General Flavor Theory

"flavor 2019: (still) puzzling and great opportunity”

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symmetry, and symmetry-breaking (the same yet not the same)
PART I
Flavor in the SM
renormalizable QFT in 3+1 Minkowski space w. local symmetry

\[ SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{em} \]

\[ \mathcal{L}_{SM} = -\frac{1}{4} F^2 + \bar{\psi} i \not{D} \psi + \frac{1}{2} (D\Phi)^2 - \bar{\psi} Y \Phi \psi + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \]

Yukawa interact.

\[ \psi: \text{fermions (quarks and leptons)} \]

\[ D_\mu = \partial_\mu - ieA_\mu + \ldots \]

\[ F_{\mu\nu}: \text{gauge bosons } g^a, \gamma, Z^0, W^\pm \]

\[ \Phi: \text{Higgs doublet} \]

Known fundamental matter comes in generations

\[ \psi \rightarrow \psi_i, \ i = 1, 2, 3, \]

subject to identical gauge transformations. "universality", hard-wired.
They are not the same

Fields in representations under the SM $SU(3)_C \times SU(2)_L \times U(1)_Y$

Higgs: $\Phi(1, 2, 1/2)$
hypercharge $Y = Q - T_3$
quarks: $Q_L(3, 2, 1/6)_i, D_R(3, 1, -1/3)_i, U_R(3, 1, 2/3)_i$

leptons: $L_L(1, 2, -1/2)_i, E_R(1, 1, -1)_i$
L: doublet, R:singlet under $SU(2)_L$

labelled with increasing mass, distinguished by 'flavor' (mass and mixing, i.e. CKM (quarks), PMNS (leptons))

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}, U_R = u_R, c_R, t_R, \quad D_R = d_R, s_R, b_R$$

$$L_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}, E_R = e_R, \mu_R, \tau_R$$
Flavor in the SM is sourced only by the Yukawa interactions:

\[ \mathcal{L}_{SM} = \sum_{\psi=Q,U,D,L,E} \bar{\psi}_i \mathcal{D}_i \psi_i 
- \bar{Q}_L (Y_u)_{ij} \Phi^C U_R j 
- \bar{Q}_L (Y_d)_{ij} \Phi D_R j 
- \bar{L}_L (Y_e)_{ij} \Phi E_R j 
+ \mathcal{L}_{higgs} + \mathcal{L}_{gauge}, \quad \Phi^C = i\sigma^2\Phi^* \]

\(Y_{u,d,e}\): Yukawa matrices \((3 \times 3, \text{complex})\)
different Yukawa \(\rightarrow\) different mass from vev \(\langle \Phi \rangle\) etc
off-diagonal entries open up possibility to mix generations
direct link with Higgs physics: \(h \rightarrow \psi \bar{\psi}\) probes \(Y_{u,d,e}\), flavor!
Program: determine flavor parameters (in SM)
Length $[m]$

- $10^{-18}$
- $10^{-15}$
- $10^{-12}$
- nano $10^{-9}$
- visible light $10^{-6}$

Energy
- $\Lambda_{EWK}$
- $\Lambda_{QCD}$
- $E_{\text{Deuteron}}$
- $E_{\text{Rydberg}}$
- $2 \text{ eV}$

Mass
- $t$
- $b$
- $\tau$
- $c$
- $\mu$
- $s$
- $u$
- $d$
- $e$

Particle physics

- atomic, nuclear physics

$m_u$ (2 GeV) 2.8 ± 0.6 MeV

$m_d$ (2 GeV) 5.0 ± 1.0 MeV

$m_s$ (2 GeV) 95 ± 15 MeV

$m_c$ ($m_c$) 1.28 ± 0.05 GeV

$m_b$ ($m_b$) 4.22 ± 0.05 GeV

$m_t$ ($m_t$) 163 ± 3 GeV

Spectrum spans five orders of magnitude.
Experimentally (if rotated solely to up-sector)

\[
Y_u \sim \begin{pmatrix}
10^{-5} & -0.002 & 0.008 + i 0.003 \\
10^{-6} & 0.007 & -0.04 \\
10^{-8} + i 10^{-7} & 0.0003 & 0.94
\end{pmatrix}
\]

\[
Y_d \sim \text{diag}\left(10^{-5}, 5 \cdot 10^{-4}, 0.025\right)
\]

\[
Y_e \sim \text{diag}\left(10^{-6}, 6 \cdot 10^{-4}, 0.01\right)
\]

\[
\text{diag}\left(a, b\right) = \begin{pmatrix}
a & 0 \\
0 & b
\end{pmatrix}
\]

Quark mixing is hierarchal, as well as quark and charged lepton masses.

Why? ideas, no "SM" nowhere near, needs BSM to progress
Flavor change

\[ \psi \rightarrow \psi_i, i = 1, 2, 3 \] sounds simple but implies rich phenomenology: flavor change (within same and other generations), CP violation, meson mixing, ...

SM flavor change only through weak interaction among left-handed fermions:

\[ V_{ij} = (V_{CKM})_{ij} \]. Product of up and down unitary matrices \( V = V_u V_d^\dagger \), derived from \( Y_{u,d} \) (diagonalization). Not physical each in SM.
Masses from spontaneous breaking of electroweak symmetry
\[ \Phi^T(x) \rightarrow 1/\sqrt{2}(0, v + h(x)), \] 
Higgs vev \( \langle \Phi \rangle = v/\sqrt{2} \approx 174 \text{ GeV} \)

\[ \mathcal{L}^{\text{yukawa}}_{\text{SM}} = -\bar{Q}_L Y_u \Phi^C U_R - \bar{Q}_L Y_d \Phi D_R - \bar{L}_L Y_e \Phi E_R \]

Want mass eigenstates rather than gauge eigenstates \( QUDLE: \)
perform unitary trasfos on quark fields \( Q_L = (U_L, D_L), U_R, D_R \)
\[ \tilde{q}_A(\text{mass}) = V_{A,q} q_A(\text{gauge}) \quad \text{with} \quad V_{A,q} V_{A,q}^\dagger = 1, \quad A = L, R, q = u, d. \]

The 4 unitary matrices \( V_{A,q} \) are determined by
\[
\text{diag}(m_u, m_c, m_t) = \langle \Phi \rangle \cdot \text{diag}(y_u, y_c, y_t) = \langle \Phi \rangle \cdot V_{L,u} Y_u V_{R,u}^\dagger \\
\text{diag}(m_d, m_s, m_b) = \langle \Phi \rangle \cdot \text{diag}(y_d, y_s, y_b) = \langle \Phi \rangle \cdot V_{L,d} Y_d V_{R,d}^\dagger 
\]
i.e., they diagonalize the Yukawas.
CKM from scratch

insert unit matrices

\[ \mathcal{L}^{\text{yukawa}}_{SM} = -\bar{U}_L V_{L,u}^\dagger V_{L,u} Y_u \Phi^C \quad V_{R,u}^\dagger V_{R,u} U_R + \text{down quarks.} \]

then rewrite term and split up

\[ \mathcal{L}^{\text{up-mass}}_{SM} = -\bar{U}_L V_{L,u}^\dagger \quad V_{L,u} Y_u V_{R,u}^\dagger \Phi^C \quad V_{R,u} U_R = -\tilde{U}_L m_{ui} \Phi^C \tilde{U}_R. \]

The tilde basis are mass eigenstates, down quarks analogously.

What else happened under the basis change in \( \mathcal{L}_{SM} \)?

Lets look at the charged currents \( W^\pm \):

\[ \bar{U}_L \gamma^\mu W_\mu^+ D_L = \bar{U}_L (V_{L,u}^\dagger V_{L,u}) \gamma^\mu W_\mu^+ (V_{L,d}^\dagger V_{L,d}) D_L \]

\[ = \tilde{U}_L \gamma^\mu W_\mu^+ V_{L,u} V_{L,d}^\dagger \tilde{D}_L; \quad \text{Since quarks mix, } V_{L,u} \neq V_{L,d}. \]
The charged current interaction gets a flavor structure by $V$

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \left( \bar{U}_L \gamma^\mu W_\mu^+ V D_L + \bar{D}_L \gamma^\mu W_\mu^- V^\dagger \bar{U}_L \right).$$

$$V = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}, \quad V_{13} = V_{ub} \ etc$$

Via $W$ exchange is the only way to change flavor in the SM. Since CKM is known (more or less precise, see talks), in SM every quark flavor changing process can be predicted! (TH uncertainties play an important role, see talks on hadronic matrix elements).

Overconstrain, fits (see talks!)
The Standard Model: CKM properties

$V$ is unitary, is in general complex, and induces CP violation
$V$ has 4 physical parameters, 3 angles and 1 phase.

"PDG" parametrization (exact, fully general)

$$V = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}$$

$s_{ij} \equiv \sin \Theta_{ij}$, $c_{ij} \equiv \cos \Theta_{ij}$. $\delta$ is the CP violating phase.
In Nature, $\delta \sim \mathcal{O}(1)$ and $V$ is hierarchical $\Theta_{13} \ll \Theta_{23} \ll \Theta_{12} \ll 1$.
Very different – large mixing angles for leptons (PMNS-Matrix):

$$\Theta_{23} \sim 45^\circ, \Theta_{12} \sim 35^\circ, \Theta_{13} \sim \mathcal{O}(10^\circ) \quad \text{all } \mathcal{O}(1) - \text{ anarchy?}$$
CP is violated!.. together with Quark Flavor

Quark mixing matrix has 1 physical CP violating phase $\delta_{CKM}$.

Verified in $B\bar{B}$ mixing

$$\sin 2\beta = 0.672 \pm 0.023$$

HFAG Aug 2010

$\delta_{CKM}$ is large, $O(1)$!

CPX also observed in $B$-decay

$$A_{CP}(B \rightarrow K^{\pm}\pi^{\mp}) = -0.098 \pm 0.013$$

HFAG Aug 2010

$$\Gamma(B \rightarrow K^{+}\pi^{-}) \neq \Gamma(\bar{B} \rightarrow K^{-}\pi^{+})$$
Charm is the new Beauty

\[
\Delta A_{CP}^\pi = \frac{-18.2 \pm 3.2 \text{ (stat.)} \pm 0.9 \text{ (syst.)}}{} \times 10^{-4}
\]

\[
\Delta A_{CP}^\mu = \frac{-9 \pm 8 \text{ (stat.)} \pm 5 \text{ (syst.)}}{} \times 10^{-4}
\]

- Compatible with previous LHCb results and the WA
- Combination with LHCb Run 1 gives:

\[
\Delta A_{CP} = (-15.4 \pm 2.9) \times 10^{-4}
\]

\[
\Delta A_{CP} = A_{CP}(D^0 \rightarrow K^+K^-) - A_{CP}(D^0 \rightarrow \pi^+\pi^-)
\]

Betti

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Moriond EW 2019 - 21/03/2019
The CKM numerics you can memorize

$V$ is hierarchical $\Theta_{13} \ll \Theta_{23} \ll \Theta_{12} \ll 1$. Wolfenstein parametrization: expansion in $\lambda = \sin \Theta_C \sim 0.2$, $A, \rho, \eta \sim \mathcal{O}(1)$

\[
V = \begin{pmatrix}
1 - \lambda^2/2 & +\lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & +A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4)
\]

Beyond lowest order $\bar{\rho} = \rho(1 - \lambda^2/2)$ and $\bar{\eta} = \eta(1 - \lambda^2/2)$

$V_{CKM}V_{CKM}^\dagger = 1$

General Flavor Theory SSI 2019
Neutral currents – a window beyond SM

Neutral current gauge interactions $\gamma, Z, g$ remain unaffected by misalignement between gauge and mass bases in SM because of $\psi \rightarrow \psi_i$ (universality) since they involve the same fields, for instance:

$$\bar{U}_L \gamma^\mu A_\mu U_L = \bar{U}_L (V^\dagger_{L,u} V_{L,u}) \gamma^\mu A_\mu (V^\dagger_{L,u} V_{L,u}) U_L$$

$$= \bar{\tilde{U}}_L \gamma^\mu A_\mu V_{L,u} V^\dagger_{L,u} \tilde{U}_L = \bar{\tilde{U}}_L \gamma^\mu A_\mu \tilde{U}_L$$

nothing has happened!

Imagine a BSM $U(1)'$ with generation-dependent charges $g_i$:

$$\bar{U}_i g_i \gamma^\mu A'_\mu U_i = \bar{\tilde{U}}_i \gamma^\mu A'_\mu (V_u)_{ij} g_j (V^\dagger_u)_{jk} \tilde{U}_k$$

Consider 2 generations, $V_u$ orthogonal with mixing angle $\vartheta_u$ between 1st and 2nd generation $V_u = \begin{pmatrix} \cos \vartheta_u & \sin \vartheta_u \\ -\sin \vartheta_u & \cos \vartheta_u \end{pmatrix}$

We obtain FCNC ”flavor changing neutral current” amplitudes

$$\cos \vartheta_u \sin \vartheta_u (g_1 - g_2) \bar{\tilde{U}}_1 \gamma^\mu A'_\mu \tilde{U}_2$$
FCNC "flavor changing neutral current" $U_2 \rightarrow U_1$ drop the tilde for mass eigenstates

$$\cos \vartheta_u \sin \vartheta_u (g_1 - g_2) \bar{U}_1 \gamma^\mu A'_\mu U_2$$

i) no tree-level FCNC in the SM or other models with universality, where $g_1 = g_2$ etc holds.

individual rotations $\vartheta_u, \vartheta_d$ in SM not physical, only $V = V_{Lu} V_{Ld}^\dagger$, no constraint on $V_{Ru}, V_{Rd}$.

ii) FCNCs have sensitivity to a) BSM physics and b) origins of flavor (up versus down, it could be that mixing comes only from up-sector and $\vartheta_d = 0$)
Exploring Physics at Highest Energies

Length [m]

- $10^{-18}$
- $10^{-15}$
- $10^{-12}$
- $10^{-9}$
- $10^{-6}$

Energy

- $\Lambda_{\text{QCD}}$
- $E_{\text{Deuteron}}$
- $E_{\text{Rydberg}}$

SM/BSM ?

- $\Lambda_{\text{EWK}}$
- $h$
- $W, Z$
- $t, b, \tau, c, \mu, s$
- $u, d, e$

particle physics

atomic, nuclear physics

2 eV

visible light

Bohr-radius

nano
In SM neutral currents conserve flavor. However, charged currents induce FCNCs through quantum loops.

\[ W^\pm \]

The upper figure shows an FCNC with flavor number changing in units of one, \( \Delta F = 1 \), as in decays, meson mixing has \( \Delta F = 2 \).
Testing the SM with flavor

Different sectors/couplings presently probed with FCNCs:

\[ s \rightarrow d: \quad K^0 - \bar{K}^0, \quad K \rightarrow \pi \nu \bar{\nu} \]

\[ c \rightarrow u: \quad D^0 - \bar{D}^0, \quad \Delta A_{CP}, \quad D \rightarrow \pi(\pi)\mu\mu, \quad D \rightarrow \rho\gamma \]

\[ b \rightarrow d: \quad B^0 - \bar{B}^0, \quad B \rightarrow \rho\gamma, \quad b \rightarrow d\gamma, \quad B \rightarrow \pi\mu\mu \]

\[ b \rightarrow s: \quad B_s - \bar{B}_s, \quad b \rightarrow s\gamma, \quad B \rightarrow K_s\pi^0\gamma, \quad b \rightarrow sll, \quad B \rightarrow K(\ast)ll \text{ (precision, angular analysis, universality)}, \quad B_s \rightarrow \mu\mu \]

\[ t \rightarrow c, u, l \rightarrow l': \quad \text{not observed} \]

in red: hot topics

"leaving no stone unturned" (see talks & lectures)
PART II
Testing the SM with FCNCs
Let's discuss a generic SM FCNC $b \to s$ amplitude

\[ A(b \to s)_{\text{SM}} = V_{ub} V_{us}^* A_u + V_{cb} V_{cs}^* A_c + V_{tb} V_{ts}^* A_t \]

quantum loop effect induced by weak interaction. $A_q = A(m_q^2/m_W^2)$.

with CKM unitarity $VV^\dagger = 1$, specifically $\sum_i V_{ib} V_{is}^* = V_{ub} V_{us}^* + V_{cb} V_{cs}^* + V_{tb} V_{ts}^* = 0$:

$A(b \to s)_{\text{SM}} = V_{tb} V_{ts}^* (A_t - A_c) + V_{ub} V_{us}^* (A_u - A_c)$

$A$ vanishes for trivial CKM matrix or if quarks in loop are degenerate.

$A(b \to s)_{\text{SM}} = \underbrace{V_{tb} V_{ts}^* (A_t - A_c)}_{\lambda^2} + \underbrace{V_{ub} V_{us}^* (A_u - A_c)}_{\lambda^4}$

$A$ dominated by first term because of lesser CKM suppression, and
\[ A(b \to s)_{\text{SM}} = V_{tb}V_{ts}^*(A_t - A_c) + V_{ub}V_{us}^*(A_u - A_c), \quad A_q = A(m_q^2/m_W^2). \]

GIM (Glashow Iliopoulos Maiani) suppression inactive for tops.

We probe top properties with rare \( b \)-decays despite of \( m_t \gg m_b \).

CP violation requires interference between the two terms with different phases; for \( b \to s \), this is small, \( O(\lambda^2) \).

Generic SM FCNC \( c \to u \) amplitude:

\[ A(c \to u)_{\text{SM}} = \begin{aligned} \left| V_{cd}V_{ud} \right|A_d &+ \left| V_{cs}V_{us} \right|A_s + \left| V_{cb}V_{ub} \right|A_b, \\ A_q &= A(m_q^2/m_W^2). \end{aligned} \]

\[ A(c \to u)_{\text{SM}} = \begin{aligned} V_{cd}^*V_{ud}(A_d - A_b) &+ V_{cs}^*V_{us}(A_s - A_b) \\ -\lambda &+\lambda \end{aligned} \]

\( A \) is GIM-suppressed; CP violation is suppressed by \( \frac{V_{cb}^*V_{ub}}{V_{cd}^*V_{ud}} \sim \lambda^4 \).

Very sensitive to corrections beyond the SM. How about \( \Delta A_{CP}^{LHCb} \approx -1.5 \cdot 10^{-3} \)?
i FCNCs are induced by the weak interaction thru loops.

ii FCNCs require $V 
eq 1$.

iii FCNCs vanish for degenerate intermediate quarks. Since mass splitting among up-quarks is larger than for down quarks, GIM suppression is larger with external up-type than down-type quarks.

$$B(b \to s\gamma) = 3 \cdot 10^{-4} \quad (E_\gamma > 1.6 \text{ GeV})$$

$$B(b \to sl^+l^-) = 4 \cdot 10^{-6} \quad (m_{ll}^2 > 0.04 \text{ GeV}^2)$$

SM: $B(t \to cg) \sim 10^{-10}$, $B(t \to c\gamma) \sim 10^{-12}$, $B(t \to cZ) \sim 10^{-13}$,
$$B(t \to ch) \lesssim 10^{-13}$$ Eilam, Hewett, Soni ’91/99

Lepton flavor violation (LFV) in SM arises through finite neutrino masses which is so small that LFV observables are ”null tests”.

Lepton Flavor Violation (LFV) in SM arises through finite neutrino masses which is so small that LFV observables are "null tests".
Flavor Changing Neutral Currents in SM

We see that 3 mechanisms suppress FCNCs in SM: CKM, GIM and absence at tree level. New physics, which doesn't need to share these features, competes with small SM background!

FCNCs feel physics in the loops from energies much higher than the ones actually involved in the real process.

They are very useful to look for new physics, in fact, we already now a lot about new physics from FCNCs!

\[
\Lambda_{\text{NP}} \ [\text{TeV}] \begin{array}{|c|cccc|} 
\hline
 & K^0 \bar{K}^0 & D^0 \bar{D}^0 & B_d^0 \bar{B}_d^0 & B_s^0 \bar{B}_s^0 \\
\hline
2 \cdot 10^5 & 5 \cdot 10^3 & 2 \cdot 10^3 & 3 \cdot 10^2 \\
\hline
\end{array}
\]

Table 1: The lower bounds on the scale of new physics from FCNC mixing data in TeV for arbitrary new physics at 95 % C.L.
Besides statistics, BSM reach is limited by theoretical uncertainties, mostly dominated by hadronic physics.

Use approximate symmetries of SM to bypass precision barrier:
(useful in beauty $\Lambda/m_b \simeq O(0.1)$, key in charm $\Lambda/m_c \sim 0.6$

- GIM $D \to \pi \pi \mu^+ \mu^-$

- Isospin, or U-spin $\Delta A_{CP}$

- CP $\Delta A_{CP}$, $B_s$-mixing

- V-A, helicity $P'_{5} \ (B \to K^* (\to K\pi)\ell^+\ell^-)$

- LFV

- universality $R_K, R_{K^*}$
Physics with angular distributions –

\[ B \rightarrow K^*(\rightarrow K\pi)\ell^+\ell^- \] precision

\[ D \rightarrow \pi\pi\mu\mu \] null tests
Model-independent analysis of $b \rightarrow s l^+ l^-$

Test the SM and BSM with FCNCs using effective theory (dim 6)

$$\mathcal{H}_{\text{eff}} = -4 \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i(\mu) O_i(\mu), \quad G_F/\sqrt{2} = g^2/(8m_W^2)$$

operators $O$ describe interactions "effectively" at low energy, here below $m_W$. The $W$ and all other heavier SM particles, $Z, t, h$ are removed from theory as propagating fields. Their net-effect is encoded in the Wilson coefficients $C$. E.g., in SM $C_{10}$ is function of $m_t^2/m_W^2$. 
The effect of BSM particles, which are also heavy, such as leptoquarks, SUSY, .. is taken into account as well:  \( C = C^{\text{SM}} + C^{\text{NP}} \).
Consider all possible operators up to dimension 6 (a fermion counts as 3/2, a scalar as 1 etc), that are consistent with gauge and Lorentz invariance. Any type of BSM particles, which are also heavy, is included in a "model-independent way":

Program: express observables thru Wilson coefficients \( C_i \) and extract the latter from data.
$b \rightarrow s \ell \ell$ FCNCs model-independently

All 4-fermion $b s \ell \ell$-operators at dim 6:

V,A operators $\mathcal{O}_9 = [\bar{s} \gamma_\mu P_L b] [\bar{\ell} \gamma^\mu \ell]$, $\mathcal{O}_9' = [\bar{s} \gamma_\mu P_R b] [\bar{\ell} \gamma^\mu \ell]$

$\mathcal{O}_{10} = [\bar{s} \gamma_\mu P_L b] [\bar{\ell} \gamma^\mu \gamma_5 \ell]$, $\mathcal{O}_{10}' = [\bar{s} \gamma_\mu P_R b] [\bar{\ell} \gamma^\mu \gamma_5 \ell]$

S,P operators $\mathcal{O}_S = [\bar{s} P_R b] [\bar{\ell} \ell]$, $\mathcal{O}_S' = [\bar{s} P_L b] [\bar{\ell} \ell]$, \textbf{ONLY $\mathcal{O}_9$, $\mathcal{O}_{10}$ are SM, all other BSM}

$\mathcal{O}_P = [\bar{s} P_R b] [\bar{\ell} \gamma_5 \ell]$, $\mathcal{O}_P' = [\bar{s} P_L b] [\bar{\ell} \gamma_5 \ell]$

and tensors $\mathcal{O}_T = [\bar{s} \sigma_{\mu\nu} b] [\bar{\ell} \sigma^{\mu\nu} \ell]$, $\mathcal{O}_{T5} = [\bar{s} \sigma_{\mu\nu} b] [\bar{\ell} \sigma^{\mu\nu} \gamma_5 \ell]$

\textbf{lepton specific} $C_i \mathcal{O}_i \rightarrow C_i^{\ell} \mathcal{O}_i^{\ell}$, $\ell = e, \mu, \tau$

\textbf{30 complex coefficients} $C_i^{\ell} + \text{LFV ones!}$ model-independent analysis: in principle.

in practice, not fully doable, too many operators $\rightarrow$ to many Wilson coeffs to fit! make working assumptions, say, fit to SM operators plus those either motivated by concrete model or high sensitivity
Dilepton Mass Spectra in $B \to K^* \mu^+ \mu^-$

Different TH for low $q^2$ (QCDF) and high $q^2$/low recoil. Binned data needed. Plot Moriond’11
Full $M \rightarrow P_1 P_2 l^+ l^-$ angular distribution– de-mystified

\[ d^5 \Gamma = \frac{1}{2\pi} \left[ \sum c_i(\theta_l, \varphi) I_i(q^2, p^2, \cos \theta_{P1}) \right] dq^2 dp^2 d \cos \theta_{P1} d \cos \theta_l d \varphi , \]

$I_i$; angular observables, SM tests, $c_i(\theta_l, \varphi)$ known trigon. functions "$Y_{lm}$"

$H_a$: transversity amplitudes, $L, R$: lepton current handedness

\[
\begin{align*}
I_1 &= \frac{1}{16} \left[ |H_0^L|^2 + (L \rightarrow R) + \frac{3}{2} \sin^2 \theta_{P1} \{ |H_\perp^L|^2 + |H_\parallel^L|^2 + (L \rightarrow R) \} \right] , \\
I_2 &= -\frac{1}{16} \left[ |H_0^L|^2 + (L \rightarrow R) - \frac{1}{2} \sin^2 \theta_{P1} \{ |H_\perp^L|^2 + |H_\parallel^L|^2 + (L \rightarrow R) \} \right] , \\
I_3 &= \frac{1}{16} \left[ |H_\perp^L|^2 - |H_\parallel^L|^2 + (L \rightarrow R) \right] \sin^2 \theta_{P1} , \\
I_4 &= -\frac{1}{8} \left[ \text{Re}(H_0^L H_\parallel^L*) + (L \rightarrow R) \right] \sin \theta_{P1} , \\
I_5 &= -\frac{1}{4} \left[ \text{Re}(H_0^L H_\perp^L*) - (L \rightarrow R) \right] \sin \theta_{P1} , \\
I_6 &= \frac{1}{4} \left[ \text{Re}(H_\parallel^L H_\perp^L*) - (L \rightarrow R) \right] \sin^2 \theta_{P1} , \\
I_7 &= -\frac{1}{4} \left[ \text{Im}(H_0^L H_\parallel^L*) - (L \rightarrow R) \right] \sin \theta_{P1} , \\
I_8 &= -\frac{1}{8} \left[ \text{Im}(H_0^L H_\perp^L*) + (L \rightarrow R) \right] \sin \theta_{P1} , \\
I_9 &= \frac{1}{8} \left[ \text{Im}(H_\parallel^L H_\perp^L*) + (L \rightarrow R) \right] \sin^2 \theta_{P1} .
\end{align*}
\]
Full $M \rightarrow P_1 P_2 l^+ l^-$ angular distribution– de-mystified

$P_5^l$: $I_5$ normalized to other $I_i$ to reduce TH uncertainties ”optimized observables”. Ongoing discussion about residual SM background.

The distribution looks of course identical for charm $D \rightarrow \pi \pi \mu \mu$ FCNCs.

In charm, due to GIM, dynamics dominated by $SU(3)_C \times U(1)_{em}$: all couplings to leptons vector-like ”$H_L = H_R$”. (no need to calculate).

null tests: $I_{5,6,7}^{SM} = 0$ (proportional to $C_{10}^{(l)}$, $O_{10}^{(l)} = [\bar{s} \gamma_\mu P_L/R b] \, [\bar{\ell} \gamma^\mu \gamma_5 \ell]$)

Counterintuitively, things are simpler than in $B$-decays because of the resonances $\rho, \varphi, ..$.)
charm: Resonance contributions vs BSM

resonance vs non-resonant SM (blue) BSM windows in branching ratios only in $D \rightarrow \pi \mu^+ \mu^-$ (left) at high $q^2_{1510.00311}$; $D \rightarrow \pi^+ \pi^- \mu \mu$ (mid), $D \rightarrow K^+ K^- \mu \mu$ (right), $1805.08516, 1705.05891$
Null tests of the SM based on its peculiarities, such as powerful GIM suppression, and allow to probe BSM physics.

$I_6 \propto A_{FB}$ already measured LHCb talk by D.Mitzel at CHARM 2018 (grey: NS) model-independent BSM effects up to few $\%_0$
Lepton Non-Universality?

\[
\begin{pmatrix}
\nu_e \\
\mu \\
\tau
\end{pmatrix},
\begin{pmatrix}
\nu_{\mu} \\
\mu \\
\tau
\end{pmatrix},
\begin{pmatrix}
\nu_{\tau} \\
\tau
\end{pmatrix}
\]

Anomalies in semileptonic $B$-meson decays:

\[R_K = \frac{\mathcal{B}(B \to K \mu\mu)}{\mathcal{B}(B \to K ee)} \approx 2.6\sigma \quad \text{(LHCb'14,19)}\]

\[R_{K^*} = \frac{\mathcal{B}(B \to K^* \mu\mu)}{\mathcal{B}(B \to K^* ee)} \approx 2.6\sigma \quad \text{(LHCb'17)}\]

\[R_{D(*)} = \frac{\mathcal{B}(B \to D(*)_{\tau\nu\tau})}{\mathcal{B}(B \to D(*)_{\ell\nu\ell})} \approx 2.7\sigma \ (D^*), \approx 2\sigma \ (D) \quad \text{(LHCb'15, B-factories)}\]
Lepton Non-Universality in $b \rightarrow s$ FCNCs

$$R_H = \frac{\mathcal{B}(B \rightarrow H\mu\mu)}{\mathcal{B}(B \rightarrow Hee)}, \quad H = K, K^*, X_s, \Phi, \ldots$$

same cuts for $e, \mu$

In models with lepton universality (incl. SM): $R_H = 1 + \text{tiny null test}$

| $\mathcal{B}(B \rightarrow K\mu\mu)$ [1,6] | $\mathcal{B}(B \rightarrow Kee)$ [1,6] | $R_K | [1,6]$ |
|--------------------------------------|--------------------------------------|----------------|
| (1.21 \pm 0.09 \pm 0.07) \cdot 10^{-7} | (1.56^{+0.19+0.06}_{-0.15-0.04}) \cdot 10^{-7} | 0.745 \pm 0.090_{0.074} \pm 0.036 |

<table>
<thead>
<tr>
<th>LHCb’14</th>
<th>SM</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.75^{+0.60}_{-0.29}) \cdot 10^{-7}</td>
<td>same</td>
</tr>
</tbody>
</table>

$R_K \approx 1$
Lepton Non-Universality tests in \( c \to u \)

<table>
<thead>
<tr>
<th>branching ratio</th>
<th>( D^0 \to \pi^+ \pi^- \mu^+ \mu^- )</th>
<th>( D^0 \to K^+ K^- \mu^+ \mu^- )</th>
<th>( D^0 \to \pi^+ \pi^- e^+ e^- )</th>
<th>( D^0 \to K^+ K^- e^+ e^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHCb 17</td>
<td>((9.64 \pm 1.20) \times 10^{-7})</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
</tr>
<tr>
<td>BESIII 18</td>
<td>(-)</td>
<td>(-)</td>
<td>(&lt; 0.7 \times 10^{-5})</td>
<td>(&lt; 1.1 \times 10^{-5})</td>
</tr>
<tr>
<td>resonant</td>
<td>(\sim 1 \times 10^{-6})</td>
<td>(\sim 1 \times 10^{-7})</td>
<td>(\sim 10^{-6})</td>
<td>(\sim 10^{-7})</td>
</tr>
<tr>
<td>non-resonant</td>
<td>(10^{-10} - 10^{-9})</td>
<td>(\mathcal{O}(10^{-10}))</td>
<td>(10^{-10} - 10^{-9})</td>
<td>(\mathcal{O}(10^{-10}))</td>
</tr>
</tbody>
</table>

\[
R^D_{P_1 P_2} = \frac{\int_{q_{2\min}^2}^{q_{2\max}^2} dB/dq^2(D \to P_1 P_2 \mu^+ \mu^-)}{\int_{q_{2\min}^2}^{q_{2\max}^2} dB/dq^2(D \to P_1 P_2 e^+ e^-)} \quad \text{with same cuts } q_{2\min}^2 \geq 4m_\mu^2
\]

<table>
<thead>
<tr>
<th>full ( q^2 )</th>
<th>SM</th>
<th>BSM</th>
<th>LQ</th>
<th>hi ( q^2 ) SM</th>
<th>LQs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^D_{\pi \pi} )</td>
<td>1.00 ± (\mathcal{O}(%))</td>
<td>0.85 ... 0.99</td>
<td>SM-like</td>
<td>1.00 ± (\mathcal{O}(%))</td>
<td>0.7 ... 4.4</td>
</tr>
<tr>
<td>( R^D_{KK} )</td>
<td>1.00 ± (\mathcal{O}(%))</td>
<td>SM-like</td>
<td>SM-like</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

\( O(1) \) BSM effects in \( R^D_{\pi \pi} \) above \( \Phi \); small BSM effects in \( R^D_{KK} \) below \( \eta \).

Naive ratios \( \bar{R}^{D_{\pi^+ \pi^-}}_\text{exp} \gtrsim 0.1 \), \( \bar{R}^{D_{K^+ K^-}}_\text{exp} \gtrsim 0.01 \) based on different cuts and about one order of magnitude away from SM, are model-dependent. opportunity to test SM in up-sector
\( \psi_i \) may be more different than we thought

**model space 2002**

2002: top-down models  plot from hep-ph/0207121

2018,19: \( U(1)' \)-extensions, leptoquarks,...

theory activities how to get these from UV-models 1708.06450, 1708.06350, 1706.05033, 1808.00942 .. see talks during this school!
The situation

Take $R_K, R_{K^*}$ data, both $\sim 2.6\sigma$ hints of new physics in $b \to sll$, non-universality between $e$’s and $\mu$’s at face value. What do we learn, and how do we go on?

$$R_H = \frac{B(\bar{B} \to H\mu\mu)}{B(B \to Hee)}, \text{ same cuts } e \text{ and } mu, \quad H = K, K^*, X_s, ...$$

Lepton-universal models (incl. SM): $R_H = 1 + \text{tiny} \quad \text{hep-ph/0310219}$

1. Which operators are responsible for the deviation? 1411.4773

2. BSM in electrons, or muons, or in both?

3. Side effects from flavor: LFV, $\tau$’s, or $SU(2)$: $\nu$’s 1411.0565, 1412.7164, 1503.01084 Charm and Kaons

4. Collider implications (leptoquarks!)
$R_H < 1$: too few muons, or too many electrons, or combination thereof. Lepton specific measurements needed.

Data on $B \to K, K^* \mu\mu, B_s \to \mu\mu$ ("global $b \to s$ fit") exhibit presently an anomaly, that even can point to the same direction as $R_{K,K^*}$

fits from Hurth, Mahmoudi, Neshatpour 18

e vs mu: don’t know for sure yet; no BSM in electrons needed; needs $B \to K, K^* ee$ cross checks
Large effect, approximately 20% of standard model amplitude (loop-induced)

Leptoquark model with tree level contribution to $R_K$:

$$\mathcal{L} = -\lambda_{d\ell} \varphi \bar{d}\ell$$

with scalar leptoquark $\varphi(3, 2)_{1/6}$ with mass $M$. 
Implications: $\phi$–decays $\phi^{2/3} \rightarrow b \, e^+$, $\phi^{-1/3} \rightarrow b \, \nu$ ($SU(2)$-doublet)

$\phi$- $SU(2)$-triplet $\phi(3, 3)_{-1/3}$, dominant decay modes

- $\phi^{2/3} \rightarrow t \, \nu$
- $\phi^{-1/3} \rightarrow b \, \nu$, $t \, \mu^-$
- $\phi^{-4/3} \rightarrow b \, \mu^-$

Mass range $1...50$ TeV
Correlations are beneficial between $R_K$ and $R_K^*$. SM-like chirality operators are the dominant source behind the anomalies. Prediction: $R_{X_s} \simeq 0.73 \pm 0.07$ inclusive decays 1704.05444 todo at Belle II?

Green band: $R_K \ 1\sigma$ LHCb, blue band $R_K^* \ 1\sigma$ LHCb. Different BSM scenarios are red dashed: pure $C_{LL}$ (LQ triplet). Black solid: $C_{LL} = -2C_{RL}$. Blue: $C_{RL}$ (LQ doublet)/disfavored as dominant source of LNU. Orange: data from $B \rightarrow X_s \ell\ell$. $R_{X_s}^{Belle'09} = 0.42 \pm 0.25$, $R_{X_s}^{BaBar'13} = 0.58 \pm 0.19$. 

General Flavor Theory

SSI 2019

Slide 46
Why are muons different from electrons? → Flavor physics, in standard model and beyond

Leptoquark coupling matrix: \( \lambda_{ql} \equiv \begin{pmatrix} \lambda_{q_1e} & \lambda_{q_1\mu} & \lambda_{q_1\tau} \\ \lambda_{q_2e} & \lambda_{q_2\mu} & \lambda_{q_2\tau} \\ \lambda_{q_3e} & \lambda_{q_3\mu} & \lambda_{q_3\tau} \end{pmatrix} \)

columns = leptons
rows = quarks, new structures not present in standard model!
columns = leptons, discrete non-abelian flavor symmetries (sub-groups of $SU(3)$), e.g., $A_4$ "zeros and ones"

Rows = quarks, hierarchical, $U(1)$-Froggatt-Nielsen-Symmetry

$1 \gg \rho \gg \rho_d$

We can use these symmetries to explain quark and lepton properties. Then predict the leptoquark coupling, e.g.,

$$\lambda_{ql} \sim \begin{pmatrix} \rho_d & \rho_d & \rho_d \\ \rho & \rho & \rho \\ 1 & 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & \rho_d & 0 \\ 0 & \rho & 0 \\ 0 & 1 & 0 \end{pmatrix}, \ldots$$

second matrix can explain $R_K$ – leptoquark couples to muons only.
$R_{K,K^*}: \frac{Y_{b\mu}Y_{s\mu}^* - Y_{be}Y_{se}^*}{M^2} \simeq \frac{1.1}{(35 \text{ TeV})^2}, (S_3)$ LQ scalar triplet

red: explains $R_K, R_{K^*}$, blue: allowed by $B_s - \bar{B}_s$-mixing, green: flavor model prediction

$Y_{q_3\ell} \sim c_{\ell}, \quad Y_{q_2\ell} \sim c_{\ell}\lambda^2, \quad q_3 = b, t, \quad q_2 = s, c, \quad \lambda, c_{\ell} \lesssim 0.2$ points to TeV-ish mass $M$!

Model-independent upper limit by $B_s$-mixing $\propto \lambda^4/M^2$ at 40 TeV.
Pair production, e.g. recent works 1706.05033, 1710.06363 1801.07641

\[ \sigma(pp \rightarrow \varphi^+ \varphi^-) \propto \alpha_s^2 \]

Single LQ production from $b$-anomalies 1801.09399 in association with a lepton \( \sigma(pp \rightarrow \varphi \ell) \propto |\lambda_{q\ell}|^2 \alpha_s \) depends on flavor
Links with $b$-physics and flavor

LHCb-data: \( \frac{\lambda_{b\mu}\lambda_{s\mu}^* - \lambda_{be}\lambda_{se}^*}{M^2} \approx \frac{1}{(35 \text{ TeV})^2} \)

\[ R_K, R_{K^*} : \lambda_{q\ell} = \begin{pmatrix} * & * & * \\ * & \lambda_{q2\ell2} & * \\ * & \lambda_{q3\ell2} & * \end{pmatrix} \text{ or } \begin{pmatrix} * & * & * \\ \lambda_{q2\ell1} & * & * \\ \lambda_{q3\ell1} & * & * \end{pmatrix} , \begin{pmatrix} * & * & * \\ \lambda_{q2\ell1} & \lambda_{q2\ell2} & * \\ \lambda_{q3\ell1} & \lambda_{q3\ell2} & * \end{pmatrix} \]

Hierarchies present in SM \( m_b \gg m_s \), explain with symmetry, assume same mechanism for LQs:

\( \lambda_{s\ell i} \sim (m_s/m_b) \lambda_{b\ell i} \rightarrow \text{third generation quarks dominant, } M/11.6 \text{ TeV} \lesssim \lambda_{b\ell} \lesssim M/3.9 \text{ TeV} \)
Producing LQs at the LHC

red: $R_{K,K^*}$ with flavor $M/11.6 \text{ TeV} \lesssim \lambda_{b\ell} \lesssim M/3.9 \text{ TeV}$

left plot: green: flavor model prediction points to multi-TeV mass; yellow: narrow width

other plots: magenta, yellow, blue: $\lambda_{d\mu} = 1$, $\lambda_{s\mu} = 1$, $\lambda_{b\mu} = 1$, black: no-loss reach with 3 ab$^{-1}$

green curve: pair production (LO Madgraph) 1801.093999

– Beauty wins over PDF if $\lambda_{ql}$ follow quark mass hierarchies. Inverted hierarchies $\lambda_{sl} > \lambda_{bl}$ would be surprising from a symmetry-based flavor model perspective and suggests means beyond.
Figure 10. Projected sensitivity of future colliders to single LQ production that decays to a muon and a jet, for the luminosities and centre of mass energies given in the legend. We also show the cross-section times branching ratio for some future collider scenarios by the curves labelled $\sigma_y \times BR$, where $y$ is the scalar LQ coupling to $b\mu$, set equal to the coupling to $s\mu$. Shaded parts of the curve indicate the conservative extrapolation method at low masses that underestimates the actual limit.

$\lambda_{b\mu} = \lambda_{s\mu}$ Allanach, Gripaios, You ’17
$R_K, R_K^*$ anomaly points to $(V - A) \times (V - A)$ -type BSM:

$$\lambda_{q_l} = \begin{pmatrix}
* & * & * \\
* & \lambda_{q_2 \ell_2} & * \\
* & \lambda_{q_3 \ell_2} & *
\end{pmatrix}$$
affects doublets: $\ell_2 = \mu, \nu_\mu$, $q_2 = s, c$, $q_3 = b, t$

$S_3$- dominant decay modes

$S_3^{+2/3} \rightarrow t\nu$

$S_3^{-1/3} \rightarrow b\nu, t\mu^-$ ($SU(2)$-triplet, scalar)

$S_3^{-4/3} \rightarrow b\mu^-$

$V_1$- dominant decay modes

$V_1^{+2/3} \rightarrow b\mu^+, t\nu$ ($SU(2)$-singlet, vector)

tagging useful to identify LQ-type (electric charge), e.g.,

$V_1^{-2/3} \rightarrow \bar{b}\mu^-$ vs $S_3^{-4/3} \rightarrow b\mu^-$
Producing leptoquarks at the LHC

Leptoquarks related to $R_{K,K^*}$ can be in reach of direct searches at the LHC – but no guarantees 1710.06363, 1801.07641, 1801.09399

matrix and lower limits from arXiv:1706.05033, Zhong, Schmaltz ’18

From $R_{K,K^*}$ perspective: $b\mu$ final states ”vanilla”, dropping a) the global $b \to s \mu \mu$ fit suggests also $be$, or b) flavor hierarchies: $j \mu$; additional modes $b\nu, t\mu, t\nu$ by $SU(2)$; more flavor: $\tau$’s $\to$ whole matrix