Theory of B Decays

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SLAC Summer Institute 2019,
Menu of Flavors: Quarks, Charged Leptons & Neutrinos

August 14, 2019

Outline of the Lectures

1. Theory of B Decays (today)
   - Intro: Classification of B Decays
   - $B \to D^{(*)}\ell\nu$ and $R_{D^{(*)}}$
   - $B_s \to \mu^+\mu^-$
   - $B \to K^*\ell^+\ell^-$ and $R_{K^{(*)}}$

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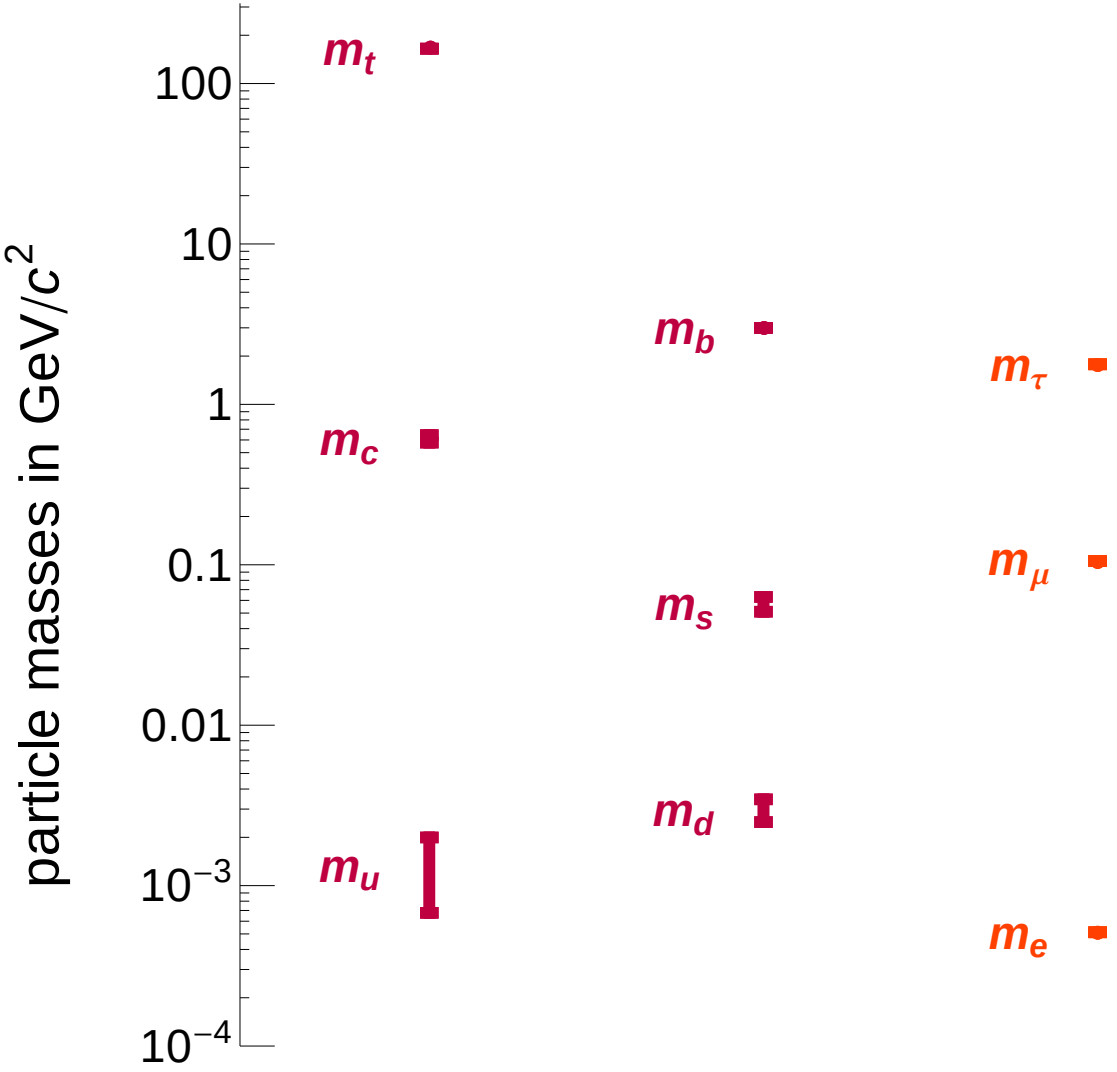
   - $B_s \to \mu^+\mu^-$
   - $B \to K^*\ell^+\ell^-$ and $R_{K^{(*)}}$
2. Effective Field Theory Fits (tomorrow)
   - B Decays as Probes of New Physics
   - The Flavor Anomalies
   - Fitting the $b \to c\ell\nu$ Anomalies
   - Fitting the $b \to s\ell\ell$ Anomalies

The b Quark



$$\overline{m}_b(\overline{m}_b) = 4.18^{+0.03}_{-0.02}\ \text{GeV}$$

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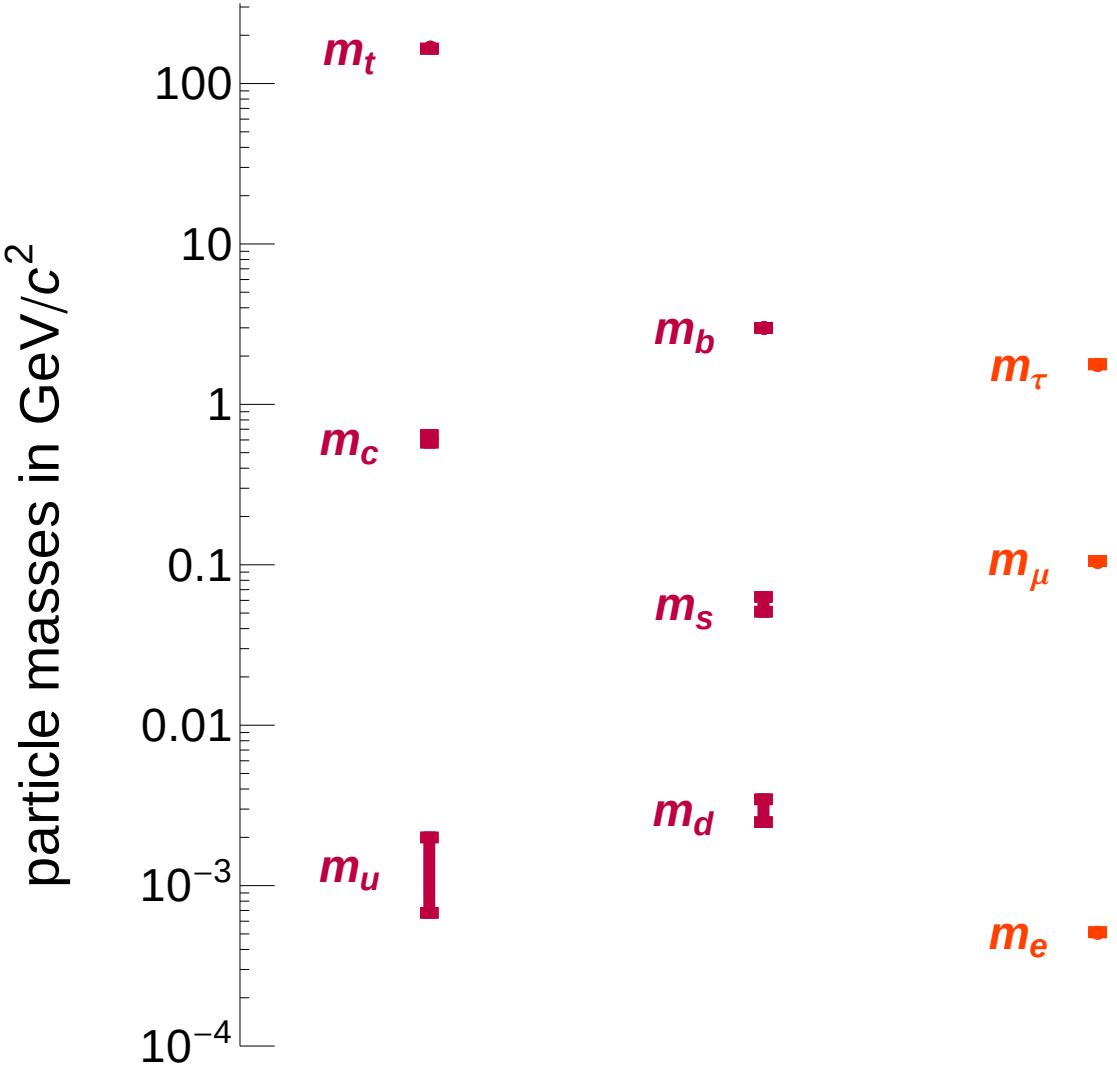

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- bottomonia: $\Upsilon(1s)$, $\Upsilon(2s)$, ...

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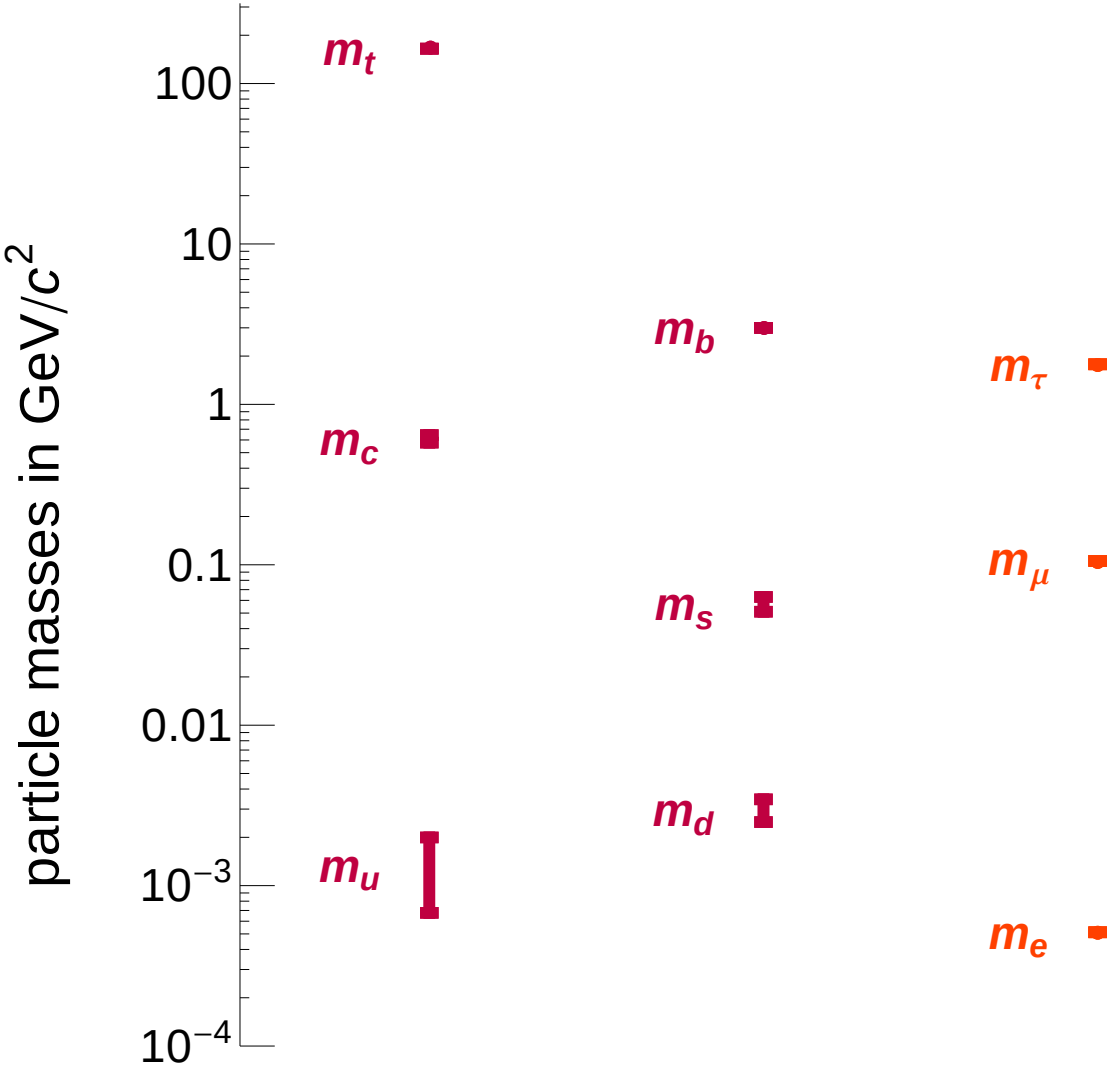

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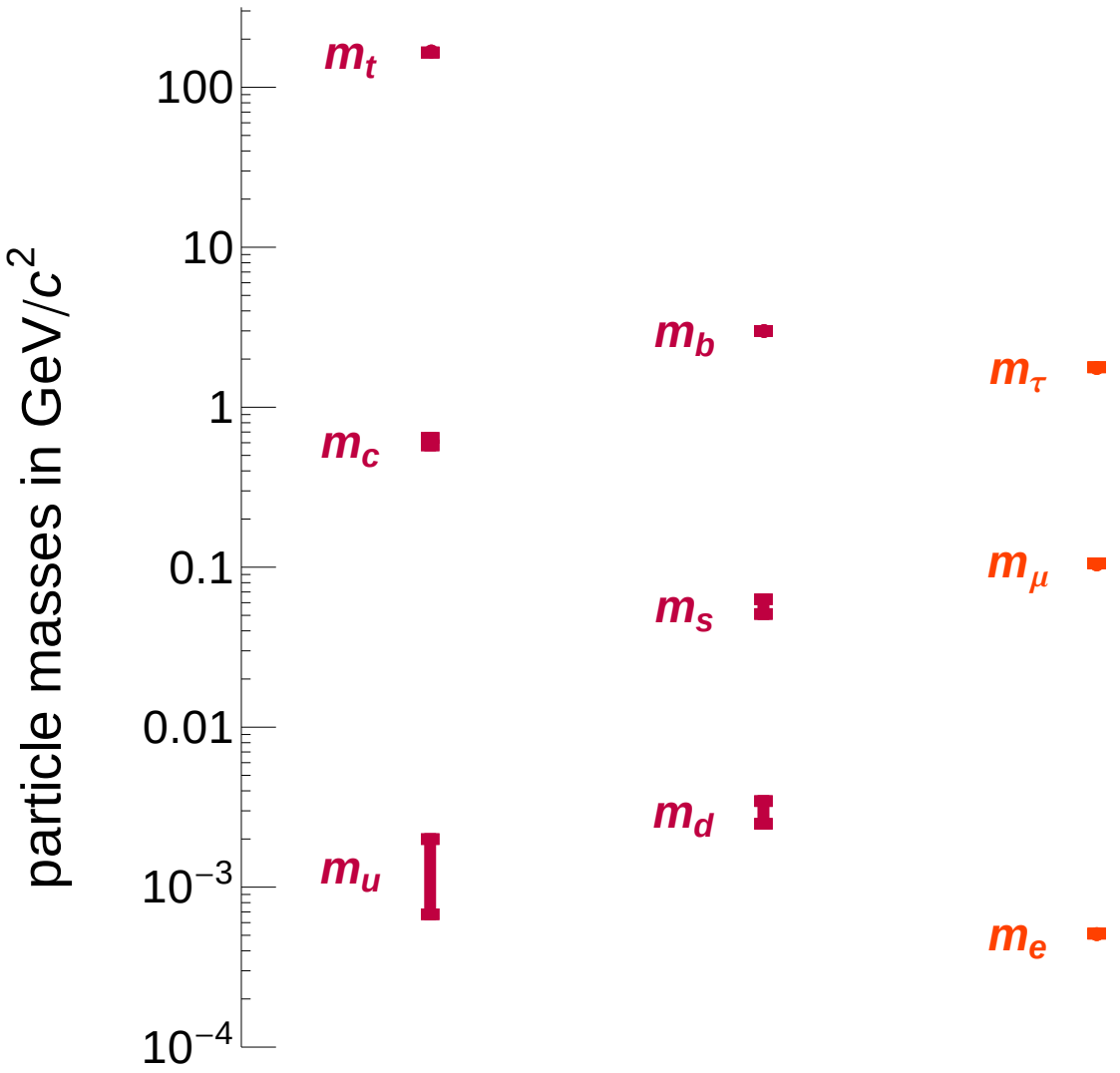

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forms bound states

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- B baryons: Λ_b, ...

Lifetime

- Decay of b quarks proceeds through the weak interactions
- Exchange of a heavy virtual W boson

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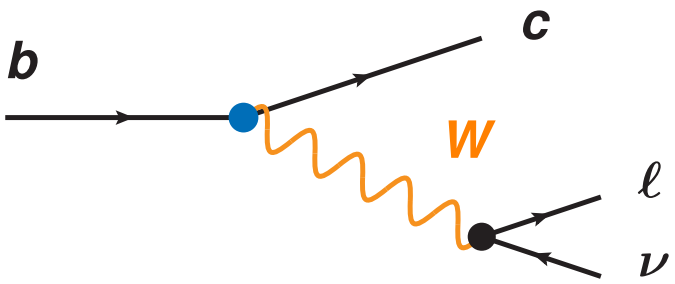
- Estimate the decay width

$$\Gamma(b \to c\ell\nu) \sim \frac{G_F^2}{192\pi^3} m_b^5 |V_{cb}|^2$$



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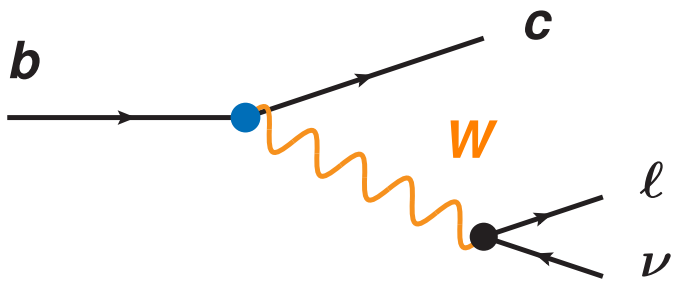
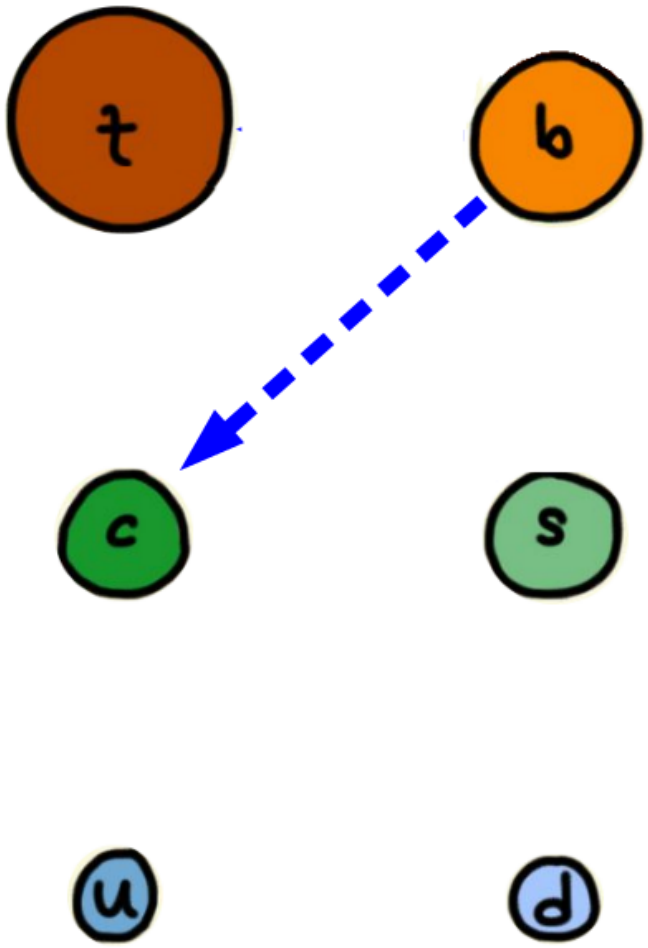


$$\Rightarrow \quad \tau = \frac{1}{\Gamma_{\text{tot}}} \sim \mathcal{O}(10^{-12} s)$$

- small decay width $\Rightarrow$ sizable lifetime
- high sensitivity to new physics effects

Charged Current Decays



- arise at tree level through W exchange

$$A(b \to c) \sim V_{cb} \sim 4 \times 10^{-2}$$

$$(\text{e.g. } B \to D\mu\nu)$$

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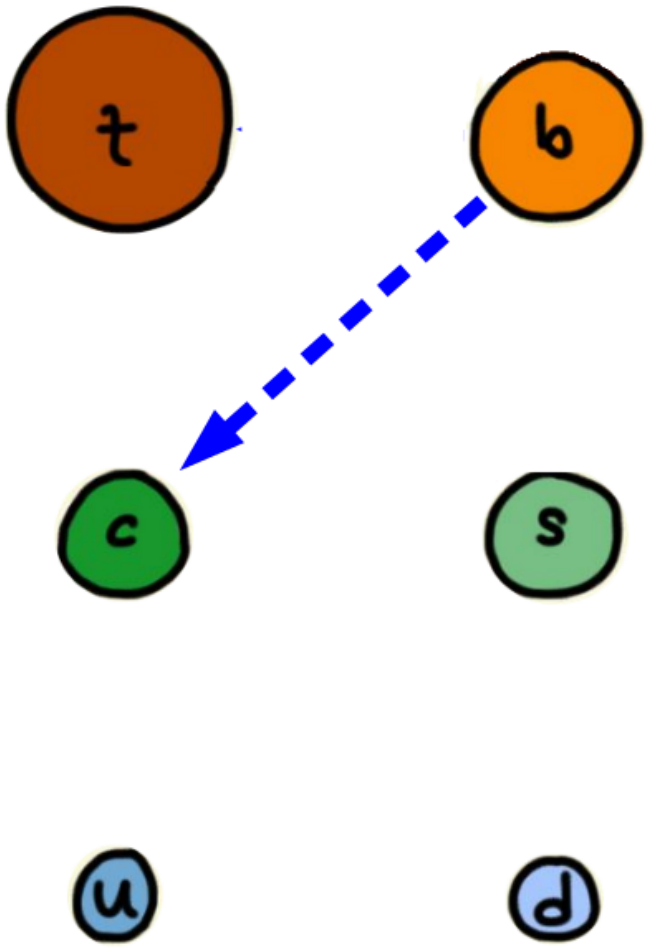

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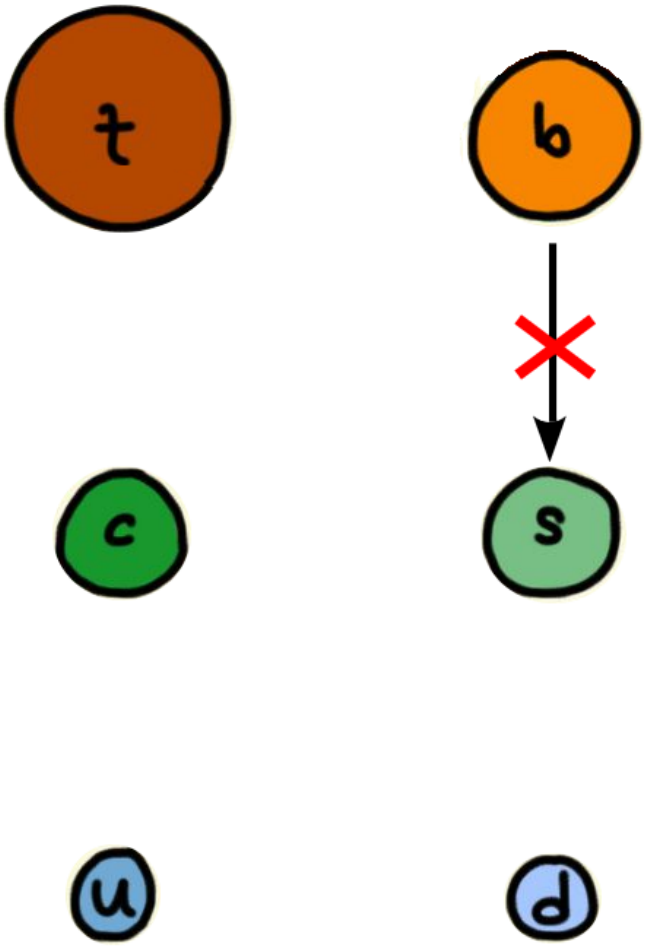
(e.g. $B \to D\mu\nu$)

$$A(b \to u) \sim V_{ub} \sim 4 \times 10^{-3}$$

(e.g. $B \to \pi\mu\nu$)

Flavor Changing Neutral Current Decays



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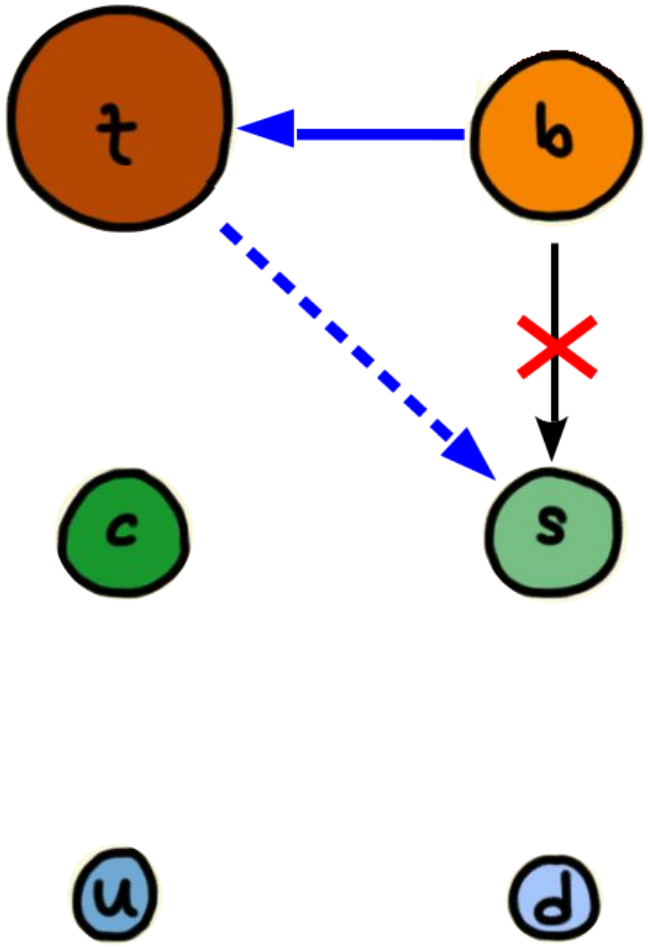

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- arise at the 1-loop level

$$A(b \to s) \sim \frac{1}{16\pi^2} V_{ts}^* V_{tb} \sim 2.5 \times 10^{-4}$$

(e.g. $B \to K^* \mu^+ \mu^-$)

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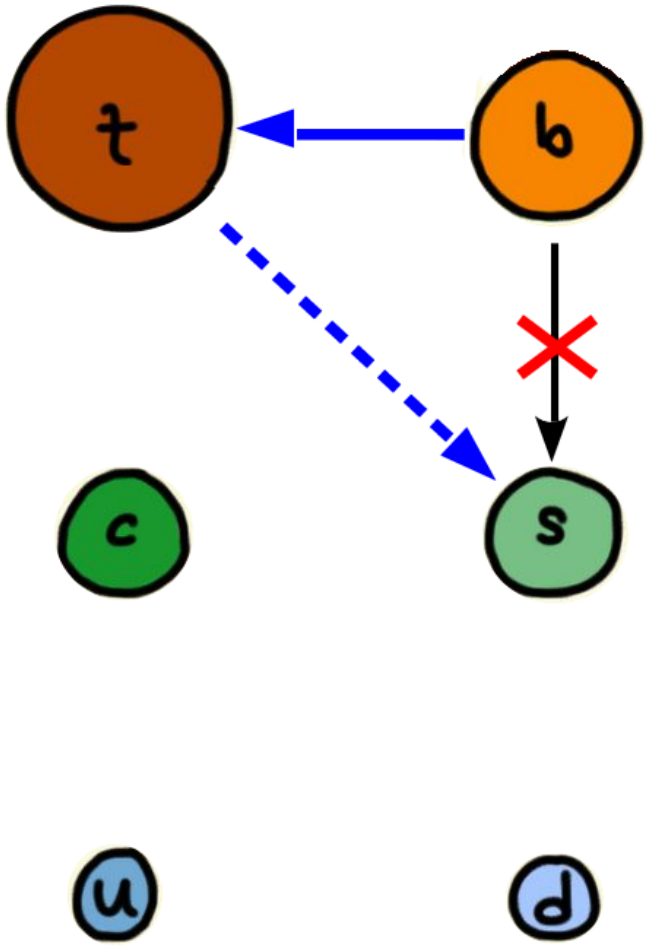

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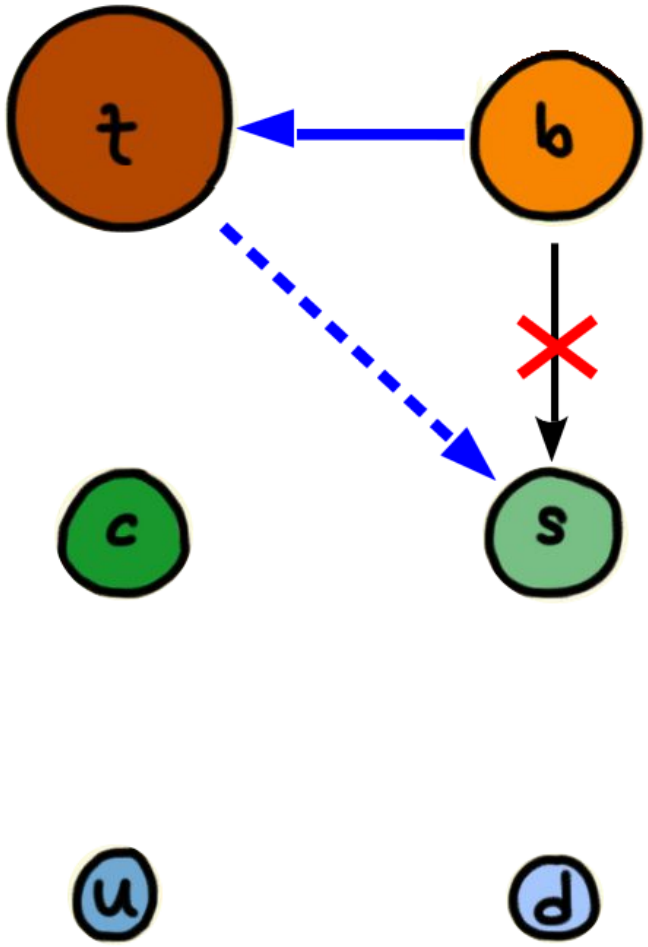

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(e.g. $B \to \pi \mu^+ \mu^-$)

"rare decays"

Classification of Charged Current Decays

- Semi-leptonic decay modes
  (both charged and neutral B mesons)

  exclusive: e.g. $B \to D\tau\nu$, $B \to D^*\mu\nu$, $B \to \pi e\nu$ ...
  inclusive: e.g. $B \to X_c\tau\nu$, $B \to X_c\mu\nu$, $B \to X_u e\nu$ ...

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- Purely hadronic decay modes
  (both charged and neutral B mesons)

  hundreds of possible final states

Classification ys

- Semi-leptonic
  (both charged

  exclusive: e.g.
  inclusive: e.g.

- Purely leptonic
  (only charged

  e.g. $B \to \tau\nu$, B

- Purely hadroni
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  hundreds of p

Γ_{97}	$D^0 K^*(892)^+$		$(3.1 \pm 1.6) \times 10^{-6}$
Γ_{98}	$\overline{D}^0 K^+ \pi^+ \pi^-$		$(5.2 \pm 2.1) \times 10^{-4}$
Γ_{99}	$[\, K^+ \pi^- \,]_D \; K^+ \pi^- \pi^+$		
Γ_{100}	$[\, K^- \pi^+ \,]_D \; K^+ \pi^- \pi^+$		
Γ_{101}	$D_{CP(+1)} K^+ \pi^- \pi^+$		
Γ_{102}	$\overline{D}^0 K^+ \overline{K}^0$		$(5.5 \pm 1.6) \times 10^{-4}$
Γ_{103}	$\overline{D}^0 K^+ \overline{K}^*(892)^0$		$(7.5 \pm 1.7) \times 10^{-4}$
Γ_{104}	$\overline{D}^0 \pi^+ \pi^+ \pi^-$		$(5.6 \pm 2.1) \times 10^{-3}$
Γ_{105}	$[\, K^- \pi^+ \,]_D \; \pi^+ \pi^- \pi^+$		
Γ_{106}	$\overline{D}^0 \pi^+ \pi^+ \pi^-$ nonresonant		$(5 \pm 4) \times 10^{-3}$
Γ_{107}	$\overline{D}^0 \pi^+ \rho^0$		$(4.2 \pm 3.0) \times 10^{-3}$
Γ_{108}	$\overline{D}^0 a_1(1260)^+$		$(4 \pm 4) \times 10^{-3}$
Γ_{109}	$\overline{D}^0 \omega \pi^+$		$(4.1 \pm 0.9) \times 10^{-3}$
Γ_{110}	$D^*(2010)^- \pi^+ \pi^+$		$(1.35 \pm 0.22) \times 10^{-3}$
Γ_{111}	$D^*(2010)^- K^+ \pi^+$		$(8.2 \pm 1.4) \times 10^{-5}$
Γ_{112}	$\overline{D}_1(2420)^0 \pi^+ \,,\; \overline{D}_1^0 \to D^*(2010)^- \pi^+$		$(5.2 \pm 2.2) \times 10^{-4}$
Γ_{113}	$D^- \pi^+ \pi^+$		$(1.07 \pm 0.05) \times 10^{-3}$
Γ_{114}	$D^- K^+ \pi^+$		$(7.7 \pm 0.5) \times 10^{-5}$
Γ_{115}	$D_0^*(2300)^0 K^+ \,,\; D_0^{*0} \to D^- \pi^+$		$(6.1 \pm 2.4) \times 10^{-6}$
Γ_{116}	$D_2^*(2460)^0 K^+ \,,\; D_2^{*0} \to D^- \pi^+$		$(2.32 \pm 0.23) \times 10^{-5}$
Γ_{117}	$D_1^*(2760)^0 K^+ \,,\; D_1^{*0} \to D^- \pi^+$		$(3.6 \pm 1.2) \times 10^{-6}$
Γ_{118}	$D^+ K^0$		$< 2.9 \times 10^{-6}$
Γ_{119}	$D^+ K^+ \pi^-$		$(5.6 \pm 1.1) \times 10^{-6}$
Γ_{120}	$D_2^*(2460)^0 K^+ \,,\; D_2^{*0} \to D^+ \pi^-$		$< 6.3 \times 10^{-7}$
Γ_{121}	$D^+ K^{*0}$		$< 4.9 \times 10^{-7}$
Γ_{122}	$D^+ \overline{K}^{*0}$		$< 1.4 \times 10^{-6}$
Γ_{123}	$\overline{D}^*(2007)^0 \pi^+$		$(4.90 \pm 0.17) \times 10^{-3}$
Γ_{124}	$\overline{D}^{*0}_{CP(+1)} \pi^+$	[4]	$(2.7 \pm 0.6) \times 10^{-3}$
Γ_{125}	$D^{*0}_{CP(-1)} \pi^+$	[4]	$(2.4 \pm 0.9) \times 10^{-3}$
Γ_{126}	$\overline{D}^*(2007)^0 \omega \pi^+$		$(4.5 \pm 1.2) \times 10^{-3}$
Γ_{127}	$\overline{D}^*(2007)^0 \rho^+$		$(9.8 \pm 1.7) \times 10^{-3}$
Γ_{128}	$\overline{D}^*(2007)^0 K^+$		$(3.97^{+0.31}_{-0.28}) \times 10^{-4}$
Γ_{129}	$\overline{D}^{*0}_{CP(+1)} K^+$	[4]	$(2.60 \pm 0.33) \times 10^{-4}$
Γ_{130}	$\overline{D}^{*0}_{CP(-1)} K^+$	[4]	$(2.19 \pm 0.30) \times 10^{-4}$
Γ_{131}	$D^*(2007)^0 K^+$		$(7.8 \pm 2.2) \times 10^{-6}$
Γ_{132}	$\overline{D}^*(2007)^0 K^*(892)^+$		$(8.1 \pm 1.4) \times 10^{-4}$
Γ_{133}	$\overline{D}^*(2007)^0 K^+ \overline{K}^0$		$< 1.06 \times 10^{-3}$
Γ_{134}	$\overline{D}^*(2007)^0 K^+ \overline{K}^*(892)^0$		$(1.5 \pm 0.4) \times 10^{-3}$
Γ_{135}	$\overline{D}^*(2007)^0 \pi^+ \pi^+ \pi^-$		$(1.03 \pm 0.12)\%$
Γ_{136}	$\overline{D}^*(2007)^0 a_1(1260)^+$		$(1.9 \pm 0.5)\%$
Γ_{137}	$\overline{D}^*(2007)^0 \pi^- \pi^+ \pi^+ \pi^0$		$(1.8 \pm 0.4)\%$
Γ_{138}	$\overline{D}^{*0}\, 3\, \pi^+ 2\, \pi^-$		$(5.7 \pm 1.2) \times 10^{-3}$
Γ_{139}	$D^*(2010)^+ \pi^0$		$< 3.6 \times 10^{-6}$

Classification of FCNC Decays (Rare Decays)

- Radiative decay modes
  (both charged and neutral B mesons)

  exclusive: e.g. $B \to K^* \gamma$, $B \to \rho\gamma$, ...
  inclusive: e.g. $B \to X_s \gamma$, ...

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- Semi-leptonic decay modes
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  inclusive: e.g. $B \to X_s \mu^+ \mu^-$, ...

- Purely leptonic decay modes
  (only neutral B mesons)

  e.g. $B_s \to \mu^+ \mu^-$, $B_d \to \tau^+ \tau^-$, ...

Application (0th order picture)

- Charged Current Decays:

  determination of CKM matrix elements

  (see talk by E. Lunghi on Friday)

- Rare Decays:

  search for new physics

Application (0th order picture)

- Charged Current Decays:

  determination of CKM matrix elements

  (see talk by E. Lunghi on Friday)

  (but can also be used to probe new physics, if the new physics is “strong” enough to compete with tree level W exchange)

- Rare Decays:

  search for new physics

  (but can also be used to determine CKM parameters, if one assumes that the decays are free of new physics)

Can’t discuss all the decay modes.

Will focus on a few examples in more detail:

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2) $B_s \to \mu^+\mu^-$
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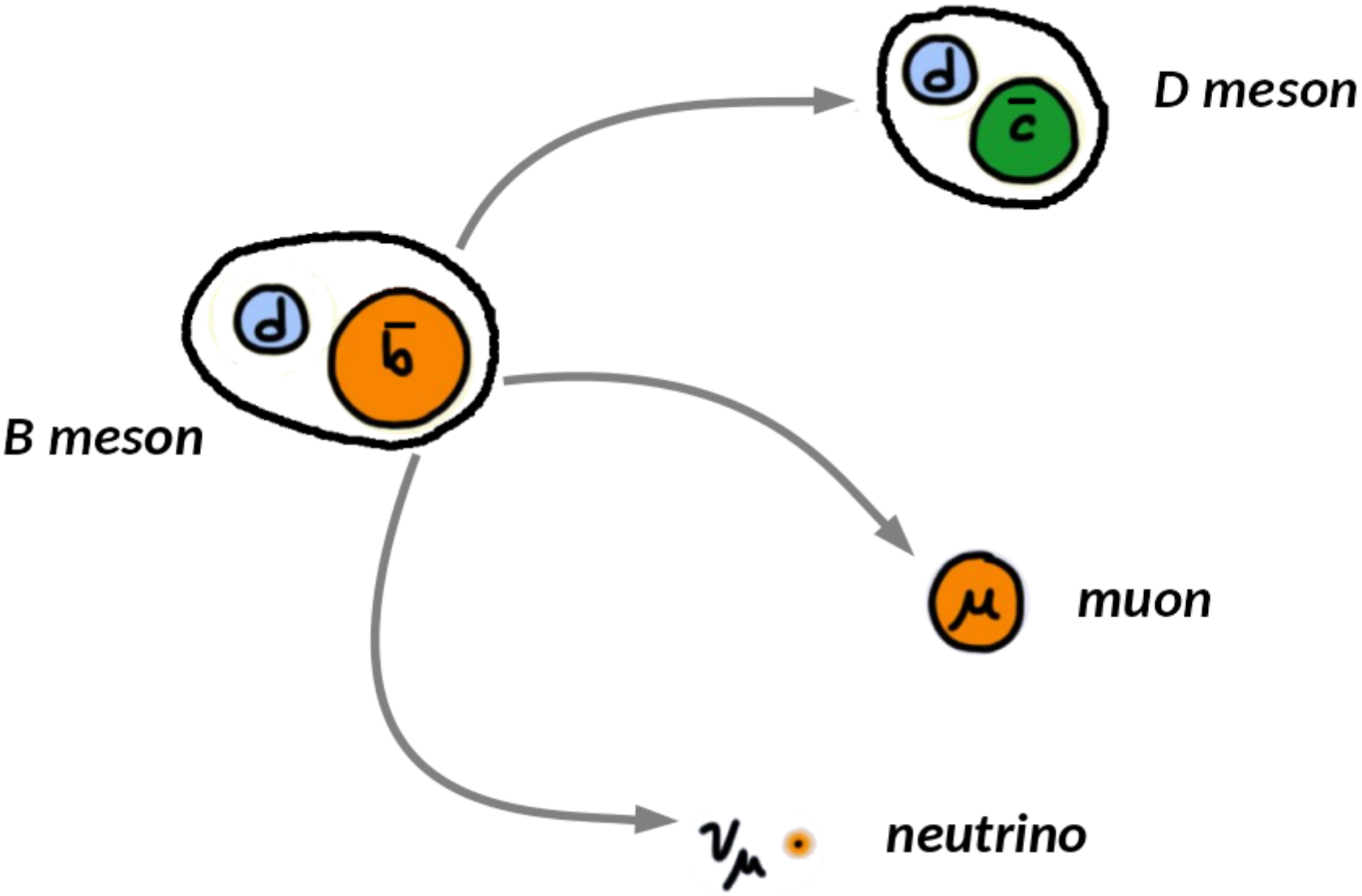
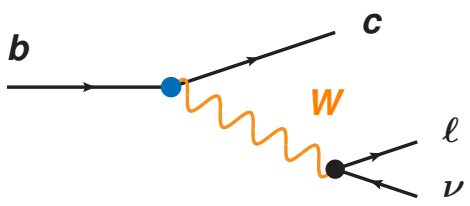
$B \to D^{(*)}\ell\nu$ and $R_{D^{(*)}}$

The $B \to D^{(*)}\ell\nu$ Decays



Effective Hamiltonian for $B \to D^{(*)}\ell\nu$ in the SM

- Charged current decays
- Induced by tree level exchange of W bosons



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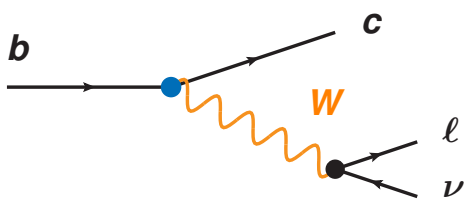

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- characteristic energy scale of weak interactions: $\mathcal{O}(m_W) \gg O(m_B)$

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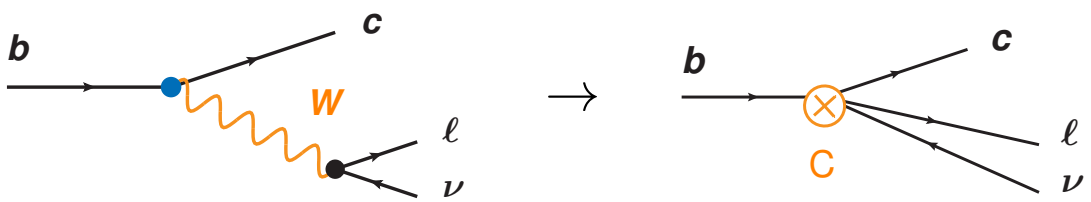

- characteristic energy scale of B decays: $\mathcal{O}(m_B)$
- characteristic energy scale of weak interactions: $\mathcal{O}(m_W) \gg O(m_B)$
- decays can be described by an effective Hamiltonian ("integrate out the W boson")

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb}\, C\, (\bar{c}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu P_L \nu_\ell)$$

Wilson coefficient

4-fermion contact interaction

Hadronic Matrix Elements

$$\langle D^{(*)}\ell\nu|\mathcal{H}_{\text{eff}}|B\rangle =$$

Hadronic Matrix Elements

$$\langle D^{(*)}\ell\nu|\mathcal{H}_{\text{eff}}|B\rangle = \frac{4G_F}{\sqrt{2}}V_{cb}C\langle\ell\bar{\nu}|(\bar{\ell}\gamma^\mu P_L\nu)|0\rangle\langle D^{(*)}|(\bar{c}\gamma_\mu P_L b)|B\rangle$$

Hadronic Matrix Elements

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Parameterization in terms of form factors

$$\left\langle D \middle| \bar{c}\gamma^\mu b \middle| \bar{B} \right\rangle \equiv f_+(q^2)(p_B + p_D)^\mu + [f_0(q^2) - f_+(q^2)] \frac{m_B^2 - m_D^2}{q^2} q^\mu$$

$$\left\langle D^* \middle| \bar{c}\gamma^\mu b \middle| \bar{B} \right\rangle \equiv -ig(q^2)\, \epsilon^{\mu\nu\rho\sigma}\, \varepsilon^*_\nu\, (p_B + p_{D^*})_\rho\, q_\sigma\,,$$

$$\left\langle D^* \middle| \bar{c}\gamma^\mu\gamma^5 b \middle| \bar{B} \right\rangle \equiv \varepsilon^{*\mu} f(q^2) + a_+(q^2)\, \varepsilon^* \cdot p_B\, (p_B + p_{D^*})^\mu + a_-(q^2)\, \varepsilon^* \cdot p_B\, q^\mu$$

Form factors are non-perturbative objects.
Can be determined e.g. on the lattice
(see talks by A. El-Khadra)

Expressions for the Decay Rates

$$\frac{\mathrm{d}\Gamma(\overline{B} \to D l \bar{\nu})}{\mathrm{d}w} = \frac{G_F^2 |V_{cb}|^2 \, \eta_{\mathrm{EW}}^2 \, m_B^5}{48\pi^3} \, (w^2-1)^{3/2} \, r_D^3 \, (1+r_D)^2 \, \mathcal{G}(w)^2 \, ,$$

$$\frac{\mathrm{d}\Gamma(\overline{B} \to D^* l \bar{\nu})}{\mathrm{d}w} = \frac{G_F^2 |V_{cb}|^2 \, \eta_{\mathrm{EW}}^2 \, m_B^5}{48\pi^3} \, (w^2-1)^{1/2} \, (w+1)^2 \, r_{D^*}^3 (1-r_{D^*})^2 \times \left[1 + \frac{4w}{w+1} \frac{1-2wr_{D^*}+r_{D^*}^2}{(1-r_{D^*})^2} \right] \mathcal{F}(w)^2 \, ,$$

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- η_{EW}: electroweak corrections (known and very small)

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if $\mathcal{G}, \mathcal{F}$ are known, can use experimental data on the decay rates to determine the CKM element V_{cb}

Lepton Flavor Universality Ratios

Take ratios of branching ratios with different leptons in the final state

$$R_{D^{(*)}} = \frac{BR(B \to D^{(*)}\tau\nu)}{BR(B \to D^{(*)}\ell\nu)}$$

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$$R_{D^{(*)}} = \frac{BR(B \to D^{(*)}\tau\nu)}{BR(B \to D^{(*)}\ell\nu)}$$

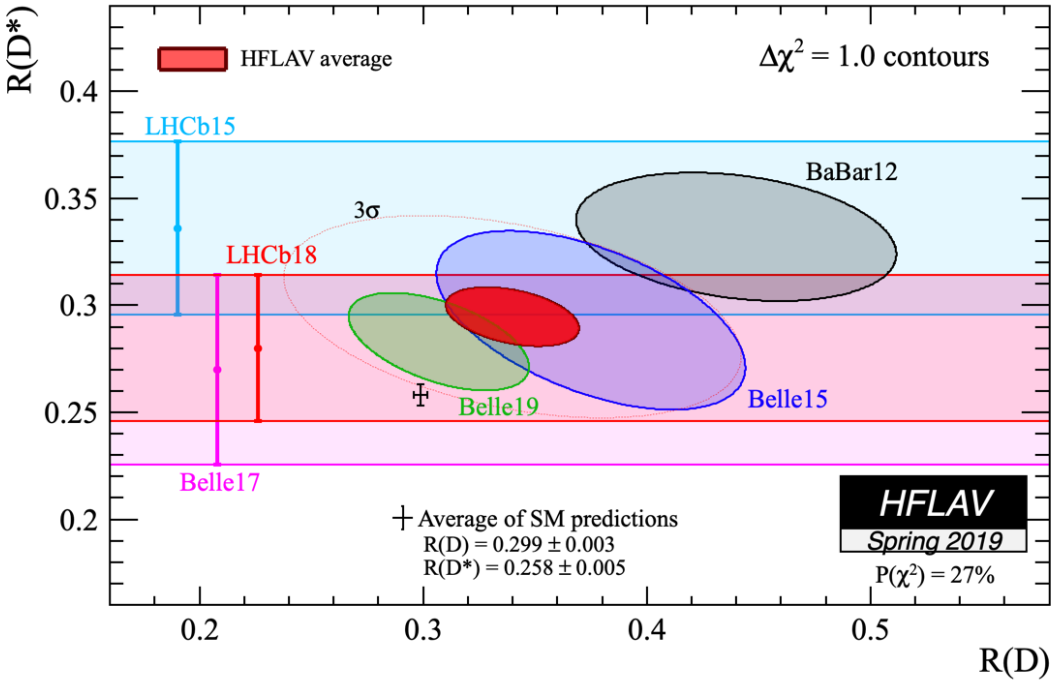
- LFU ratios do not depend on the CKM matrix elements
- Have reduced dependence on form factors
- can be predicted in the SM with high precision

$$R_D^{\text{SM}} = 0.299 \pm 0.003 \quad , \quad R_{D^*}^{\text{SM}} = 0.257 \pm 0.003$$

Bernlochner, Ligeti, Papucci, Robinson 1703.05330

The $R_{D^{(*)}}$ Anomalies

world average from the heavy flavor averaging group



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$\ell = \mu, e$ (BaBar/Belle)
$\ell = \mu$ (LHCb)

$$R_D^{\text{exp}} = 0.340 \pm 0.027 \pm 0.013 \quad , \qquad R_{D^*}^{\text{exp}} = 0.295 \pm 0.011 \pm 0.008$$

combined discrepancy with the SM of 3.1σ

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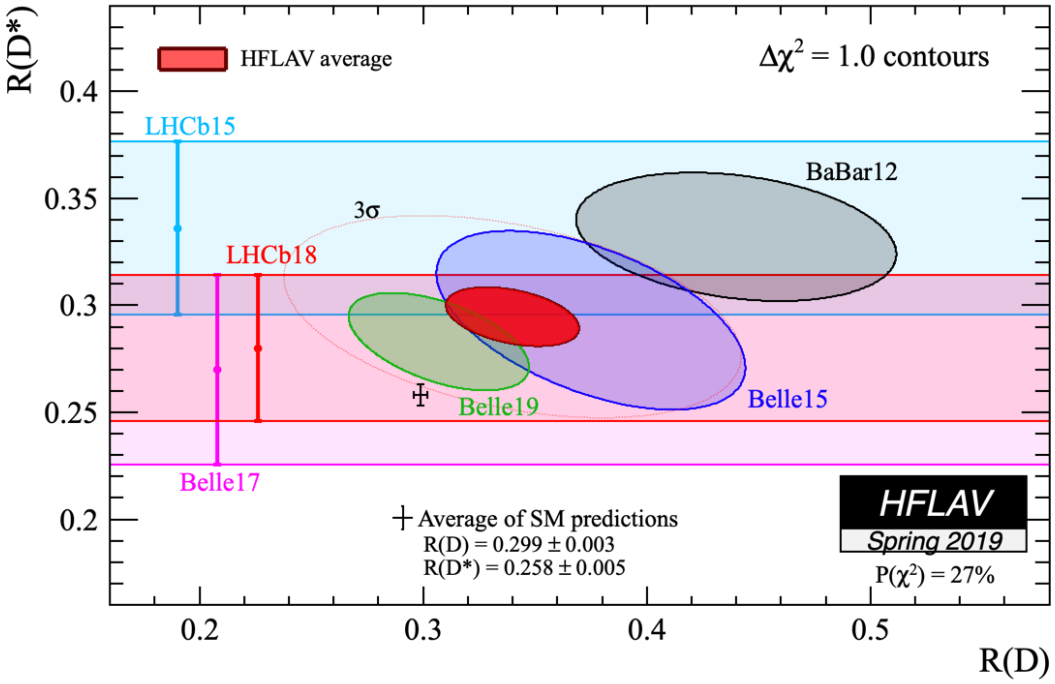

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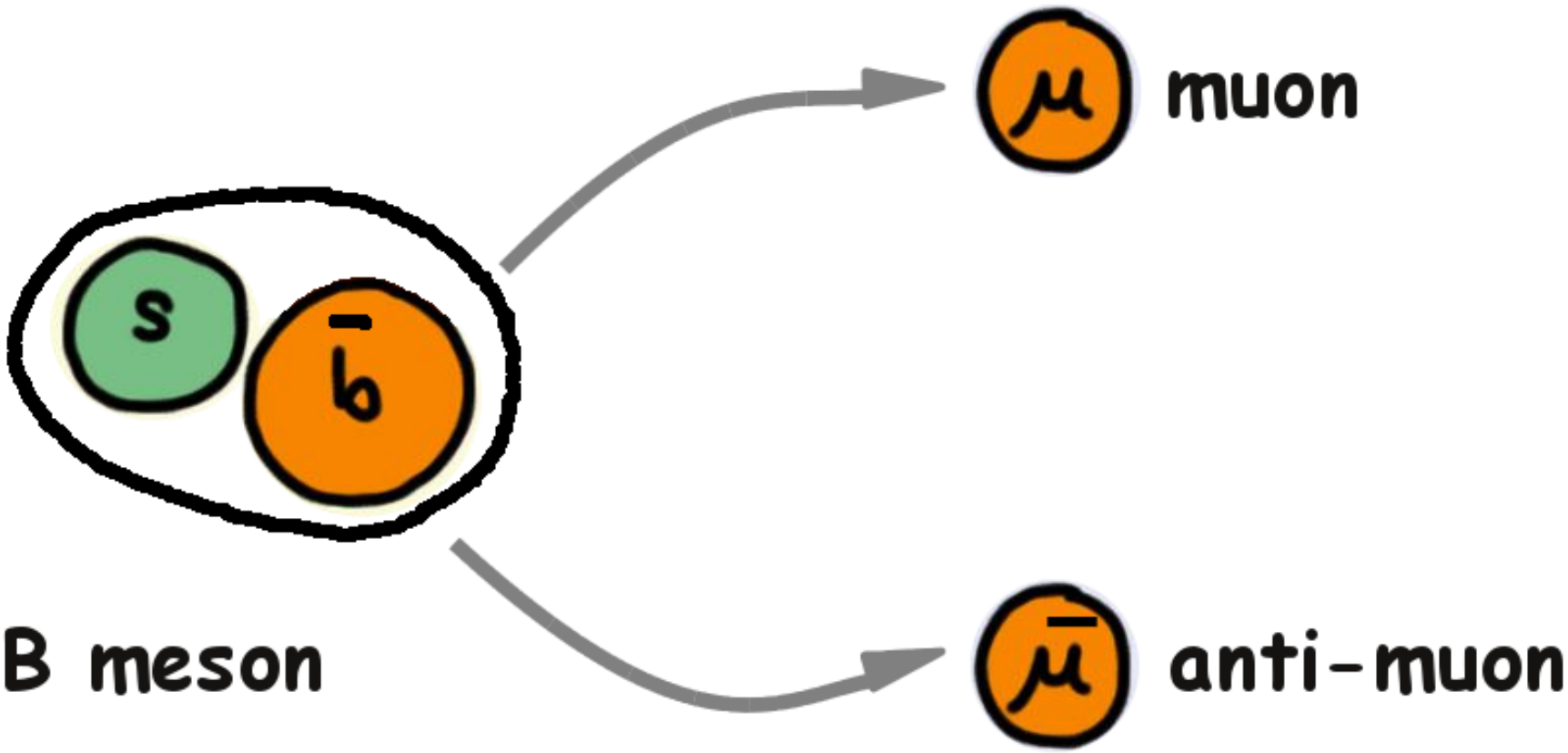
Implications? See talks tomorrow by WA and B. Grinstein.

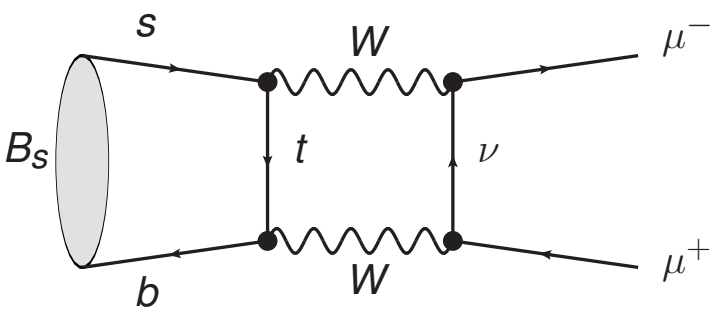
$$B_s \to \mu^+\mu^-$$

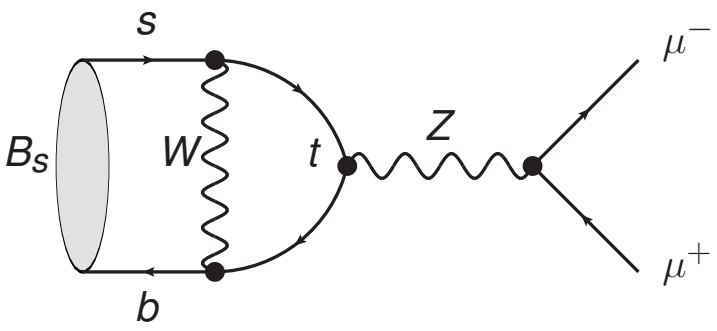
The $B_s \to \mu^+\mu^-$ Decay



SM Contribution

- Flavor changing neutral current process
- induced by Boxes and Z penguins





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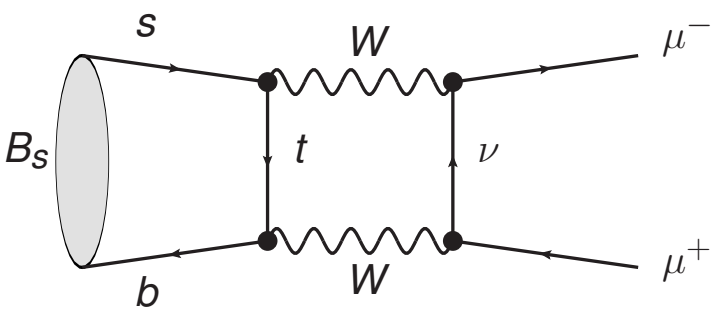
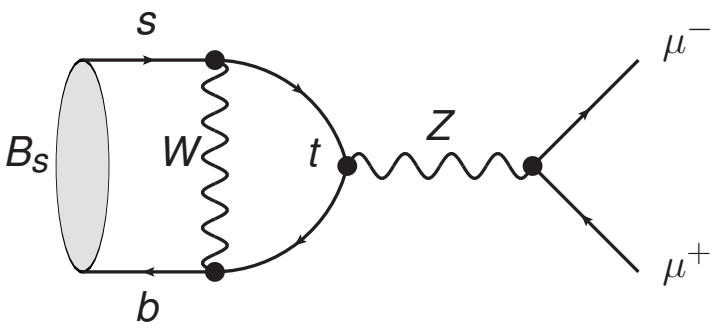


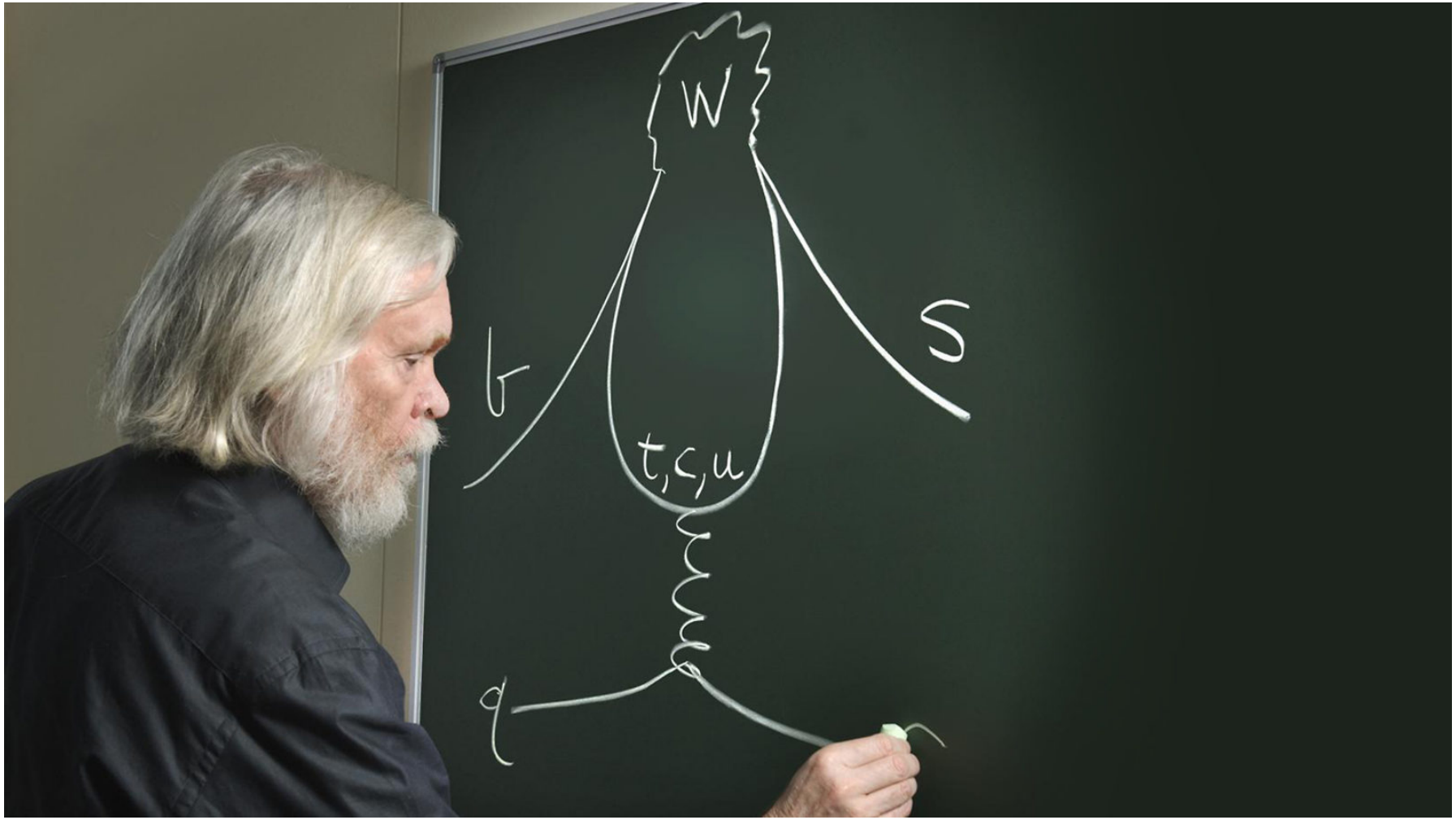


- helicity suppressed decay (similar to pion decay):

  B meson is spin 0, muons spin 1/2
  $\rightarrow$ one muon has to be left-handed, other one right-handed

  electroweak interactions only give muons of the same handedness
  $\rightarrow$ branching ratio is helicity suppressed by m_μ^2/m_B^2

Penguin Diagrams



https://www.symmetrymagazine.org/article/june-2013/the-march-of-the-penguin-diagrams

Effective Hamiltonian for $B_s \to \mu^+\mu^-$ in the SM

- Integrate out the top, W, Z, ... to arrive at an effective Hamiltonian that describes the $B_s \to \mu^+\mu^-$ decay

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \; C_{10} (\bar{s}\gamma_\alpha P_L b)(\bar{\mu}\gamma^\alpha \mu)$$

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- In the SM there is a single Wilson coefficient that is relevant

$$C_{10} = \frac{1}{s_W^2} Y(x_t) = \frac{1}{s_W^2} \left(Y_0(x_t) + \frac{\alpha_s}{4\pi} Y_1(x_t) + \dots \right)$$

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- s_W is the sine of the weak mixing angle
- Y_0 and Y_1 are loop functions that depend on $x_t = m_t^2/m_W^2$
- known at NNLO in QCD and NLO in the electroweak interactions

The Hadronic Matrix Element

$$\langle \mu^+\mu^- | \mathcal{H}_{\text{eff}} | B_s \rangle = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \; C_{10} \langle \mu^+\mu^- | (\bar{\mu}\gamma^\alpha \mu) | 0 \rangle \langle 0 | (\bar{s}\gamma_\alpha P_L b) | B_s \rangle$$

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- Hadronic matrix element is given by the B_s meson decay constant

$$\langle 0 | (\bar{s}\gamma^\alpha b) | B_s \rangle = 0$$

$$\langle 0 | (\bar{s}\gamma^\alpha \gamma_5 b) | B_s \rangle = i f_{B_s} p_{B_s}^\alpha$$

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- decay constants can be determined on the lattice

$f_{B_s} = (230.3 \pm 1.3)\text{MeV}$, $f_{B_d} = (190.0 \pm 1.3)\text{MeV}$ (FLAG 1902.08191)

percent level precision!

Branching Ratio Prediction

decay constant

Effect of lifetime difference of Bs and Bs-bar

$$BR(B_s \to \mu^+\mu^-)_{\rm SM} = \tau_{B_s} \frac{G_F^2}{\pi} m_{B_s} f_{B_s}^2 m_\mu^2 \frac{\alpha^2}{16\pi^2} |V_{ts}^* V_{tb}|^2 \frac{Y(x_t)^2}{s_W^4} \frac{1}{1-y_s}$$

helicity suppression

loop suppression

CKM suppression

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helicity suppression

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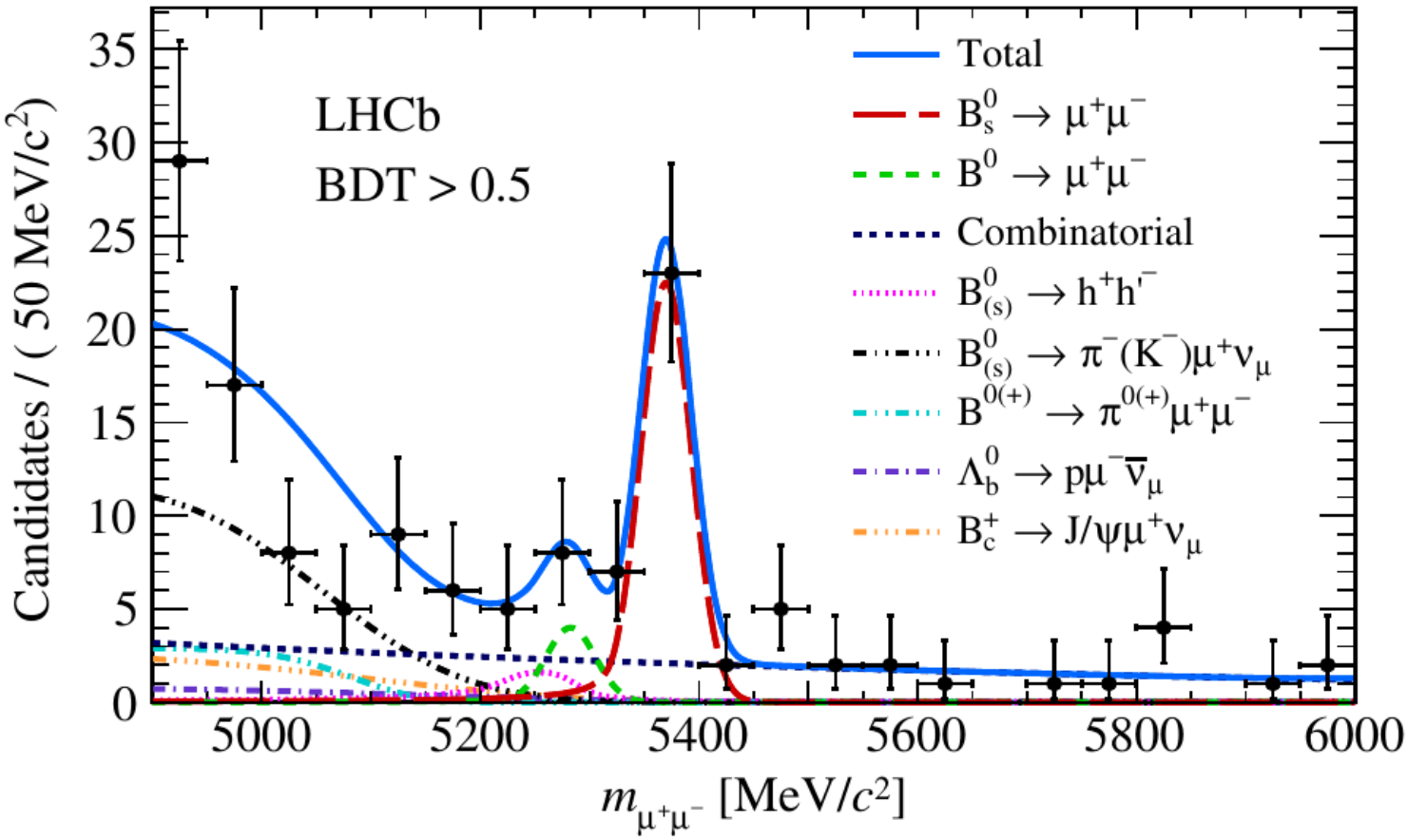
CKM suppression

$$BR(B_s \to \mu^+\mu^-)_{\rm SM} = (3.65 \pm 0.23) \times 10^{-9}$$ (Bobeth et al. 1311.0903)

a truly rare decay!

Observation of $B_s \to \mu^+\mu^-$

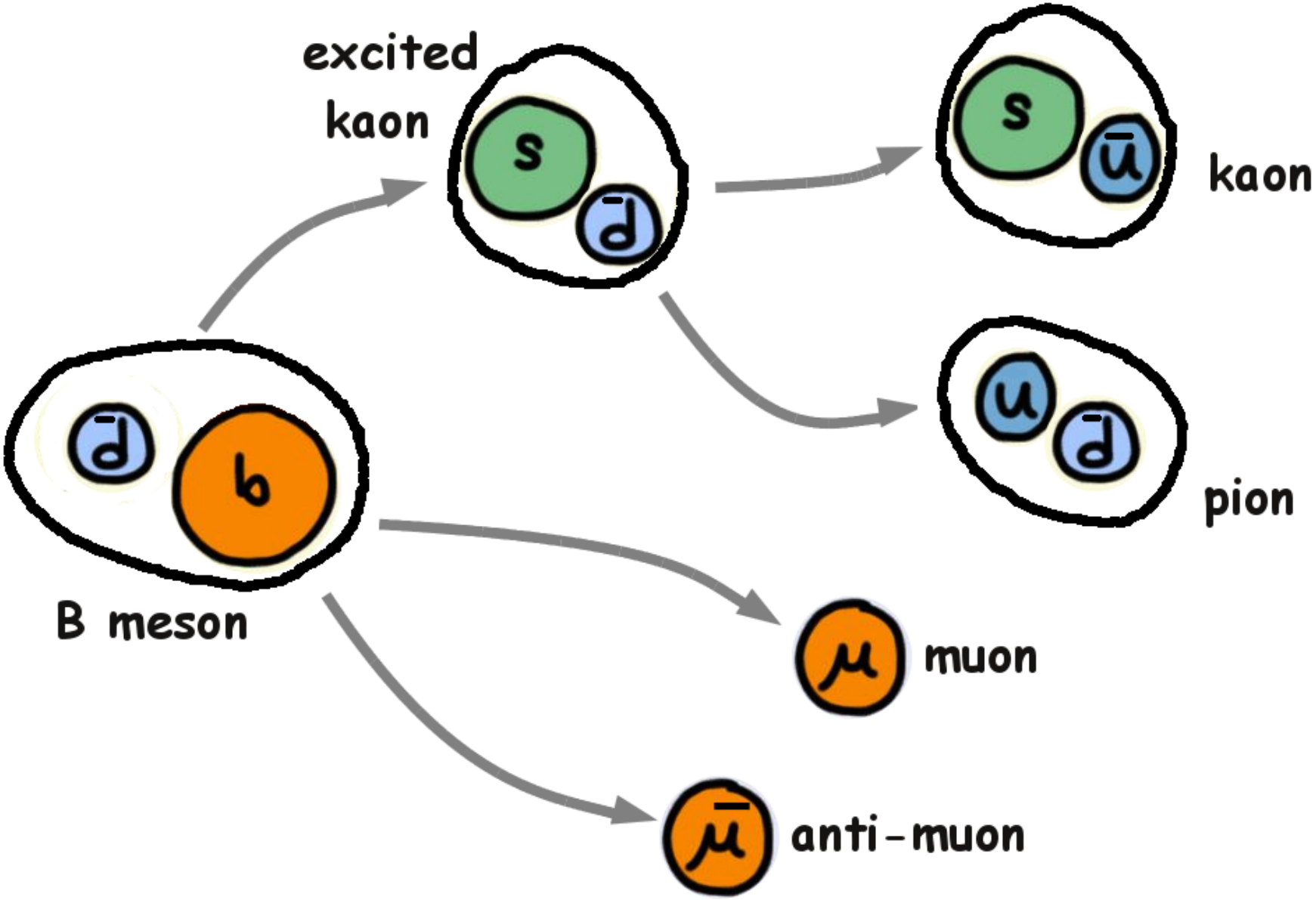


$$BR(B_s \to \mu^+\mu^-)_{\text{exp}} = (3.0 \pm 0.6^{+0.3}_{-0.2}) \times 10^{-9}$$ (LHCb 1703.05747)

Slightly on the low side of the SM prediction ...

$B \to K^* \ell^+ \ell^-$ and $R_{K^{(*)}}$

The $B \to K^*(\to K\pi)\mu^+\mu^-$ Decay



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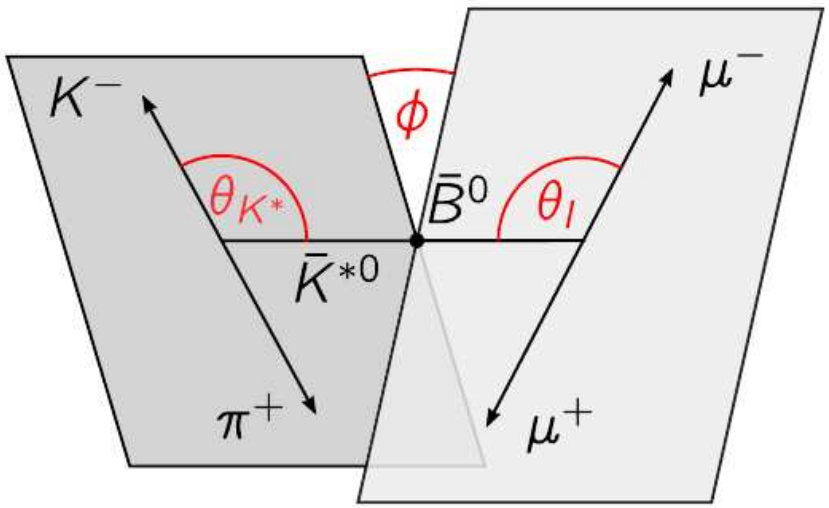
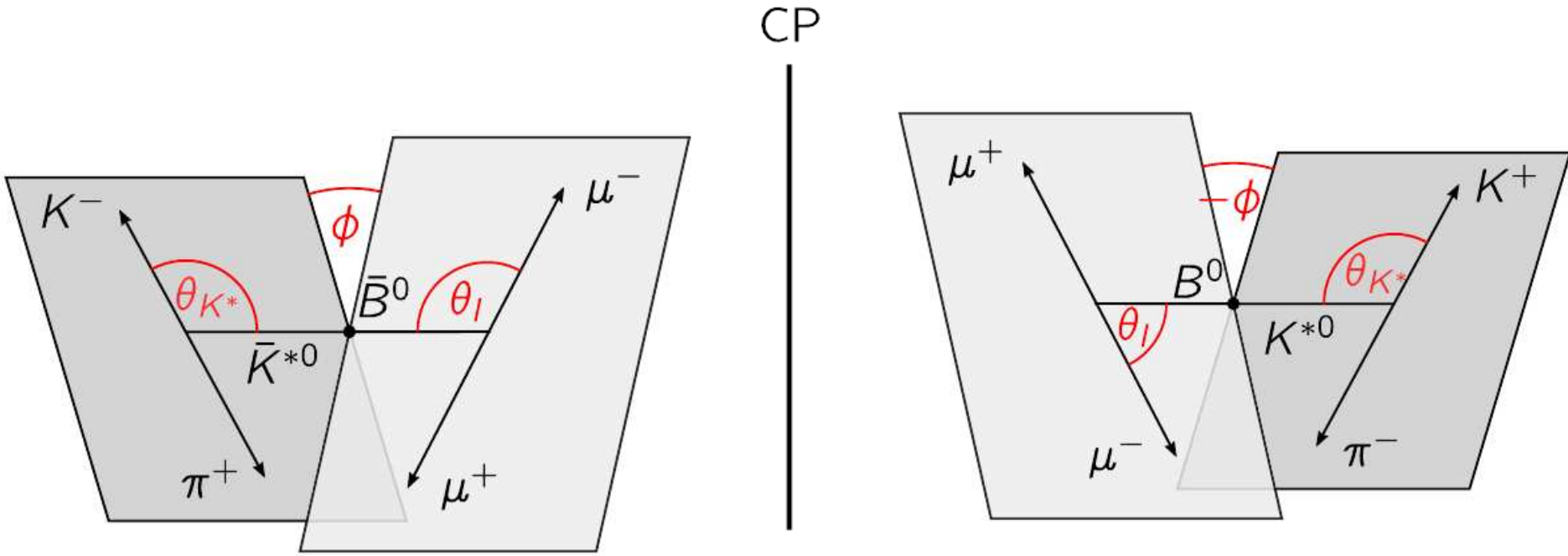


- kinematics described by 4 variables

  invariant mass squared of the two muons: q^2
  three angles: $0 < \theta_{K^*} < \pi$, $0 < \theta_\ell < \pi$, $-\pi < \phi < \pi$

  $\to$ many observables accessible from the angular distribution

The $B \to K^*(\to K\pi)\mu^+\mu^-$ Decay

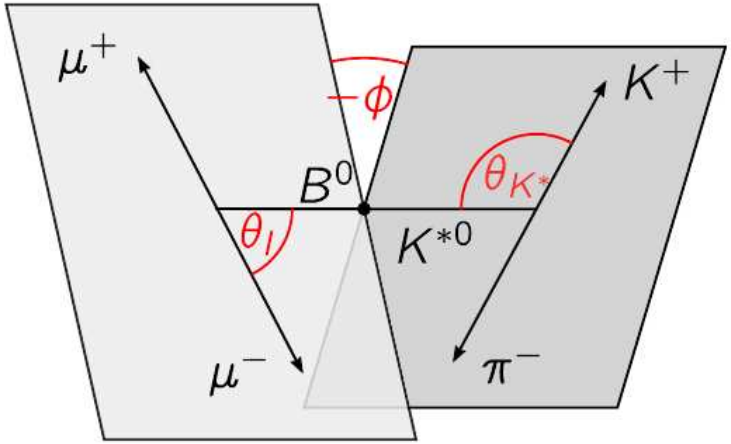


- self tagging:

  $K^+\pi^-$ final state for B^0
  $K^-\pi^+$ final state for $\bar{B}^0$

  $\to$ in principle easy access to CP asymmetries

The $B \to K^* \mu^+ \mu^-$ Angular Decay Distribution



$$\frac{d^4\bar{\Gamma}}{dq^2\, d\cos\theta_\ell\, d\cos\theta_{K^*}\, d\phi} =$$

$$= \frac{9}{32\pi}\bar{I}(q^2, \theta_\ell, \theta_{K^*}, \phi)$$

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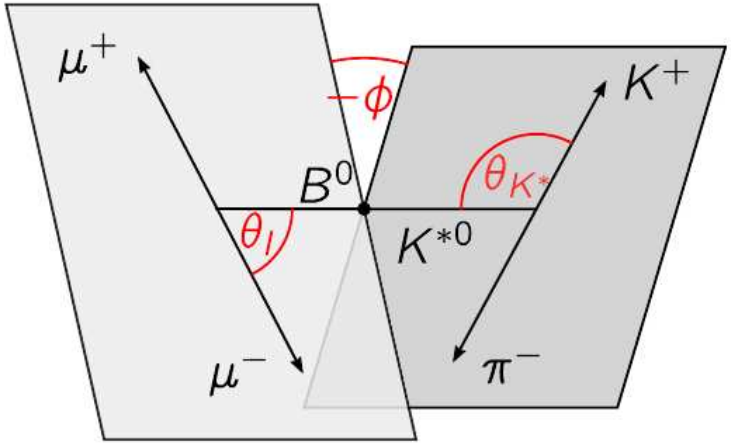

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$$\begin{aligned}
&\bar{I}(q^2, \theta_\ell, \theta_{K^*}, \phi) = \\
&= \bar{I}_1^s \sin^2\theta_{K^*} + \bar{I}_1^c \cos^2\theta_{K^*} + (\bar{I}_2^s \sin^2\theta_{K^*} + \bar{I}_2^c \cos^2\theta_{K^*}) \cos 2\theta_\ell \\
&+ \bar{I}_3 \sin^2\theta_{K^*} \sin^2\theta_\ell \cos 2\phi + \bar{I}_4 \sin 2\theta_{K^*} \sin 2\theta_\ell \cos\phi \\
&- \bar{I}_5 \sin 2\theta_{K^*} \sin\theta_\ell \cos\phi \\
&- (\bar{I}_6^s \sin^2\theta_{K^*} + \bar{I}_6^c \cos^2\theta_{K^*}) \cos\theta_\ell + \bar{I}_7 \sin 2\theta_{K^*} \sin\theta_\ell \sin\phi \\
&- \bar{I}_8 \sin 2\theta_{K^*} \sin 2\theta_\ell \sin\phi - \bar{I}_9 \sin^2\theta_{K^*} \sin^2\theta_\ell \sin 2\phi
\end{aligned}$$

The I's are moments of the angular distribution.

Effective Hamiltonian for $B \to K^* \ell^+ \ell^-$ in the SM

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_{i=7,9,10} C_i \mathcal{O}_i + \ldots$$

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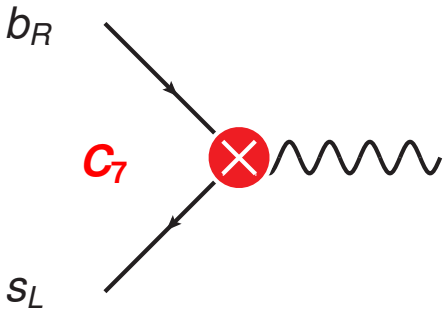
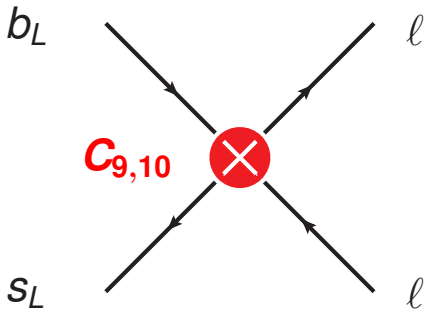
magnetic dipole operators



$C_7 (\bar{s} \sigma_{\mu\nu} P_R b) F^{\mu\nu}$

semileptonic operators



$C_9 (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu \ell)$

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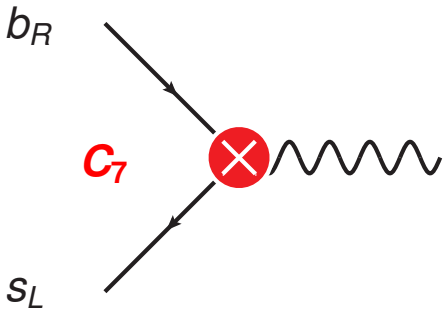
magnetic dipole operators



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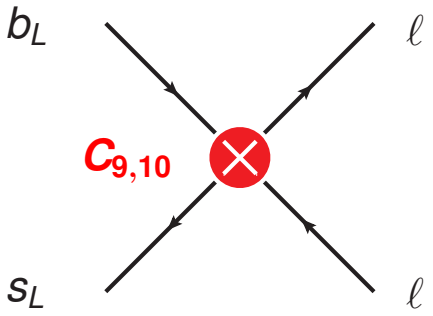

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$C_{10} (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\mu \gamma_5 \ell)$

(We'll come back to the dots in a couple of slides)

Form Factors

Hadronic matrix elements are parameterized in terms of form factors

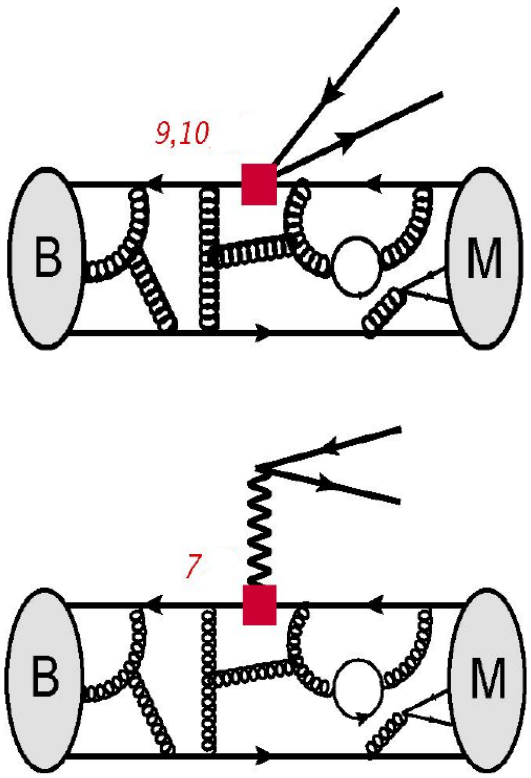
$$\langle \bar{K}^*(k)|\bar{s}\gamma_\mu(1-\gamma_5)b|\bar{B}(p)\rangle = -i\epsilon^*_\mu(m_B+m_{K^*})A_1(q^2) + i(2p-q)_\mu(\epsilon^*\cdot q)\,\frac{A_2(q^2)}{m_B+m_{K^*}}$$
$$+\, iq_\mu(\epsilon^*\cdot q)\,\frac{2m_{K^*}}{q^2}\,\left[A_3(q^2)-A_0(q^2)\right] + \epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho k^\sigma\,\frac{2V(q^2)}{m_B+m_{K^*}}$$

$$\langle \bar{K}^*(k)|\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|\bar{B}(p)\rangle = i\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho k^\sigma\, 2T_1(q^2)$$
$$+\, T_2(q^2)\left[\epsilon^*_\mu(m_B^2-m_{K^*}^2) - (\epsilon^*\cdot q)\,(2p-q)_\mu\right] + T_3(q^2)(\epsilon^*\cdot q)\left[q_\mu - \frac{q^2}{m_B^2-m_{K^*}^2}\,(2p-q)_\mu\right]$$

Predictions exist from lattice QCD and
other non-perturbative methods (light cone sum rules)

Non-Factorizable Effects

So far we discussed the “factorizable” contributions

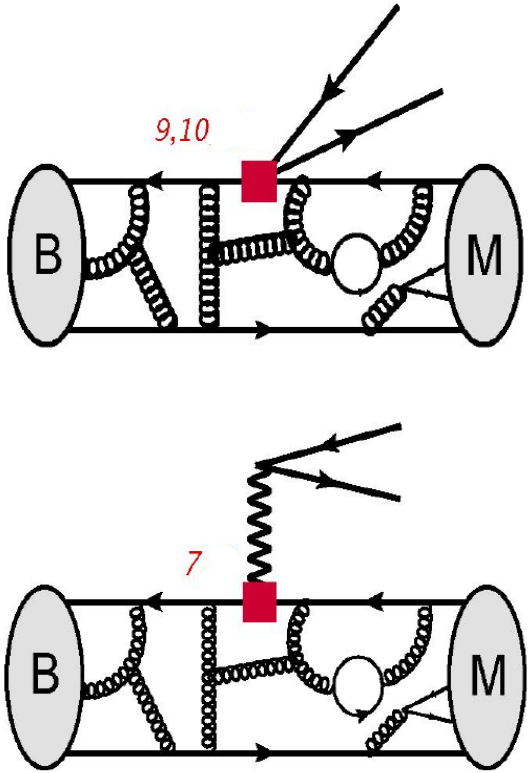


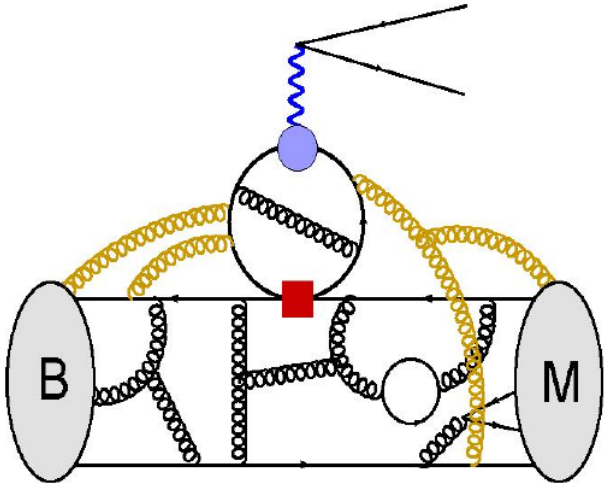
(illustrations by Danny van Dyk)

Non-Factorizable Effects

So far we discussed the “factorizable” contributions

there are also non-factorizable effects coming from 4-quark operators



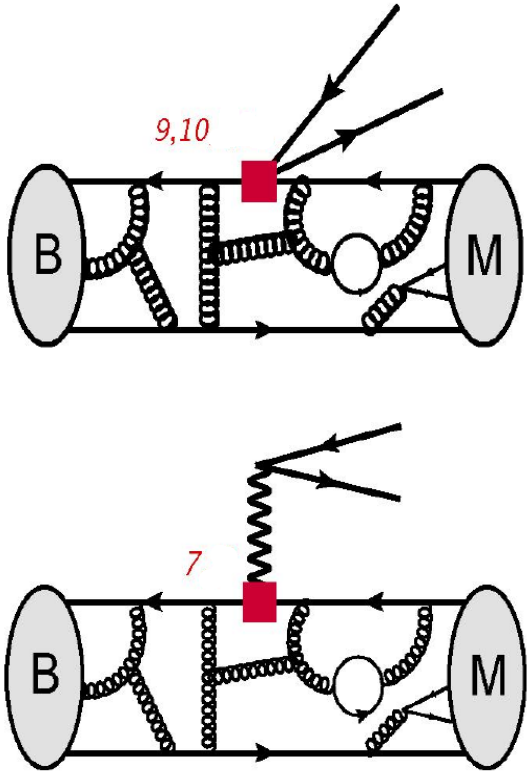


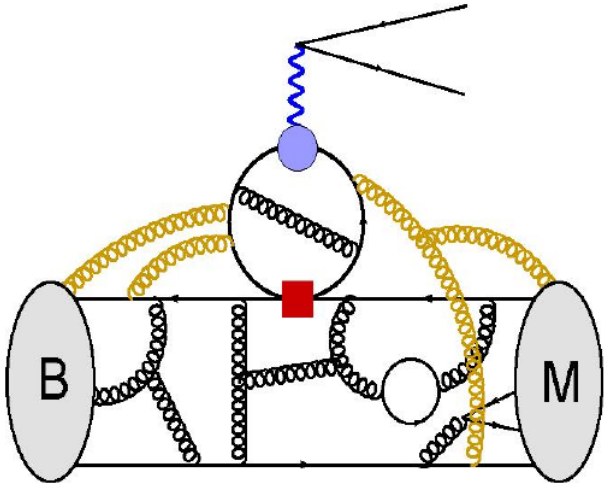
(illustrations by Danny van Dyk)

Non-Factorizable Effects

So far we discussed the “factorizable” contributions

there are also non-factorizable effects coming from 4-quark operators

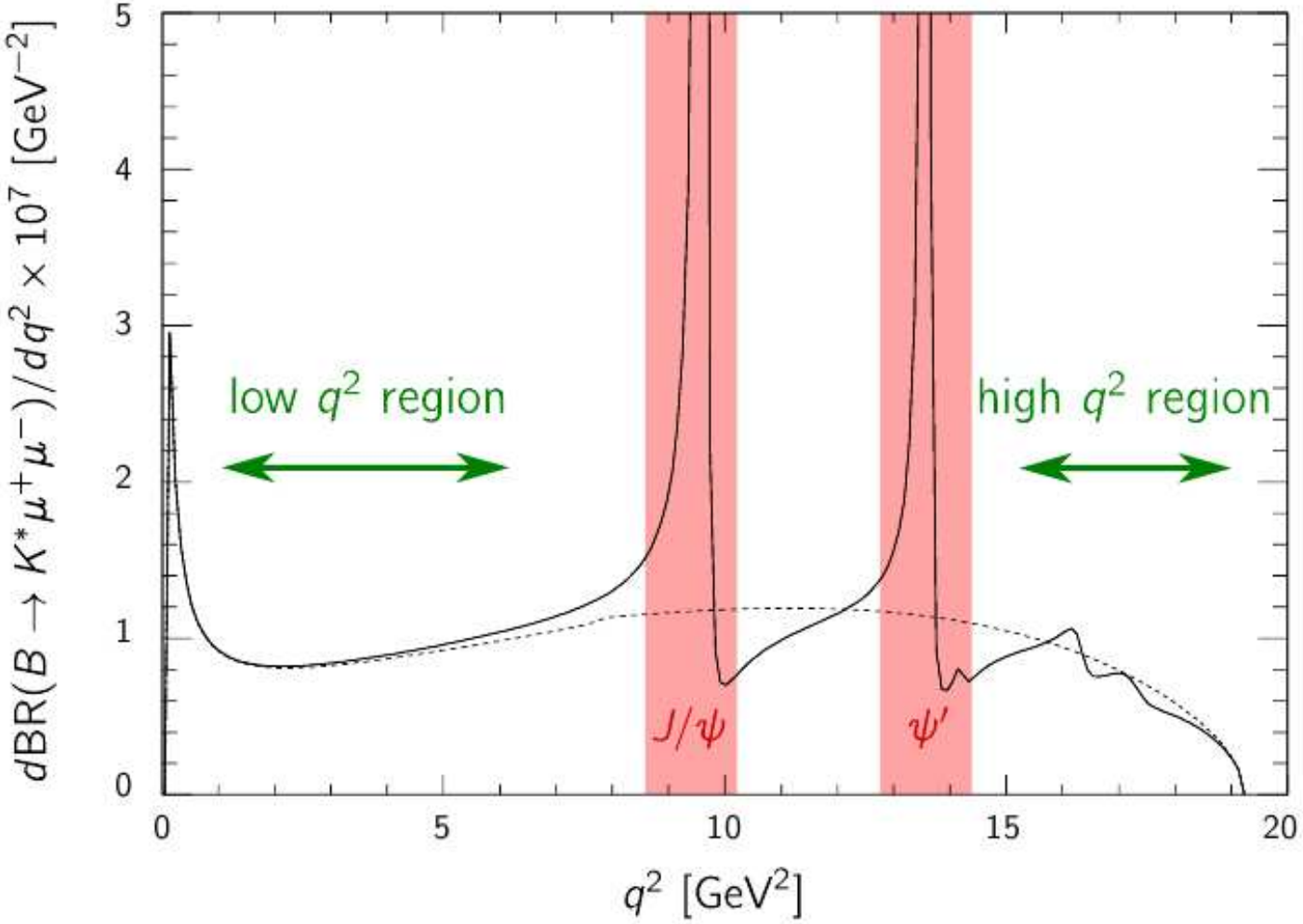




(illustrations by Danny van Dyk)

Size the of the non-factorizable effects is not very well understood.
Major source of uncertainty in the theory predictions.

The q^2 Spectrum



Angular Observables

take sums and differences of the angular moments
and normalize to the differential decay width

- CP-Asymmetries

$$A_i^{(a)}(q^2) = \left(I_i^{(a)}(q^2) - \bar{I}_i^{(a)}(q^2)\right) \Big/ \frac{d\left(\Gamma + \bar{\Gamma}\right)}{dq^2}$$

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$$P_4' = \frac{2S_4}{\sqrt{F_L(1-F_L)}} \quad , \quad P_5' = \frac{S_5}{\sqrt{F_L(1-F_L)}}$$

$$P_6'^{\mathrm{CP}} = \frac{-A_7}{\sqrt{F_L(1-F_L)}} \quad , \quad P_8'^{\mathrm{CP}} = \frac{-2A_8}{\sqrt{F_L(1-F_L)}}$$

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- minimize dependence on formfactors
- normalize such that the form factors cancel out exactly in the heavy quark limit

  Descotes-Genon, Hurth, Matias, Virto 1303.5794; Matias, Mescia, Ramon, Virto 1202.4266;

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- still subject to factorizable and non-factorizable power corrections
- P_5' will return in tomorrow’s talk

Lepton Flavor Universality Ratios

- Hadronic uncertainties drop out almost entirely in lepton flavor universality ratios

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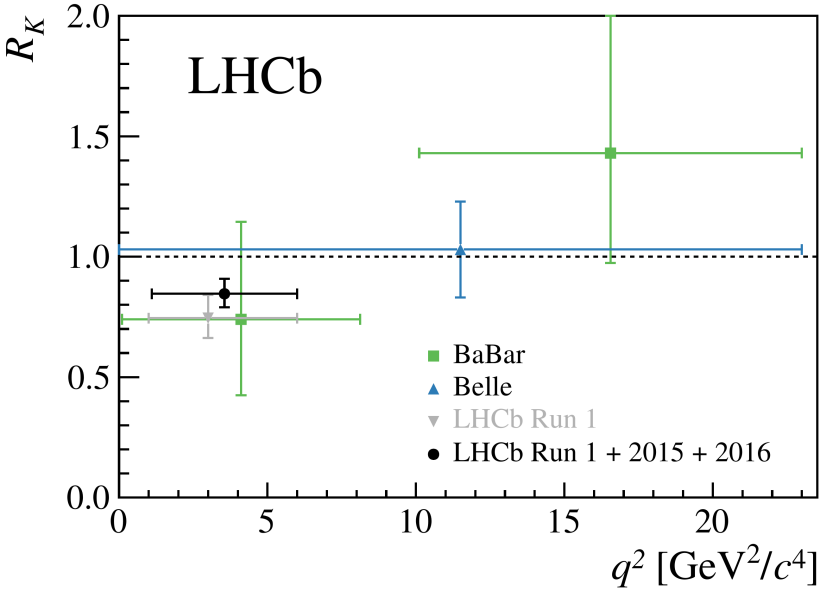
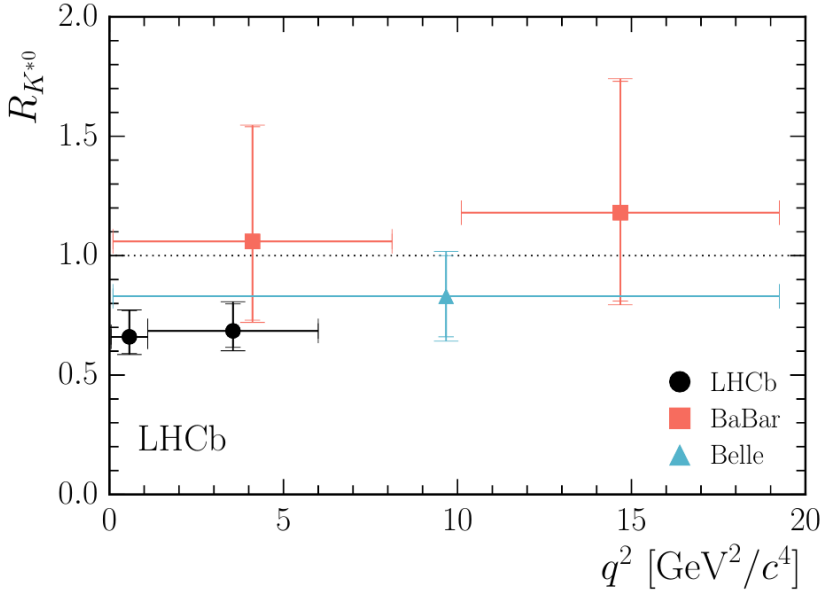
- Standard Model Predictions Bordone, Isidori, Pattori 1605.07633

$$R_K^{[1,6]} = 1.00 \pm 0.01 \ , \quad R_{K^*}^{[1.1,6]} = 1.00 \pm 0.01 \ , \quad R_{K^*}^{[0.045,1.1]} = 0.91 \pm 0.03$$

(The numbers in square brackets indicate the q^2 region)

The $R_{K^{(*)}}$ Anomalies





$$R_K^{[1,6]} = 0.846^{+0.060\,+0.016}_{-0.054\,-0.014}$$

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3 observables
deviating by $\sim 2\sigma - 2.5\sigma$
from the SM predictions

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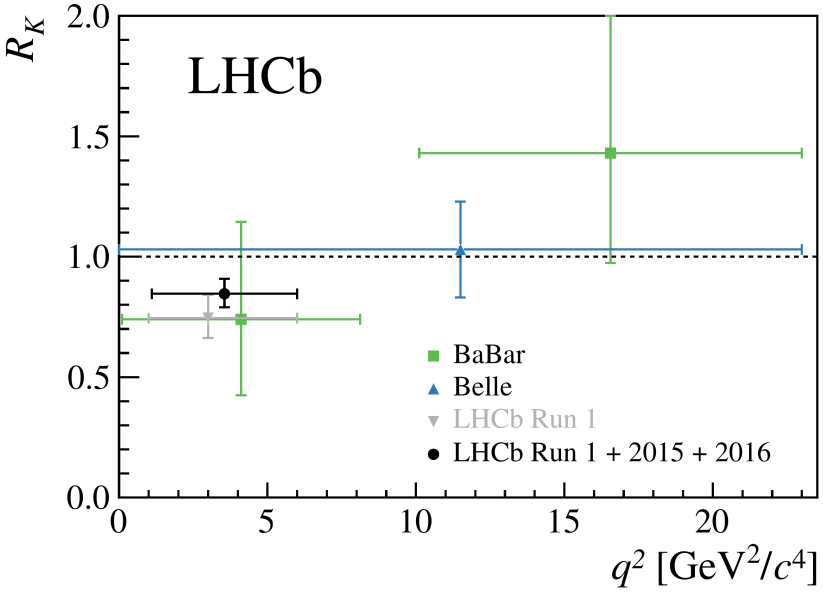

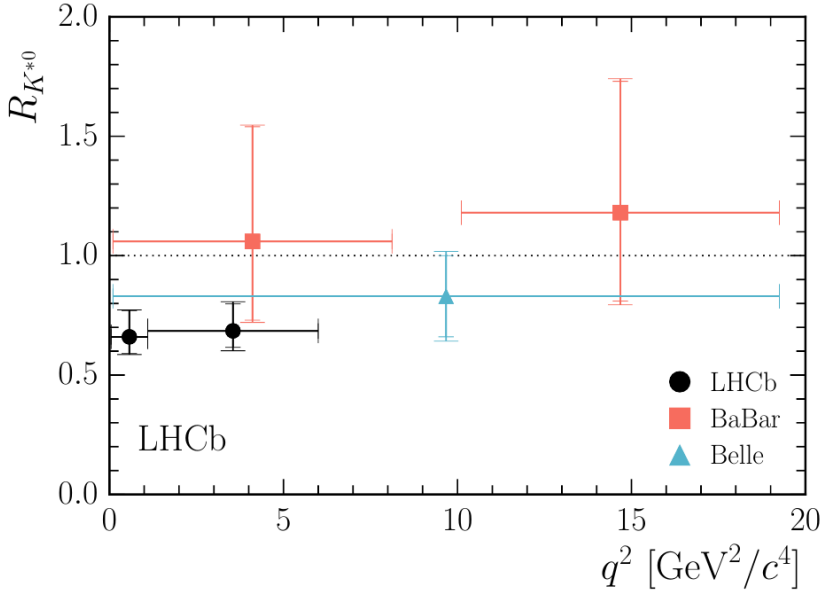

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Implications?
See talks tomorrow
by WA and B. Grinstein

Summary

- B decays are a rich laboratory for testing the flavor structure of the Standard Model and extensions
- Set of intriguing discrepancies in the current data:

  What are the implications?