Kaon mixing and CP violation

Ulrich Nierste

Karlsruhe Institute of Technology
Institute for Theoretical Particle Physics

Karlsruhe Institute of Technology

SLAC Summer Institute 2019:
Menu of Flavors: Quarks, Charged Leptons & Neutrinos
14 Aug 2019

Some flavored mesons:
charged:

$$K^+ \sim \overline{s}u, \quad D^+ \sim c\overline{d}, \quad D_s^+ \sim c\overline{s}, \quad B^+ \sim \overline{b}u, \quad B_c^+ \sim \overline{b}c,$$
$$K^- \sim s\overline{u}, \quad D^- \sim \overline{c}d, \quad D_s^- \sim \overline{c}s, \quad B^- \sim b\overline{u}, \quad B_c^- \sim b\overline{c},$$

neutral:

$$K \sim \overline{s}d, \qquad D \sim c\overline{u}, \qquad B_d \sim \overline{b}d, \qquad B_s \sim \overline{b}s,$$
$$\overline{K} \sim s\overline{d}, \qquad \overline{D} \sim \overline{c}u, \qquad \overline{B}_d \sim b\overline{d}, \qquad \overline{B}_s \sim b\overline{s},$$

In flavor physics only the ground-state hadrons which decay weakly rather than strongly are interesting.
Weakly decaying baryons are less interesting, because they are produced in smaller rates and are theoretically harder to cope with.

Appetizers: Why Kaons?

Some flavored mesons:
charged:

$$K^+ \sim \bar{s}u, \quad D^+ \sim c\bar{d}, \quad D_s^+ \sim c\bar{s}, \quad B^+ \sim \bar{b}u, \quad B_c^+ \sim \bar{b}c,$$
$$K^- \sim s\bar{u}, \quad D^- \sim \bar{c}d, \quad D_s^- \sim \bar{c}s, \quad B^- \sim b\bar{u}, \quad B_c^- \sim b\bar{c},$$

neutral:

$$K \sim \bar{s}d, \qquad D \sim c\bar{u}, \qquad B_d \sim \bar{b}d, \qquad B_s \sim \bar{b}s,$$
$$\bar{K} \sim s\bar{d}, \qquad \bar{D} \sim \bar{c}u, \qquad \bar{B}_d \sim b\bar{d}, \qquad \bar{B}_s \sim b\bar{s},$$

The neutral K, D, B_d and B_s mesons mix with their antiparticles, $\bar{K}$, $\bar{D}$, $\bar{B}_d$ and $\bar{B}_s$ thanks to the weak interaction (quantum-mechanical two-state systems).

Appetizers: Why Kaons?

Some flavored mesons:

charged:

$$K^+ \sim \bar{s}u, \quad D^+ \sim c\bar{d}, \quad D_s^+ \sim c\bar{s}, \quad B^+ \sim \bar{b}u, \quad B_c^+ \sim \bar{b}c,$$
$$K^- \sim s\bar{u}, \quad D^- \sim \bar{c}d, \quad D_s^- \sim \bar{c}s, \quad B^- \sim b\bar{u}, \quad B_c^- \sim b\bar{c},$$

neutral:

$$K \sim \bar{s}d, \qquad D \sim c\bar{u}, \qquad B_d \sim \bar{b}d, \qquad B_s \sim \bar{b}s,$$
$$\bar{K} \sim s\bar{d}, \qquad \bar{D} \sim \bar{c}u, \qquad \bar{B}_d \sim b\bar{d}, \qquad \bar{B}_s \sim b\bar{s},$$

The neutral K, D, B_d and B_s mesons mix with their antiparticles, $\bar{K}$, $\bar{D}$, $\bar{B}_d$ and $\bar{B}_s$ thanks to the weak interaction (quantum-mechanical two-state systems).

$\Rightarrow$ gold mine for fundamental parameters

This talk: Kaons

Kaons have a mass of 0.5 GeV.

How can a particle decay at such a low energy be interesting to probe physics beyond the Standard Model (SM)?

This talk: Kaons

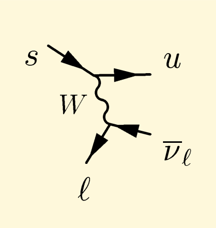
Kaons have a mass of 0.5 GeV.

How can a particle decay at such a low energy be interesting to probe physics beyond the Standard Model (SM)?

SM: Weak decay amplitude $\mathcal{A}$ of any hadron is suppressed by $G_F \propto 1/M_W^2$

New physics with particle mass $M_{\rm NP} \gg M_W$ contributes $1/M_{\rm NP}^2$ to $\mathcal{A}$:

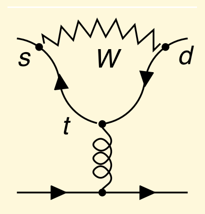
$$\mathcal{A} = \mathcal{A}_{\rm SM}\left[1 + \left(\frac{M_W^2}{M_{\rm NP}^2}\right)\right]$$

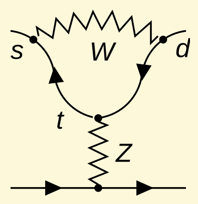


$\Rightarrow$ The game is even between B, D, and K!

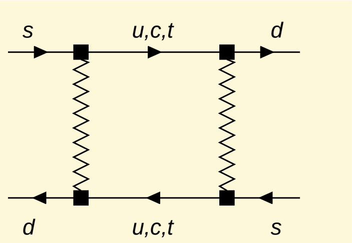
Flavor-changing neutral current (FCNC) processes:

$\Delta|S| = 1$ transitions (Kaon decays):





$\Delta|S| = 2$ transitions (Kaon mixing):



S is the strangeness:

s quark, $\overline{K}^0 \sim s\bar{d}$, $K^- \sim s\bar{u}$ have $S = -1$

$\bar{s}$ quark, $K^0 \sim \bar{s}d$, $K^+ \sim \bar{s}u$ have $S = +1$

GIM

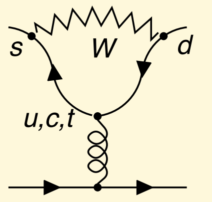
Consider $|\Delta S| = 1$ FCNC amplitude: internal quark can be $u, c,$ or t.

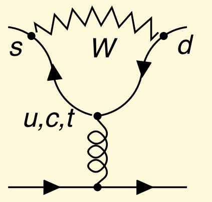
$$\mathcal{A} = \sum_{j=u,c,t} V_{js}^* V_{jd} \, f\left(\frac{m_j^2}{M_W^2}\right)$$
$$= V_{cs}^* V_{cd} \left[f\left(\frac{m_c^2}{M_W^2}\right) - f\left(\frac{m_u^2}{M_W^2}\right)\right] + V_{ts}^* V_{td} \left[f\left(\frac{m_t^2}{M_W^2}\right) - f\left(\frac{m_u^2}{M_W^2}\right)\right]$$

Here $V_{js}^* V_{jd} f\left(\frac{m_t^2}{M_W^2}\right)$ denotes the result of the loop diagram with internal quark j.

Essential: Unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix:

$$V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0$$
$$\Rightarrow \quad V_{cs}^* V_{cd} = -V_{us}^* V_{ud} - V_{ts}^* V_{td}$$





Set $m_u = 0$ and expand in m_c^2/M_W^2 to first order:

$$f\left(\frac{m_c^2}{M_W^2}\right) - f\left(\frac{m_u^2}{M_W^2}\right) = A\,\frac{m_c^2}{M_W^2}\,\log\frac{m_c^2}{M_W^2} + B\,\frac{m_c^2}{M_W^2} + \ldots$$

$$\begin{aligned}\mathcal{A} &= V_{cs}^* V_{cd}\left[f\left(\frac{m_c^2}{M_W^2}\right) - f\left(\frac{m_u^2}{M_W^2}\right)\right] + V_{ts}^* V_{td}\left[f\left(\frac{m_t^2}{M_W^2}\right) - f\left(\frac{m_u^2}{M_W^2}\right)\right] \\ &\simeq V_{cs}^* V_{cd}\,\frac{m_c^2}{M_W^2}\left[A\,\log\frac{m_c^2}{M_W^2} + B\right] + V_{ts}^* V_{td}\left[f\left(\frac{m_t^2}{M_W^2}\right) - f\,(0)\right]\end{aligned}$$

GIM

$$\mathcal{A} \simeq \underbrace{V_{cs}^* V_{cd}}_{\mathcal{O}(-0.2)} \underbrace{\frac{m_c^2}{M_W^2}\left[A \log \frac{m_c^2}{M_W^2} + B\right]}_{\mathcal{O}(-0.002)} + \underbrace{V_{ts}^* V_{td}}_{\mathcal{O}(3\cdot 10^{-4})} \underbrace{\left[f\left(\frac{m_t^2}{M_W^2}\right) - f(0)\right]}_{\mathcal{O}(1)}$$

Glashow-Iliopoulos-Maiani (GIM) suppression of the CKM-favored term

Both terms are similar in size.

$\Rightarrow$ Kaon FCNCs are sensitive to virtual top quark effects.

CP violating $s \to d$ transitions involve $\mathrm{Im}\, V_{ts}^* V_{td} = 1.5 \cdot 10^{-4}$

Compare this with $\bar{b} \to \bar{d}$ transition: $\mathrm{Im}\, V_{tb}^* V_{td} = -3.6 \cdot 10^{-3}$

... or $\bar{b} \to \bar{s}$ transition: $\mathrm{Im}\, V_{tb}^* V_{ts} = -8.2 \cdot 10^{-4}$

CP violating $s \to d$ transitions involve $\mathrm{Im}\, V_{ts}^* V_{td} = 1.5 \cdot 10^{-4}$

Compare this with $\bar{b} \to \bar{d}$ transition: $\mathrm{Im}\, V_{tb}^* V_{td} = -3.6 \cdot 10^{-3}$

…or $\bar{b} \to \bar{s}$ transition: $\mathrm{Im}\, V_{tb}^* V_{ts} = -8.2 \cdot 10^{-4}$

$\Rightarrow$ Kaon FCNCs are best to probe NP with generic flavor structure!

“Generic” means “unrelated to CKM mechanism of the SM”.

Opposite concept: “minimally flavor violating (MFV)” meaning “governed by CKM mechanism”.

Parity transformation P: $\vec{x} \to -\vec{x}$

Charge conjugation C: Exchange particles and antiparticles, e.g. $e^- \leftrightarrow e^+$

Time reversal T: $t \to -t$

1954/1955: CPT is a symmetry of every Lorentz-invariant quantum field theory.

1954/1955: CPT is a symmetry of every Lorentz-invariant quantum field theory.

1956/1957: P is not a symmetry of the microscopic laws of nature!

C and P

1954/1955:	CPT is a symmetry of every Lorentz-invariant quantum field theory.
1956/1957:	P is not a symmetry of the microscopic laws of nature!
1964:	CP is not a symmetry of the microscopic laws of nature!

1954/1955:	CPT is a symmetry of every Lorentz-invariant quantum field theory.
1956/1957:	P is not a symmetry of the microscopic laws of nature!
1964:	CP is not a symmetry of the microscopic laws of nature!
$\Rightarrow$	Also the T symmetry must be violated, there is a microscopic arrow of time!

1973: Explanation of CP violation by postulating
a third fermion generation.

K and M

1973: Explanation of CP violation by postulating
a third fermion generation.

Makoto Kobayashi and Toshihide Maskawa:
CP Violation in the Renormalizable Theory of Weak Interaction,
Prog.Theor.Phys.49:652-657,1973,

1973: Explanation of CP violation by postulating a third fermion generation.

Makoto Kobayashi and Toshihide Maskawa:
CP Violation in the Renormalizable Theory of Weak Interaction,
Prog.Theor.Phys.49:652-657,1973,
$\sim$9782 citations (ranks 6th in elementary particle physics).

Nobel Prize 2008.

K and M

1973: Explanation of CP violation by postulating a third fermion generation.

Makoto Kobayashi and Toshihide Maskawa:
CP Violation in the Renormalizable Theory of Weak Interaction,
Prog.Theor.Phys.49:652-657,1973,
$\sim$9782 citations (ranks 6th in elementary particle physics).

Nobel Prize 2008.

Strong interaction

The QCD lagrangian permits a term which violates P, CP, and T, but experimentally the corresponding coefficient θ is found to be smaller than 10^{-11}.

The QCD lagrangian permits a term which violates P, CP, and T, but experimentally the corresponding coefficient θ is found to be smaller than 10^{-11}.

$\Rightarrow$ The strong interaction essentially respects C, P, and therefore T,

$$[H_{\mathrm{strong}}, \mathrm{P}] = [H_{\mathrm{strong}}, \mathrm{C}] = [H_{\mathrm{strong}}, \mathrm{T}] = 0$$

$\Rightarrow$ We can assign C and P quantum numbers, which can be $+1$ or -1, to hadrons.

The QCD lagrangian permits a term which violates P, CP, and T, but experimentally the corresponding coefficient θ is found to be smaller than 10^{-11}.

$\Rightarrow$ The strong interaction essentially respects C, P, and therefore T,

$$\left[H_{\text{strong}}, \mathrm{P}\right] = \left[H_{\text{strong}}, \mathrm{C}\right] = \left[H_{\text{strong}}, \mathrm{T}\right] = 0$$

$\Rightarrow$ We can assign C and P quantum numbers, which can be $+1$ or -1, to hadrons.

Example: A π^0 meson has $P = -1$ and $C = +1$. A π^+ has $P = -1$, but is no eigenstate of C, because $C|\pi^+\rangle = |\pi^-\rangle$.

Strong interaction

The QCD lagrangian permits a term which violates P, CP, and T, but experimentally the corresponding coefficient θ is found to be smaller than 10^{-11}.

$\Rightarrow$ The strong interaction essentially respects C, P, and therefore T,

$$\left[H_{\text{strong}}, \mathrm{P}\right] = \left[H_{\text{strong}}, \mathrm{C}\right] = \left[H_{\text{strong}}, \mathrm{T}\right] = 0$$

$\Rightarrow$ We can assign C and P quantum numbers, which can be $+1$ or -1, to hadrons.

Example: A π^0 meson has $P = -1$ and $C = +1$. A π^+ has $P = -1$, but is no eigenstate of C, because $C|\pi^+\rangle = |\pi^-\rangle$.

Also QED respects C,P, and T.

1956: $\theta - \tau$ *puzzle*:
A seemingly degenerate pair (θ, τ) of two mesons with P$= +1$ and P$= -1$, weakly decaying as

$$\begin{array}{lllll} \text{“}\theta\text{”} & \to & \pi\pi & & \mathrm{P} = +1 \\ \text{“}\tau\text{”} & \to & \pi\pi\pi & & \mathrm{P} = -1 \end{array}$$

1956: $\theta - \tau$ *puzzle*:
A seemingly degenerate pair (θ, τ) of two mesons with P$= +1$ and P$= -1$, weakly decaying as

$$\begin{aligned} \text{“}\theta\text{”} \quad &\rightarrow \quad \pi\pi \qquad\qquad \mathrm{P} = +1 \\ \text{“}\tau\text{”} \quad &\rightarrow \quad \pi\pi\pi \qquad\quad \mathrm{P} = -1 \end{aligned}$$

Explanation by Lee and Yang:
“θ” and “τ” are the same particle, instead the weak interaction violates parity.

1956: $\theta - \tau$ *puzzle*:
A seemingly degenerate pair (θ, τ) of two mesons with P$= +1$ and P$= -1$, weakly decaying as

$$\begin{array}{lllr} \text{“}\theta\text{”} & \to & \pi\pi & \mathrm{P} = +1 \\ \text{“}\tau\text{”} & \to & \pi\pi\pi & \mathrm{P} = -1 \end{array}$$

Explanation by Lee and Yang:
“θ” and “τ” are the same particle, instead the weak interaction violates parity.

$$K = \text{“}\theta\text{”} = \text{“}\tau\text{”}.$$

Maximal P violation

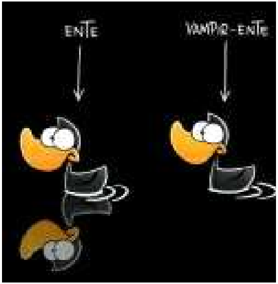
In the SM only left-handed fields feel the charged weak interaction, no couplings of the W-boson to u_R^j, d_R^j, and e_R^j.

Early monograph on parity violation:

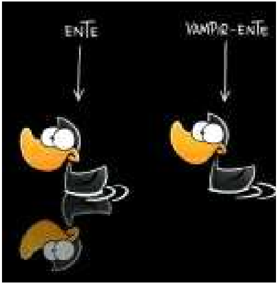
Early monograph on parity violation:

Lewis Carroll:
Alice through the looking glass

Maximal parity violation



Maximal parity violation



Charge conjugation C maps left-handed (particle) fields on right-handed (antiparticle) fields and vice versa:

$$\psi_L \stackrel{C}{\longleftrightarrow} \psi_L^C, \qquad \text{where} \quad \psi_L^C \equiv (\psi^C)_R \text{ is right-handed.}$$

$\Rightarrow$ The weak interaction also violates C!

Charge conjugation C maps left-handed (particle) fields on right-handed (antiparticle) fields and vice versa:

$$\psi_L \overset{C}{\longleftrightarrow} \psi_L^C, \qquad \text{where} \quad \psi_L^C \equiv (\psi^C)_R \text{ is right-handed.}$$

$\Rightarrow$ The weak interaction also violates C!

But: Nothing prevents CP and T from being good symmetries...

Charge conjugation C maps left-handed (particle) fields on right-handed (antiparticle) fields and vice versa:

$$\psi_L \stackrel{C}{\longleftrightarrow} \psi_L^C, \qquad \text{where} \quad \psi_L^C \equiv (\psi^C)_R \text{ is right-handed.}$$

$\Rightarrow$ The weak interaction also violates C!

But: Nothing prevents CP and T from being good symmetries. . .

. . . except experiment!

Neutral K mesons:

K_L and K_S (linear combinations of K and $\overline{K}$).

Dominant decay channels:

$$K_L \to \pi\pi\pi \qquad \mathrm{CP} = -1$$
$$K_S \to \pi\pi \qquad \mathrm{CP} = +1$$

Neutral K mesons:

K_L and K_S (linear combinations of K and $\overline{K}$).

Dominant decay channels:

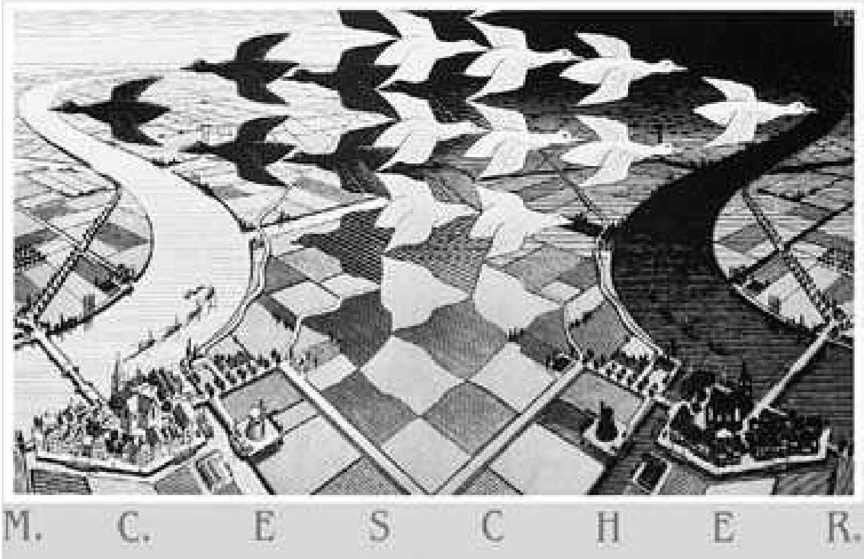
$$K_L \to \pi\pi\pi \qquad \mathrm{CP} = -1$$
$$K_S \to \pi\pi \qquad \mathrm{CP} = +1$$

1964: Christenson, Cronin, Fitch and Turlay observe

$$K_L \to \pi\pi$$

and therefore discover CP violation.

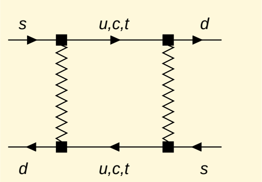
$$\epsilon_K \equiv \frac{\langle(\pi\pi)_{I=0}|H_{weak}|K_L\rangle}{\langle(\pi\pi)_{I=0}|H_{weak}|K_S\rangle} = (2.229 \pm 0.010) \cdot 10^{-3} e^{i0.97\pi/4}.$$


M. C. E S C H E R.

Time evolution of $|K^+\rangle$ state (in rest frame):

$$|K^+(t)\rangle = e^{-iMt} e^{-\Gamma t/2} |K^+\rangle$$

$$\Rightarrow \quad \frac{\langle K^+(t)|K^+(t)\rangle}{\langle K^+|K^+\rangle} = e^{-\Gamma t}$$



Straightforward generalization to a two-state system with $|1\rangle = |K^0\rangle$, $|2\rangle = |\bar{K}^0\rangle$:

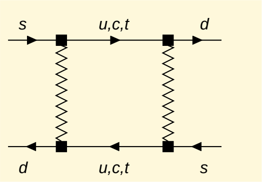
2×2 mass matrix M

2×2 decay matrix Γ

with $M = M^\dagger$ and $\Gamma = \Gamma^\dagger$.

$$\begin{pmatrix} |K^0(t)\rangle \\ |\bar{K}^0(t)\rangle \end{pmatrix} = \exp\left(-iMt - \frac{1}{2}\Gamma t\right) \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix}$$

The decay matrix element Γ_{12} is made up of all decays $K^0 \to f$, $\bar{K}^0 \to f$ into final states f common to K^0 and $\bar{K}^0$.



$$\Gamma_{12} = \frac{1}{2m_K} \sum_f \langle K^0|f\rangle\langle f|\overline{K}^0\rangle$$
$$\simeq \frac{1}{2m_K} \left[\langle K^0|\pi^+\pi^-\rangle\langle\pi^+\pi^-|\overline{K}^0\rangle + \langle K^0|\pi^0\pi^0\rangle\langle\pi^0\pi^0|\overline{K}^0\rangle \right].$$

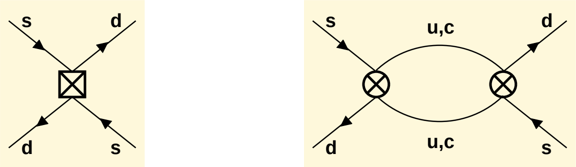
Here e.g. $\langle K^0|\pi^+\pi^-\rangle$ is short for $\langle K^0|H^{|\Delta S|=1}|\pi^+\pi^-\rangle$ with the effective $|\Delta S| = 1$ hamiltonian

$$H^{|\Delta S|=1} = \frac{G_F}{\sqrt{2}} V_{us} V_{ud}^* \sum_{j=1}^{2} C_j Q_j + \mathcal{O}(V_{ts} V_{td}^*) + \text{h.c.}$$

→ see Gudrun Hiller's lecture

The interesting short-distance physics (probing high scales and being sensitive to new physics) is encoded in M_{12}:

$$M_{12} = \frac{1}{2m_K} \langle K^0 | H^{|\Delta S|=2} | \overline{K}^0 \rangle + \text{smaller term with } H^{|\Delta S|=1}$$



Effective $|\Delta S| = 2$ hamiltonian (calculated from box diagram)

$$H^{|\Delta S|=2} = \frac{G_F^2}{16\pi^2} \left[(V_{cs} V_{cd}^*)^2 \widetilde{C}^c + V_{cs} V_{cd}^* V_{ts} V_{td}^* \widetilde{C}^{ct} + (V_{ts} V_{td}^*)^2 \widetilde{C}^t \right] \widetilde{Q} + \text{h.c.}$$

with $\widetilde{Q} = \bar{d} \gamma_\mu (1 - \gamma_5) s \; \bar{d} \gamma^\mu (1 - \gamma_5) s$

To find exponentially decaying states, the mass eigenstates, we must diagonalize $M - \frac{i}{2}\Gamma$.
CPT theorem: $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$.

$\Rightarrow$ Even the smallest $M_{12} \neq 0$ or $\Gamma_{12} \neq 0$ leads to near-maximal mixing.

lighter, short-lived eigenstate $|K_S\rangle$ decays dominantly as $K_S \to \pi\pi$
heavier, long-lived eigenstate $|K_L\rangle$ decays dominantly as $K_L \to \pi\pi\pi$

$$|K_L(t)\rangle = e^{-iM_L t}\, e^{-\Gamma_L t/2}\, |K_L\rangle$$
$$|K_S(t)\rangle = e^{-iM_S t}\, e^{-\Gamma_S t/2}\, |K_S\rangle$$

where $M_{L,S} - \frac{i}{2}\Gamma_{L,S}$ are the eigenvalues of $M - \frac{i}{2}\Gamma$ and $M_{L,S}$ and $\Gamma_{L,S}$ are mass and decay constant of $|K_{L;S}\rangle$, respectively.

mass difference $\Delta m = M_L - M_S > 0$ and
width difference $\Delta\Gamma = \Gamma_S - \Gamma_L > 0$

Experiment:

$$\begin{aligned} \Delta m &= (0.5301 \pm 0.0014) \cdot 10^{10} s^{-1}, \\ \Delta\Gamma &= (1.1174 \pm 0.0010) \cdot 10^{10} s^{-1} \;\approx 2\Delta m \end{aligned}$$

$\mathrm{K}-\overline{\mathrm{K}}$ mixing involves three quantities which are independent of phase conventions:

$$|M_{12}|, \quad |\Gamma_{12}|, \quad \phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

$\phi \neq 0$ leads to CP violation in mixing.

Solve the eigenvalue problem by expanding to lowest non-vanishing order in ϕ:

Ansatz:

$$|K_S\rangle = p\,|K^0\rangle + q\,|\bar{K}^0\rangle$$
$$|K_L\rangle = p\,|K^0\rangle - q\,|\bar{K}^0\rangle$$

Find:

$$|M_{12}| = \frac{\Delta m}{2} + \mathcal{O}\left(\phi^2\right), \qquad |\Gamma_{12}| = \frac{\Delta\Gamma}{2} + \mathcal{O}\left(\phi^2\right),$$
$$\left|\frac{q}{p}\right| = 1 - \phi\,\frac{\Delta\Gamma/2}{\Delta m + i\,\Delta\Gamma/2} + \mathcal{O}\left(\phi^2\right).$$

Note: $\phi \neq 0 \Leftrightarrow \left|\frac{q}{p}\right| \neq 1.$

Strong isospin: Instead of U and D use (I, I_3):

Fundamental doublets ($I = \frac{1}{2}$): $\begin{pmatrix} u \\ d \end{pmatrix}$ and $\begin{pmatrix} \overline{d} \\ -\overline{u} \end{pmatrix}$.

For $m_u = m_d$ the QCD lagrangian is invariant under SU(2) rotations of $\begin{pmatrix} u \\ d \end{pmatrix}$ and $\begin{pmatrix} \overline{d} \\ -\overline{u} \end{pmatrix}$.

Strong isospin: Instead of U and D use (I, I_3):

Fundamental doublets ($I = \frac{1}{2}$): $\begin{pmatrix} u \\ d \end{pmatrix}$ and $\begin{pmatrix} \overline{d} \\ -\overline{u} \end{pmatrix}$.

For $m_u = m_d$ the QCD lagrangian is invariant under SU(2) rotations of $\begin{pmatrix} u \\ d \end{pmatrix}$ and $\begin{pmatrix} \overline{d} \\ -\overline{u} \end{pmatrix}$.

"QCD cannot distinguish up and down"

Strong isospin: Instead of U and D use (I, I_3):

Fundamental doublets ($I = \frac{1}{2}$): $\begin{pmatrix} u \\ d \end{pmatrix}$ and $\begin{pmatrix} \overline{d} \\ -\overline{u} \end{pmatrix}$.

For $m_u = m_d$ the QCD lagrangian is invariant under SU(2) rotations of $\begin{pmatrix} u \\ d \end{pmatrix}$ and $\begin{pmatrix} \overline{d} \\ -\overline{u} \end{pmatrix}$.

"QCD cannot distinguish up and down"

Owing to $m_d - m_u \ll \Lambda_{\text{had}} \sim 400\,\text{MeV}$, strong isospin holds to $\sim 2\%$ accuracy.

Isospin triplet:

$$\pi^+ \sim u\bar{d}, \qquad \pi^0 \sim \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}, \qquad \pi^- \sim -d\bar{u}.$$

Compare with spin triplet

$$\uparrow\uparrow, \qquad \frac{\uparrow\uparrow + \downarrow\downarrow}{\sqrt{2}}, \qquad \downarrow\downarrow$$

Measurements

Define

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | K_L\rangle}{\langle \pi^+\pi^- | K_S\rangle}, \qquad \eta_{00} = \frac{\langle \pi^0\pi^0 | K_L\rangle}{\langle \pi^0\pi^0 | K_S\rangle}.$$

Their moduli and phases are measured as

$$|\eta_{+-}| = (2.285 \pm 0.019)\cdot 10^{-3}, \qquad \phi_{+-} = 43.5^\circ \pm 0.6^\circ,$$
$$|\eta_{00}| = (2.275 \pm 0.019)\cdot 10^{-3}, \qquad \phi_{00} = 43.4^\circ \pm 1.0^\circ$$

Introduce isospin states

$$|\pi^0\pi^0\rangle = \sqrt{\frac{1}{3}}\,|\,(\pi\pi)_{I=0}\rangle - \sqrt{\frac{2}{3}}\,|\,(\pi\pi)_{I=2}\rangle$$
$$|\pi^+\pi^-\rangle = \sqrt{\frac{2}{3}}\,|\,(\pi\pi)_{I=0}\rangle + \sqrt{\frac{1}{3}}\,|\,(\pi\pi)_{I=2}\rangle$$

Isospin amplitudes:

$$A_I = \langle (\pi\pi)_I | K^0 \rangle, \qquad \overline{A}_I \;=\; \langle (\pi\pi)_I | \overline{K}^0 \rangle, \qquad I = 0, 2.$$

The complex phase of any $K^0 \to \pi\pi$ amplitude is the sum of a CP conserving phase δ and a CP violating phase Φ. The CP conjugate decay $\bar{K}^0 \to \pi\pi$ instead involves the phases δ and $-\Phi$.

Isospin amplitudes:

$$A_I = \langle (\pi\pi)_I | K^0 \rangle, \qquad \overline{A}_I = \langle (\pi\pi)_I | \overline{K^0} \rangle, \qquad I = 0, 2.$$

The complex phase of any $K^0 \to \pi\pi$ amplitude is the sum of a CP conserving phase δ and a CP violating phase Φ. The CP conjugate decay $\bar{K}^0 \to \pi\pi$ instead involves the phases δ and $-\Phi$.

Φ is also called the weak phase, it is calculable in terms of the KM phase and sensitive to NP contributions.

δ is also called strong phase, it originates from the rescattering of the final state, dominated by the strong interaction.

$\pi^+\pi^-$ scatters into $\pi^0\pi^0$ and vice versa. This rescattering is described by a unitary 2×2 matrix.

Benefit of isospin states: The isospin symmetry of QCD forbids the rescattering

$$(\pi\pi)_{I=0} \leftrightarrow (\pi\pi)_{I=2}$$

as well as the rescattering into three-pion states. Thus $(\pi\pi)_{I=0,2}$ just pick up phase factors $e^{I\Phi_{0,2}}$.

Hence we can write:

$$A_I = |A_I| e^{i\Phi_I} e^{i\delta_I}, \qquad \overline{A}_I = -|A_I| e^{-i\Phi_I} e^{i\delta_I},$$

where I have used the following convention for C and CP:

$$C|K^0\rangle = |\bar{K}^0\rangle, \qquad CP|K^0\rangle = -|\bar{K}^0\rangle$$

Recall

$$A_I = |A_I| e^{i\Phi_I} e^{i\delta_I}, \qquad \overline{A}_I = -|A_I| e^{-i\Phi_I} e^{i\delta_I}.$$

Direct CP violation (or CP violation in decay) means

$$|\langle \pi\pi | K^0 \rangle| \neq |\langle \pi\pi | \bar{K}^0 \rangle|$$

For isospin states we therefore find no direct CP violation:

$$|\langle (\pi\pi)_I | K^0 \rangle| = |A_I| = |\langle (\pi\pi)_I | \bar{K}^0 \rangle|$$

Indirect CP violation stems from the phase $\phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$.

Recall

$$A_I = |A_I| e^{i\Phi_I} e^{i\delta_I}, \qquad \overline{A}_I \ = \ -|A_I| e^{-i\Phi_I} e^{i\delta_I}.$$

Direct CP violation (or CP violation in decay) means

$$|\langle \pi\pi | K^0 \rangle| \neq |\langle \pi\pi | \bar{K}^0 \rangle|$$

For isospin states we therefore find no direct CP violation:

$$|\langle (\pi\pi)_I | K^0 \rangle| = |A_I| = |\langle (\pi\pi)_I | \bar{K}^0 \rangle|$$

Indirect CP violation stems from the phase $\phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$. Seek a clever linear combination of the measured $\eta_{+-} = \langle \pi^+\pi^- | K_L \rangle / \langle \pi^+\pi^- | K_S \rangle$ and $\eta_{00} = \langle \pi^0\pi^0 | K_L \rangle / \langle \pi^0\pi^0 | K_S \rangle$ from which indirect CP violation drops out.
Simplification from the empirical "$\Delta I = 1/2$ rule" $|A_0| \simeq 22\,|A_2|$:

$$\epsilon_K \simeq \frac{\eta_{00} + 2\,\eta_{+-}}{3} \ = \ \frac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle} \left[1 + \mathcal{O}\left(\frac{A_2^2}{A_0^2} \right) \right]$$

Plugging everything in:

$$\epsilon_K = \frac{\phi}{2} \frac{\Delta m_K}{\sqrt{(\Delta m_K)^2 + (\Delta\Gamma_K/2)^2}} \, e^{i\phi_\epsilon} + \mathcal{O}\left(\phi^2, \frac{A_2^2}{A_0^2}, \frac{\Gamma_L}{\Gamma_S}\right)$$

with $\phi_\epsilon = \arctan \frac{\Delta m_K}{\Delta\Gamma_K/2}$. Experiment:

$$\epsilon_K = (2.28 \pm 0.01) \cdot 10^{-3} \cdot e^{i\,(0.97 \pm 0.02)\,\pi/4}.$$

Thus

$$\phi = (6.63 \pm 0.03) \cdot 10^{-3}.$$

Predict ϵ_K in the SM

Need

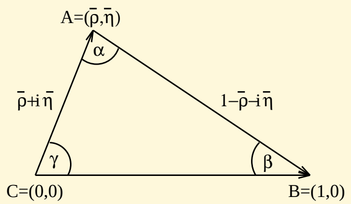
$$\phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right) \simeq \frac{\mathrm{Im}\, M_{12}}{|M_{12}|} = 2\frac{\mathrm{Im}\, M_{12}}{\Delta m}$$

where I have used that $-\Gamma_{12}$ is essentially proportional to $V_{us}V_{ud}^*$ and real and positive.

Now: $\mathrm{Im}\, M_{12} = \frac{1}{2m_K}\mathrm{Im}\,\langle K^0|H^{|\Delta S|=2}|\overline{K}^0\rangle$ and recall the hamiltonian

$$H^{|\Delta S|=2} = \frac{G_F^2}{16\pi^2}\left[(V_{cs}V_{cd}^*)^2\widetilde{C}^c + V_{cs}V_{cd}^*V_{ts}V_{td}^*\widetilde{C}^{ct} + (V_{ts}V_{td}^*)^2\widetilde{C}^t\right]\widetilde{Q} + \text{h.c.}$$

Express the CKM factors $\mathrm{Im}\,(V_{cs}V_{cd}^*)^2$, $\mathrm{Im}\, V_{cs}V_{cd}^*V_{ts}V_{td}^*$, $\mathrm{Im}\,(V_{ts}V_{td}^*)^2$, in terms of $|V_{cb}|$ and $\bar{\rho}, \bar{\eta}$.



Further need

$$\langle K^0|\widetilde{Q}|\overline{K}^0\rangle \propto \widehat{B}_K$$

Lattice QCD result

$$\widehat{B}_K = 0.7567 \pm 0.0021_{\rm stat} \pm 0.0123_{\rm syst}$$

The constraint on $(\bar{\rho}, \bar{\eta})$ from ϵ_K the reads

$$9.6 \cdot 10^{-7} = \widehat{B}_K\, \bar{\eta}\, |V_{cb}|^2 \left[|V_{cb}|^2 (1-\bar{\rho}) + 4.7 \cdot 10^{-4} \right]$$

Thus ϵ_K defines a hyperbola in the $(\bar{\rho}, \bar{\eta})$ plane.
The constraint suffers from the experimental error on $|V_{cb}|$ because of the $|V_{cb}|^4$ dependence.

Promise: Flavor-changing neutral current (FCNC) transitions of Kaons probe virtual effects of very high mass scales.

How about the 1964 discovery of Kaon CP violation in $\mathrm{K}-\overline{\mathrm{K}}$ mixing? Today we know that ϵ_K is dominated by loops with top quarks.

$\Rightarrow$ experiment at 0.5 GeV probes physics at 173 GeV

Direct CP violation is characterized by:

$$\epsilon'_K = \frac{\eta_{00} - \eta_{+-}}{3} = \frac{\epsilon_K}{\sqrt{2}} \left[\frac{\langle (\pi\pi)_{I=2} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_L \rangle} - \frac{\langle (\pi\pi)_{I=2} | K_S \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle} \right] \left[1 + \mathcal{O}\left(\frac{A_2}{A_0} \right) \right].$$

Experiment:

$$\epsilon'_K = (16.6 \pm 2.3) \cdot 10^{-4} \cdot \epsilon_K$$

discovered in 1999

Experimentally well-known:

$$\mathrm{Re}A_0 = (3.3201 \pm 0.0018) \times 10^{-7}\ \text{GeV},$$
$$\mathrm{Re}A_2 = (1.4787 \pm 0.0031) \times 10^{-8}\ \text{GeV}.$$

$\uparrow$

assumes PDG convention for CKM elements

Master equation for ϵ'_K:

$$\frac{\epsilon'_K}{\epsilon_K} = \frac{\omega_+}{\sqrt{2}|\epsilon_K^{\rm exp}|{\rm Re}A_0^{\rm exp}} \left\{ \frac{{\rm Im}A_2}{\omega_+} - \left(1 - \hat{\Omega}_{\rm eff}\right) {\rm Im}A_0 \right\}.$$

Here:

$$\omega_+ \simeq \frac{{\rm Re}A_2}{{\rm Re}A_0} = (4.53 \pm 0.02) \cdot 10^{-2}$$

$\hat{\Omega}_{\rm eff} = (14.8 \pm 8.0) \cdot 10^{-2}$ quantifies isospin breaking.

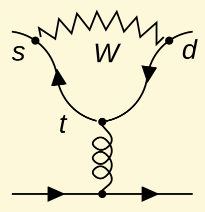
Important theoretical ingredients: ${\rm Im}A_0$ and ${\rm Im}A_2$, calculated from the effective $|\Delta S| = 1$ hamiltonian describing $s \to dq\overline{q}$ decays.

The enhanced sensitivity to $\Delta I = 3/2$ transitions (such as electroweak penguins and boxes) is a special feature of ϵ'_K.

$\mathrm{Im} A_0$ is dominated by gluon penguins:

Operator: $Q_6 = \overline{s}^j_L \gamma_\mu d^k_L \sum_q \overline{q}^k_R \gamma^\mu q^j_R$

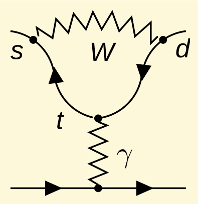
Matrix element: $\langle (\pi\pi)_{I=0} | Q_6 | K^0 \rangle$

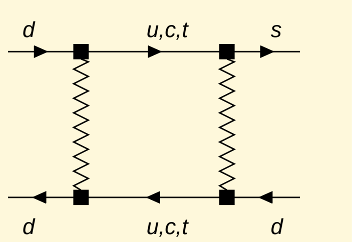


$\mathrm{Im} A_2$ is dominated by photon penguin and box diagrams:

Operator: $Q_8 = \frac{3}{2} \overline{s}^j_L \gamma_\mu d^k_L \sum_q e_q \overline{q}^k_R \gamma^\mu q^j_R$

Matrix element: $\langle (\pi\pi)_{I=2} | Q_8 | K^0 \rangle$





$$\frac{\epsilon'_K}{\epsilon_K} = (16.6 \pm 2.3) \times 10^{-4} \qquad \text{(experiments: NA62, KTeV)}$$

$$\frac{\epsilon'_K}{\epsilon_K} = \left(1.1 \pm 4.7_{\text{lattice}} \pm 1.9_{\text{NNLO}} \pm 0.6_{\text{isosp. br.}} \pm 0.2_{m_t}\right) \times 10^{-4} \qquad \text{(SM)}$$

Kitahara,UN,Tremper, JHEP 1612 (2016) 078

The prediction uses the lattice-QCD results from RBC-UKQCD, Phys. Rev. Lett. **115** 212001 (2015).

Discrepancy with a significance of 2.8σ!

Our prediction uses the methodology of Buras et al. (JHEP 1511 (2015) 202) (taking $\mathrm{Re}A_{0,2}$ from data), NLO formulae from Buras et al., and a new formula for the RG evolution.

Standard Model:
Cabibbo-Kobayashi-Maskawa (CKM) factor:

$$\tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} \sim (1.5 - 0.6i) \cdot 10^{-3}$$

$$\epsilon_K^{\prime\,\mathrm{SM}} \propto \mathrm{Im}\,\tau \qquad \text{and} \qquad \epsilon_K^{\mathrm{SM}} \propto \mathrm{Im}\,\tau^2.$$

Generic loop-induced new physics:
some flavor-violating parameter δ with $|\delta| \gg |\tau|$ to compensate for suppression from heavy new-physics mass

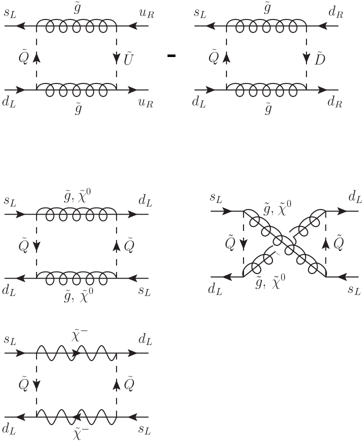
$$\epsilon_K^{\prime\,\mathrm{NP}} \propto \mathrm{Im}\,\delta \qquad \text{and} \qquad \epsilon_K^{\mathrm{NP}} \propto \mathrm{Im}\,\delta^2.$$

$\Rightarrow$ If $\epsilon_K^{\prime\,\mathrm{NP}} \sim \epsilon_K^{\prime\,\mathrm{SM}}$, expect $\epsilon_K^{\mathrm{NP}} \gg \epsilon_K^{\mathrm{SM}}$.

$\Rightarrow$ Need clever ideas to suppress ϵ_K^{NP}.

The MSSM has a mechanism

- to enhance $\mathrm{Re}A_2$, because it permits strong-isospin violation through splittings between right-handed up-squark and down-squark masses (Trojan penguins),
  Grossman, Kagan, Neubert 1999.
- to suppress the $\mathrm{K}-\overline{\mathrm{K}}$ mixing amplitude thanks to the Majorana nature of the gluinos, with negative interference of two box diagrams. Crivellin, Davidkov 2010

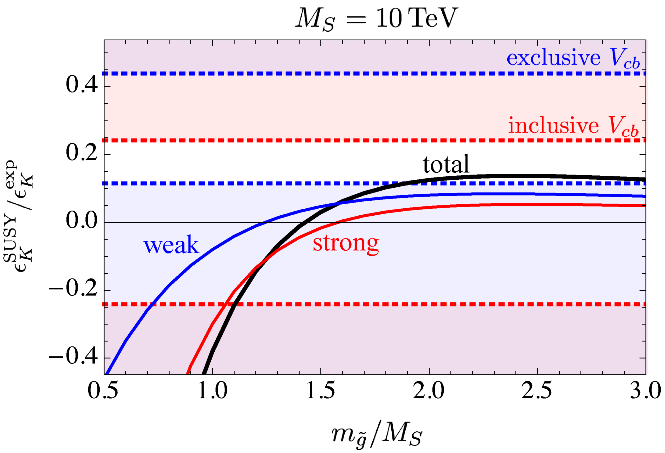


The second feature makes the MSSM contribution to $\mathrm{K}-\overline{\mathrm{K}}$ mixing vanish for $M_{\tilde{g}} \sim 1.5 M_{\tilde{q}}$, it stays small for $M_{\tilde{g}} > 1.5 M_{\tilde{q}}$.

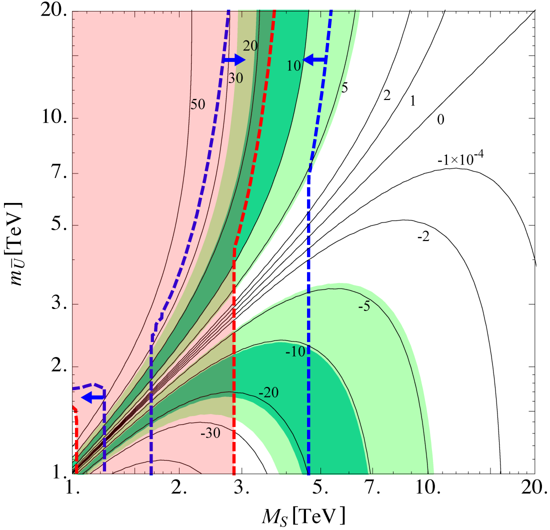
Teppei Kitahara, UN, Paul Tremper, Phys. Rev. Lett. 117 (2016) 091802

Choose:

Sparticle masses $M_S \sim 10$ TeV, $M_{\tilde{g}} > 1.5 M_S$, flavor mixing in down-squark mass matrix only with $\arg \Delta_{sd}^{LL} = \pi/4$.



Explain ϵ'_K



x-axis: generic sparticle mass, $M_{\tilde{g}} = 1.5 M_S$

y-axis: right-handed up-squark mass

red region: excluded by ϵ_K if $|V_{cb}|$ from inclusive decays is correct

blue dashes: delimit allowed region, if $|V_{cb}|$ from exclusive decays is correct

Dessert: $K \to \pi\nu\bar{\nu}$

The (near) future of Kaon physics:

$B(K^+ \to \pi^+\nu\bar{\nu}) \stackrel{\text{SM}}{=} (8.3 \pm 0.3) \cdot 10^{-11}$ for NA62 (CERN)

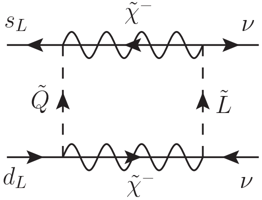
$B(K_L \to \pi^0\nu\bar{\nu}) \stackrel{\text{SM}}{=} (2.9 \pm 0.2) \cdot 10^{-11}$ for KØTØ (J-PARC)

These branching ratios are theoretically extremely clean and can discriminate between different NP explanations of ϵ'_k.

$K_L \to \pi^0\nu\bar{\nu}$ is CP violating!

$$B(K_L \to \pi^0\nu\bar{\nu}) \stackrel{\text{SM}}{\propto} \bar{\eta}^2$$

In our MSSM scenario:
Contributions from wino-like chargino box:



Giancarlo D'Ambrosio, Andreas Crivellin, Teppei Kitahara, UN, 1703.05786

If you allow for at most 10% (fine-)tuning in ϵ_K, you find (for GUT relations between $M_{1,2,3}$):

$$\frac{B(K^+ \to \pi^+\nu\bar{\nu})}{B(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM}} \leq 1.1 \qquad \text{and} \qquad \frac{B(K_L \to \pi^0\nu\bar{\nu})}{B(K_L \to \pi^0\nu\bar{\nu})_{\rm SM}} \leq 1.2.$$

$\longrightarrow$ need upgrade KØTØ–step2, aiming at $\mathcal{O}(100)$ events.

Furthermore: if the new-physics contribution to ϵ'_K is positive (as indicated by present data), find

$$\text{sgn}\left[B(K_L \to \pi^0\nu\overline{\nu}) - B^{\text{SM}}(K_L \to \pi^0\nu\overline{\nu})\right] = \text{sgn}\left(m_{\overline{U}} - m_{\overline{D}}\right)$$

Here $\overline{U}$ and $\overline{D}$ denote the right-handed up and down squarks, respectively.

Could collider experiments ever achieve this?

Kaon revival: Why now?

(i) theoretical control of ϵ_K increases steadily (hadronic matrix elements and NNLO QCD under good control, $\epsilon_K \propto |V_{cb}|^4$ issue improving),

(ii) ϵ'_K now tractable with lattice QCD,

(iii) upcoming measurements of theoretically clean branching ratios $B(K^+ \to \pi^+ \nu\bar{\nu})$ (by NA62) and $B(K_L \to \pi^0 \nu\bar{\nu})$ (by KØTØ),

(iv) no new particles found at LHC:

$\Rightarrow$ weaker rationale for Minimal Flavor Violation (MFV)

If the flavor structure of new physics is unrelated to the SM Yukawa sector, one expects the largest effects in Kaon (and $\mu \to e$) FCNC processes.

Summary

- ϵ_K has profited from an enormous progress in the lattice calculation of the hadronic matrix element $\langle K^0|\widetilde{Q}|\overline{K^0}\rangle$ and gives a precise constraint on $(|V_{cb}|, \bar{\rho}, \bar{\eta})$.
- To efficiently explore new physics with ϵ_K we need a better measurement of $|V_{cb}|$.
- The 1999 measurement of ϵ'_K becomes a powerful NP analyzer only now, with emerging precise lattice predictions for $\langle(\pi\pi)_{I=0}|Q_6|K^0\rangle$.
- The new lattice results for the matrix element $\langle(\pi\pi)_{I=0}|Q_6|K^0\rangle$ from RBC-UKQCD points to a tension between the experimental value of ϵ'_K and the Standard-Model prediction.
- $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ are clean probes of NP.

Summary

- If new physics enters through loops, a sizable effect in ϵ'_K requires a new source of flavor violation which is much larger then the CKM factor $\mathrm{Im}\,\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*} \sim 6 \cdot 10^{-4}$. But then the effect on ϵ_K will typically be too big.
- In the MSSM one can simultaneously enhance ϵ'_K and suppress the new-physics contributions to ϵ_K. This requires flavor mixing among left-handed squarks, masses of right-handed up-type squarks different from those of the down-type squarks, and a gluino mass above 1.5 times the mass of the left-handed squarks.
- $B(K \to \pi\nu\overline{\nu})$ data will test our scenario. $B(K_L \to \pi^0\nu\overline{\nu})$ can determine, whether the right-handed up squark is heavier or lighter than the right-handed down squark.

Penguins: Wake-up call for new physics?

Backup

Sensitivity to new physics

Standard Model:

Cabibbo-Kobayashi-Maskawa (CKM) factor:

$$\tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} \sim (1.5 - 0.6i) \cdot 10^{-3}$$

$$\epsilon_K'^{\mathrm{SM}} \propto \mathrm{Im}\, \frac{\tau}{M_W^2} \qquad \text{and} \qquad \epsilon_K^{\mathrm{SM}} \propto \mathrm{Im}\, \frac{\tau^2}{M_W^2}.$$

Generic new physics:

Some flavor-violating parameter: δ

$$\epsilon_K'^{\mathrm{NP}} \propto \mathrm{Im}\, \frac{\delta}{M^2} \qquad \text{and} \qquad \epsilon_K^{\mathrm{NP}} \propto \mathrm{Im}\, \frac{\delta^2}{M^2}.$$

with new heavy particle mass $M \gg M_W$.

But data require $|\epsilon_K^{\rm NP}| \leq |\epsilon_K^{\rm SM}|$, so that

$$\left|\frac{\epsilon_K'^{\,\rm NP}}{\epsilon_K'^{\,\rm SM}}\right| \leq \frac{\left|\epsilon_K'^{\,\rm NP}/\epsilon_K'^{\,\rm SM}\right|}{\left|\epsilon_K^{\rm NP}/\epsilon_K^{\rm SM}\right|} = \mathcal{O}\left(\frac{\mathrm{Re}\,\tau}{\mathrm{Re}\,\delta}\right).$$

If new physics enters through loops, we need $|\delta| \gg |\tau|$ to compensate for $M \gg M_W$, but then ϵ_K prohibits large effects in ϵ_K'.

But data require $|\epsilon_K^{\mathrm{NP}}| \leq |\epsilon_K^{\mathrm{SM}}|$, so that

$$\left|\frac{\epsilon_K'^{\,\mathrm{NP}}}{\epsilon_K'^{\,\mathrm{SM}}}\right| \leq \frac{\left|\epsilon_K'^{\,\mathrm{NP}}/\epsilon_K'^{\,\mathrm{SM}}\right|}{\left|\epsilon_K^{\mathrm{NP}}/\epsilon_K^{\mathrm{SM}}\right|} = \mathcal{O}\left(\frac{\mathrm{Re}\,\tau}{\mathrm{Re}\,\delta}\right).$$

If new physics enters through loops, we need $|\delta| \gg |\tau|$ to compensate for $M \gg M_W$, but then ϵ_K prohibits large effects in ϵ_K'.

Solutions in the literature:

- tree-level new physics (e.g. Z') with $|\delta| \sim |\tau|$
- fine-tuning of the CP phase to get $\mathrm{Re}\,\delta \sim 0$
- exploit special features of supersymmetry (this talk).