Theories of Quark and Lepton Masses

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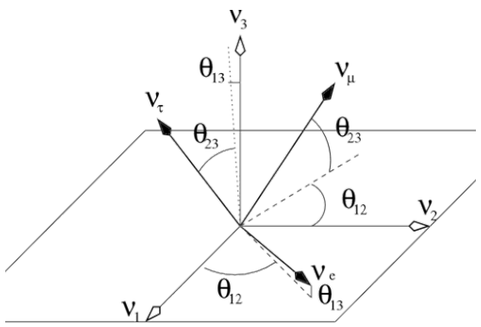
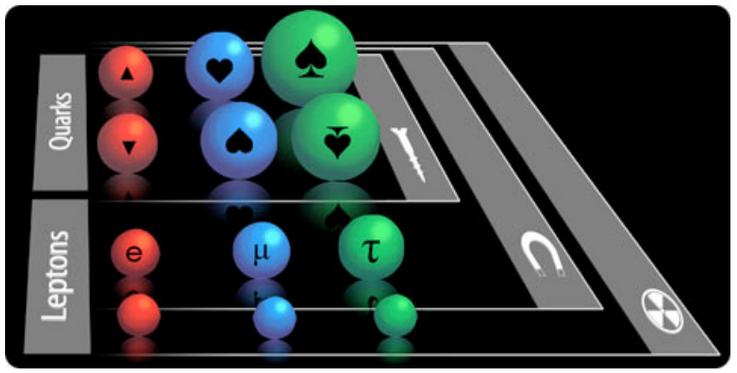
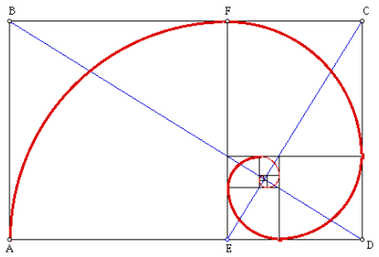
SLAC Summer Institute 2019







(image credits: Wikipedia, E. Palti)

The Mystery of Mass

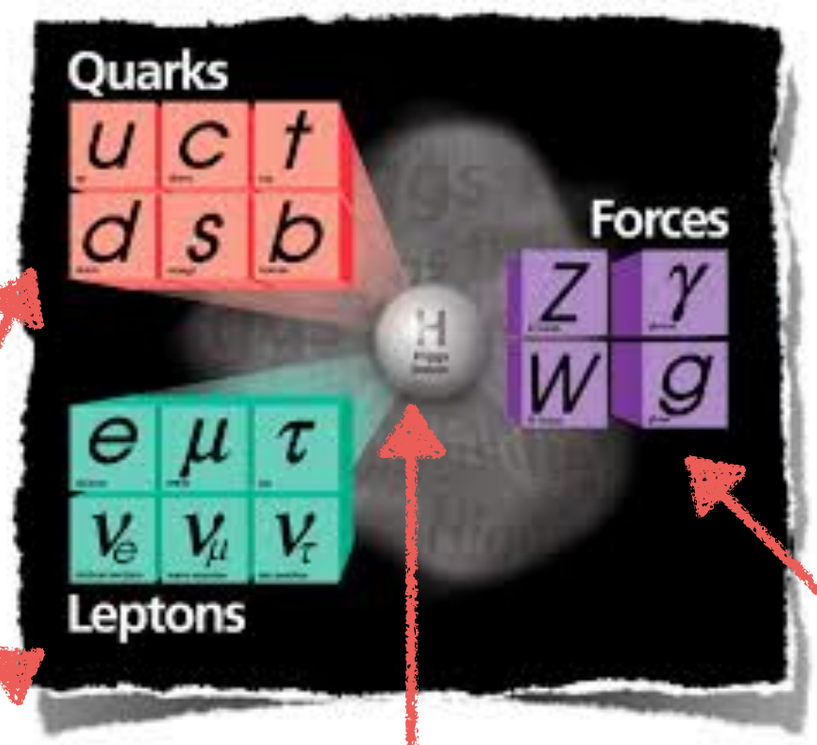
The Standard Model:

spectacularly successful — but many unanswered questions, including

Glashow Salam Weinberg

Nobel Prize, 1979

What is the origin of the masses of all of the elementary particles?



(image credit: Fermilab)

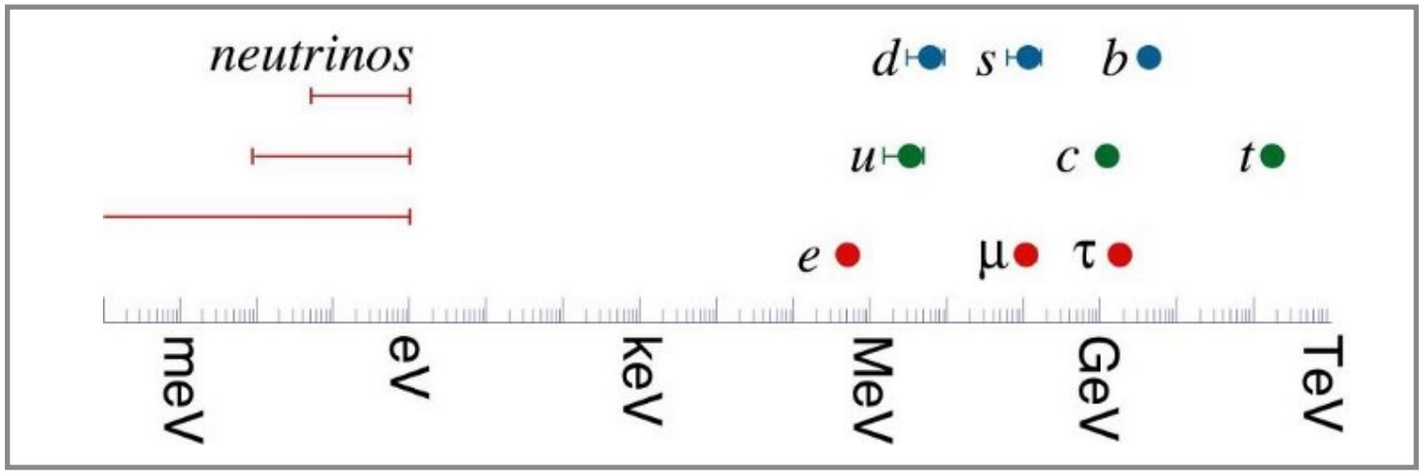
mysterious

mysterious

“understood”

gauge symmetry,
Higgs mechanism

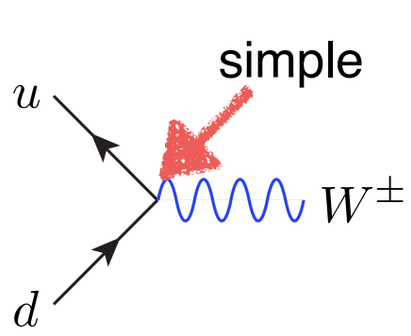
Focus here on masses/mixing of the SM fermions:

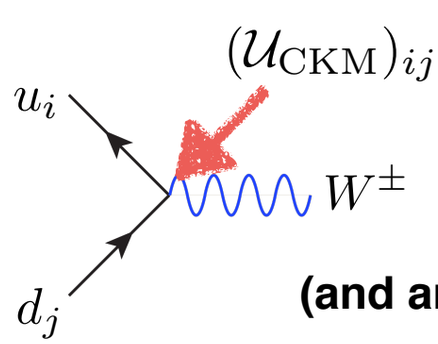


(image credit: H. Murayama)

Masses and intergenerational mixings:

charged current:
gauge v. mass eigenstates





(and analogously for leptons)

SM parameter count:

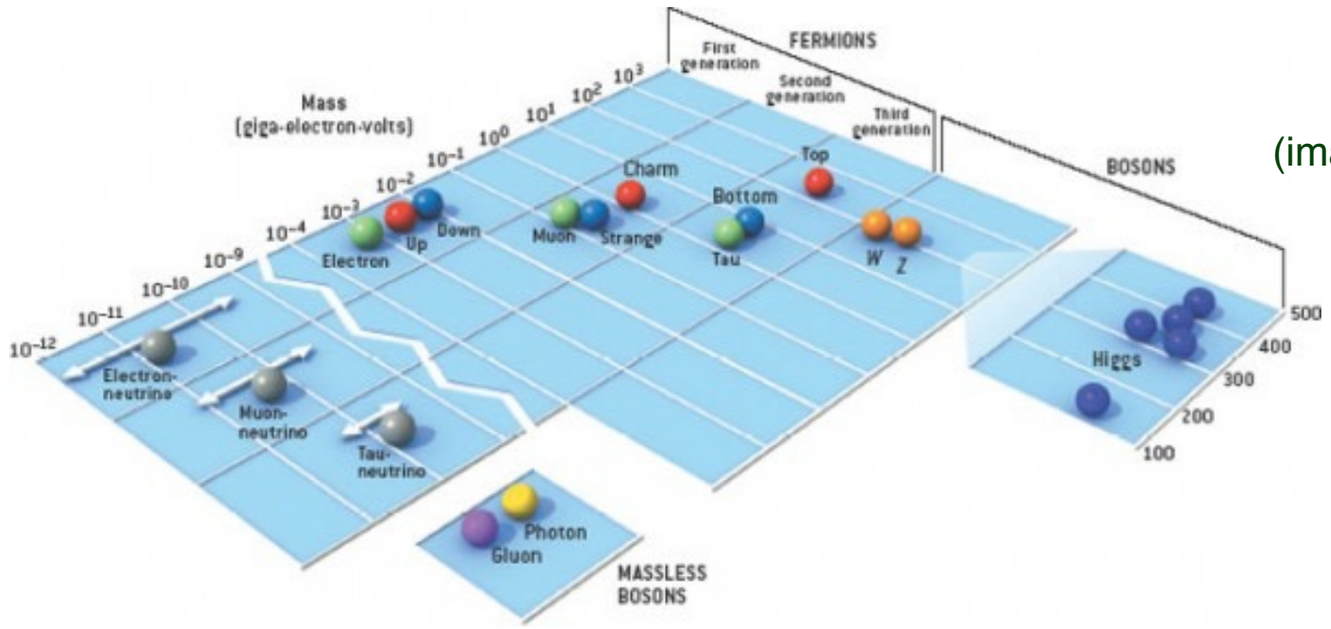
traditional SM definition: no neutrino mass **13 out of 19 SM parameters**

(nontrivial SM gauge reps + renormalizable operators only)

With neutrino masses: model-dependent, but at least 7 more (9 more if Majorana)

"SM flavor puzzle" → **can we explain these parameters?**

The quest to answer this question both predates and runs in parallel with the theoretical development of the SM itself, as well as its extensions.



(image credit: Scientific American/G. Kane)

Some of the key elements for SM:

V-A interactions	**Marshak, Sudarshan (1957), Feynman, Gell-Mann (1958),…**
Cabibbo angle and weak interactions universality	**Cabibbo (1963),…**
GIM mechanism and FCNC suppression	**Glashow, Iliopoulos, Maiani (1970),…**
quark sector CP violation	**Kobayashi, Maskawa (1973),…**

No attempt here to characterize this rich and interwoven history fully

This quest has resulted in its own rich history, and multitude of ideas.

Again, impossible to be fully comprehensive. (Consider referencing accordingly!)

Instead, the aim here is to provide a brief guide and review of ideas for addressing the SM Flavor Puzzle, with two main caveats:

- **Put aside (for the most part) the "why 3 generations?" question**

  A profound question. But one that likely requires physics beyond EFT's.
  We'll instead tacitly assume three generations and work in that framework.

- **Concentrate mainly on responses to a subset of experimental breakthroughs**

  ✓ **discovery of top quark** — CDF, D0 (1995)

  ✓ **precise determination of quark mixing matrix entries** — many exps/analyses

  ✓ **discovery of neutrino oscillations** — SuperK (1998)

  and subsequent two decades of lepton mixing measurements — many others (some details shortly)

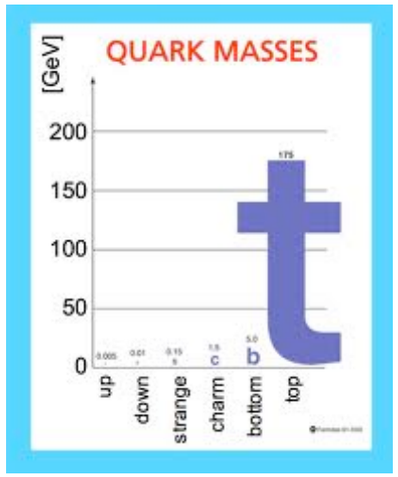
  ✓ **Higgs discovery and coupling measurements to fermions** — ATLAS, CMS (2012)

Each has had an important and substantial impact on the SM flavor puzzle!

But first, a (brief) summary of the data

(image credit: D0 Single Top group)

- **Quark and charged lepton masses**



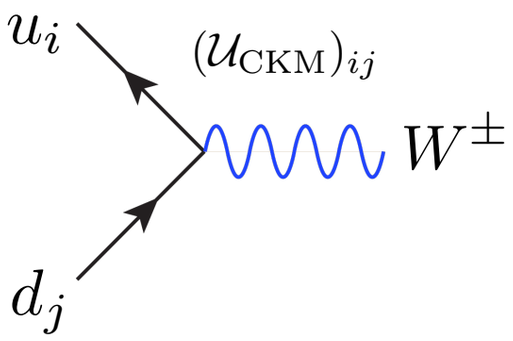
$$m_u \simeq 2-3\ \text{MeV} \qquad m_c \simeq 1.3\ \text{GeV} \qquad m_t \simeq 173\ \text{GeV}$$

$$m_d \simeq 4-6\ \text{MeV} \qquad m_s \simeq 90-100\ \text{MeV} \qquad m_b \simeq 4\ \text{GeV}$$

$$m_e = 0.511\ \text{MeV} \qquad m_\mu \simeq 106\ \text{MeV} \qquad m_\tau \simeq 1.8\ \text{GeV}$$

PDG (2019)

- **Quark mixing**



$$\mathcal{U}_{\rm CKM} = \mathcal{R}_1(\theta_{23}^{\rm CKM})\mathcal{R}_2(\theta_{13}^{\rm CKM}, \delta_{\rm CKM})\mathcal{R}_3(\theta_{12}^{\rm CKM})$$

"standard" PDG parametrization

$$s_{13} \ll s_{23} \ll s_{12}$$

Wolfenstein parametrization: $\lambda = \sin\theta_c$ **(Cabibbo angle)**

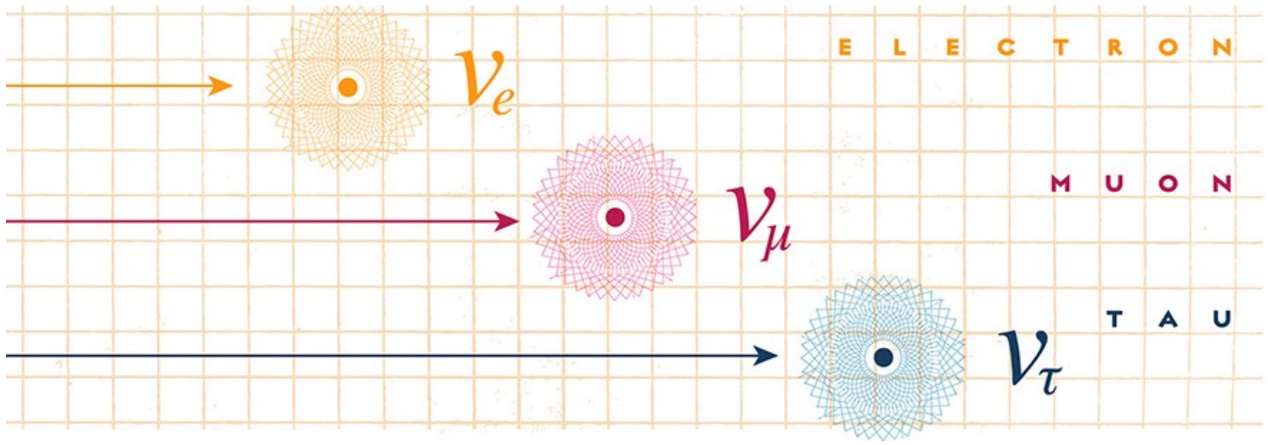
(Cabibbo expansion)

$$s_{12} = \lambda \quad s_{23} = A\lambda^2 \quad s_{13} = A\lambda^3(\rho + i\eta) = \frac{A\lambda^3(\bar\rho + i\bar\eta)\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}(1 - A^2\lambda^4(\bar\rho + i\bar\eta))}$$

$$\lambda \simeq 0.225 \qquad A \simeq 0.83 \qquad \bar\rho \simeq 0.1 \quad \bar\eta \simeq 0.35$$

PDG (2018)

- **Neutrino masses** SM → νSM



(image credit: C. Wiens)

Nobel Prize, 2015

neutrino oscillations discovery!

SuperK ’98…

2 decades+ of results since…

Some highlights:

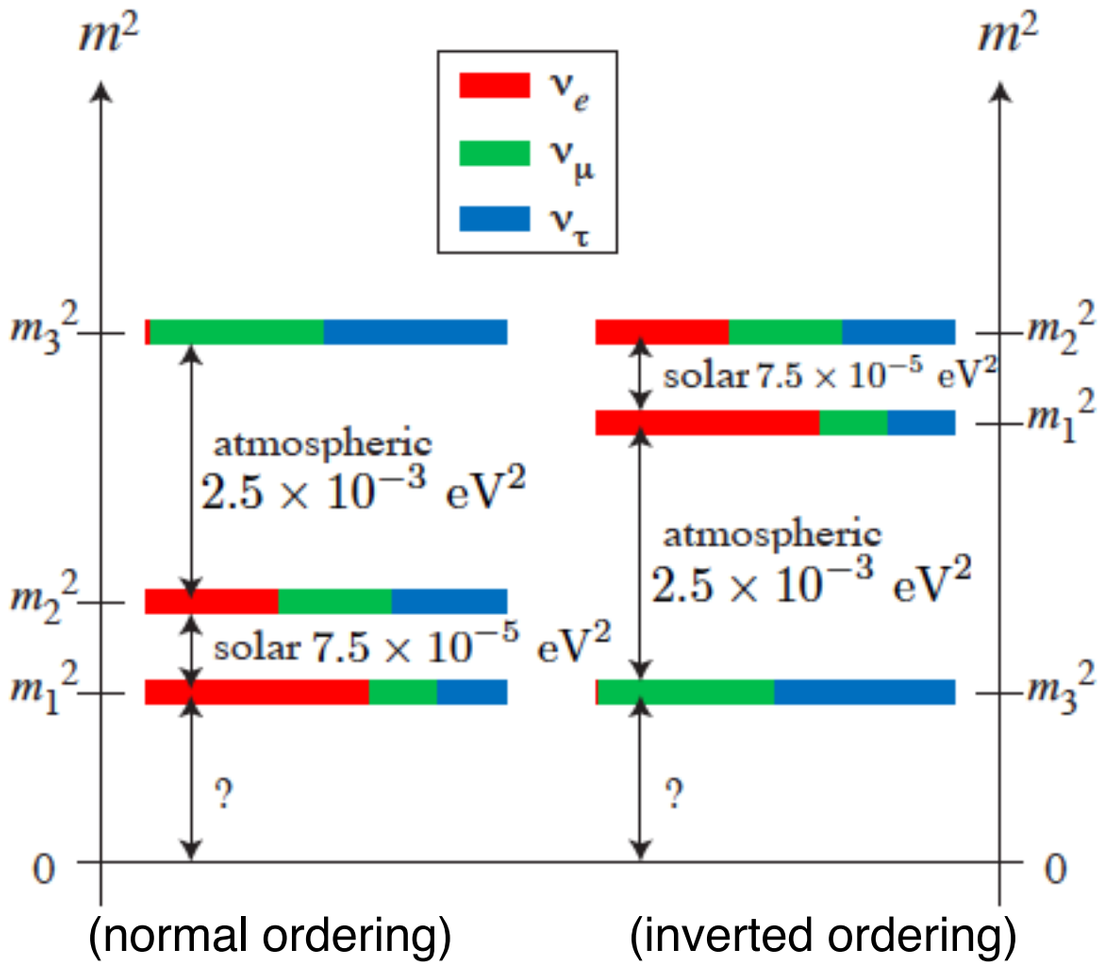
- 1998: atmospheric ν_μ disappearance (SK)
- 2002: solar ν_e disappearance (SK)
- 2002: solar ν_e appear as ν_μ, ν_τ (SNO)
- 2004: reactor $\overline{\nu}_e$ oscillations (KamLAND)
- 2004: accelerator ν_μ disappearance (K2K)
- 2006: accelerator ν_μ disappearance (MINOS)
- 2011: accel. ν_μ appear as ν_e (T2K,MINOS)
- 2012: reactor $\overline{\nu}_e$ disappear (Daya Bay, reactor angle measured! RENO)
- 2014: CP violation hint? (T2K)
- 2015: normal hierarchy hint? (SK, T2K, NOvA)
- 2016: non-maximal atm hint? (NOvA)
- 2018: CP cons disfavored at 2σ (T2K)

Lectures here by Messier

The emergent picture…

a (seemingly) robust 3-neutrino mixing scheme

✓ **mass-squared differences** $\Delta m^2_{ij} \equiv m^2_i - m^2_j$



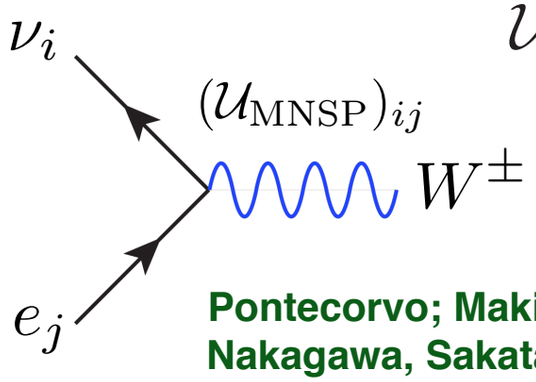
(image credits: King, Luhn)

✓ **individual masses: limits from direct searches, cosmology**

$m_{\bar{\nu}_e} < 2$ eV $\qquad \sum m_\nu < O(1$ eV$)$

- **lepton mixing**

diagonal phase matrix
(if Majorana neutrinos)



Pontecorvo; Maki, Nakagawa, Sakata

$$\mathcal{U}_{\rm MNSP} = \mathcal{R}_1(\theta_{23})\mathcal{R}_2(\theta_{13},\delta)\mathcal{R}_3(\theta_{12})\mathcal{P}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{-i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathcal{P}$$

Global Fits:

Forero et al., ’17 Capozzi et al.,’18

Gonzalez-Garcia et al., (www.nu-fit.org)

NuFit, Nov 2018

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 4.7$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK-atm	$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
	$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$
	$\sin^2\theta_{23}$	$0.580^{+0.017}_{-0.021}$	$0.418 \to 0.627$	$0.584^{+0.016}_{-0.020}$	$0.423 \to 0.629$
	$\theta_{23}/^\circ$	$49.6^{+1.0}_{-1.2}$	$40.3 \to 52.4$	$49.8^{+1.0}_{-1.1}$	$40.6 \to 52.5$
	$\sin^2\theta_{13}$	$0.02241^{+0.00065}_{-0.00065}$	$0.02045 \to 0.02439$	$0.02264^{+0.00066}_{-0.00066}$	$0.02068 \to 0.02463$
	$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.22 \to 8.99$	$8.65^{+0.13}_{-0.13}$	$8.27 \to 9.03$
	$\delta_{\rm CP}/^\circ$	215^{+40}_{-29}	$125 \to 392$	284^{+27}_{-29}	$196 \to 360$
	$\frac{\Delta m^2_{21}}{10^{-5}\ {\rm eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ {\rm eV}^2}$	$+2.525^{+0.033}_{-0.032}$	$+2.427 \to +2.625$	$-2.512^{+0.034}_{-0.032}$	$-2.611 \to -2.412$

Note

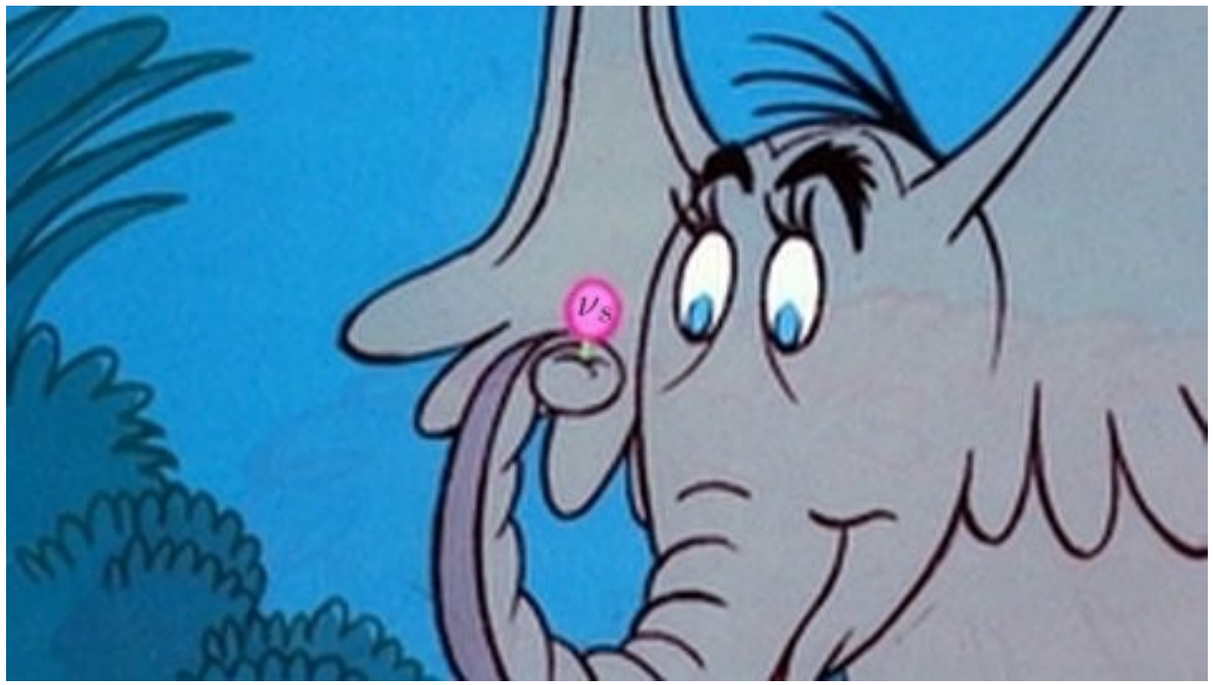
here: “MNSP”

more often: “PMNS”

(apologies for non-standard notation!)

Caveat: sterile neutrino(s)?

(image credit: ParticleBites)



Anomalies:

1995: $\overline{\nu}_e$ appearance (LSND)

2007: $\overline{\nu}_e$ appearance (MiniBooNE)

2012: ν_e appearance (MiniBooNE)

1995: ν_e disappearance (Gallium)

2011: ν_e disappearance (Reactor)

[lots of results, investigation in the interim…]

[well-documented tension between appearance and disappearance data]

See: Huber's IPA 2017 talk for "scorecard" (and lectures here)
Maltoni's talk at Neutrino 2018

Restrict focus here to 3 active light families only.

A plethora of interesting results to (try to) explain!

Immediate impression:

- **Quark and lepton sectors look very different!**

  ✓ **Quarks: hierarchical masses, small mixings, O(1) CP violation**

  ✓ **Leptons: hierarchical charged lepton masses**
  hierarchy apparently "milder" for neutrino masses
  two large mixing angles (or more)**

→ **implications for quark-lepton unification and other BSM physics**

These lectures:

- **Start with charged fermions.**
  key historical developments, standard paradigms for hierarchy/small mixing

- **Next: neutrino masses, lepton sector. Very rich framework.**
  new approaches, implications for connections (or not) to quark sector

(Charged) Fermion Masses

- **Key feature of SM:** **chiral charged fermions**

  ✓ **disallows bare "vectorlike" masses**

  ✓ **Mass generation must proceed via Yukawa interactions w/Higgs:**

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \qquad Q_{Li} = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}, L_{Li} = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix} \qquad u_{Ri},\, d_{Ri},\, e_{Ri} \qquad i,j = 1,2,3$$

(family indices)

(Lorentz and gauge indices suppressed)

Yukawa couplings →

$$Y_{ij} H \cdot \bar{\psi}_{Li} \psi_{Rj}$$

↑ **Higgs**

→ **Dirac masses upon electroweak symmetry breaking (EWSB)**

$$\mathcal{M}_{ij}^{\text{Dirac}} \equiv Y_{ij} \langle H \rangle$$

Dirac

see FB "The Same Oddly Asymmetric Picture of Paul Dirac Each Day"

$$Y_{ij} H \cdot \bar{\psi}_{Li} \psi_{Rj}$$

$$\mathcal{M}_{ij}^{\text{Dirac}} \equiv Y_{ij} \langle H \rangle$$

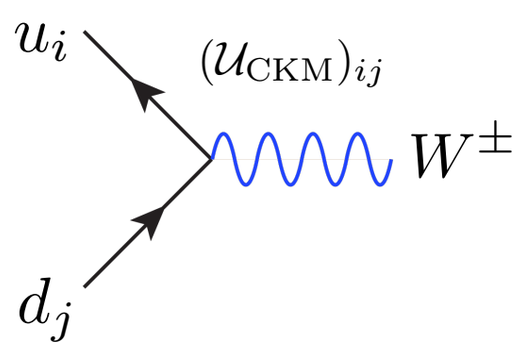
Diagonalize via bi-unitary transformation:

$$\mathcal{U}_{fL}^{\dagger} \mathcal{M}_f^{\text{Dirac}} \mathcal{U}_{fR} = \mathcal{M}_f^{\text{diag}}$$

$$\mathcal{U}_{fL}^{\dagger} \mathcal{M}_f \mathcal{M}_f^{\dagger} \mathcal{U}_{fL} = (\mathcal{M}_f^{\text{diag}})^2 \qquad \mathcal{U}_{fR}^{\dagger} \mathcal{M}_f^{\dagger} \mathcal{M}_f \mathcal{U}_{fR} = (\mathcal{M}_f^{\text{diag}})^2$$

CKM: combination of left-handed rotations

$$\mathcal{U}_{\text{CKM}} = \mathcal{U}_{uL}^{\dagger} \mathcal{U}_{dL}$$



(right-handed rotations unobservable in SM)

Neglecting neutrino masses (for now): lepton mixing unobservable

With one Higgs doublet (SM):**

size of charged fermion masses → **size of Yukawa couplings**

$$Y \sim 10^{-6}$$

hierarchy!

$$Y \sim 1$$

No guiding principle for Yukawa couplings in the SM

(each a priori a complex 3x3 matrix in family space)

many small dimensionless numbers (mass ratios, mixings) to explain!

****>1 Higgs doublet: additional possibilities due to distribution of electroweak VEV**

(= more on this later)**

Options

- **Bottom-up approach:** **special (simplified) Yukawa structures**

(over)constrain → **mixing angles as functions of mass ratios**

Famous example: Gatto-Sartori-Tonin relation

$$\sin\theta_c = \sqrt{m_d/m_s}$$

Gatto, Sartori, Tonin ’68

✓ **early two-family implementation:**

down-type quark mass matrix symmetric

vanishing 1-1 entry

(up-type quark diagonal basis)

Weinberg ’77

Wilczek, Zee ’77

Fritzsch ’77

✓ **Generalization to three families** → **“texture zeros”**

(basis-dependent statement)

One canonical early example: **Fritzsch ansatz** Fritzsch ’78

Fritzsch texture:

Fritzsch '78

- ✓ **symmetric mass matrices (up to phases)**
- ✓ **four texture zeros in each**

$$\mathcal{M}_{u,d,e} = P_{u,d,e} \begin{pmatrix} 0 & a_{u,d,e} & 0 \\ a_{u,d,e} & 0 & b_{u,d,e} \\ 0 & b_{u,d,e} & c_{u,d,e} \end{pmatrix} Q_{u,d,e}$$

P, Q diagonal phase matrices

2 phases relevant for q mixing: (ψ, ϕ)

- ✓ **Eigenvalue hierarchy:** $a \ll b \ll c$
- ✓ **yields GST relation for quarks**

→ **But ruled out for quarks:**

combination of top quark mass + $(\mathcal{U}_{\rm CKM})_{cb}$ **CKM measurement**

$$|\mathcal{U}_{cb}| = \left| \sqrt{m_s/m_b} - e^{i\phi} \sqrt{m_c/m_t} \right|$$

Another canonical example: Georgi-Jarlskog texture

Georgi, Jarlskog '79

✓ **symmetric/Hermitian mass matrices**

✓ **different structures for *u*, *d*, *e* (inspired by GUT embedding**):**

$$\mathcal{M}_u = \begin{pmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & c \end{pmatrix}$$ **(Fritzsch-like)**

$$\mathcal{M}_d = \begin{pmatrix} 0 & de^{i\phi} & 0 \\ de^{-i\phi} & f & 0 \\ 0 & 0 & g \end{pmatrix} \qquad \mathcal{M}_e = \begin{pmatrix} 0 & d & 0 \\ d & -3f & 0 \\ 0 & 0 & g \end{pmatrix}$$ **(five texture zeros)**

✓ **7 parameters for 13 observables: 6 predictions**

→ **their specific realization ruled out for quarks:** $|\mathcal{U}_{cb}| \simeq |\sqrt{m_c/m_t}|$

✓ **What persists: GJ mass relations (assumed to hold at high scale)**

$$m_b = m_\tau \qquad m_\mu = 3m_s \qquad m_d = 3m_e$$

Chanowitz et al. '77
Buras et al. '78

factors of 3: # of colors (RG effects)

Continued exploration of texture zero framework: a few highlights below

"Stitching the Yukawa Quilt"

Ramond, Roberts, Ross '93

✓ **Hermitian mass matrices**

✓ **4 or 5+ texture zeros**

→ **Parametrization via Cabibbo expansion a la Wolfenstein**

Highly predictive!

Found five then-viable scenarios with 5 texture zeros, more with 4

Systematic review of broader set of predictive possibilities

Fritzsch, Xing '99

→ **How do these structures fare today?**

Predictive scenarios (few parameters) ruled out. Need more general forms.

✓ **survey of more general possibilities** — Ludl, Grimus '15

✓ **"Occam's razor" approach:** — Tanimoto, Yanagida '16

asymmetric matrices, 3 texture zeros

Fritzsch ’84; Jarlskog ’86, Koide ’89…

- **Bottom-up approach:** **“flavor democracy”**

Start with “democratic” rank 1 mass matrix:

enhanced symmetry point: $(S_3)_L \times (S_3)_R$

$$\mathcal{M}_{u,d} = \frac{m_{t,b}}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

One nonzero eigenvalue (3rd gen), 2 zero eigenvalues (1st and 2nd)

→ **3rd generation eigenvector: trimaximal** $\frac{1}{\sqrt{3}}(1,1,1)$

(thus can be diagonalized by tribimaximal mixing matrix)

Generate masses for light generations:

symmetry breaking (two stages)

Fritzsch, Holtmanspotter ’94

Xing ’97

Immediate questions:

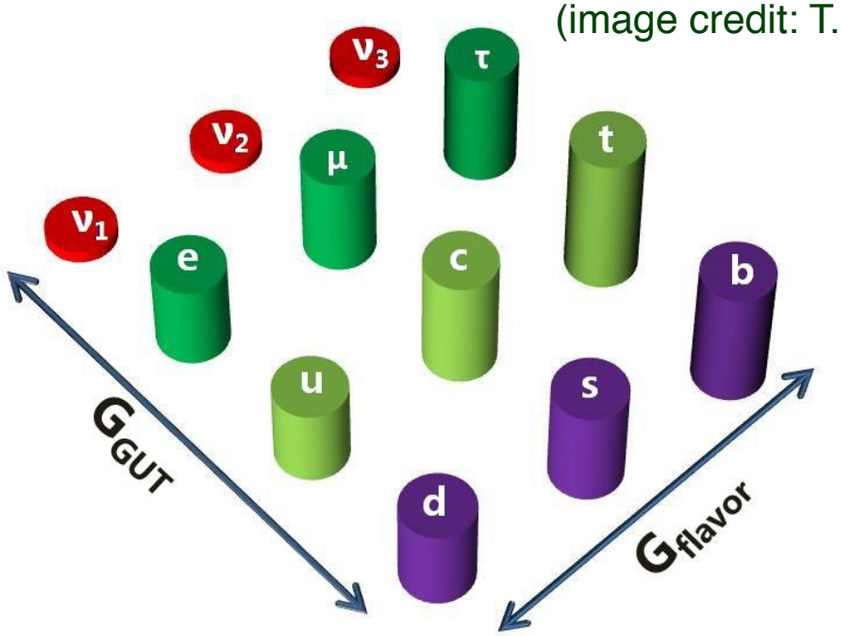
These structures are *ad hoc* →

can they be explained within a broader theory?

are they stable?

- ✓ **Leads to the notion of family (“horizontal”) symmetry**
- ✓ **Often also explored within grand unified (“vertical”) symmetry**

(image credit: T. Ohlsson, KTH)



- ✓ **Both often explored within SUSY**

(“natural” for GUTs, control of high scale)

Family Symmetry (also called Horizontal or Flavor Symmetry)

- **Postulate symmetry G_f that distinguishes generations**

  G_f spontaneously broken by "flavon" fields $\{\varphi_a\}$

$$Y_{ij} H \cdot \bar{\psi}_{Li} \psi_{Rj} \longrightarrow \left(\frac{\langle\varphi\rangle}{\Lambda}\right)^{n_{ij}} H \cdot \bar{\psi}_{Li} \psi_{Rj}$$

$$\epsilon = \left(\frac{\langle\varphi\rangle}{\Lambda}\right) \qquad \Lambda > \langle\varphi\rangle > \langle H\rangle \qquad \epsilon \ll 1$$

Froggatt, Nielsen '79

Idea: small numbers given by ratio of mass scales

Introduce heavy Froggatt-Nielsen (FN) fermion fields

with renormalizable couplings to SM and flavons

integrate out heavy states

Heavy sector >> TeV **(avoid too-large flavor-changing neutral currents)**

Many reviews: e.g. Martin ’97, Chung et al. ’03,…

- **Often explored in framework of *N=1* softly broken SUSY**

  ✓ **logical choice given high scales involved**

  ✓ **consistency with gauge coupling unification, etc.**

Several specific advantages of SUSY in this context:

  ✓ **holomorphy of superpotential** →

better theoretical control of texture zeros (“supersymmetric zeros”)

caveat: Kahler potential corrections (canonical normalization effects)

  ✓ **two Higgs doublets: more flexibility in G_f charge assignments**

- **Immediate consequence: SUSY flavor/CP problem**

We will not focus on this important and relevant issue here.

Many authors (including local experts!)…

- **Possibilities for G_f:**

canonical example: $\boxed{U(1)}$ **family symmetries** Froggatt, Nielsen '79

Have guidance from enhanced symmetry for vanishing Yukawas:

✓ **SM (no neutrino masses):** $U(3)^5$

$$U(3)_Q \otimes U(3)_{u^c} \otimes U(3)_{d^c} \otimes U(3)_L \otimes U(3)_{e^c}$$

$\longrightarrow$ G_f **subgroup of** $U(3)^5$

Key input: ***O(1)*** **top quark Yukawa coupling**

$$U(3)_Q \otimes U(3)_{u^c} \xrightarrow{m_t} U(2)_Q \otimes U(2)_{u^c}$$

Hall lecture, TASI '97

Many examples! continuous Abelian, non-Abelian, discrete non-Abelian

One canonical example: anomalous $U(1)_A$ family symmetry

anomalies cancelled via universal Green-Schwarz mechanism

Dine-Seiberg-Witten (SUSY framework) Green, Schwarz '84

Dine, Seiberg, Witten '87

$$\frac{C_i}{k_i} = \frac{C_{\rm grav}}{12} = \frac{C_F}{k_F}$$

✓ **prediction of Weinberg mixing angle**

Ibanez, Ross '94
Jain, Schrock '94
Binetruy, Ramond '95
Binetruy, Lavignac, Ramond '96
...

✓ **calculable** ϵ

$$\epsilon \sim O(\lambda)$$

$\lambda = \sin\theta_c$ **(Cabibbo angle)**

Prototypical example of Cabibbo expansion for quark masses and mixing:

$$\frac{m_u}{m_t} \sim \lambda^8, \quad \frac{m_c}{m_t} \sim \lambda^4 \qquad \frac{m_d}{m_b} \sim \lambda^4, \quad \frac{m_s}{m_b} \sim \lambda^2$$

valid at high scales

(evolve parameters to weak scale via RGE)

$$\mathcal{U}_{\rm CKM} = \mathcal{U}_{uL}^\dagger \mathcal{U}_{dL} = 1 + O(\lambda)$$

unique theoretical starting point!**

Another canonical example: $U(2)$ **family symmetry** **(SUSY framework)**

non-Abelian symmetry: how to embed the quark families in G_f

natural choice: $\mathbf{2} \oplus \mathbf{1}$

1st, 2nd generations **3rd generation (renormalizable top quark coupling)**

Concrete $U(2)$ **case:**

$$U(2) \xrightarrow{\epsilon} U(1) \xrightarrow{\epsilon'} 0$$

"Effective flavons"

$$\varphi/\Lambda \sim \epsilon$$
$$S^{ab}/\Lambda \sim \epsilon$$
$$A^{ab}/\Lambda \sim \epsilon'$$

$$Y_{u,d,e} \sim \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix}$$

$$\epsilon \sim 0.02 \sim O(\lambda^2)$$
$$\epsilon' \sim 0.001 \sim O(\lambda^4)$$

GST relation obtained!

qualitatively good agreement with data

Barbieri, Dvali, Hall '96
Barbieri, Hall, Raby, Romanino '96
Barbieri, Hall, Romanino '97
...

(Supersymmetric) Grand Unification

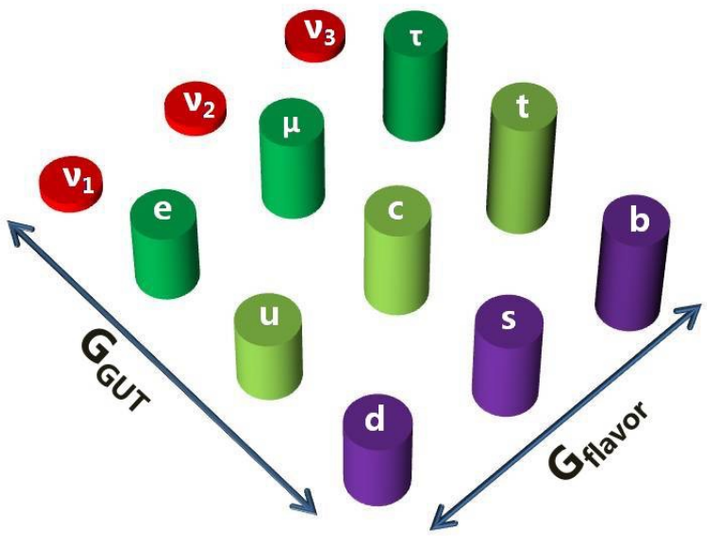
- **Postulate "vertical" symmetry** $G_{\rm GUT}$

Canonical examples:

$$G_{\rm GUT} = SU(5)$$ **Georgi, Glashow '74**

$$G_{\rm GUT} = SO(10)$$ **Georgi '75, Fritzsch, Minkowski '75**

$$G_{\rm GUT} = SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$$ **Pati, Salam '73**



(image credit: T. Ohlsson, KTH)

Many well-known advantages!

✓ **SM quantum numbers**

✓ **gauge coupling unification (SUSY)**

This context:

consequences for SM flavor puzzle

→ **Yukawa unification**

SUSY GUT Basics:

Standard embedding of SM generations and minimal Higgs content

$$SU(5): \quad \bar{\mathbf{5}} \ \{d^c, L\} \quad \mathbf{10} \quad \{Q, u^c, e^c\} \quad (\mathbf{5_H}, \bar{\mathbf{5}}_\mathbf{H})$$

$$SO(10): \quad \mathbf{16} \qquad \mathbf{16} = \mathbf{10} \oplus \bar{\mathbf{5}} \oplus \mathbf{1} \quad \mathbf{10_H}$$

Tree-level GUT relations: **Yukawa unification**

$$SU(5): \quad \mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}_\mathrm{H} \quad \mathbf{10}\,\mathbf{10}\,\mathbf{5}_\mathrm{H} \qquad \text{partial unification: } Y_d = Y_e^T$$

$$SO(10): \quad \mathbf{16}\,\mathbf{16}\,\mathbf{10}_\mathrm{H}$$

Thus maximal G_f **is given by** $SU(5): \ U(3)^2 \quad SO(10): U(3)$

Yukawa unification works reasonably well for 3rd generation (large $\tan\beta$**)**

Many authors. see e.g. Carena et al. '94

but catastrophic for lighter generations! (better: approximate GJ mass relations)

Fixing the problem:

✓ **tree-level relations for third generation only**

✓ **higher-dimensional Higgs representations, G_f assignments for first and second generations**

Many examples!**

Many authors. Key players: Raby, Mohapatra, Babu,…

example: $U(2)$ family symmetry in $SU(5)$ and $SO(10)$

Barbieri, Hall, Raby, Romanino ’96
Barbieri, Hall, Romanino ’97, ’98

need to suppress 22, 21 entries of Y_u

nontrivial $SU(5)$ reps for “effective” flavons

$$\{S^{ab}(\mathbf{75}), A^{ab}(\mathbf{1}), \varphi(\mathbf{1},\mathbf{24}), \Sigma(\mathbf{24})\}$$

✓ **explicit Froggatt-Nielsen realization (including flavon dynamics)**

stronger up-type mass hierarchies

✓ **Georgi-Jarlskog mass relations (approximate)**

****Return to this topic after discussing lepton sector**

More options

- **Radiative mass generation**

Idea: small numbers explained via loop effects

✓ third generation: tree level

✓ each successive generation suppressed by loop factor

Barr, Zee ’77, Ibanez ’82, Balakrishna et al. ’88, Babu, Ma ’88

He et al. ‘90, Babu, Mohapatra ’90; Dobrescu et al. ’08,..

- **Extra (warped) dimensions**

Randall, Sundrum ’99

Idea: small numbers explained via warping, localization

Arkani-Hamed, Schmaltz ’00

Huber, Shafi ’01

Huber ’03, Agashe et al. ’04,…

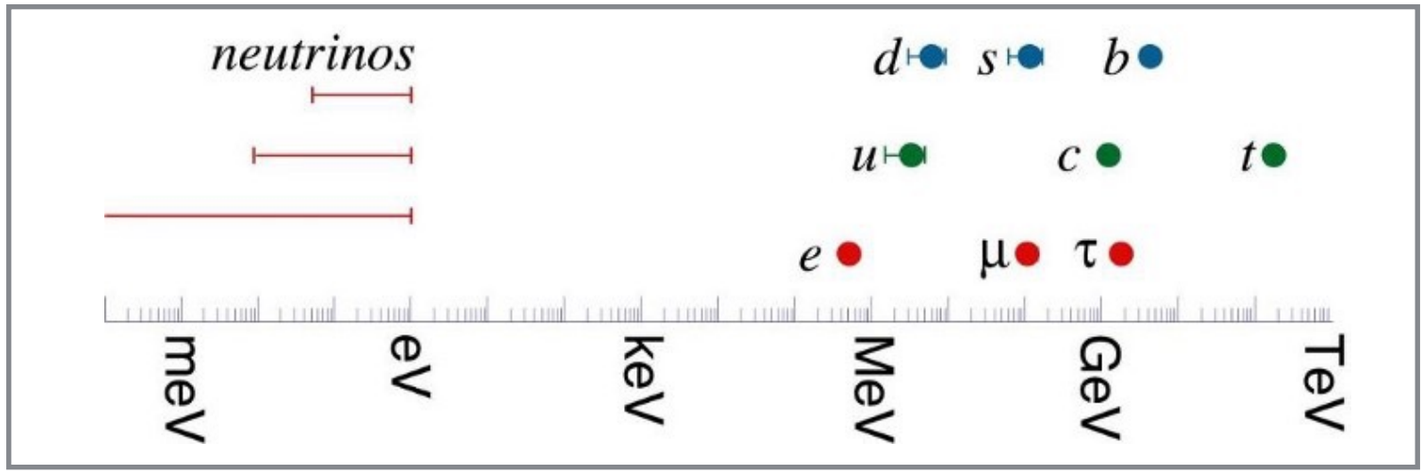
✓ third generation and Higgs: near TeV brane

✓ lighter generation: closer Planck brane

Neutrino Sector

- **neutrino masses** **SM** → ν **SM**

(image credit: H. Murayama)



Many new aspects for SM flavor puzzle:

- ✓ **neutrinos Majorana or Dirac?**
- ✓ **origin of overall mass suppression**

} **discuss together**

- ✓ **origin of observed lepton mixing angle pattern**
- ✓ **connection (or not) to quark sector**

(image credits: FB, Wikipedia)

Mass Generation

- **Unlike SM, variety of theoretical starting points for ν SM**

Main question: **origin of neutrino mass suppression**

Options: Dirac

$$\Delta L = 0$$

~ charged fermion sector

Majorana

$$\Delta L = 2$$

lepton number-violating

Critically important question, to be settled by experiment.

First step in constructing models of neutrino masses:

which to choose?

Majorana neutrinos

- **lepton # accidental symmetry of SM**
  intriguing consequences (e.g. $0\nu\beta\beta$,…) if violated

SM at NR level: **Weinberg dimension 5 operator**

$$\frac{\lambda_{ij}}{\Lambda} L_i H L_j H \qquad\qquad \Delta L = 2$$

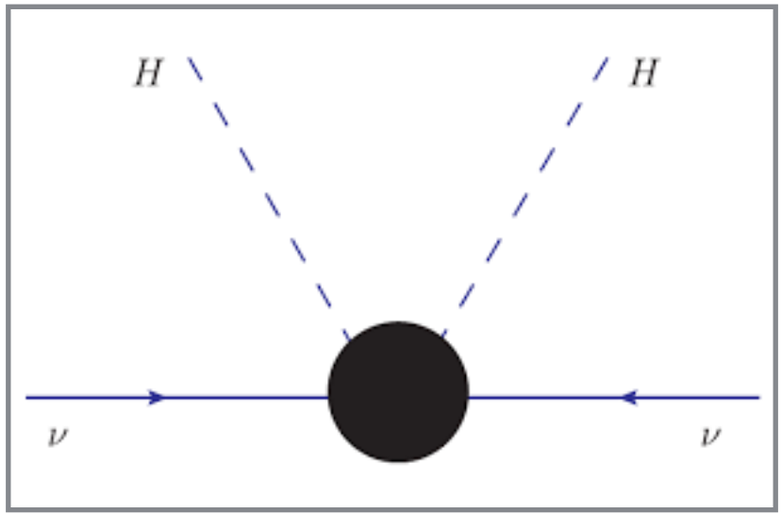
→ **results in Majorana neutrino masses upon EWSB**

$$\mathcal{M}_\nu^{\rm Maj} \simeq \frac{\lambda_{ij} \langle H \rangle^2}{\Lambda}$$

if $\lambda \sim O(1) \quad \Lambda \gg \langle H \rangle \sim O(100\ \mathrm{GeV})$ →

ν **mass suppression via ratio of EW to heavy scale**
but could have lighter scale, other means of suppression

- **Underlying mechanism for Weinberg operator** $\dfrac{\lambda_{ij}}{\Lambda} L_i H L_j H$



(image credit: Dinh et al.)

→ **3 tree-level possibilities**

✓ **Type I seesaw** ν_R **(fermion singlet)**

✓ **Type II seesaw** Δ **(electroweak triplet scalar)**

✓ **Type III seesaw** Σ **(electroweak triplet fermion)**

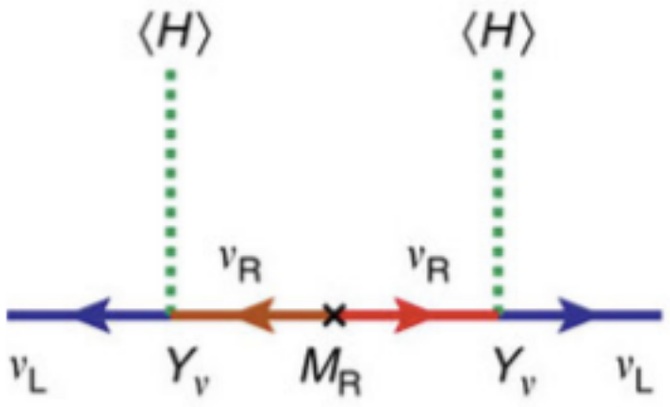
(and combinations)

see e.g. Ma, ’98

Type I seesaw

Minkowski ’77, Gell-Mann, Ramond, Slansky ’79, Yanagida ’79
Mohapatra, Senjanovic ’80, Schecter, Valle ’80…

✓ **introduce right-handed neutrinos**



(image credit: T. Ohlsson et al., Nat. Comm.)

(forbidden by EW symm)

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$$

$$m_1 \sim \frac{m^2}{M} \qquad m_2 \sim M \gg m_1$$

$$\nu_{1,2} \sim \nu_{L,R} + \frac{m}{M}\nu_{R,L}$$

$$Y_{ij} L_i \nu_{Rj} H + M_{R\,ij} \nu_{Ri} \nu^c_{Rj}$$

(unrelated to EW scale)

$$\boxed{\mathcal{M}_\nu \sim \langle H \rangle^2 Y M_R^{-1} Y^T}$$

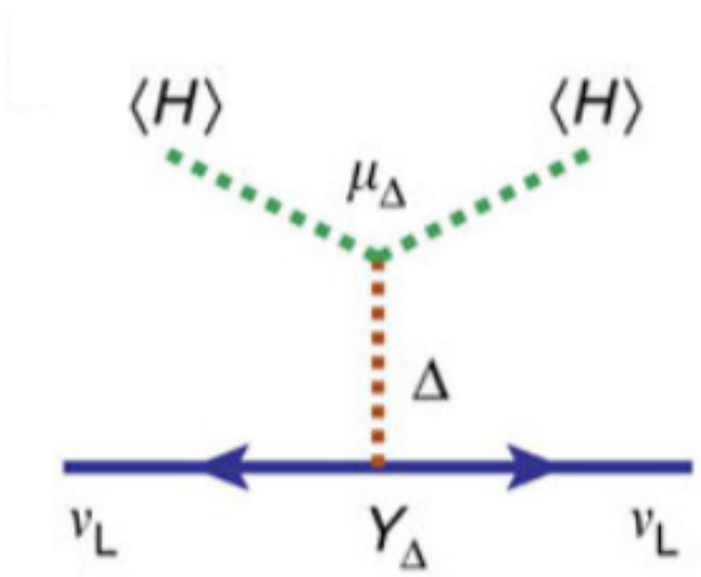
Advantages: economical, connection to grand unification, leptogenesis

Disadvantages: testability without model assumptions

Type II seesaw

Magg, Wetterich '80, Lazarides, Shafi, Wetterich '81,
Mohapatra, Senjanovic '81, Cheng, Li '80,...

✓ **introduce triplet Higgs scalar**



(image credit: T. Ohlsson et al., Nat. Comm.)

$$\Delta \sim (\mathbf{3}, 2)$$
$$(SU(2)_L, U(1)_Y)$$

$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$$

$$(Y_\Delta)_{ij} L_i L_j \Delta + \mu_\Delta H H \Delta$$

$$\boxed{\mathcal{M}_\nu \sim \langle H \rangle^2 Y_\Delta \mu_\Delta / M_\Delta^2}$$

can have clean LHC signatures of lepton # violation via decays of H^+, H^{++}

if $M_\Delta \leq O(\mathrm{TeV})$ Fileviez Perez et al. '08, Gavela et al. '09,...

(also LFV)

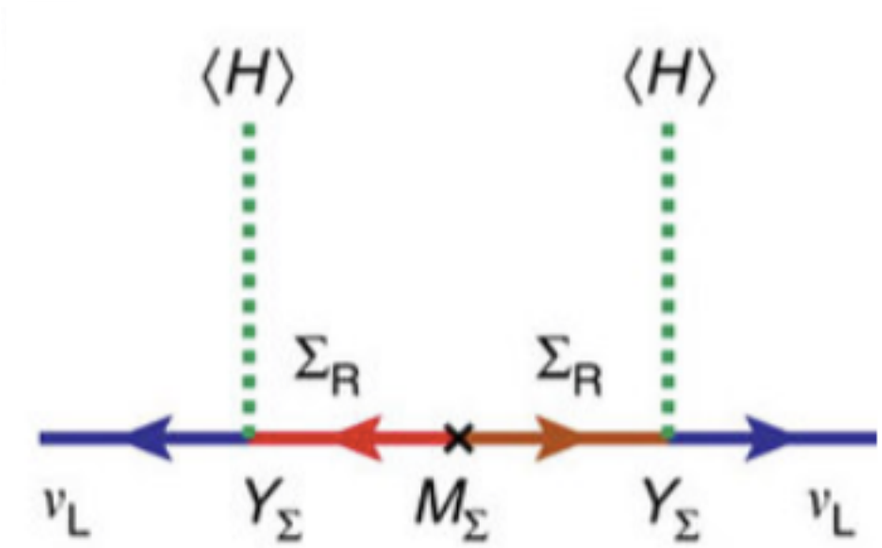
Advantages: testability (charged Higgs states probed at LHC)

Disadvantages: not as economical/minimal as Type I

Type III seesaw

Foot, He, Joshi ’89, Ma ’98,…

✓ **introduce electroweak triplet fermions**



(image credit: T. Ohlsson et al., Nat. Comm.)

$$\Sigma \sim (\mathbf{3}, 0)$$
$$(SU(2)_L, U(1)_Y)$$

$$\Sigma_i = \left(\Sigma_i^0, \Sigma_i^{\pm}\right)$$ **(3 of them)**

$$(Y_\Sigma)_{ij} L_i \Sigma_j H + (M_\Sigma)_{ij} \Sigma_i \Sigma_j$$

$$\mathcal{M}_\nu \sim \langle H \rangle^2 Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T$$

can have clean LHC signatures via mixing w/charged leptons

if $M_\Sigma \sim O(\mathrm{TeV})$ Franchesini, Hambye, Strumia ’08,…

also highly predictive pattern of LFV signals Abada et al. ’07, Gavela et al. ’09,…

Advantages: testability (new charged states probed at LHC, LFV)

Disadvantages: not as economical/minimal as Type I

Zee ’86, Babu ’88, Ma ’98,…

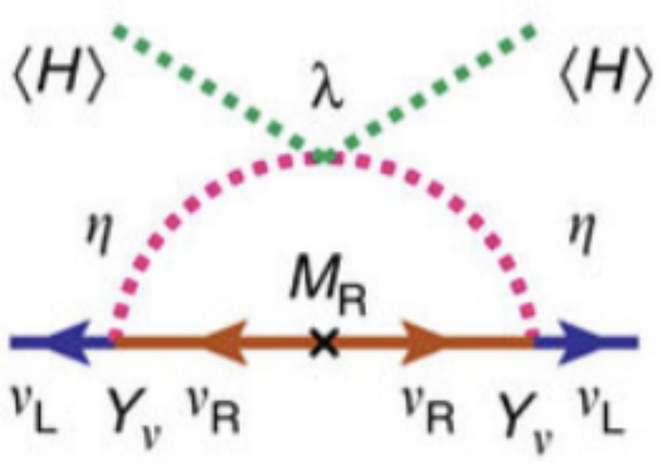
Radiative generation of Majorana masses

→ **complete Weinberg operator via loops**

radiative seesaw models

Canonical example: “scotogenic” model

Ma ’06



(image credit: T. Ohlsson et al., Nat. Comm.)

introduce right-handed neutrinos

new EW doublets (can be DM), Z_2 symmetry

$$\mathcal{M}_\nu \sim \lambda \frac{\langle H \rangle^2}{16\pi^2} Y M_R^{-1} Y^T$$

“radiative Type I seesaw”

analogous construction with fermion triplet: “radiative Type III seesaw”

Ma, Suematsu ’09

“radiative Type II seesaw”: forbid $L_i L_j \Delta$ via (softly broken) symm

$$L \to (-1)^L$$

Fraser, Kownacki, Ma, Popov ’15

Zee ’86, Babu ’88, Ma ’98,…

Radiative generation of Majorana masses

→ **alternatives to Weinberg operator**

Many other NR operators in SM with $\Delta L = 2$

(odd mass dimension d>5)

Babu and Leung ’01

de Gouvea and Jenkins ’07

Bonnet et al. ’12

Classification

d=7	**d=9**	**…**
$LLLe^cH$	$LLLe^cLe^c$ (Zee-Babu)	
$LLQd^cH$	$LLQd^cQd^c$	
$LL\overline{Q}\overline{u}^cH$	+ many others…	
$L\overline{e}^c\overline{u}^cd^cH$		

new physics scale can be accessible at LHC (subject to LFV bounds)

Many explicit realizations!

See e.g. excellent review:

Cai, Herrero-Garcia, Schmidt, Vicente, Volkas ’17

Generic advantage of radiative models: testability

potential connections to other, possibly accessible, new physics

✓ **possible connections to dark matter**

symmetry for dark matter stability → radiative ν mass

potential DM candidates in loops Ma ’15

see e.g. scotogenic case, many other examples

many authors!

✓ **possible connections to flavor physics anomalies**

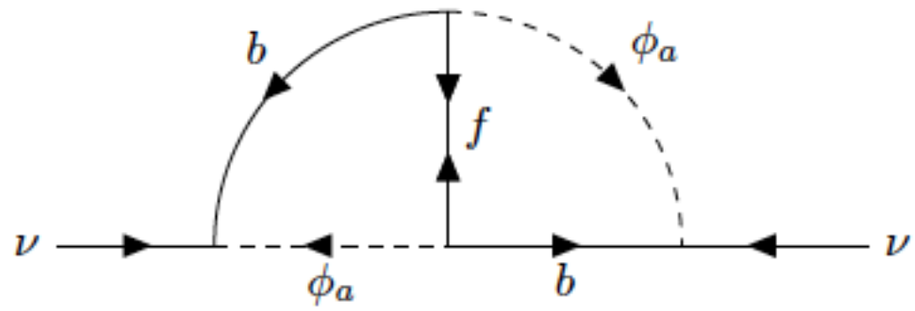
Päs, Schumacher ’15, Deppisch et al. ’16,…

One way leptoquarks can manifest themselves:

Two-loop representative example:

scalar leptoquark $\phi \sim (\mathbf{3}, \mathbf{1}, -1/3)$

+ octet fermion $f \sim (\mathbf{8}, \mathbf{1}, 0)$



Angel, Cai, Rodd, Schmidt, Volkas ’13

Cai, Gargalones, Schmidt, Volkas ’17

Many other ideas for Majorana masses

✓ **more complicated seesaws** → **lower scales**

e.g. double/inverse seesaw: 3 ν_R and 3 new singlet fermions

$$Y_{ij} L_i (\nu_R)_j H + M_{ij} S_i (\nu_R)_j + \mu_{ij} S_i S_j$$

Mohapatra, Valle ’86
…

$$\mu, m \ll M$$

$$\mathcal{M}_\nu \sim \begin{pmatrix} 0 & m & 0 \\ m & 0 & M \\ 0 & M & \mu \end{pmatrix} \qquad M_{\rm eff} = M^T \mu^{-1} M$$

$$\boxed{\mathcal{M}_\nu \sim \langle H \rangle^2 (Y^T M_{\rm eff}^{-1} Y)}$$

and extended versions…

Xing et al. ’09

✓ **SUSY with R-parity violation**

neutrino-neutralino mixing
loop effects

Aulakh, Mohapatra ’82, Hall, Suzuki ’84,…

see e.g. Borzumati et al. ’96, Grossman et al. ’03

✓ **Warped extra dimensions**

see e.g. Csaki, Delauney, Grojean, Grossman ’08
…

lepton number violation → **Majorana ν masses**

Dirac neutrinos

$\Delta L = 0$

- **Analogous to charged fermions,**
  but much stronger suppression $Y_\nu \sim 10^{-14}$

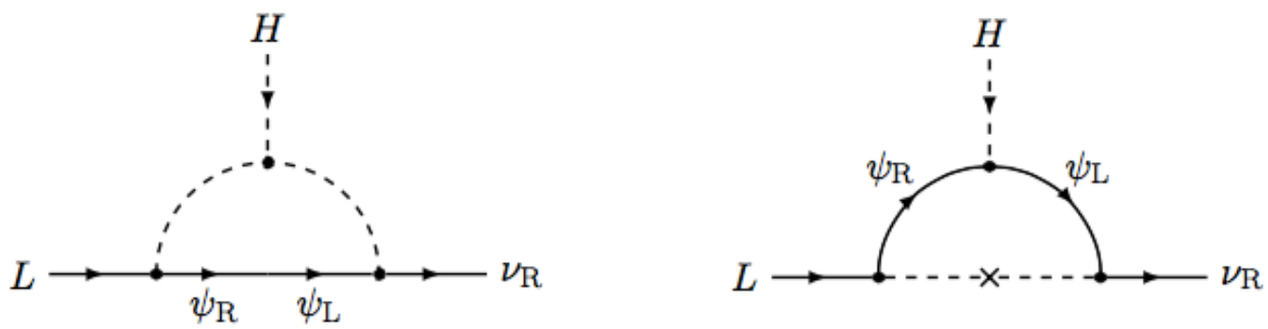
→ **Must forbid both types of "bare" mass terms**
Less intuitive, but mechanisms exist. Some examples:

✓ **texture zeros/Froggatt-Nielsen** — Hagedorn, Rodejohann '05,…

✓ **radiative mass generation** — Cheng, Li '78, Mohapatra '87, '88; Balakrishna, Mohaptra '89, Rajpoot '01; …

→ **e.g. discrete symmetry: ν_R nontrivial**
one-loop: 2 topologies

Ma, Popov '16; Wang et al. '16, '17; Review: Cai et al. '17



variety of possibilities for new states

(many studies in GUT contexts)

other options: higher loops, loop-induced vev,…

Dirac neutrinos (cont.)

✓ **extended gauge sectors**

non-singlet ν_R → **forbids "bare" terms, simplest seesaws**

higher-dimensional operators: e.g. $U(1)'$ **symmetry**

see e.g. Langacker '11 for review

✓ **SUSY breaking**

symm+holomorphy forbids superpotential

allows Kahler potential contributions

Arkani-Hamed et al. '00
Borzumati et al. '00,'01
Demir, LE, Langacker '08
…

✓ **string constructions**

exponentially suppressed interactions from stringy instanton effects…

Cvetic et al. '08,…
Langacker review, '11
…

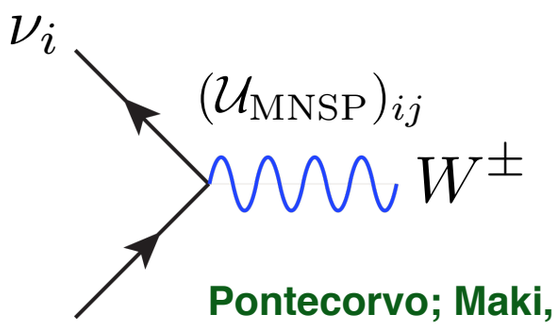
…

General theme for neutrino masses

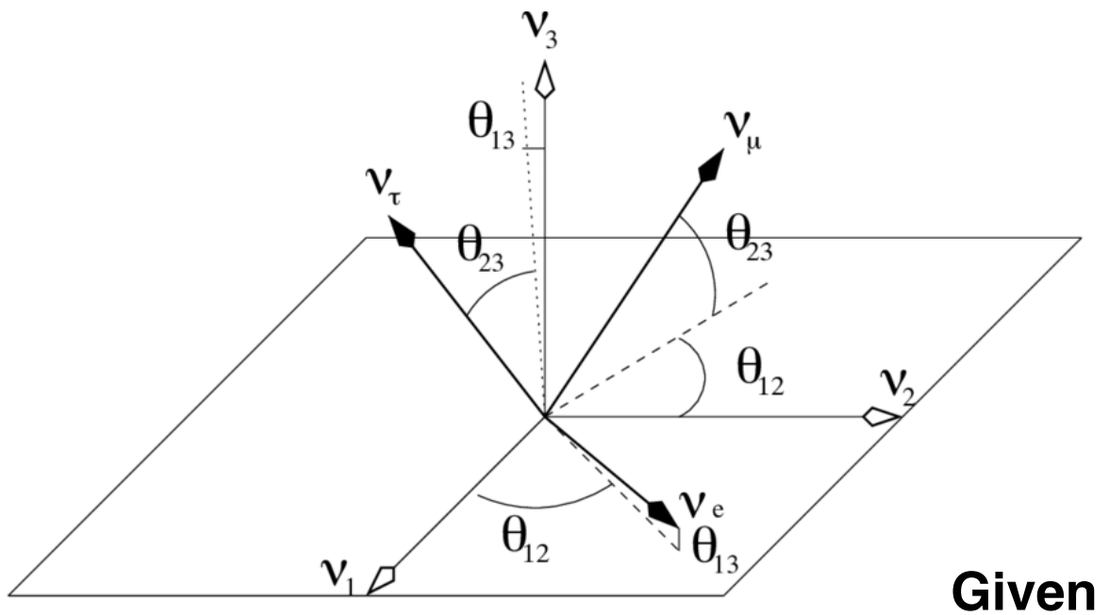
Much richer than charged fermion cases →

but trade-off between simplicity and testability

Lepton mixing pattern



Pontecorvo; Maki, Nakagawa, Sakata



(image credit: S. King)

(Dirac or Majorana neutrinos)

$$\mathcal{U}_\nu^T \mathcal{M}_\nu^{\rm Maj} \mathcal{U}_\nu = \mathcal{M}_\nu^{\rm diag}$$

$$\mathcal{U}_{fL}^\dagger \mathcal{M}_f^{\rm Dirac} \mathcal{U}_{fR} = \mathcal{M}_f^{\rm diag}$$

$$\mathcal{U}_{\rm MNSP} = \mathcal{U}_{eL}^\dagger \mathcal{U}_{\nu L}$$

$$\mathcal{U}_{\rm MNSP} = \mathcal{R}_1(\theta_{23}) \mathcal{R}_2(\theta_{13}, \delta) \mathcal{R}_3(\theta_{12}) \mathcal{P}$$

Given great progress in measuring MNSP parameters:

tremendous excitement among BSM theorists to shed light on the origin of the mixing pattern

(can work within any framework for mass suppression. vast majority: Type I seesaw)

The MNSP is quite different from the CKM! (a surprise!)

Two large mixing angles: $\theta_{23},\ \theta_{12}$

CP violation

Dirac phase: important goal of experimental program

Majorana phases: unlikely to know anytime soon

A basic question: is θ_{13} **"large" or "small"?**

large reactor angle **vs.** **small reactor angle**

the case for anarchy **the case for structure**

Anarchy

(character: Watterson)

Hall, Murayama, Weiner '99

anarchy hypothesis

$\mathcal{U}_\nu$ **from a random draw of unbiased distribution of 3x3 unitary matrices**

statistical tests: lower bound on $|\mathcal{U}_{e3}|^2$

basis independence:

distribution invariant upon unitary transformations

→ **flat in Haar measure**

Haba, Murayama '00

Post-reactor angle measurement: renewed focus

de Gouvea, Murayama '12
Altarelli et al. '12
Bai and Torroba '12
…

Some recent highlights:

RG analysis

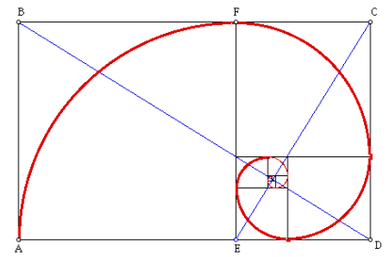
Brdar, Konig, Kopp '15

model-building + quark sector

Babu et al. '16,…
Fortin et al. '17

Note: anarchy hypothesis alone does not provide information on Δm^2

Structure



Standard assumption: structure from symmetry

Usual paradigm (recall charged fermions)

G_f **spontaneously broken at scale M**

dimensionless numbers governed by $\epsilon \sim \langle\varphi\rangle/M$ φ **= "flavon"**

But quite different from the quark sector!

Recall there, a natural identification:

$$\epsilon \sim O(\lambda) \qquad \lambda = \sin\theta_c$$

Cabibbo angle (or some power) as a flavor expansion parameter

In this context, a unique theoretical starting point:

$\mathcal{U}_{\rm CKM} \sim 1 + O(\lambda)$ **approximately the identity as** $\lambda \to 0$

- **For the leptons: not as straightforward**

In the basis where $\mathcal{M}_e$ is diagonal, $\mathcal{M}_\nu$ is not diagonal:

$\mathcal{M}_\nu$ diagonalization: 1 small, 2 large mixing angles

Arguably the most challenging* pattern: **(* for 3 families)**

3 small angles → **$\sim$ recall quarks**

2 small, 1 large → **$\sim$ $\mathrm{Rank}\mathcal{M}_\nu < 3$**

3 large angles → **anarchical $\mathcal{M}_\nu$**

relatively straightforward at leading order

2 large, 1 small → **fine-tuning, non-Abelian**

→ **A model-building opportunity!**

Family symmetry approach

✓ *spontaneously broken G_f

typical choice: →

discrete non-Abelian group

*But see recent interesting work in *symmetric* limit

Reyiumaji and Romanino, ’18

✓ Difference from quarks:

No unique theoretical starting point for “Cabbibo-like” expansion

$$\mathcal{U}_{\rm MNSP} \sim \mathcal{W} + O(\lambda') \qquad \lambda' \ll 1$$

↑

“bare” mixing angles (diagonal charged lepton basis)

$(\theta^{\nu}_{12}, \theta^{\nu}_{23}, \theta^{\nu}_{13})$

First stage: symmetry breaking to generate nontrivial $\mathcal{W}$

different unbroken subgroups for neutrinos, charged leptons

→ large mixing angles

Next stage: corrections as expansion in λ'

"Bare" mixing angles generically shift due to $O(\lambda')$ corrections

✓ ***A priori,*** **expansions in quark and lepton sectors unrelated**

Option **unification paradigm (broad sense): set $\lambda' = \lambda$**

ideas of quark-lepton complementarity and "Cabibbo haze"

$$\theta_{23} = \theta_{12} + \theta_c$$

(empirical)

Raidal '04, Minakata+Smirnov '04,...

("haze" terminology from Datta, L.E., Ramond '05)

not an unreasonable approach given the data

$$\theta_{13} \sim O(\lambda)$$

pre-measurement, idea that θ_{13} is a Cabibbo effect:

$$\theta_{13}^{\nu} = 0 \qquad \theta_{13} = \lambda/\sqrt{2}$$

Vissiani '98, '01

Ramond '04

Possible theoretical starting points:

✓ **Most studied: maximal atm, zero reactor** $\sin^2\theta^\nu_{23} = 1/2 \quad \sin^2\theta^\nu_{13} = 0$

→ **classify scenarios by bare solar angle**

tri-bimaximal mixing: $\sin^2\theta^\nu_{12} = 1/3$ — Harrison, Perkins, Scott ’02; Xing ’02, He, Zee ’02, Ma ’03…

bimaximal mixing: $\sin^2\theta^\nu_{12} = 1/2$ — Petcov ’82, Vissiani ’97, Barger et al. ’98, Baltz et al. ’98

golden ratio (A) mixing: $\sin^2\theta^\nu_{12} = 1/(2+r) \sim 0.276$, $r = (1+\sqrt{5})/2$ — Datta, Ling, Ramond ’03; Kajiyama, Raidal, Strumia ’08;…

golden ratio (B) mixing: $\sin^2\theta^\nu_{12} = (3-r)/4 \sim 0.345$ — Rodejohann ’09,…

hexagonal mixing: $\sin^2\theta^\nu_{12} = 1/4$ — Albright, Duecht, Rodejohann ’10, Kimand, Seo ’11,…

✓ **can also study nonzero reactor:** $\sin^2\theta^\nu_{13} \neq 0$ — Lam ’13, Holthausen et al. ’12,… …

All can be obtained via SSB of discrete non-Abelian family symmetries

Family symmetry models

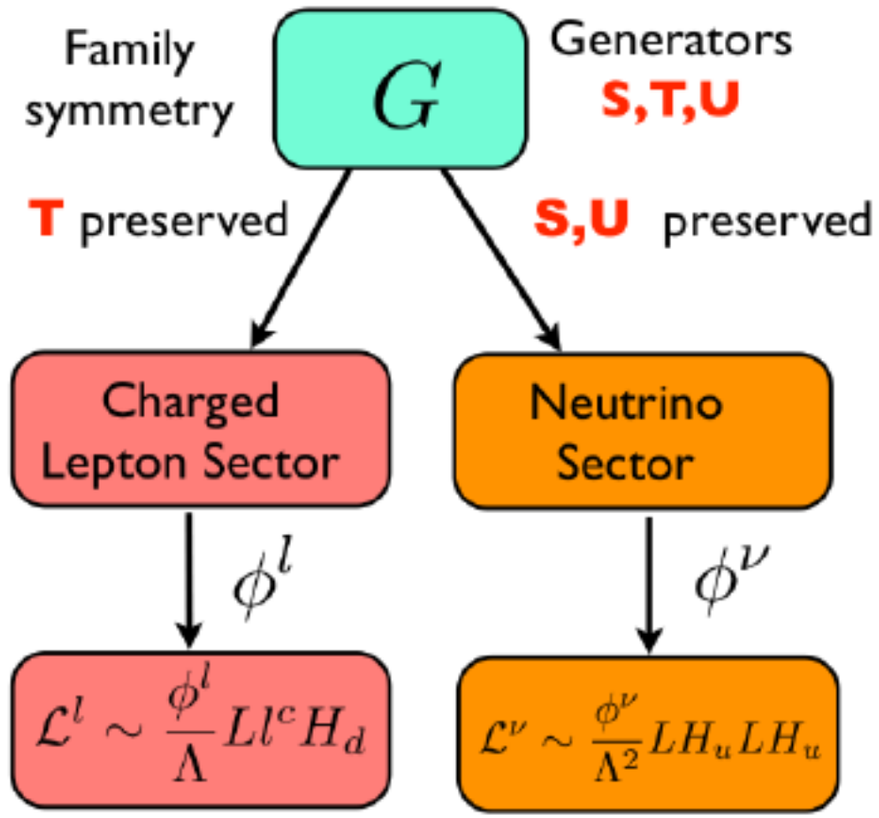
✓ **usual choices:** $SU(3),\ SO(3)$ **subgroups:**

$$\mathcal{A}_4 \quad \mathcal{S}_4 \quad \mathcal{A}_5 \quad \Delta(3n^2) \quad \Delta(6n^2) \quad \mathcal{D}_n \quad \mathcal{T}' \quad \mathcal{I}' \quad \ldots$$

"Platonic solid" groups + double covers

example (Majorana):

(image credit: King, Luhn)



see reviews by King, Luhn '13, King '17

Flavons:

$$\phi^l, \phi^\nu$$

Residual symmetries:

$$T\langle\phi^l\rangle \approx \langle\phi^l\rangle$$
$$S,U\langle\phi^\nu\rangle \approx \langle\phi^\nu\rangle$$

(or broken further, e.g. only *S* or *U* unbroken)

Many papers and authors! Some authors (not comprehensive):

Babu, Chen, Ding, L.E., Feruglio, Grimus, Hagedorn, King, Lam, Luhn, Ma, Merle, Ohlsson, Rodejohann, Stuart,…

Residual Symmetries

Lam ’08, ’09, Grimus et al. ’09, Ge et al. ’11, Toroop et al. ’11, He et al. ’12, Hernandez et al. ’12,’13, Holthausen et al. ’12, King et al. ’13, Hagedorn et al. ’14, Lavoura, Ludl ’14, Fonseca, Grimus ’14 …

- **model-independent approach:**

determine rows and columns in $\mathcal{U}_{\rm MNSP}$

as pure numbers, independent of masses,

depending on preserved subgroups of finite group G_f

$$T^\dagger \mathcal{M}_e \mathcal{M}_e^\dagger T = \mathcal{M}_e \mathcal{M}_e^\dagger \qquad S^\dagger \mathcal{M}_\nu S = \mathcal{M}_\nu$$

$$G_f \to G_e,\ T \in G_e \qquad G_f \to G_\nu,\ S \in G_\nu$$

$$\mathcal{U}_{eL}^\dagger T \mathcal{U}_{eL} = T^{\rm diag} \qquad \mathcal{U}_\nu^\dagger S \mathcal{U}_\nu = S^{\rm diag}$$

Majorana: $G_\nu \supseteq Z_2 \times Z_2$ **(Klein group)**

systematic classification of possible mixing matrices Fonseca, Grimus ’14

Very different from texture zeros (mixing angles as ratios of masses)!

Corrections

✓ **charged lepton corrections**

source the reactor angle: $\mathcal{U}_{eL} \sim 1 + O(\lambda)$

correlations among observables → **"sum rules"**

example: $\mathcal{U}_{eL} \sim \mathcal{R}_1(\theta^e_{23}, \delta^e_{23})\mathcal{R}_3(\theta^e_{12}, \delta^e_{12})$

Prediction for Dirac phase δ !

Ge, Dicus, Repko '11, Hanlon et al. '12, Marzocca et al. '13, Petcov '14, Girardi et al. '14-16, Ballet et al. '14

$$\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\cos 2\theta^{\nu}_{12} + \left(\sin^2\theta_{12} - \cos^2\theta^{\nu}_{12}\right)\left(1 - \cot^2\theta_{23}\sin^2\theta_{13}\right)\right]$$

1 model parameter!

guideline for "distinguishing power" needed from data

Marzocca et al. '13, Petcov '14, Girardi et al. '14-16

✓ **canonical normalization** (Kahler potential corrections)

Many authors. King '17 (review)

✓ **RG effects** more significant for IO, heavy neutrino masses (can be significant for sum rule analysis)

Antusch et al. '03 … Gehrlein et al. '16

Example: tri-bimaximal mixing (TBM/HPS)

(Type I seesaw)

$$\mathcal{U}_{\rm MNSP}^{\rm (HPS)} = \begin{pmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(~Clebsch-Gordan coeffs!) Meshkov, Zee…

✓ **Many models predated reactor angle measurements** see e.g. Albright et al. ’10

→ **data now requires Cabibbo-sized corrections**

$L_i \sim \mathbf{3}$ $\mathcal{A}_4$ $\mathcal{S}_4$ $\mathcal{T}'$ **(typically SUSY/SUSY-GUT)** Many authors!!

Aranda, Carone, Lebed ’00, Ma et al. ’03,
Altarelli, Feruglio ’05, Chen et al. ’07 …

“minimal” flavor group (contains *S,T,U* generators) Lam ’11

✓ **Residual symmetries:** $\mathcal{Z}_3$ ~*T* $\mathcal{Z}_2 \times \mathcal{Z}_2$ ~*S,U,SU*

Can further break down Klein symmetry:

1 column only of HPS matrix preserved

see e.g. King ’17 for review

Specific implementation:

asymmetric charged lepton corrections to TBM/HPS Rahat, Ramond, Xu '18

(with a dash of grand unification)

✓ ***SU(5)*, *SO(10)* GUT-inspired relations:**

<u>symmetric</u> Yukawas → **insufficient corrections to** θ_{13}

Kile, Perez, Ramond, Zhang '14

<u>asymmetric</u> Yukawas → **possible for specific corrections** Y_e **(via** $Y_{\bar{5}}$**)**

✓ **discrete non-Abelian family symmetry embedding** Perez et al. '19

$$\mathcal{T}_{13} = Z_{13} \rtimes Z_3$$

notable (often-found) feature:

phase required in $\mathcal{U}_\nu \sim \mathcal{U}^{(\mathrm{HPS})}$ **for consistency with mixing angle data**

numerical example: $\delta \simeq \pm 1.3\pi, \quad J \simeq \mp 0.03$

CP Violation

- **spontaneous CP violation — calculable phases**

Generalized CP

$$X^T \mathcal{M}_\nu X = \mathcal{M}_\nu^* \qquad Y^\dagger \mathcal{M}_e \mathcal{M}_e^\dagger Y = (\mathcal{M}_e \mathcal{M}_e^\dagger)^*$$

"ordinary" CP has $X = Y = 1$ **Branco, Lavoura, Rebelo '86…**

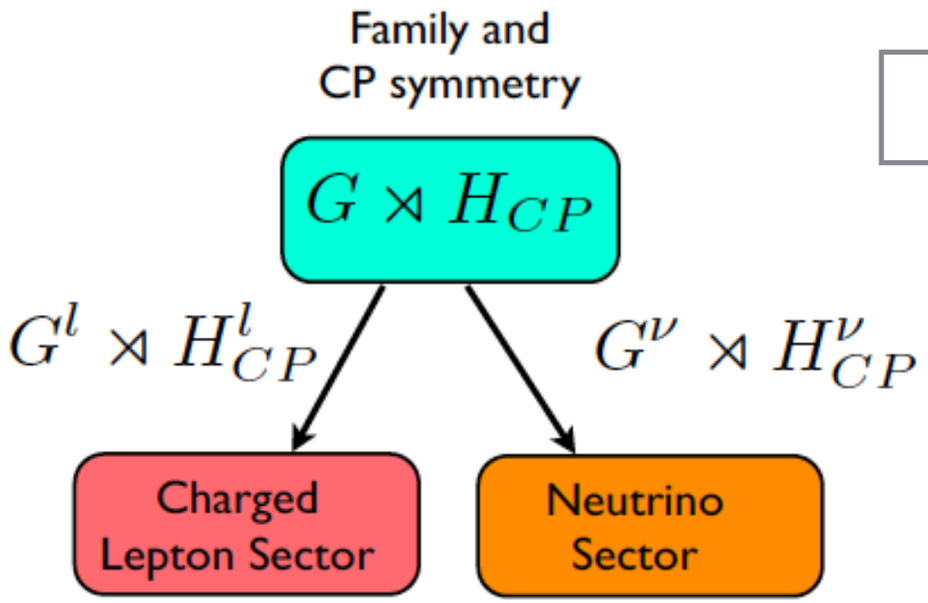
automorphisms of discrete family symmetry

Grimus, Rebelo '95
Holthausen et al. '12, Feruglio et al. '12, Chen et al. '14, Ding et al. '14, Branco et al. '15…

$$X \rho(g)^* X^{-1} = \rho(g')$$

consistency condition

family symmetry



many recent papers! see King '17 for review

Residual/generalized CP symmetries

existence of "CP basis" — Holthausen, et al. '12

group classification — Chen et al. '14

bottom-up approach (Klein symm preserved) — Feruglio et al. '12; L.E., Garon, Stuart '15; L.E., Stuart '16

…

Connection (or not) to quark sector

- **not particularly straightforward (subjective!)**

✓ **discrete non-Abelian symmetry models:**

quarks can require alternate embeddings

e.g. often $L_i \sim \mathbf{3}$ **but** $Q_i \sim \mathbf{2} \oplus \mathbf{1}$

→ **groups with both doublets and triplets**

(larger groups, double covers $\mathcal{T}'$, $\mathcal{I}'$ **)**

✓ **work explicitly in SUSY GUT framework**

$SO(10)$ **with family symmetry:**

$\mathcal{D}_3 \times U(1) \times Z_3 \times Z_3$ $\quad \mathbf{2} \oplus \mathbf{1}$

Dermisek et al. ’05, ’06

Poh, Raby ’15

14 fermion sector inputs → **6 fermion sector predictions**

(Type I seesaw)

Pati-Salam version (lighter superpartners)

Poh, Raby, Wang ’17

24-26 parameter models

Top-down

- **String constructions:**

Many authors!
see e.g. Langacker'11 for review

✓ **variety of possibilities for mass suppression** →

higher-dimensional operators (field theoretic)

geometric suppression (braneworlds) $Y \sim e^{-A}$

worldsheet instantons (nonperturbative)

✓ **Yukawa unification often not retained even in GUT scenarios**

✓ **not necessarily just minimal Type I seesaw** →

ν_R **candidates often not pure gauge singlets**

explorations in heterotic orbifolds Giedt et al. '05, Buchmuller et al.'07…

"Mixed" scenarios (e.g. seesaw + R-parity violation)

e.g. G2 models Acharya et al. '16…

F theory GUTs Heckman, Vafa '08,…

different framework from "usual" EFTs

Concluding remarks

- provided an introductory tour of ideas for solving SM flavor puzzle
  (far from fully comprehensive, hopefully a useful starting point)

Much effort and many intriguing ideas, but still seeking

compelling, complete, testable theories

More data in this sector has helped and will help enormously!

Stay tuned!