Theory of B Decays

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SLAC Summer Institute 2019,
Menu of Flavors: Quarks, Charged Leptons & Neutrinos
August 14, 2019
Theory of B Decays (today)

- Intro: Classification of B Decays
- $B \rightarrow D^{(*)}\ell\nu$ and $R_{D^(*)}$
- $B_s \rightarrow \mu^+\mu^-$
- $B \rightarrow K^*\ell^+\ell^-$ and $R_{K^(*)}$
Outline of the Lectures

1. Theory of B Decays (today)
   - Intro: Classification of B Decays
   - $B \rightarrow D^{(*)} \ell \nu$ and $R_{D(*)}$
   - $B_s \rightarrow \mu^+ \mu^-$
   - $B \rightarrow K^* \ell^+ \ell^-$ and $R_{K(*)}$

2. Effective Field Theory Fits (tomorrow)
   - B Decays as Probes of New Physics
   - The Flavor Anomalies
   - Fitting the $b \rightarrow c \ell \nu$ Anomalies
   - Fitting the $b \rightarrow s \ell \ell$ Anomalies
The b Quark

\[ \overline{m}_b(m_b) = 4.18^{+0.03}_{-0.02} \text{ GeV} \]

Particle masses in GeV:
- \( m_t \)
- \( m_c \)
- \( m_b \)
- \( m_s \)
- \( m_d \)
- \( m_u \)
- \( m_\tau \)
- \( m_\mu \)
- \( m_e \)
The b Quark

Particle masses in GeV/$c^2$

$m_t$  
$m_b$  
$m_c$  
$m_s$  
$m_d$  
$m_u$  
$m_e$  
$m_\mu$  
$m_\tau$

$\bar{m}_b(m_b) = 4.18^{+0.03}_{-0.02}$ GeV

Forms bound states

- Bottomonia: $\Upsilon(1s), \Upsilon(2s), ...$

- B mesons: $B^\pm, B^0, B_s, B_c, ...$

- B baryons: $\Lambda_b, ...$
The $b$ Quark

particle masses in GeV/c^2

$m_t$ ▐

$m_c$ ▐

$m_b$ ▐

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The b Quark

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- B mesons: \( B^\pm, B^0, B_s, B_c, \ldots \)
- B baryons: \( \Lambda_b, \ldots \)
Decay of $b$ quarks proceeds through the weak interactions.

Exchange of a heavy virtual $W$ boson.

Estimate the decay width $\Gamma(b \rightarrow c\ell\nu) \sim \frac{G_F^2}{192\pi^3} m_b^5 |V_{cb}|^2 \Rightarrow \tau = \frac{1}{\Gamma_{\text{tot}}} \sim O(10^{-12})\text{ s}$.

Small decay width $\Rightarrow$ sizable lifetime.

High sensitivity to new physics effects.
Lifetime

- Decay of b quarks proceeds through the **weak interactions**
- Exchange of a heavy **virtual W boson**
- Estimate the decay width

\[ \Gamma(b \to c\ell\nu) \sim \frac{G_F^2}{192\pi^3} m_b^5 |V_{cb}|^2 \]

\[ \tau = \frac{1}{\Gamma_{\text{tot}}} \sim O(10^{-12}) \text{ s} \]

\[ \text{small decay width} \Rightarrow \text{sizable lifetime} \]

\[ \text{high sensitivity to new physics effects} \]

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Lifetime

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$$\Gamma(b \to c\ell\nu) \sim \frac{G_F^2}{192\pi^3} m_b^5 |V_{cb}|^2$$

$$\Rightarrow \tau = \frac{1}{\Gamma_{tot}} \sim \mathcal{O}(10^{-12} \text{s})$$

- small decay width $\Rightarrow$ sizable lifetime
- high sensitivity to new physics effects
Charged Current Decays

► arise at tree level through $W$ exchange

$$A(b \rightarrow c) \sim V_{cb} \sim 4 \times 10^{-2}$$

(e.g. $B \rightarrow D_{\mu\nu}$)
Charged Current Decays

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\[ A(b \rightarrow c) \sim V_{cb} \sim 4 \times 10^{-2} \]

(e.g. $B \rightarrow D_{\mu\nu}$)

\[ A(b \rightarrow u) \sim V_{ub} \sim 4 \times 10^{-3} \]

(e.g. $B \rightarrow \pi_{\mu\nu}$)
Flavor Changing Neutral Current Decays

absent in the SM at tree level (GIM mechanism)

\[
A(b \to s) \sim \frac{1}{16\pi^2} V^*_{ts} V_{tb} \sim 2.5 \times 10^{-4}
\]

(e.g. \(B \to K^* \mu^+ \mu^-\))

\[
A(b \to d) \sim \frac{1}{16\pi^2} V^*_{td} V_{tb} \sim 5 \times 10^{-5}
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(e.g. \(B \to \pi \mu^+ \mu^-\))

"rare decays"
absent in the SM at tree level (GIM mechanism)

arise at the 1-loop level

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"rare decays"
Classification of Charged Current Decays

- **Semi-leptonic decay modes**
  (both charged and neutral B mesons)

  exclusive: e.g. $B \to D\tau\nu$, $B \to D^*\mu\nu$, $B \to \pi e\nu$ ...

  inclusive: e.g. $B \to X_c\tau\nu$, $B \to X_c\mu\nu$, $B \to X_u e\nu$ ...

- **Purely leptonic decay modes**
  (only charged B mesons)

- **Purely hadronic decay modes**
  (both charged and neutral B mesons)

  hundreds of possible final states
Classification of Charged Current Decays

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  hundreds of possible final states
Classification of Charged Current Decays

▶ Semi-leptonic decay modes

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exclusive: e.g. $B \rightarrow D \tau \nu$

inclusive: e.g. $B \rightarrow X_c \tau \nu$

▶ Purely leptonic decay modes

(only charged B mesons)

e.g. $B \rightarrow \tau \nu, \mu \nu$

▶ Purely hadronic decay modes

(both charged and neutral B mesons)

hundreds of possible final states
Radiative decay modes
(both charged and neutral B mesons)

exclusive: e.g. $B \rightarrow K^* \gamma$, $B \rightarrow \rho \gamma$, ...
inclusive: e.g. $B \rightarrow X_s \gamma$, ...
Radiative decay modes
(both charged and neutral B mesons)
exclusive: e.g. $B \rightarrow K^* \gamma$, $B \rightarrow \rho \gamma$, ...  
inclusive: e.g. $B \rightarrow X_s \gamma$, ...

Semi-leptonic decay modes
(both charged and neutral B mesons)
exclusive: e.g. $B \rightarrow K \mu^+ \mu^-$, $B_s \rightarrow \phi e^+ e^-$, $B \rightarrow K^* \nu \bar{\nu}$, ...
inclusive: e.g. $B \rightarrow X_s \mu^+ \mu^-$, ...
Classification of FCNC Decays (Rare Decays)

- **Radiative decay modes**
  (both charged and neutral B mesons)
  
  exclusive: e.g. \( B \rightarrow K^* \gamma \), \( B \rightarrow \rho \gamma \), ...
  
  inclusive: e.g. \( B \rightarrow X_s \gamma \), ...

- **Semi-leptonic decay modes**
  (both charged and neutral B mesons)
  
  exclusive: e.g. \( B \rightarrow K \mu^+ \mu^- \), \( B_s \rightarrow \phi e^+ e^- \), \( B \rightarrow K^* \nu \bar{\nu} \), ...
  
  inclusive: e.g. \( B \rightarrow X_s \mu^+ \mu^- \), ...

- **Purely leptonic decay modes**
  (only neutral B mesons)
  
  e.g. \( B_s \rightarrow \mu^+ \mu^- \), \( B_d \rightarrow \tau^+ \tau^- \), ...

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Theory of B Decays

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► Charged Current Decays:

determination of CKM matrix elements
(see talk by E. Lunghi on Friday)

► Rare Decays:

search for new physics
Charged Current Decays:

determination of CKM matrix elements
(see talk by E. Lunghi on Friday)
(but can also be used to probe new physics, if the new physics is "strong" enough to compete with tree level W exchange)

Rare Decays:

search for new physics
(but can also be used to determine CKM parameters, if one assumes that the decays are free of new physics)
Can’t discuss all the decay modes.

Will focus on a few examples in more detail:

1) $B \rightarrow D(\ast) \ell \nu$ and $R_D(\ast)$
2) $B_s \rightarrow \mu^+ \mu^-$
3) $B \rightarrow K(\ast) \ell^+ \ell^-$ and $R_K(\ast)$
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1) \( B \rightarrow D^{(*)} \ell \nu \) and \( R_{D^{(*)}} \)
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Will focus on a few examples in more detail:

1) $B \rightarrow D^{(*)}\ell\nu$ and $R_{D^{(*)}}$

2) $B_s \rightarrow \mu^+\mu^-$

3) $B \rightarrow K^*\ell^+\ell^-$ and $R_{K^{(*)}}$
$B \rightarrow D^{(*)} \ell \nu$ and $R_{D^{(*)}}$
The $B \rightarrow D^{(*)}\ell\nu$ Decays
Effective Hamiltonian for $B \to D^{(*)}\ell\nu$ in the SM

- Charged current decays
- Induced by tree level exchange of W bosons
Effective Hamiltonian for $B \rightarrow D(\ast) \ell \nu$ in the SM

- Charged current decays
- Induced by tree level exchange of W bosons

characteristic energy scale of B decays: $\mathcal{O}(m_B)$
characteristic energy scale of weak interactions: $\mathcal{O}(m_W) \gg O(m_B)$
Effective Hamiltonian for $B \rightarrow D^{(*)}\ell\nu$ in the SM

- Charged current decays
  - Induced by tree level exchange of W bosons

- characteristic energy scale of B decays: $\mathcal{O}(m_B)$
- characteristic energy scale of weak interactions: $\mathcal{O}(m_W) \gg \mathcal{O}(m_B)$
- decays can be described by an effective Hamiltonian ("integrate out the W boson")

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} C \left( \bar{c} \gamma_\mu P_L b \right) \left( \bar{\ell} \gamma^\mu P_L \nu_\ell \right)$$

Wilson coefficient
4-fermion contact interaction
\[ \langle D^{(*)} \ell \nu | \mathcal{H}_{\text{eff}} | B \rangle = \]

Parameterization in terms of form factors.
Form factors are non-perturbative objects.
Can be determined e.g. on the lattice (see talks by A. El-Khadra).
\[ \langle D^{}(\ell \nu) | H_{\text{eff}} | B \rangle = \frac{4G_F}{\sqrt{2}} V_{cb} C \langle \ell \bar{\nu} | (\bar{\ell} \gamma^\mu P_L \nu) | 0 \rangle \langle D^{}(\bar{c} \gamma^\mu P_L b) | B \rangle \]
\[ \langle D^{(*)} \ell \nu | \mathcal{H}_\text{eff} | B \rangle = \frac{4 G_F}{\sqrt{2}} V_{cb} C \langle \ell \bar{\nu} | (\bar{\ell} \gamma^\mu P_L \nu) | 0 \rangle \langle D^{(*)} | (\bar{c} \gamma_\mu P_L b) | B \rangle \]

Parameterization in terms of form factors

\[ \langle D | \bar{c} \gamma^\mu b | \bar{B} \rangle \equiv f_+(q^2) (p_B + p_D)^\mu + \left[ f_0(q^2) - f_+(q^2) \right] \frac{m_B^2 - m_D^2}{q^2} q^\mu \]

\[ \langle D^* | \bar{c} \gamma^\mu b | \bar{B} \rangle \equiv -ig(q^2) \varepsilon^{\mu \nu \rho \sigma} \varepsilon_\nu^* (p_B + p_{D^*})_\rho q_\sigma, \]

\[ \langle D^* | \bar{c} \gamma^\mu \gamma^5 b | \bar{B} \rangle \equiv \varepsilon^* \mu f(q^2) + a_+(q^2) \varepsilon^* \cdot p_B (p_B + p_{D^*})^\mu + a_-(q^2) \varepsilon^* \cdot p_B q^\mu \]

Form factors are non-perturbative objects.
Can be determined e.g. on the lattice
(see talks by A. El-Khadra)
Expressions for the Decay Rates

\[
\frac{d\Gamma(B \to D l\bar{\nu})}{dw} = \frac{G_F^2 |V_{cb}|^2 \eta_{\text{EW}}^2 m_B^5}{48\pi^3} (w^2 - 1)^{3/2} r_D^3 (1 + r_D)^2 G(w)^2 ,
\]

\[
\frac{d\Gamma(B \to D^* l\bar{\nu})}{dw} = \frac{G_F^2 |V_{cb}|^2 \eta_{\text{EW}}^2 m_B^5}{48\pi^3} (w^2 - 1)^{1/2} (w + 1)^2 r_{D^*}^3 (1 - r_{D^*})^2 
\times \left[ 1 + \frac{4w}{w + 1} \frac{1 - 2wr_{D^*} + r_{D^*}^2}{(1 - r_{D^*})^2} \right] \mathcal{F}(w)^2 ,
\]
Expressions for the Decay Rates

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\times \left[ 1 + \frac{4w}{w + 1} \frac{1 - 2wr_D^{*} + r_D^{*2}}{(1 - r_D^{*})^2} \right] F(w)^2 ,
\]

- \( \eta_{EW} \): electroweak corrections (known and very small)
Expressions for the Decay Rates

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- \( \eta_{EW} \): electroweak corrections (known and very small)
- \( \omega \): “recoil parameter” \( \omega = V_B \cdot V_{D(*)} \)
Expressions for the Decay Rates

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\frac{d\Gamma(\bar{B} \rightarrow Dl\bar{\nu})}{dw} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW}^2 m_B^5}{48\pi^3} (w^2 - 1)^{3/2} r_D^3 (1 + r_D)^2 G(w)^2 ,
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\times \left[ 1 + \frac{4w - 2wr_D^* + r_D^{*2}}{w + 1} \right] \mathcal{F}(w)^2 ,
\]

- \( \eta_{EW} \): electroweak corrections (known and very small)
- \( \omega \): “recoil parameter” \( \omega = V_B \cdot V_{D(*)} \)
- \( r_D^{(*)} = m_D^{(*)}/m_B \)
Expressions for the Decay Rates

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\times \left[ 1 + \frac{4w}{w + 1} \frac{1 - 2wr_{D^*} + r_{D^*}^2}{(1 - r_{D^*})^2} \right] \mathcal{F}(w)^2,
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- \( \eta_{EW} \): electroweak corrections (known and very small)
- \( \omega \): “recoil parameter” \( \omega = V_B \cdot V_{D(*)} \)
- \( r_{D(*)} = m_{D(*)}/m_B \)
- \( \mathcal{G}, \mathcal{F} \): combinations of form factors
Expressions for the Decay Rates

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- \( r_{D(*)} = m_{D(*)}/m_B \)
- \( \mathcal{G}, \mathcal{F} \): combinations of form factors

if \( \mathcal{G}, \mathcal{F} \) are known, can use experimental data on the decay rates to determine the CKM element \( V_{cb} \).
Lepton Flavor Universality Ratios

Take ratios of branching ratios with different leptons in the final state

\[ R_{D^{(*)}} = \frac{BR(B \to D^{(*)}\tau\nu)}{BR(B \to D^{(*)}\ell\nu)} \]
Lepton Flavor Universality Ratios

Take ratios of branching ratios with different leptons in the final state

$$R_D^{(*)} = \frac{BR(B \to D^{(*)}\tau\nu)}{BR(B \to D^{(*)}\ell\nu)}$$

- LFU ratios do not depend on the CKM matrix elements
- Have reduced dependence on form factors
- can be predicted in the SM with high precision

$$R_D^{SM} = 0.299 \pm 0.003 \quad , \quad R_D^{SM \ (*)} = 0.257 \pm 0.003$$

Bernlochner, Ligeti, Papucci, Robinson 1703.05330
The $R_{D(*)}$ Anomalies

world average from the heavy flavor averaging group

$$R_{D(*)} = \frac{BR(B \rightarrow D^{(*)}\tau\nu)}{BR(B \rightarrow D^{(*)}\ell\nu)}$$

$$\ell = \mu, e \quad \text{(BaBar/Belle)}$$

$$\ell = \mu \quad \text{(LHCb)}$$

$$R_{D}^{\exp} = 0.340 \pm 0.027 \pm 0.013 \quad , \quad R_{D^{*}}^{\exp} = 0.295 \pm 0.011 \pm 0.008$$

combined discrepancy with the SM of $3.1\sigma$
The $R_{D(*)}$ Anomalies

The world average from the heavy flavor averaging group

\[ R_{D(*)} = \frac{BR(B \to D^{(*)} \tau \nu)}{BR(B \to D^{(*)} \ell \nu)} \]

where $\ell = \mu, e$ (BaBar/Belle) and $\ell = \mu$ (LHCb).

The experimental results are:

\[ R_{D}^{exp} = 0.340 \pm 0.027 \pm 0.013 \quad \text{and} \quad R_{D*}^{exp} = 0.295 \pm 0.011 \pm 0.008 \]

This indicates a combined discrepancy with the SM of $3.1\sigma$.

Implications? See talks tomorrow by WA and B. Grinstein.
$B_s \rightarrow \mu^+ \mu^-$
The $B_s \rightarrow \mu^+\mu^-$ Decay
Flavor changing neutral current process

induced by Boxes and Z penguins
Flavor changing neutral current process

induced by Boxes and Z penguins

helicity suppressed decay (similar to pion decay):

B meson is spin 0, muons spin 1/2
→ one muon has to be left-handed, other one right-handed
electroweak interactions only give muons of the same handedness
→ branching ratio is helicity suppressed by $m^2_\mu/m^2_B$


Integrate out the top, W, Z, ... to arrive at an effective Hamiltonian that describes the $B_s \rightarrow \mu^+ \mu^-$ decay

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} C_{10} (\bar{s}\gamma_\alpha P_L b)(\bar{\mu}\gamma^\alpha \mu)$$
Integrate out the top, W, Z, ... to arrive at an effective Hamiltonian that describes the $B_s \rightarrow \mu^+ \mu^-$ decay

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In the SM there is a single Wilson coefficient that is relevant

$$C_{10} = \frac{1}{s_W^2} Y(x_t) = \frac{1}{s_W^2} \left( Y_0(x_t) + \frac{\alpha_s}{4\pi} Y_1(x_t) + \ldots \right)$$
Integrate out the top, W, Z, ... to arrive at an effective Hamiltonian that describes the $B_s \rightarrow \mu^+ \mu^-$ decay

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$s_W$ is the sine of the weak mixing angle

$Y_0$ and $Y_1$ are loop functions that depend on $x_t = m_t^2 / m_W^2$

known at NNLO in QCD and NLO in the electroweak interactions
The Hadronic Matrix Element

\[ \langle \mu^+ \mu^- | H_{\text{eff}} | B_s \rangle = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} C_{10} \langle \mu^+ \mu^- | (\bar{\mu} \gamma^\alpha \mu) | 0 \rangle \langle 0 | (\bar{s} \gamma^\alpha P_L b) | B_s \rangle \]
The Hadronic Matrix Element

\[ \langle \mu^+ \mu^- | H_{\text{eff}} | B_s \rangle = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} C_{10} \langle \mu^+ \mu^- | (\overline{\mu} \gamma^\alpha \mu) | 0 \rangle \langle 0 | (\overline{s} \gamma^\alpha P_L b) | B_s \rangle \]

- Hadronic matrix element is given by the \( B_s \) meson decay constant

\[ \langle 0 | (\overline{s} \gamma^\alpha b) | B_s \rangle = 0 \]

\[ \langle 0 | (\overline{s} \gamma^\alpha \gamma_5 b) | B_s \rangle = if_{B_s} p_{B_s}^\alpha \]
The Hadronic Matrix Element

\[ \langle \mu^+ \mu^- | H_{\text{eff}} | B_s \rangle = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} C_{10} \langle \mu^+ \mu^- | (\bar{\mu} \gamma^\alpha \mu) | 0 \rangle \langle 0 | (\bar{s} \gamma^\alpha P_L b) | B_s \rangle \]

- Hadronic matrix element is given by the $B_s$ meson decay constant

\[ \langle 0 | (\bar{s} \gamma^\alpha b) | B_s \rangle = 0 \]

\[ \langle 0 | (\bar{s} \gamma^\alpha \gamma_5 b) | B_s \rangle = i f_{B_s} p_{B_s}^\alpha \]

- Decay constants can be determined on the lattice

\[ f_{B_s} = (230.3 \pm 1.3) \text{MeV} \quad , \quad f_{B_d} = (190.0 \pm 1.3) \text{MeV} \]

(FLAG 1902.08191)

Percent level precision!
Branching Ratio Prediction

\[ BR(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} = \tau_{B_s} \frac{G_F^2}{\pi} m_{B_s} f_{B_s}^2 m_{\mu}^2 \frac{\alpha^2}{16\pi^2} |V_{ts}^* V_{tb}|^2 \frac{Y(x_t)^2}{S_W^4} \frac{1}{1 - y_s} \]

- decay constant
- Effect of lifetime difference of Bs and Bs-bar
- helicity suppression
- loop suppression
- CKM suppression

Wolfgang Altmannshofer
Theory of B Decays
SSI 2019
Branching Ratio Prediction

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} = \tau_{B_s} \frac{G_F^2}{\pi} m_{B_s} f_{B_s}^2 m_{\mu} \frac{\alpha^2}{16 \pi^2} |V_{ts}^* V_{tb}|^2 \frac{Y(x_t)^2}{S_W^4} \frac{1}{1 - y_s}$$

- decay constant
- Effect of lifetime difference of Bs and Bs-bar
- helicity suppression
- loop suppression
- CKM suppression

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} = (3.65 \pm 0.23) \times 10^{-9} \quad (\text{Bobeth et al. 1311.0903})$$

a truly rare decay!
Observation of $B_s \rightarrow \mu^+ \mu^-$

$BR(B_s \rightarrow \mu^+ \mu^-)_{\text{exp}} = \left(3.0 \pm 0.6^{+0.3}_{-0.2}\right) \times 10^{-9}$ (LHCb 1703.05747)

Slightly on the low side of the SM prediction ...
$B \rightarrow K^* \ell^+ \ell^−$ and $R_K(\ast)$
The $B \rightarrow K^*(\rightarrow K\pi)\mu^+\mu^-$ Decay
The $B \to K^*(\to K\pi)\mu^+\mu^-$ Decay

- Kinematics described by 4 variables

  - Invariant mass squared of the two muons: $q^2$
  - Three angles: $0 < \theta_{K^*} < \pi$, $0 < \theta_\ell < \pi$, $-\pi < \phi < \pi$

- Many observables accessible from the angular distribution
The $B \to K^*(\to K\pi)\mu^+\mu^-$ Decay

- self tagging:

  $K^+\pi^-$ final state for $B^0$
  $K^-\pi^+$ final state for $\bar{B}^0$

  $\rightarrow$ in principle easy access to CP asymmetries
The $B \to K^* \mu^+ \mu^-$ Angular Decay Distribution

\[
\frac{d^4\hat{\Gamma}}{dq^2 \, d\cos \theta_\ell \, d\cos \theta_K \, d\phi} = \frac{9}{32\pi} \bar{I}(q^2, \theta_\ell, \theta_K^*, \phi)
\]
The $B \rightarrow K^* \mu^+ \mu^-$ Angular Decay Distribution

\[
\frac{d^4 \bar{\Gamma}}{dq^2 \, d \cos \theta_\ell \, d \cos \theta_{K^*} \, d \phi} = \frac{9}{32\pi} \bar{l}(q^2, \theta_\ell, \theta_{K^*}, \phi)
\]

\[
\bar{l}(q^2, \theta_\ell, \theta_{K^*}, \phi) = \\
= \bar{l}_1 \sin^2 \theta_{K^*} + \bar{l}_1 \cos^2 \theta_{K^*} + \left( \bar{l}_2 \sin^2 \theta_{K^*} + \bar{l}_2 \cos^2 \theta_{K^*} \right) \cos 2\theta_\ell \\
+ \bar{l}_3 \sin^2 \theta_{K^*} \sin^2 \theta_\ell \cos 2\phi + \bar{l}_4 \sin 2\theta_{K^*} \sin 2\theta_\ell \cos \phi \\
- \bar{l}_5 \sin 2\theta_{K^*} \sin \theta_\ell \cos \phi \\
- \left( \bar{l}_6 \sin^2 \theta_{K^*} + \bar{l}_6 \cos^2 \theta_{K^*} \right) \cos \theta_\ell + \bar{l}_7 \sin 2\theta_{K^*} \sin \theta_\ell \sin \phi \\
- \bar{l}_8 \sin 2\theta_{K^*} \sin 2\theta_\ell \sin \phi - \bar{l}_9 \sin^2 \theta_{K^*} \sin^2 \theta_\ell \sin 2\phi
\]

The $l$'s are moments of the angular distribution.
\[ \mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_{i=7,9,10} C_i \mathcal{O}_i + \ldots \]
Effective Hamiltonian for $B \rightarrow K^* \ell^+ \ell^-$ in the SM

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_{i=7,9,10} C_i \mathcal{O}_i + \ldots$$

- **Magnetic dipole operators**
  - $C_7$ (\(\bar{s}\sigma_{\mu\nu} P_R b\)) $F^{\mu\nu}$
  - $C_9$ (\(\bar{s}\gamma_{\mu} P_L b\))(\(\bar{\ell}\gamma^\mu \ell\))
  - $C_{10}$ (\(\bar{s}\gamma_{\mu} P_L b\))(\(\bar{\ell}\gamma^\mu \gamma_5 \ell\))

- **Semileptonic operators**
  - $b_R$
  - $s_L$
  - $b_L$
  - $s_L$
  - $\ell$
  - $\ell$
Effective Hamiltonian for $B \rightarrow K^* \ell^+ \ell^-$ in the SM

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_{i=7,9,10} C_i \mathcal{O}_i + \ldots$$

magnetic dipole operators

$$C_7 (\bar{s} \sigma_{\mu\nu} P_R b) F^{\mu\nu}$$

semileptonic operators

$$C_9 (\bar{s} \gamma_{\mu} P_L b) (\bar{\ell} \gamma^\mu \ell)$$
$$C_{10} (\bar{s} \gamma_{\mu} P_L b) (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

(We’ll come back to the dots in a couple of slides)
Hadronic matrix elements are parameterized in terms of form factors

\[
\langle \bar{K}^*(k) | \bar{s} \gamma_\mu (1 - \gamma_5) b | \bar{B}(p) \rangle = -i \epsilon^*_\mu (m_B + m_{K^*}) A_1(q^2) + i(2p - q)_\mu (\epsilon^* \cdot q) \frac{A_2(q^2)}{m_B + m_{K^*}}
\]

\[
+ i q_\mu (\epsilon^* \cdot q) \frac{2m_{K^*}}{q^2} \left[ A_3(q^2) - A_0(q^2) \right] + \epsilon_{\mu \nu \rho \sigma} \epsilon^{* \nu} p^\rho k^\sigma \frac{2V(q^2)}{m_B + m_{K^*}}
\]

\[
\langle \bar{K}^*(k) | \bar{s} \sigma_{\mu \nu} q^\nu (1 + \gamma_5) b | \bar{B}(p) \rangle = i \epsilon_{\mu \nu \rho \sigma} \epsilon^{* \nu} p^\rho k^\sigma 2T_1(q^2)
\]

\[
+ T_2(q^2) \left[ \epsilon^*_\mu (m_B^2 - m_{K^*}^2) - (\epsilon^* \cdot q) (2p - q)_\mu \right] + T_3(q^2) (\epsilon^* \cdot q) \left[ q_\mu - \frac{q^2}{m_B^2 - m_{K^*}^2} (2p - q)_\mu \right]
\]

Predictions exist from lattice QCD and other non-perturbative methods (light cone sum rules)
So far we discussed the “factorizable” contributions

(illustrations by Danny van Dyk)
So far we discussed the “factorizable” contributions there are also non-factorizable effects coming from 4-quark operators.

(illustrations by Danny van Dyk)
So far we discussed the “factorizable” contributions there are also non-factorizable effects coming from 4-quark operators.

Size the of the non-factorizable effects is not very well understood. **Major source of uncertainty** in the theory predictions.
The $q^2$ Spectrum

![Graph showing $dBR(B \to K^* \mu^+ \mu^-)/dq^2 \times 10^7$ with $q^2$ range from 0 to 20 GeV$^2$ and distinct peaks at low and high $q^2$ regions. Peaks are labeled $J/\psi$ and $\psi'$ with shaded regions highlighting these areas.].
Angular Observables

take sums and differences of the angular moments and normalize to the differential decay width

▶ CP-Asymmetries

\[ A_i^{(a)}(q^2) = \left( I_i^{(a)}(q^2) - \bar{I}_i^{(a)}(q^2) \right) \frac{d \left( \Gamma + \bar{\Gamma} \right)}{dq^2} \]
Angular Observables

take sums and differences of the angular moments
and normalize to the differential decay width

▶ CP-Asymmetries

\[ A_i^{(a)}(q^2) = \left( I_i^{(a)}(q^2) - \bar{I}_i^{(a)}(q^2) \right) \frac{d(\Gamma + \bar{\Gamma})}{dq^2} \]

▶ CP-averaged angular coefficients ("Symmetries")

\[ S_i^{(a)}(q^2) = \left( I_i^{(a)}(q^2) + \bar{I}_i^{(a)}(q^2) \right) \frac{d(\Gamma + \bar{\Gamma})}{dq^2} \]
Angular Observables

take sums and differences of the angular moments and normalize to the differential decay width

▶ CP-Asymmetries

\[ A_i^{(a)}(q^2) = \left( I_i^{(a)}(q^2) - \bar{I}_i^{(a)}(q^2) \right) \left/ \frac{d(\Gamma + \bar{\Gamma})}{dq^2} \right. \]

▶ CP-averaged angular coefficients (“Symmetries”)

\[ S_i^{(a)}(q^2) = \left( I_i^{(a)}(q^2) + \bar{I}_i^{(a)}(q^2) \right) \left/ \frac{d(\Gamma + \bar{\Gamma})}{dq^2} \right. \]

▶ normalization reduces dependence on form factors


\[
\begin{align*}
P'_4 &= \frac{2S_4}{\sqrt{F_L(1 - F_L)}}, & P'_5 &= \frac{S_5}{\sqrt{F_L(1 - F_L)}} \\
P'_{6\text{CP}} &= \frac{-A_7}{\sqrt{F_L(1 - F_L)}}, & P'_{8\text{CP}} &= \frac{-2A_8}{\sqrt{F_L(1 - F_L)}}
\end{align*}
\]

\[\cdots\]

- minimize dependence on formfactors
- normalize such that the form factors cancel out exactly in the heavy quark limit

Descotes-Genon, Hurth, Matias, Virto 1303.5794; Matias, Mescia, Ramon, Virto 1202.4266;
\[
P'_4 = \frac{2S_4}{\sqrt{F_L(1 - F_L)}}, \quad P'_5 = \frac{S_5}{\sqrt{F_L(1 - F_L)}}
\]
\[
P'^{\text{CP}}_6 = \frac{-A_7}{\sqrt{F_L(1 - F_L)}}, \quad P'^{\text{CP}}_8 = \frac{-2A_8}{\sqrt{F_L(1 - F_L)}}
\]

- Minimize dependence on formfactors
- Normalize such that the form factors cancel out exactly in the heavy quark limit

- Still subject to factorizable and non-factorizable power corrections

Descotes-Genon, Hurth, Matias, Virto 1303.5794; Matias, Mescia, Ramon, Virto 1202.4266;
\[ P_4' = \frac{2S_4}{\sqrt{F_L(1 - F_L)}} \]  
\[ P_5' = \frac{S_5}{\sqrt{F_L(1 - F_L)}} \]  
\[ P_{6\text{CP}}' = \frac{-A_7}{\sqrt{F_L(1 - F_L)}} \]  
\[ P_{8\text{CP}}' = \frac{-2A_8}{\sqrt{F_L(1 - F_L)}} \]  
\[ \ldots \]

- minimize dependence on formfactors
- normalize such that the form factors cancel out exactly in the heavy quark limit
- still subject to factorizable and non-factorizable power corrections
- \( P_5' \) will return in tomorrow’s talk

Descotes-Genon, Hurth, Matias, Virto 1303.5794; Matias, Mescia, Ramon, Virto 1202.4266;
Hadronic uncertainties drop out almost entirely in lepton flavor universality ratios

\[ R_{K^*} = \frac{BR(B \to K^* \mu^+ \mu^-)}{BR(B \to K^* e^+ e^-)} \]
Hadronic uncertainties drop out almost entirely in lepton flavor universality ratios

\[ R_{K^*} = \frac{BR(B \to K^* \mu^+ \mu^-)}{BR(B \to K^* e^+ e^-)} \]

Analogously for the \( B \to K \ell^+ \ell^- \) decays

\[ R_K = \frac{BR(B \to K \mu^+ \mu^-)}{BR(B \to K e^+ e^-)} \]
Hadronic uncertainties drop out almost entirely in lepton flavor universality ratios.

\[ R_{K^*} = \frac{BR(B \rightarrow K^* \mu^+ \mu^-)}{BR(B \rightarrow K^* e^+ e^-)} \]

Analogously for the \( B \rightarrow K \ell^+ \ell^- \) decays:

\[ R_K = \frac{BR(B \rightarrow K \mu^+ \mu^-)}{BR(B \rightarrow Ke^+ e^-)} \]

Standard Model Predictions (Bordone, Isidori, Pattori 1605.07633)

\begin{align*}
R_K^{[1,6]} &= 1.00 \pm 0.01, & R_{K^*}^{[1.1,6]} &= 1.00 \pm 0.01, & R_{K^*}^{[0.045,1.1]} &= 0.91 \pm 0.03 \end{align*}

(The numbers in square brackets indicate the \( q^2 \) region)
The $R_{K(*)}$ Anomalies

\[ R_K^{[1,6]} = 0.846^{+0.060+0.016}_{-0.054-0.014} \]

\[ R_{K^*}^{[0.045,1.1]} = 0.66^{+0.11}_{-0.07} \pm 0.03 \]

\[ R_{K^*}^{[1.1,6]} = 0.69^{+0.11}_{-0.07} \pm 0.05 \]

3 observables deviating by $\sim 2\sigma - 2.5\sigma$ from the SM predictions
The $R_K(*)$ Anomalies

$R_K^{[1,6]} = 0.846^{+0.060+0.016}_{-0.054-0.014}$

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$R_K^{[1.1,6]} = 0.69^{+0.11}_{-0.07} \pm 0.05$

3 observables deviating by $\sim 2\sigma - 2.5\sigma$ from the SM predictions

Implications?
See talks tomorrow by WA and B. Grinstein
B decays are a rich laboratory for testing the flavor structure of the Standard Model and extensions.

Set of intriguing discrepancies in the current data:

What are the implications?