Theories of Quark and Lepton Masses

Lisa L. Everett
University of Wisconsin-Madison

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The Standard Model:
spectacularly successful — but many unanswered questions, including

What is the origin of the masses of all of the elementary particles?

(mysterious)

“understood”
gauge symmetry, Higgs mechanism

(image credit: Fermilab)

Nobel Prize, 1979

Glashow, Salam, Weinberg
Focus here on masses/mixing of the SM fermions:

Masses and intergenerational mixings:

charged current: 
gauge v. mass eigenstates

SM parameter count:

traditional SM definition: no neutrino mass
(nontrivial SM gauge reps + renormalizable operators only)

With neutrino masses: model-dependent, but at least 7 more (9 more if Majorana)

“SM flavor puzzle” can we explain these parameters?
The quest to answer this question both predates and runs in parallel with the theoretical development of the SM itself, as well as its extensions.

Some of the key elements for SM:
- V-A interactions
- Cabibbo angle and weak interactions universality
- GIM mechanism and FCNC suppression
- quark sector CP violation

No attempt here to characterize this rich and interwoven history fully

This quest has resulted in its own rich history, and multitude of ideas. Again, impossible to be fully comprehensive. (Consider referencing accordingly!)
Instead, the aim here is to provide a brief guide and review of ideas for addressing the SM Flavor Puzzle, with two main caveats:

- Put aside (for the most part) the “why 3 generations?” question
  A profound question. But one that likely requires physics beyond EFT’s. We’ll instead tacitly assume three generations and work in that framework.

- Concentrate mainly on responses to a subset of experimental breakthroughs
  ✓ discovery of top quark
  ✓ precise determination of quark mixing matrix entries
  ✓ discovery of neutrino oscillations and subsequent two decades of lepton mixing measurements
  ✓ Higgs discovery and coupling measurements to fermions

Each has had an important and substantial impact on the SM flavor puzzle!
But first, a (brief) summary of the data

**Quark and charged lepton masses**

- \( m_u \simeq 2 - 3 \text{ MeV} \)  
- \( m_c \simeq 1.3 \text{ GeV} \)  
- \( m_t \simeq 173 \text{ GeV} \)  
- \( m_d \simeq 4 - 6 \text{ MeV} \)  
- \( m_s \simeq 90 - 100 \text{ MeV} \)  
- \( m_b \simeq 4 \text{ GeV} \)  
- \( m_e = 0.511 \text{ MeV} \)  
- \( m_\mu \simeq 106 \text{ MeV} \)  
- \( m_\tau \simeq 1.8 \text{ GeV} \)  

PDG (2019)

**Quark mixing**

\[
U_{\text{CKM}} = R_1(\theta_{23}^{\text{CKM}})R_2(\delta_{\text{CKM}}, \theta_{13}^{\text{CKM}})R_3(\theta_{12}^{\text{CKM}})
\]

\[
s_{13} \ll s_{23} \ll s_{12}
\]

Wolfenstein parametrization: \( \lambda = \sin \theta_c \)  
(Cabibbo angle)

(Cabibbo expansion)

\[
s_{12} = \lambda \quad s_{23} = A\lambda^2 \quad s_{13} = A\lambda^3(\rho + i\eta) = \frac{A\lambda^3(\bar{\rho} + i\bar{\eta})\sqrt{1 - A^2\lambda^4}}{\sqrt{1 - \lambda^2(1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta}))}}
\]

\[
\lambda \simeq 0.225 \quad A \simeq 0.83 \quad \bar{\rho} \simeq 0.1 \quad \bar{\eta} \simeq 0.35
\]

PDG (2018)
Neutrino masses

Some highlights:

1998: atmospheric $\nu_\mu$ disappearance (SK)
2002: solar $\nu_e$ disappearance (SK)
2002: solar $\nu_e$ appear as $\nu_\mu, \nu_\tau$ (SNO)
2004: reactor $\bar{\nu}_e$ oscillations (KamLAND)
2004: accelerator $\nu_\mu$ disappearance (K2K)
2006: accelerator $\nu_\mu$ disappearance (MINOS)
2011: accel. $\nu_\mu$ appear as $\nu_e$ (T2K, MINOS)
2012: reactor $\bar{\nu}_e$ disappear (Daya Bay, RENO)
2014: CP violation hint? (T2K)
2015: normal hierarchy hint? (SK, T2K, NOvA)
2016: non-maximal atm hint? (NOvA)
2018: CP cons disfavored at $2\sigma$ (T2K)

Lectures here by Messier
The emergent picture...

a (seemingly) robust 3-neutrino mixing scheme

✓ mass-squared differences

\[ \Delta m_{ij}^2 \equiv m_i^2 - m_j^2 \]

✓ individual masses: limits from direct searches, cosmology

\[ m_{\bar{\nu}_e} < 2 \text{ eV} \quad \sum m_{\nu} < O(1 \text{ eV}) \]

(image credits: King, Luhn)
**lepton mixing**

\[
U_{\text{MNSP}} = R_1(\theta_{23}) R_2(\theta_{13}, \delta) R_3(\theta_{12}) \mathcal{P}
\]

Diagonal phase matrix (if Majorana neutrinos)

\[
W^\pm = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta_{23} & \sin \theta_{23} \\
0 & -\sin \theta_{23} & \cos \theta_{23}
\end{pmatrix}
\begin{pmatrix}
\cos \theta_{13} & 0 & \sin \theta_{13} e^{i\delta} \\
0 & 1 & 0 \\
-\sin \theta_{13} e^{-i\delta} & 0 & \cos \theta_{13}
\end{pmatrix}
\begin{pmatrix}
\cos \theta_{12} & \sin \theta_{12} & 0 \\
-\sin \theta_{12} & \cos \theta_{12} & 0 \\
0 & 0 & 1
\end{pmatrix} \mathcal{P}
\]

Global Fits:

Forero et al., '17
Capozzi et al., '18
Gonzalez-Garcia et al., (www.nu-fit.org)

NuFit, Nov 2018

<table>
<thead>
<tr>
<th></th>
<th>Normal Ordering (best fit)</th>
<th>Inverted Ordering ($\Delta \chi^2 = 4.7$)</th>
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</thead>
<tbody>
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<td></td>
<td>bfp $\pm 1\sigma$</td>
<td>$3\sigma$ range</td>
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<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>$0.310^{+0.013}_{-0.012}$</td>
<td>$0.275 \rightarrow 0.350$</td>
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<td>$\theta_{12}/^\circ$</td>
<td>$33.82^{+0.78}_{-0.76}$</td>
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<tr>
<td>$\sin^2 \theta_{23}$</td>
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<td>$\theta_{23}/^\circ$</td>
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<tr>
<td>$\sin^2 \theta_{13}$</td>
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<td>$0.02045 \rightarrow 0.02439$</td>
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<tr>
<td>$\theta_{13}/^\circ$</td>
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<td>$\delta_{\text{CP}}/^\circ$</td>
<td>$215^{+40}_{-29}$</td>
<td>$125 \rightarrow 392$</td>
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<tr>
<td>$\Delta m_{21}^2/^{10^{-5}} eV^2$</td>
<td>$7.39^{+0.21}_{-0.20}$</td>
<td>$6.79 \rightarrow 8.01$</td>
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<tr>
<td>$\Delta m_{3\ell}^2/^{10^{-3}} eV^2$</td>
<td>$+2.526^{+0.033}_{-0.032}$</td>
<td>$+2.427 \rightarrow +2.625$</td>
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Note

here: “MNSP”
more often: “PMNS”

(apologies for non-standard notation!)
Caveat: sterile neutrino(s)?

Anomalies:

1995: $\bar{\nu}_e$ appearance (LSND)
2007: $\bar{\nu}_e$ appearance (MiniBooNE)
2012: $\nu_e$ appearance (MiniBooNE)
1995: $\nu_e$ disappearance (Gallium)
2011: $\nu_e$ disappearance (Reactor)

[lots of results, investigation in the interim…]

[well-documented tension between appearance and disappearance data]

See: Huber’s IPA 2017 talk for “scorecard” (and lectures here)
Maltoni’s talk at Neutrino 2018

Restrict focus here to 3 active light families only.
A plethora of interesting results to (try to) explain!

Immediate impression:

- Quark and lepton sectors look very different!

  - Quarks: hierarchical masses, small mixings, $O(1)$ CP violation

  - Leptons: hierarchical charged lepton masses
    hierarchy apparently “milder” for neutrino masses
    two large mixing angles (or more**)

  implications for quark-lepton unification and other BSM physics

These lectures:

- Start with charged fermions.
  key historical developments, standard paradigms for hierarchy/small mixing

  new approaches, implications for connections (or not) to quark sector
(Charged) Fermion Masses

- Key feature of SM: **chiral charged fermions**
  
  ✓ disallows bare “vectorlike” masses
  
  ✓ Mass generation must proceed via Yukawa interactions with Higgs:

\[
\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \quad Q_{Li} = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}, \quad L_{Li} = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix}, \quad u_{Ri}, d_{Ri}, e_{Ri} \quad i, j = 1, 2, 3
\]

(family indices)

(Lorentz and gauge indices suppressed)

\[
Y_{ij} H \cdot \bar{\psi}_{Li} \psi_{Rj}
\]

Yukawa couplings

Dirac masses upon electroweak symmetry breaking (EWSB)

\[
\mathcal{M}_{ij}^{\text{Dirac}} \equiv Y_{ij} \langle H \rangle
\]

see FB “The Same Oddly Asymmetric Picture of Paul Dirac Each Day”
Diagonalize via bi-unitary transformation:

\[
U^\dagger_f M^\text{Dirac}_f U_f = M^\text{diag}_f
\]

\[
U^\dagger_f M_f M^\dagger_f U_f = (M^\text{diag}_f)^2
\]

\[
U^\dagger_R M_f M_f U_R = (M^\text{diag}_f)^2
\]

CKM: combination of left-handed rotations

\[
U_{CKM} = U^\dagger_{uL} U_{dL}
\]

Neglecting neutrino masses (for now): lepton mixing unobservable

(right-handed rotations unobservable in SM)
With one Higgs doublet (SM)**:

- size of charged fermion masses
- size of Yukawa couplings

\[ Y \sim 10^{-6} \quad \text{hierarchy!} \]

No guiding principle for Yukawa couplings in the SM
(each a priori a complex 3x3 matrix in family space)

many small dimensionless numbers (mass ratios, mixings) to explain!

**>1 Higgs doublet: additional possibilities due to distribution of electroweak VEV

(***= more on this later)
Options

- Bottom-up approach: special (simplified) Yukawa structures

(over)constrain mixing angles as functions of mass ratios

Famous example: Gatto-Sartori-Tonin relation

\[
\sin \theta_c = \sqrt{\frac{m_d}{m_s}}
\]

Gatto, Sartori, Tonin ’68

✓ early two-family implementation:

down-type quark mass matrix symmetric
vanishing 1-1 entry
(up-type quark diagonal basis)

✓ Generalization to three families “texture zeros”
(basis-dependent statement)

One canonical early example: Fritzsch ansatz

Fritzsch ’78
Fritzsch texture:

- symmetric mass matrices (up to phases)
- four texture zeros in each

\[ M_{u,d,e} = P_{u,d,e} \begin{pmatrix} 0 & a_{u,d,e} & 0 \\ a_{u,d,e} & 0 & b_{u,d,e} \\ 0 & b_{u,d,e} & c_{u,d,e} \end{pmatrix} Q_{u,d,e} \]

- Eigenvalue hierarchy: \( a \ll b \ll c \)
- yields GST relation for quarks

But ruled out for quarks:

combination of top quark mass + \((U_{\text{CKM}})_{cb}\) CKM measurement

\[ |U_{cb}| = \left| \sqrt{m_s/m_b} - e^{i\phi} \sqrt{m_c/m_t} \right| \]
Another canonical example: **Georgi-Jarlskog texture**

- symmetric/Hermitian mass matrices
- different structures for $u$, $d$, $e$ (inspired by GUT embedding**):

\[
\mathcal{M}_u = \begin{pmatrix}
0 & a & 0 \\
a & 0 & b \\
0 & b & c \\
\end{pmatrix}
\]  
(Fritzsch-like)

\[
\mathcal{M}_d = \begin{pmatrix}
0 & de^{-i\phi} & 0 \\
d e^{i\phi} & f & 0 \\
0 & 0 & g \\
\end{pmatrix}
\]

\[
\mathcal{M}_e = \begin{pmatrix}
0 & d & 0 \\
d & -3f & 0 \\
0 & 0 & g \\
\end{pmatrix}
\]  
(five texture zeros)

- 7 parameters for 13 observables: 6 predictions
- their specific realization ruled out for quarks: $|U_{cb}| \simeq |\sqrt{m_c/m_t}|$
- What persists: **GJ mass relations** (assumed to hold at high scale)

\[
m_b = m_\tau \quad m_\mu = 3m_s \quad m_d = 3m_e
\]

factors of 3: # of colors (RG effects)

Chanowitz et al. '77
Buras et al. '78
Continued exploration of texture zero framework: a few highlights below

“Stitching the Yukawa Quilt”

- Hermitian mass matrices
- 4 or 5+ texture zeros
  \textit{Parametrization via Cabibbo expansion a la Wolfenstein}
  Highly predictive!

Found five then-viable scenarios with 5 texture zeros, more with 4

Systematic review of broader set of predictive possibilities

- How do these structures fare today?

Predictive scenarios (few parameters) ruled out. Need more general forms.

- survey of more general possibilities
- “Occam’s razor” approach:
  asymmetric matrices, 3 texture zeros
• Bottom-up approach: “flavor democracy”

Start with “democratic” rank 1 mass matrix:

\[ M_{u,d} = \frac{m_{t,b}}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \]

One nonzero eigenvalue (3rd gen), 2 zero eigenvalues (1st and 2nd)

3rd generation eigenvector: trimaximal \( \frac{1}{\sqrt{3}} (1, 1, 1) \)

(thus can be diagonalized by tribimaximal mixing matrix)

Generate masses for light generations:

symmetry breaking (two stages)
Immediate questions:

These structures are *ad hoc* can they be explained within a broader theory? are they stable?

✓ Leads to the notion of family ("horizontal") symmetry

✓ Often also explored within grand unified ("vertical") symmetry

✓ Both often explored within SUSY ("natural" for GUTs, control of high scale)
Family Symmetry  
(also called Horizontal or Flavor Symmetry)

- Postulate symmetry $G_f$ that distinguishes generations
  
  $G_f$ spontaneously broken by “flavon” fields $\{\varphi_a\}$

\[
Y_{ij} H \cdot \bar{\psi}_{Li} \psi_{Rj} \rightarrow \left(\frac{\langle \varphi \rangle}{\Lambda}\right)^{n_{ij}} H \cdot \bar{\psi}_{Li} \psi_{Rj}
\]

\[
\epsilon = \left(\frac{\langle \varphi \rangle}{\Lambda}\right) \quad \Lambda > \langle \varphi \rangle > \langle H \rangle \quad \epsilon \ll 1
\]

Idea: small numbers given by ratio of mass scales

Introduce heavy Froggatt-Nielsen (FN) fermion fields
  
  with renormalizable couplings to SM and flavons

\[\text{integrate out heavy states}\]

Heavy sector $>>$ TeV  
(avoid too-large flavor-changing neutral currents)
Often explored in framework of $N=1$ softly broken SUSY

- logical choice given high scales involved
- consistency with gauge coupling unification, etc.

Several specific advantages of SUSY in this context:

- holomorphy of superpotential
  - better theoretical control of texture zeros ("supersymmetric zeros")
  - caveat: Kahler potential corrections (canonical normalization effects)

- two Higgs doublets: more flexibility in $G_f$ charge assignments

Immediate consequence: SUSY flavor/CP problem

We will not focus on this important and relevant issue here.

Many authors (including local experts!)...
• Possibilities for $G_f$:

 canonical example: $\text{U}(1)$ family symmetries

Have guidance from enhanced symmetry for vanishing Yukawas:

✓ SM (no neutrino masses): $\text{U}(3)^5$

$$\text{U}(3)_Q \otimes \text{U}(3)_{u^c} \otimes \text{U}(3)_{d^c} \otimes \text{U}(3)_L \otimes \text{U}(3)_{e^c}$$

$G_f$ subgroup of $\text{U}(3)^5$

Key input: $O(1)$ top quark Yukawa coupling

$$\text{U}(3)_Q \otimes \text{U}(3)_{u^c} \xrightarrow{m_t} \text{U}(2)_Q \otimes \text{U}(2)_{u^c}$$

Many examples! continuous Abelian, non-Abelian, discrete non-Abelian

Froggatt, Nielsen ’79

Hall lecture, TASI ’97
One canonical example: anomalous $U(1)_A$ family symmetry

Anomalies cancelled via universal Green-Schwarz mechanism

Dine-Seiberg-Witten (SUSY framework)  

$$\frac{C_i}{k_i} = \frac{C_{\text{grav}}}{12} = \frac{C_F}{k_F}$$

✓ prediction of Weinberg mixing angle

✓ calculable $\epsilon$

$$\epsilon \sim O(\lambda)$$  

$\lambda = \sin \theta_C$  

(Cabibbo angle)

Prototypical example of Cabibbo expansion for quark masses and mixing:

$$\frac{m_u}{m_t} \sim \lambda^8, \quad \frac{m_c}{m_t} \sim \lambda^4 \quad \frac{m_d}{m_b} \sim \lambda^4, \quad \frac{m_s}{m_b} \sim \lambda^2$$

valid at high scales  

(evolve parameters to weak scale via RGE)

$$U_{\text{CKM}} = U_{UL}^\dagger U_{DL} = 1 + O(\lambda)$$

unique theoretical starting point!**
Another canonical example: $U(2)$ family symmetry (SUSY framework)

non-Abelian symmetry: how to embed the quark families in $G_f$

natural choice: $2 \oplus 1$

Concrete $U(2)$ case:

$U(2) \xrightarrow{\epsilon} U(1) \xrightarrow{\epsilon'} 0$

$Y_{u,d,e} \sim \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix}$

GST relation obtained!

"Effective flavons"

$\varphi/\Lambda \sim \epsilon$
$S^{ab}/\Lambda \sim \epsilon$
$A^{ab}/\Lambda \sim \epsilon'$

$\epsilon \sim 0.02 \sim O(\lambda^2)$
$\epsilon' \sim 0.001 \sim O(\lambda^4)$

qualitatively good agreement with data

Barbieri, Dvali, Hall '96
Barbieri, Hall, Raby, Romanino '96
Barbieri, Hall, Romanino '97

...
(Supersymmetric) Grand Unification

- Postulate “vertical” symmetry $G_{GUT}$

Canonical examples:

$G_{GUT} = SU(5)$

$G_{GUT} = SO(10)$

$G_{GUT} = SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$

Georgi, Glashow ’74

Georgi ’75, Fritzsch, Minkowski ’75

Pati, Salam ’73

Many well-known advantages!

- SM quantum numbers
- Gauge coupling unification (SUSY)

This context:

consequences for SM flavor puzzle

Yukawa unification

(image credit: T. Ohlsson, KTH)
SUSY GUT Basics:

Standard embedding of SM generations and minimal Higgs content

\[ SU(5) : \begin{array}{c} 5 \{ d^c, L \} \end{array} 10 \begin{array}{c} \{ Q, u^c, e^c \} \end{array} (5_H, \bar{5}_H) \]

\[ SO(10) : 16 \begin{array}{c} \bar{5} \oplus 1 \end{array} 10_H \]

Tree-level GUT relations: Yukawa unification

\[ SU(5) : \begin{array}{c} 10 \bar{5}_H \end{array} 10 5_H \quad \text{partial unification: } Y_d = Y_e^T \]

\[ SO(10) : \begin{array}{c} 16 16 10_H \end{array} \]

Thus maximal \( G_f \) is given by \( SU(5) : U(3)^2 \quad SO(10) : U(3) \)

Yukawa unification works reasonably well for 3rd generation (large \( \tan \beta \))

Many authors. see e.g. Carena et al. ’94

but catastrophic for lighter generations! (better: approximate GJ mass relations)
Fixing the problem:

- tree-level relations for third generation only
- higher-dimensional Higgs representations, $G_f$ assignments for first and second generations

Many examples!**

Example: $U(2)$ family symmetry in $SU(5)$ and $SO(10)$

Need to suppress 22, 21 entries of $Y_u$

Nontrivial $SU(5)$ reps for “effective” flavons

\{ $S^{ab}(75)$, $A^{ab}(1)$, $\varphi(1, 24)$, $\Sigma(24)$ \}

- explicit Froggatt-Nielsen realization (including flavon dynamics)
- **Georgi-Jarlskog mass relations (approximate)**

**Return to this topic after discussing lepton sector**
Radiative mass generation

Idea: small numbers explained via loop effects

✓ third generation: tree level
✓ each successive generation suppressed by loop factor

Barr, Zee ’77, Ibanez ’82, Balakrishna et al. ’88, Babu, Ma ’88
He et al. ‘90, Babu, Mohapatra ’90; Dobrescu et al. ’08,..

Extra (warped) dimensions

Idea: small numbers explained via warping, localization

✓ third generation and Higgs: near TeV brane
✓ lighter generation: closer Planck brane

Randall, Sundrum ’99
Arkani-Hamed, Schmaltz ’00
Huber, Shafi ’01
Huber ’03, Agashe et al. ’04,..
Neutrino Sector

- neutrino masses

SM $\rightarrow \nu$ SM

Many new aspects for SM flavor puzzle:

- neutrinos Majorana or Dirac?
- origin of overall mass suppression
- origin of observed lepton mixing angle pattern
- connection (or not) to quark sector

(image credit: H. Murayama)
First step in constructing models of neutrino masses:

Main question: origin of neutrino mass suppression

Options:
- Dirac: $\Delta L = 0$
  - charged fermion sector
- Majorana: $\Delta L = 2$
  - lepton number-violating

Unlike SM, variety of theoretical starting points for $\nu$ SM

Critically important question, to be settled by experiment.

Which to choose?
Majorana neutrinos

- lepton # accidental symmetry of SM
  intriguing consequences (e.g. $0\nu\beta\beta$, ...) if violated

SM at NR level: Weinberg dimension 5 operator

$$\frac{\lambda_{ij}}{\Lambda} L_i H L_j H \quad \Delta L = 2$$

→ results in Majorana neutrino masses upon EWSB

$$\mathcal{M}_\nu^{\text{Maj}} \sim \frac{\lambda_{ij} \langle H \rangle^2}{\Lambda}$$

if $\lambda \sim O(1) \quad \Lambda \gg \langle H \rangle \sim O(100 \text{ GeV})$

$\nu$ mass suppression via ratio of EW to heavy scale
but could have lighter scale, other means of suppression
Underlying mechanism for Weinberg operator

\[ \frac{\lambda_{ij}}{\Lambda} L_i H L_j H \]

3 tree-level possibilities

- **Type I seesaw** \( \nu_R \) (fermion singlet)
- **Type II seesaw** \( \Delta \) (electroweak triplet scalar)
- **Type III seesaw** \( \Sigma \) (electroweak triplet fermion)

(and combinations)

(image credit: Dinh et al.)

see e.g. Ma, '98
Type I seesaw

✓ introduce right-handed neutrinos

(image credit: T. Ohlsson et al., Nat. Comm.)

(forbidden by EW symm)

\[ \mathcal{M}_\nu = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \]

\[ m_1 \sim \frac{m^2}{M} \quad m_2 \sim M \gg m_1 \]

\[ \nu_{1,2} \sim \nu_{L,R} + \frac{m}{M} \nu_{R,L} \]

\[ Y_{ij} L_i \nu_{R,j} H + M_{R \ ij} \nu_{R i} \nu_{R j}^c \]

\[ \mathcal{M}_\nu \sim \langle H \rangle^2 Y M_R^{-1} Y^T \]

Advantages: economical, connection to grand unification, leptogenesis

Disadvantages: testability without model assumptions
Type II seesaw

✓ introduce triplet Higgs scalar

\[
\begin{align*}
\Delta & \sim (3, 2) \\
& \left( SU(2)_L, U(1)_Y \right) \\
\Delta &= \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix} \\
(Y_\Delta)_{ij} L_i L_j \Delta + \mu_\Delta H H \Delta \\
\mathcal{M}_\nu & \sim \langle H \rangle^2 Y_\Delta \mu_\Delta / M_\Delta^2
\end{align*}
\]

can have clean LHC signatures of lepton # violation via decays of \( H^+, H^{++} \) if \( M_\Delta \leq O(\text{TeV}) \)

Fileviez Perez et al. '08, Gavela et al. '09,... (also LFV)

Advantages: testability (charged Higgs states probed at LHC)
Disadvantages: not as economical/minimal as Type I
Type III seesaw

- introduce electroweak triplet fermions

\[
\Sigma \sim (3, 0) \\
(SU(2)_L, U(1)_Y)
\]

\[
\Sigma_i = (\Sigma_i^0, \Sigma_i^\pm) \quad (3 \text{ of them})
\]

\[
(Y_\Sigma)_{ij} L_i \Sigma_j H + (M_\Sigma)_{ij} \Sigma_i \Sigma_j
\]

\[
\mathcal{M}_\nu \sim \langle H \rangle^2 Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T
\]

- can have clean LHC signatures via mixing w/charged leptons if \( M_\Sigma \sim O(\text{TeV}) \)

- also highly predictive pattern of LFV signals

Advantages: testability (new charged states probed at LHC, LFV)
Disadvantages: not as economical/minimal as Type I
Radiative generation of Majorana masses

- complete Weinberg operator via loops
- radiative seesaw models

Canonical example: “scotogenic” model

introduce right-handed neutrinos
new EW doublets (can be DM), $Z_2$ symmetry

$\mathcal{M}_\nu \sim \lambda \frac{\langle H \rangle^2}{16\pi^2} Y M_R^{-1} Y^T$

“radiative Type I seesaw”

analogous construction with fermion triplet: “radiative Type III seesaw”

“radiative Type II seesaw”: forbid $L_i^T L_i^T \Delta$ via (softly broken) symm

$L \to (-1)^L$

(image credit: T. Ohlsson et al., Nat. Comm.)
Radiative generation of Majorana masses

alternatives to Weinberg operator

Many other NR operators in SM with $\Delta L = 2$
(odd mass dimension $d>5$)

Classification

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<th>$d=7$</th>
<th>$d=9$</th>
<th>…</th>
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<tr>
<td>$LLQ\bar{u}^c H$</td>
<td></td>
<td>+ many others…</td>
</tr>
<tr>
<td>$L\bar{e}^c \bar{u}^c d^c H$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

new physics scale can be accessible at LHC (subject to LFV bounds)

Many explicit realizations!

See e.g. excellent review:
Cai, Herrero-Garcia, Schmidt, Vicente, Volkas '17
One way leptoquarks can manifest themselves:

Scalar leptoquark $\phi \sim (3, 1, -1/3)$

+ Octet fermion $f \sim (8, 1, 0)$

Possible connections to dark matter:

- Symmetry for dark matter stability
- Radiative $\nu$ mass
- Potential DM candidates in loops
  
  See e.g. scotogenic case, many other examples

Possible connections to flavor physics anomalies:

- Potential DM candidates in loops
  
  Many authors!

Generic advantage of radiative models: testability

Potential connections to other, possibly accessible, new physics

Ma '15

Angel, Cai, Rodd, Schmidt, Volkas '13

Cai, Gargalones, Schmidt, Volkas '17

Päsch, Schumacher '15, Deppisch et al. '16,...
Many other ideas for Majorana masses

- more complicated seesaws → lower scales
  
  e.g. double/inverse seesaw: $3 \nu_R$ and 3 new singlet fermions
  
  \[ Y_{ij} L_i (\nu_R)_j H + M_{ij} S_i (\nu_R)_j + \mu_{ij} S_i S_j \]
  
  $\mu, m \ll M$

  \[ M_{\nu} \sim \begin{pmatrix} 0 & m & 0 \\ m & 0 & M \\ 0 & M & \mu \end{pmatrix} \]

  \[ M_{\text{eff}} = M^{T} \mu^{-1} M \]

  \[ M_{\nu} \sim \langle H \rangle^2 (Y^T M_{\text{eff}}^{-1} Y) \]

  and extended versions…

- SUSY with R-parity violation
  
  neutrino-neutralino mixing
  
  loop effects

- Warped extra dimensions

lepton number violation → Majorana $\nu$ masses
Dirac neutrinos

\[ \Delta L = 0 \]

- Analogous to charged fermions, but much stronger suppression

\[ Y_\nu \sim 10^{-14} \]

Must forbid both types of “bare” mass terms

Less intuitive, but mechanisms exist. Some examples:

- texture zeros/Froggatt-Nielsen
- radiative mass generation

\[ e.g. \text{discrete symmetry: } \nu_R \text{ nontrivial} \]
\[ \text{one-loop: 2 topologies} \]

- other options: higher loops, loop-induced vev,

Hagedorn, Rodejohann ’05,

Cheng, Li ’78, Mohapatra ’87, ’88
Balakrishna, Mohapatra ’89, Rajpoot ’01

Ma, Popov ’16
Wang et al. ’16, ’17
Review: Cai et al. ’17

variety of possibilities for new states

(many studies in GUT contexts)
Dirac neutrinos (cont.)

- extended gauge sectors
  - non-singlet $\nu_R$ forbids “bare” terms, simplest seesaws
  - higher-dimensional operators: e.g. $U(1)'$ symmetry

- SUSY breaking
  - symm+holomorphy forbids superpotential
  - allows Kahler potential contributions

- string constructions
  - exponentially suppressed interactions from stringy instanton effects…

General theme for neutrino masses

- Much richer than charged fermion cases
  - but trade-off between simplicity and testability
Lepton mixing pattern

\( \nu_i \)

\((\mathcal{U}_{\text{MNSP}})_{ij}\)

\(W^\pm\)

Pontecorvo; Maki, Nakagawa, Sakata

(Dirac or Majorana neutrinos)

\[
\mathcal{U}_\nu^T \mathcal{M}_\nu^{\text{Maj}} \mathcal{U}_\nu = \mathcal{M}_\nu^{\text{diag}}
\]

\[
\mathcal{U}_{fL}^\dagger \mathcal{M}_f^{\text{Dirac}} \mathcal{U}_{fR} = \mathcal{M}_f^{\text{diag}}
\]

\[
\mathcal{U}_{\text{MNSP}} = \mathcal{U}_{eL}^\dagger \mathcal{U}_{\nu L}
\]

\[
\mathcal{U}_{\text{MNSP}} = \mathcal{R}_1(\theta_{23}) \mathcal{R}_2(\theta_{13}, \delta) \mathcal{R}_3(\theta_{12}) \mathcal{P}
\]

Given great progress in measuring MNSP parameters:

tremendous excitement among BSM theorists to shed light on the origin of the mixing pattern

(can work within any framework for mass suppression. vast majority: Type I seesaw)
The MNSP is quite different from the CKM! (a surprise!)

Two large mixing angles: $\theta_{23}, \theta_{12}$

CP violation

Dirac phase: important goal of experimental program
Majorana phases: unlikely to know anytime soon

A basic question: is $\theta_{13}$ “large” or “small”?

large reactor angle vs. small reactor angle

the case for anarchy vs. the case for structure
Post-reactor angle measurement: renewed focus

Some recent highlights:

- RG analysis
- model-building + quark sector

Note: anarchy hypothesis alone does not provide information on $\Delta m^2$
Standard assumption: structure from symmetry

Usual paradigm (recall charged fermions)

\[ G_f \] spontaneously broken at scale \( M \)
dimensionless numbers governed by \( \epsilon \sim \langle \varphi \rangle / M \)
\( \varphi = \) “flavon”

But quite different from the quark sector!

Recall there, a natural identification:

\[ \epsilon \sim O(\lambda) \]
\( \lambda = \sin \theta_c \)

Cabibbo angle (or some power) as a flavor expansion parameter

In this context, a unique theoretical starting point:

\[ U_{\text{CKM}} \sim 1 + O(\lambda) \]
approximately the identity as \( \lambda \to 0 \)
• For the leptons: not as straightforward

In the basis where $M_e$ is diagonal, $M_\nu$ is not diagonal:

$M_\nu$ diagonalization: 1 small, 2 large mixing angles

Arguably the most challenging* pattern:  

\[
\begin{align*}
3 \text{ small angles} & \quad \Rightarrow \quad \sim \, \text{recall quarks} \\
2 \text{ small, 1 large} & \quad \Rightarrow \quad \sim \, \text{Rank} M_\nu < 3 \\
3 \text{ large angles} & \quad \Rightarrow \quad \text{anarchical } M_\nu \\
2 \text{ large, 1 small} & \quad \Rightarrow \quad \text{fine-tuning, non-Abelian}
\end{align*}
\]

\* for 3 families

A model-building opportunity!
Family symmetry approach

*spontaneously broken $G_f$

typical choice: discrete non-Abelian group

But see recent interesting work in symmetric limit
Reyiumaji and Romanino, '18

Difference from quarks:
No unique theoretical starting point for “Cabbibo-like” expansion

\[ U_{\text{MNSP}} \sim W + O(\lambda') \quad \lambda' \ll 1 \]

“bare” mixing angles (diagonal charged lepton basis)
\[ (\theta_{12}^\nu, \theta_{23}^\nu, \theta_{13}^\nu) \]

First stage: symmetry breaking to generate nontrivial $W$
different unbroken subgroups for neutrinos, charged leptons

Next stage: corrections as expansion in $\lambda'$
“Bare” mixing angles generically shift due to \( O(\lambda') \) corrections

\( \checkmark \) A priori, expansions in quark and lepton sectors unrelated

Option unification paradigm (broad sense): set \( \lambda' = \lambda \)

ideas of quark-lepton complementarity and “Cabibbo haze”

\[ \theta_{23} = \theta_{12} + \theta_c \]  
(empirical)

not an unreasonable approach given the data

\[ \theta_{13} \sim O(\lambda) \]

pre-measurement, idea that \( \theta_{13} \) is a Cabibbo effect:

\[ \theta_{13}^\nu = 0 \quad \theta_{13} = \lambda / \sqrt{2} \]

Raidal ’04, Minakata+Smirnov ’04,…  
(“haze” terminology from  
Datta, L.E., Ramond ’05)

Vissiani ’98, ’01  
Ramond ’04
Most studied: maximal atm, zero reactor

classify scenarios by bare solar angle

tribi-maximal mixing: \[ \sin^2 \theta_{12} = \frac{1}{3} \]

bimaximal mixing: \[ \sin^2 \theta_{12} = \frac{1}{2} \]

golden ratio (A) mixing: \[ \sin^2 \theta_{12} = \frac{1}{2 + r} \sim 0.276 \]
\[ r = \frac{1 + \sqrt{5}}{2} \]

golden ratio (B) mixing: \[ \sin^2 \theta_{12} = \frac{3 - r}{4} \sim 0.345 \]

hexagonal mixing: \[ \sin^2 \theta_{12} = \frac{1}{4} \]

Can also study nonzero reactor: \[ \sin^2 \theta_{13} \neq 0 \]

All can be obtained via SSB of discrete non-Abelian family symmetries
Family symmetry models

![Diagram](image)

**usual choices:** $SU(3)$, $SO(3)$ subgroups:

- $A_4$
- $S_4$
- $A_5$
- $\Delta(3n^2)$
- $\Delta(6n^2)$
- $D_n$
- $T'$
- $I'$

“Platonic solid” groups + double covers

**example (Majorana):**

Flavons:

$$\phi^l, \phi^\nu$$

Residual symmetries:

$$T\langle\phi^l\rangle \approx \langle\phi^l\rangle$$
$$S, U\langle\phi^\nu\rangle \approx \langle\phi^\nu\rangle$$

(or broken further, e.g. only $S$ or $U$ unbroken)

Many papers and authors! Some authors (not comprehensive):

Babu, Chen, Ding, L.E., Feruglio, Grimus, Hagedorn, King, Lam, Luhn, Ma, Merle, Ohlsson, Rodejohann, Stuart,...
Residual Symmetries

- model-independent approach:

  determine rows and columns in $U_{\text{MNSP}}$

  as pure numbers, independent of masses,

  depending on preserved subgroups of finite group $G_f$

\[
T^\dagger M_e M_e^\dagger T = M_e M_e^\dagger \quad S^\dagger M_\nu S = M_\nu
\]

$G_f \to G_e, \quad T \in G_e$ \quad $G_f \to G_\nu, \quad S \in G_\nu$

\[
U_{eL}^\dagger T U_{eL} = T^{\text{diag}} \quad U_{\nu}^\dagger S U_{\nu} = S^{\text{diag}}
\]

Majorana: \quad $G_\nu \supseteq Z_2 \times Z_2$ \quad (Klein group)

systematic classification of possible mixing matrices

Very different from texture zeros (mixing angles as ratios of masses)!

Lam '08, '09, Grimus et al. '09, Ge et al. '11, Toroop et al. '11, He et al. '12, Hernandez et al. '12,'13, Holthausen et al. '12, King et al. '13, Hagedorn et al. '14, Lavoura, Ludl '14, Fonseca, Grimus '14

...
charged lepton corrections

source the reactor angle: \( U_{eL} \sim 1 + O(\lambda) \)
correlations among observables

example: \( U_{eL} \sim R_1(\theta_{23}^e, \delta_{23}^e) R_3(\theta_{12}^e, \delta_{12}^e) \)

Prediction for Dirac phase \( \delta \)!

\[
\cos \delta = \frac{\tan \theta_{23}}{\sin 2\theta_{12} \sin \theta_{13}} \left[ \cos 2\theta_{12}^e + \left( \sin^2 \theta_{12} - \cos^2 \theta_{12}^e \right) \left( 1 - \cot^2 \theta_{23} \sin^2 \theta_{13} \right) \right]
\]

1 model parameter!

guideline for “distinguishing power” needed from data

canonical normalization (Kahler potential corrections)

RG effects

more significant for IO, heavy neutrino masses
(can be significant for sum rule analysis)
Example: tri-bimaximal mixing (TBM/HPS) (Type I seesaw)

\[ u_{\text{MNSP}}^{(\text{HPS})} = \begin{pmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \] (~Clebsch-Gordan coeffs!)

Meshkov, Zee...

✓ Many models predated reactor angle measurements
data now requires Cabibbo-sized corrections

\[ L_i \sim 3 \quad A_4 \quad S_4 \quad T' \] (typically SUSY/SUSY-GUT)

Many authors!!

Aranda, Carone, Lebed '00, Ma et al. '03, Altarelli, Feruglio '05, Chen et al. '07 ...

✓ “minimal” flavor group (contains \(S,T,U\) generators)

Many authors!!

Aranda, Carone, Lebed '00, Ma et al. '03, Altarelli, Feruglio '05, Chen et al. '07 ...

✓ Residual symmetries: \(\mathbb{Z}_3 \sim T\) \(\mathbb{Z}_2 \times \mathbb{Z}_2 \sim S,U,SU\)

Many authors!!

Aranda, Carone, Lebed '00, Ma et al. '03, Altarelli, Feruglio '05, Chen et al. '07 ...

Can further break down Klein symmetry:

1 column only of HPS matrix preserved

see e.g. King ’17 for review
Specific implementation:

asymmetric charged lepton corrections to TBM/HPS (with a dash of grand unification)  

Rahat, Ramond, Xu ’18

\[ SU(5), \ SO(10) \ GUT\text{-}inspired \ relations: \]

\[
\begin{align*}
\text{symmetric Yukawas} & \quad \rightarrow \quad \text{insufficient corrections to } \theta_{13} \\
\text{asymmetric Yukawas} & \quad \rightarrow \quad \text{possible for specific corrections } Y_e \ (\text{via } Y_{\bar{5}})
\end{align*}
\]

Kile, Perez, Ramond, Zhang ’14

\[ \tau_{13} = Z_{13} \rtimes Z_3 \]

Perez et al. ’19

notable (often-found) feature:

phase required in \( U_\nu \sim U^{(HPS)} \) for consistency with mixing angle data

numerical example: \( \delta \simeq \pm 1.3\pi, \ J \simeq \mp 0.03 \)
spontaneous CP violation — calculable phases

\[ X^T M_\nu X = M_\nu^* \quad Y^\dagger M_e M_e^\dagger Y = (M_e M_e^\dagger)^* \]

“ordinary” CP has \( X = Y = 1 \)  
Branco, Lavoura, Rebelo '86...

automorphisms of discrete family symmetry

\[ X \rho(g)^* X^{-1} = \rho(g') \]
consistency condition

family symmetry

Residual/generalized CP symmetries

existence of “CP basis”  
group classification

bottom-up approach  
(Klein symm preserved)

many recent papers! see King ’17 for review

Grimus, Rebelo ’95
Holthausen et al. ’12, Feruglio et al. ’12, Chen et al. ’14, Ding et al. ’14, Branco et al. ’15...
Connection (or not) to quark sector

- not particularly straightforward (subjective!)

  ✓ discrete non-Abelian symmetry models:
  
  quarks can require alternate embeddings
  
  e.g. often $L_i \sim 3$ but $Q_i \sim 2 \oplus 1$

  \[ \text{groups with both doublets and triplets} \]
  
  \[ \text{(larger groups, double covers } T', T') \]

  ✓ work explicitly in SUSY GUT framework

\[ SO(10) \] with family symmetry:

\[ \mathcal{D}_3 \times U(1) \times Z_3 \times Z_3 \]  \[ 2 \oplus 1 \]

14 fermion sector inputs \[ \rightarrow \] 6 fermion sector predictions

(Type I seesaw)

Pati-Salam version (lighter superpartners)

24-26 parameter models

Dermisek et al. '05, '06

Poh, Raby '15

Poh, Raby, Wang '17
String constructions:

- variety of possibilities for mass suppression
  - higher-dimensional operators (field theoretic)
  - geometric suppression (braneworlds) $Y \sim e^{-A}$
  - worldsheets instantons (nonperturbative)
- Yukawa unification often not retained even in GUT scenarios
- not necessarily just minimal Type I seesaw
  - $\nu_R$ candidates often not pure gauge singlets
- explorations in heterotic orbifolds Giedt et al. ’05, Buchmuller et al.’07…
- “Mixed” scenarios (e.g. seesaw + R-parity violation)
  - e.g. G2 models Acharya et al. ’16…
  - F theory GUTs Heckman, Vafa ’08,…

Top-down

Many authors! see e.g. Langacker’11 for review
Concluding remarks

- provided an introductory tour of ideas for solving SM flavor puzzle (far from fully comprehensive, hopefully a useful starting point)

Much effort and many intriguing ideas, but still seeking compelling, complete, testable theories

More data in this sector has helped and will help enormously!

Stay tuned!