to final states accessible only to \(B^0\) or \(\bar{B}^0\)

\[ |\beta| \neq |\bar{\beta}| \iff \left| \frac{q}{p} \right|^2 \neq 1 \]

- An example is the neutral semileptonic decay asymmetry to wrong sign leptons:

\[ a_{sl}(t) = \frac{\Gamma(\bar{B}^0(t) \to \ell^+\nu X) - \Gamma(B^0(t) \to \ell^-\bar{\nu} X)}{\Gamma(\bar{B}^0(t) \to \ell^+\nu X) + \Gamma(B^0(t) \to \ell^-\bar{\nu} X)} \]
Only 2 amplitudes do not vanish:

\[ A = \langle \ell^+ \nu X | \mathcal{H}^d | B^0 \rangle \quad \bar{A} = \langle \ell^- \bar{\nu} X | \mathcal{H}^d | \bar{B}^0 \rangle \]

Their magnitude is the same: \( |A|^2 = |\bar{A}|^2 \)

The time dependent rates are:

\[ \Gamma(\bar{B}^0(t) \to \ell^+ \nu X) = \left| \frac{p}{q} g_-(t) A \right|^2 \]
\[ \Gamma(B^0(t) \to \ell^- \bar{\nu} X) = \left| \frac{q}{p} g_-(t) \bar{A} \right|^2 \]

The asymmetry is:

\[ a_{sl}(t) = \frac{1 - |q/p|^4}{1 + |q/p|^4} \approx \text{Im} \frac{\Gamma_{12}}{M_{12}} \]
CP violation in decay

- It stems from the difference between the rates for \( B \to f \) and \( \bar{B} \to \bar{f} \).

- We need two amplitudes with different weak and strong phases:

\[
A_f = A_1 e^{i(\delta_1 + \phi_1)} + A_2 e^{i(\delta_2 + \phi_2)}
\]

\[
\bar{A}_f = A_1 e^{i(\delta_1 - \phi_1)} + A_2 e^{i(\delta_2 - \phi_2)}
\]

\[
\mathcal{A} = \frac{\Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f})}{\Gamma(B \to f) + \Gamma(\bar{B} \to \bar{f})}
\]

\[
= -\frac{2A_1 A_2 \sin(\delta_2 - \delta_1) \sin(\phi_2 - \phi_1)}{A_1^2 + A_2^2 + 2A_1 A_2 \cos(\delta_2 - \delta_1) \cos(\phi_2 - \phi_1)}
\]

- We \( A_1, A_2 \) and \( \delta_1 - \delta_2 \): all these quantities are sensitive to non-perturbative QCD.
Interference in decays with and without and mixing

- We look at the time dependent CP asymmetries of neutral B decays to final states that are CP eigenstate:

\[ A_{f_{CP}} = \frac{\Gamma(\bar{B}^0(t) \rightarrow f_{CP}) - \Gamma(B^0(t) \rightarrow f_{CP})}{\Gamma(\bar{B}^0(t) \rightarrow f_{CP}) + \Gamma(B^0(t) \rightarrow f_{CP})} \]

- Same as CPV in decay, the only difference is that \( |\bar{f}\rangle = \eta_f |f\rangle \)

- We need two amplitudes, with different weak and strong phases

- More amplitudes usually implies non-perturbative trouble...

- Even if \( |A(B^0 \rightarrow f)| = |A(\bar{B}^0 \rightarrow f)| \) we can use \( B^0 \rightarrow f_{CP} \) and \( B^0 \rightarrow \bar{B}^0 \rightarrow f_{CP} \)

- \( B^0 \rightarrow f_{CP} \) and \( \bar{B}^0 \rightarrow f_{CP} \) with \( A_f \) vs \( i\frac{q_p}{p} \bar{A}_f \)
Interference in decays with and without and mixing

- \[ A_f = A e^{i\delta + i\phi} \]

\[ i q p A_f = A |q_p| e^{i[\delta + \frac{\pi}{2}]} + i[\arg \frac{q}{p} - \phi] = iA_f \underbrace{\frac{q A_f}{p A_f}}_{\equiv \lambda_f} \]

- In formulae:

\[ A_{f_{CP}} = \frac{\Gamma(\bar{B}^0(t) \rightarrow f) - \Gamma(B^0(t) \rightarrow f)}{\Gamma(\bar{B}^0(t) \rightarrow f) + \Gamma(B^0(t) \rightarrow f)} \]

\[ = \frac{2\text{Im}\lambda_f}{1 + |\lambda_f|^2} \sin(\Delta m t) - \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos(\Delta m t) \]

\[ = S_f - C_f \]

- If \(|\lambda_f| = 1\) we get:

\[ A_{f_{CP}} = \text{Im}\lambda_f \sin(\Delta m t) \]
The three types of CP violation

- There are three quantities that drive CPV effects:

\[ \left| \frac{q}{p} \right|, \quad \left| \frac{\bar{A}_f}{A_f} \right|, \quad \lambda_f = \frac{q \bar{A}_f}{p A_f} \]

- CPV in mixing: \( |q/p| \neq 1 \)

- CPV in decay: \( |\bar{A}_f/A_f| \neq 1 \)

- CPV in the interference between decay and mixing: \( \lambda_f \neq \pm 1 \)
Calculation of the amplitudes ($A_f$)

- We are strongly affected by non-perturbative QCD effects

- In general we must take matrix elements of a decay effective Hamiltonian:

\[
A_f = A(B \rightarrow f) = \langle f|\mathcal{H}^d|B\rangle = \sum C_i \langle f|O_i|B\rangle \\
\bar{A}_f = A(\bar{B} \rightarrow \bar{f}) = \langle \bar{f}|\mathcal{H}^d|\bar{B}\rangle = \sum C_i^* \langle \bar{f}|O_i^\dagger|\bar{B}\rangle
\]

- If only one operator contributes ($C = |C|e^{i\varphi}$):

\[
\frac{\bar{A}_f}{A_f} = \frac{C^*}{C} = e^{-2i\varphi}
\]

- If all the Wilson coefficients come with the same phase ($\varphi$):

\[
\frac{\bar{A}_f}{A_f} = e^{-2i\varphi} \frac{\sum |C_i| \langle \bar{f}|O_i^\dagger|\bar{B}\rangle}{\sum |C_i| \langle f|O_i|B\rangle} = e^{-2i\varphi}
\]
Calculation of $M_{12}$

- $\Delta B=2$ transitions need to go through a W loop: short distance physics is involved

As usual we write an effective Hamiltonian that captures the SD physics:

$$H_{\text{eff}} = \frac{G_F^2 m_W^2}{4\pi^2} (V_{tb}^* V_{tq})^2 S_0 \left( \frac{m_t^2}{m_W^2} \right) \eta_B b_B(\mu) (\bar{q}_L \gamma_\mu b_L)(\bar{q}_L \gamma^\mu b_L) + \text{h.c.}$$

- NLO Wilson coefficient: $C(\mu)$
- Local operator: $Q(\mu)$
Calculation of $M_{12}$ cont’d

- Now we must specify the external states:
  \[ M_{12} = C(\mu) \langle B^0 | Q(\mu) | \bar{B}^0 \rangle \]

- From lattice-QCD we obtain:
  \[ \langle B^0 | Q(\mu) | \bar{B}^0 \rangle = \frac{2}{3} f_B^2 \frac{\hat{B}_B}{b_B(\mu)} \]

- Putting everything together:
  \[
  M_{12} = \frac{\langle B^0 | H_{eff} | \bar{B}^0 \rangle}{2 m_B} = \frac{G_F m_W^2}{12 \pi^2} \eta_B m_B (f_B^2 \hat{B}_B) S_0 \left( \frac{m_t^2}{m_W^2} \right) (V_{tb} V_{tq}^*)^2
  \]

- Introduced to render $\hat{B}_B$ scale independent. Needs to be calculated order by order in perturbation theory.

- Includes NLO perturbative effects.

- Dominant theory uncertainty.

- $f_{B_d} \sqrt{\hat{B}_{B_d}} = 225(9)$ MeV

- $f_{B_s} \sqrt{\hat{B}_{B_s}} = 274(8)$ MeV
Calculation of $M_{12}$ cont’d

- Mass difference:

\[
\Delta m_B \quad = \quad 2|M_{12}|
\]

\[
= \quad \frac{G_F^2 m_W^2}{6 \pi^2} \eta_B m_B \left(f_B^2 \hat{B}_B\right) S_0 \left(\frac{m_t^2}{m_W^2}\right)|V_{tb}^* V_{tq}|^2
\]

- Mixing parameter:

\[
\frac{q}{p} \quad = \quad -\frac{M_{12}^*}{|M_{12}|} \quad = \quad -\frac{(V_{tb}^* V_{tq})^2}{|V_{tb}^* V_{tq}|^2} \quad = \quad \begin{cases} 
-e^{-2i\beta} & \text{q=d} \\
-e^{-2i\beta_s} & \text{q=s}
\end{cases}
\]

up to corrections proportional to $\Gamma_{12}/M_{12}$
\begin{equation}
\left\{ \begin{array}{ll}
\Delta m_{B_q} & \text{from experiment} \\
\hat{f}_{B_q}^2 \hat{B}_{Bq} & \text{from lattice-QCD}
\end{array} \right.
\end{equation}
Time dependent CP asymmetries
Time dependent CP asymmetry in $B \to J/\psi K_S$

- This process is mediated by both tree and penguin operators:

\[
A = (T_{c\bar{c}s} + P_s^c - P_s^t) V_{cb}V_{cs}^* + (P_s^u - P_s^t) V_{ub}V_{us}^*
\]

The last term is doubly suppressed:

\[
\frac{V_{ub}V_{us}^*}{V_{cb}V_{cs}^*} \approx 10^{-2}
\]

\[
P/T \approx O(0.1)
\]
Time dependent CP asymmetry in $B \to J/\psi K_S$

- There is a problem: $B^0 \to K^0$ and $\bar{B}^0 \to \bar{K}^0$

- $K_s$ is the lighter mass eigenstate: $|K_s\rangle = p_K|K^0\rangle + q_K|\bar{K}^0\rangle$

- Interference between $B^0 \to J/\psi K_s$ and $\bar{B}^0 \to J/\psi K_s$ is only possible through K mixing:

  $$\frac{A(B^0 \to J/\psi K_s)}{A(\bar{B}^0 \to J/\psi K_s)} = \frac{A(B^0 \to J/\psi K^0)}{A(\bar{B}^0 \to J/\psi \bar{K}^0)} \frac{p_K}{q_K} = -\eta_{\psi K_s} \frac{V_{cb}V_{cs}^*V_{cs}V_{cd}^*}{V_{cb}^*V_{cs}V_{cs}^*V_{cd}}$$

- Putting everything together:

  $$\lambda_{\psi K_s} = \frac{q}{p} \frac{A_{\psi K^0}}{A_{\bar{\psi} \bar{K}^0}} \frac{p_K}{q_K} = \eta_{\psi K_s} \frac{V_{tb}^*V_{td}V_{cb}V_{cs}^*V_{cs}V_{cd}^*}{V_{tb}V_{td}^*V_{cb}^*V_{cs}V_{cs}^*V_{cd}} = \eta_{\psi K_s} e^{-2i\beta}$$

- and the time dependent CP asymmetry is

  $$\mathcal{A}_{\psi K_s} = \frac{\Gamma(\bar{B}^0(t) \to J/\psi K_s) - \Gamma(B^0(t) \to J/\psi K_s)}{\Gamma(\bar{B}^0(t) \to J/\psi K_s) + \Gamma(B^0(t) \to J/\psi K_s)}$$

  $$= -\eta_{\psi K_s} \sin(2\beta) \sin(\Delta m_{B_d} t)$$
Time dependent CP asymmetry in $B \to J/\psi K_S$

\[ a_{\psi K_S}^{\exp} = 0.699 \pm 0.017 \]
Time dependent CP asymmetry in $B \rightarrow \phi K_S$ (and related modes)

- This process is mediated by penguin operators only:

$$A = (P_s^c - P_s^t) V_{cb} V_{cs}^* + (P_s^u - P_s^t) V_{ub} V_{us}^*$$

- The last term is only CKM suppressed:

$$\frac{V_{ub} V_{us}^*}{V_{cb} V_{cs}^*} \sim 10^{-2} \quad \frac{A(B^0 \rightarrow \phi K^0)}{A(\bar{B}^0 \rightarrow \phi \bar{K}^0)} \sim \frac{V_{cb} V_{cs}^*}{V_{cb} V_{cs}^*} + O(??)$$
Time dependent CP asymmetry in $B \rightarrow \phi K_S$ (and related modes)

\[
\sin(2\beta_{\text{eff}}) \equiv \sin(2\phi_{1\text{eff}}) \]

<table>
<thead>
<tr>
<th>Decay Mode</th>
<th>Value (90% CL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \rightarrow ccs$</td>
<td>World Average</td>
</tr>
<tr>
<td>$\phi K^0$</td>
<td>0.39 ± 0.17</td>
</tr>
<tr>
<td>$\eta^* K^0$</td>
<td>0.61 ± 0.07</td>
</tr>
<tr>
<td>$K_S K_S K_S$</td>
<td>0.58 ± 0.20</td>
</tr>
<tr>
<td>$\pi^0 K_S$</td>
<td>0.38 ± 0.19</td>
</tr>
<tr>
<td>$\rho^0 K_S$</td>
<td>0.61 ±0.25-0.27</td>
</tr>
<tr>
<td>$\omega K_S$</td>
<td>0.48 ± 0.24</td>
</tr>
<tr>
<td>$f_0 K^0$</td>
<td>0.85 ± 0.07</td>
</tr>
<tr>
<td>$\pi^0 \pi^0 K_S$</td>
<td>Average</td>
</tr>
<tr>
<td>$K^+ K^0 K^0$</td>
<td>0.73 ± 0.10</td>
</tr>
</tbody>
</table>

2007
Time dependent CP asymmetry in $B \rightarrow \phi K_S$ (and related modes)

\[
\sin(2\beta^{\text{eff}}) \equiv \sin(2\phi_1^{\text{eff}})
\]

<table>
<thead>
<tr>
<th>Mode</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \rightarrow c\bar{c}s$ World Average</td>
<td>0.70 ± 0.02</td>
</tr>
<tr>
<td>$\phi K^0$ Average</td>
<td>0.74 ± 0.11</td>
</tr>
<tr>
<td>$\eta^+ K^0$ Average</td>
<td>0.63 ± 0.06</td>
</tr>
<tr>
<td>$K_S K_S K_S$ Average</td>
<td>0.72 ± 0.19</td>
</tr>
<tr>
<td>$\pi^0 K^0$ Average</td>
<td>0.57 ± 0.17</td>
</tr>
<tr>
<td>$\rho^0 K_S$ Average</td>
<td>0.54 ± 0.10</td>
</tr>
<tr>
<td>$\omega K_S$ Average</td>
<td>0.71 ± 0.21</td>
</tr>
<tr>
<td>$f_0 K_S$ Average</td>
<td>0.69 ± 0.12</td>
</tr>
<tr>
<td>$f_2 K_S$ Average</td>
<td>0.48 ± 0.53</td>
</tr>
<tr>
<td>$f_0 K_S$ Average</td>
<td>0.20 ± 0.53</td>
</tr>
<tr>
<td>$\pi^0 \pi^0 K_S$ Average</td>
<td>0.66 ± 0.28</td>
</tr>
<tr>
<td>$\phi \pi^0 K_S$ Average</td>
<td>0.97 ± 0.03</td>
</tr>
<tr>
<td>$\pi^- \pi^+ K_S$ Average</td>
<td>0.01 ± 0.33</td>
</tr>
<tr>
<td>$K^+ K^- K^0$ Average</td>
<td>0.68 ± 0.09</td>
</tr>
</tbody>
</table>

2018
Time dependent CP asymmetry in $B \to \pi\pi$

- This process is mediated by both tree and penguin operators:

\[
A = T_{u\bar{u}d} V_{ub} V_{ud}^* + P_{d}^u V_{ub} V_{ud}^* + P_{d}^c V_{cb} V_{cd}^* + P_{d}^t V_{tb} V_{td}^*
\]

\[
= (T_{u\bar{u}d} + P_{d}^u - P_{d}^c) V_{ub} V_{ud}^* + (P_{d}^t - P_{d}^c) V_{tb} V_{td}^*
\]

\[
= T_{\pi\pi} V_{ub} V_{ud}^* + P_{\pi\pi} V_{tb} V_{td}^*
\]

\[
= T_{\pi\pi} V_{ub} V_{ud}^* \left(1 + r_{\pi\pi} \frac{V_{tb} V_{td}^*}{V_{ub} V_{ud}^*}\right) = T_{\pi\pi} V_{ub} V_{ud}^* (1 + r_{\pi\pi} \kappa) \quad \text{penguin pollution}
\]

- Is the last term suppressed?

\[
\left|\frac{V_{tb} V_{td}^*}{V_{ub} V_{ud}^*}\right| \sim 2
\]

\[
r_{\pi\pi} \sim O(0.1)
\]

\[
\frac{A(B^0 \to \pi\pi)}{A(\bar{B}^0 \to \pi\pi)} \sim \frac{V_{ub} V_{ud}^*}{V_{ub} V_{ud}^*} \frac{1 + r_{\pi\pi} \kappa}{1 + r_{\pi\pi} \kappa^*}
\]
The time-dependent CP asymmetry is:

\[ \lambda_{\pi\pi} = \frac{q}{p} \frac{A_{\pi\pi}}{\bar{A}_{\pi\pi}} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{ub} V_{ud}^*}{V_{ub}^* V_{ud}} \frac{1 + r_{\pi\pi} \kappa}{1 + r_{\pi\pi} \kappa^*} = e^{2i\alpha} \frac{1 + r_{\pi\pi} \kappa}{1 + r_{\pi\pi} \kappa^*} \]

\[ e^{-2i\beta} \quad e^{-2i\gamma} \]

The time-dependent CP asymmetry is:

\[ A_{\pi\pi} = -C_{\pi\pi} \cos(\Delta m_{B_d} t) + S_{\pi\pi} \sin(\Delta m_{B_d} t) \]

\[ S_{\pi\pi} = \sin(2\alpha) + O(r_{\pi\pi}) \]

\[ C_{\pi\pi} = O(r_{\pi\pi}) \]

What do we get from experiments (Belle & BaBar)?

\[ S_{\pi\pi}^{\exp} = -0.59 \pm 0.09 \]

\[ C_{\pi\pi}^{\exp} = -0.39 \pm 0.07 \]
Time dependent CP asymmetry in $B \to \pi \pi$

- Solutions:
  - use effective theories to calculate $r_{\pi \pi}$
  - use SU(3) flavor symmetry to relate $B \to K\pi$ and $B \to \pi\pi$
  - isospin analysis [Gronau, London]

- Up to isospin breaking corrections we can describe

\[ A(B^+ \to \pi^+\pi^0), \ A(B^0 \to \pi^+\pi^-), \ A(B^0 \to \pi^0\pi^0) \]

in terms of two isospin amplitudes

\[ A(B \to [\pi\pi]_0), \ A(B \to [\pi\pi]_2) \]

- The implied constraint is:

\[ \frac{1}{\sqrt{2}} A(B^0 \to \pi^+\pi^-) + A(B^0 \to \pi^0\pi^0) = A(B^+ \to \pi^+\pi^0) \]

- From the measurements of the branching ratios ($BR_{+,00}, BR_{00}$) and of some CP violating parameters ($C_{+,00}, S_{+,00}$) it is possible to extract $\alpha$ without hadronic uncertainties (but up to isospin corrections)
Overall determination of $\alpha$
Time dependent CP asymmetry in $B \rightarrow J/\psi K_S$
Fits galore
First row CKM unitarity

\[ K^0 \rightarrow \pi^- \ell^+ \nu_\ell \]

[See A. El-Khadra’s lecture]
B_d Unitarity Triangle (present)

\[ \gamma \]

\[ \Delta m_d \quad \Delta m_s \]

\[ \varepsilon_K \]

\[ \sin 2\beta \]

\[ \beta \]

\[ \gamma \]

\[ \rho \]

\[ |V_{ub}| \]

\[ \eta \]

Excluded area has CL > 0.95

Solution w/ \( \cos 2\beta < 0 \) (excl. at CL > 0.95)
**B_d Unitarity Triangle (phase 1)**

Phase 1:

\[
\begin{align*}
&\text{LHCb} & 27 \text{ fb}^{-1} \\
&\text{ATLAS/CMS} & 300 \text{ fb}^{-1}
\end{align*}
\]
B_d Unitarity Triangle (phase 2)

Phase 2: \[
\begin{align*}
&\text{LHCb} & 300 \text{ fb}^{-1} \\
&\text{ATLAS/CMS} & 3000 \text{ fb}^{-1}
\end{align*}
\]
B_d Unitarity Triangle: anatomy

- **CP-allowed only**
- **CP-violating only**
- **Tree only**
- **Loop only**
B_d Unitarity Triangle: anatomy (phase 1)
B_d Unitarity Triangle: anatomy (phase 2)

\[ \begin{aligned} \text{LHCb} & \quad 300 \text{ fb}^{-1} \\
\text{ATLAS/CMS} & \quad 3000 \text{ fb}^{-1} \end{aligned} \]
"Kaon" Unitarity Triangle

The Kaon UT fit involves $\varepsilon_K, \varepsilon'/\varepsilon, K^+ \to \pi^+\nu\bar{\nu}$.

SUT = Standard UT fit
KUT = Kaon UT fit

[Lehner, EL, Soni, 1508.01801]
The Kaon UT fit involves \( \epsilon_K, \frac{\epsilon'}{\epsilon}, K^+ \to \pi^+ \nu \bar{\nu} \).

Future scenario in which:
\[
\delta[\text{Im}A_2]_{\text{lat}} = 5\%, \quad \delta[\text{Im}A_0]_{\text{lat}} = 18\%, \quad \delta\text{BR}(K^+ \to \pi^+ \nu \bar{\nu}) = 7\% 
\]
“Kaon” Unitarity Triangle

- The Kaon UT fit involves $\varepsilon_K$, $\varepsilon'/\varepsilon$, $K^+ \rightarrow \pi^+\nu\bar{\nu}$
- Future scenario in which:
  \[
  \delta[\text{Im} A_2]_{\text{lat}} = 5\% , \ \delta[\text{Im} A_0]_{\text{lat}} = 18\% , \ \delta\text{BR}(K^+ \rightarrow \pi^+\nu\bar{\nu}) = 7\%
  \]
- Impact of a measurement of $K_L \rightarrow \pi^0\nu\bar{\nu}$ at the 10% level

SUT = Standard UT fit
KUT = Kaon UT fit

[Lehner, EL, Soni, 1508.01801]
New Physics effects
High Intensity vs High Energy

**High Energy Physics** finds new physics around 1 TeV:

\[ g \times f_T(M_{NP}) \]

**High Intensity Physics** will tell us FC couplings and loop structure:

\[ g_{ij} \times f_L(M_{NP}) \]

If **direct searches** don’t find new physics (besides a standard Higgs), **indirect searches** can push the search to much higher scales under the assumptions of not too small flavor changing new couplings.

Hadronic uncertainties might be large enough to hide a possible new physics signal.
The delicate structure

- The SM pattern of Flavor Changing Neutral Currents and CP violation follows directly from the choice of using one single Higgs doublet to give masses to all the fermions.

- The two Yukawa Matrices (that contain 36 real parameters) break almost completely the flavor symmetry of the SM: $U(3)_Q \times U(3)_U \times U(3)_D \rightarrow U(1)_B$. **26 of the 36 parameters are unphysical.**

- The introduction of additional fields capable of interacting with quarks, lead often to new flavor changing interactions. The latter cannot be usually rotated away since we already used all the freedom allowed by the $U(3)^3$ symmetry to simplify the Yukawas.

- **The SM has a very delicate structure: almost any addition to it will result in a proliferation of flavor changing interactions and CP violating phases.**
Minimal Flavor Violation (MFV)

- In the SM the quark Yukawas are:
  \[ Y_U = V_{\text{CKM}} D_L Y_U^{\text{diag}} U_R \]
  \[ Y_D = D_L Y_U^{\text{diag}} D_R \]

  and the three unitary matrices \( D_L \), \( D_R \) and \( U_R \) are unobservable.

- In several new physics models (type-III 2HDM, MSSM, ...) these matrices become observable.

- The MFV principle requires these matrices to remain unobservable.
Minimal Flavor Violation (MFV)

- This means that all flavor changing interactions are controlled by the CKM matrix.

- For instance:

\[ \delta_{ts} f(m_\tilde{t}/m_\tilde{\chi}) \]

\[ V_{tb}^* V_{ts} f(m_\tilde{t}/m_\tilde{\chi}) \]
Once we decide to walk the MFV road, the structure of new physics contributions to rare decays and CP violating observables is very constrained.

MFV contributions can be absorbed in the following parameters:

\[
M_{12}^{d} \rightarrow M_{12}^{d,SM} \left(1 + h_d e^{2i\sigma_d}\right) = M_{12}^{d,SM} r_d^2 e^{2i\theta_d}
\]

\[
M_{12}^{s} \rightarrow M_{12}^{s,SM} \left(1 + h_s e^{2i\sigma_s}\right) = M_{12}^{s,SM} r_s^2 e^{2i\theta_s}
\]

\[
M_{12}^{K} \rightarrow M_{12}^{K,SM} \left(1 + h_K e^{2i\sigma_K}\right) = M_{12}^{K,SM} r_K^2 e^{2i\theta_K}
\]

Some predictions for several observables change:

\[
\Delta m_{B_d} \rightarrow \Delta m_{B_d}^{SM} r_d^2
\]

\[
\frac{\Delta m_{B_s}}{\Delta m_{B_d}} \rightarrow \left(\frac{\Delta m_{B_s}}{\Delta m_{B_d}}\right)^{SM} r_s^2
\]

\[
a_{\psi K_s} \rightarrow \sin 2(\beta + \theta_d)
\]
Model Independent Analysis: $b \rightarrow d$ sector ($B_d$)

past (2003)

near future: LHCb@7fb$^{-1}$
BelleII@5ab$^{-1}$

present

far future: LHCb@50fb$^{-1}$
BelleII@50ab$^{-1}$
Model Independent Analysis: $b \rightarrow s$ sector ($B_s$)

**Past:**

**Present:**

**Near future:**
- LHCb@7fb$^{-1}$
- BelleII@5ab$^{-1}$

**Far future:**
- LHCb@50fb$^{-1}$
- BelleII@50ab$^{-1}$
Model Independent Analysis: scale of new physics

[UTfit collaboration, 1710.09644]

- Operator basis for generic contributions to $B_d$ mixing:
  \[
  Q_1 = (\bar{b}_L \gamma_\mu d_L)(\bar{b}_L \gamma_\mu d_L) \\
  Q_2 = (\bar{d}_R d_L)(\bar{b}_R d_L) \\
  Q_3 = (\bar{b}_R^\alpha d_L \beta)(\bar{b}_R^\beta d_L^\alpha) \\
  Q_4 = (\bar{b}_R d_L)(\bar{b}_L d_R) \\
  Q_5 = (\bar{b}_R^\alpha d_L \beta)(\bar{b}_L^\beta d_R^\alpha)
  \]

- The $B_s$, $K$ and $D$ operators are obtained via $(b, d) \rightarrow (b, s), (s, d), (c, u)$

- Parameterization of the Wilson coefficients ($B_d$ case):
  \[
  H_{\text{eff}} = \sum C_i Q_i \\
  C_i = \begin{cases} 
  \frac{\alpha}{\Lambda^2} |V_{td} V_{tb}^*|^2 e^{i\delta} & \text{NMFV} \\
  \frac{\alpha}{\Lambda^2} e^{i\delta} & \text{General}
  \end{cases}
  \]

- NP has the same CKM dependence as SM, arbitrary mass scale and phase
- NP violates the CKM paradigm with an arbitrary mass scale and phase
Under the right circumstances, flavor observables allow to probe scales inaccessible to the present and near future generation of collider searches.
What do we learn?

- The Unitarity Triangle fit is a tremendous check of the CKM mechanism

- Arbitrary new physics is strongly constrained

- If new physics is MFV (i.e. shows proper respect to the CKM) the UT fit constraints are partially lifted: in these models the scale at which new physics enters (read: the masses of the new particles) can be reasonably low.

- Complementarity between direct and indirect searches
  - NP scale is identified
  - Little information on Flavor changing structure
  - NP scale is poorly constrained
  - Complete information on flavor changing interactions

Both necessary to pin down BSM physics
Extra slides
Express review of K mixing

- The basic formulae are identical to the B mixing case but the hierarchy between the various entries in the mixing Hamiltonian is very different:

\[ \Delta M_K^{\text{exp}} = (3.491 \pm 0.009) \times 10^{-15} \text{ GeV} \]
\[ \Delta \Gamma_K^{\text{exp}} = -7.4 \times 10^{-15} \text{ GeV} \simeq -2 \Delta M_K \]

- Note that in the SM both quantities suffer from very large hadronic uncertainties and are very hard to use to constrain the CKM

- The mass eigenstates are conventionally written as:

\[ K_{L,S} = \frac{(1 + \bar{\varepsilon})}{\sqrt{2(1 + |\bar{\varepsilon}|^2)}} K^0 \pm \frac{(1 - \bar{\varepsilon})}{\sqrt{2(1 + |\bar{\varepsilon}|^2)}} \bar{K}^0 \]

where \( \bar{\varepsilon} \) is a small complex parameter of order \( 10^{-3} \)

\[ \frac{q_K}{p_K} = \frac{1 - \bar{\varepsilon}}{1 + \bar{\varepsilon}} = \sqrt{\frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}}} \ll 1 \quad \left\{ \begin{array}{l}
\text{Im} M_{12} \ll \text{Re} M_{12} \\
\text{Im} \Gamma_{12} \ll \text{Re} \Gamma_{12}
\end{array} \right. \]
Express review of K mixing cont’d

Note that kaon CP and mass eigenstates are different:

\[
\begin{align*}
K_1 &= \frac{1}{\sqrt{2}} (K^0 - \bar{K}^0) \quad \text{CP} = 1 \\
K_2 &= \frac{1}{\sqrt{2}} (K^0 + \bar{K}^0) \quad \text{CP} = -1
\end{align*}
\]

\[
\begin{align*}
K_S &= \frac{1}{\sqrt{1+|\epsilon|^2}} (K_1 + \bar{\epsilon} K_2) \\
K_L &= \frac{1}{\sqrt{1+|\epsilon|^2}} (K_2 + \bar{\epsilon} K_1)
\end{align*}
\]

Since CP(\(\pi\pi\))=1 and CP(\(\pi\pi\pi\))=-1 we find immediately that the dominant decays of the kaon mass eigenstates are:

\(K_S \rightarrow 2\pi\) (via \(K_1\))

\(K_L \rightarrow 3\pi\) (via \(K_2\))

The phase space suppression of the 3 pion mode is responsible for the large disparity in their lifetimes (i.e. for the very large \(\Delta \Gamma\))

Decays to “wrong number” of pions are possible:

\(K_L \propto K_2 + \bar{\epsilon} K_1\)

**Diagram:**

- **Indirect:** \(\epsilon\) → \(\pi \pi\)
- **Direct:** \(\epsilon'\) → \(\pi \pi\)
Express review of K mixing cont'd

- One starts from the isospin decomposition of $K \rightarrow \pi \pi$ amplitudes:

  
  $A(K^+ \rightarrow \pi^+ \pi^0) = \sqrt{\frac{3}{2}} A_2 e^{i\delta_2}$

  $A(K^0 \rightarrow \pi^+ \pi^-) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}} A_2 e^{i\delta_2}$

  $A(K^0 \rightarrow \pi^0 \pi^0) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} - 2\sqrt{\frac{1}{3}} A_2 e^{i\delta_2}$

  where $A_{0,2} = A(K \rightarrow (\pi\pi)_{I=0,2})$ and $\delta_{0,2}$ are strong phases that can be measured. The amplitudes $A_{0,2}$ can be calculated up to several matrix elements that have been calculated on the lattice only very recently.

- The two CP violating quantities are given as:

  
  \[
  \varepsilon = \frac{e^{i\pi/4}}{\sqrt{2\Delta M_K^{\text{exp}}}} \left( \text{Im}M_{12} + 2\frac{\text{Im}A_0}{\text{Re}A_0} \text{Re}M_{12} \right) \\
  \varepsilon' = \frac{1}{\sqrt{2}} \text{Im} \left( \frac{A_2}{A_0} \right) e^{i(\pi/2+\delta_2-\delta_0)}
  \]

  mixing

  decay
Express review of K mixing cont’d

- Experiments measure:

\[ \eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \varepsilon - \frac{2\varepsilon'}{1 - \sqrt{\text{Re}A_2/\text{Re}A_0}} \approx \varepsilon - 2\varepsilon' \]

\[ \eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \varepsilon + \frac{\varepsilon'}{1 + \frac{1}{2}\text{Re}A_2/\text{Re}A_0} \approx \varepsilon + \varepsilon' \]

\[ \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 \approx 1 - 6 \text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) \]

- The imaginary part of \( M_{12} \) is controlled by perturbative QCD:

\[ M_{12} = \frac{G_F^2}{12\pi^2} F_K^2 \hat{B}_K m_K M_W^2 \left[ \lambda_c^* \eta_1 S_0(x_c) + \lambda_t^* \eta_2 S_0(x_t) + \lambda_c^* \lambda_t^* \eta_3 S_0(x_c, x_t) \right] \]

where \( \lambda_q = V_{qs}^* V_{qd} \), \( x_q = (m_q/M_W)^2 \) and \( \hat{B}_K = 0.7625(97) \)