Kaon mixing and CP violation

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SLAC Summer Institute 2019:
Menu of Flavors: Quarks, Charged Leptons & Neutrinos
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Appetizers: Why Kaons?

Intermediate course: discrete symmetries

Main course: CP violation in $K \rightarrow \pi\pi$

Dessert: $K \rightarrow \pi\nu\bar{\nu}$

Summary
Appetizers: Why Kaons?

Some flavored mesons:
charged:

\[ K^+ \sim \bar{s}u, \quad D^+ \sim \bar{c}d, \quad D_s^+ \sim c\bar{s}, \quad B^+ \sim \bar{b}u, \quad B_c^+ \sim \bar{b}c, \]

\[ K^- \sim s\bar{u}, \quad D^- \sim \bar{c}d, \quad D_s^- \sim c\bar{s}, \quad B^- \sim b\bar{u}, \quad B_c^- \sim b\bar{c}, \]

neutral:

\[ K \sim \bar{s}d, \quad D \sim \bar{c}u, \quad B_d \sim \bar{b}d, \quad B_s \sim \bar{b}s, \]

\[ \bar{K} \sim s\bar{d}, \quad \bar{D} \sim \bar{c}u, \quad \bar{B}_d \sim b\bar{d}, \quad \bar{B}_s \sim b\bar{s}, \]

In flavor physics only the ground-state hadrons which decay weakly rather than strongly are interesting. Weakly decaying baryons are less interesting, because they are produced in smaller rates and are theoretically harder to cope with.
Appetizers: Why Kaons?

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\[ K^- \sim s\bar{u}, \quad D^- \sim \bar{c}d, \quad D_s^- \sim \bar{c}s, \quad B^- \sim b\bar{u}, \quad B_c^- \sim b\bar{c}, \]
neutral:

\[ K \sim \bar{s}d, \quad D \sim \bar{c}u, \quad B_d \sim \bar{b}d, \quad B_s \sim \bar{b}s, \]
\[ K \sim s\bar{d}, \quad D \sim \bar{c}u, \quad B_d \sim \bar{b}d, \quad B_s \sim b\bar{s}, \]

The neutral \( K, D, B_d \) and \( B_s \) mesons mix with their antiparticles, \( \bar{K}, \bar{D}, \bar{B}_d \) and \( \bar{B}_s \) thanks to the weak interaction (quantum-mechanical two-state systems).
Some flavored mesons:

charged:

\[
\begin{align*}
K^+ & \sim su, \\
D^+ & \sim cd, \\
D_s^+ & \sim cs, \\
B^+ & \sim bu, \\
B_c^+ & \sim bc, \\
K^- & \sim su, \\
D^- & \sim cd, \\
D_s^- & \sim cs, \\
B^- & \sim bu, \\
B_c^- & \sim bc,
\end{align*}
\]

neutral:

\[
\begin{align*}
K & \sim sd, \\
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B_d & \sim bd, \\
B_s & \sim bs, \\
\bar{K} & \sim sd, \\
\bar{D} & \sim cu, \\
\bar{B}_d & \sim bd, \\
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The neutral $K$, $D$, $B_d$ and $B_s$ mesons mix with their antiparticles, $\bar{K}$, $\bar{D}$, $\bar{B}_d$ and $\bar{B}_s$ thanks to the weak interaction (quantum-mechanical two-state systems).

$\Rightarrow$ gold mine for fundamental parameters
This talk: **Kaons**

Kaons have a mass of 0.5 GeV.

How can a particle decay at such a low energy be interesting to probe physics beyond the Standard Model (SM)?
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Kaons have a mass of $0.5 \text{ GeV}$.

How can a particle decay at such a low energy be interesting to probe physics beyond the Standard Model (SM)?

**SM:** Weak decay amplitude $\mathcal{A}$ of any hadron is suppressed by $G_F \propto 1/M_W^2$

New physics with particle mass $M_{NP} \gg M_W$ contributes $1/M_{NP}^2$ to $\mathcal{A}$:

$$\mathcal{A} = \mathcal{A}_{SM} \left[ 1 + \left( \frac{M_W^2}{M_{NP}^2} \right) \right]$$

⇒ The game is even between $B$, $D$, and $K$!
Flavor-changing neutral current (FCNC) processes:

$\Delta|S| = 1$ transitions (Kaon decays):

$\Delta|S| = 2$ transitions (Kaon mixing):

$S$ is the strangeness:

$s$ quark, $\bar{K}^0 \sim \bar{s}d$, $K^- \sim s\bar{u}$ have $S = -1$

$\bar{s}$ quark, $K^0 \sim \bar{s}d$, $K^+ \sim s\bar{u}$ have $S = +1$
Consider $|\Delta S| = 1$ FCNC amplitude: internal quark can be $u, c, \text{or } t$.

$$\mathcal{A} = \sum_{j=u,c,t} V_{js}^* V_{jd} f \left( \frac{m_j^2}{M_W^2} \right)$$

$$= V_{cs}^* V_{cd} \left[ f \left( \frac{m_c^2}{M_W^2} \right) - f \left( \frac{m_u^2}{M_W^2} \right) \right] + V_{ts}^* V_{td} \left[ f \left( \frac{m_t^2}{M_W^2} \right) - f \left( \frac{m_u^2}{M_W^2} \right) \right]$$

Here $V_{js}^* V_{jd} f \left( \frac{m_j^2}{M_W^2} \right)$ denotes the result of the loop diagram with internal quark $j$.

**Essential:** Unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix:

$$V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0$$

$$\Rightarrow V_{cs}^* V_{cd} = -V_{us}^* V_{ud} - V_{ts}^* V_{td}$$
Set $m_u = 0$ and expand in $m_c^2/M_W^2$ to first order:

$$f\left(\frac{m_c^2}{M_W^2}\right) - f\left(\frac{m_u^2}{M_W^2}\right) = A \frac{m_c^2}{M_W^2} \log \frac{m_c^2}{M_W^2} + B \frac{m_c^2}{M_W^2} + \ldots$$

$$\mathcal{A} = V_{cs}^* V_{cd} \left[ f\left(\frac{m_c^2}{M_W^2}\right) - f\left(\frac{m_u^2}{M_W^2}\right) \right] + V_{ts}^* V_{td} \left[ f\left(\frac{m_t^2}{M_W^2}\right) - f\left(\frac{m_u^2}{M_W^2}\right) \right]$$

$$\simeq V_{cs}^* V_{cd} \frac{m_c^2}{M_W^2} \left[ A \log \frac{m_c^2}{M_W^2} + B \right] + V_{ts}^* V_{td} \left[ f\left(\frac{m_t^2}{M_W^2}\right) - f(0) \right]$$
\[ A \simeq V_{cs}^* V_{cd} \left( \frac{m_c^2}{M_W^2} \right) \left[ A \log \frac{m_c^2}{M_W^2} + B \right] + V_{ts}^* V_{td} \left[ f \left( \frac{m_t^2}{M_W^2} \right) - f(0) \right] \]

\[ \mathcal{O}(-0.2) \quad \mathcal{O}(-0.002) \quad \mathcal{O}(3 \cdot 10^{-4}) \quad \mathcal{O}(1) \]

Glashow-Iliopoulos-Maiani (GIM) suppression of the CKM-favored term

Both terms are similar in size.

⇒ Kaon FCNCs are sensitive to virtual top quark effects.
CP violating $s \rightarrow d$ transitions involve $\text{Im } V_{ts}^* V_{td} = 1.5 \cdot 10^{-4}$

Compare this with $\bar{b} \rightarrow \bar{d}$ transition: $\text{Im } V_{tb}^* V_{td} = -3.6 \cdot 10^{-3}$

... or $\bar{b} \rightarrow \bar{s}$ transition: $\text{Im } V_{tb}^* V_{ts} = -8.2 \cdot 10^{-4}$
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$\Rightarrow$ Kaon FCNCs are best to probe NP with generic flavor structure!

“Generic” means “unrelated to CKM mechanism of the SM”.

Opposite concept: “minimally flavor violating (MFV)” meaning “governed by CKM mechanism”.
Parity transformation $P$:  \[ \vec{x} \rightarrow -\vec{x} \]

Charge conjugation $C$: Exchange particles and antiparticles, e.g. $e^- \leftrightarrow e^+$

Time reversal $T$:  \[ t \rightarrow -t \]
1954/1955: \textbf{CPT} is a symmetry of every Lorentz-invariant quantum field theory.
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1964: CP is not a symmetry of the microscopic laws of nature!

⇒ Also the T symmetry must be violated, there is a microscopic arrow of time!
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Strong interaction

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⇒ The strong interaction essentially respects $C$, $P$, and therefore $T$,

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[H_{\text{strong}}, P] = [H_{\text{strong}}, C] = [H_{\text{strong}}, T] = 0
\]

⇒ We can assign $C$ and $P$ quantum numbers, which can be $+1$ or $-1$, to hadrons.
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Example: A \( \pi^0 \) meson has \( P = -1 \) and \( C = +1 \). A \( \pi^+ \) has \( P = -1 \), but is no eigenstate of \( C \), because \( C |\pi^+\rangle = |\pi^-\rangle \).
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Example: A $\pi^0$ meson has $P = -1$ and $C = +1$. A $\pi^+$ has $P = -1$, but is no eigenstate of $C$, because $C|\pi^+\rangle = |\pi^-\rangle$.

Also QED respects $C, P$, and $T$. 
1956: $\theta - \tau$ puzzle:
A seemingly degenerate pair $(\theta, \tau)$ of two mesons with $P = +1$ and $P = -1$, weakly decaying as

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\begin{align*}
\text{"}$\theta$" & \rightarrow \pi\pi & P = +1 \\
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\[ K = \text{“}\theta\text{”} = \text{“}\tau\text{”}. \]
Maximal P violation

In the **SM** only left-handed fields feel the charged weak interaction, no couplings of the **W-boson** to $u^j_R$, $d^j_R$, and $e^j_R$. 
Early monograph on parity violation:
Early monograph on parity violation:

Lewis Carroll:  
*Alice through the looking glass*
Maximal parity violation
Maximal parity violation

Warum Vampire so schlechte Zähne haben.

ENTE → VAMPIR-ENTE
Charge conjugation $C$ maps left-handed (particle) fields on right-handed (antiparticle) fields and vice versa:

$$\psi_L \xrightarrow{C} \psi_L^C,$$

where $\psi_L^C \equiv (\psi^C)_R$ is right-handed.

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⇒ The weak interaction also violates $C$!

But: Nothing prevents $CP$ and $T$ from being good symmetries. . .

. . . except experiment!
Neutral $K$ mesons:

$K_L$ and $K_S$ (linear combinations of $K$ and $\bar{K}$).

Dominant decay channels:

$K_L \rightarrow \pi\pi\pi$  \hspace{1cm} CP $= -1$

$K_S \rightarrow \pi\pi$  \hspace{1cm} CP $= +1$
Neutral $K$ mesons: $K_L$ and $K_S$ (linear combinations of $K$ and $\bar{K}$).

Dominant decay channels:

$$K_L \rightarrow \pi\pi\pi \quad \text{CP} = -1$$
$$K_S \rightarrow \pi\pi \quad \text{CP} = +1$$

1964: Christenson, Cronin, Fitch and Turlay observe

$$K_L \rightarrow \pi\pi$$

and therefore discover CP violation.

$$\epsilon_K \equiv \frac{\langle (\pi\pi)_{I=0} | H_{\text{weak}} | K_L \rangle}{\langle (\pi\pi)_{I=0} | H_{\text{weak}} | K_S \rangle} = (2.229 \pm 0.010) \cdot 10^{-3} e^{i0.97\pi/4}. $$
Time evolution of $|K^+\rangle$ state (in rest frame):

$$|K^+(t)\rangle = e^{-iMt}e^{-\Gamma t/2}|K^+\rangle$$

$$\Rightarrow \frac{\langle K^+(t)|K^+(t)\rangle}{\langle K^+|K^+\rangle} = e^{-\Gamma t}$$

Straightforward generalization to a two-state system with $|1\rangle = |K^0\rangle$, $|2\rangle = |\bar{K}^0\rangle$:

2 $\times$ 2 mass matrix $M$

2 $\times$ 2 decay matrix $\Gamma$

with $M = M^\dagger$ and $\Gamma = \Gamma^\dagger$.

$$\begin{pmatrix} |K^0(t)\rangle \\ |\bar{K}^0(t)\rangle \end{pmatrix} = \exp \left(-iMt - \frac{1}{2}\Gamma t\right) \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix}$$
The decay matrix element $\Gamma_{12}$ is made up of all decays $K^0 \to f$, $\bar{K}^0 \to f$ into final states $f$ common to $K^0$ and $\bar{K}^0$.

$$\Gamma_{12} = \frac{1}{2m_K} \sum_f \langle K^0 | f \rangle \langle f | \bar{K}^0 \rangle$$

$$\simeq \frac{1}{2m_K} \left[ \langle K^0 | \pi^+ \pi^- \rangle \langle \pi^+ \pi^- | \bar{K}^0 \rangle + \langle K^0 | \pi^0 \pi^0 \rangle \langle \pi^0 \pi^0 | \bar{K}^0 \rangle \right].$$

Here e.g. $\langle K^0 | \pi^+ \pi^- \rangle$ is short for $\langle K^0 | H|\Delta S|=1 | \pi^+ \pi^- \rangle$ with the effective $|\Delta S| = 1$ Hamiltonian

$$H|\Delta S|=1 = \frac{G_F}{\sqrt{2}} V_{us} V_{ud}^* \sum_{j=1}^{2} C_j Q_j + O(V_{ts} V_{td}^*) + h.c.$$
The interesting short-distance physics (probing high scales and being sensitive to new physics) is encoded in $M_{12}$:

$$M_{12} = \frac{1}{2m_K} \langle K^0 | H^{\Delta S=2} | \bar{K}^0 \rangle + \text{smaller term with } H^{\Delta S=1}$$

Effective $|\Delta S| = 2$ hamiltonian (calculated from box diagram)

$$H^{\Delta S=2} = \frac{G_F^2}{16\pi^2} \left[ (V_{cs} V_{cd}^*)^2 \tilde{C}^c + V_{cs} V_{cd}^* V_{ts} V_{td}^* \tilde{C}^{ct} + (V_{ts} V_{td}^*)^2 \tilde{C}^t \right] \tilde{Q} + \text{h.c.}$$

with $\tilde{Q} = \bar{d} \gamma_\mu (1 - \gamma_5) s \bar{d} \gamma^{\mu} (1 - \gamma_5) s$
To find exponentially decaying states, the mass eigenstates, we must diagonalize $M - \frac{i}{2}\Gamma$.

**CPT** theorem: $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$.

$\Rightarrow$ Even the smallest $M_{12} \neq 0$ or $\Gamma_{12} \neq 0$ leads to near-maximal mixing.

lighter, short-lived eigenstate $|K_S\rangle$ decays dominantly as $K_S \rightarrow \pi\pi$

heavier, long-lived eigenstate $|K_L\rangle$ decays dominantly as $K_L \rightarrow \pi\pi\pi$

$$|K_L(t)\rangle = e^{-iM_Lt} e^{-\Gamma_Lt/2} |K_L\rangle$$

$$|K_S(t)\rangle = e^{-iM_St} e^{-\Gamma_St/2} |K_S\rangle$$

where $M_{L,S} - \frac{i}{2}\Gamma_{L,S}$ are the eigenvalues of $M - \frac{i}{2}\Gamma$ and $M_{L,S}$ and $\Gamma_{L,S}$ are mass and decay constant of $|K_{L,S}\rangle$, respectively.
mass difference $\Delta m = M_L - M_S > 0$ and width difference $\Delta \Gamma = \Gamma_S - \Gamma_L > 0$

Experiment:

\[
\begin{align*}
\Delta m &= (0.5301 \pm 0.0014) \cdot 10^{10} \text{s}^{-1}, \\
\Delta \Gamma &= (1.1174 \pm 0.0010) \cdot 10^{10} \text{s}^{-1} \approx 2\Delta m
\end{align*}
\]
$\bar{K}-K$ mixing involves three quantities which are independent of phase conventions:

$$|M_{12}|, \quad |\Gamma_{12}|, \quad \phi = \arg \left( -\frac{M_{12}}{\Gamma_{12}} \right)$$

$\phi \neq 0$ leads to CP violation in mixing.

Solve the eigenvalue problem by expanding to lowest non-vanishing order in $\phi$:

Ansatz:

$$|K_S\rangle = p |K^0\rangle + q |\bar{K}^0\rangle$$

$$|K_L\rangle = p |K^0\rangle - q |\bar{K}^0\rangle$$

Find:

$$|M_{12}| = \frac{\Delta m}{2} + O \left( \phi^2 \right), \quad |\Gamma_{12}| = \frac{\Delta \Gamma}{2} + O \left( \phi^2 \right),$$

$$\left| \frac{q}{p} \right| = 1 - \phi \frac{\Delta \Gamma / 2}{\Delta m + i \Delta \Gamma / 2} + O \left( \phi^2 \right).$$

Note: $\phi \neq 0 \Leftrightarrow \left| \frac{q}{p} \right| \neq 1.$
Strong isospin: Instead of $U$ and $D$ use $(I, I_3)$:

Fundamental doublets ($I = \frac{1}{2}$): $(u \ d)$ and $(\bar{d} \ -\bar{u})$.

For $m_u = m_d$ the QCD lagrangian is invariant under SU(2) rotations of $(u \ d)$ and $(\bar{d} \ -\bar{u})$. 
Strong isospin: Instead of $U$ and $D$ use $(I, I_3)$:

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“QCD cannot distinguish up and down”
Strong isospin: Instead of $U$ and $D$ use $(l, l_3)$:

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"QCD cannot distinguish up and down"

Owing to $m_d - m_u \ll \Lambda_{\text{had}} \sim 400 \text{ MeV}$, strong isospin holds to $\sim 2\%$ accuracy.
Isospin triplet:

\[ \pi^+ \sim ud, \quad \pi^0 \sim \frac{uu - dd}{\sqrt{2}}, \quad \pi^- \sim -d\bar{u}. \]

Compare with spin triplet

\[ \uparrow\uparrow, \quad \frac{\uparrow\uparrow + \downarrow\downarrow}{\sqrt{2}}, \quad \downarrow\downarrow \]
Measurements

Define

\[ \eta_{+-} = \frac{\langle \pi^+\pi^- | K_L \rangle}{\langle \pi^+\pi^- | K_S \rangle}, \quad \eta_{00} = \frac{\langle \pi^0\pi^0 | K_L \rangle}{\langle \pi^0\pi^0 | K_S \rangle}. \]

Their moduli and phases are measured as

\[ |\eta_{+-}| = (2.285 \pm 0.019) \cdot 10^{-3}, \quad \phi_{+-} = 43.5^\circ \pm 0.6^\circ, \]
\[ |\eta_{00}| = (2.275 \pm 0.019) \cdot 10^{-3}, \quad \phi_{00} = 43.4^\circ \pm 1.0^\circ. \]

Introduce isospin states

\[ |\pi^0\pi^0\rangle = \sqrt{\frac{1}{3}} |(\pi\pi)_{I=0}\rangle - \sqrt{\frac{2}{3}} |(\pi\pi)_{I=2}\rangle, \]
\[ |\pi^+\pi^-\rangle = \sqrt{\frac{2}{3}} |(\pi\pi)_{I=0}\rangle + \sqrt{\frac{1}{3}} |(\pi\pi)_{I=2}\rangle. \]
Isospin amplitudes:

\[ A_I = \langle (\pi\pi)_I | K^0 \rangle, \quad \bar{A}_I = \langle (\pi\pi)_I | \bar{K}^0 \rangle, \quad I = 0, 2. \]

The complex phase of any \( K^0 \rightarrow \pi\pi \) amplitude is the sum of a CP conserving phase \( \delta \) and a CP violating phase \( \Phi \). The CP conjugate decay \( \bar{K}^0 \rightarrow \pi\pi \) instead involves the phases \( \delta \) and \(-\Phi\).
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The complex phase of any \( K^0 \rightarrow \pi\pi \) amplitude is the sum of a CP conserving phase \( \delta \) and a CP violating phase \( \Phi \). The CP conjugate decay \( \bar{K}^0 \rightarrow \pi\pi \) instead involves the phases \( \delta \) and \( -\Phi \).

\( \Phi \) is also called the weak phase, it is calculable in terms of the KM phase and sensitive to NP contributions.

\( \delta \) is also called strong phase, it originates from the rescattering of the final state, dominated by the strong interaction.

\( \pi^+\pi^- \) scatters into \( \pi^0\pi^0 \) and vice versa. This rescattering is described by a unitary \( 2 \times 2 \) matrix.
Benefit of isospin states: The isospin symmetry of QCD forbids the rescattering

\[(\pi\pi)_{I=0} \leftrightarrow (\pi\pi)_{I=2}\]

as well as the rescattering into three-pion states. Thus \((\pi\pi)_{I=0,2}\) just pick up phase factors \(e^{i\Phi_{0,2}}\).

Hence we can write:

\[A_I = |A_I|e^{i\Phi_I}e^{i\delta_I}, \quad \bar{A}_I = -|A_I|e^{-i\Phi_I}e^{i\delta_I},\]

where I have used the following convention for \(C\) and \(CP\):

\[C|K^0\rangle = |\bar{K}^0\rangle, \quad CP|K^0\rangle = -|\bar{K}^0\rangle\]
Recall

\[ A_I = |A_I| e^{i\Phi_I} e^{i\delta_I}, \quad \bar{A}_I = -|A_I| e^{-i\Phi_I} e^{i\delta_I}. \]

Direct CP violation (or CP violation in decay) means

\[ |\langle \pi\pi | K^0 \rangle| \neq |\langle \pi\pi | \bar{K}^0 \rangle| \]

For isospin states we therefore find no direct CP violation:

\[ |\langle (\pi\pi)_I | K^0 \rangle| = |A_I| = |\langle (\pi\pi)_I | \bar{K}^0 \rangle| \]

Indirect CP violation stems from the phase \( \phi = \text{arg} \left( -\frac{M_{12}}{\Gamma_{12}} \right) \).
Recall

\[ A_I = |A_I|e^{i\Phi_I}e^{i\delta_I}, \quad \bar{A}_I = -|A_I|e^{-i\Phi_I}e^{i\delta_I}. \]

Direct CP violation (or CP violation in decay) means

\[ |\langle \pi\pi | K^0 \rangle| \neq |\langle \pi\pi | \bar{K}^0 \rangle| \]

For isospin states we therefore find no direct CP violation:

\[ |\langle (\pi\pi)_I | K^0 \rangle| = |A_I| = |\langle (\pi\pi)_I | \bar{K}^0 \rangle| \]

Indirect CP violation stems from the phase \( \phi = \arg \left( -\frac{M_{12}}{\Gamma_{12}} \right) \). Seek a clever linear combination of the measured

\[ \eta_{+-} = \frac{\langle \pi^+\pi^- | K_L \rangle}{\langle \pi^+\pi^- | K_S \rangle} \]

and

\[ \eta_{00} = \frac{\langle \pi^0\pi^0 | K_L \rangle}{\langle \pi^0\pi^0 | K_S \rangle} \]

from which indirect CP violation drops out.

Simplification from the empirical “\( \Delta I = 1/2 \) rule” \(|A_0| \approx 22 |A_2|\):

\[ \varepsilon_K \approx \frac{\eta_{00} + 2\eta_{+-}}{3} = \frac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle} \left[ 1 + \mathcal{O} \left( \frac{A_2^2}{A_0^2} \right) \right] \]
Plugging everything in:

\[
\epsilon_K = \frac{\phi}{2} \frac{\Delta m_K}{\sqrt{(\Delta m_K)^2 + (\Delta \Gamma_K/2)^2}} e^{i \phi_\epsilon} + O \left( \phi^2, \frac{A_2^2}{A_0^2}, \frac{\Gamma_L}{\Gamma_S} \right)
\]

with \( \phi_\epsilon = \arctan \frac{\Delta m_K}{\Delta \Gamma_K/2} \). Experiment:

\[
\epsilon_K = (2.28 \pm 0.01) \cdot 10^{-3} \cdot e^{i (0.97 \pm 0.02) \pi/4}.
\]

Thus

\[
\phi = (6.63 \pm 0.03) \cdot 10^{-3}.
\]
Predict $\epsilon_K$ in the SM

Need

$$\phi = \arg \left( -\frac{M_{12}}{\Gamma_{12}} \right) \approx \frac{\text{Im} \ M_{12}}{|M_{12}|} = 2 \frac{\text{Im} \ M_{12}}{\Delta m}$$

where I have used that $-\Gamma_{12}$ is essentially proportional to $V_{us} V_{ud}$ and real and positive.

Now: $\text{Im} \ M_{12} = \frac{1}{2m_K} \text{Im} \langle K^0 | H | \Delta S = 2 | \bar{K}^0 \rangle$ and recall the hamiltonian

$$H|\Delta S = 2 = \frac{G_F^2}{16\pi^2} \left[ (V_{cs} V_{cd}^*)^2 \tilde{C}^c + V_{cs} V_{cd} V_{ts} V_{td}^* \tilde{C}^{ct} + (V_{ts} V_{td}^*)^2 \tilde{C}^t \right] \tilde{Q} + \text{h.c.}$$

Express the CKM factors $\text{Im} (V_{cs} V_{cd}^*)^2$, $\text{Im} V_{cs} V_{cd}^* V_{ts} V_{td}^*$, $\text{Im} (V_{ts} V_{td}^*)^2$, in terms of $|V_{cb}|$ and $\bar{\rho}, \bar{\eta}$.
Further need

$$\langle K^0 | \tilde{Q} | K^0 \rangle \propto \hat{B}_K$$

Lattice QCD result

$$\hat{B}_K = 0.7567 \pm 0.0021_{\text{stat}} \pm 0.0123_{\text{syst}}$$

The constraint on $$(\tilde{\rho}, \tilde{\eta})$$ from $\epsilon_K$ reads

$$9.6 \cdot 10^{-7} = \hat{B}_K \tilde{\eta} |V_{cb}|^2 \left[ |V_{cb}|^2 (1 - \tilde{\rho}) + 4.7 \cdot 10^{-4} \right]$$

Thus $\epsilon_K$ defines a hyperbola in the $$(\tilde{\rho}, \tilde{\eta})$$ plane.

The constraint suffers from the experimental error on $|V_{cb}|$ because of the $|V_{cb}|^4$ dependence.
Promise: **Flavor-changing neutral current (FCNC)** transitions of Kaons probe virtual effects of **very high mass scales**.

How about the **1964 discovery of Kaon CP violation in $K - \bar{K}$ mixing**? Today we know that $\epsilon_K$ is dominated by loops with **top quarks**.

⇒ experiment at **0.5 GeV** probes physics at **173 GeV**
Direct CP violation in $K \rightarrow \pi\pi$

Direct CP violation is characterized by:

$$
\epsilon'_K = \frac{\eta_{00} - \eta_{+-}}{3} = \frac{\epsilon_K}{\sqrt{2}} \left[ \frac{\langle (\pi\pi)_{I=2} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_L \rangle} - \frac{\langle (\pi\pi)_{I=2} | K_S \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle} \right] \left[ 1 + \mathcal{O} \left( \frac{A_2}{A_0} \right) \right].
$$

Experiment:

$$
\epsilon'_K = (16.6 \pm 2.3) \cdot 10^{-4} \cdot \epsilon_K \quad \text{discovered in 1999}
$$
Experimentally well-known:

\[ \text{Re}A_0 = (3.3201 \pm 0.0018) \times 10^{-7} \text{ GeV}, \]
\[ \text{Re}A_2 = (1.4787 \pm 0.0031) \times 10^{-8} \text{ GeV}. \]

↑ assumes PDG convention for CKM elements
Master equation for $\varepsilon'_K$:

$$\frac{\varepsilon'_K}{\varepsilon_K} = \frac{\omega_+}{\sqrt{2}|\varepsilon_K^{\exp}|\text{Re}A_0^{\exp}} \left\{ \frac{\text{Im}A_2}{\omega_+} - \left( 1 - \hat{\Omega}_{\text{eff}} \right) \text{Im}A_0 \right\}. $$

Here:

$$\omega_+ \simeq \frac{\text{Re}A_2}{\text{Re}A_0} = (4.53 \pm 0.02) \cdot 10^{-2}$$

$$\hat{\Omega}_{\text{eff}} = (14.8 \pm 8.0) \cdot 10^{-2}$$ quantifies isospin breaking.

Important theoretical ingredients: $\text{Im}A_0$ and $\text{Im}A_2$, calculated from the effective $|\Delta S| = 1$ hamiltonian describing $s \rightarrow dq\bar{q}$ decays.

The enhanced sensitivity to $\Delta I = 3/2$ transitions (such as electroweak penguins and boxes) is a special feature of $\varepsilon'_K$. 
\( \text{Im} A_0 \) is dominated by gluon penguins:

Operator: \( Q_6 = \bar{s}_L^{i} \gamma_\mu d_L^{k} \sum_q \bar{q}_R^k \gamma^{\mu} q_R^{i} \)

Matrix element: \( \langle (\pi\pi)_{I=0} | Q_6 | K^0 \rangle \)

\( \text{Im} A_2 \) is dominated by photon penguin and box diagrams:

Operator: \( Q_8 = \frac{3}{2} \bar{s}_L^{i} \gamma_\mu d_L^{k} \sum_q e_q \bar{q}_R^k \gamma^{\mu} q_R^{i} \)

Matrix element: \( \langle (\pi\pi)_{I=2} | Q_8 | K^0 \rangle \)
\[
\frac{\epsilon'_K}{\epsilon_K} = (16.6 \pm 2.3) \times 10^{-4} \quad \text{(experiments: NA62, KTeV)}
\]

\[
\frac{\epsilon'_K}{\epsilon_K} = (1.1 \pm 4.7_{\text{lattice}} \pm 1.9_{\text{NNLO}} \pm 0.6_{\text{isosp. br.}} \pm 0.2_{m_t}) \times 10^{-4} \quad \text{(SM)}
\]

Kitahara, UN, Tremper, JHEP 1612 (2016) 078


Discrepancy with a significance of 2.8\sigma!

Our prediction uses the methodology of Buras et al. (JHEP 1511 (2015) 202) (taking \(\text{Re}A_{0,2}\) from data), NLO formulae from Buras et al., and a new formula for the RG evolution.
Sensitivity to new physics

Standard Model:
Cabibbo-Kobayashi-Maskawa (CKM) factor:
\[ \tau = -\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*} \sim (1.5 - 0.6i) \cdot 10^{-3} \]

\[ \epsilon'_K^{\text{SM}} \propto \text{Im} \, \tau \quad \text{and} \quad \epsilon'^2_K^{\text{SM}} \propto \text{Im} \, \tau^2. \]

Generic loop-induced new physics:
some flavor-violating parameter \( \delta \) with \( |\delta| \gg |\tau| \) to compensate for suppression from heavy new-physics mass

\[ \epsilon'_K^{\text{NP}} \propto \text{Im} \, \delta \quad \text{and} \quad \epsilon'^2_K^{\text{NP}} \propto \text{Im} \, \delta^2. \]

⇒ If \( \epsilon'_K^{\text{NP}} \sim \epsilon'_K^{\text{SM}} \), expect \( \epsilon'_K^{\text{NP}} \gg \epsilon'^2_K^{\text{SM}} \).
⇒ Need clever ideas to suppress \( \epsilon'_K^{\text{NP}} \).
The MSSM has a mechanism

- to enhance $\text{Re}A_2$, because it permits strong-isospin violation through splittings between right-handed up-squark and down-squark masses (Trojan penguins),
- to suppress the $K - \bar{K}$ mixing amplitude thanks to the Majorana nature of the gluinos, with negative interference of two box diagrams. Crivellin, Davidkov 2010

The second feature makes the MSSM contribution to $K - \bar{K}$ mixing vanish for $M_{\tilde{g}} \sim 1.5M_{\tilde{q}}$, it stays small for $M_{\tilde{g}} > 1.5M_{\tilde{q}}$. 
Choose:
Sparticle masses $M_S \sim 10$ TeV, $M_{\tilde{g}} > 1.5M_S$, flavor mixing in down-squark mass matrix only with $\arg \Delta_{sd}^{LL} = \pi/4$. 

![Graph showing $M_S = 10$ TeV with various values of $m_{\tilde{g}}/M_S$]
Explain $\epsilon'_K$

**x-axis:** generic sparticle mass, $M_{\tilde{g}} = 1.5M_S$

**y-axis:** right-handed up-squark mass

**red region:** excluded by $\epsilon_K$ if $|V_{cb}|$ from inclusive decays is correct

**blue dashes:** delimit allowed region, if $|V_{cb}|$ from exclusive decays is correct
The (near) future of Kaon physics:

\[ B(K^+ \to \pi^+ \nu \bar{\nu}) \overset{\text{SM}}{=} (8.3 \pm 0.3) \cdot 10^{-11} \quad \text{for NA62 (CERN)} \]

\[ B(K_L \to \pi^0 \nu \bar{\nu}) \overset{\text{SM}}{=} (2.9 \pm 0.2) \cdot 10^{-11} \quad \text{for KØTØ (J-PARC)} \]

These branching ratios are theoretically extremely clean and can discriminate between different NP explanations of \( \epsilon'_K \).

\( K_L \to \pi^0 \nu \bar{\nu} \) is CP violating!

\[ B(K_L \to \pi^0 \nu \bar{\nu}) \overset{\text{SM}}{\propto} \eta^2 \]
In our MSSM scenario:
Contributions from wino-like chargino box:

Giancarlo D’Ambrosio, Andreas Crivellin, Teppei Kitahara, UN, 1703.05786
If you allow for at most 10% (fine-)tuning in $\epsilon_K$, you find (for GUT relations between $M_{1,2,3}$):

\[
\frac{B(K^+ \to \pi^+ \nu \bar{\nu})}{B(K^+ \to \pi^+ \nu \bar{\nu})_{SM}} \leq 1.1 \quad \text{and} \quad \frac{B(K_L \to \pi^0 \nu \bar{\nu})}{B(K_L \to \pi^0 \nu \bar{\nu})_{SM}} \leq 1.2.
\]

\[\rightarrow \quad \text{need upgrade KØTØ–step2, aiming at} \quad \mathcal{O}(100) \quad \text{events.}\]
Furthermore: if the new-physics contribution to $\epsilon'_K$ is positive (as indicated by present data), find

$$\text{sgn} \left[ B(K_L \to \pi^0 \nu \bar{\nu}) - B^{\text{SM}}(K_L \to \pi^0 \nu \bar{\nu}) \right] = \text{sgn} \left( m_{\bar{U}} - m_{\bar{D}} \right)$$

Here $\bar{U}$ and $\bar{D}$ denote the right-handed up and down squarks, respectively.

Could collider experiments ever achieve this?
(i) theoretical control of $\epsilon_K$ increases steadily (hadronic matrix elements and NNLO QCD under good control, $\epsilon_K \propto |V_{cb}|^4$ issue improving),

(ii) $\epsilon'_K$ now tractable with lattice QCD,

(iii) upcoming measurements of theoretically clean branching ratios $B(K^+ \to \pi^+ \nu \bar{\nu})$ (by NA62) and $B(K_L \to \pi^0 \nu \bar{\nu})$ (by KØTØ),

(iv) no new particles found at LHC:  
\[ \Rightarrow \text{weaker rationale for Minimal Flavor Violation (MFV)} \]

If the flavor structure of new physics is unrelated to the SM Yukawa sector, one expects the largest effects in Kaon (and $\mu \to e$) FCNC processes.
\( \epsilon_K \) has profited from an enormous progress in the lattice calculation of the hadronic matrix element \( \langle K^0 | \tilde{Q} | K^0 \rangle \) and gives a precise constraint on \((|V_{cb}|, \bar{\rho}, \bar{\eta})\).

To efficiently explore new physics with \( \epsilon_K \) we need a better measurement of \(|V_{cb}|\).

The 1999 measurement of \( \epsilon'_K \) becomes a powerful NP analyzer only now, with emerging precise lattice predictions for \( \langle (\pi\pi)_{I=0} | Q_6 | K^0 \rangle \).

The new lattice results for the matrix element \( \langle (\pi\pi)_{I=0} | Q_6 | K^0 \rangle \) from RBC-UKQCD points to a tension between the experimental value of \( \epsilon'_K \) and the Standard-Model prediction.

\( K^+ \to \pi^+ \nu \bar{\nu} \) and \( K_L \to \pi^0 \nu \bar{\nu} \) are clean probes of NP.
If *new physics* enters through loops, a sizable effect in $\epsilon'_K$ requires a new source of flavor violation which is much larger than the CKM factor $\text{Im} \frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} \sim 6 \cdot 10^{-4}$. But then the effect on $\epsilon_K$ will typically be too big.

In the **MSSM** one can simultaneously enhance $\epsilon'_K$ and suppress the new-physics contributions to $\epsilon_K$. This requires flavor mixing among *left-handed squarks*, masses of right-handed *up-type* squarks different from those of the *down-type* squarks, and a gluino mass above $1.5$ times the mass of the left-handed squarks.

$B(K \rightarrow \pi \nu \overline{\nu})$ data will test our scenario. $B(K_L \rightarrow \pi^0 \nu \overline{\nu})$ can determine, whether the *right-handed up squark* is heavier or lighter than the *right-handed down squark*. 
Penguins: Wake-up call for new physics?
Backup
Sensitivity to new physics

Standard Model:
Cabibbo-Kobayashi-Maskawa (CKM) factor:
\[ \tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} \sim (1.5 - 0.6i) \cdot 10^{-3} \]

**Im \( \tau \)** \( \frac{M^2_W}{M^2_W} \)

and

**Im \( \tau^2 \)** \( \frac{M^2_W}{M^2_W} \)

Generic new physics:
Some flavor-violating parameter: \( \delta \)

**Im \( \delta \)** \( \frac{M^2}{M^2} \)

and

**Im \( \delta^2 \)** \( \frac{M^2}{M^2} \)

with new heavy particle mass \( M \gg M_W \).
But data require $|\epsilon^\text{NP}_K| \leq |\epsilon^\text{SM}_K|$, so that

$$\left| \frac{\epsilon^\prime_\text{NP}_K}{\epsilon^\prime_\text{SM}_K} \right| \leq \frac{|\epsilon^\prime_\text{NP}_K/\epsilon^\prime_\text{SM}_K|}{|\epsilon^\prime_\text{NP}_K/\epsilon^\prime_\text{SM}_K|} = \mathcal{O} \left( \frac{\text{Re } \tau}{\text{Re } \delta} \right).$$

If new physics enters through loops, we need $|\delta| \gg |\tau|$ to compensate for $M \gg M_W$, but then $\epsilon_K$ prohibits large effects in $\epsilon'_K$. 
But data require $|\epsilon'_K^\text{NP}| \leq |\epsilon'_K^\text{SM}|$, so that

$$\left| \frac{\epsilon'_K^\text{NP}}{\epsilon'_K^\text{SM}} \right| \leq \frac{|\epsilon'_K^\text{NP} / \epsilon'_K^\text{SM}|}{|\epsilon^\text{NP} / \epsilon^\text{SM}|} = \mathcal{O} \left( \frac{\text{Re} \, \tau}{\text{Re} \, \delta} \right).$$

If new physics enters through loops, we need $|\delta| \gg |\tau|$ to compensate for $M \gg M_W$, but then $\epsilon_K$ prohibits large effects in $\epsilon'_K$.

Solutions in the literature:

- tree-level new physics (e.g. $Z'$) with $|\delta| \sim |\tau|$
- fine-tuning of the CP phase to get $\text{Re} \, \delta \sim 0$
- exploit special features of supersymmetry (this talk).